Extracting the gluon momentum fraction of the nucleon via the gradient flow

A report on arXiv:2602.14260

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HadStruc Collaboration

QUARK-GLUON TOMOGRAPHY COLLABORATION

Standard Model parameters and observables from gradient flow
University of Edinburgh 2026

HadStruc

Outline

Motivation: Nuclear physics from the Standard Model

Extracting the average gluon momentum fraction: the HadStruc approach

Results from one lattice spacing

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Work led by Alex Sturzu at William & Mary

Motivation

Grounding nuclear physics in the Standard Model is a central goal for nuclear theory

Average gluon momentum fraction of the nucleon a key quantity for nuclear physics

- Input into mass and spin decomposition of the proton
  - Directly connected to energy-momentum tensor and the origin of mass
- Benchmark quantity for lattice calculations of gluons
  - Directly connected to gluon PDF of the nucleon
  - Increasingly important in new era of x-dependent hadron structure calculations
- More technically, serves as normalisation in reduced pseudo-distribution approach

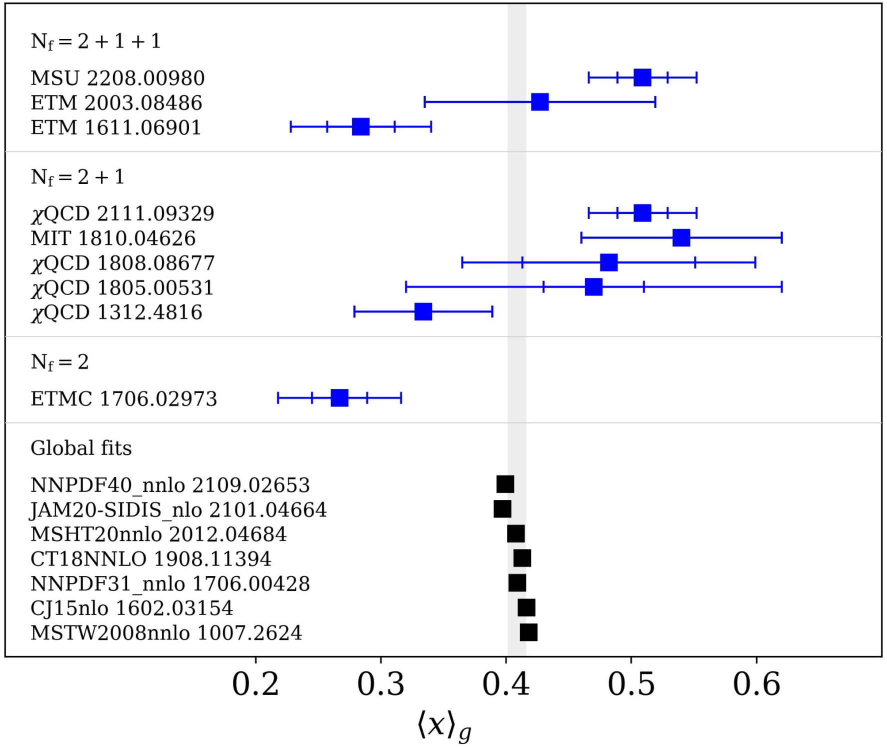
LHC physics!

One alternative to moment calculations

See R. Harlander's and A. Shindler's talks

Gluon momentum fraction of the nucleon

As a benchmark, we have work to do...



See also A. Shindler's talk

Gluon momentum fraction of the ~~nucleon~~ pion

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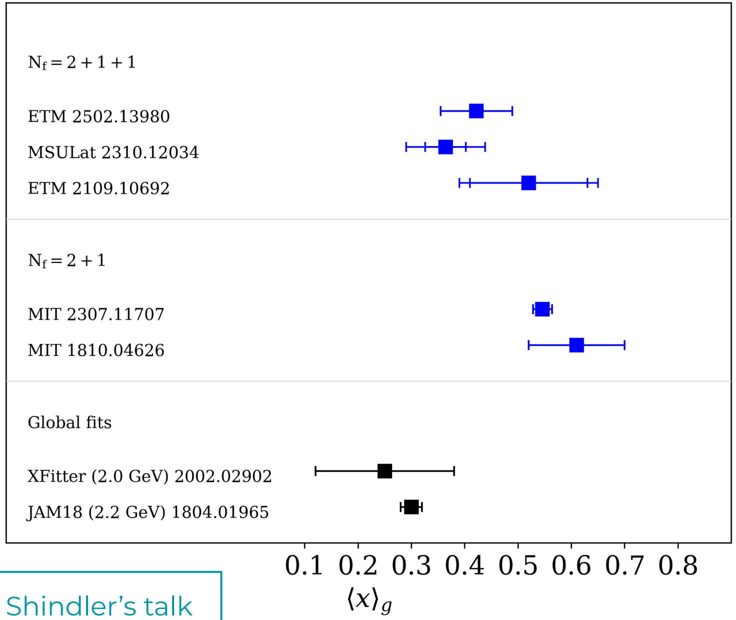

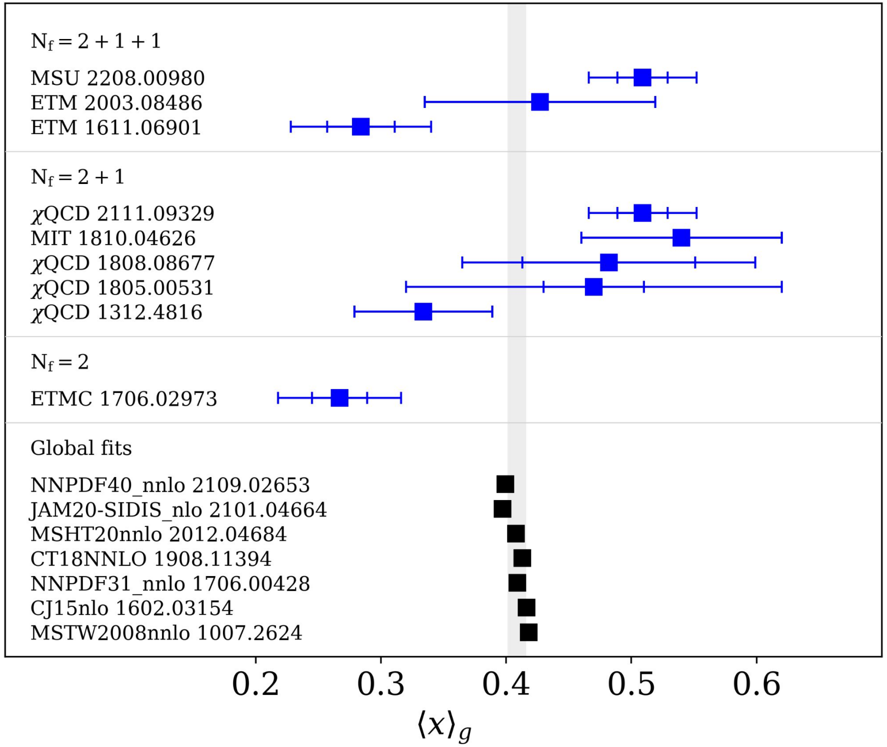

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Approaches to the average gluon momentum fraction

Defined as the first Mellin moment of the gluon parton distribution function

$$\langle x \rangle_g(\mu^2) = \frac{1}{2}\int_{-1}^{1} \mathrm{d}x\, x f_{g/H}(x,\mu^2)$$

Through an operator product expansion, moments are related to local twist-2 operators

$$2\langle x \rangle_g(\mu^2)[P_{\mu_1}P_{\mu_2}\cdots P_{\mu_n} - \mathrm{traces}] = \frac{1}{2}\sum_{s=1}^{2}\langle P,s|\mathcal{O}^g_{\{\mu_1\cdots\mu_{n+1}\}}|P,s\rangle$$

$$\mathcal{O}^g_{\{\mu_1\cdots\mu_{n+1}\}} = -\frac{i^{n-2}}{2}\left[G_{\{\mu_1\alpha}D_{\mu_2}\cdots D_{\mu_{n-1}}G^{\ \alpha}_{\mu_n\}}\right] - \mathrm{traces}$$

See A. Shindler's talk

Given a sufficient number of moments, one can reconstruct the original function

$$f_{g/H}(x,\mu^2) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\frac{\mathrm{d}n}{x^{n+1}}\langle x^n\rangle_g(\mu^2)$$

Moments and distribution functions

This approach has (surmountable) practical challenges

- Power-divergent mixing on the lattice

Solved by the gradient flow!
Note that gluon moments can be extracted without ratios

- Definition of a “sufficient number of moments” for reconstruction is unclear

Higher moments nevertheless provide important constraints or inputs for global fits

- Experimental data (and x-dependent lattice methods) do not cover full kinematic range

Potentially solved by “window moments”
Karpie, CJM, Orginos, & Zafeiropoulos PRL 135 (2025) 191901

$$\langle x^n \rangle_i(x_1, x_2; \mu^2) = \int_{x_1}^{x_2} \mathrm{d}x\, x^{n-1} f_{i/H}(x, \mu^2)$$

Extracting the gluon momentum fraction

Gluon momentum fraction related to the energy-momentum tensor

$$T_g^{\{\mu\nu\}} = \frac{1}{4} g^{\mu\nu} G_{\alpha\beta} G^{\alpha\beta} - G^{\mu\alpha} G^{\nu}{}_{\alpha}$$

In the following we neglect mixing with isoscalar quark contributions

Extracted via two possible operators

$$\mathcal{O}_{Ai} = 2\mathrm{Tr}\left[G_{i\alpha} G_{4\alpha}\right]$$
$$\mathcal{O}_{B} = 2\mathrm{Tr}\left[G_{4\alpha} G_{4\alpha} - G_{j\alpha} G_{j\alpha}\right]$$

with hadronic matrix elements

$$\langle P | \mathcal{O}_{Ai} | P \rangle = 4 i E_N P_i \langle x \rangle_g$$
$$\langle P | \mathcal{O}_{B} | P \rangle = -\left(E_N^2 + \frac{2}{3} \mathbf{P}^2 \right) \langle x \rangle_g$$

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Tests with both operators indicate general agreement, but (unsurprisingly) significantly improved statistical precision at zero hadron momentum.

Best et al., PRD 56 (1997) 2743; ZeRo, EPJC 40 (2005) 69; ETM, PRD 96 (2017) 054503

Extracting the gluon momentum fraction

Extract finite gluon momentum fraction at finite flow time

$$\langle x\rangle_{g/N}(\tau) = -\frac{1}{E_N^2}\langle E_N|\mathcal{O}_B(\tau)|E_N\rangle$$

Relate to the result in the MS-bar scheme via the (inverse) short flow-time expansion

$$\langle x\rangle_{g/N}(\mu^2) \approx C_g^{-1}(\mu^2\tau)\langle x\rangle_{g/N}(\tau) + \mathcal{O}(\tau)$$

,

whose coefficients are known to NNLO in perturbation theory

Practically, but not necessarily conceptually, equivalent to the gradient flowed/smeared OPE

Makino & Suzuki, PTEP2014 (2014) 063B02
Harlander et al., EPJC 78 (2018) 944

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$$\begin{aligned}\mathcal{O}_B(\mu^2) =& 2\left[\mathcal{O}_{1,44}(\mu^2) - \sum_{j=1}^{3}\mathcal{O}_{j,44}(\mu^2)\right] \\ \approx& 2\left[Z_{1k}(\mu^2)H_{kl}\zeta_{lm}^{-1}(\tau)\mathcal{O}_{m,44}(\tau) - Z_{1k}(\mu^2)H_{kl}\zeta_{lj}^{-1}(\tau)\mathcal{O}_{j,44}(\tau)\right]\end{aligned}$$

$k, l, m \in 1, 2$ $\quad$ $j \in 1, 2, 3$

Work led by Alex Sturzu (W&M)
arXiv:2602.14260

Lattice configurations

We present results from a single ensemble of 2+1 clover-improved Wilson fermions with tree-level tadpole-improved Symanzik glue generated by the JLab-LANL-MIT-WM collaboration

ID	a (fm)	m_π (MeV)	$L^3 \times T$	N_{cfg}	N_{srcs}	N_{D}
$a094m358$	0.094(1)	358(3)	$32^3 \times 64$	1121	64	64

In addition to gradient flow (which serves as regulator *and* improves signal-to-noise) we use:

- Distillation to construct hadron interpolators
- Summed GEVP method
- Bayesian model averaging for data analysis

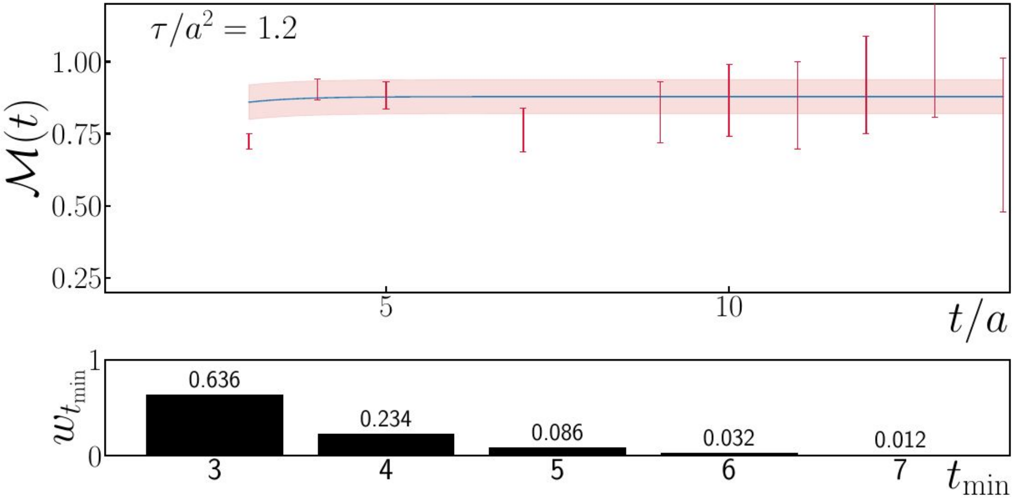
Peardon et al., PRD 80 (2009) 054506
Bulava et al., JHEP 2012 (140) 2012
Jay & Neil, PRD 103 (2021) 114502

Gluon momentum fraction: flow-time dependence

Relevant matrix element extracted from ratio of three- and two-point functions

$$\mathcal{M}(t;\tau) = A(\tau) + B(\tau)te^{-\Delta E t}$$

Recall: t is Euclidean time and τ the flow time



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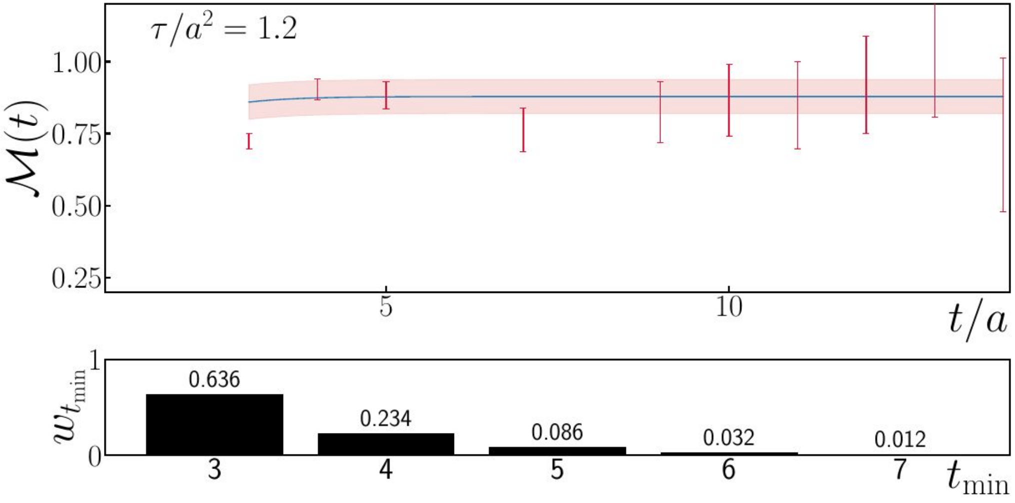
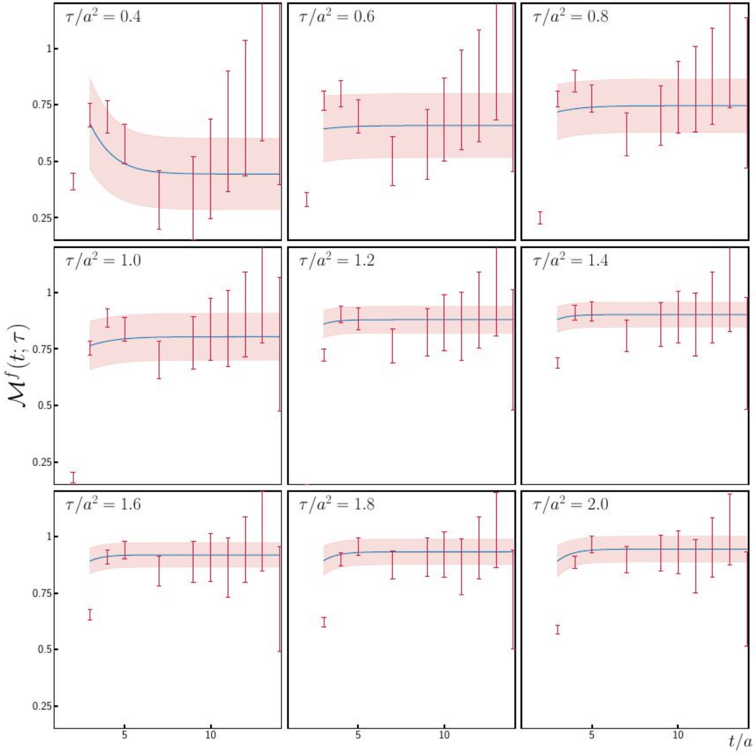
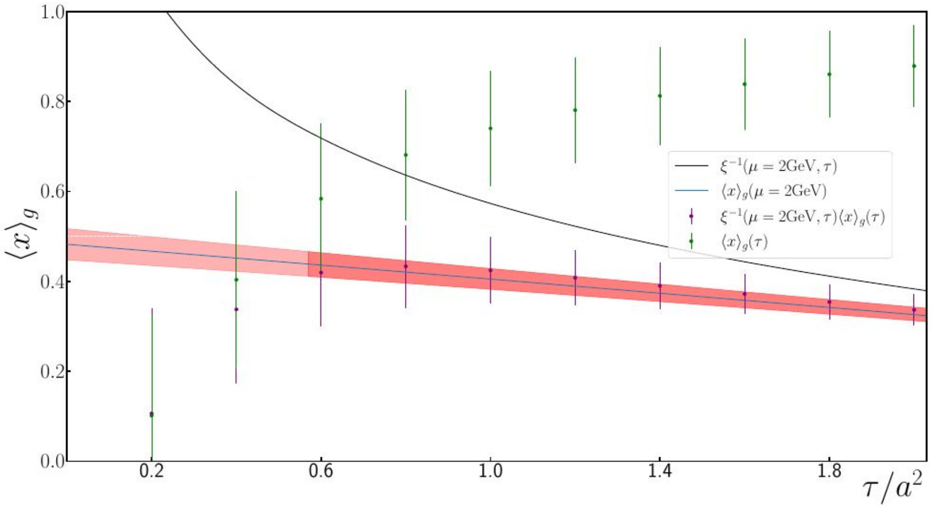
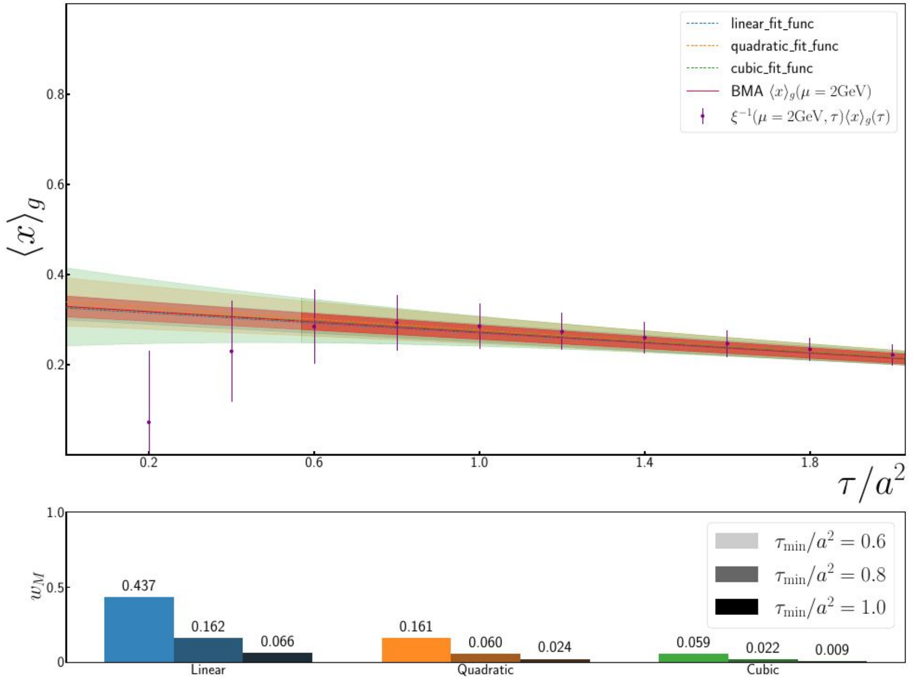




Gluon momentum fraction: flow-time dependence

Fix renormalisation scale and extrapolate to vanishing flow time

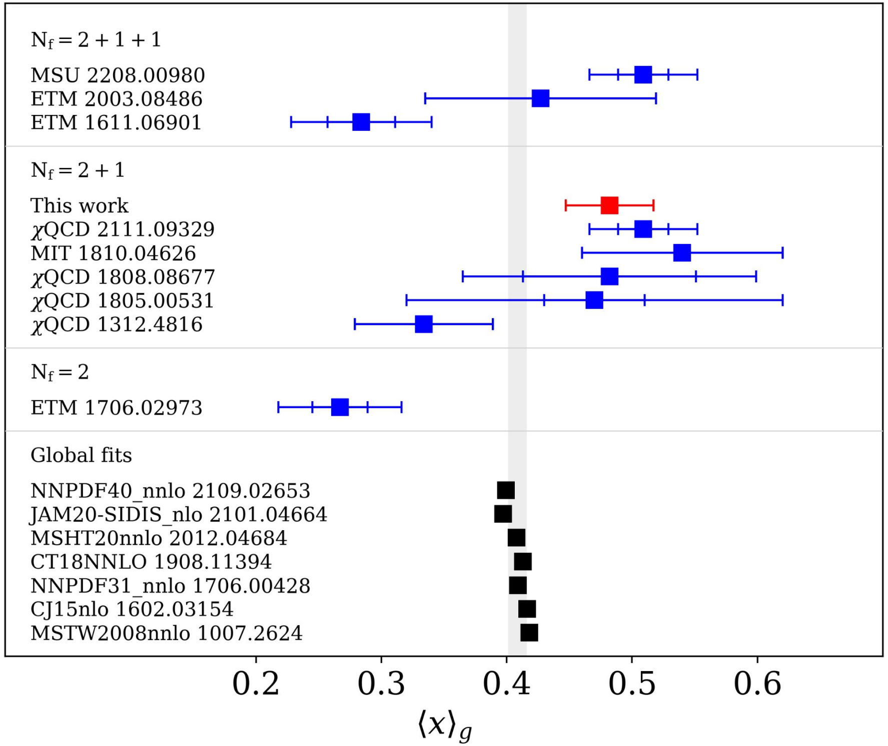




Gluon momentum fraction of the nucleon

We obtain

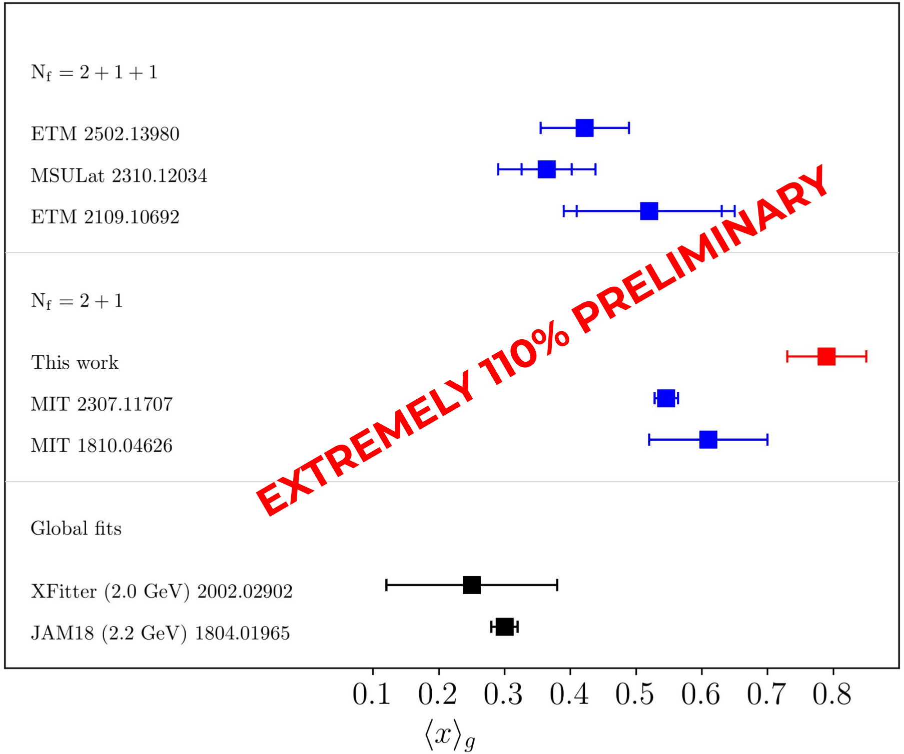
$$\langle x \rangle_{g/N}(\mu = 2\,\mathrm{GeV}) = 0.482(35)$$



Note:

- single lattice spacing
- unphysical pion mass
- no mixing with singlet quark sector

Preliminary gluon momentum fraction of the pion



N.B. 4 time sources on 460 configurations
[vs. 64 time sources on 1121 configurations for the nucleon]

Summary

QCD connects the Standard Model to nuclear physics

Extracting PDFs from QCD is a key challenge

- Relevant wherever protons serve as our probe of the physics of interest
  - LHC
  - neutrino-nucleus scattering
- Central experimental focus for future Electron-Ion Collider

Mellin moments of PDFs provide one approach

- provide complementary information to x-dependent method
- important benchmark for lattice calculations

Gluon moments computationally challenging

- no need for ratios (unless you are specifically interested in RGI quantities)

Thank you!

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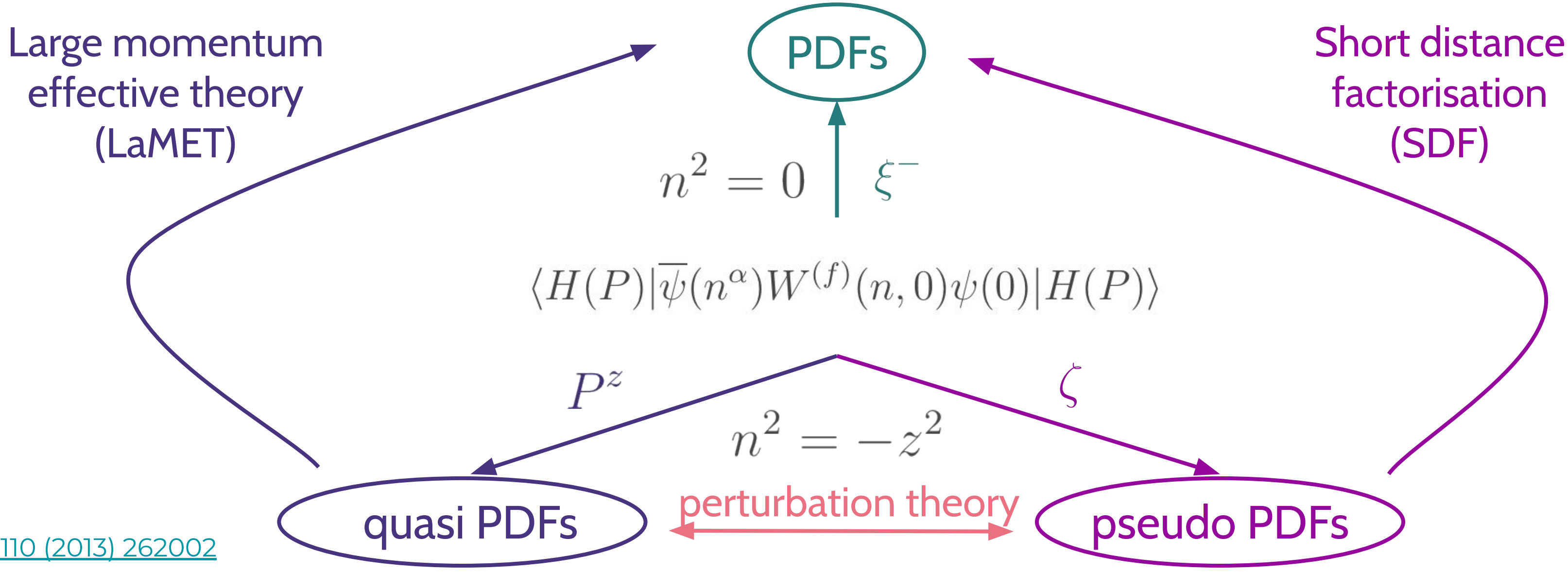
Gradient flow applied in: Monahan & Orginos JHEP 03 (2017) 116
Monahan, PRD 97 (2018) 054507
Shindler, PRD 110 (2024) L051503
Shindler, arXiv:2410.00198

x-dependent parton distributions

$$f_{q/H}^{(0)}(x) = \int_{-1}^{1} \frac{\mathrm{d}\xi^-}{4\pi} e^{-i\xi^- x P^+} \langle H(P)|\overline{\psi}(n^\alpha)W^{(f)}(n,0)\psi(0)|H(P)\rangle$$



Ji, PRL 110 (2013) 262002

$$\widetilde{f}_{j/H}^{(0)}(\xi, P^z) = \int_{-\infty}^{\infty} \frac{\mathrm{d}z}{4\pi} e^{i\xi P_z z} \langle H(P)\left|\overline{\psi}(0,z,\mathbf{0}_\mathrm{T})W(z,0)\Gamma_j\psi(0)\right| H(P)\rangle$$

$$\widetilde{p}_{j/H}^{(0)}(\xi, z^2) = \int_{-\infty}^{\infty} \frac{\mathrm{d}\zeta}{4\pi} e^{i\xi\zeta} \langle H(P)\left|\overline{\psi}(0,z,\mathbf{0}_\mathrm{T})W(z,0)\Gamma_j\psi(0)\right| H(P)\rangle$$

Radyushkin, PRD 96 (2017) 034025

Three-point functions

Principle correlators defined by

$$C(t)v^{\alpha}(t,t_0) = \lambda^{\alpha}(t,t_0)C(t_0)v^{\alpha}(t,t_0)$$

Define subtracted three-point functions

$$C^{i}_{3\mathrm{pt}}(t,t_g) = \left\langle \left(C^{i}_{2\mathrm{pt}}(t) - \langle C_{2\mathrm{pt}}(t)\rangle\right)\left(\mathcal{O}^{i}_{\mathrm{B}}(t_g) - \langle \mathcal{O}_{\mathrm{B}}(t_g)\rangle\right)\right\rangle$$

Sum over operator insertions

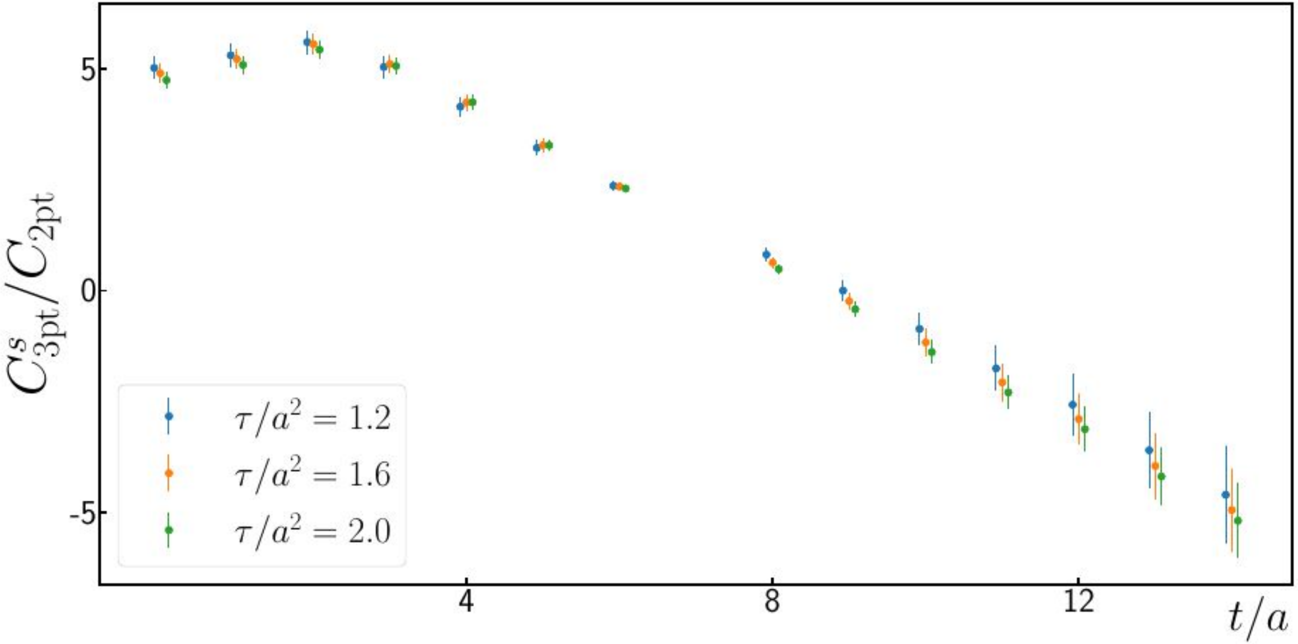
$$C^{i,\mathrm{s}}_{3\mathrm{pt}}(t) = \sum_{t_g=1}^{t-1} C^{i}_{3\mathrm{pt}}(t,t_g)$$

Rotate to the GEVP basis and isolate state of interest

$$\mathcal{M}^{i}(t,t_0) = -\partial_t \left\{ \frac{v_n^{\dagger}\left(C^{i,\mathrm{s}}_{3\mathrm{pt}}(t)\lambda^{-1}_{i,0}(t,t_0) - C^{i,\mathrm{s}}_{3\mathrm{pt}}(t_0)\right)v_n}{v_n^{\dagger}\left(C^{i}_{2\mathrm{pt}}(t_0)\right)v_n} \right\}$$

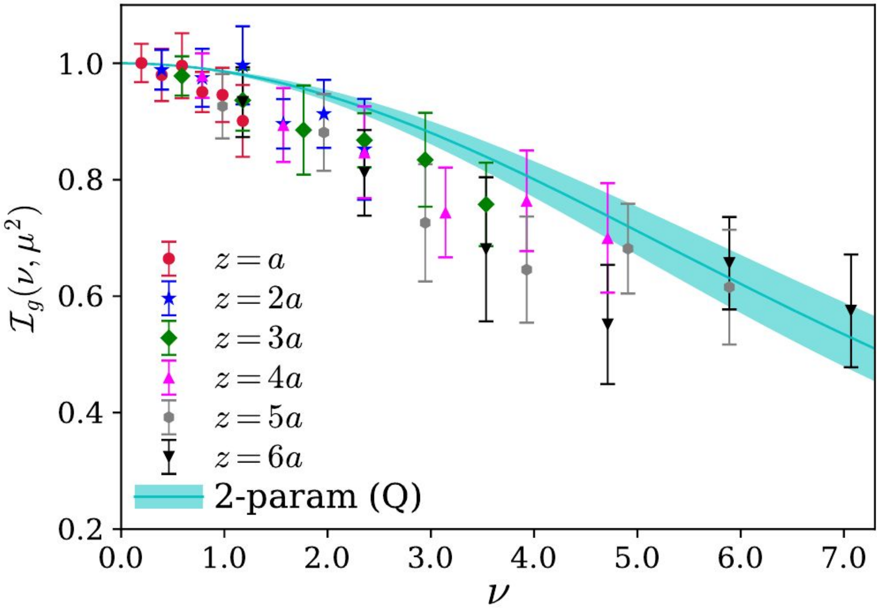
Three-point functions

Ratio of summed three-point functions to two-point functions



HadStruc calculations

$\nu = P \cdot n$



Khan et al., PRD 104 (2021) 094516

Short flow-time expansion: open questions*

*These may simply reflect my ignorance

What is known of the properties to all orders in perturbation theory?

Can the radius of convergence be shown to be non-zero?

To what extent is this really (or really analogous to) an operator product expansion?

Does the validity of the expansion in bare operators hold beyond perturbation theory?

Can one analyse the window problem other than through numerical experiment?

What defines small flow time and can practical lattice calculations overcome the window problem in high-precision scenarios (at the 0.2 to 0.5% level)?