Gradient-flow coupling in QED_3 and QED_4

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The gradient-flow coupling of three- and four-dimensional QED

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Abstract

We evaluate the QED coupling in the gradient-flow scheme in three and four space-time dimensions. Our general result applies to any theory with a U(1) gauge field coupled to arbitary other fields via arbitrary interactions. As an example, we consider QED with n_f flavors in three and four space-time dimensions and evaluate the corresponding β functions. In four dimensions, we find that the perturbative expansion of the β function behaves much better than the corresponding expression in QCD. In three dimensions, we recover both the ultraviolet as well as the infrared fixed points of the QED coupling in the large-n_f limit.

Institute for Theoretical Particle Physics and Cosmology

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Gradient-flow scheme

- See GF as regularization scheme:

➡ No flowed gluon renormalization
➡ No operator mixing

[Lüscher ’10; Lüscher, Weisz ‘11]

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- Current status:

perturbation theory

$$\alpha_{\mathrm{GF}}(t) = \alpha_{\mathrm{s}}\left[1 + c_1(t)\alpha_{\mathrm{s}} + c_2(t)\alpha_{\mathrm{s}}^2 + c_3(t)\alpha_{\mathrm{s}}^3 + \dots\right]$$

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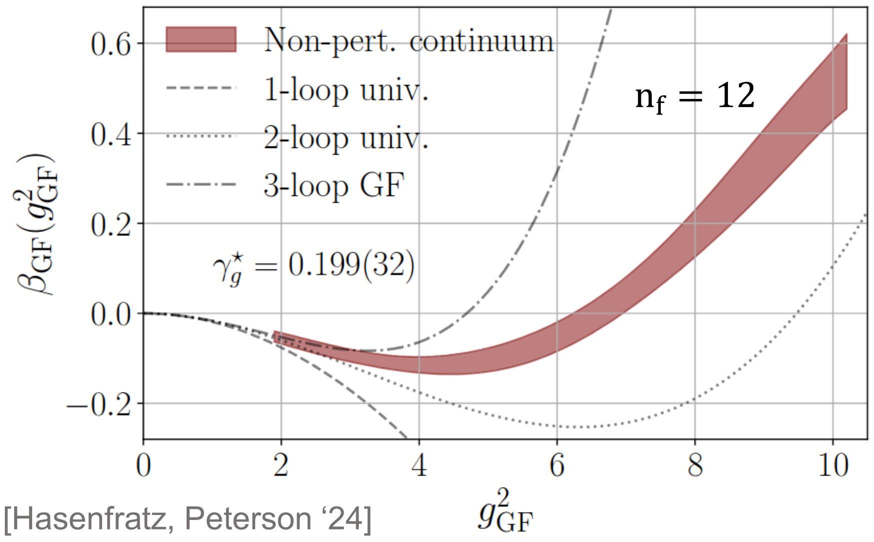

[Hasenfratz, Peterson '24]

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[Lüscher '10] [Harlander, Neumann '16]

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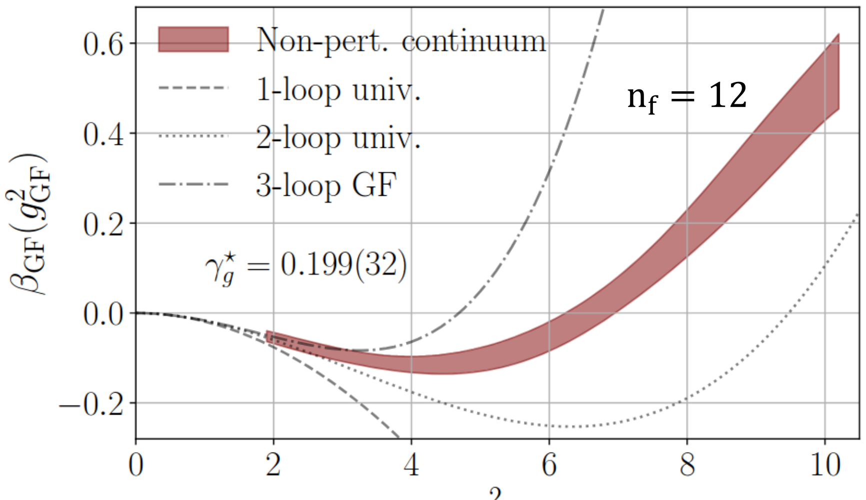
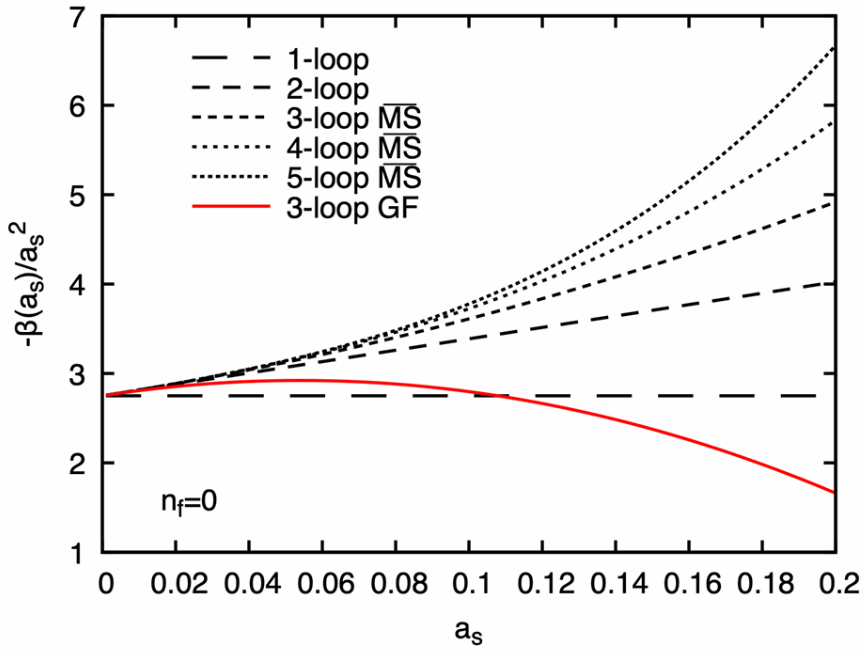
[Lüscher ’10] [Harlander, Neumann ’16] Unknown



[Hasenfratz, Peterson ‘24]



[Harlander, Lattice meets continuum slides ‘24]

Perturbative gradient flow

[Lüscher '10; Lüscher, Weisz '11]

$$\mathcal{L}_{flow} = \mathcal{L}_{QCD} + \int dt \left[L_\mu \frac{\partial}{\partial t} B_\mu - L_\mu \mathcal{D}_\nu G_{\nu\mu} \right]$$

$$G_{\mu\nu}(t) = \partial_\mu B_\nu - \partial_\nu B_\mu + g_0 \left[B_\mu, B_\nu \right]$$

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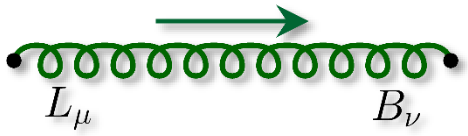
[Lüscher '10; Lüscher, Weisz '11]

Drawn with FeynGame



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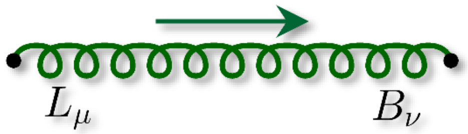

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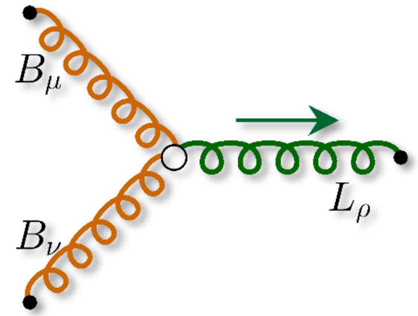

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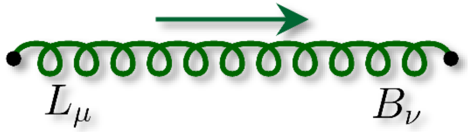

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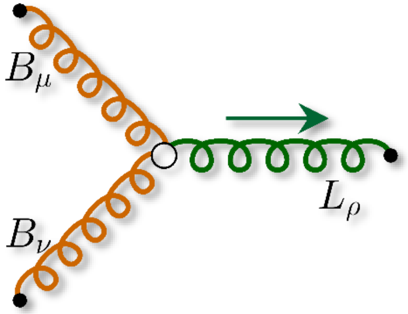

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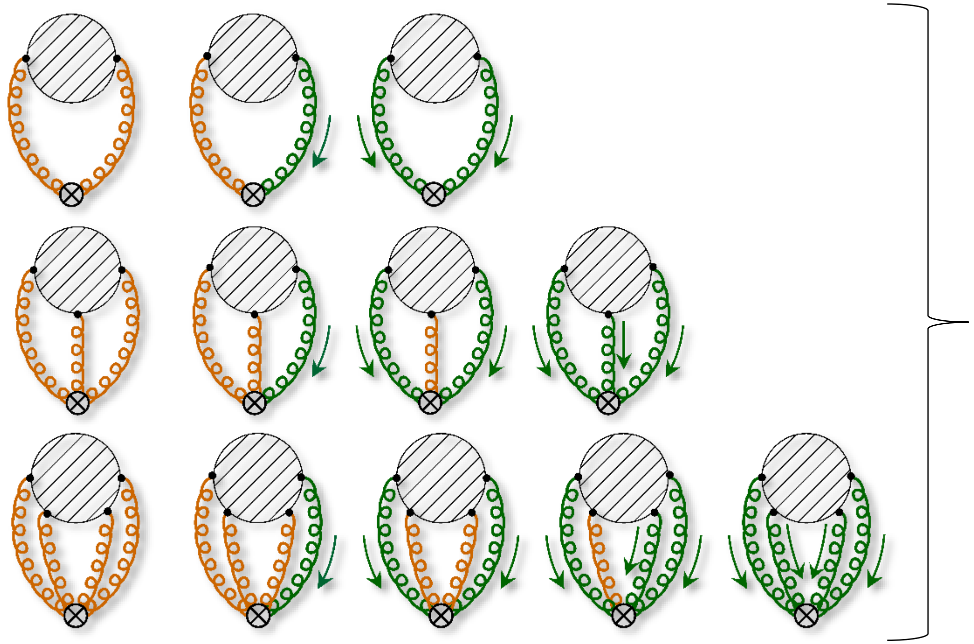

Possible at 4-loop?

~ 5.500 Diagrams

~ 500.000 Integrals

- Computational heavy

- In progress

$$G_{\mu\nu}(t) = \partial_\mu B_\nu - \partial_\nu B_\mu + g_0 \left[B_\mu, B_\nu \right]$$

Digression: ftint

[Harlander, Nellopoulos, Olsson, Wesle '24]

$$I\Big(\{c_1,\ldots,c_N\},\{a_1,\ldots,a_9\},\{b_1,\ldots,b_9\}\Big) = \left(\prod_{j=1}^{N}\int_0^1 \mathrm{d}u_j\, u_j^{c_j}\right)\int_{p_1,\ldots,p_4} \frac{\exp\left[-t\sum_{i=1}^{9} a_i(u) P_i^2\right]}{\left(P_1^2\right)^{b_1}\ldots\left(P_9^2\right)^{b_9}}$$

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$$\frac{1}{(p^2)^b}\sim\int_0^\infty \mathrm{d}x\, x^{b-1}e^{-xp^2}\quad\left(\overset{\text{map}}{\to}\int_0^1\mathrm{d}x\dots\right)$$

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Sector decomposition:
→ pySecDec
[Borowka et al. '17]

$$\sim\sum_{l=1}^{\alpha}\sum_{n=-r}^{p}\mathcal{I}_{l,n}\frac{1}{\epsilon^n}+\mathcal{O}(\epsilon^{r+1})$$

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Numerical integration:
→ pySecDec/Disteval
[Borowka et al. '17]

$$= \sum_{n=-r}^{p} d_n\frac{1}{\epsilon^n} + \mathcal{O}(\epsilon^{r+1})$$

Digression: ftint at 4-loop

[Harlander, Nellopoulos, Olsson, Wesle '24]

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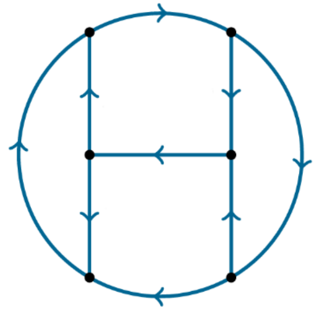
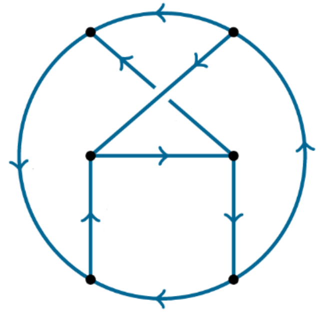


(a) **H**



(b) **X**

Digression: ftint at 4-loop

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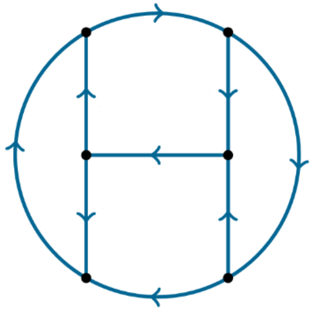
Example:

$$I\Big(\{\},\{2,0,\dots,0\},\{2,1,\dots,1,-1,-1\},H\Big)$$



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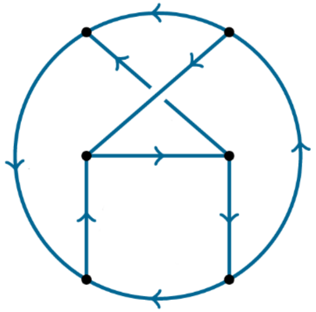

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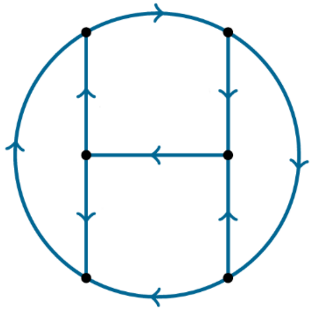
Example:

$$I\Big(\{\},\{2,0,\ldots,0\},\{2,1,\ldots,1,-1,-1\},H\Big)$$

$< 1\,\mathrm{min}$



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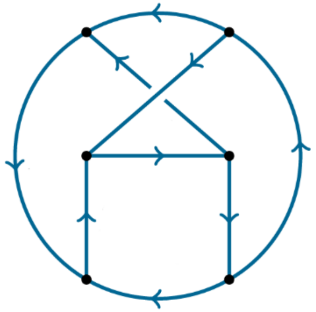

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[Harlander, Nellopoulos, Olsson, Wesle '24]

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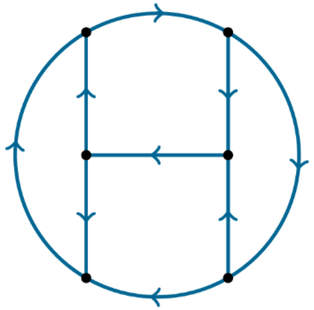
$< 1\,\mathrm{min}$ →

$$\sim \left(\prod_{j=1}^{N}\int_0^1 \mathrm{d}u_j\, u_j^{c_j}\right)\left(\prod_{i=1}^{9}\int_0^1 \mathrm{d}x_i\, x_i^{b_i-1}\right)\left[\det A(x,u)\right]^{-D/2}$$

$$\sim \sum_{l=1}^{\alpha}\sum_{n=-r}^{p}\mathcal{I}_{l,n}\frac{1}{\epsilon^n}+\mathcal{O}(\epsilon^{r+1})$$

$\sim 2\,\mathrm{min}$ →

$$= \sum_{n=-r}^{p} d_n\frac{1}{\epsilon^n}+\mathcal{O}(\epsilon^{r+1})$$



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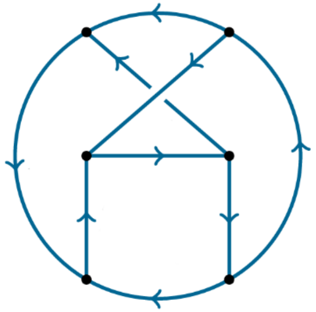

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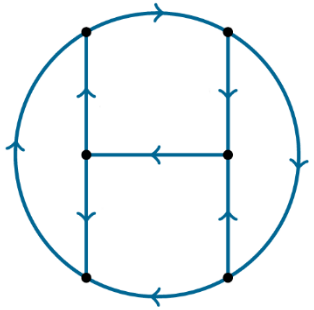
Digression: ftint at 4-loop

[Harlander, Nellopoulos, Olsson, Wesle '24]

$$I\Big(\{c_1,\ldots,c_N\},\{a_1,\ldots,a_9\},\{b_1,\ldots,b_9\}\Big) = \left(\prod_{j=1}^{N}\int_0^1 \mathrm{d}u_j\, u_j^{c_j}\right)\int_{p_1,\ldots,p_4}\frac{\exp\left[-t\sum_{i=1}^{9}a_i(u)P_i^2\right]}{(P_1^2)^{b_1}\ldots(P_9^2)^{b_9}}$$



(a) **H**

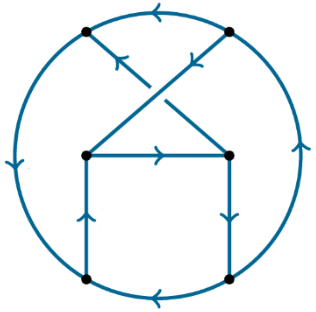
Example:

$$I\Big(\{\},\{2,0,\ldots,0\},\{2,1,\ldots,1,-1,-1\},H\Big)$$

$$\sim\left(\prod_{j=1}^{N}\int_0^1 \mathrm{d}u_j\, u_j^{c_j}\right)\left(\prod_{i=1}^{9}\int_0^1 \mathrm{d}x_i\, x_i^{b_i-1}\right)\int_{p_1,\ldots,p_4}\exp\left[\mathbf{p}^T A(x,u)\mathbf{p}\right]$$

$< 1\,\mathrm{min}$ ➡

$$\sim\left(\prod_{j=1}^{N}\int_0^1 \mathrm{d}u_j\, u_j^{c_j}\right)\left(\prod_{i=1}^{9}\int_0^1 \mathrm{d}x_i\, x_i^{b_i-1}\right)\left[\det A(x,u)\right]^{-D/2}$$



(b) **X**

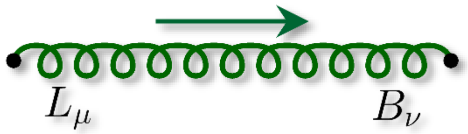
$\sim 56\,\mathrm{hours}$ ➡

$$\sim\sum_{l=1}^{\alpha}\sum_{n=-r}^{p}\mathcal{I}_{l,n}\frac{1}{\epsilon^n}+\mathcal{O}(\epsilon^{r+1})$$

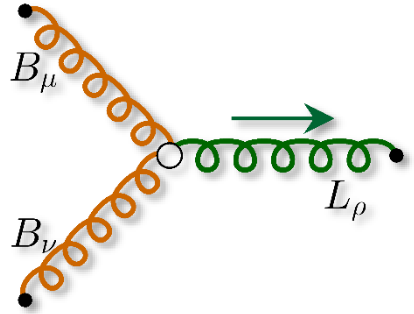
$\sim 2\,\mathrm{min}$ ➡

$$=\sum_{n=-r}^{p}d_n\frac{1}{\epsilon^n}+\mathcal{O}(\epsilon^{r+1})$$

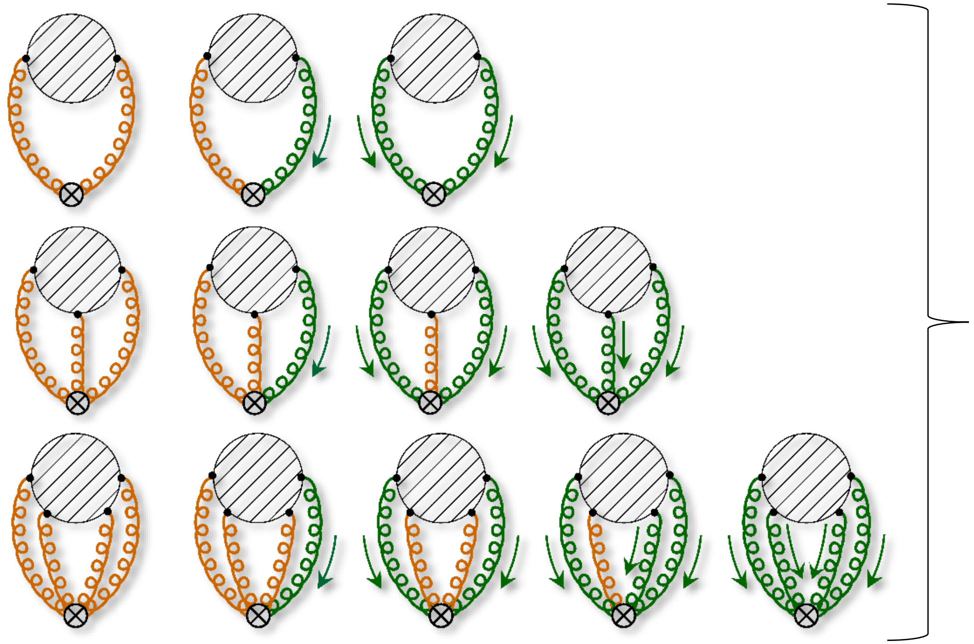
Perturbative gradient flow



$$\mathcal{L}_{flow} = \mathcal{L}_{QCD} + \int dt \left[L_\mu \frac{\partial}{\partial t} B_\mu - L_\mu \mathcal{D}_\nu G_{\nu\mu} \right]$$



$$\alpha_{\mathrm{GF}}(t) = \frac{4\pi t^2}{3} \left\langle G_{\mu\nu}(t) G^{\mu\nu}(t) \right\rangle$$



Possible at 4-loop?

~ 5.500 Diagrams

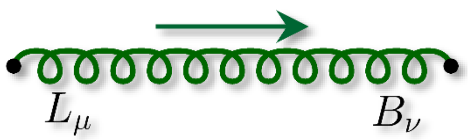
~ 500.000 Integrals

- Computational heavy

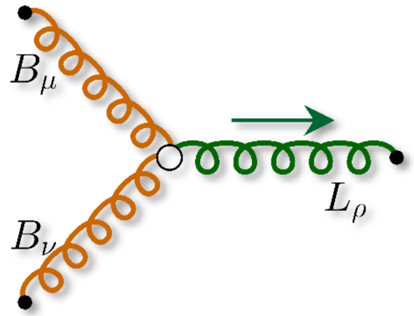
- In progress

$$G_{\mu\nu}(t) = \partial_\mu B_\nu - \partial_\nu B_\mu + g_0 \left[B_\mu, B_\nu \right]$$

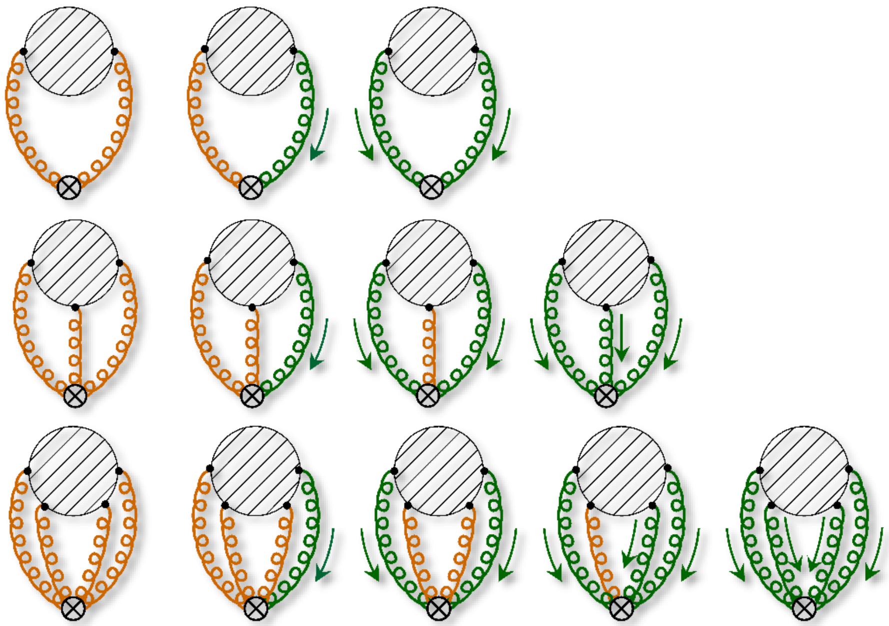
Perturbative gradient flow



$$\mathcal{L}_{flow} = \mathcal{L}_{QCD} + \int dt \left[L_\mu \frac{\partial}{\partial t} B_\mu - L_\mu \Box B_\mu \right]$$



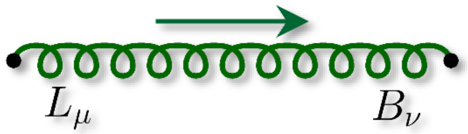
$$\alpha_{\mathrm{GF}}(t) = \frac{4\pi t^2}{3} \left\langle F_{\mu\nu}(t) F^{\mu\nu}(t) \right\rangle$$



Simpler Coupling: QED

$$F_{\mu\nu}(t) = \partial_\mu B_\nu - \partial_\nu B_\mu + g_0 [B_\mu, B_\nu]$$

Perturbative gradient flow



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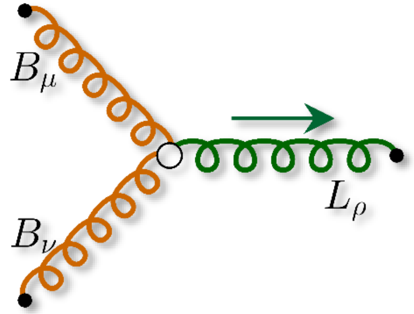

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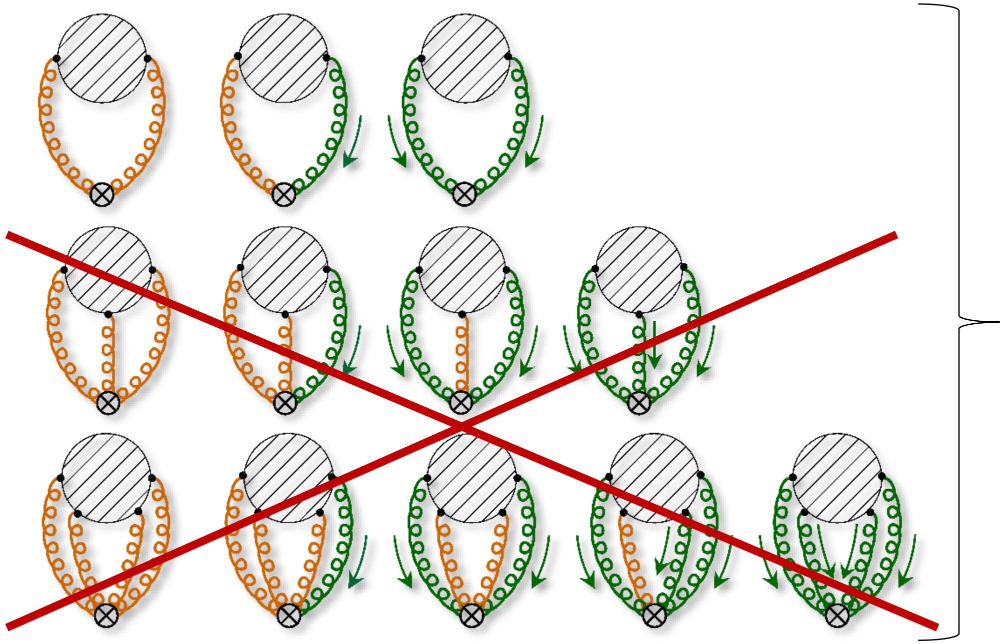
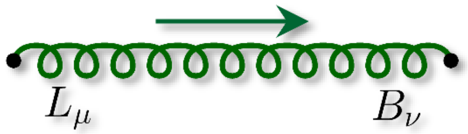


Simpler Coupling: QED

- $F_{\mu\nu}$ is linear in B_μ

$$F_{\mu\nu}(t) = \partial_\mu B_\nu - \partial_\nu B_\mu + g_0 [B_\mu, B_\nu]$$

Perturbative gradient flow



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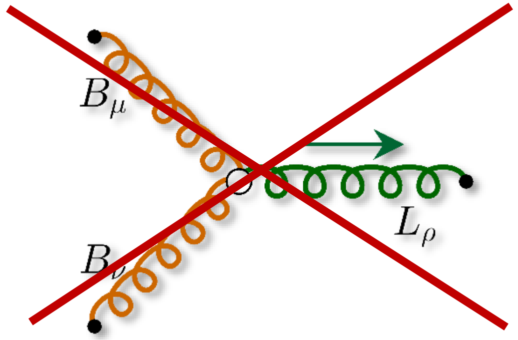

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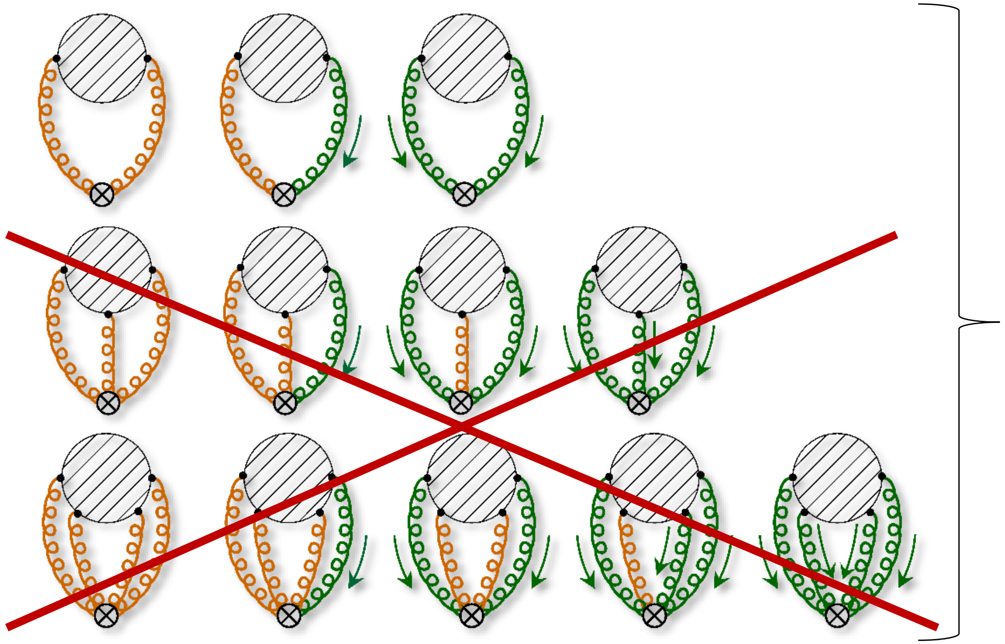


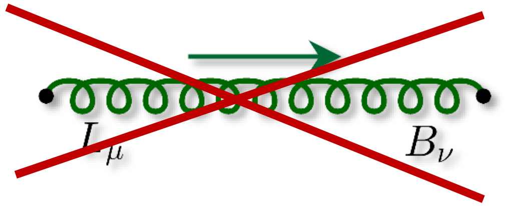
Simpler Coupling: QED

- $F_{\mu\nu}$ is linear in B_μ

- No flow-vertices

$$F_{\mu\nu}(t) = \partial_\mu B_\nu - \partial_\nu B_\mu + g_0 [B_\mu, B_\nu]$$

Perturbative gradient flow



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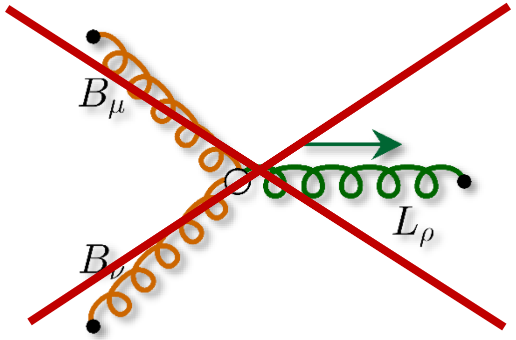

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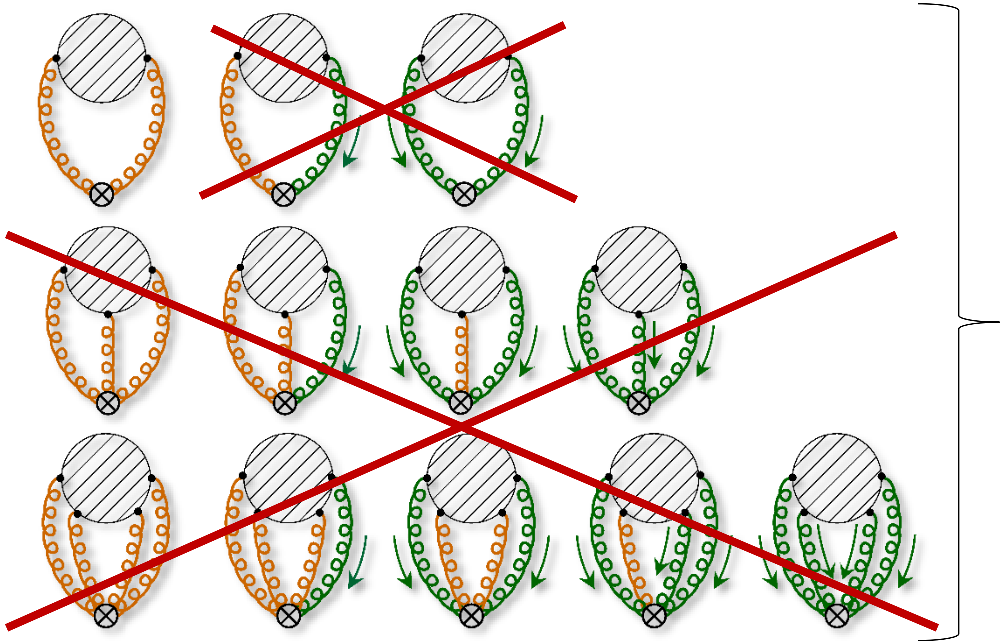

Simpler Coupling: QED

- $F_{\mu\nu}$ is linear in B_μ

- No flow-vertices

➡ No photon flow-lines

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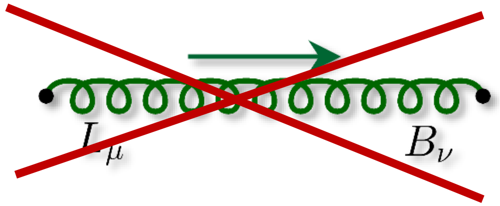
Perturbative gradient flow



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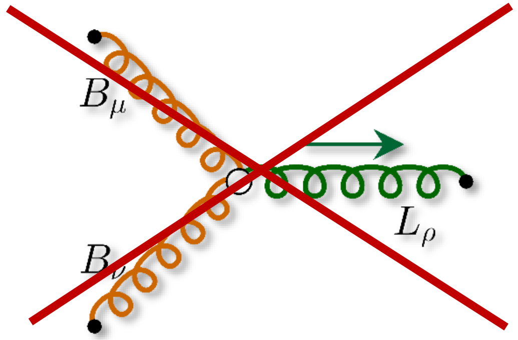

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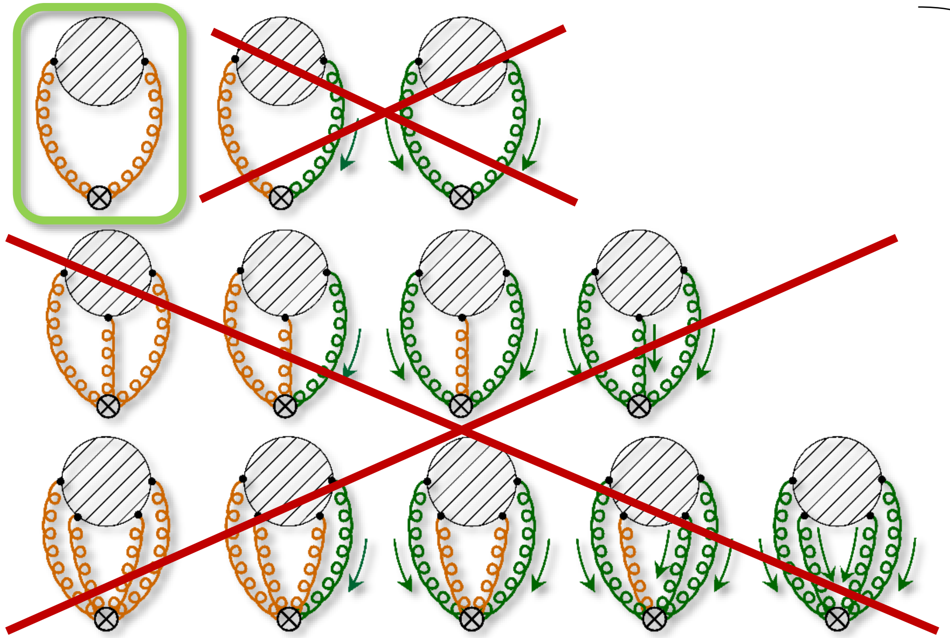


Simpler Coupling: QED

- $F_{\mu\nu}$ is linear in B_μ

- No flow-vertices

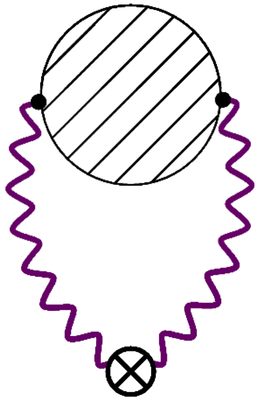
➡ No photon flow-lines

- Solution to flow-equation is exact

$$F_{\mu\nu}(t) = \partial_\mu B_\nu - \partial_\nu B_\mu + g_0 [B_\mu, B_\nu]$$

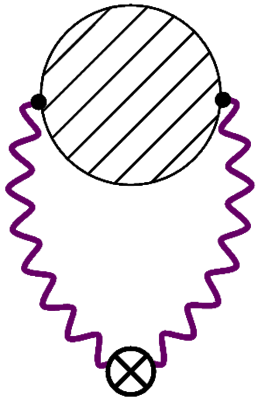
QED coupling in the gradient flow

$$\alpha_{\rm e}^{\rm GF}(t) =$$



$$= \alpha_{\rm e}(8\pi t)^{D/2} \int \frac{d^D p}{(2\pi)^D} \frac{e^{-2tp^2}}{1+\Pi_{\rm R}(p)}$$

QED coupling in the gradient flow

$\alpha_e^{GF}(t) =$  $= \alpha_e (8\pi t)^{D/2} \int \frac{d^D p}{(2\pi)^D} \frac{e^{-2tp^2}}{1 + \Pi_R(p)}$

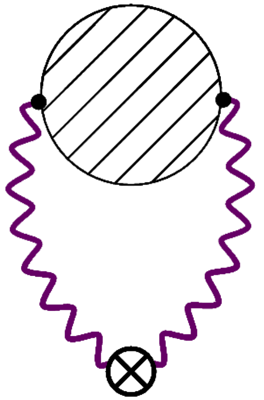
Photon polarization function of the <u>unflowed</u> theory

Not restricted to QED contributions

➔ QCD contr.

➔ BSM contr.

QED coupling in the gradient flow



$$\alpha_e^{GF}(t) = \quad = \alpha_e (8\pi t)^{D/2} \int \frac{d^D p}{(2\pi)^D} \frac{e^{-2tp^2}}{1 + \Pi_R(p)}$$

Photon polarization function of the unflowed theory

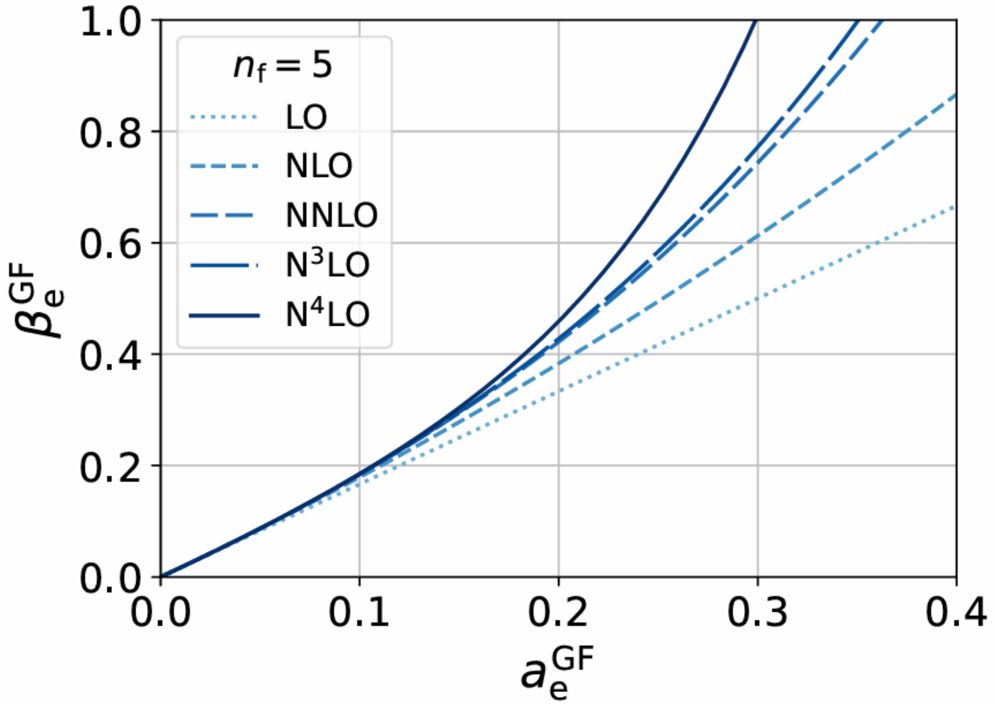
Not restricted to QED contributions
➔ QCD contr.
➔ BSM contr.

First example: QED_4 contr.
→ Photon polarization function known through N^4LO

[Baikov, Chetyrkin, Kühn, Sturm '13]

- First 4-loop GF results

- Testing ground for QCD



QED in (2+1) dimensions

- **Phenomenological relevance**:

QED in (2+1) dimensions

- **Phenomenological relevance**:

  - Effective modeling of 2D materials

Two-loop fermion self-energy in reduced quantum electrodynamics and application to the ultrarelativistic limit of graphene #1

A.V. Kotikov (Dubna, JINR), S. Teber (Paris, LPTHE) (Dec 9, 2013)

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Regarding confinement and dynamical chiral symmetry breaking in QED3
A. Bashir (IFM-UMSNH, Michoacan), A. Raya (IFM-UMSNH, Michoacan), I.C. Cloet (Argonne, PHY), C.D. Roberts (Argonne, PHY) (Jun, 2008)
Published in: *Phys.Rev.C* 78 (2008) 055201 • e-Print: 0806.3305 [hep-ph]

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    $\longrightarrow$ Lattice

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A. Bashir (IFM-UMSNH, Michoacan), A. Raya (IFM-UMSNH, Michoacan), I.C. Cloet (Argonne, PHY), C.D. Roberts (Argonne, PHY) (Jun, 2008)
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pdf DOI cite claim reference search 89 citations

A Computer Simulation of Chiral Symmetry Breaking in (2+1)-Dimensional QED with N Flavors #3
Elbio Dagotto (Illinois U., Urbana), John B. Kogut (Illinois U., Urbana), Aleksandar Kocic (Arizona U.) (Aug, 1988)
Published in: *Phys.Rev.Lett.* 62 (1989) 1083
pdf DOI cite claim reference search 180 citations

GF-coupling in QED$_3$

$$\alpha_{\mathrm{e}}^{\mathrm{GF}}(t) = \alpha_{\mathrm{e}}(8\pi t)^{D/2} \int \frac{d^D p}{(2\pi)^D} \frac{e^{-2tp^2}}{1+\Pi_{\mathrm{R}}(p)}$$

- Ingredient: Photon polarization function $\Pi(p)$

$$\Pi_{\mathrm{R}}(p) = c_1 \left(\frac{\alpha_{\mathrm{e}}}{p}\right) + c_2 \left(\frac{\alpha_{\mathrm{e}}}{p}\right)^2 + c_3 \left(\frac{\alpha_{\mathrm{e}}}{p}\right)^3 + \ldots$$

- Highly unstable in the IR!!
- α_{e} no viable expansion parameter

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- Highly unstable in the IR!!
- $\alpha_{\rm e}$ no viable expansion parameter

Large$-n_f$:

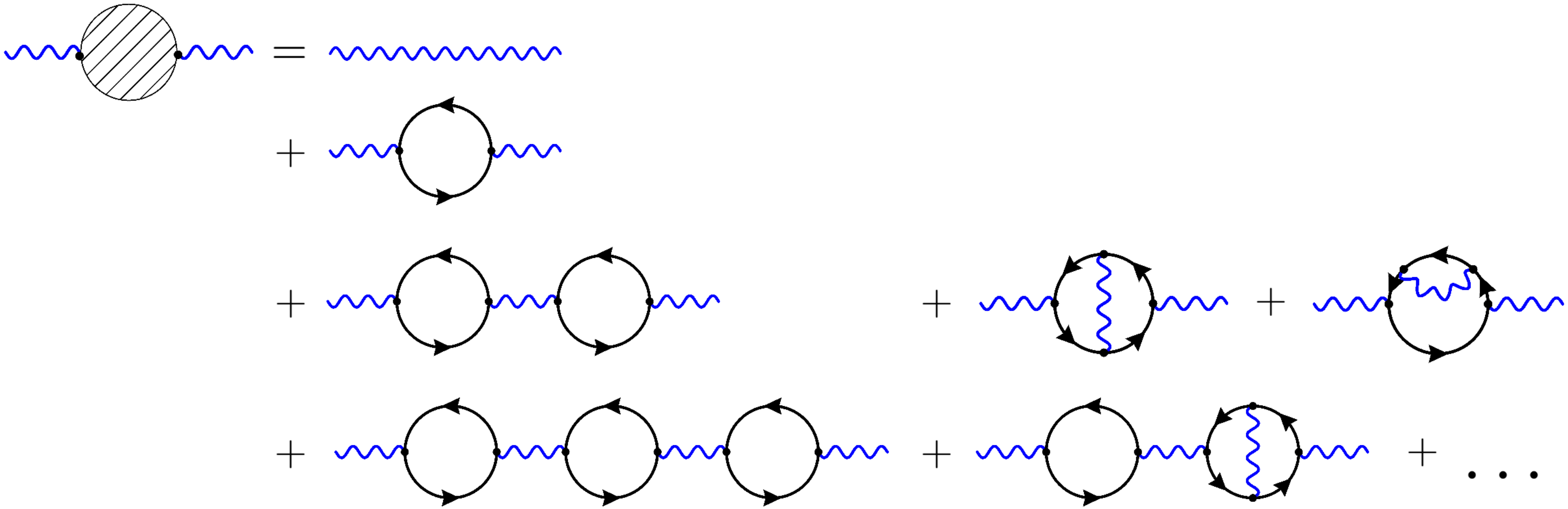
$$\Pi_{\rm R}(p) = d_0 \left(\frac{\alpha_{\rm e}}{p}\right) \left[1 + d_1 \frac{1}{n_{\rm f}} + \dots\right]$$

[Gusynin, Hams, Reenders '01]

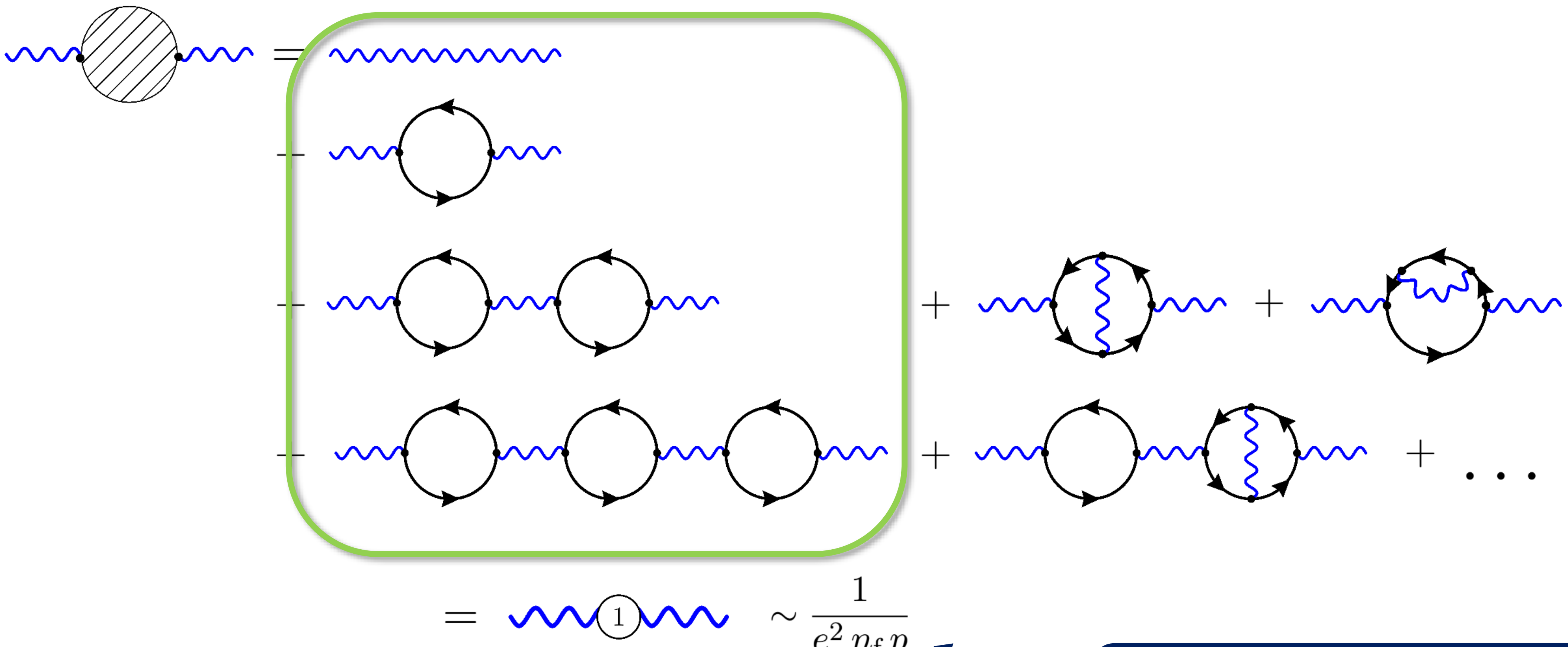
- IR-softened
- Non-trivial IR fixed point

[Appelquist, Bowick, Karabali, Wijewardhana '86]

Large-n_f expansion



Large-n_f expansion

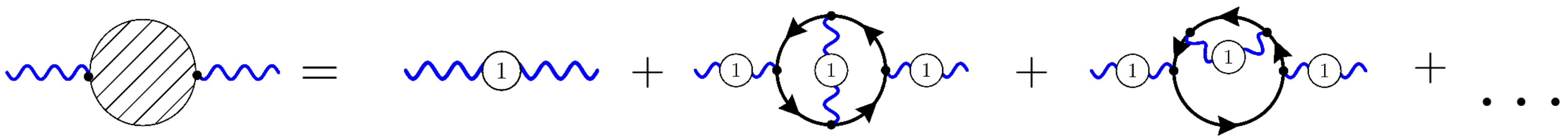


$$\sim \frac{1}{e^2\, n_{\mathrm{f}}\, p}$$

LO-softened photon propagator

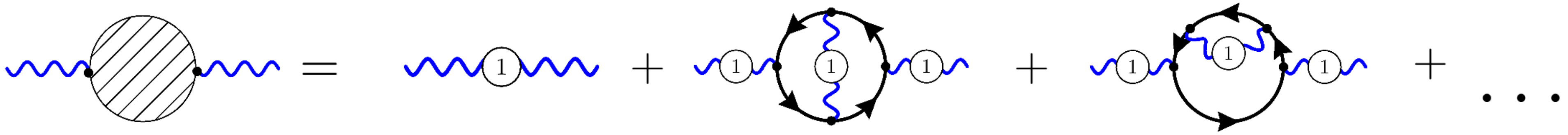
Large-n_f expansion

For more details: [Gracey '18] & [Metayer, Teber '23]



Large-n_f expansion

For more details: [Gracey '18] & [Metayer, Teber '23]



$$\Pi_{\mathrm{R}}(p) = \left(\frac{\alpha_{\mathrm{e}}}{p}\right) h_f \qquad \textbf{with} \qquad h_f = \frac{\pi\, n_{\mathrm{f}}}{2}\left[1 + \left(\frac{184}{9\pi^2} - 2\right)\frac{1}{n_{\mathrm{f}}} + \ldots\right]$$

[Gusynin, Hams, Reenders '01]

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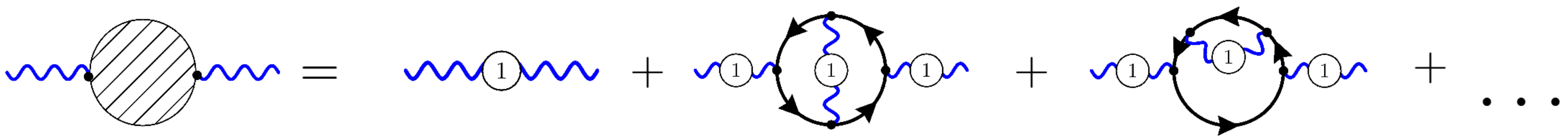

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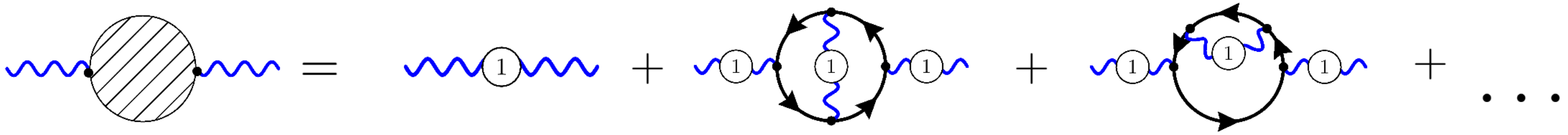
[Gusynin, Hams, Reenders ‘01]

- Polarization function is IR-softened

Large-n_f expansion

For more details: [Gracey '18] & [Metayer, Teber '23]



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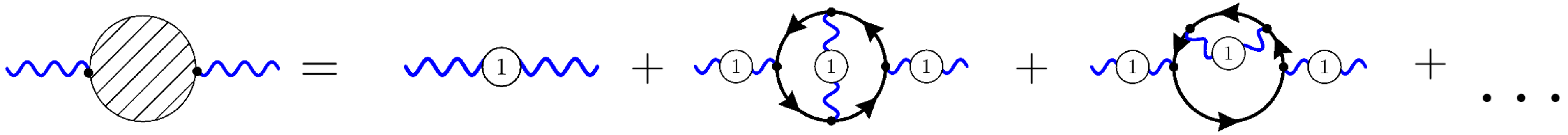

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- Polarization function is IR-softened
- Define rescaled dimensionless coupling: $\hat{\alpha}_{\mathrm{e}} \sim h_f \alpha_{\mathrm{e}} \sqrt{t}$
- Determine $\hat{\alpha}_{\mathrm{e}}^{\mathrm{GF}}$ analytically

$$\alpha_{\mathrm{e}}^{\mathrm{GF}}(t) = \alpha_{\mathrm{e}}(8\pi t)^{D/2} \int \frac{d^D p}{(2\pi)^D} \frac{e^{-2tp^2}}{1 + \Pi_{\mathrm{R}}(p)}$$

Results for QED_3

$$\hat{\alpha}_{\rm e}^{\rm GF}(t) = \hat{\alpha}_{\rm e}\left\{1 - \hat{\alpha}_{\rm e} + \frac{\pi}{2}\hat{\alpha}_{\rm e}^2 - \frac{\pi}{4}\hat{\alpha}_{\rm e}^3 e^{-\frac{\pi}{4}\hat{\alpha}_{\rm e}^2}\left[\Gamma\left(0, -\frac{\pi}{4}\hat{\alpha}_{\rm e}^2\right) + \sqrt{\pi} i \Gamma\left(\frac{1}{2}, -\frac{\pi}{4}\hat{\alpha}_{\rm e}^2\right)\right]\right\}$$

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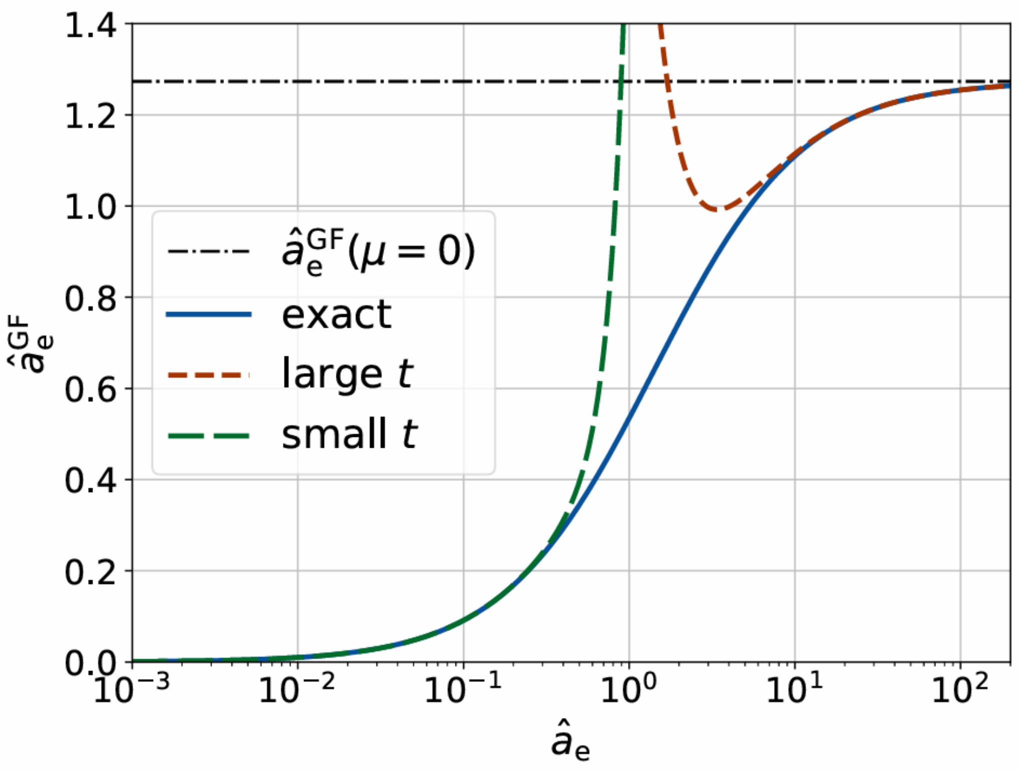

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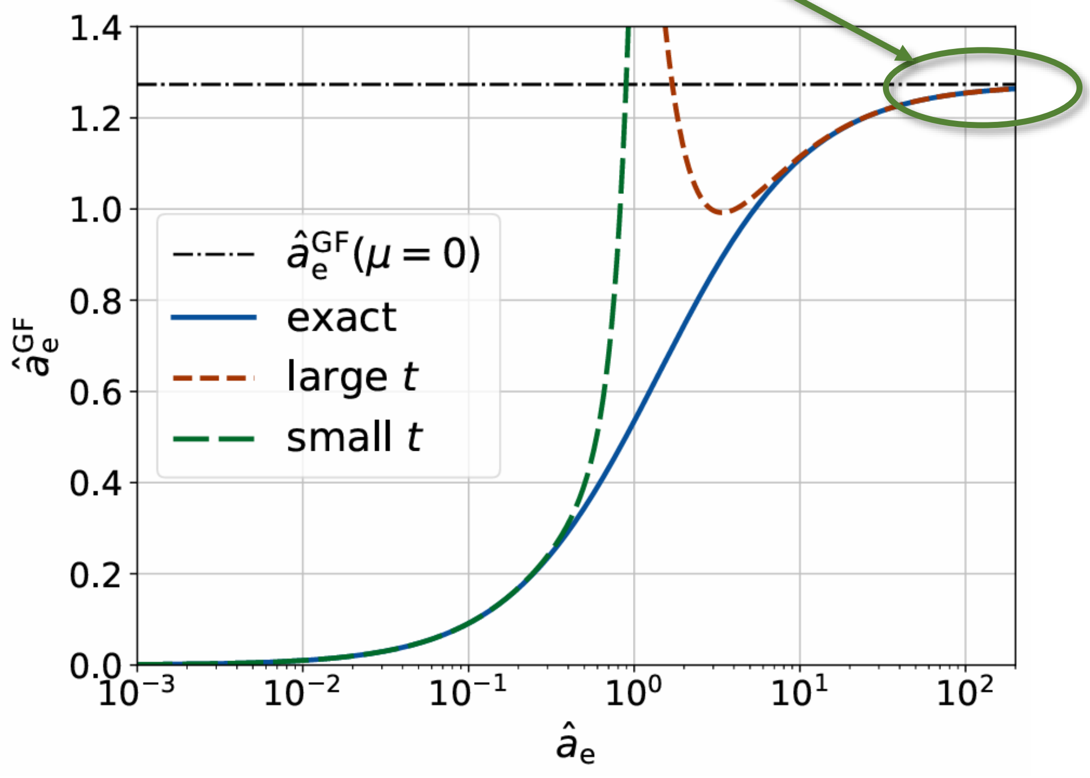
$$\hat{\alpha}_{\mathrm{e},*}^{\mathrm{GF}} = \frac{4}{\pi}$$



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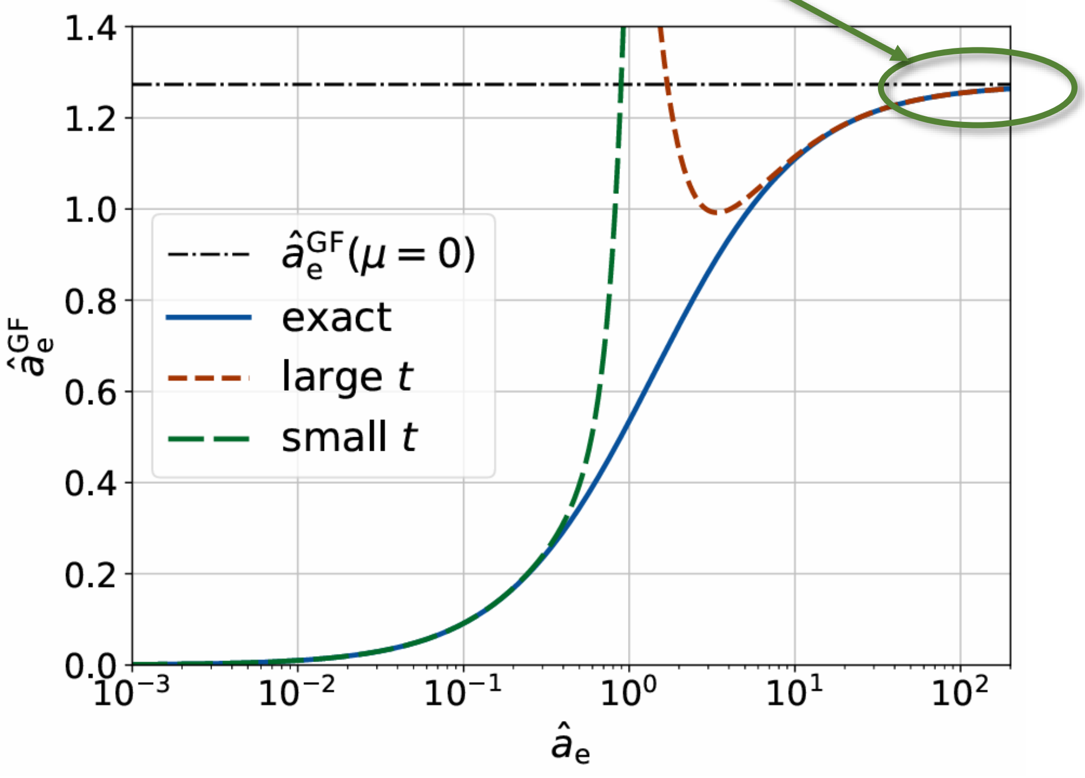
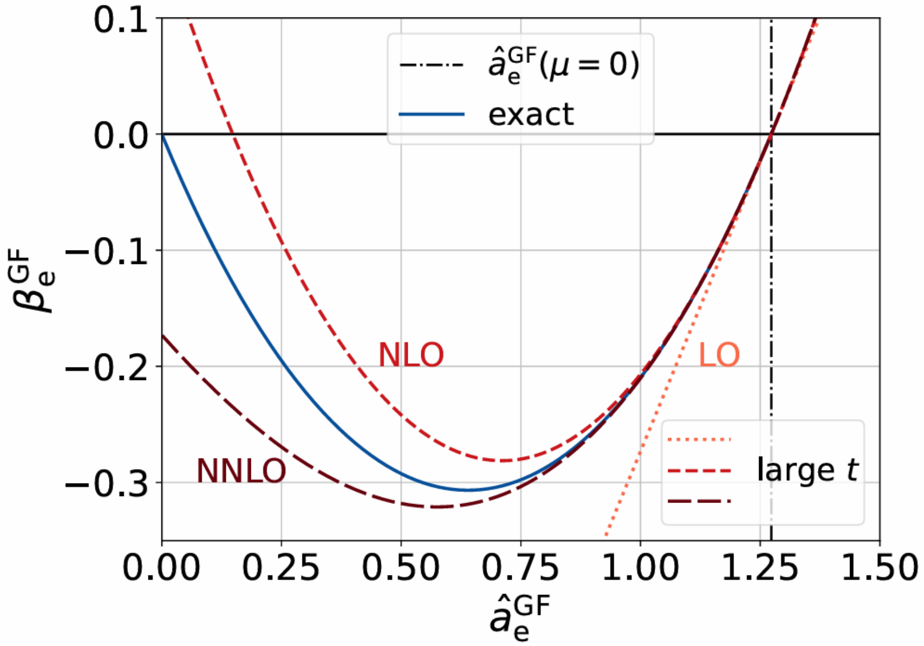


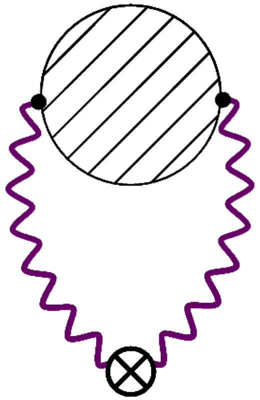
- Recover Infra-red fixed point in the GF scheme predicted by [Appelquist et al. '86]
- Scheme independent critical exponent matches: $\beta'(\hat{\alpha}_{e,*}) = 1$



Summary & Conclusion

- QED gradient-flow coupling can be expressed in terms of the unflowed theory

$$\alpha_{\mathrm{e}}^{\mathrm{GF}}(t) =$$



$$= \alpha_{\mathrm{e}}(8\pi t)^{D/2} \int \frac{d^D p}{(2\pi)^D} \frac{e^{-2tp^2}}{1 + \Pi_{\mathrm{R}}(p)}$$

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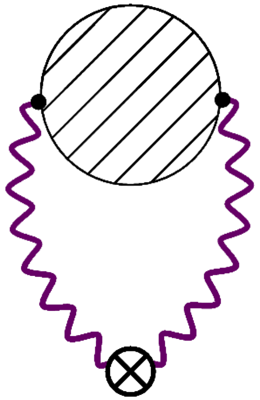

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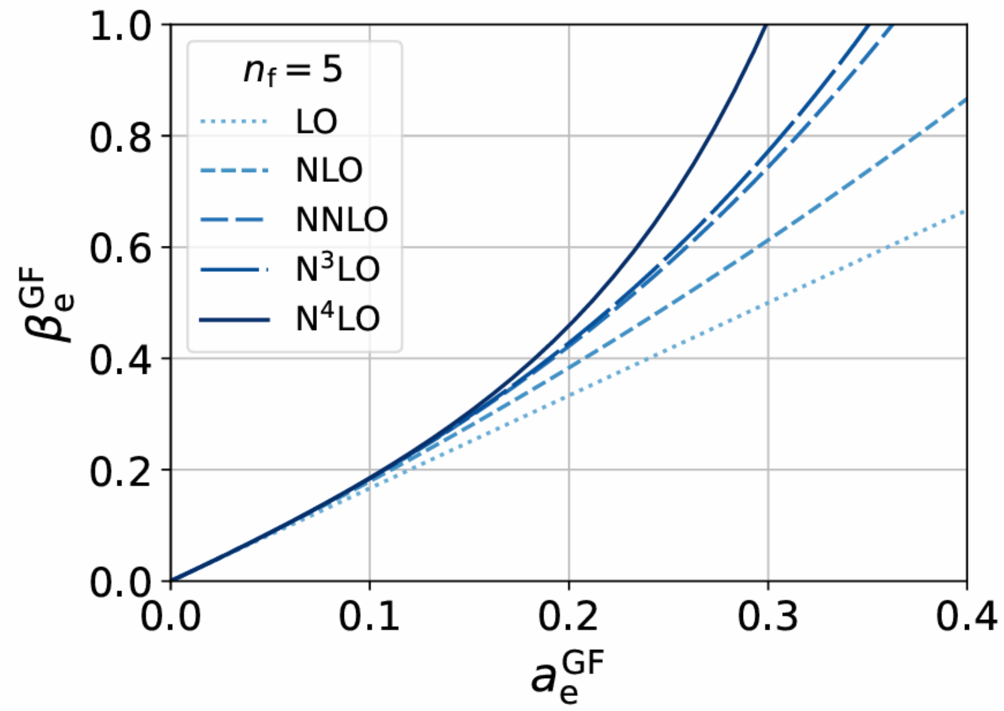

Summary & Conclusion

- QED gradient-flow coupling can be expressed in terms of the unflowed theory
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- This provides first steps toward 4-loop gradient-flow coupling in QCD

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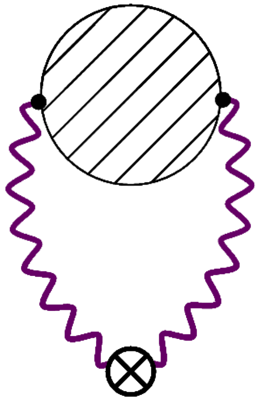

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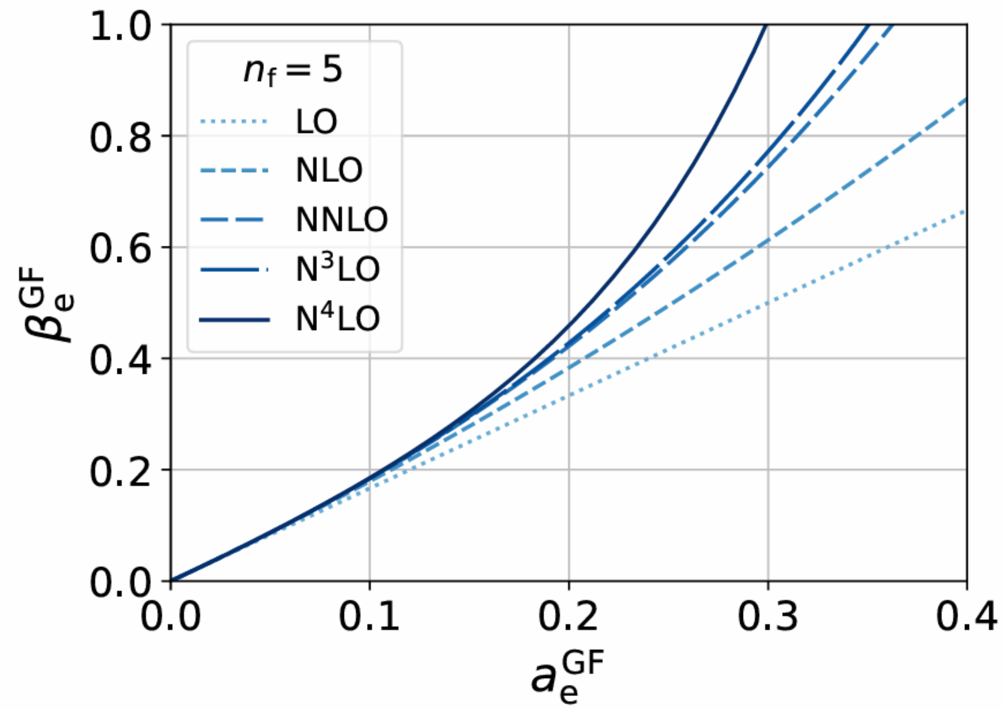

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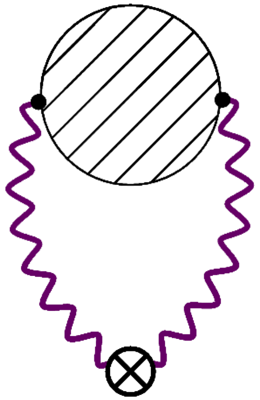

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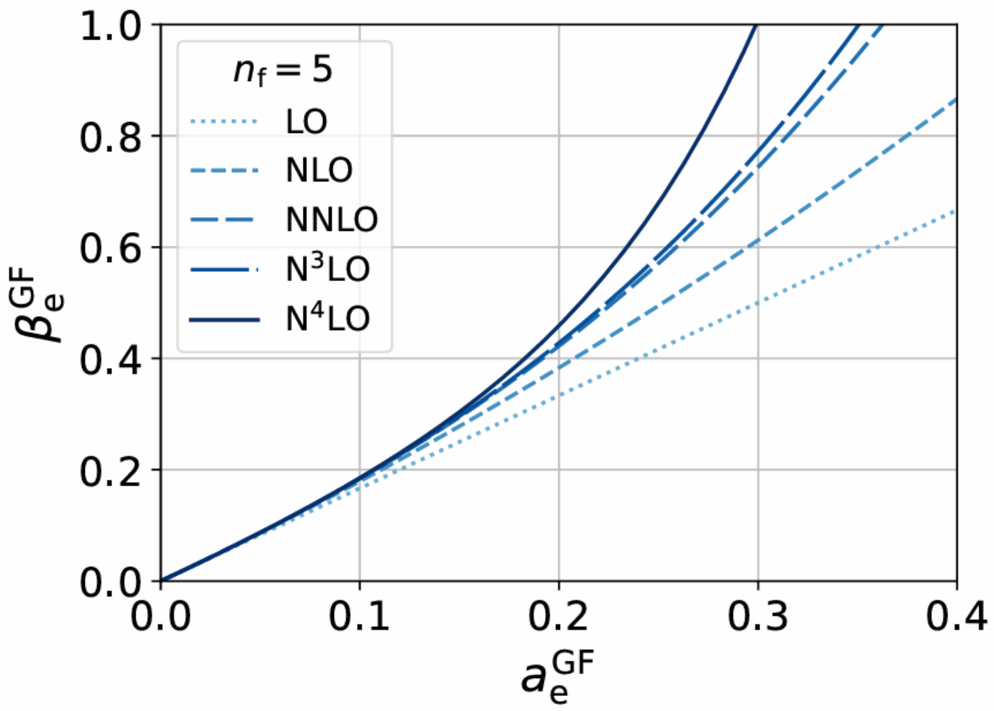

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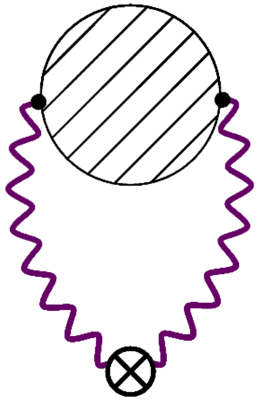

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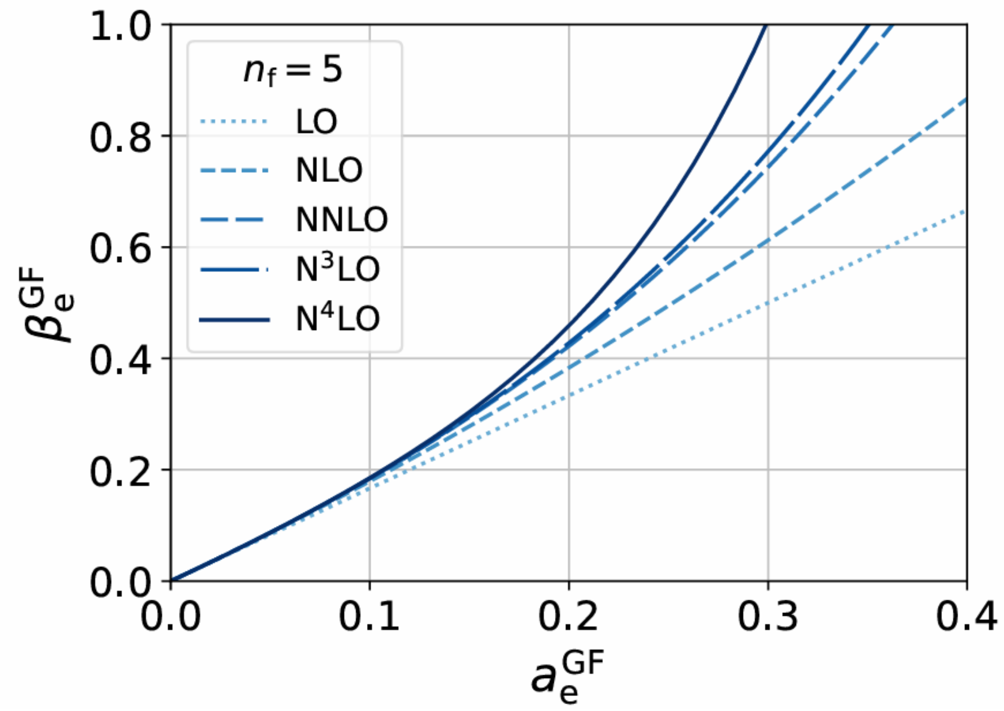

Summary & Conclusion

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- This provides first steps toward 4-loop gradient-flow coupling in QCD
- QED_3 as QCD-like test theory
- Studying QED_3 can offer insight into non-perturbative QCD-like dynamics
- Reproduced known IR fixed point in the gradient-flow scheme

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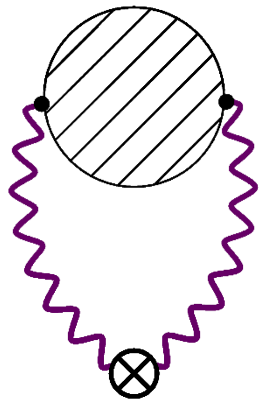

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