The Perturbative Ricci Flow In Gravity

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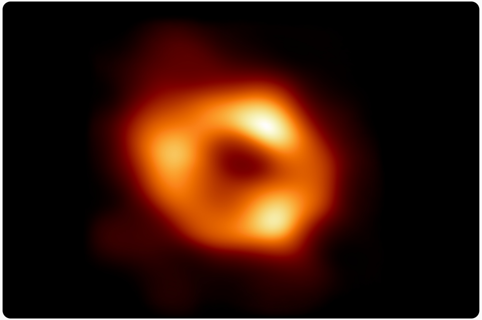
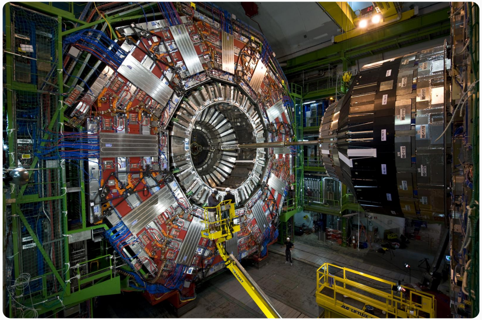
In collaboration with: R. Harlander, Y. Kluth, and J. Kohnen

The Current Landscape of Physics



General Relativity
[EHT, 2019]



Standard Model
[CMS]

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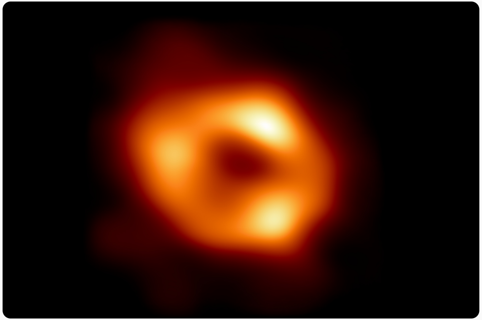

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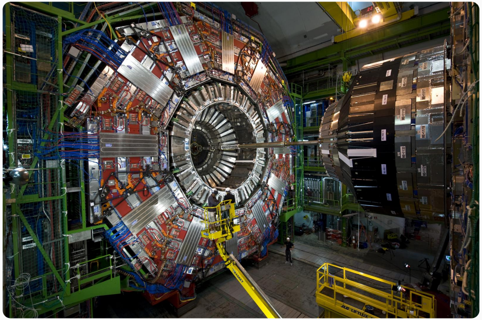

Standard Model
[CMS]

Theory of Quantum Gravity?

Non-renormalizability and Predictivity Crisis

Perturbative Quantisation:

$$S = \frac{1}{16\pi G_N}\int d^d x\sqrt{-g}(R - 2\Lambda)$$

Classical GR, $[G_N] = -2$

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$$+ \int d^dx\sqrt{-g}\left[c_1 R^2 + c_2 R_{\mu\nu}R^{\mu\nu}\right]$$

On-shell finite ←

['t Hooft & Veltman, 1974]

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$$+ \int d^dx\sqrt{-g}\left[c_{GS} R_{\mu\nu}{}^{\rho\sigma} R_{\rho\sigma}{}^{\alpha\beta} R_{\alpha\beta}{}^{\mu\nu} + \dots\right]$$

← On-shell divergence

[Goroff & Sagnotti, 1986]

$$+ \quad \dots$$

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On-shell divergence [Goroff & Sagnotti, 1986]

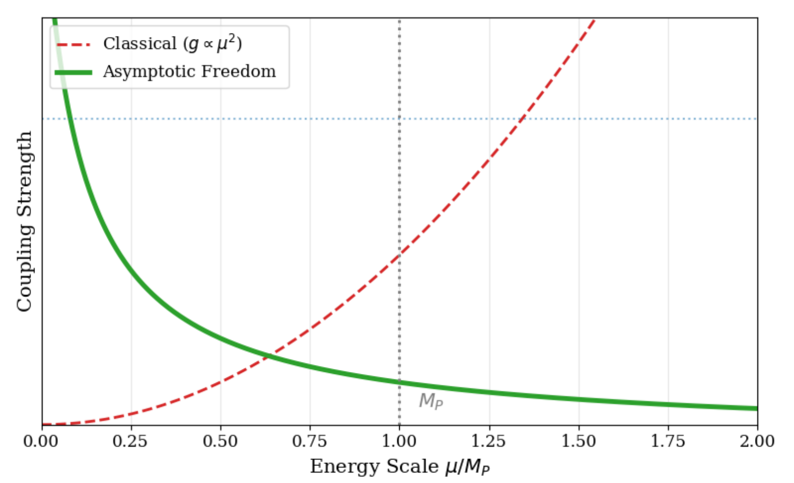
$$+ \quad \dots$$

Infinitely many couplings $\Longrightarrow$ Loss of predictivity

The Asymptotic Safety Conjecture

Conjecture: The RG flow hits a non-Gaußian fixed point (NGFP) [Weinberg, 1976]

$$g(\mu) = G_N \mu^2$$



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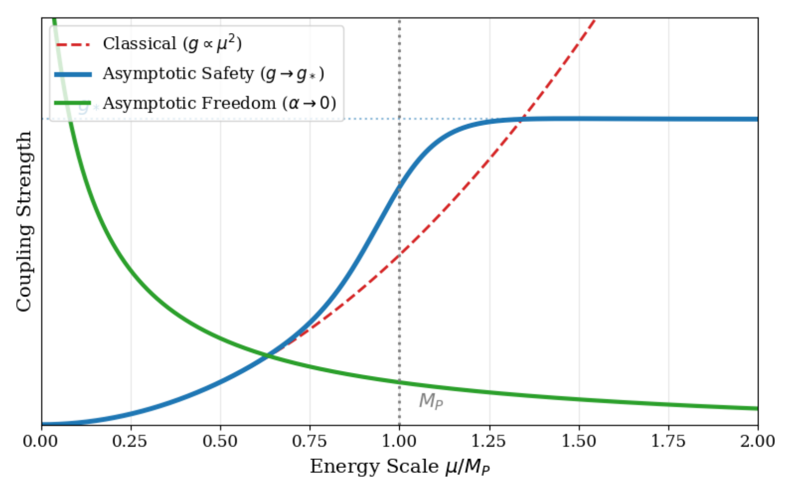
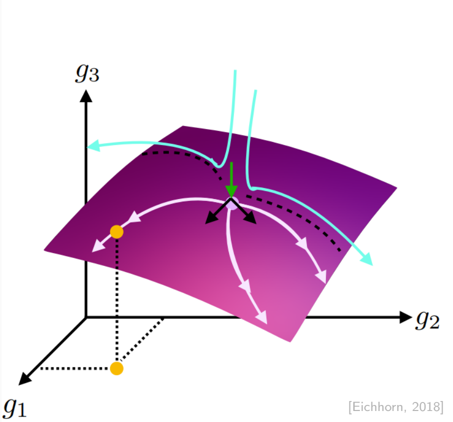


Restoring Predictivity I: The Critical Surface

- **UV Completion:**
  Require all couplings to hit the FP in the UV.

- **The Critical Surface:**
  Only RG trajectories lying on the purple surface are asymptotically safe.



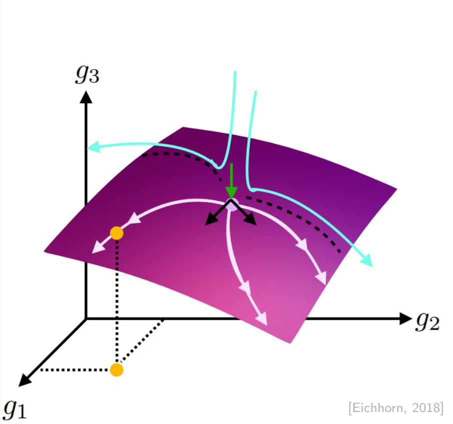
Fixed Point Condition:

$$\beta_i(g^*) = 0$$

Stability Matrix & Exponents:

$$\left.\frac{\partial \beta_i}{\partial g_j}\right|_{g^*} v_j = -\theta_i v_i$$

- **Relevant Directions**: $\theta_i > 0$
- **Irrelevant Directions**: $\theta_i < 0$



Restoring Predictivity II: Critical Exponents

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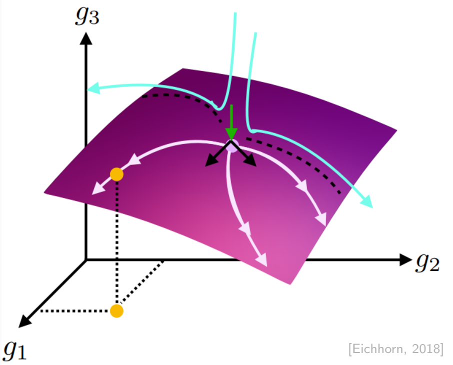


Finite relevant directions $\implies$ Predictive Theory

Could AS be Perturbative?

The Dimension 6 Truncation

$$S = \int d^dx\sqrt{g}\left[\frac{R-2\Lambda}{16\pi G_N} + c_{GS} R_{\mu\nu}{}^{\rho\sigma} R_{\rho\sigma}{}^{\alpha\beta} R_{\alpha\beta}{}^{\mu\nu}\right]$$

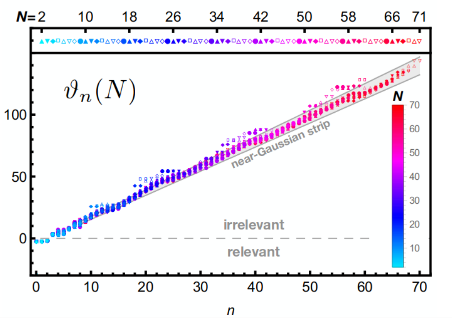
$$c^*_{GS} \propto 10^{-7}$$

order	θ_1	θ_2
$\mathcal{O}(\partial^2)$	2.239	—
$\mathcal{O}(\partial^4)$	2.228	—
$\mathcal{O}(\partial^6)$	2.225	-3.850

[Baldazzi, Falls, Kluth, Knorr, 2023]

Polynomial Truncations

$$S = \int d^dx\sqrt{g}\sum_{n=0}^{N} a_n R^n$$



[Falls, Litim, Schröder, 2019]

Perturbative?

Running of G_N and Power-Law Divergences

$$G_0 = G_N + \underbrace{\delta G_N}_{\text{1-loop}} \quad \text{where} \quad \delta G_N \sim (\mathit{scale})^2 G_N^2$$

Cutoff

Scale without Symmetry

- Captures power-law effects
- Breaks symmetry

DimReg

Symmetry without Scale

- Respects symmetry
- Only log-divergencies

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Goal: A definition of G_N that:

Maintains symmetry & captures PLD & extends to higher orders

PLD from DimReg

Idea: Subtracting poles in $d \neq 4$ is equivalent to a **mass-dependent renormalization scheme**

QCD: Threshold effects in the running coupling [Kluth, 2026]

$$\beta(\alpha) = (d-4)\alpha + \frac{\alpha^2}{4\pi}\left[-\frac{22}{3}C_A + \frac{4}{3}\sum_{q=1}^{n_F} e^{-m_q/\mu}\right]$$

Gravity: Quadratic divergencies [Kluth, 2025], [Falls, Fererro, 2025]

$$G_0 = G + A\frac{\mu^{d-2}}{d-2}G^2$$

Propagator of Gravity

$$\mathcal{P}_{\mu\nu\rho\sigma}(p) = \frac{\kappa^2}{p^2}\left(\delta_{\mu(\rho}\delta_{\sigma)\nu} - \frac{1}{d-2}\delta_{\mu\nu}\delta_{\rho\sigma}\right)$$

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Goal: A definition of G_N that:

Maintains symmetry & captures PLD & extends to higher orders & avoids gravity in $d = 2$

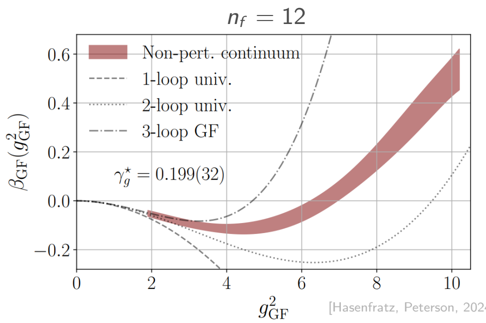
Gradientflow

Definition of α_s

$$\langle E(t)\rangle = \frac{1}{4}\langle G^a_{\mu\nu} G^a_{\mu\nu}\rangle \propto \alpha_{GF}(t)$$

Key Properties

- **UV Finiteness:** Operators at $t > 0$ are finite [Lüscher, Weisz, 2011]
- **Internal Cutoff:** e^{-tp^2} smooth momentum cutoff
- **Symmetry:** Maintains BRST/Gauge invariance



[Hasenfratz, Peterson, 2024]

Applications:

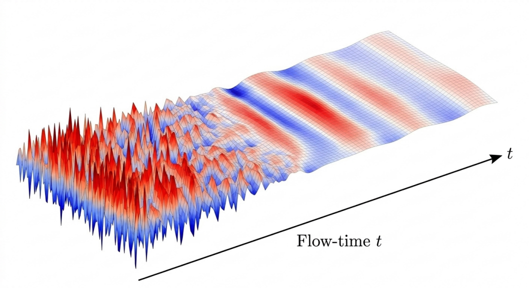
- Lattice Scale Setting.
- EMT Matrix Elements.
- Parton Densities (PDFs).
- ...

Reminder: Perturbative Gradient Flow

Evolve the gauge field $A_\mu(x)$ along an artificial flow-time t [Lüscher, 2010]:

$$\partial_t B_\mu(t,x) = D_\nu G_{\nu\mu}, \quad B_\mu(0,x) = A_\mu(x)$$

- **Diffusion :**
  $\partial_t B_\mu = \partial^2 B_\mu + \mathcal{O}(g)$
- **LO Solution:**
  $\tilde{B}_\mu(t,p) = e^{-tp^2}\tilde{A}_\mu(p) + \mathcal{O}(g)$



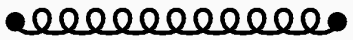
Reminder: Perturbative Gradient Flow

Trick: $S = S_{YM} + S_{flow}$

$$S_{flow} = \int_0^\infty \mathrm{d}t \int \mathrm{d}^d x \, L_\mu \left(\partial_t B_\mu - D_\nu G_{\nu\mu}\right)$$

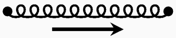
Propagator BB

$$\langle BB \rangle \sim \frac{e^{-(t+s)p^2}}{p^2}$$



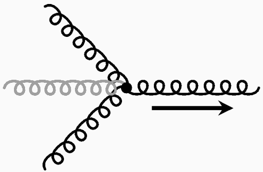
Flow Line LB

$$\langle LB \rangle \sim \theta(t-s) e^{-(t-s)p^2}$$



Vertices

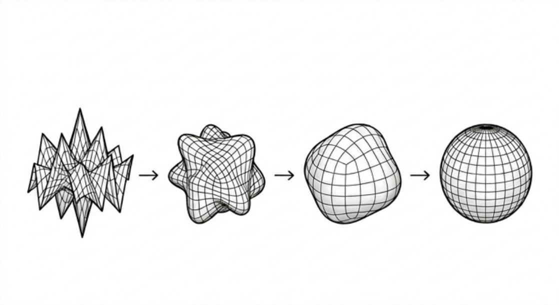
$LBB, LBBB$



The Ricci Flow in Gravity

Evolve the metric $\hat{g}_{\mu\nu}(t)$ along the Ricci flow:

$$\partial_t \hat{g}_{\mu\nu} = -2\hat{R}_{\mu\nu}, \quad \hat{g}_{\mu\nu}(0,x) = g_{\mu\nu}(x)$$



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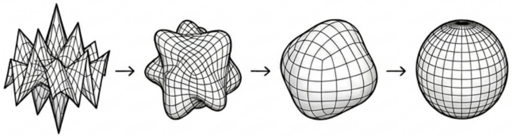
- **Perturbative Expansion:**
  $\hat{g}_{\mu\nu} = \delta_{\mu\nu} + \hat{h}_{\mu\nu}$
- **LO Solution:**
  $\tilde{\hat{h}}_{\mu\nu} \sim e^{-tp^2}\tilde{h}_{\mu\nu} + \dots$



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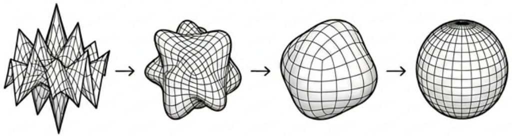
- **The Goal:**
  Calculate VEVs of invariant operators:

$$\left\langle \int d^d x \sqrt{\hat{g}} \right\rangle, \quad \left\langle \int d^d x \sqrt{\hat{g}}\hat{R} \right\rangle$$



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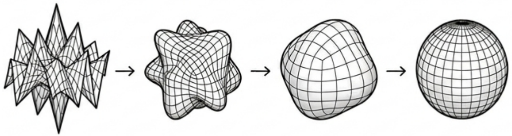
$$\left\langle \int d^d x \sqrt{\hat{g}} \right\rangle, \quad \left\langle \int d^d x \sqrt{\hat{g}}\hat{R} \right\rangle$$

and define a running coupling $G_{RF}(t)$



The $(d+1)$-Dimensional Formulation

The Strategy: $(d+1)$-dimensional QFT

$$S = S_{grav} + S_{flow}$$

$$S_{flow} = \int_0^\infty dt \int d^d x \, \hat{L}^{\mu\nu} \left[\partial_t \hat{h}_{\mu\nu} + 2\hat{R}_{\mu\nu} \right]$$

Flow Vertices

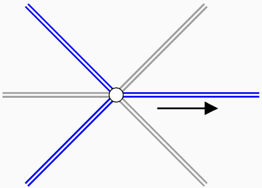
$$\hat{L}\hat{h}\hat{h}, \hat{L}\hat{h}\hat{h}\hat{h}, \ldots$$

Propagator

$$\langle \hat{h}\hat{h} \rangle \sim \frac{e^{-(t+s)p^2}}{p^2}$$

Flow Line

$$\langle \hat{L}\hat{h} \rangle \sim \theta(t-s) e^{-(t-s)p^2}$$



BRST Symmetry: Gravity vs. Flow

Standard Gravity Sector

$$S_{\text{grav}} + S_{\text{gf}} + S_{\text{gh}}$$

- **Metric Fluctuation**

$$s\,h_{\mu\nu} = c^\rho \partial_\rho g_{\mu\nu} + g_{\mu\rho}\partial_\nu c^\rho + g_{\nu\rho}\partial_\mu c^\rho$$

- $+$ ghosts

Flow Sector

$$S_{\text{flow}} + S_{\text{f,gf}} + S_{\text{f,gh}}$$

- **Flowed Fluctuation**

$$s\hat{h}_{\mu\nu} = \hat{c}^\rho \partial_\rho \hat{g}_{\mu\nu} + \hat{g}_{\mu\rho}\partial_\nu \hat{c}^\rho + \hat{g}_{\nu\rho}\partial_\mu \hat{c}^\rho$$

- **Lagrange Multiplier**

$$s\hat{L}^{\mu\nu} = \hat{c}^\rho \partial_\rho \hat{L}^{\mu\nu} - \hat{L}^{\mu\rho}\partial_\rho \hat{c}^\nu - \hat{L}^{\nu\rho}\partial_\rho \hat{c}^\mu + \hat{L}^{\mu\nu}\partial_\rho \hat{c}^\rho$$

- $+$ flowed ghosts

Renormalisation

The Localization of Divergences

- For $t > 0$, flow provides exponential damping e^{-tp^2}
- UV divergences are strictly restricted to the $t = 0$ boundary

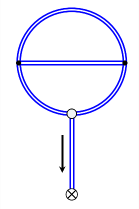
1. Standard One-Loop Gravity

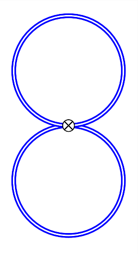
$$g_{\mu\nu} \to g_{\mu\nu} + \frac{G_N}{4\pi}\left[c_1 R\, g_{\mu\nu} + c_2 R_{\mu\nu}\right]$$

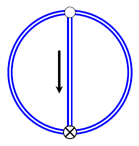
2. Ricci Flow Initial Condition

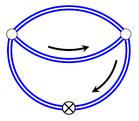
$$\hat{g}_{\mu\nu}(t = 0) = g_{\mu\nu} + \delta\hat{g}_{\mu\nu}$$

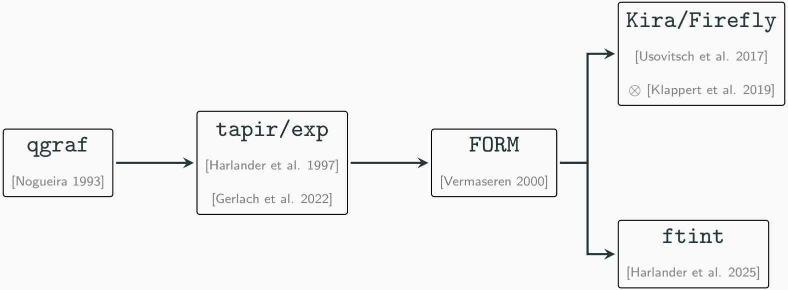
Computational Toolchain











Results and Renormalization Schemes

$$\left\langle \int d^d x \sqrt{\hat{g}} \right\rangle^{\overline{\text{MS}}} = -\frac{3G_N}{4\pi t}\left[1 + \frac{G_N}{4\pi t}\left(5\, l_{\mu t} + 9 \ln\frac{4}{3} + \frac{1231}{120}\right)\right]$$

$$\left\langle \int d^d x \sqrt{\hat{g}} \hat{R} \right\rangle^{\overline{\text{MS}}} = -\frac{3G_N}{4\pi t^2}\left[1 + \frac{G_N}{2\pi t}\left(5\, l_{\mu t} + 9 \ln\frac{4}{3} + \frac{931}{120}\right)\right]$$

Gauge parameter independence

Fixed Volume Scheme (FVS)

$$\boxed{\left\langle \int d^d x \sqrt{\hat{g}} \right\rangle^{\text{FVS}} = -\frac{3G_N}{4\pi t}}$$

$$\boxed{\left\langle \int d^d x \sqrt{\hat{g}} \hat{R} \right\rangle^{\text{FVS}} = -\frac{3G_N}{4\pi t^2}\left[1 - \frac{5G_N}{4\pi t}\right]}$$

Identification $\mu \propto 1/\sqrt{t} \rightarrow$ PLD sensitivity!

$l_{\mu t} = \ln(\mu^2 t) + \gamma_E$

Coupling and Fixed Point

Ricci-Flow Coupling Definition

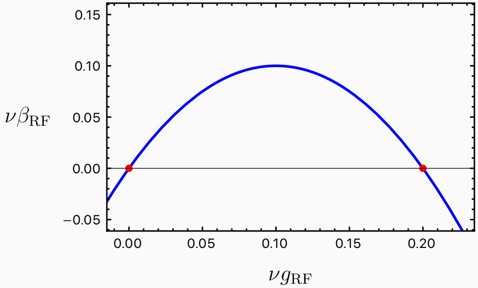
$$g_{\mathrm{RF}}(\mu) \equiv -\frac{t}{3\nu}\left\langle \int_x \sqrt{\hat{g}}\,\hat{R} \right\rangle^{\mathrm{FVS}}\Bigg|_{t=\rho/\mu^2}$$

The β-Function

$$\beta_{\mathrm{RF}} \equiv \mu\frac{\partial}{\partial\mu} g_{\mathrm{RF}} = 2g_{\mathrm{RF}}\left(1 - 5\nu g_{\mathrm{RF}}\right)$$

Fixed Points:

- **Gaußian:** $g^*_{\mathrm{RF}} = 0$
- **Non-Gaußian:** $g^*_{\mathrm{RF}} = \frac{1}{5\nu}$



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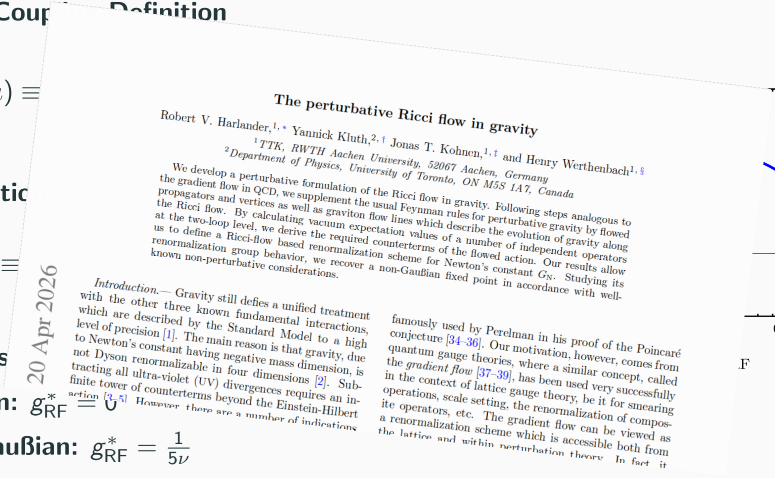

20 Apr 2026

The perturbative Ricci flow in gravity

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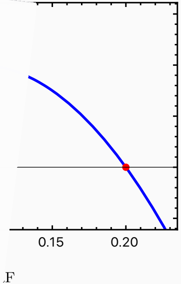
[2]Department of Physics, University of Toronto, ON M5S 1A7, Canada

We develop a perturbative formulation of the Ricci flow in gravity. Following steps analogous to the gradient flow in QCD, we supplement the usual Feynman rules for perturbative gravity by flowed propagators and vertices as well as graviton flow lines which describe the evolution of gravity along the Ricci flow. By calculating vacuum expectation values of a number of independent operators at the two-loop level, we derive the required counterterms of the flowed action. Our results allow us to define a Ricci-flow based renormalization scheme for Newton's constant G_{N}. Studying its renormalization group behavior, we recover a non-Gaußian fixed point in accordance with well-known non-perturbative considerations.

Introduction.— Gravity still defies a unified treatment with the other three known fundamental interactions, which are described by the Standard Model to a high level of precision [1]. The main reason is that gravity, due to Newton's constant having negative mass dimension, is not Dyson renormalizable in four dimensions [2]. Subtracting all ultra-violet (UV) divergences requires an infinite tower of counterterms beyond the Einstein-Hilbert action [3–5]. However, there are a number of indications

famously used by Perelman in his proof of the Poincaré conjecture [34–36]. Our motivation, however, comes from quantum gauge theories, where a similar concept, called the *gradient flow* [37–39], has been used very successfully in the context of lattice gauge theory, be it for smearing operations, scale setting, the renormalization of composite operators, etc. The gradient flow can be viewed as a renormalization scheme which is accessible both from the lattice and within perturbation theory. In fact, it



Fixed Points

- **Gaußian:** $g^*_{\mathrm{RF}} = 0$
- **Non-Gaußian:** $g^*_{\mathrm{RF}} = \frac{1}{5\nu}$

Summary & Outlook

Achievements

- **Framework:** Ricci flow formulated as a $(d+1)$-dimensional QFT
- **Renormalization:** Systematic renormalization of flowed gravity established
- **Asymptotic Safety:** Recovered via purely perturbative methods

Future Directions:

- **Operators:** Including Λ_{cc}, ...
- **Precision:** Extending to higher loops
- **SM:** Including matter and gauge fields

Summary & Outlook

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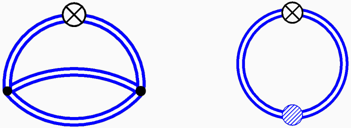
Thank you for your attention!

NNLO

Order	Independent Operators	Diagrams
LO	R	1
NLO	$R^2, R^2_{\mu\nu}$	$\sim$ 30
NNLO	$R\Box R, R_{\mu\nu}\Box R^{\mu\nu}, R^3_{\mu\nu\rho\sigma}, R^3, \ldots$ (10)	$\sim$ 400

NLO



NNLO