Classically perfect gradient flows

from machine-learned fixed-point actions

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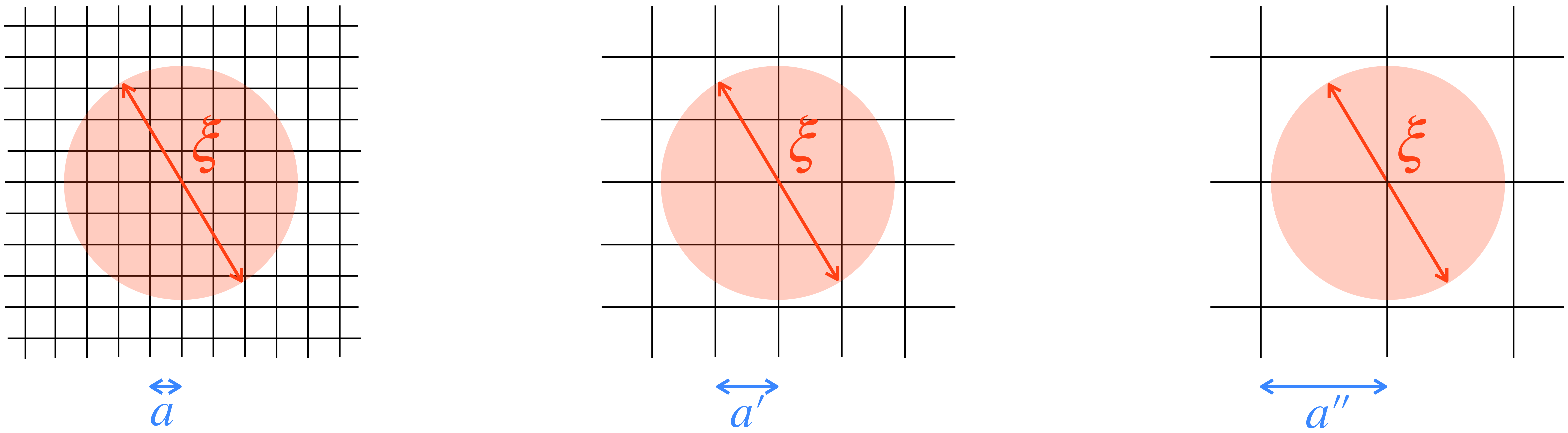
PRD 110 (2024) 7, 074502 [arXiv:2401.06481], PRL 136 (2026) 3, 031901 [arXiv:2502.03315]

Gradient Flow Workshop, 13 May 2026 - University of Edinburgh, Switzerland

Introduction

The lattice spacing a is determined by the gauge coupling: $\beta = \frac{2N_c}{g^2}$

continuum limit (2nd order phase transition $\xi/a \to \infty$)



$g \ll g' \ll g''$

lattice spacing a from dimensionless g: $\Rightarrow$ dimensional transmutation

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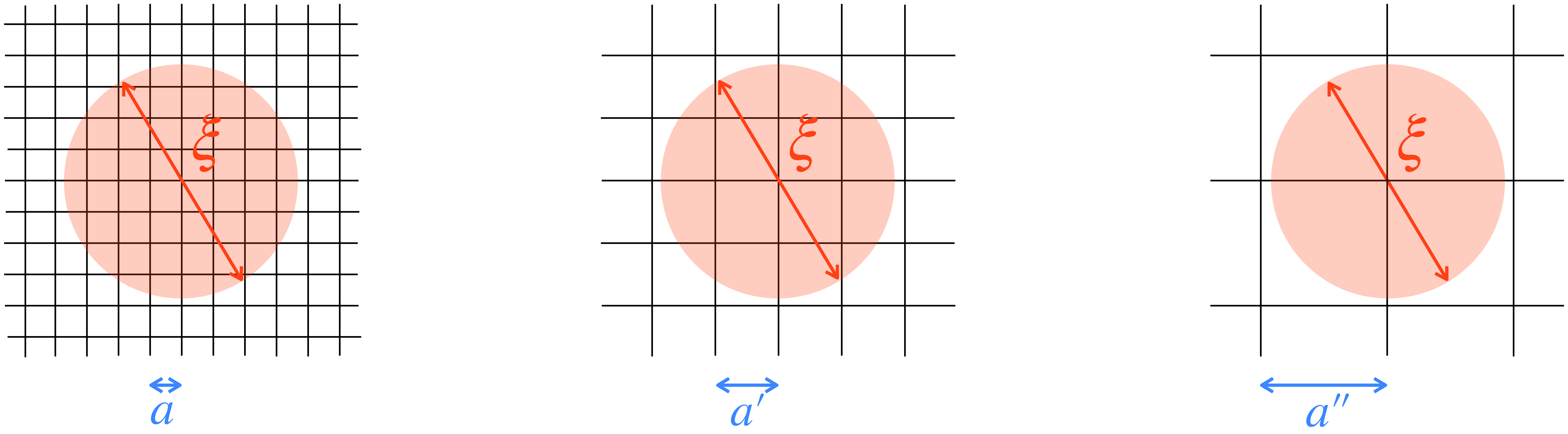


critical slowing down
(topological freezing) ?

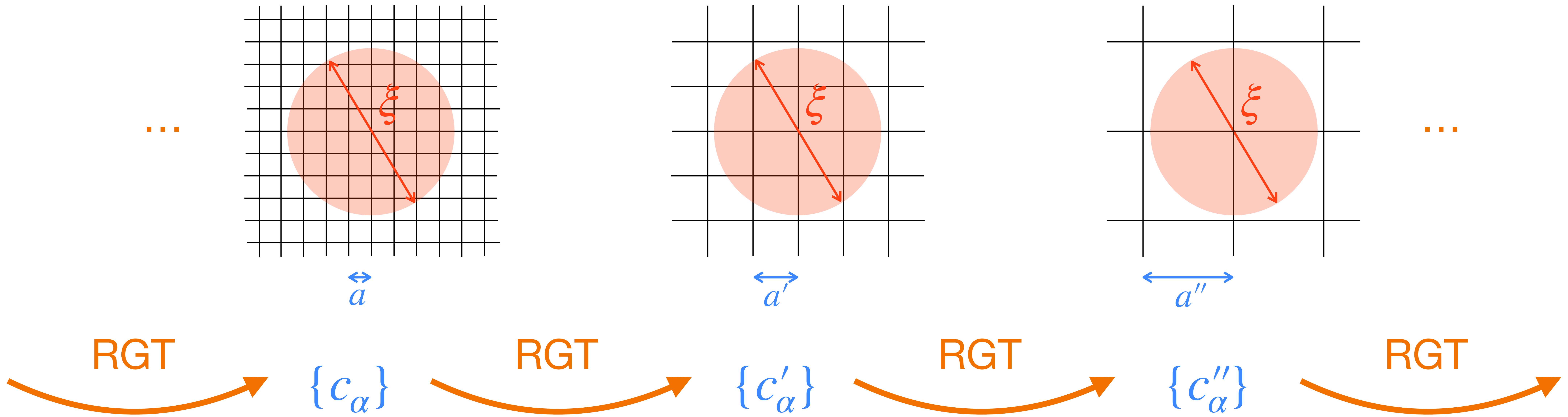
large lattice artefacts?

Renormalization group transformation

Introduce (coordinate space) renormalization group transformation (RGT):

$$\exp\left\{-\beta' A'[V]\right\} = \int \mathscr{D}U \exp\left\{-\beta\left(A[U] + T[U,V]\right)\right\}$$

The effective action $\beta' A'[V]$ is described by infinitely many couplings $\{c'_\alpha\}$:



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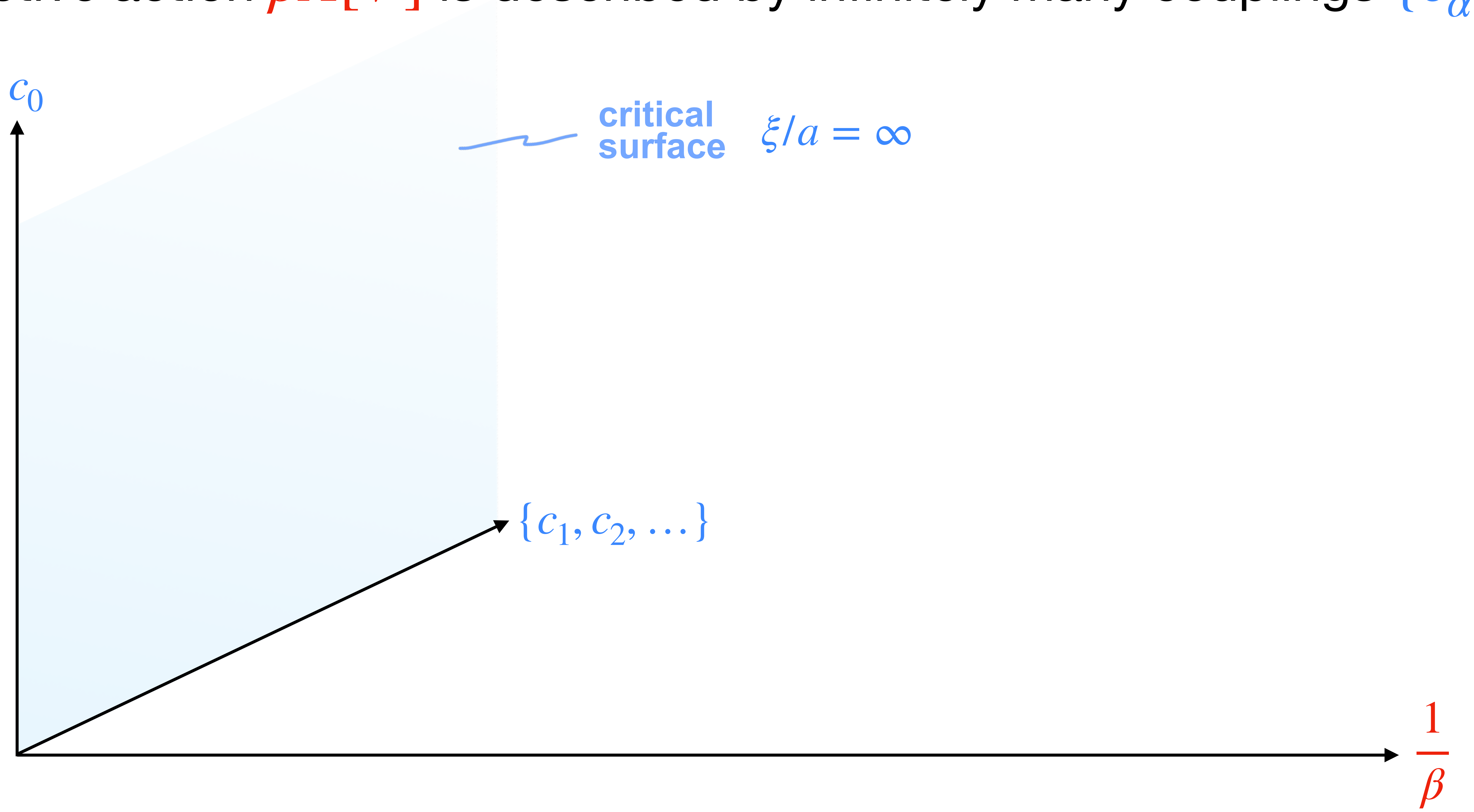

$\Rightarrow$ for asymptotically free gauge theories, there is one relevant direction $1/\beta$

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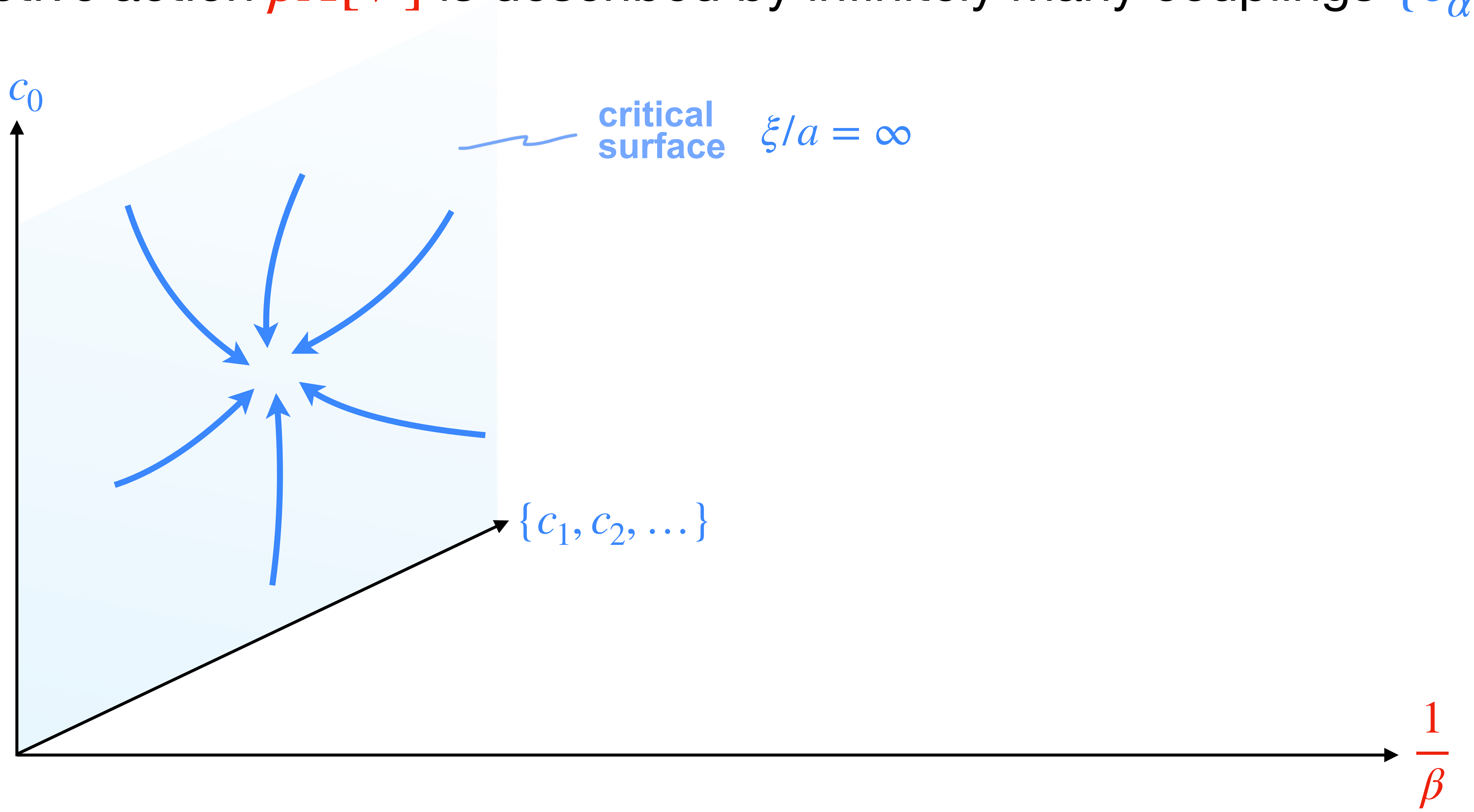

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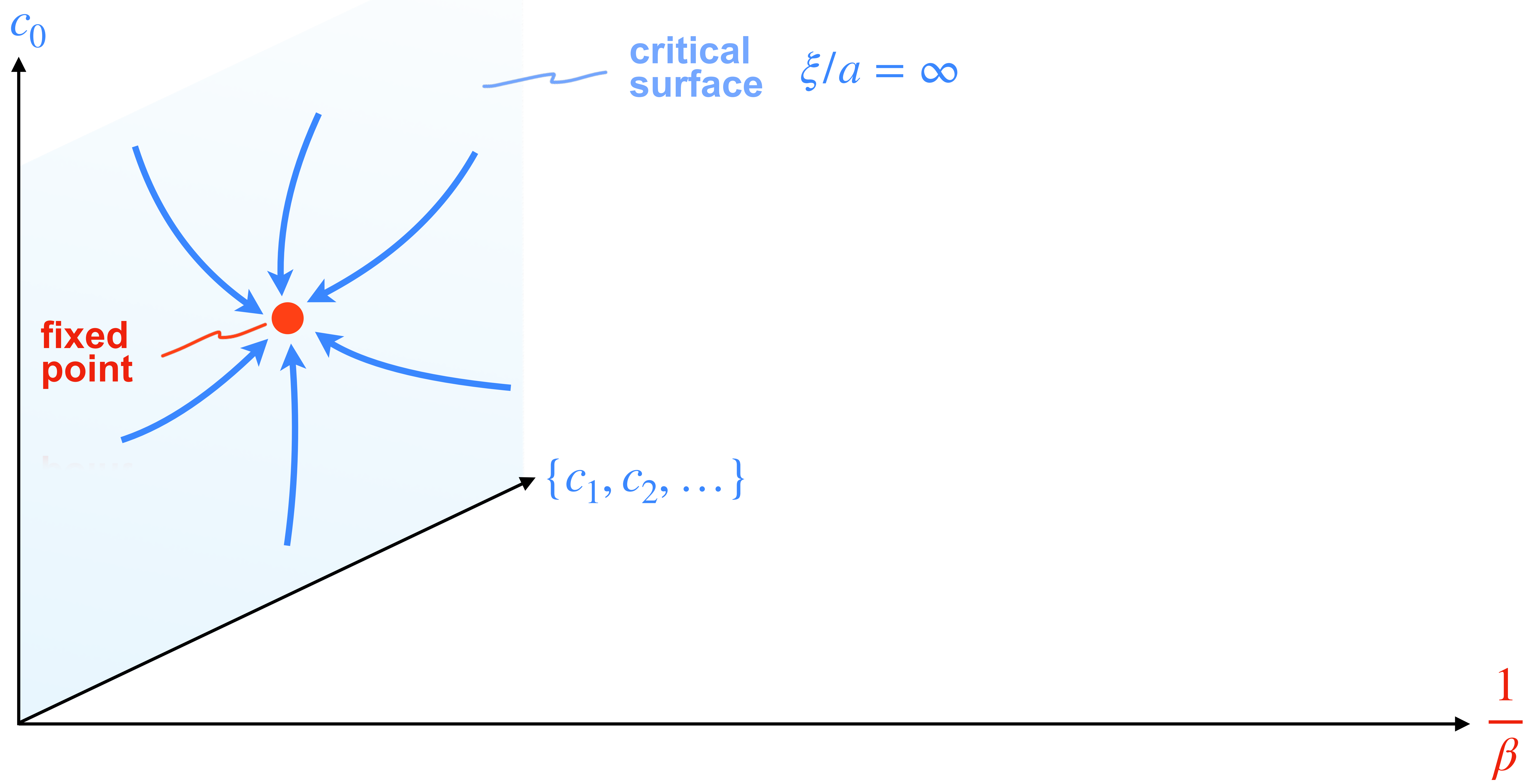

$\Rightarrow$ fixed point of RGT iterations (when $\xi/a \to \infty$): $\{c_\alpha^{FP}\} \xrightarrow{\text{RGT}} \{c_\alpha^{FP}\}$

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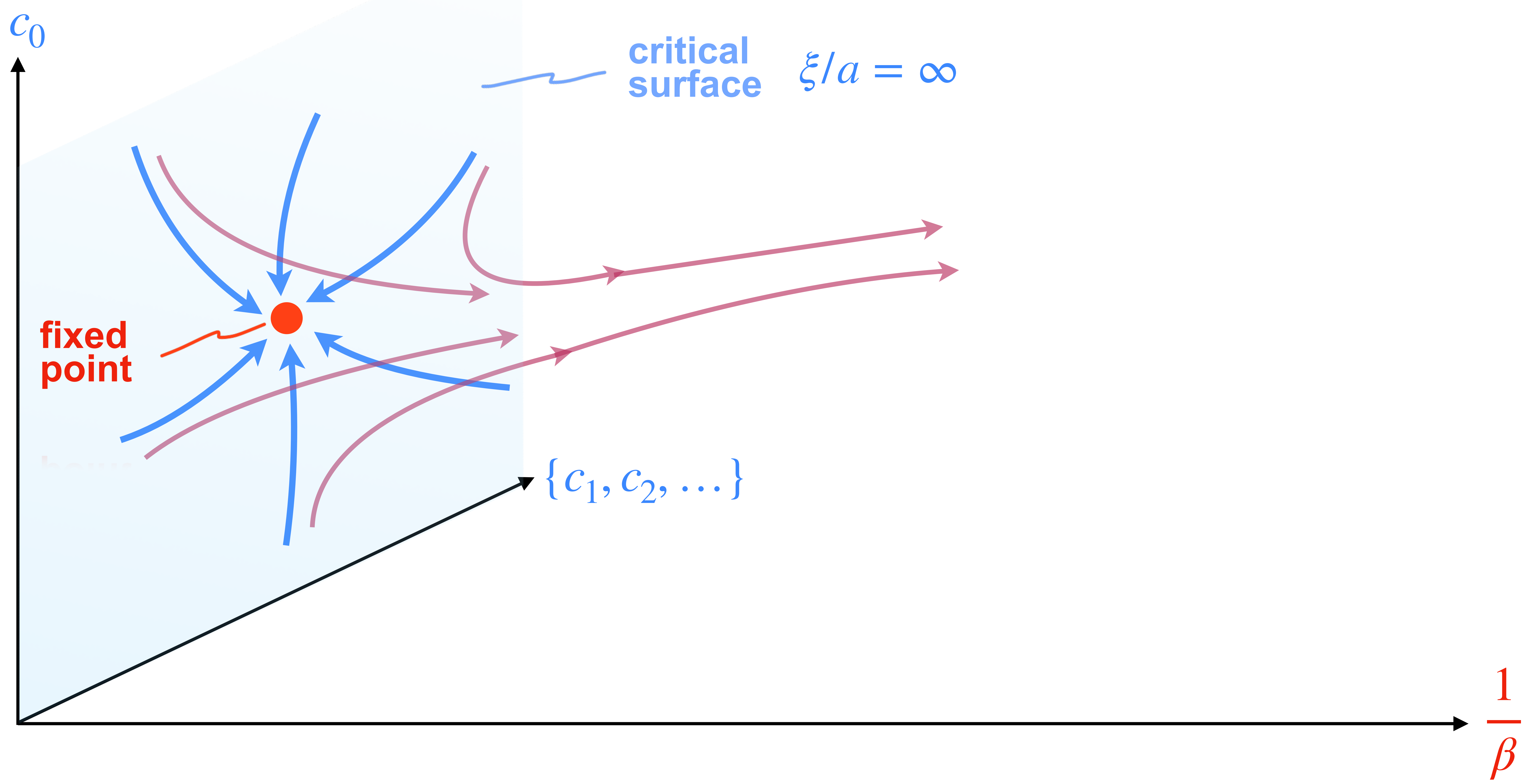

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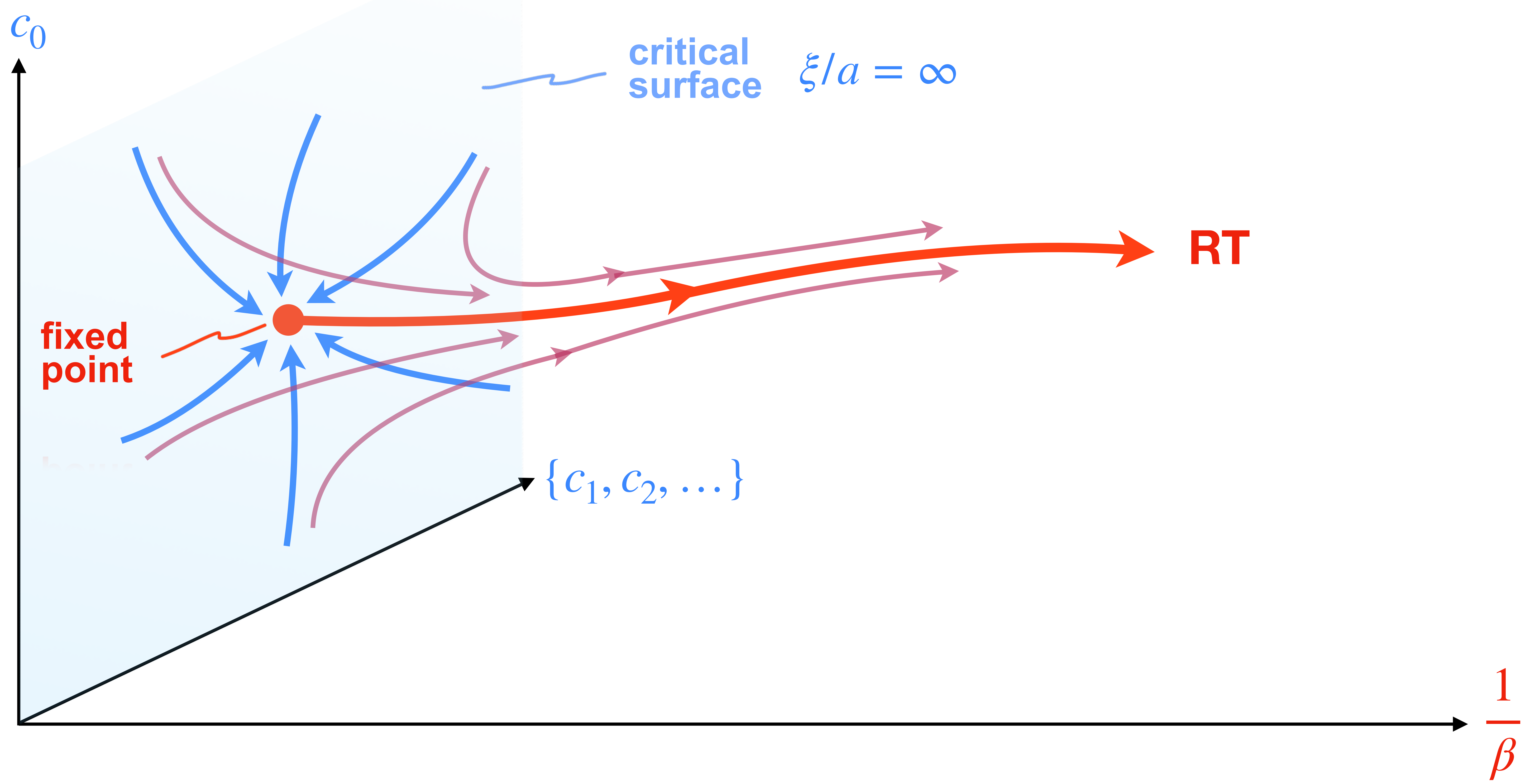

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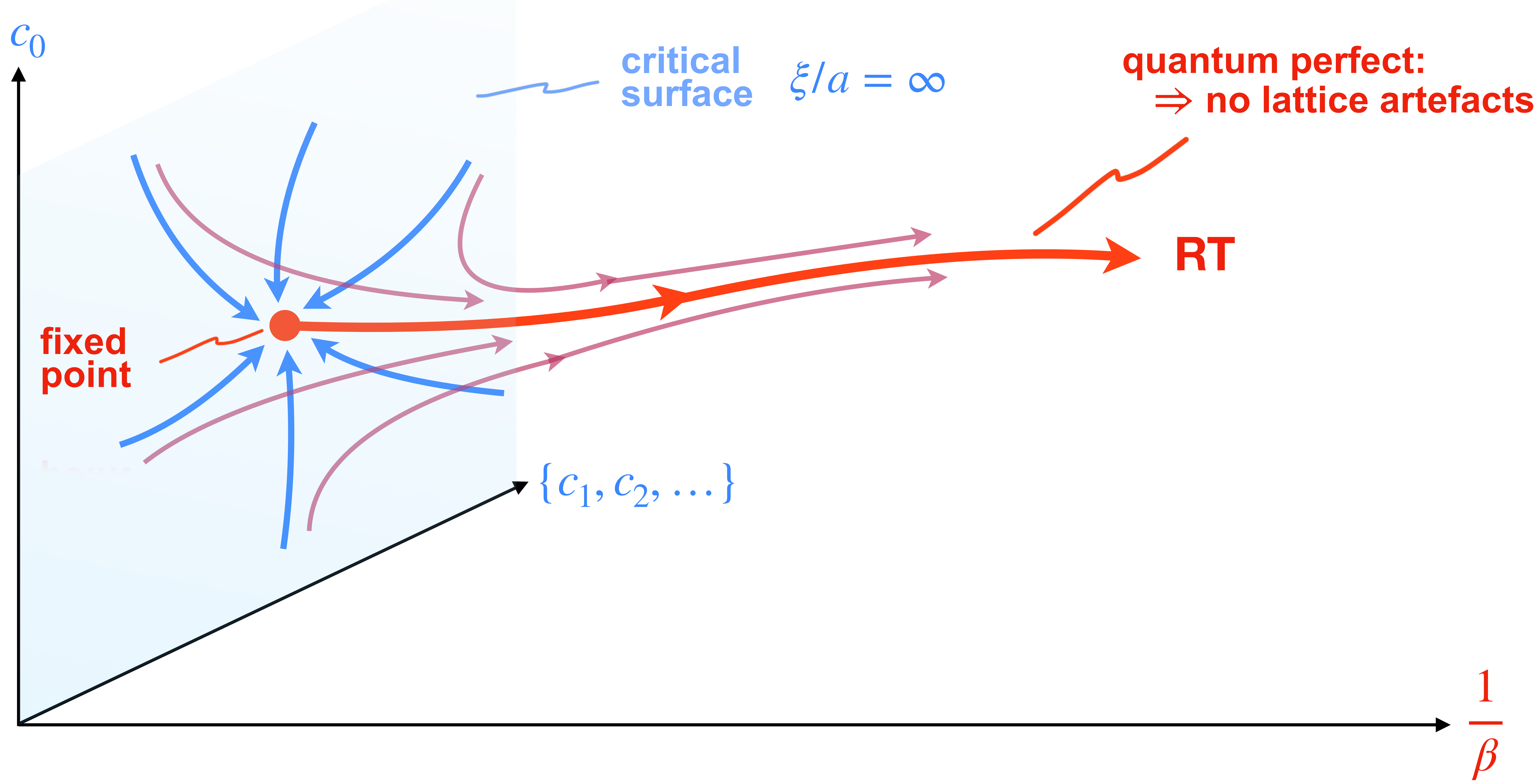

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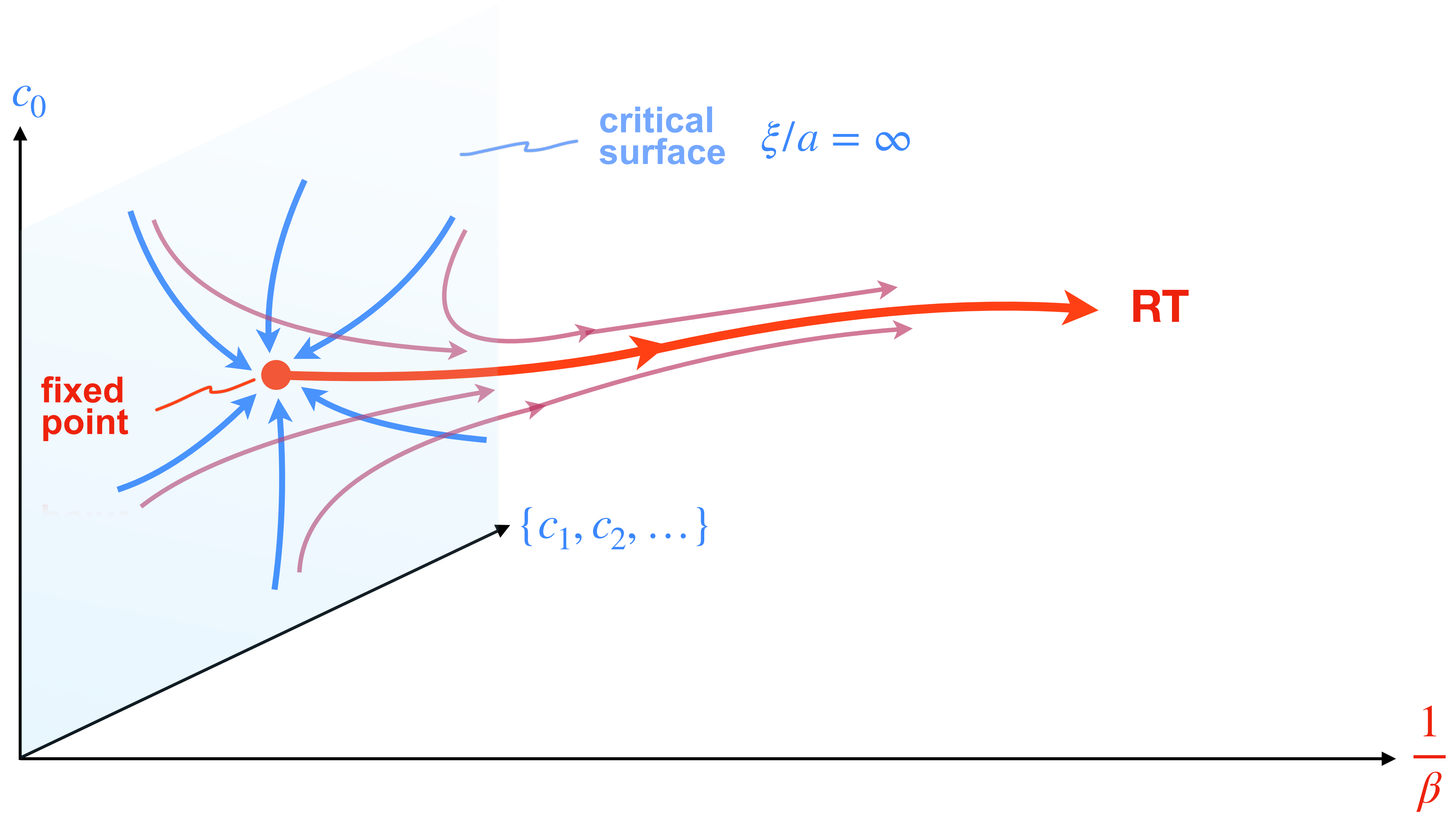

$$\exp\left\{-\beta' A'[V]\right\} = \int \mathscr{D}U \exp\left\{-\beta\left(A[U] + T[U,V]\right)\right\}$$

Two practical problems:

- how to parametrize RT, i.e., which set $\{c_\alpha\}$?
- how to determine $\{c_\alpha^{\text{RT}}\}$ or $\{c_\alpha^{\text{FP}}\}$?

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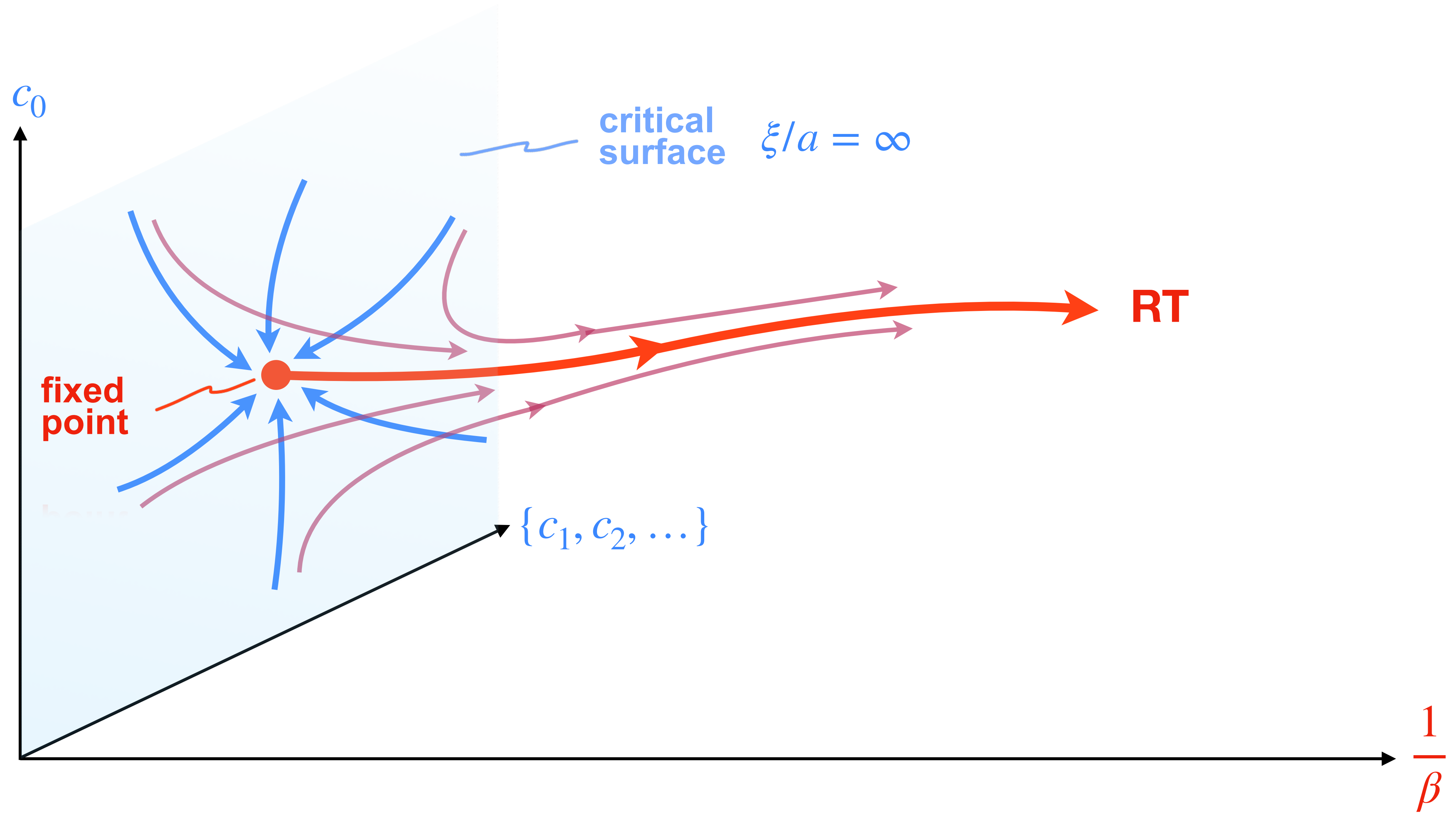

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P. Hasenfratz, F. Niedermayer [Nucl. Phys. B414 (1994) 785, hep-lat/9308004]

for $\beta \to \infty$ (on critical surface) the RGT becomes a classical saddle point problem:

$$A^{\text{FP}}[V] = \min_{\{U\}} \left\{A^{\text{FP}}[U] + T[U,V]\right\}$$

Classically perfect FP actions

The classical FP action A^{FP} defines an action for all β:

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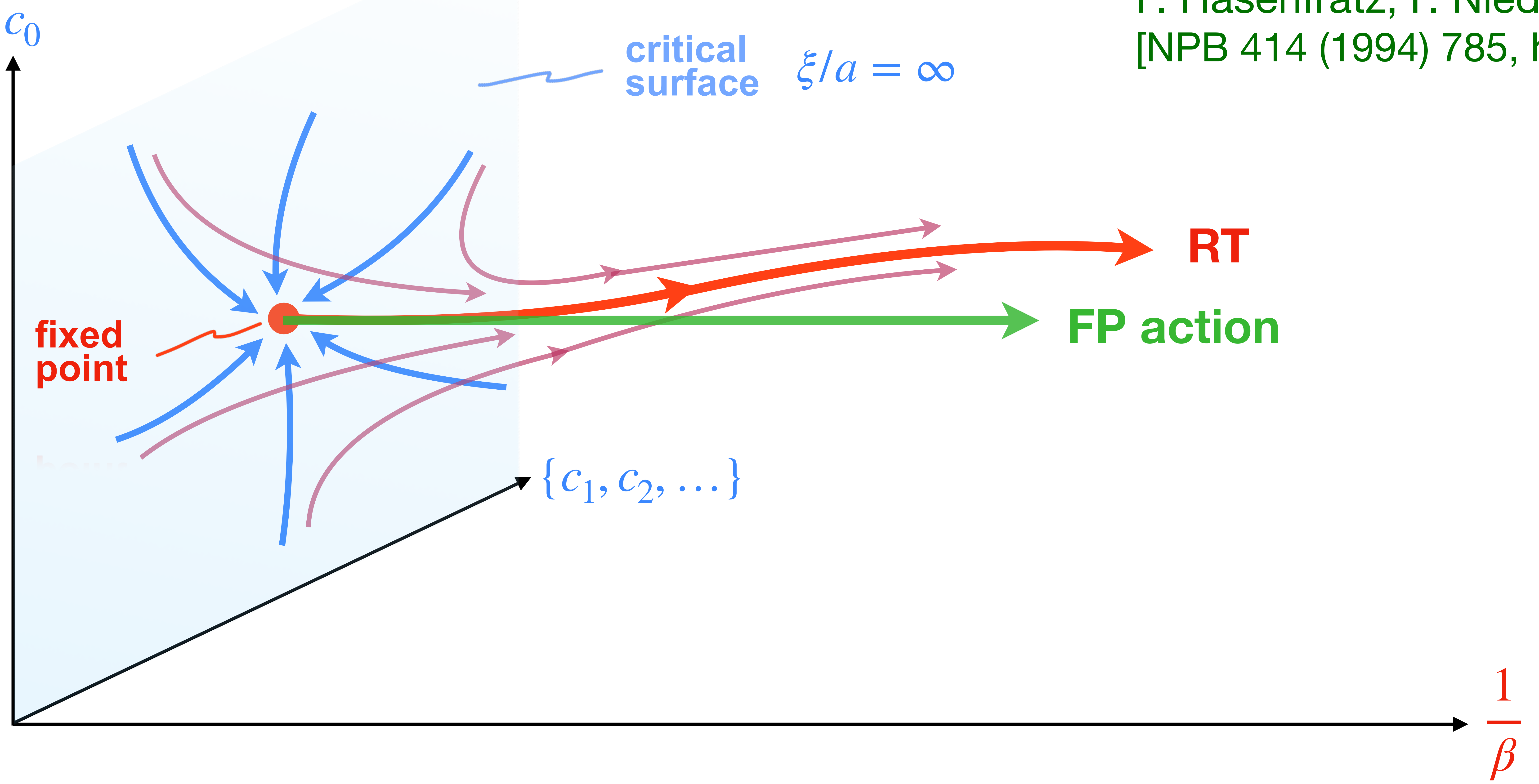

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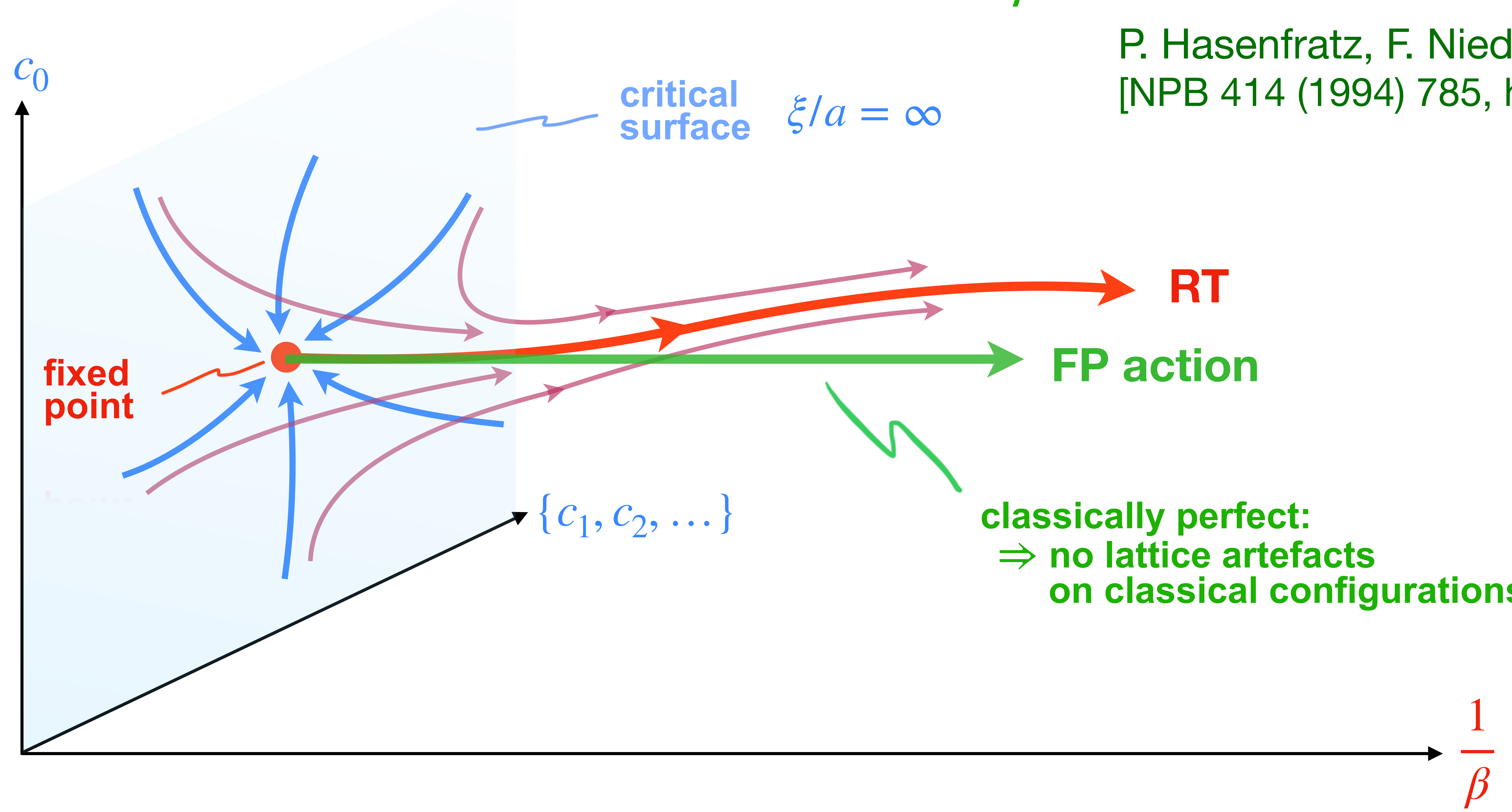

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- Proof using the chain rule:

$$\frac{\delta A^{\text{FP}}[V]}{\delta V} = \left[\frac{\delta}{\delta U}(A^{\text{FP}}[U] + T[U,V]) \frac{\delta U}{\delta V} + \frac{\delta T[U,V]}{\delta V} \right]_{U_{\text{min}}}$$

$$\Rightarrow \left. \frac{\delta T[U,V]}{\delta V} \right|_{U_{\text{min}}} = 0 \quad \text{hence} \quad T[U_{\text{min}}, V] = 0$$

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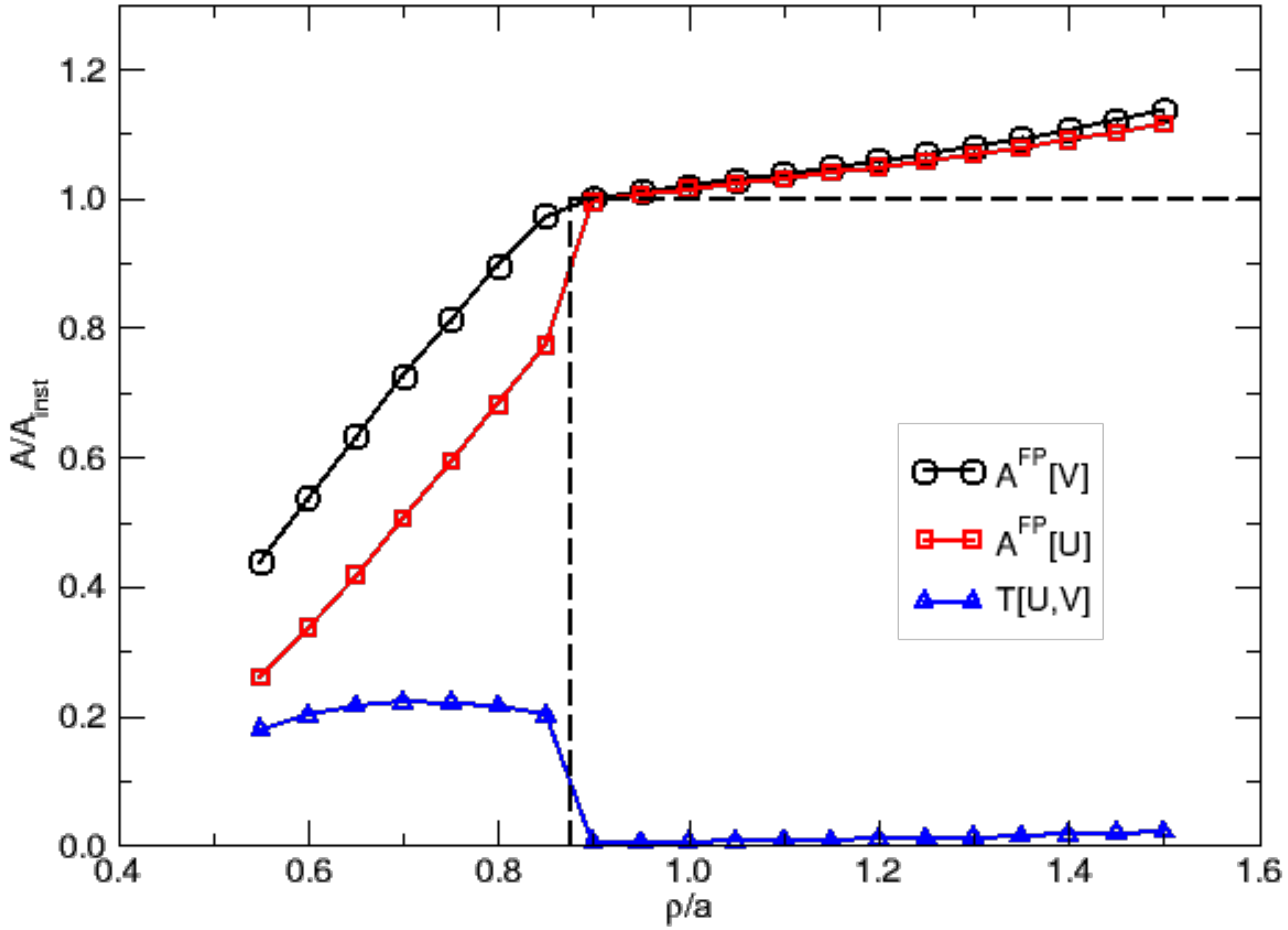

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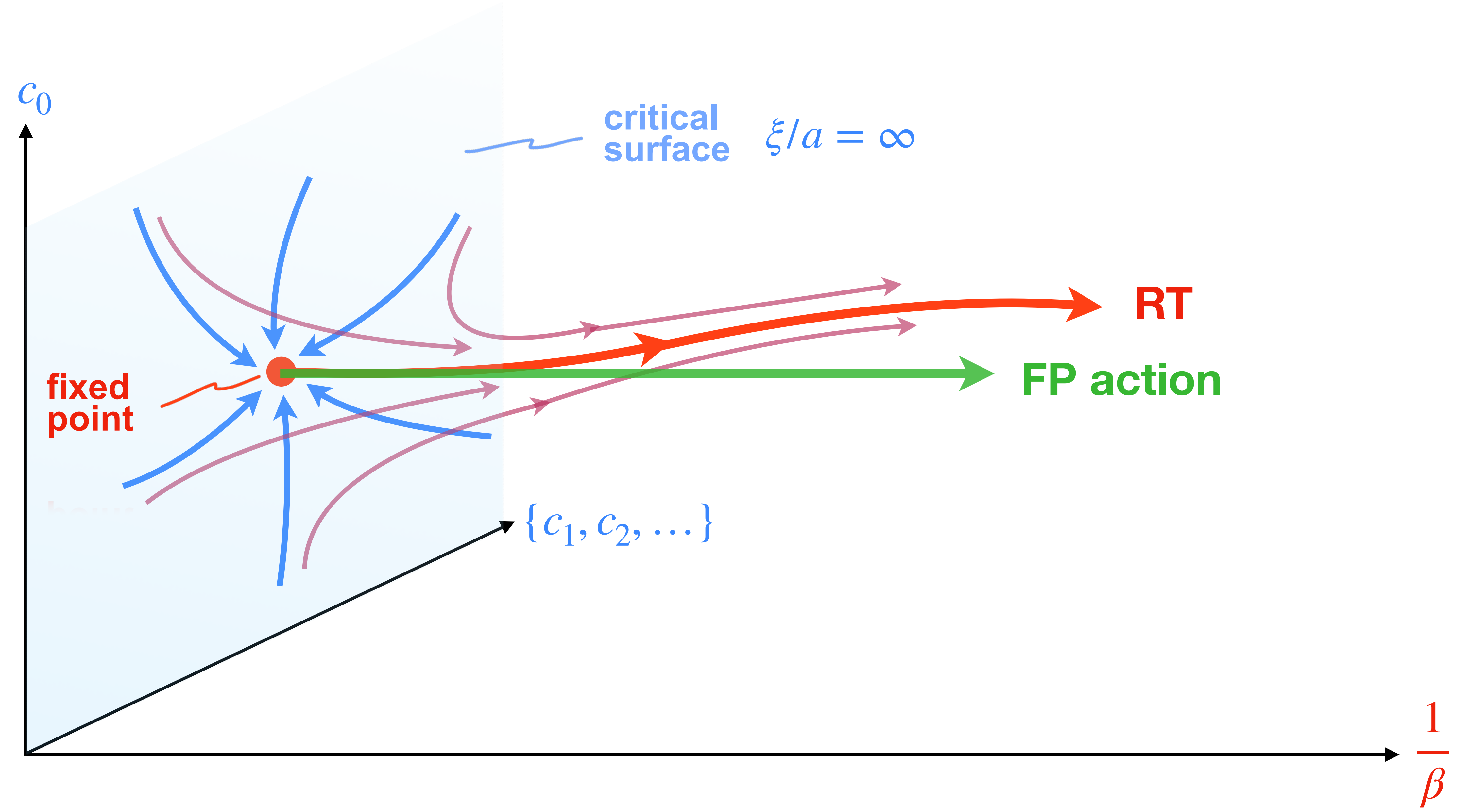

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$\Rightarrow$ $A^{\mathrm{FP}}[V]$ has scale invariant instanton solutions

- For large $\beta, A^{\mathrm{FP}}[V]$ is very close to $A^{\mathrm{RT}}[V]$:

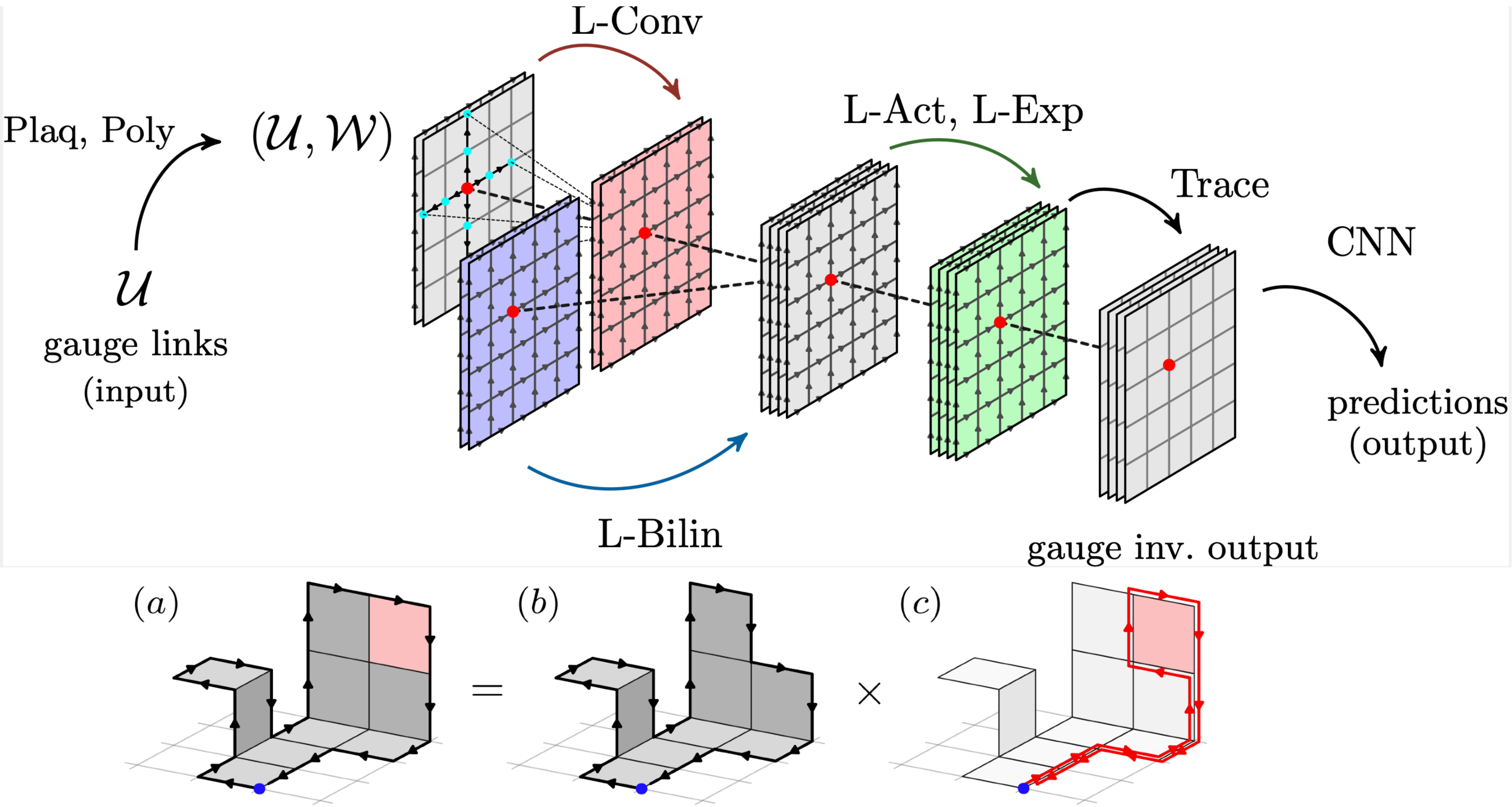
$\Rightarrow$ lattice artefacts expected to be substantially reduced:

$$\cancel{\mathcal{O}(a^{2n})}, \mathcal{O}(g^2 a^{2n}) \quad n = 1,2,\ldots$$

Machine learning the FP action

ML architecture: Lattice gauge equivariant Convolutional Neural Network (L-CNN)

[Favoni, Ipp, Müller, Schuh, PRL 128 (2022) 3, 2012.12901]



Machine learning the FP action: FP data

Use the exact FP action values for training, plus the derivatives of the FP action:

$$\frac{\delta A^{\mathrm{FP}}[V]}{\delta V^a_{x,\mu}} = \frac{\delta T[U,V]}{\delta V^a_{x,\mu}} = -\kappa\, \mathrm{Re\ Tr}(it^a V_{x,\mu} Q^\dagger_{x,\mu}) \qquad Q^\dagger_{x,\mu} = Q^\dagger_{x,\mu}[U]$$

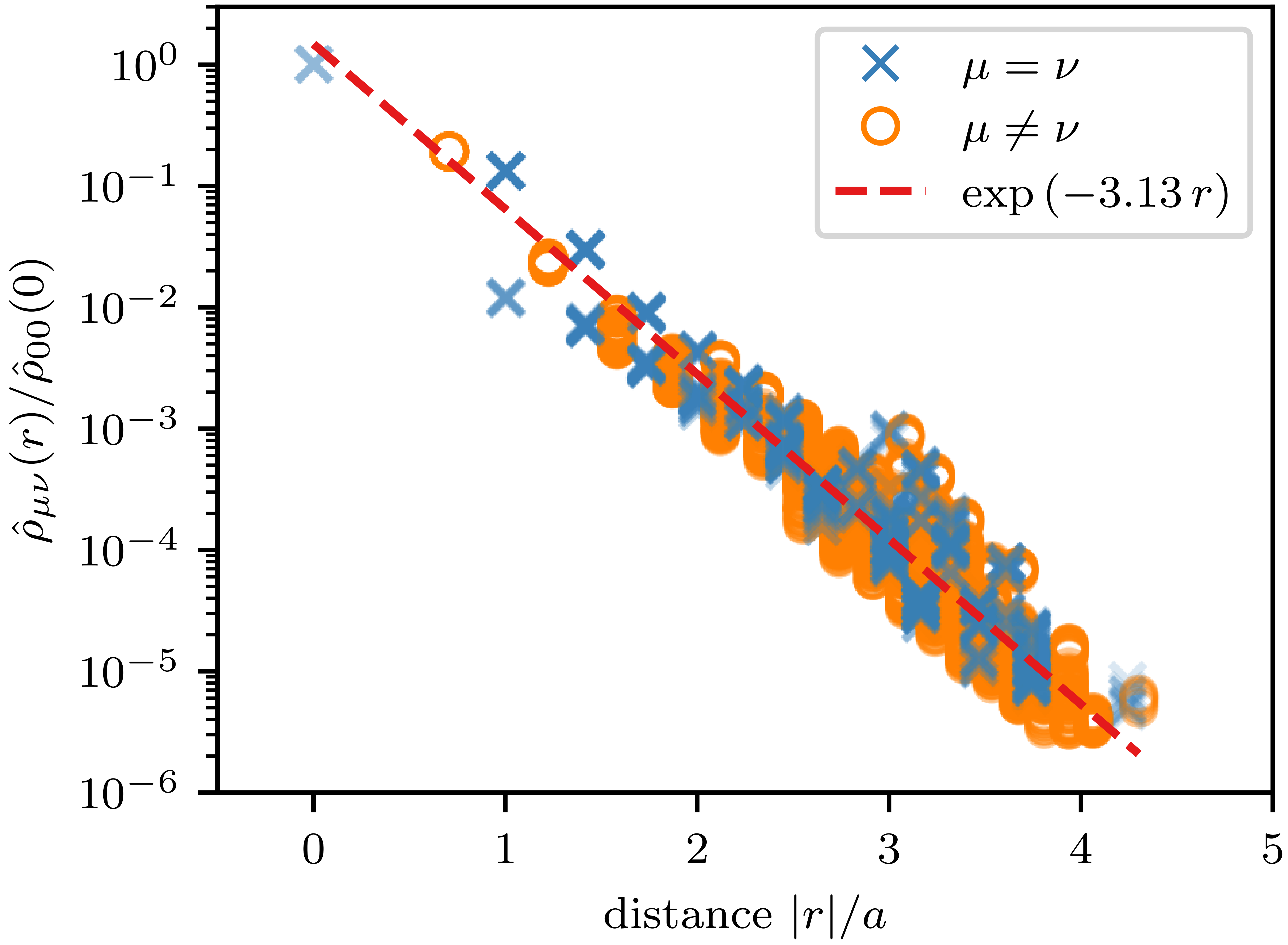
$\Rightarrow$ yields 4 x 8 x Volume (link) (color) (position) data per configuration

- FP action values
- FP action derivatives

} $\Rightarrow$ data set for supervised ML

Machine learning the FP action: Locality

Locality of L-CNN trained FP action:



$$\hat{\rho}_{\mu\nu}(r) = \frac{1}{\sqrt{N_c^2 - 1}} \sqrt{\sum_{a,b} D^{ab}_{\mu\nu}(x,y) D^{ab}_{\mu\nu}(x,y)}$$

where $D^{ab}_{\mu\nu}(x,y) = \dfrac{\delta^2 A}{\delta V^a_{x,\mu} \delta V^b_{y,\nu}}$

- couplings fall off exponentially, as desired
- even on coarse configurations

Machine learning the FP action: Symmetries

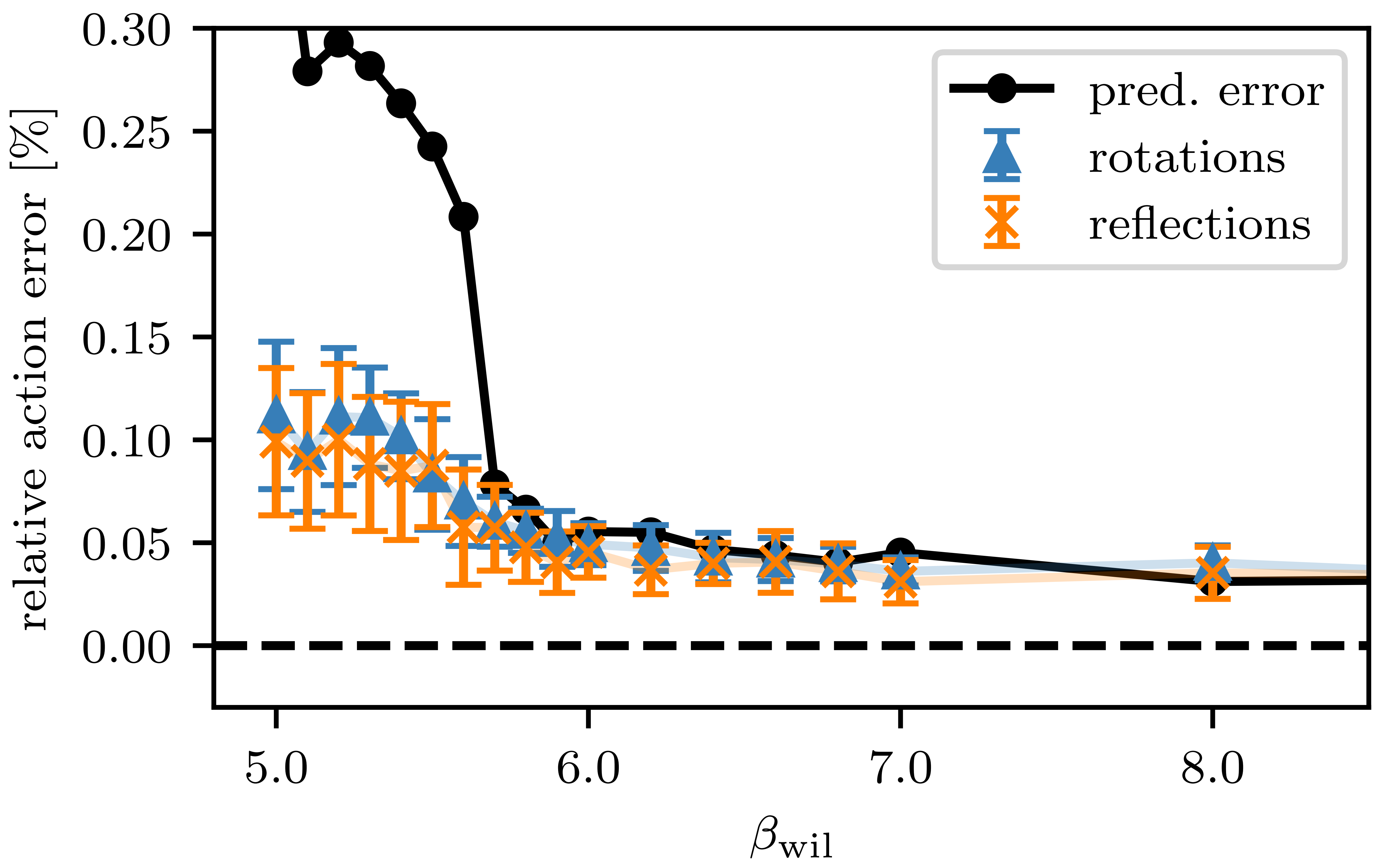
Test of lattice symmetries:

translations: $\Rightarrow A^{\text{L-CNN}}[U'_{(\text{shift})}] = A^{\text{L-CNN}}[U]$ by construction

rotations: $U \to U' = U_{(\text{rot})}$

reflections: $U \to U' = U_{(\text{refl})}$

a priori not present, but learned!



Scaling properties of FP actions

Use renormalized GF coupling as scaling quantity:

$$\frac{dA_\mu(t)}{dt} = -\frac{\delta S_{YM}}{\delta A_\mu} \qquad \langle t^2 E(t)\rangle = \frac{3(N^2-1)g^2}{128\pi^2}\left(1+O(g^2)\right) \equiv \frac{3g_{GF}^2(t)}{16\pi^2}$$

where g is the renormalised $\overline{\text{MS}}$ coupling at RG scale $\mu = 1/\sqrt{8t}$, with the corresponding β-function:

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$\Rightarrow$ turns out that GF with FP actions is classically perfect!

Gradient flow on the lattice

On the lattice, the flow of the gauge links is with a lattice flow action S^f: $$\frac{dU_\mu}{dt} = -i\frac{\delta S^f}{\delta U_\mu}U_\mu$$

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where $C(a^2/t) = 1 + \mathcal{O}(a^2/t)$ contains the tree-level lattice artifacts.

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Calculation in momentum space to quadratic order in A_μ:

$$A_\mu(p)S_{\mu\nu}(p)A_\nu(-p)$$

(with gauge fixing term $\mathcal{G}$)

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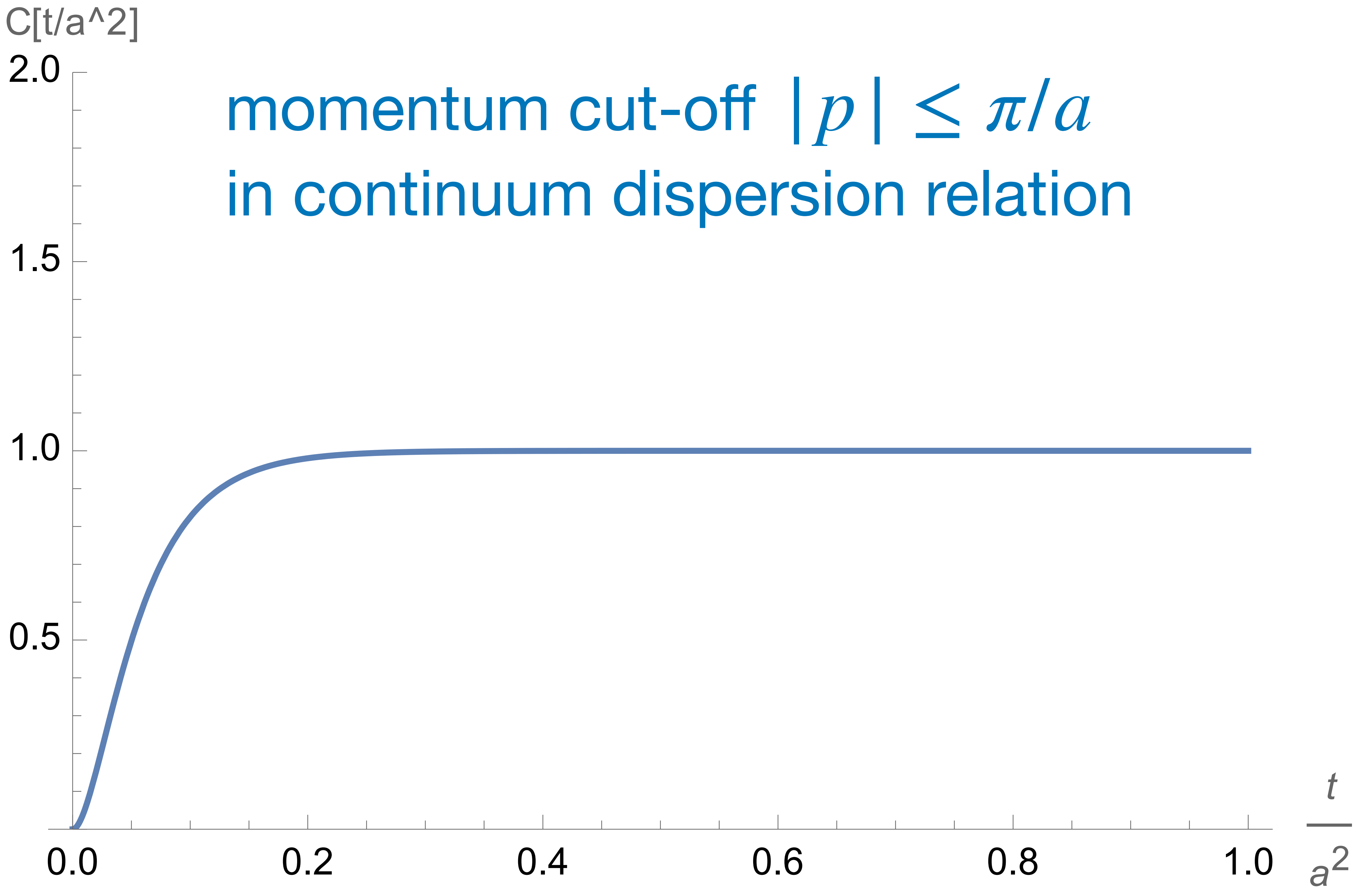

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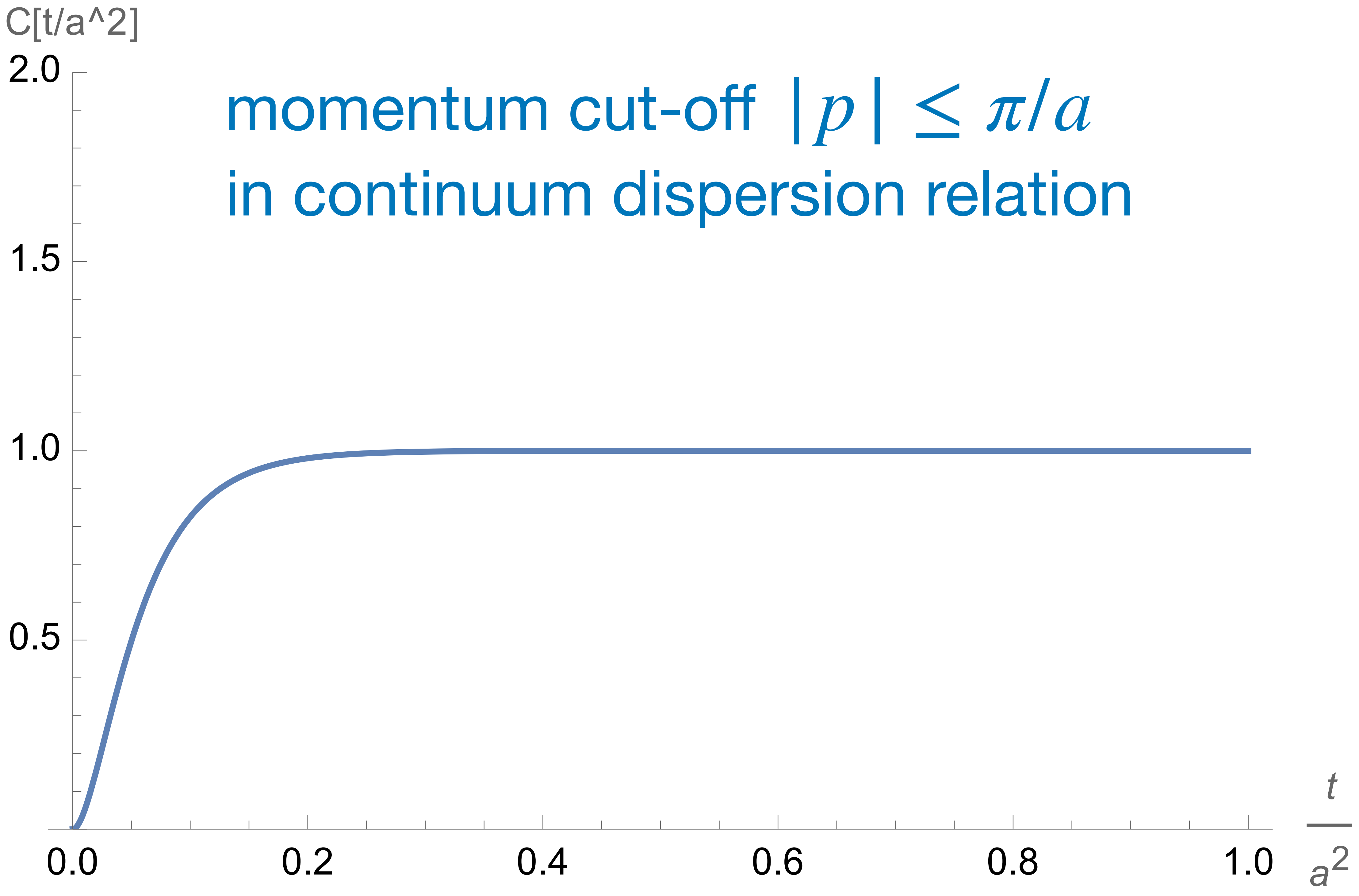

Classically perfect FP dispersion relation

Iterating the FP equation in the quadratic approximation:

$$D'_{\mu\nu}(p) = \frac{1}{16}\sum_{l=0}^{1}\left[\omega\left(\frac{p+2\pi l}{2}\right) D\left(\frac{p+2\pi l}{2}\right)\omega^{\dagger}\left(\frac{p+2\pi l}{2}\right)\right]_{\mu\nu} + \frac{1}{\kappa}\delta_{\mu\nu}$$

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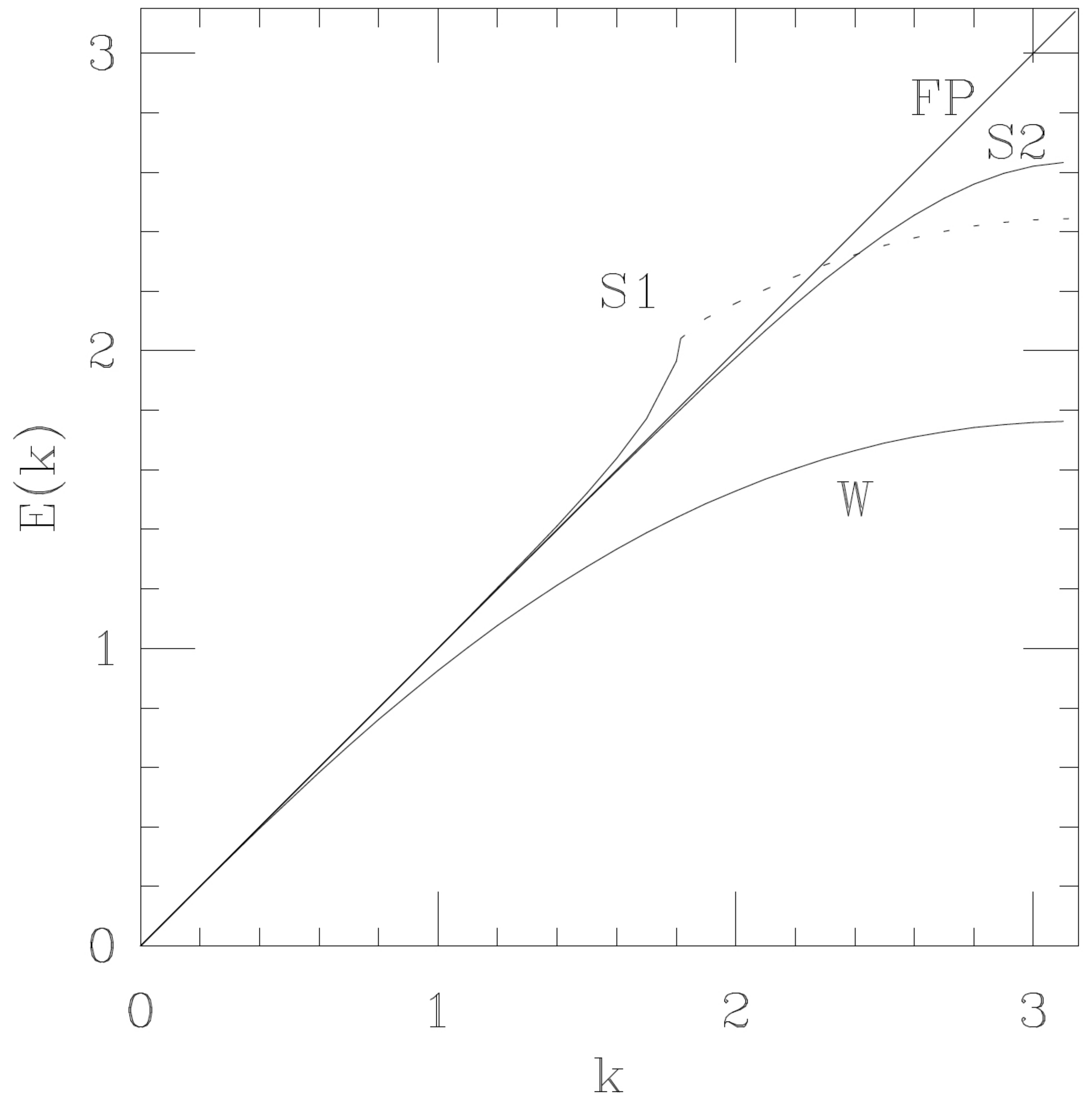
After n iterations:

$$D^{(n)}_{\mu\nu}(p) \sim \left[\Omega^{(n)}\left(\frac{p+2\pi l}{2^n}\right)\Omega^{(n)\dagger}\left(\frac{p+2\pi l}{2^n}\right)\right]_{\mu\nu}\frac{1}{\left(p+2\pi l\right)^2}$$

The poles determine the dispersion relation:



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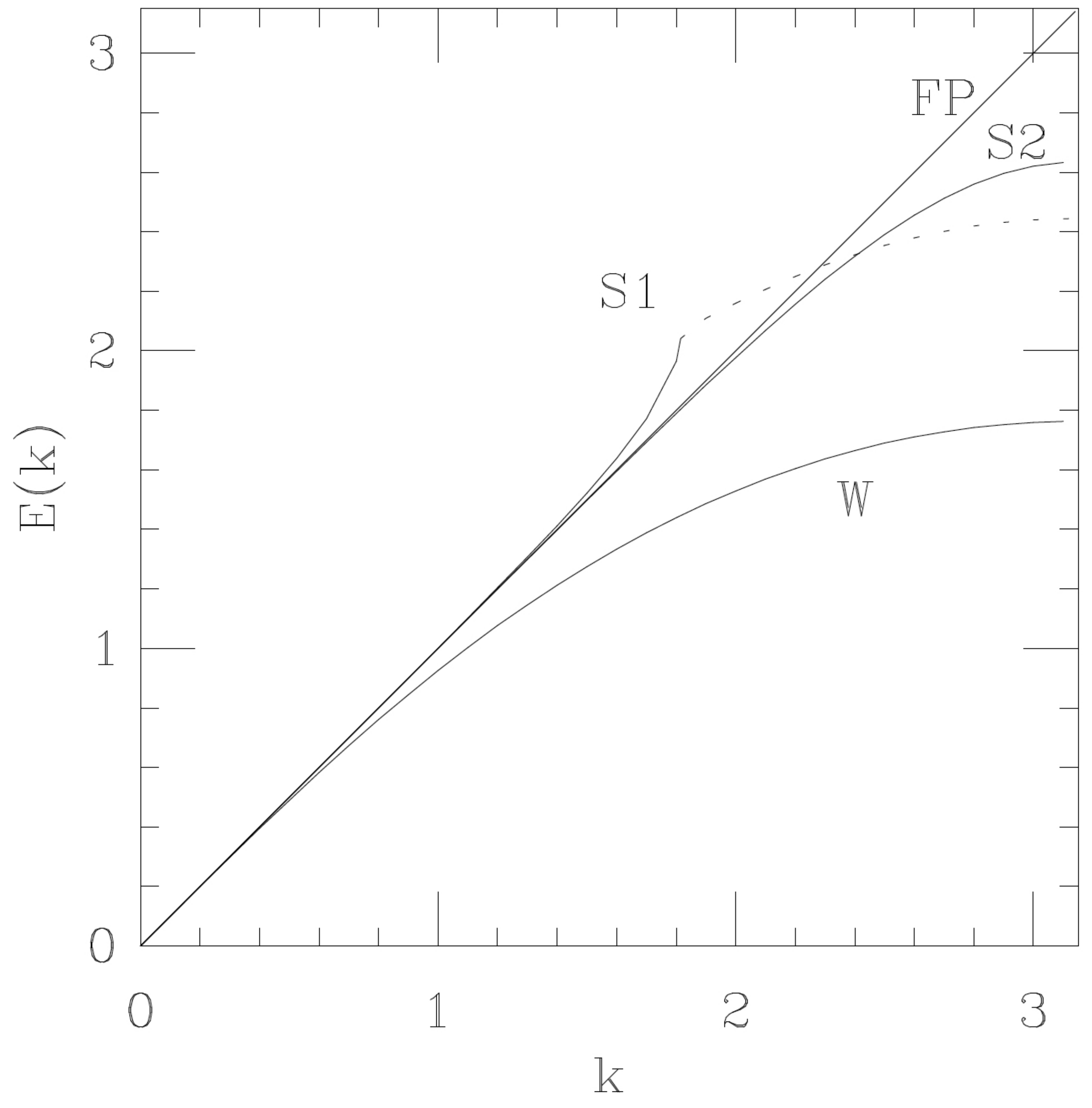

$\forall p$ sums over l generate tower of poles $\Rightarrow$ full relativistic spectrum recovered!

Classically perfect FP gradient flow

Lattice momenta restricted as usual: $-\pi/a \leq p_\mu \leq \pi/a$

but iterated RG transformations generate additional poles in the propagator:

$$(p + 2\pi l)^2 \qquad \text{for } l = 0,1,2,\ldots$$

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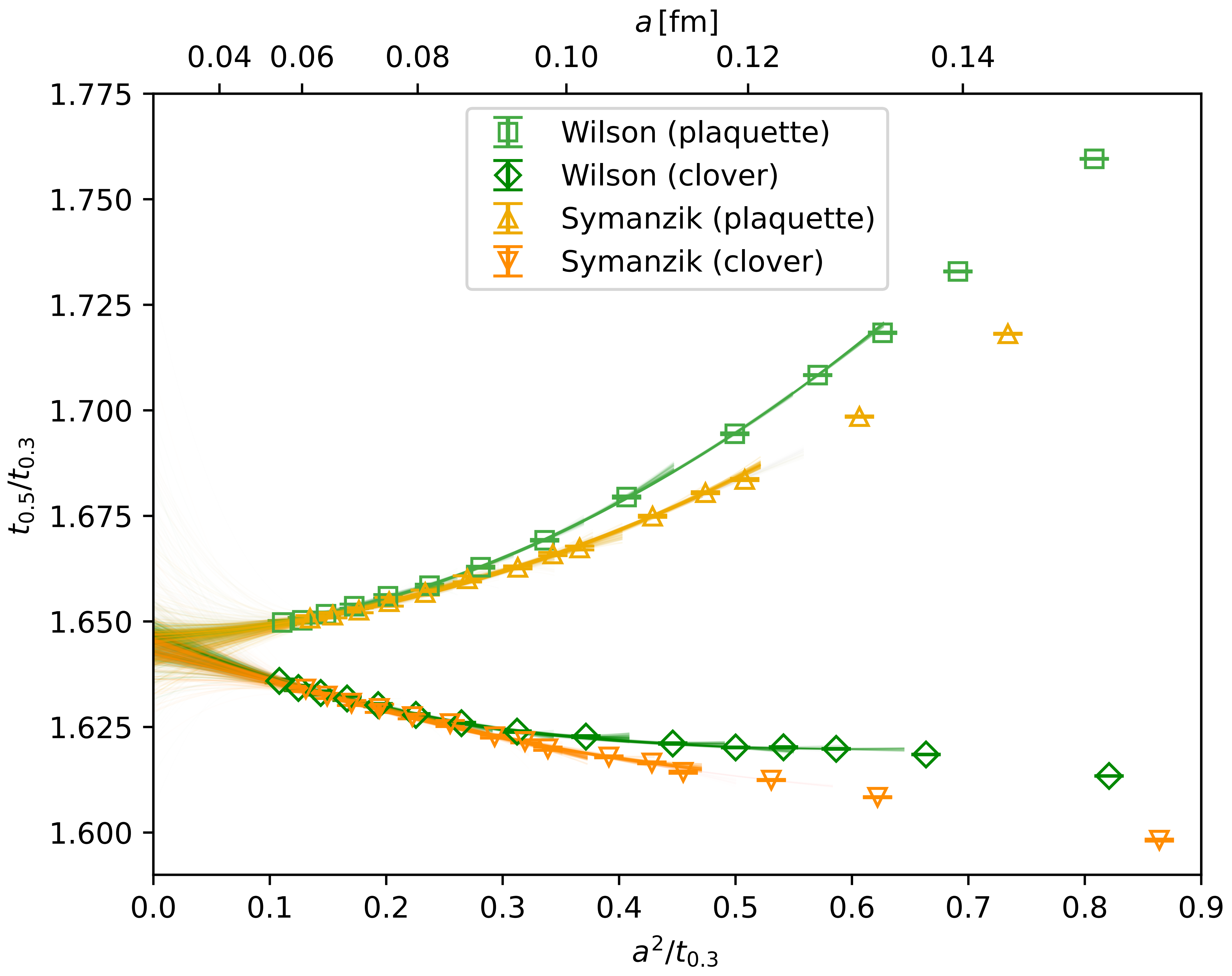
$$C^{FP}(a^2/t) = \frac{64\pi^2 t^2}{3} \cdot 3 \left(\int_{-\infty}^{+\infty} \frac{dp}{2\pi} e^{-2tp^2} \right)^4 = \frac{64\pi^2 t^2}{(\sqrt{8\pi t})^4} = 1$$

$\Rightarrow$ gradient flow with FP actions is classically perfect!

Scaling of gradient-flow scales

Physical reference scales defined through $t^2\langle E\rangle\,|_{t=t_x} = x,\qquad t\frac{d}{dt}\left(t^2\langle E\rangle\right)\Big|_{t=w_x^2} = x$

yielding dimensionless ratios t_x/w_x^2 or t_x/t_y as scaling quantities vs. $a^2/t_{0.3}$:



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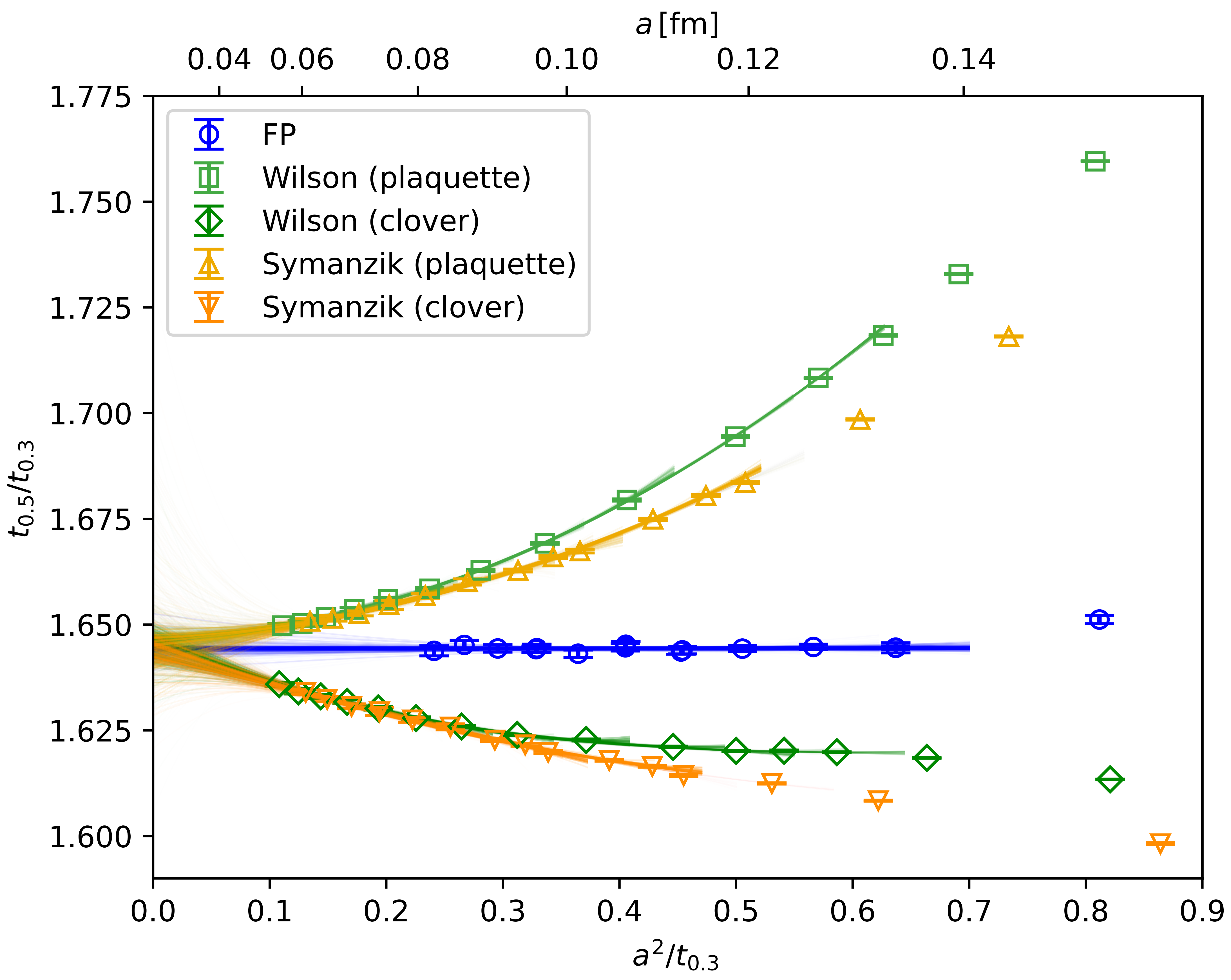

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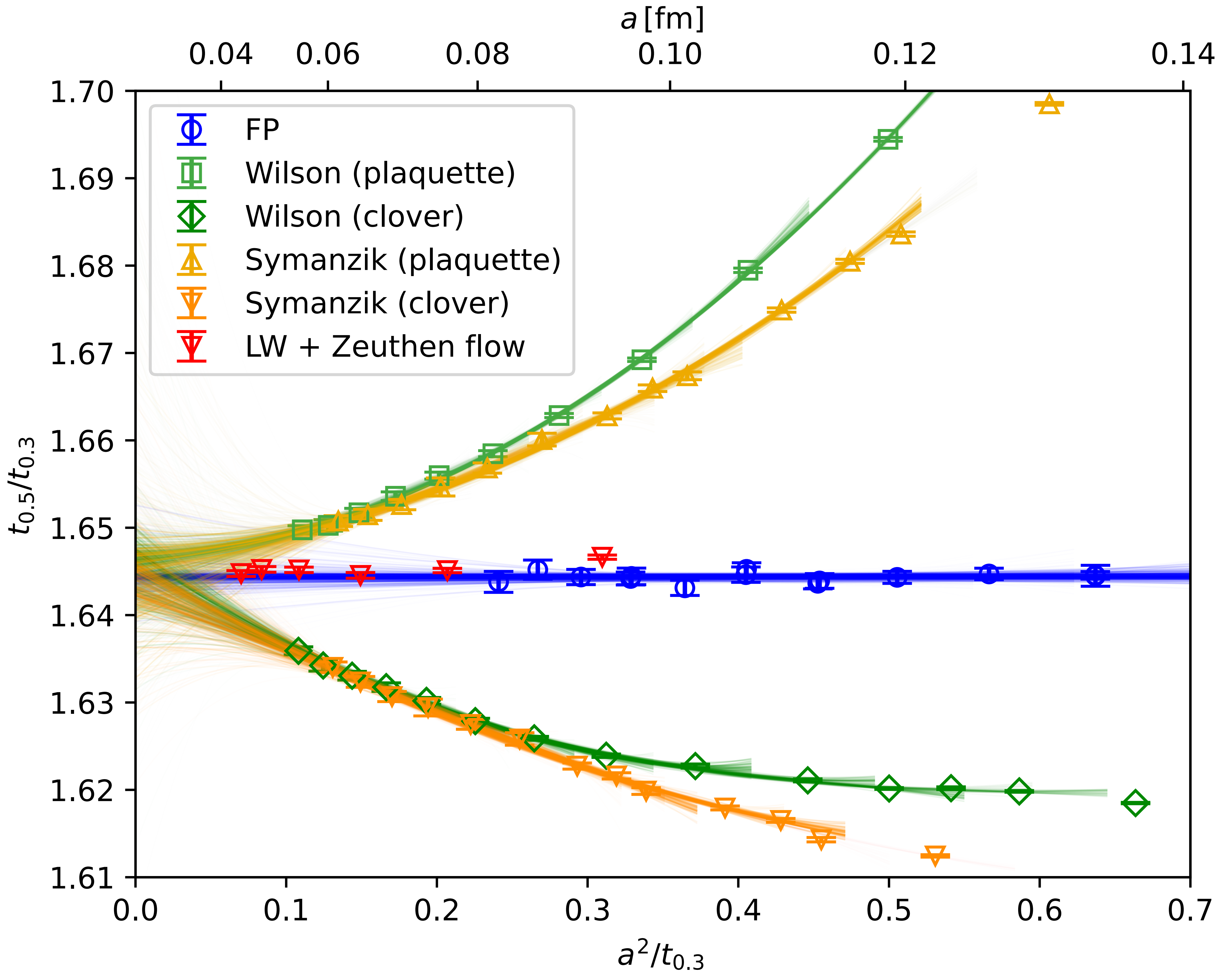

Note:

similar improvement with Zeuthen flow

[Ramos & Sint, Eur.Phys.J.C 76 (2016) 1, 15, 1508.05552 [hep-lat]]

but only up to $a \lesssim 0.08$ fm.

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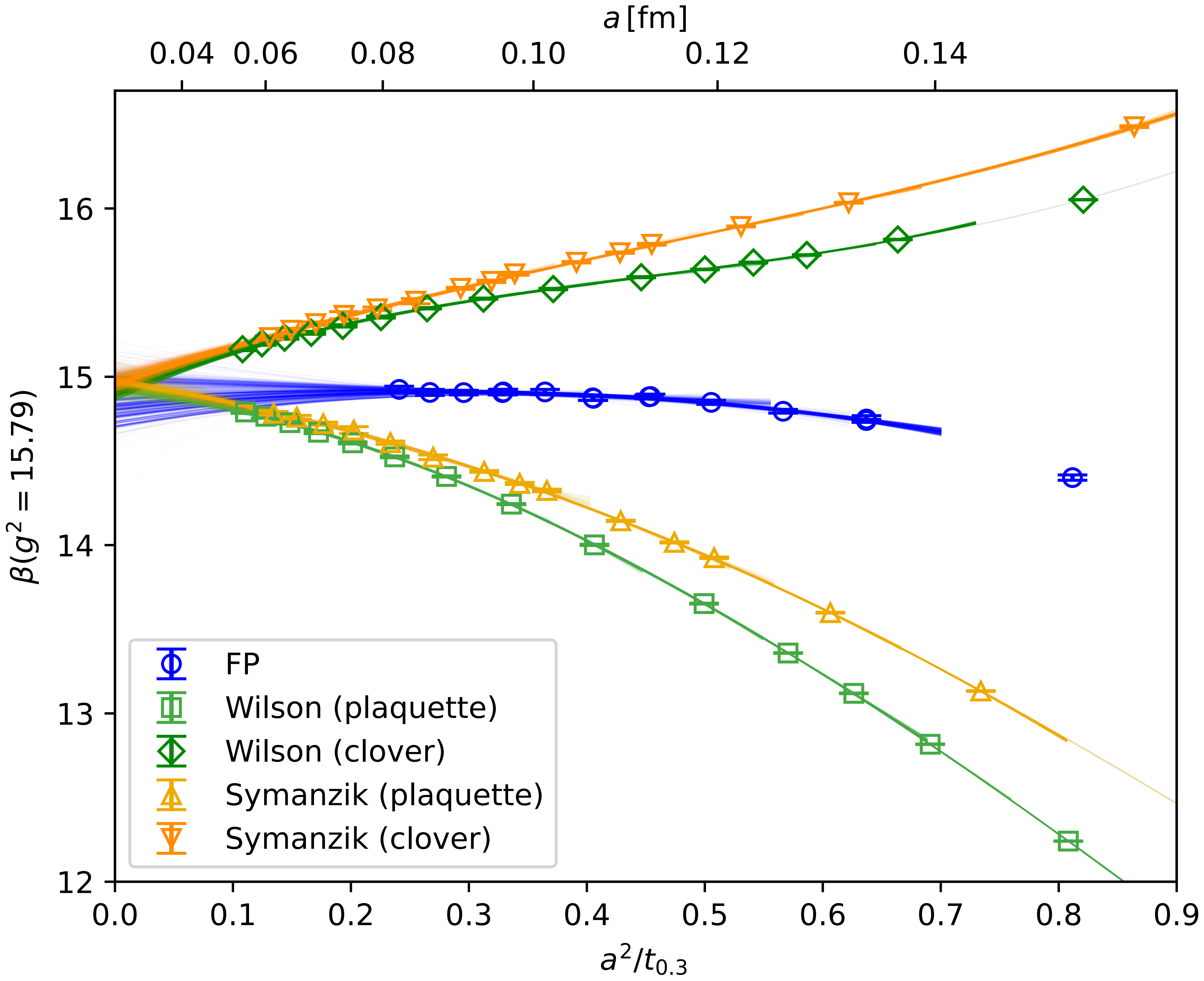
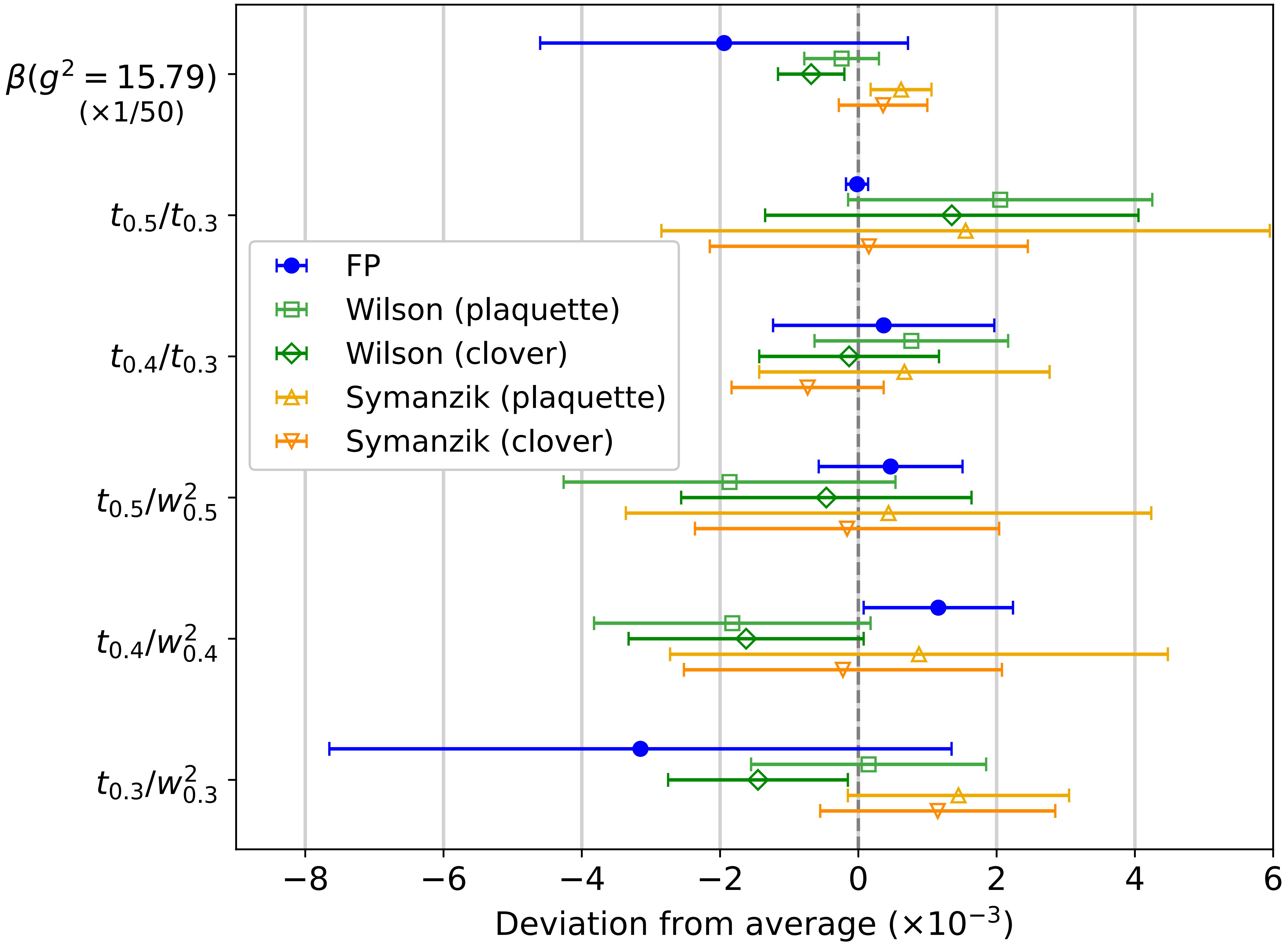
β-function at $g^2_{GF} = 15.79$:





GF with FP action: Conclusions

Three questions were addressed:

- can the FP action be parametrised sufficiently well? ✓
- is the FP action sufficiently local for truncations to work? ✓
- how good are the scaling properties of the L-CNN FP action? ✓

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Availability of derivatives from the L-CNN is the stepping stone for:

- HMC, Langevin, gradient flow
- application of exact RGT step(s)