Adjoint chromoelectric correlators with lattice QCD

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Standard Model parameters and observables from gradient flow, Edinburgh, close to England, May 14 2026

Motivation

- Chromoelectric correlators show up in effective field theory descriptions
- at finite temperature:
  - related to transport coefficients
  - perturbation theory does not converge $\rightarrow$ use lattice QCD
- Zero T:
  - related to quarkonium decay and quarkonium production
  - related to gluelumps
  - infer the non-perturbative part for the proper EFT
- the relevant EFTs: NRQCD, pNRQCD
- Why gradient flow?
  - Gradient flow renormalizes the field insertions
  - Gradient flow serves as intermediate regulator for power divergences
  - Gradient flow improves the signal-to-noise ratio
  - No lattice perturbation theory for matching is required

Related Studies (in our collaboration)

Finite T:

- Nora Brambilla, Viljami Leino, **Julian Mayer-Steudte**, Peter Petreczky
  Heavy quark diffusion coefficient with gradient flow
  Phys.Rev.D 107 (2023) 5, 054508, arXiv:2206.02861
- Nora Brambilla, Saumen Datta, Marc Janer, Viljami Leino, **Julian Mayer-Steudte**, Peter Petreczky, Antonio Vairo
  Lattice study of correlators of chromoelectric fields for heavy quarkonium dynamics in the quark-gluon plasma
  Phys.Rev.D 112 (2025) 7, 074509, arXiv:2505.16603

Zero T:

- Nora Brambilla, Viljami Leino, Panayiotis Panayiotou, Andrea Shindler, **Julian Mayer-Steudte**, Antonio Vairo, Xiang-Peng Wang
  First principle determination of inclusive P-wave quarkonium decay widths from lattice QCD and pNRQCD
  TUM-EFT 204/26

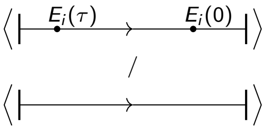
Recall: Fundamental (2206.02861)

- Related EFT: NRQCD (in HQET description)
- Describes heavy quark diffusion
- Related Euclidean correlator and relation to κ^{fund}:

$$G_E(\tau) = -\frac{1}{3}\sum_{i=1}^{3} \frac{\langle \mathrm{ReTr}[U(\beta,\tau) g E_i(\tau) U(\tau,0) g E_i(0)]\rangle}{\langle \mathrm{ReTr}(L_3)\rangle}$$

$$G_E(\tau) = \int_0^\infty \frac{d\omega}{\pi}\rho(\omega) K(\omega), \quad K(\omega) = \frac{\cosh(\beta/2-\tau)\omega}{\sin \beta\omega/2}$$

$$\kappa^{\text{fund}} = \lim_{\omega\to 0} \frac{2T}{\omega}\rho(\omega)$$



(Moore and Teaney PRC71 (2005)), (Caron-Huot and Moore JHEP02 (2008)), (Caron-Huot et.al.JHEP 04 (2009) 053)

- This operator was computed over many years by different groups and works better than J-J-correlators

Recall: Fundamental (2206.02861)

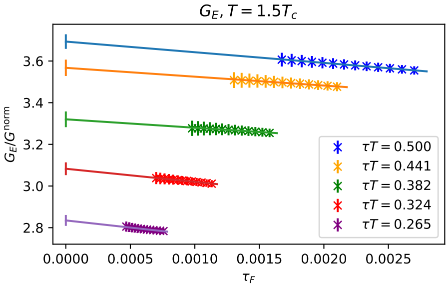
- Divergence in the static Wilson lines:

$$U(\beta,\tau).U(\tau,0). \propto e^{-\delta m/T}$$
$$L_3 \propto e^{-\delta m/T}$$

$\rightarrow$ divergence cancels in the ratio of both quantities

- Gives a linear flow time dependence
  $\rightarrow$ zero flow time limit is safe
- Extract κ^{fund}:

$$T = 1.5\,T_c : 1.7 \leq \frac{\kappa^{\text{fund}}}{T^3} \leq 3.12$$
$$T = 10^4\,T_c : 0.02 \leq \frac{\kappa^{\text{fund}}}{T^3} \leq 0.16$$



(Brambilla et.al. PRD107,054508(2023))

other Refs: (Banerjee et.al. PRD85,014510(2012)), (Francis et.al. PRD92,116003(2015)), (Brambilla et.al. PRD102,074503(2020)), (Banerjee et.al. JHEP08,128(2022)), unquenched: Altenkort et.al. PRD130,232902(2023)

Next: Adjoint (2505.16603)

- Related EFT: pNRQCD
- Describes Quarkonium dynamics: two heavy quark systems
  $\rightarrow$ Quarkonium diffusion ($\kappa^{\text{adj}} = \kappa^{\text{fund}}$? (Scheihing-Hitschfeld and Yao PRD 108 (2023) 5, 054024))
- First calculation of the correlators describing Quarkonium diffusion and needed to study the non-equilibrium evolution of Quarkonium in medium
  (Brambilla et.al. PRD96, 034021(2017)), (Brambilla et.al. JHEP05,282021 29136(2021)),(Brambilla et.al. PRD108, L011502(2023))

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- Two possible interactions:
  - Bound state: singlet state
  - Scatter state: octet state

Three possible processes:

- singlet $\rightarrow$ octet: dissociation
- octet $\rightarrow$ singlet: recombination
- octet $\rightarrow$ octet

- Construction of the open quantum system and the Lindblad equation close to publication
  (Brambilla et.al. TUM-EFT 191/24, FERMILAB-PUB-24-0451-T)
- But still related to chromoelectric correlators

$\Rightarrow$ calculate adjoint chromoelectric correlators to extract κ^{adj}

Adjoint correlator 1: Introducing G_E^{oct}

- Octet-octet diffusion described by:

(Brambilla et.al. TUM-EFT 191/24, FERMILAB-PUB-24-0451-T)

$$G_E^{\mathrm{oct}}(\tau) = -\frac{1}{3}\sum_{i=1}^{3}\frac{\langle W_{ab}(\beta,\tau)d_{bcd}E_i^d(\tau)W_{ce}(\tau,0)d_{eaf}E_i^f(0)\rangle}{\langle L_8\rangle}$$

E_i^a :(adjoint) fields, W_{ab} : adjoint Wilson lines, d_{abc} : symmetric structure constants

L_8 :adjoint Polyakov loop

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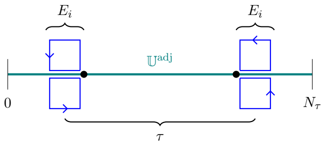
- Nominator & denominator have the same divergence $\propto e^{-\delta m_8/T}$

➡ Continuum limit and zero flow time limit of G_E^{oct} are safe

Adjoint correlator 1: Results of G_E^{oct}

- Focus on half-of-clover (2-plaquette) discretizations of the E-fields:

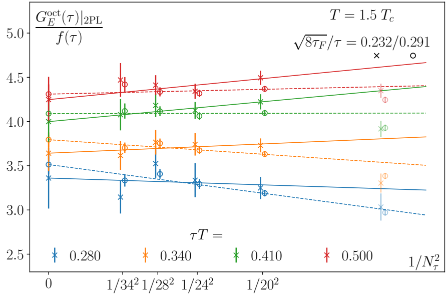


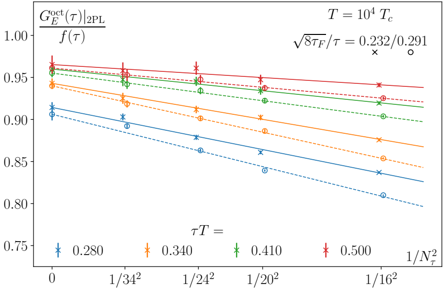
- Consider dimensionless and tree-level improved quantity $G_E^{\mathrm{Latt}}/G_{\mathrm{norm}}^{\mathrm{Latt}}$ with

$$\frac{G_{\mathrm{norm}}^{\mathrm{Latt}}(\tau T)}{T^4} = \frac{N_\tau^4}{3}\int_{-\pi}^{\pi}\frac{d^3k}{(2\pi)^3}\,\frac{\cosh zN_\tau(\frac{1}{2}-\tau T)}{\sinh zN_\tau/2}\,\frac{1}{\sinh z}\times\left(\hat{k}^2+\frac{\hat{k}^4-\left(\hat{k}^2\right)^2}{8}\right)$$

$$\hat{k}^n = \sum_i\left(2\sin\frac{k_i}{2}\right)^n, \quad \sinh\frac{z}{2} = \sqrt{\frac{\hat{k}^2}{4}}$$

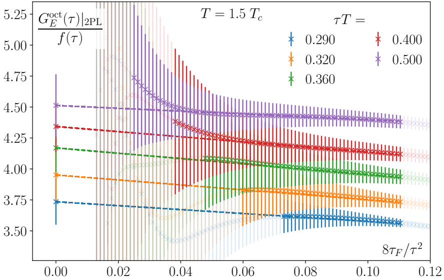
Adjoint correlator 1: Results of G_E^{oct}

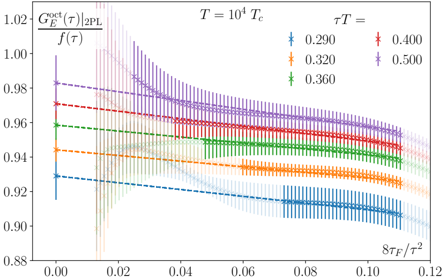




- linear in $1/N_\tau^2 \Leftrightarrow a^2$ continuum extrapolation
- $\chi^2/\mathrm{dof} = \mathcal{O}(1)$

Adjoint correlator 1: Results of $G_E^{\rm oct}$





- linear flow time dependence within a proper flow time window
  $\rightarrow$ linear zero flow time limit

Adjoint correlator 1: Results of G_E^{oct}

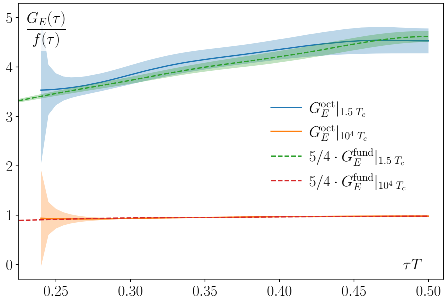
- Continuum & zero flow time extrapolated correlators show same scaling within errors compared with the fundamental correlator as at tree level

  (Brambilla et.al. PRD107,054508(2023))

- To extract κ, we already have the parameters for ρ from the fundamental

$$G_E(\tau) = (5/4)G^{\text{fund}}(\tau)$$
$$= \int_0^\infty \frac{d\omega}{\pi}(5/4)\rho(\omega)K(\omega)$$
$$\Rightarrow \kappa_{adj}^{oct} = (5/4)\kappa^{\text{fund}}$$

- Related to octet-octet diffusion



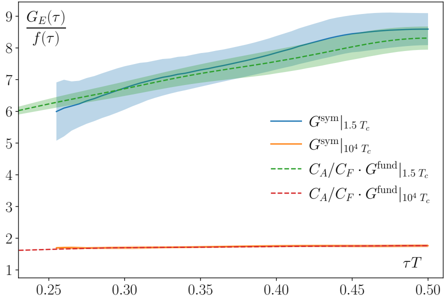
Adjoint correlator 1.2: Introducing G_E^{symm}

- Symmetric correlator:

$$G_E^{\mathrm{symm}}(\tau) = -\frac{1}{3}\sum_{i=1}^{3} \frac{\langle W_{ab}(\beta,\tau) f_{bcd} E_i^d(\tau) W_{ce}(\tau,0) f_{eaf} E_i^f(0)\rangle}{\langle L_8\rangle}$$

- Describes diffusion of a heavy quark in adjoint representation

$$G_E^{symm}(\tau) = (C_A/C_F) G^{\mathrm{fund}}(\tau)$$
$$\Rightarrow \kappa_{adj}^{symm} = (C_A/C_F)\kappa^{\mathrm{fund}}$$

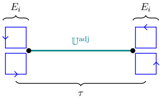


Adjoint correlator 2: Introducing non-symmetric G_E

- This correlator emerged from EFT calculations:

$$G_E(\tau) = -\frac{1}{3}\sum_{i=1}^{3}\langle E_i^a(\tau) W_{ab}(\tau,0) E_i^b(0)\rangle$$

- $G_E(\tau) = e^{-\delta m_8 \tau} G_E^r \rightarrow e^{\delta m_8 \tau} G_E(\tau) = G_E^r$



2-plaquette

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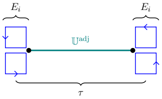
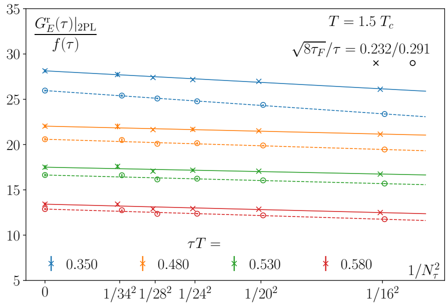
- $G_E(\tau) = e^{-\delta m_8 \tau} G_E^r \rightarrow e^{\delta m_8 \tau} G_E(\tau) = G_E^r$
- $L_8 = e^{-\delta m_8 / T} L_8^r$
  (Gupta et.al.PRD77,034503(2008))
- $\delta m_8(\tau_F) = T \log \frac{L_8^r}{L_8(\tau_F)}$
- Describes dissociation and recombination processes

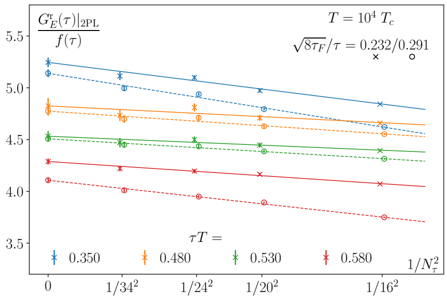


2-plaquette

$G_E^r = \left(\frac{L_8^r}{L_8(\tau_F)}\right)^{\tau T} G_E$ is a divergent free quantity

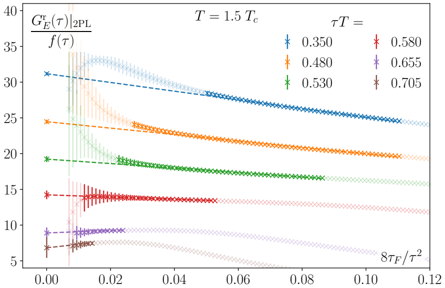
Adjoint correlator 2: Results of the non-symmetric G_E

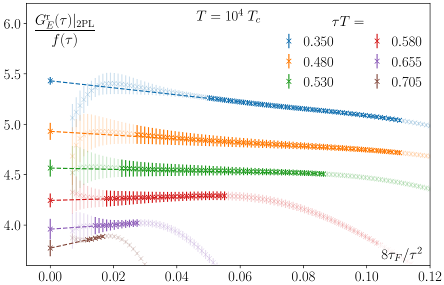




- now cubic splines with default boundary conditions
- linear in $1/N_\tau^2 \Leftrightarrow a^2$ continuum extrapolation
- $\chi^2/\mathrm{dof} = \mathcal{O}(1)$

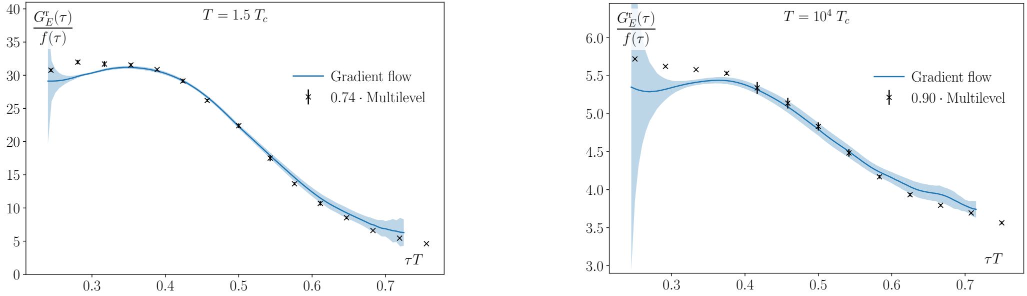
Adjoint correlator 2: Results of the non-symmetric G_E





- linear behavior in flow time
  $\rightarrow$ perform linear zero flow time limit

Adjoint correlator 2: Results of the non-symmetric G_E



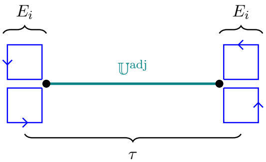
- comparison to results with ML: tadpole improvement, same Wilson line renormalization
- GF and ML agree up to a T-dependent factor
- Correlator is not symmetric: Requires further studies for κ-extraction

Adjoint correlator 2: Towards zero T calculation of G_E

- Inclusive quarkonium P-wave decay width in pNRQCD:

(Brambilla et.al. PRL 88 (2002) 012003) , (Brambilla et.al. JHEP 04 (2020) 095) , Brambilla et.al. PRD 67 (2003) 034018

$$\Gamma\left(\chi_{QJ}\to LH\right)=\frac{3N_c}{2\pi}\left|R'_{\chi_Q}(0)\right|^2\frac{16}{m_Q^4}\left[2\mathrm{Im}f_1\left({}^3P_J\right)(\Lambda)+2\mathrm{Im}f_8\left({}^3S_1\right)\frac{2T_F}{9N_C}\mathcal{E}_3(\Lambda)\right]$$



2-plaquette

- Chromoelectric part:

$$\mathcal{E}(t)\equiv\langle 0|gE^{i,a}(t)\Phi^{ab}(t,0)gE^{i,b}(0)|0\rangle$$

$$\mathcal{E}_3\left(\Lambda\right)=\frac{T_F}{N_c}\int_0^\infty \mathrm{d}t\, t^3\mathcal{E}(t)$$

(Slightly different notation in the old papers but still the same object!)

- Problem: No Polyakov loop normalization possible

Adjoint correlator 2: Towards zero T calculation of G_E

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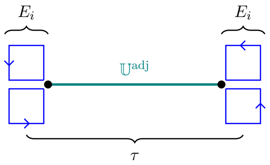
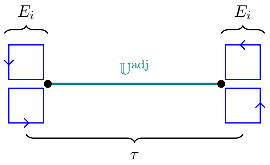
- Problem: No Polyakov loop normalization possible
- Gradient flow as intermediate regulator
- First attempt of computing $\mathcal{E}_3$ on the lattice
- Similar objects for various decay and production rates



2-plaquette

Adjoint correlator 2: Towards zero T calculation of G_E

- $G_E(\tau) = e^{-\delta m_8 \tau} G_E^{sub} \to e^{\delta m_8 \tau} G_E(\tau) = G_E^{sub}$
- $\delta m_8(a) = \frac{d'}{a}$
- $\delta m_8(\tau_F) = \frac{d}{\sqrt{8\tau_F}}$
- $\Rightarrow \log\langle L_8 \rangle = -\frac{d}{\sqrt{8\tau_F}T} +$ finite parts



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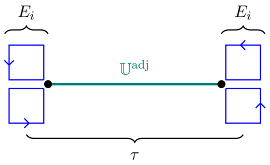

2-plaquette

Extract d directly from the flow time behavior

Adjoint correlator 2: Towards zero T calculation of G_E

- $\log\langle L_8\rangle = -\frac{d}{\sqrt{8\tau_F}T} + \text{finite parts}$
- $\langle L_8\rangle$ exists only at finite T
- UV-divergence $\to$ T-independent
- d depends only on lattice spacing
- fit d at different temperatures, but fixed lattice spacing

- Finite parts unknown
  $\to$ use a set of finite *ansätze*
- Using Bayesian sampling to extract d

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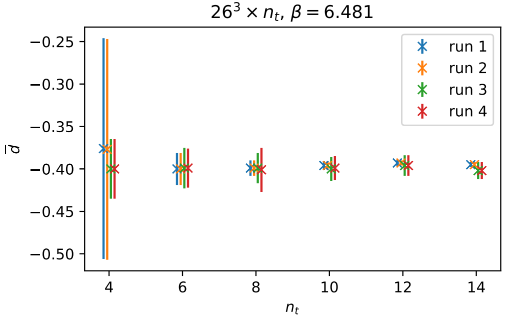
Observation: d is independent of T



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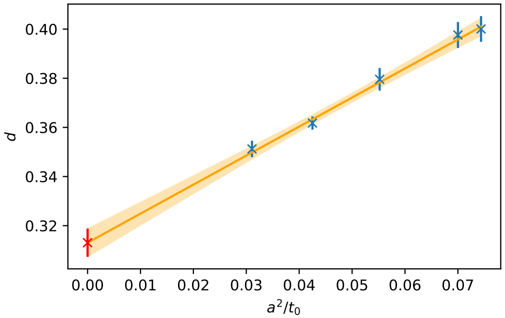
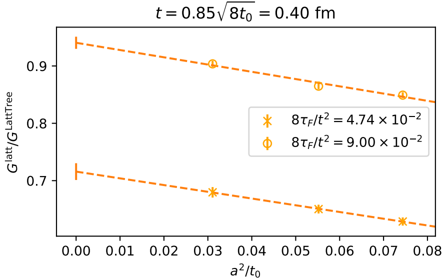
Continuum limit of d: $d = 0.315(6)$



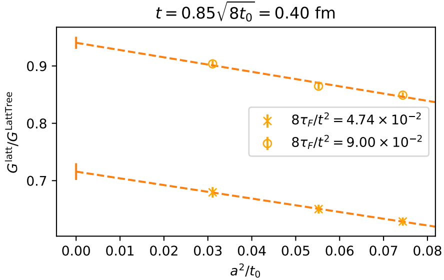
Adjoint correlator 2: Towards zero T calculation of G_E

- Next Steps:
  - Continuum limit of G_E at fixed t and fixed τ_F
    $a < \sqrt{8\tau_F} < t/3$



Adjoint correlator 2: Towards zero T calculation of G_E

- Next Steps:
  - Continuum limit of G_E at fixed t and fixed τ_F
    $a < \sqrt{8\tau_F} < t/3$
  - Substract the power divergence:
    $G_E^{sub}(t, \tau_F) = e^{\frac{d}{\sqrt{8\tau_F}} t} G_E(t, \tau_F)$
  - Matching of the E-fields: $Z(\tau_F) = 1 + \mathcal{O}(\alpha_s^2)$



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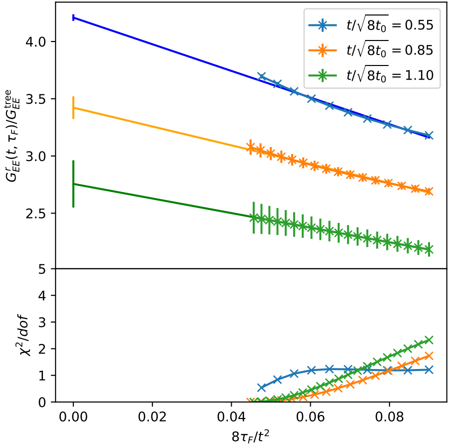
  - Matching of the E-fields: $Z(\tau_F) = 1 + \mathcal{O}(\alpha_s^2)$
  - Perform zero flow time limit at linear extrapolation in τ_F



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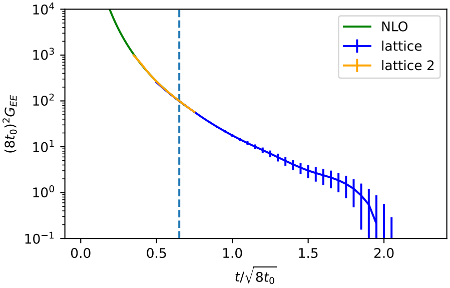
  - Matching of the E-fields: $Z(\tau_F) = 1 + \mathcal{O}(\alpha_s^2)$
  - Perform zero flow time limit at linear extrapolation in τ_F
  - Match to $\overline{\mathrm{MS}}$:
    $\mathcal{E}^{\overline{\mathrm{MS}}}(t) = e^{m_0^{\overline{\mathrm{MS}}}(\alpha_S)t} G_E(t)$
    $\mathcal{E}^{\overline{\mathrm{MS}}}$ at NLO: (Eidemüller and Jamin Phys.Lett.B 416 (1998) 415-420)
  - Compute $\mathcal{E}_3$ and Γ
    $\rightarrow$ I will leave it as a trivial task to the audience



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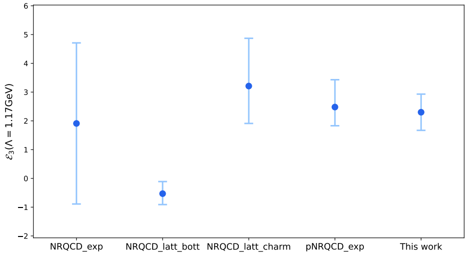
  - Match to $\overline{\mathrm{MS}}$:
    $\mathcal{E}^{\overline{\mathrm{MS}}}(t) = e^{m_0^{\overline{\mathrm{MS}}}(\alpha_S)t} G_E(t)$
    $\mathcal{E}^{\overline{\mathrm{MS}}}$ at NLO: (Eidemüller and Jamin Phys.Lett.B 416 (1998) 415-420)
  - Compute $\mathcal{E}_3$ and Γ
    $\rightarrow$ I will leave it as a trivial task to the audience
- This method allows to compute various inclusive decay widths that are not directly accessible on the lattice



Summary and Outlook

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  - Finite T: find κ for the non-symmetric correlator
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Thank you for your attention!

Sample frame title

This is some text in a sample frame. Don't waste your time and stay focused on the talks.

Knock Knock!! Who's there!?