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Factorization in QCD and Beyond Coherence

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work with Patrick Hager, Sebastian Jaskiewicz, Matthias Neubert, and Dominik Schwienbacher

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The Fog is Lifting

Factorization Breaking

Glauber Gluons

Coherence Violation

Super-Leading Logarithms

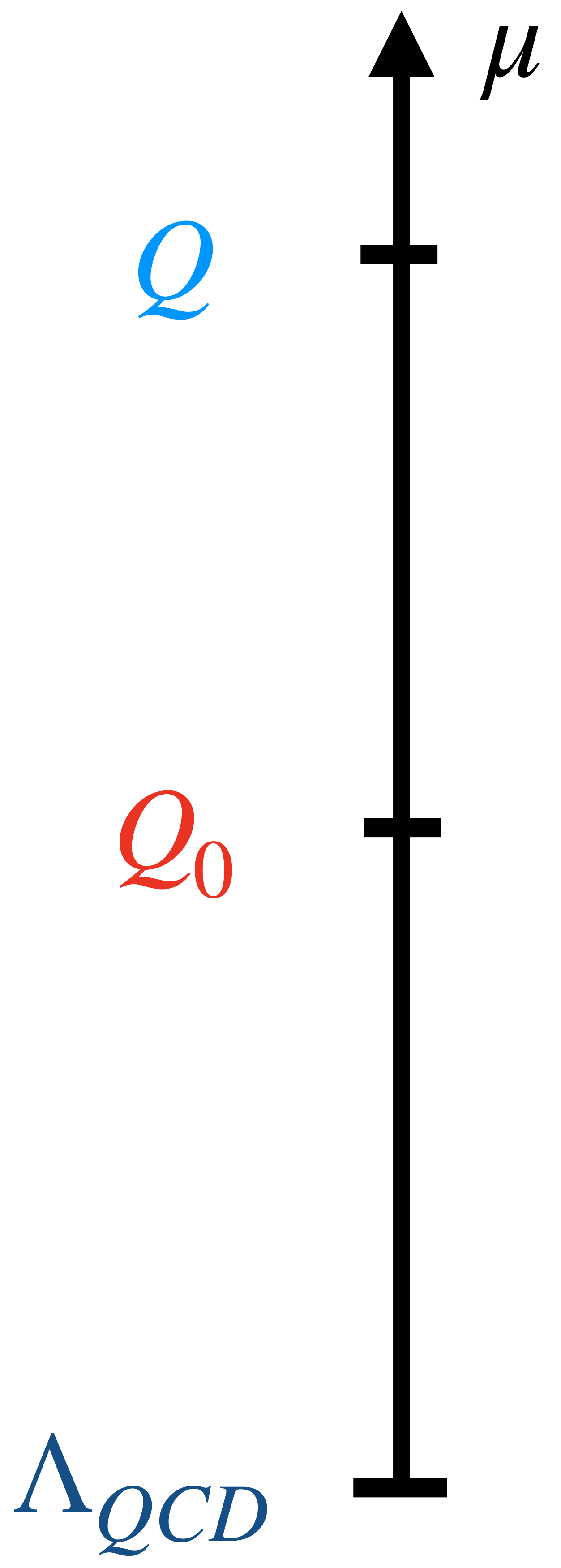
Confusion arises because the same term refers to related but different concepts, for example

- Factorization
- Glauber Gluons

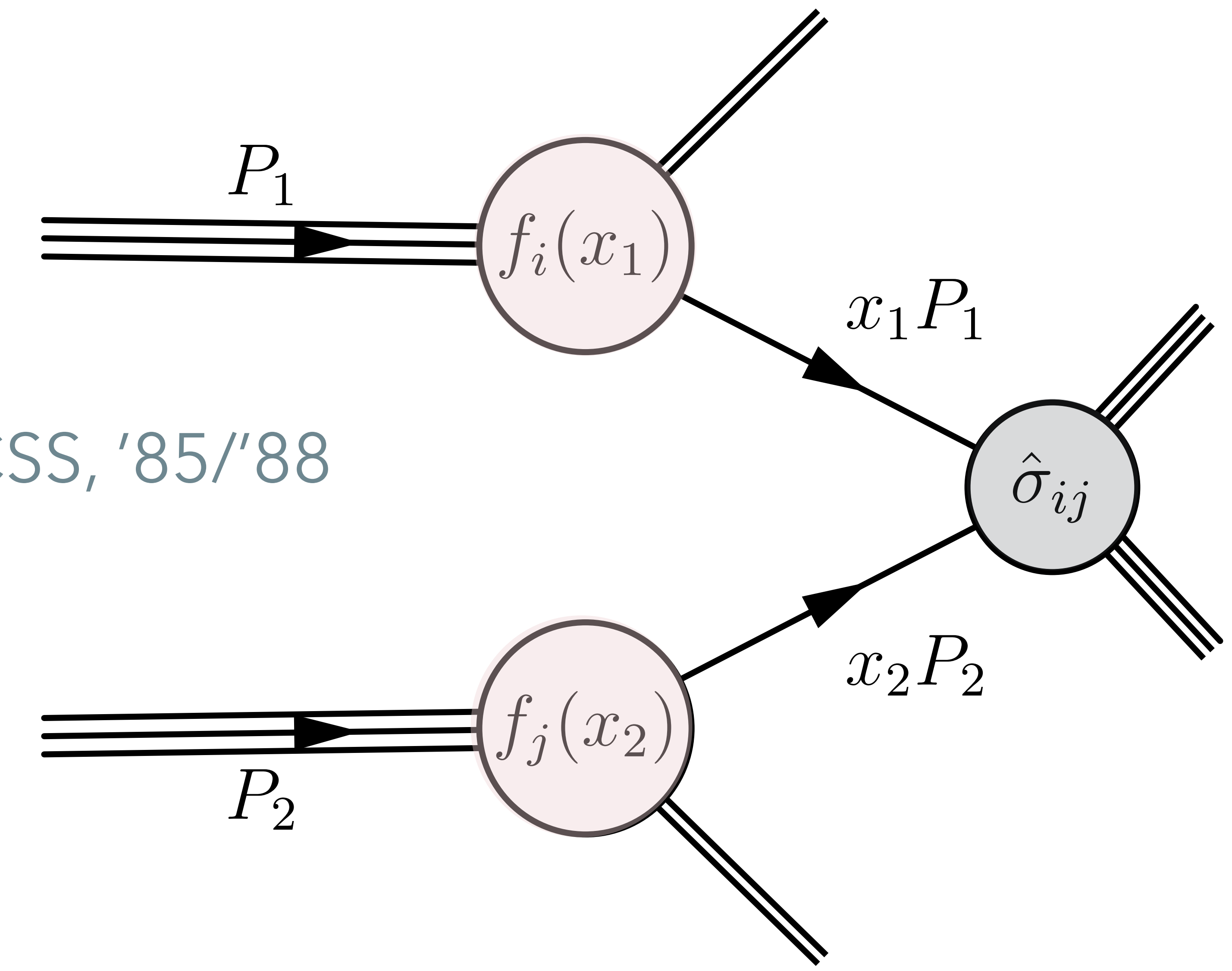
together with Dadaist language such as Super-Super-Leading Logarithms

Factorization = Scale Separation

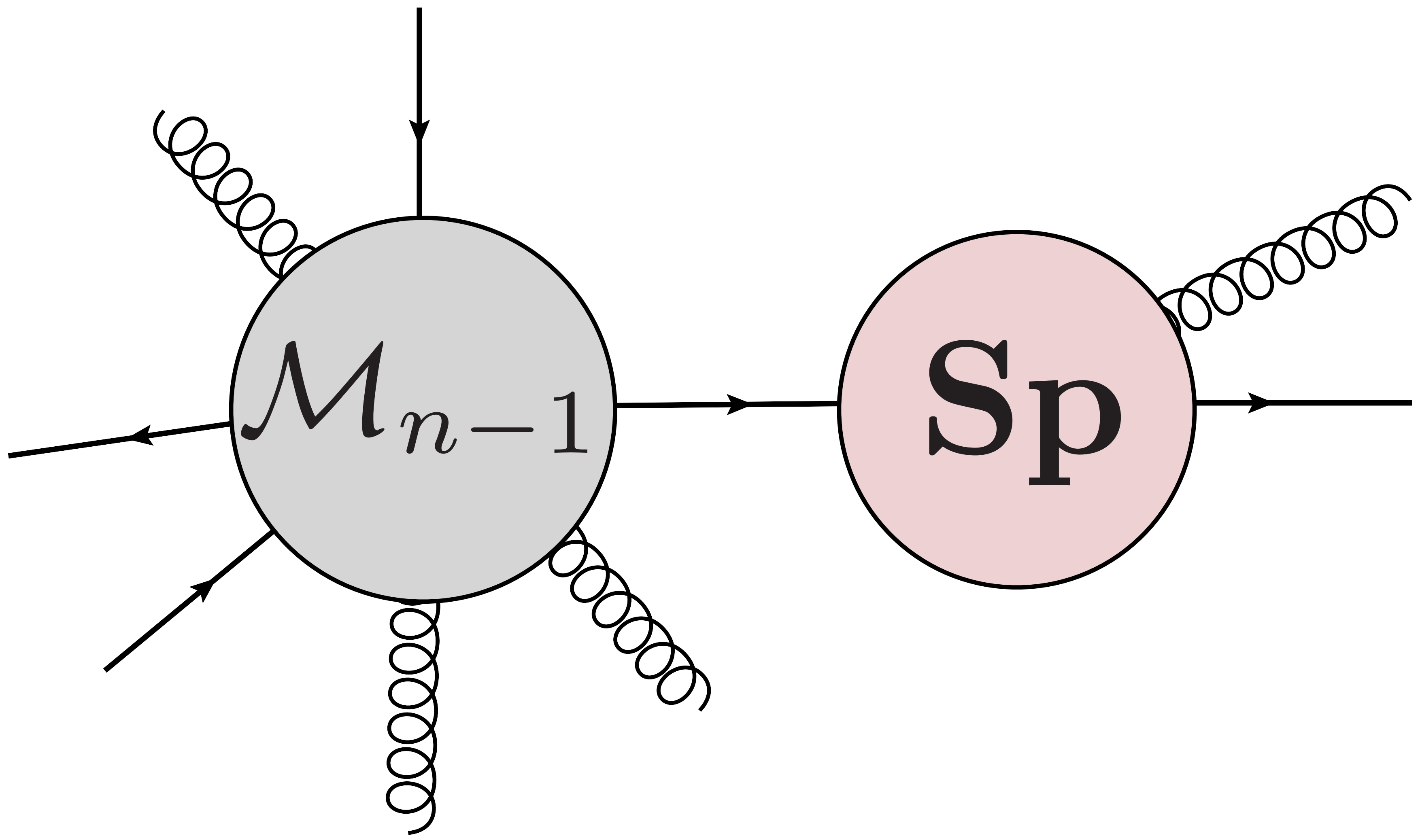
- Believe it is always possible to separate physics at disparate scales, but low-energy matrix elements can be complicated
    - Typically, soft Wilson lines along all directions of large momentum flow
- Intimately connected to the construction of low-energy effective field theories
- On a diagrammatic level captured by the method of regions (MoR) expansion Beneke, Smirnov ‘98



Factorization = Universality

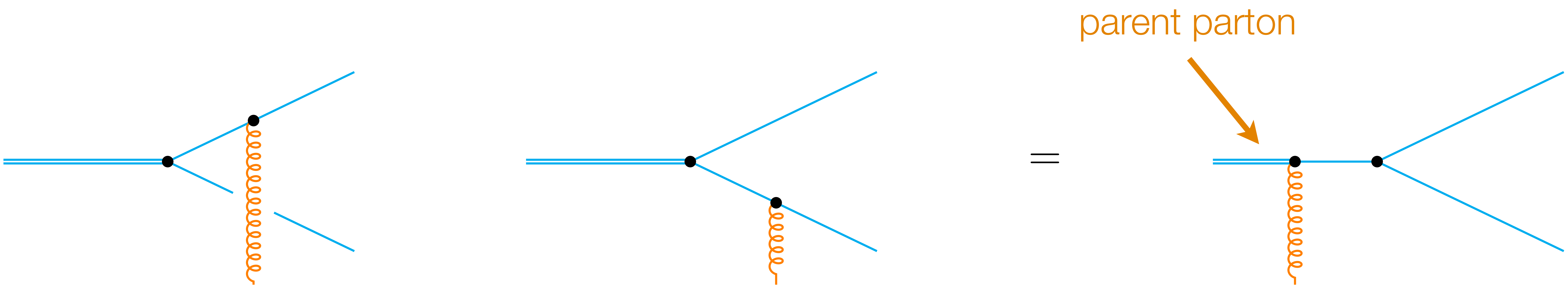


CSS, '85/'88



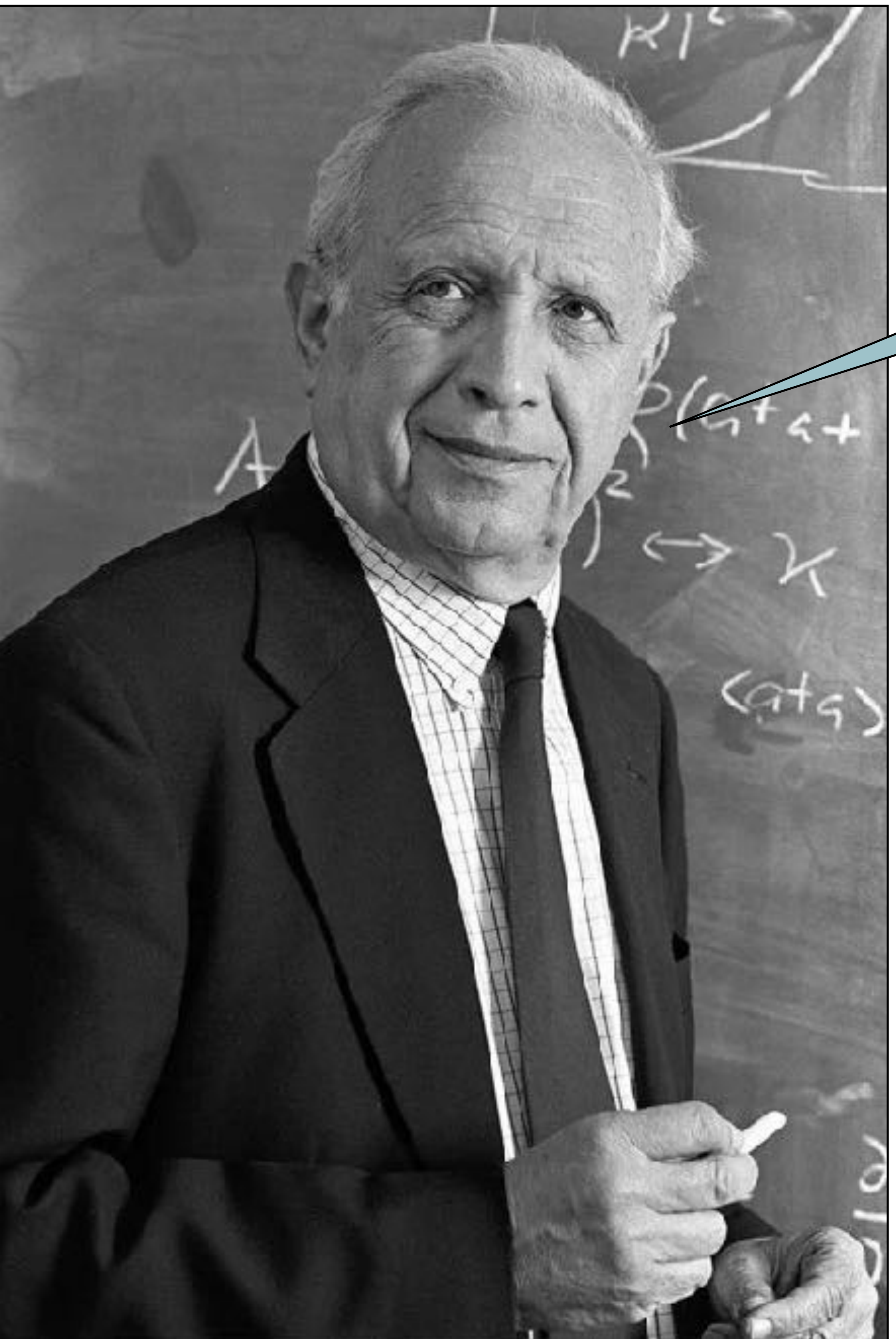
- Sometimes low-energy matrix elements are independent of the hard process
- Only if there are no soft or Glauber exchanges between different sectors

Color Coherence



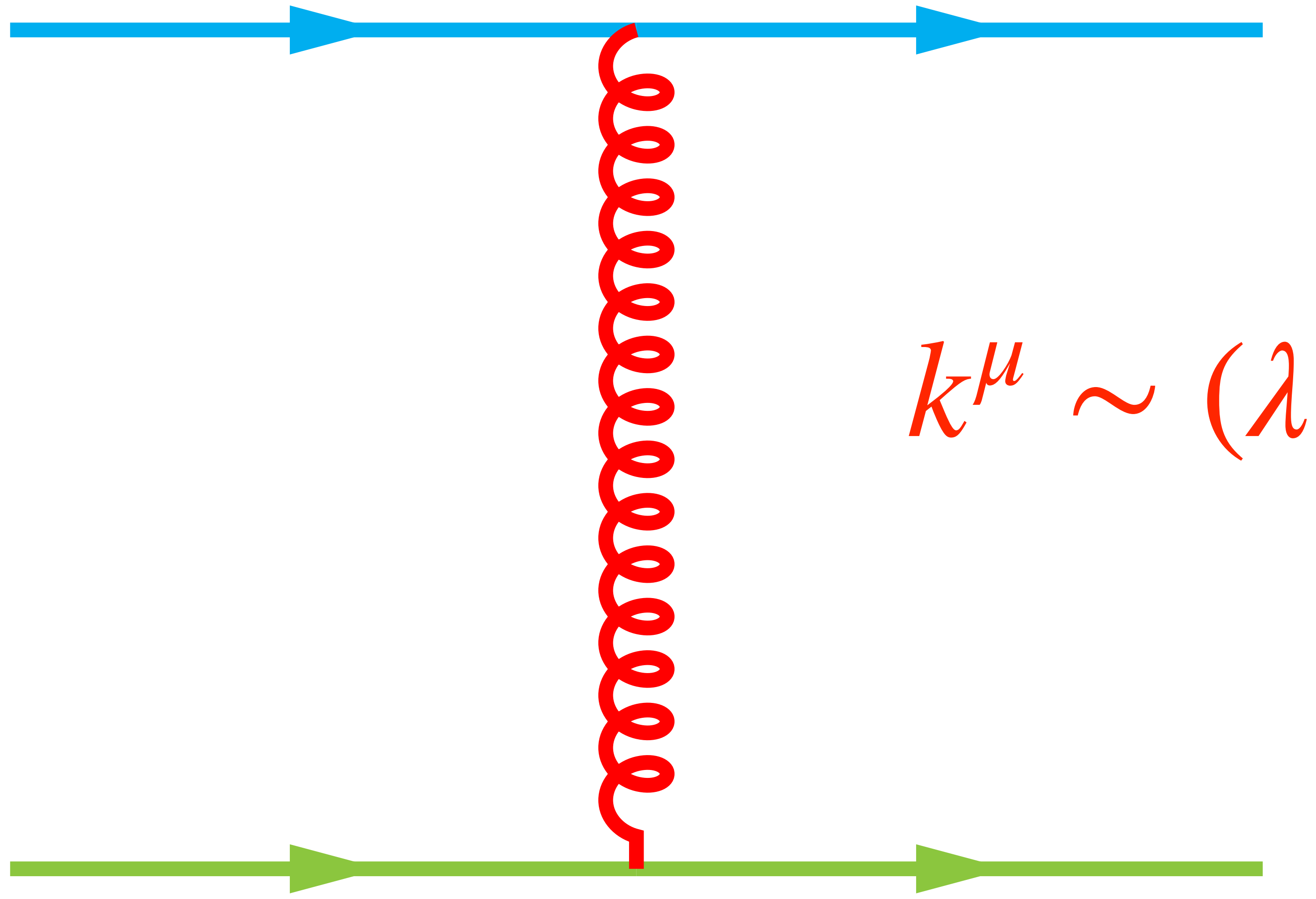
- Soft radiation is only sensitive to net charge of collinear partons
  - Only parent charge is relevant!
- Concept violated for space-like kinematics!

WTF are Glauber gluons?



Roy Glauber

Glauber Exchange



$p_c^\mu \sim (\lambda^2, 1, \lambda)$ collinear

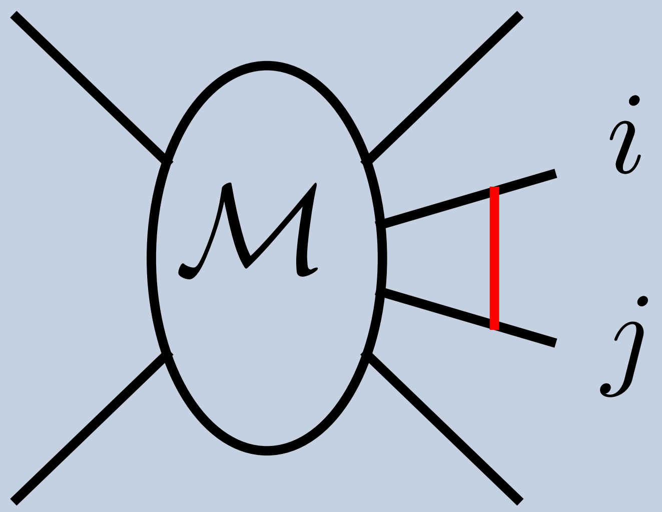
$k^\mu \sim (\lambda^2, \lambda^2, \lambda)$ Glauber

$p_{\bar{c}}^\mu \sim (1, \lambda^2, \lambda)$ anti-collinear

- Light-cone decomposition $p^\mu = (n \cdot p)\dfrac{\bar{n}^\mu}{2} + (\bar{n} \cdot p)\dfrac{n^\mu}{2} + p_\perp^\mu \equiv p_+^\mu + p_-^\mu + p_\perp^\mu$
- Expansion parameter $\lambda^2 \sim p_c^2/Q^2 \sim p_{\bar{c}}^2/Q^2$. Scaling $(p_+^\mu, p_-^\mu, p_\perp^\mu) \sim (\lambda^a, \lambda^b, \lambda^c)$

Glauber phases

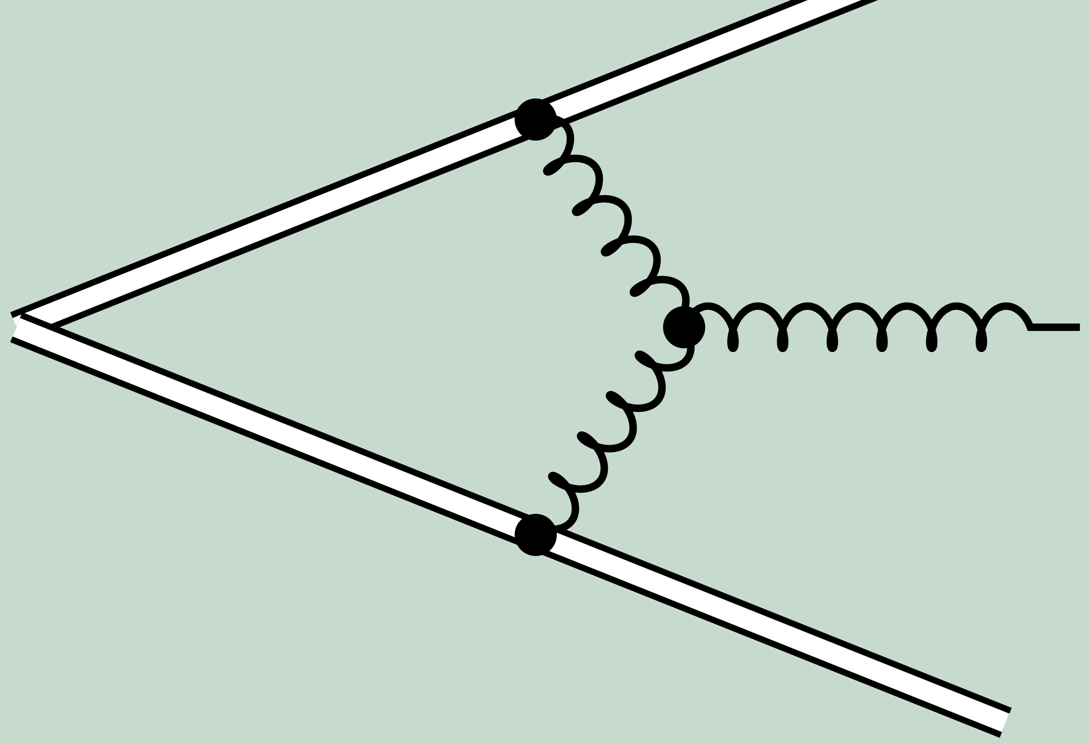
in IR divergences of hard amplitudes



$$\frac{2i\pi}{\epsilon}\mathbf{T}_i \cdot \mathbf{T}_j$$

Cheshire Glaubers*

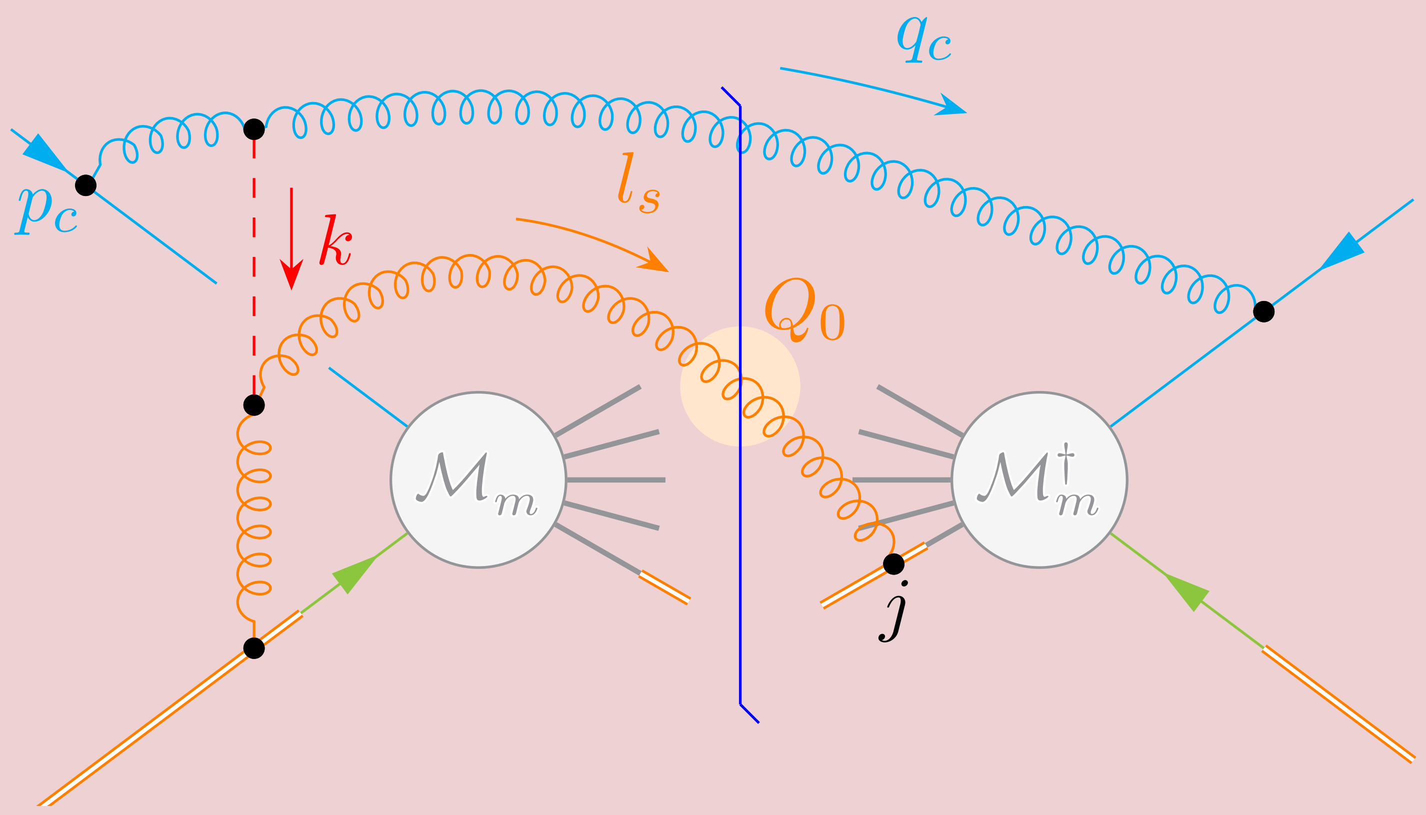
Glauber contributions in soft and collinear integrals in MoR
e.g. phase in 1-loop soft current



"You may have noticed that I'm not all there myself."

Genuine Glaubers

Exceptional (non-facet) momentum region
[mode vs. region]



* © Ira Rothstein and Iain Stewart

Glauber Computations in SCET

- Effective Glauber Lagrangian Stewart, Rothstein ‘16

- Subtract Glauber terms from soft and collinear integrals
  - Non-analytic regulator so that subtractions do not vanish
- Alternative:
  - Same Lagrangian, but strict Method of Regions. Leave soft and collinear unchanged, only genuine Glauber contributions

Outline

1) Super-leading logarithms (SLLs)
    - SLLs from RG evolution
    - PDF Factorization Restoration through Glauber Gluons
2) Collinear factorization violation in SCET
    - Collinear Wilson lines for space-like processes
3) Coherence-violating logarithms (CVLs) in *N*-jettiness

Super-Leading Logarithms

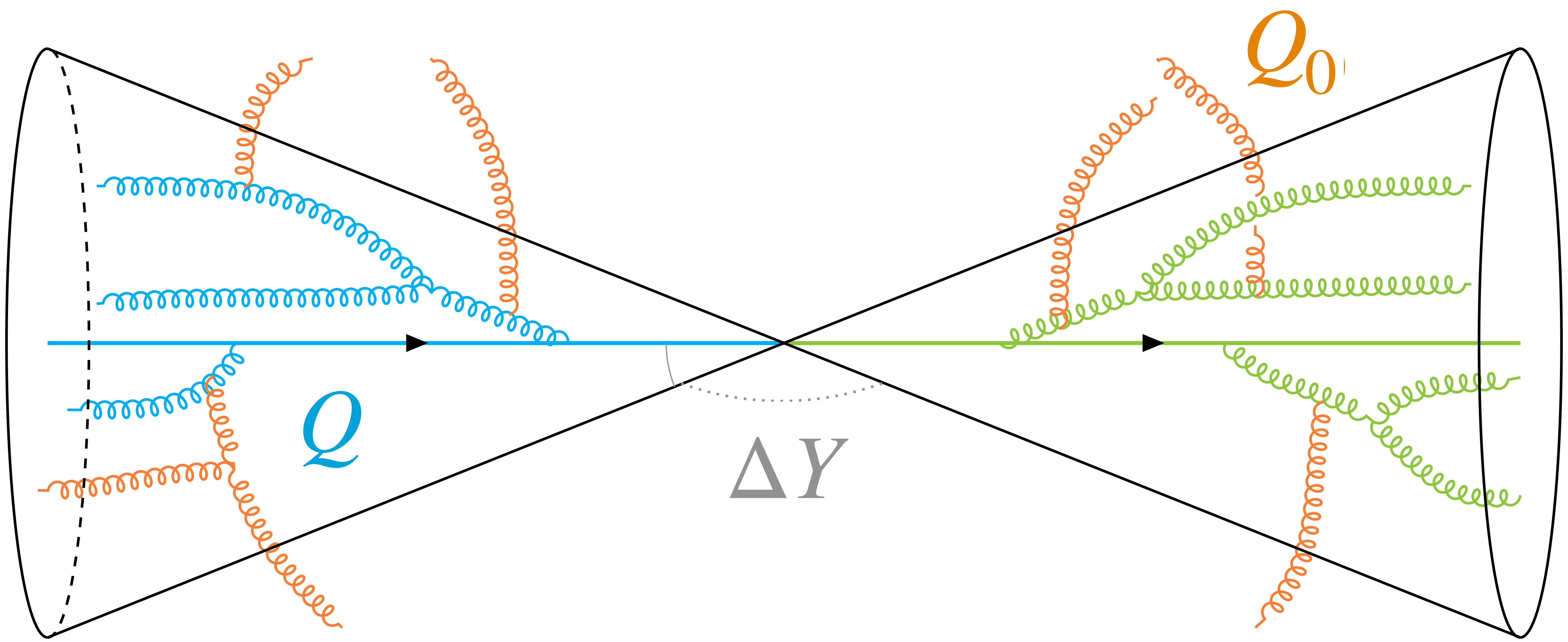
Factorization restoration through genuine Glauber contributions

TB, Hager, Jaskiewicz, Neubert, Schwienbacher

PRL 134 (2025) 6, 06190 and JHEP 01 (2025) 171

Gap-between jets

- Only soft radiation inside a gap of size ΔY between jets.
- Soft radiation veto scale $Q_0 \ll$ jet scale Q
- Prototypical non-global observable, non-global logarithms (NGLs)



Super-Leading Logarithms (SLLs)

- For $Q_0 \ll Q$ in case of hadron colliders one finds

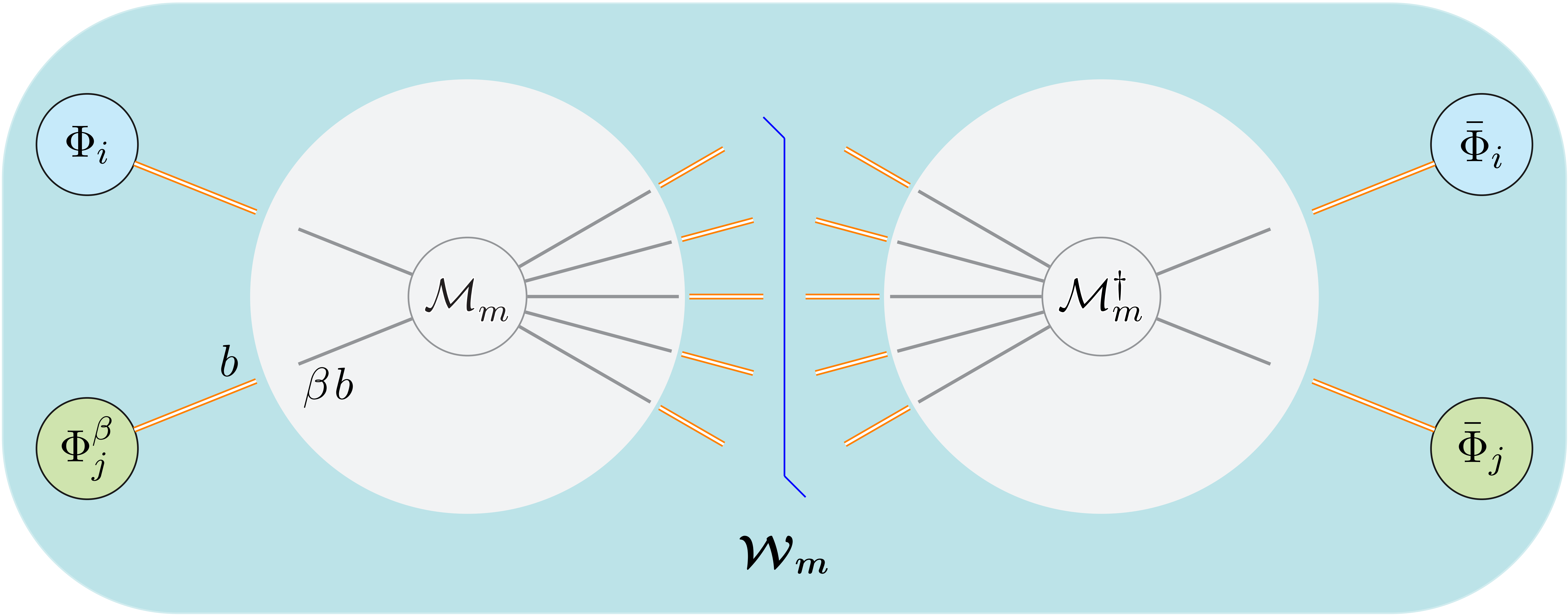
$$\sigma \sim \sigma_B(1 + \alpha_s \ln(Q/Q_0) + \alpha_s^2 \ln^2(Q/Q_0) + \ldots + \alpha_s^4 \ln^5(Q/Q_0) + \ldots)$$

Forshaw, Kyrieleis, Seymour '06 '08

- Non-cancellation of collinear effects due to Glauber phases
- Formally leading logarithmic effect but
- Suppressed in color & loop order
- Numerical impact is significant

SLLs are directly connected to factorization violation!

Factorization Theorem



For $Q_0 \ll Q$ we can derive

$$\sigma_{2\to M}(Q_0) = \int dx_1 \int dx_2 \sum_{m=2+M}^{\infty} \langle \mathcal{H}_m(\{\underline{n}\}, s, x_1, x_2, \mu) \otimes \mathcal{W}_m(\{\underline{n}\}, Q_0, x_1, x_2, \mu) \rangle$$

- Factorization between hard and soft-collinear physics

RG evolution

- Renormalized hard functions fulfill RG equation

$$\frac{d}{d\ln\mu}\mathcal{H}_m = -\sum_{l=m_0}^{m}\mathcal{H}_l\,\boldsymbol{\Gamma}^H_{lm}$$

matrix in multiplicity and color space

Virtual terms $\propto \delta_{lm}$
Real emissions $\propto \delta_{l(m+1)}$

- One-loop hard anomalous dimension:

$$\boldsymbol{\Gamma}^H = \gamma_{cusp}(\alpha_s)\left(\boldsymbol{\Gamma}^c \ln\frac{\mu^2}{Q^2} + \boldsymbol{V}^G\right) + \frac{\alpha_s}{4\pi}\overline{\boldsymbol{\Gamma}} + \boldsymbol{\Gamma}^C$$

cusp-piece soft+collinear (generates SLLs)

$\propto i\pi$ Glauber-phase

purely soft (generates NGLs)

purely collinear (color-aware DGLAP)

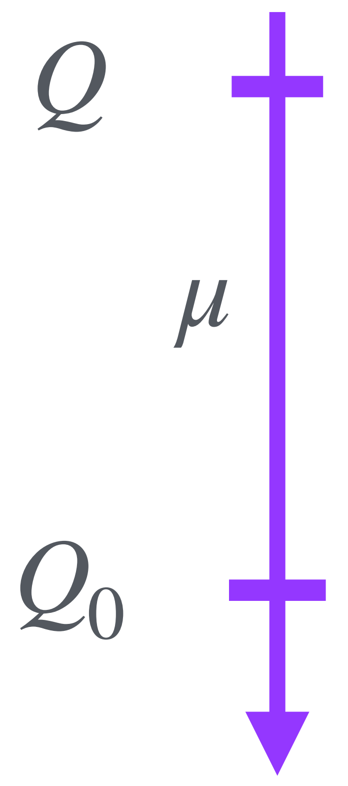
SLLs from RG evolution

Evolve hard function from $\mu_h \sim Q$ to $\mu_s \sim Q_0$

$$\sigma(Q, Q_0) = \sum_{m,l=m_0}^{\infty} \int d\xi_1 d\xi_2 \, \langle \mathcal{H}_m(Q, \mu_h) U_{ml}(\mu_h, \mu_s) \otimes \mathcal{W}_l(Q_0, \mu_s) \rangle$$

$$U(\mu_h, \mu_s) = \boldsymbol{P} \exp\left[\int_{\mu_s}^{\mu_h} \frac{d\mu}{\mu} \boldsymbol{\Gamma}^H \right]$$

$$= \boldsymbol{1} + \int_{\mu_s}^{\mu_h} \frac{d\mu_1}{\mu_1} \boldsymbol{\Gamma}^H + \int_{\mu_s}^{\mu_h} \frac{d\mu_1}{\mu_1} \int_{\mu_1}^{\mu_h} \frac{d\mu_2}{\mu_2} \boldsymbol{\Gamma}^H(\mu_1)\, \boldsymbol{\Gamma}^H(\mu_2)$$



Evolution $U(\mu_h, \mu_s)$ achieves resummation of logarithms

Resummation of SLLs

Reduce products of $\boldsymbol{\Gamma}_H$ using commutation relations and other properties:

$$\langle \ldots \boldsymbol{V}^G \rangle = \langle \ldots \boldsymbol{\Gamma}^c \rangle = 0 \qquad [\boldsymbol{\Gamma}^c, \overline{\boldsymbol{\Gamma}}] = 0 \qquad [\boldsymbol{\Gamma}^c, \boldsymbol{V}^G] \neq 0$$

Color coherence

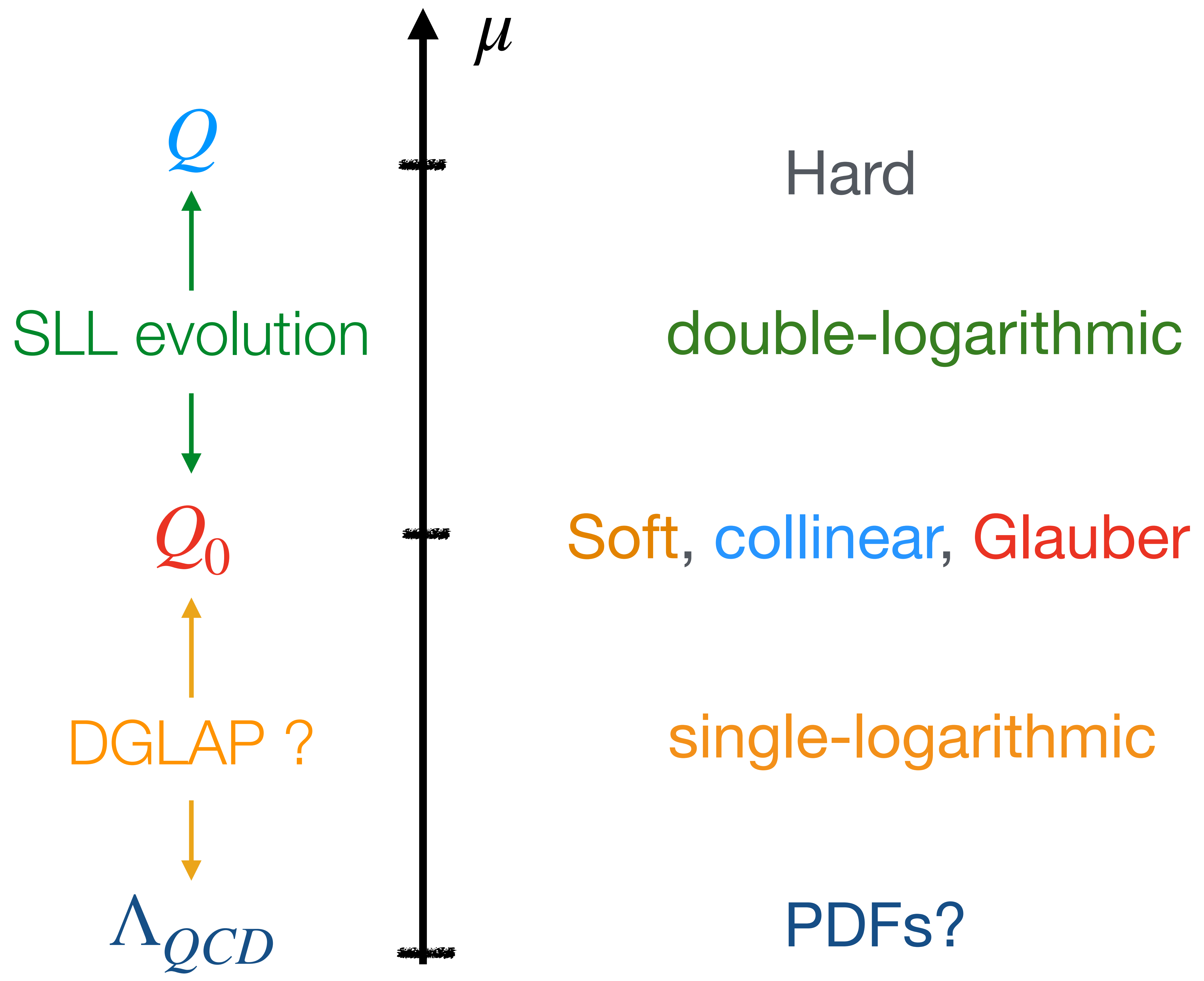
Coherence violation through Glauber phase

Leaves a small set of structures to be evaluated:

$$\sigma_{\rm SLL} \sim \langle \mathscr{H} \left(\boldsymbol{\Gamma}^c\right)^{n-r} \boldsymbol{V}^G \left(\boldsymbol{\Gamma}^c\right)^{r} \boldsymbol{V}^G \overline{\boldsymbol{\Gamma}} \rangle$$

Map products of $\boldsymbol{\Gamma}'s$ to a small basis of color structures, evaluate, and sum series. TB, Neubert, Shao, '21 + Stillger '23, Böer, Hager, Neubert, Stillger, Xu '23, '24

Scale hierarchy



Requires highly non-trivial interplay for consistency with DGLAP!

Consistency check

- *Assume* PDF factorization

$$\mathscr{W}_m(Q_0,\mu) = \mathscr{I}_m(Q_0,\mu) * f_1(\mu) f_2(\mu)$$

perturbative matching

- Know anomalous dimensions of both $\mathscr{W}_m$ and $f_i(x,\mu)$ $\rightarrow$ predict $1/\epsilon$ poles in $\mathscr{I}_m^{\text{bare}}$

$$\mathscr{I}_m^{\text{bare}} = \mathbf{1} + \frac{\alpha_s}{4\pi}\frac{\overline{\boldsymbol{\Gamma}}}{2\varepsilon} + \left(\frac{\alpha_s}{4\pi}\right)^2\left(\frac{\boldsymbol{V}^G\overline{\boldsymbol{\Gamma}}}{2\varepsilon^2} + \ldots\right) + \left(\frac{\alpha_s}{4\pi}\right)^3\left[\frac{\boldsymbol{\Gamma}^c\boldsymbol{V}^G\overline{\boldsymbol{\Gamma}}}{3\varepsilon^3}\left(\frac{11}{6\varepsilon} + \ln\frac{\mu_s^2}{Q^2} + \frac{9}{2}\ln\frac{\mu_s^2}{Q_0^2}\right) + \frac{\boldsymbol{V}^G\boldsymbol{V}^G\overline{\boldsymbol{\Gamma}}}{3\varepsilon^3} + \frac{\left[\boldsymbol{\Gamma}^C,\boldsymbol{V}^G\right]\overline{\boldsymbol{\Gamma}}}{12\varepsilon^3} + \ldots\right]$$

- Verify prediction for poles order by order, by computing partonic matrix elements $\mathscr{I}_m^{\text{bare}}$

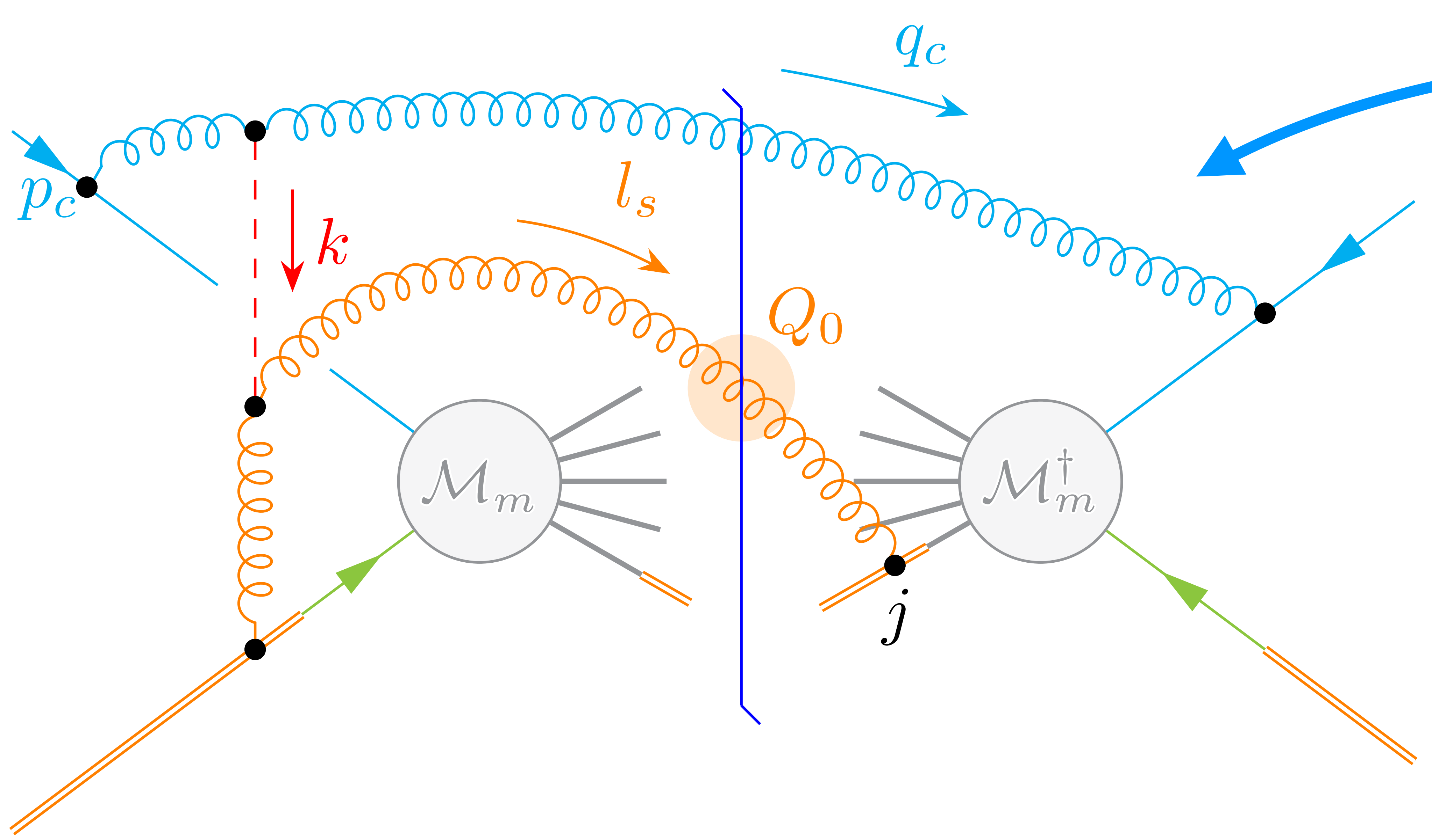
Order-by-order check

$$\mathcal{I}_m^{\text{bare}} = 1 + \frac{\alpha_s}{4\pi}\frac{\overline{\Gamma}}{2\varepsilon} + \left(\frac{\alpha_s}{4\pi}\right)^2\left(\frac{V^G\,\overline{\Gamma}}{2\varepsilon^2} + \ldots\right) + \left(\frac{\alpha_s}{4\pi}\right)^3\left[\frac{\Gamma^c V^G\,\overline{\Gamma}}{3\varepsilon^3}\left(\frac{11}{6\varepsilon} + \ln\frac{\mu_s^2}{Q^2} + \frac{9}{2}\ln\frac{\mu_s^2}{Q_0^2}\right) + \frac{V^G V^G\,\overline{\Gamma}}{3\varepsilon^3} + \frac{\left[\Gamma^C, V^G\right]\overline{\Gamma}}{12\varepsilon^3} + \ldots\right]$$

✓ ✓ ✓

- Up to two loops, all terms are purely soft contributions, obtained by taking matrix elements of soft currents Catani, Grazzini ’00, Duhr, Gehrmann ’13, Dixon, Herrmann, Yan, Zhu ’20
- Purely collinear contributions to $\mathcal{I}_m^{\text{bare}}$ vanish because they are independent of Q_0
- Remaining terms involve large scale Q and dependence on momentum fractions. Needs soft-collinear interaction! Different options
  - soft-collinear messenger mode at lower scale Q_0^2/Q ✗
  - Genuine Glauber mode at scale Q_0 + collinear anomaly with rapidity logarithm $\ln(Q)$

Genuine Glauber Diagram



q_c integral not well-defined in dim. reg. alone

Introduce rapidity regulator $\left(\frac{\nu}{q_{c-}}\right)^{\alpha}$

$$\mathscr{I}_m^{\text{bare}} \ni \frac{i\alpha_s^3}{12\pi^2\varepsilon^3}\left(\frac{\mu_s^2}{Q_0^2}\right)^{3\varepsilon} f^{abc}f^{ade}\sum_{j>2} J_j\left[\boldsymbol{T}_{2L}^d\boldsymbol{T}_{2R}^e\boldsymbol{T}_{1L}^b\boldsymbol{T}_{jR}^c\left(\frac{1}{\alpha}+\ln\frac{\nu\bar{p}_{\bar{c}}^+}{Q_0^2}-\frac{11}{6\varepsilon}+\frac{\overline{\mathscr{P}}_{q\to q}}{2}+\ldots\right)+\boldsymbol{T}_{1L}^d\boldsymbol{T}_{1R}^e\boldsymbol{T}_{2L}^b\boldsymbol{T}_{jR}^c\left(-\frac{1}{\alpha}-\ln\frac{\nu}{p_c^-}+\frac{\overline{\mathscr{P}}_{q\to q}}{2}+\ldots\right)\right]$$

- Poles in rapidity regulator α cancel between collinear and anti-collinear sectors, but leave behind $\ln(Q)$

Discussion

- Glauber loop maps onto pentagon diagram. Have verified MoR result against known full results for scalar pentagon integrals
  - Genuine Glauber loop is well defined in dimensional regularization
- Performed the consistency check for all partonic channels and also for off-diagonal splittings
- With the Glauber contribution at three-loop order, the low energy matrix $\mathscr{W}_m$ no longer obeys soft-collinear factorization. The Glauber contributions
  - transmit soft scale Q_0 into collinear sector
  - induce nontrivial spin-dependence of the form $n_{j\perp}^{\mu}\, n_{j\perp}^{\nu}$ in $\mathscr{W}_m$ for subleading poles, where n_j is the direction of soft Wilson line associated with final state parton

Factorization Restoration by Glauber Gluons

Coherence violation and collinear factorization breaking at $\mu = Q$	x	soft-collinear factorization breaking by Glauber modes at $\mu = Q_0$	=	PDF factorization for $\mu < Q_0$

- Remarkable, intricate mechanism!
- Consistent with intuitive picture that scattering becomes inclusive for $\mu < Q_0$ and that then arguments by Collins, Soper and Sterman ’86 should become applicable

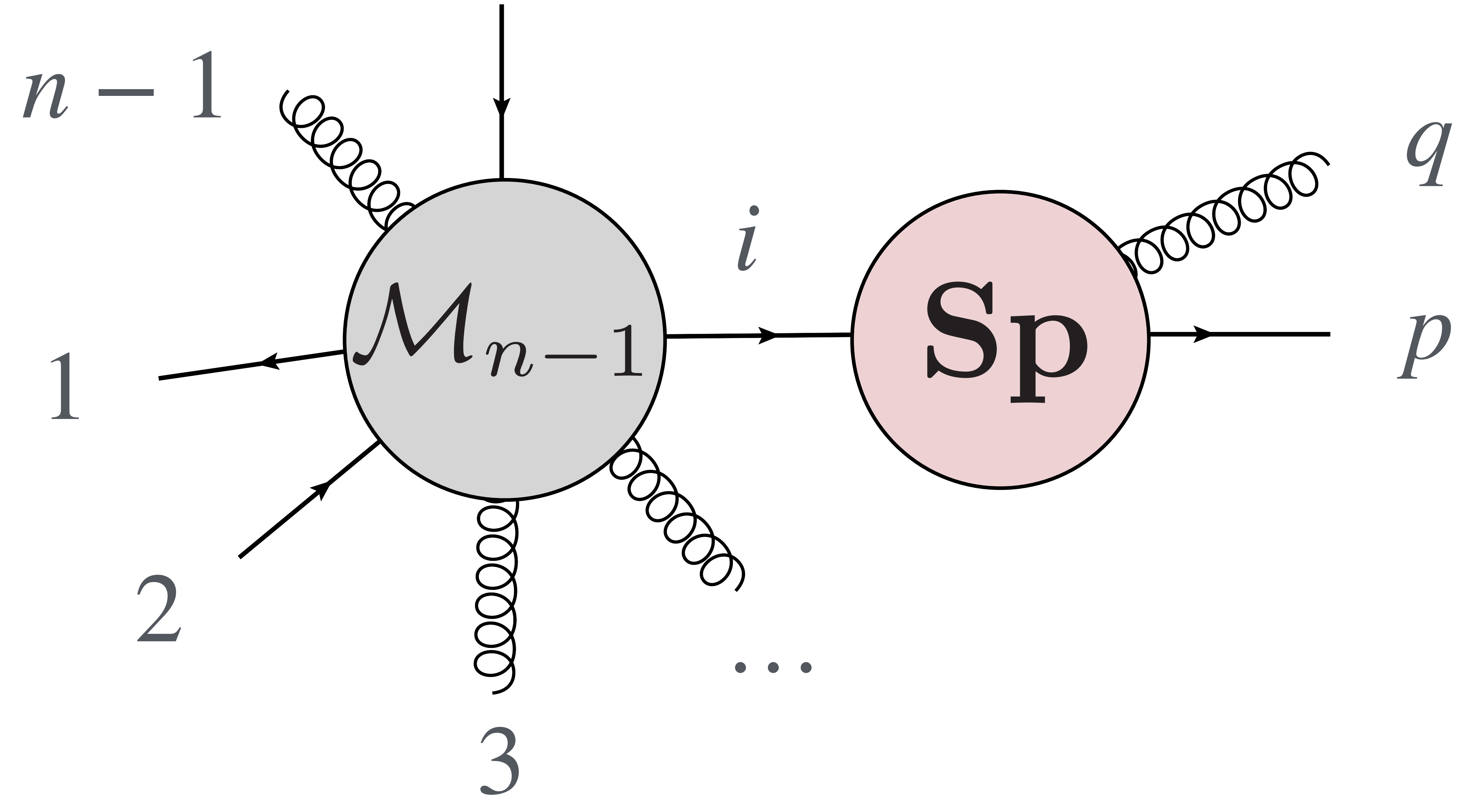
Collinear Factorization Breaking

Cheshire Glauber contributions, sensitive to $i\varepsilon$ of eikonal collinear propagators

TB, Hager, Neubert, Schwienbacher

End Matter of 2603.12383

Splitting amplitudes in SCET



- In SCET, the splitting amplitude is obtained from a matrix element of a collinear field, e.g.

$$\mathrm{Sp}(p_c\,, q_c) = \langle p_c\,, q_c \,|\, \chi_i(0) \,|\, 0 \rangle$$

Collinear Wilson line

Collinear quark field in SCET involves a Wilson line

$$\chi_i(x) = \boldsymbol{W}_i^\dagger(x)\, q_i(x)$$

This Wilson line arises as remnant of attachments of the i-collinear gluon to the hard partons in all other directions

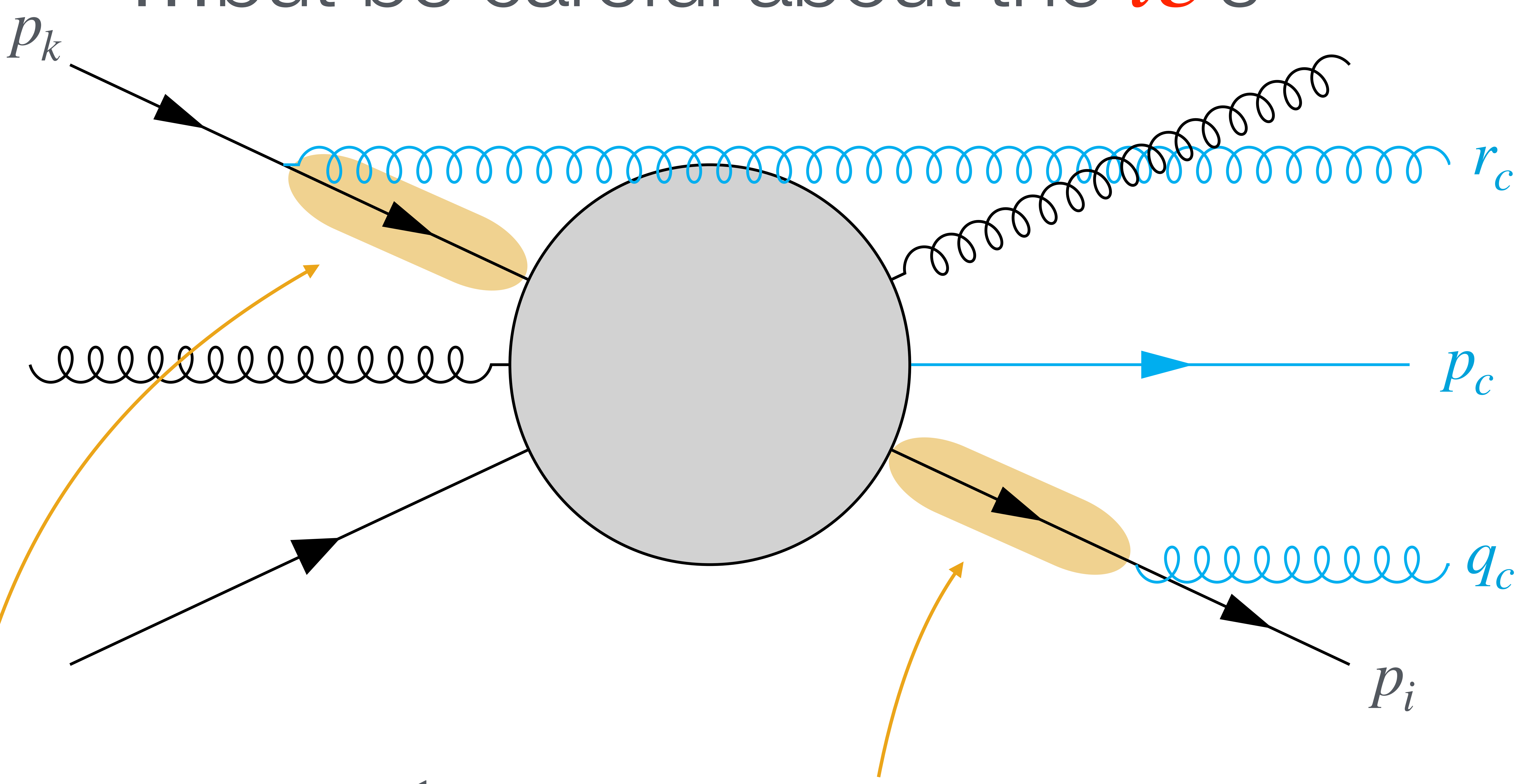
$$\boldsymbol{W}_i(x) = \mathbf{P}\exp\left[i g_s \int_{-\infty}^{0} ds\, \bar{n}_i \cdot A_i^a(x + s\bar{n}_i)\, \mathbf{T}_i^a \right]$$

treat all other partons as incoming

by color conservation

$$\mathbf{T}_i^a = -\sum_{j\neq i} \mathbf{T}_j^a$$

…but be careful about the $i\varepsilon$'s



$$\frac{1}{(p_k - r_c)^2 + i\varepsilon} \to -\frac{1}{\bar{n}\cdot r_c - i\varepsilon}$$

$$\frac{1}{(p_i + q_c)^2 + i\varepsilon} \to \frac{1}{\bar{n}\cdot q_c + i\varepsilon}$$

→ incoming Wilson line

→ outgoing Wilson line

Collinear Wilson lines revisited

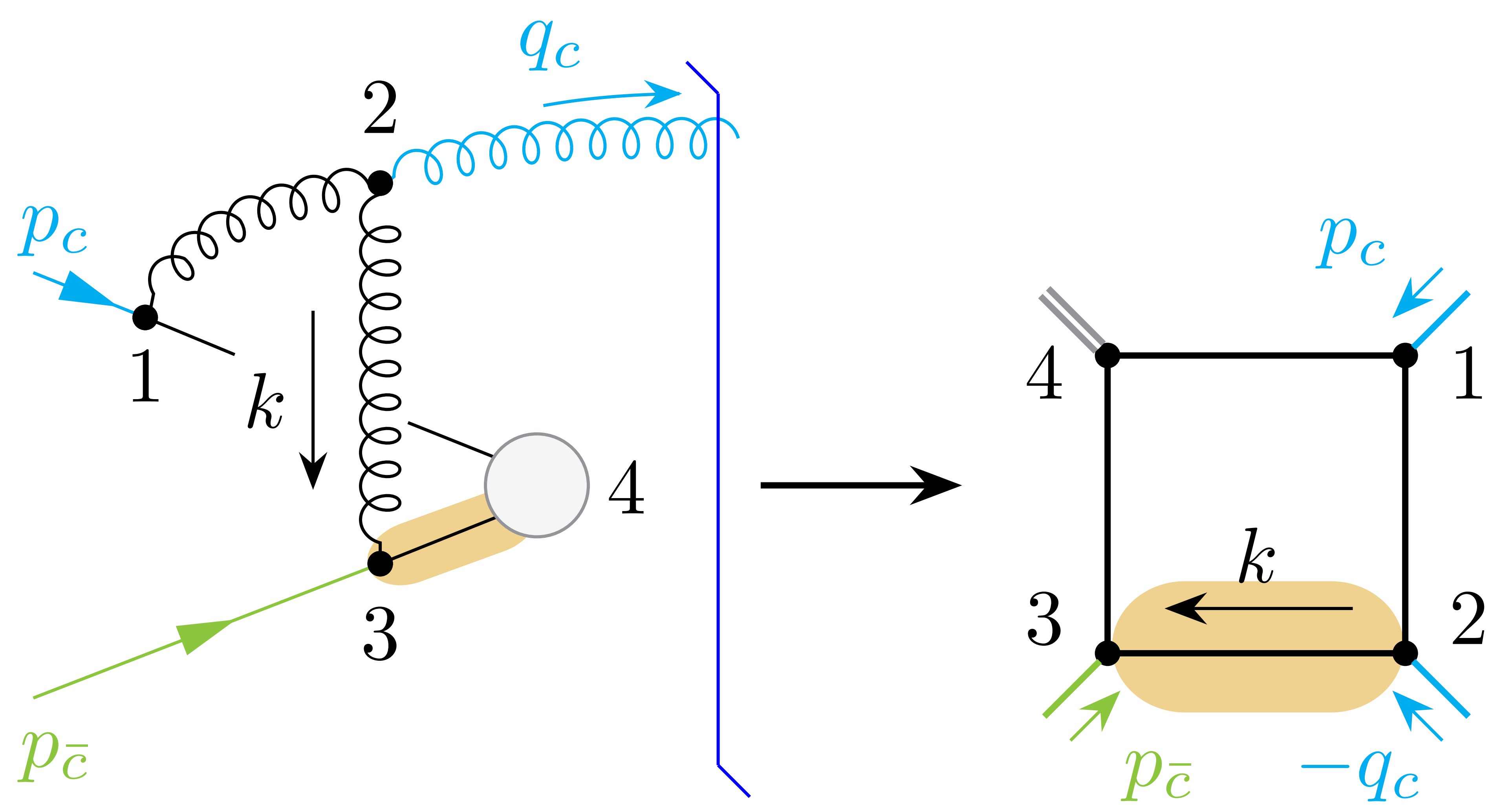
However, in processes with incoming and outgoing hard partons, MoR expansion leads to products of incoming and outgoing Wilson lines $\boldsymbol{W}^{+}$ and $\boldsymbol{W}^{-}$

$$\boldsymbol{W}_i(x) = \underbrace{\prod_{k \neq i} \boldsymbol{W}^{+}_{i,k}(x)}_{\text{incoming}} \underbrace{\prod_{l \neq i} \boldsymbol{W}^{-}_{i,l}(x)}_{\text{outgoing}}$$

To replace all Wilson lines with incoming ones one argues that the $i\varepsilon$ prescription in the eikonal propagators is irrelevant, since the propagator involves the large component of the collinear momentum.

Incorrect for space-like splittings due to Cheshire Glaubers!

Collinear Factorization Breaking



- Result for the collinear loop depends on sign of $i\varepsilon$ of the eikonal propagator
- Computing these collinear diagrams reproduces factorization-breaking terms in one-loop splitting amplitude
- Alternative: SCET computation with non-analytic Glauber regulator Schwartz, Yan, Zhu ‘17

Coherence Violation in N-Jettiness

Glauber contributions
affect even simple, global observables!

TB, Hager, Neubert, Schwienbacher 2603.12383

N-jettiness

Stewart, Tackmann, Waalewijn '10

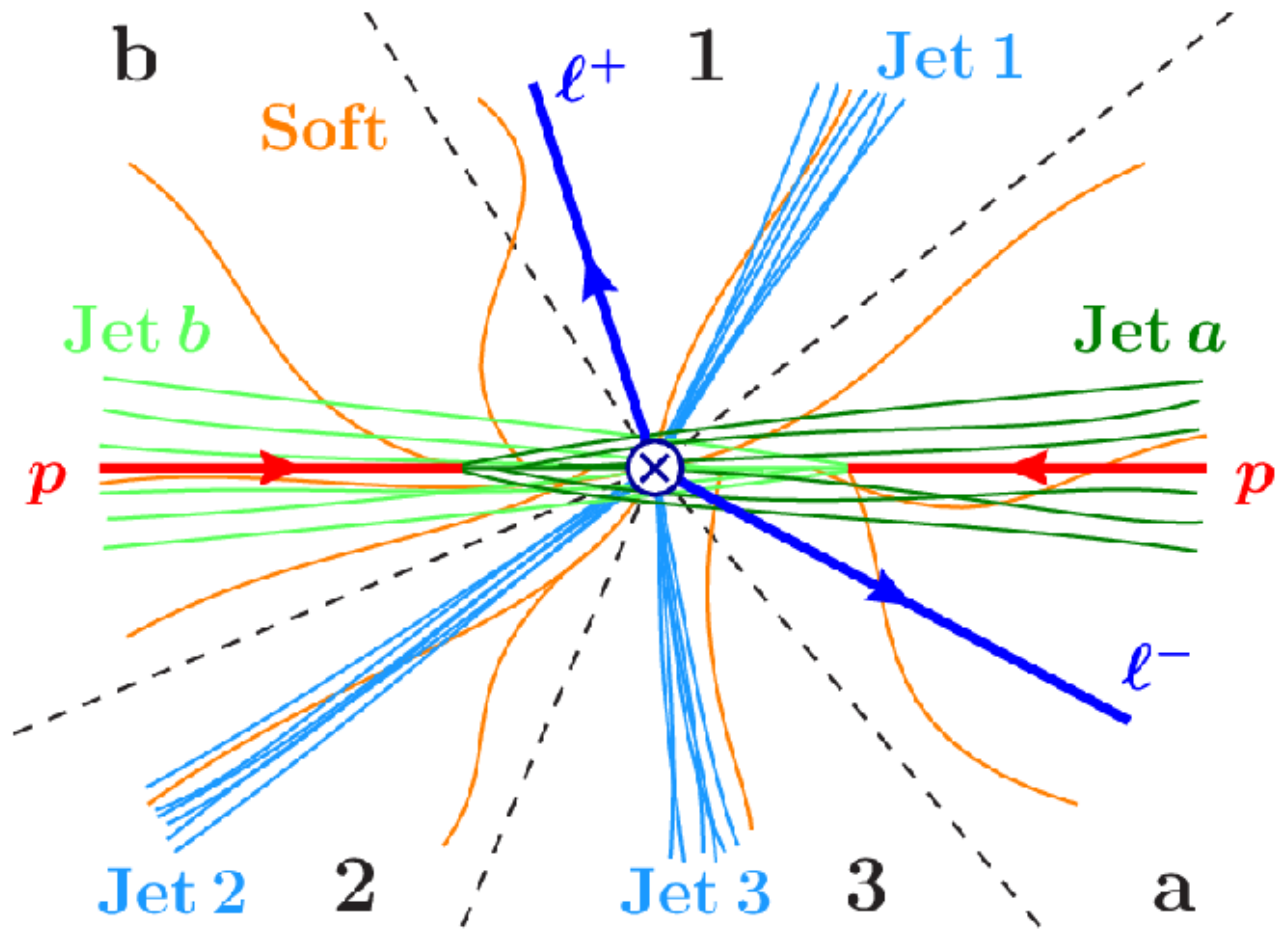
N-jettiness variable

jet directions n_i

$$\mathcal{T}_N = 2 \sum_k \min \{ n_a \cdot p_k, n_b \cdot p_k, n_1 \cdot p_k, \ldots, n_N \cdot p_k \}$$

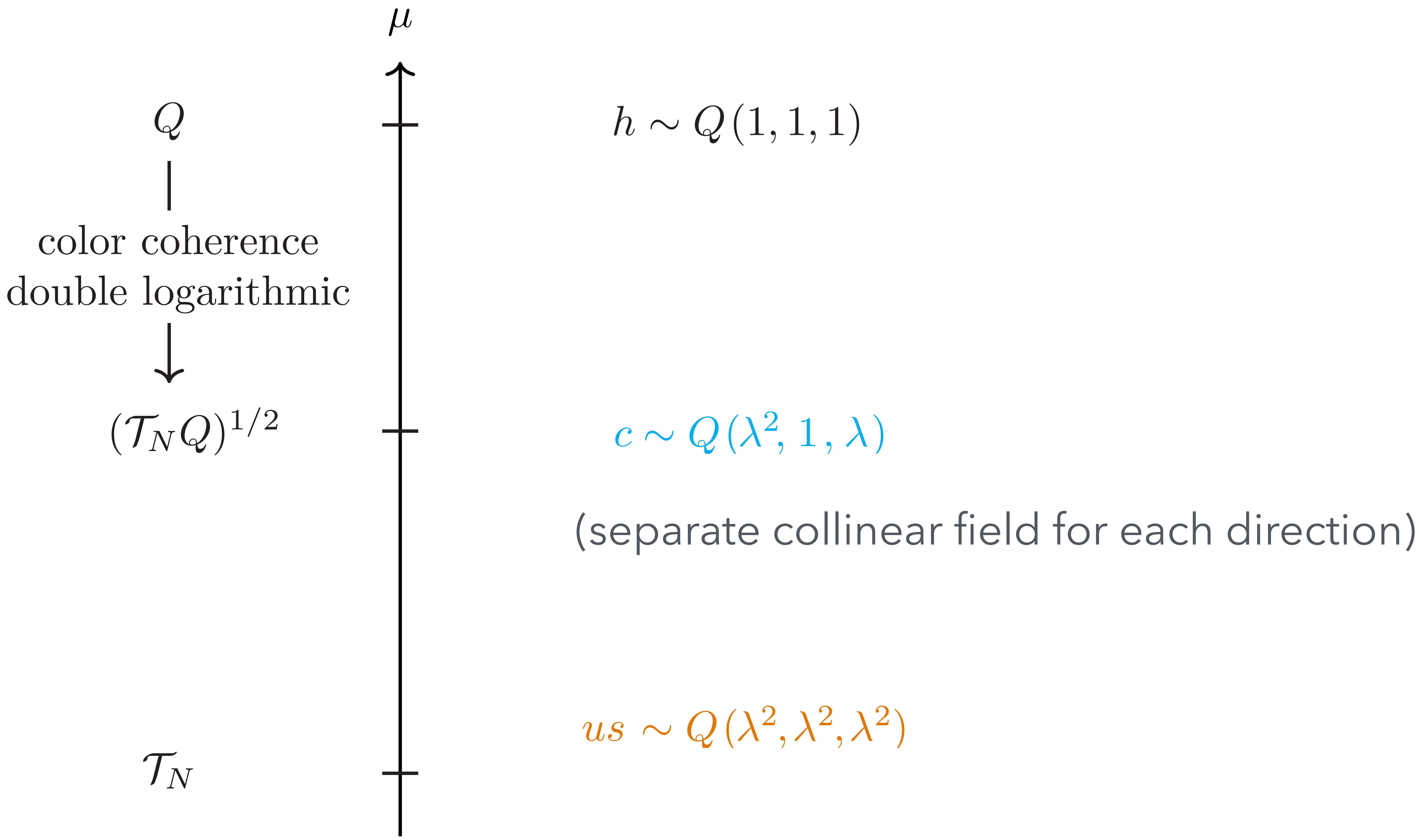
Beam directions n_a and n_b

Sum over emissions k



from Stewart, Tackmann, Waalewijn '10

Scale hierarchy for N-jettiness



(Naive) factorization theorem for N-jettiness

Jet functions J_i (color diagonal)

Soft fuctions

$$\frac{d\sigma}{d\mathcal{T}_N} = \left\langle \boldsymbol{\mathcal{H}}_N \otimes B_a \otimes B_b \otimes \left[\prod_{i=1}^{N} J_i\right] \otimes \boldsymbol{S}_N \right\rangle$$

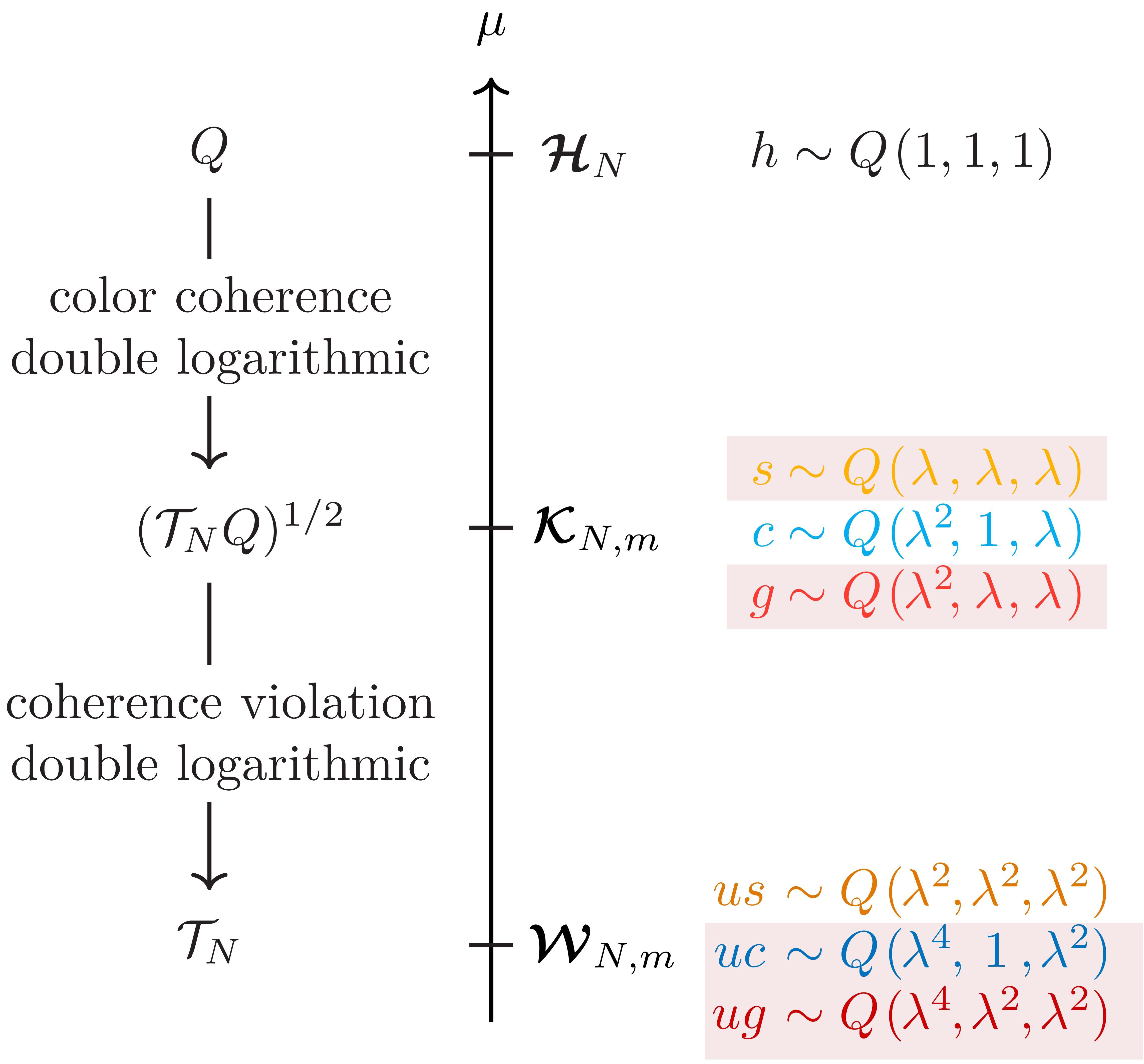
Hard functions

Beam functions $B_{a/b}$ (color diagonal)

Convolutions

Color trace

Glauber modes in N-jettiness



Argued to arise in

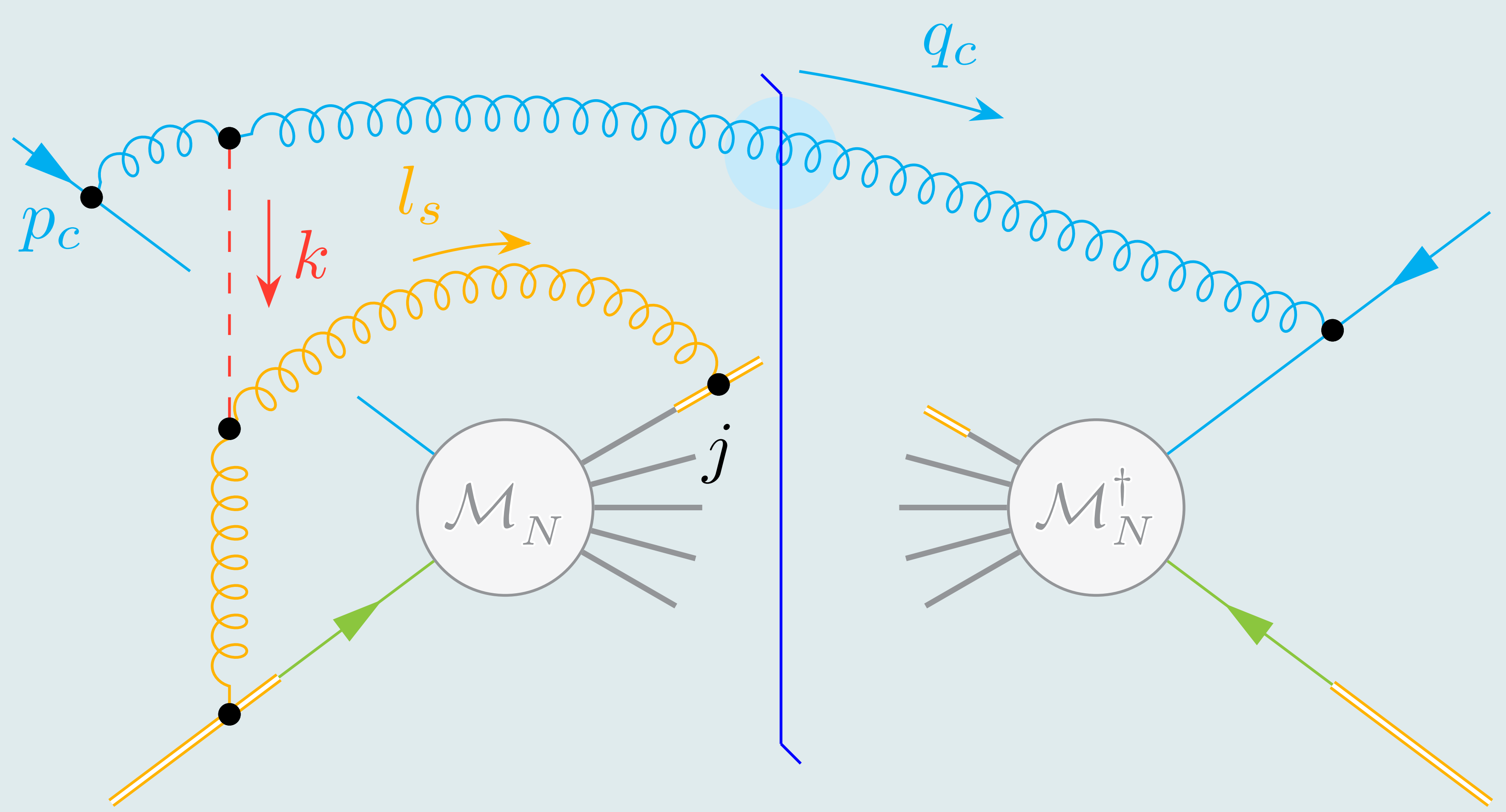
Banfi, Salam, Zanderighi '10

Forshaw, Holguin '21

New modes at 3 loops

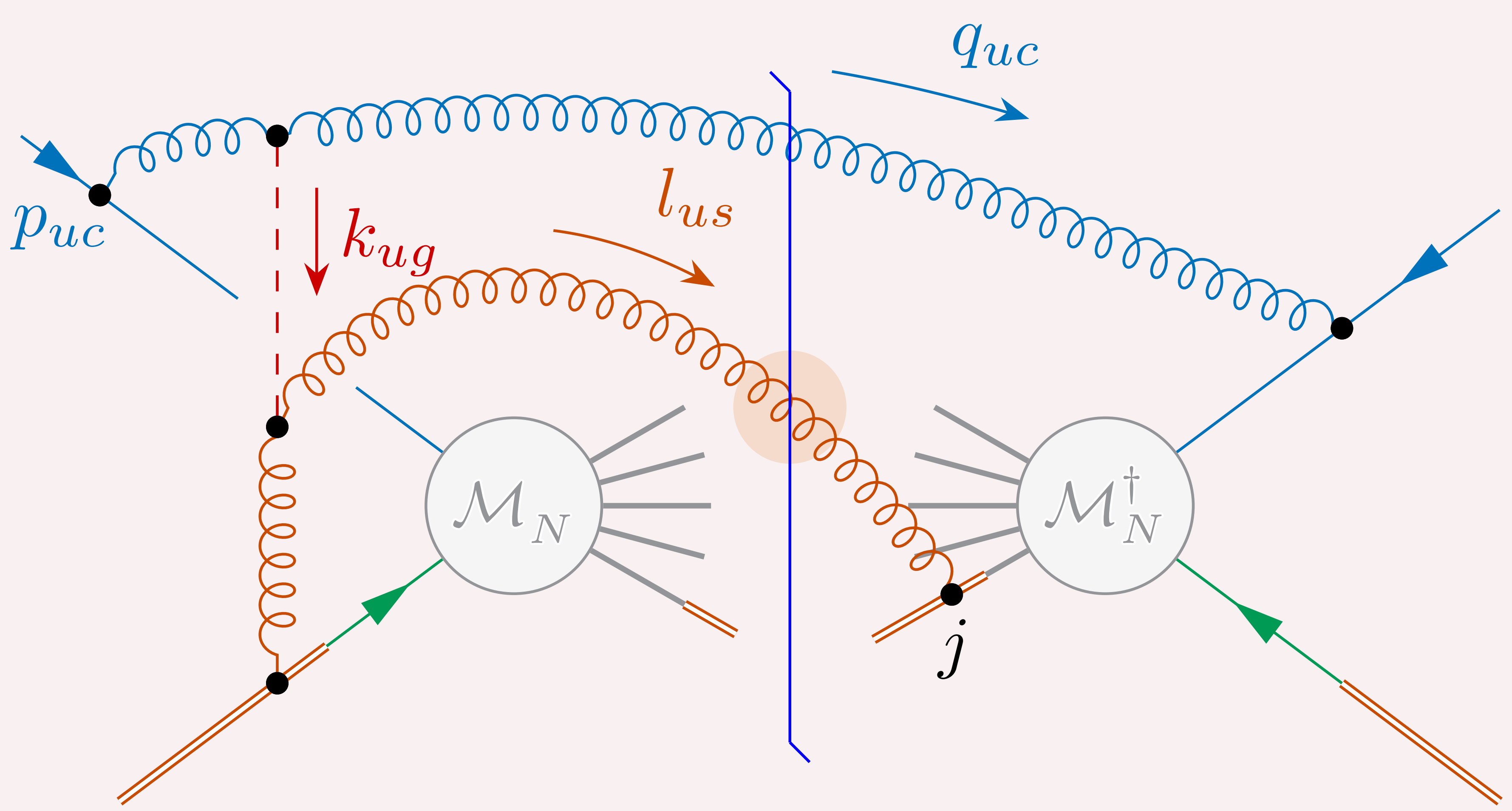
Glauber diagrams

Collinear scale $(Q\mathscr{T}_N)^{1/2}$



- Soft gluons $s \sim (\lambda, \lambda, \lambda)$ cannot be emitted, and purely virtual diagrams are scaleless
- Genuine Glauber $g \sim (\lambda^2, \lambda, \lambda)$ transmits scale from collinear to soft sector

Ultrasoft scale $\mathscr{T}_N$



- Ultra-collinear mode $uc \sim (\lambda^4, 1, \lambda^2)$ does not contribute to jettiness; purely uc diagrams are scaleless
- Ultra-Glauber $g \sim (\lambda^4, \lambda^2, \lambda^2)$ transmits us scale into the ultra-collinear sector

Factorization theorem for N-jettiness

Hard functions (same as before)

Low-energy matrix element contains ultra-soft Wilson lines, ultra-collinear modes and ultra Glaubers

$$\frac{d\sigma}{d\omega} = \sum_m \left\langle \mathcal{H}_N(Q,\mu)\, \widetilde{\mathcal{K}}_{N,m}(\sqrt{Q\omega},\mu)\, \widetilde{\mathcal{W}}_{N,m}(\omega,\mu) \right\rangle$$

Laplace-space $\mathcal{T}_N \leftrightarrow \omega$

Jet and Beam functions no longer factorize! Contain soft and Glauber modes! (also **not** color diagonal)

Color trace

Low-energy matrix element resolves colors of the individual $N + m$ partons!

Resummation

- Extract anomalous dimension of low-energy matrix elements via gluon mass
- Cancellation of gluon mass between different sectors leaves large logarithms

$$\Gamma^{\mathscr{W}}_{\text{virt}} \ni -\frac{\alpha_s}{2\pi}\sum_{i \nparallel j}\left(\boldsymbol{T}_i^L \cdot \boldsymbol{T}_j^L + \boldsymbol{T}_i^R \cdot \boldsymbol{T}_j^R\right)\ln\frac{s_{ij}}{\mu^2} - \frac{\alpha_s}{\pi}\boldsymbol{V}^G \qquad \Gamma^{\mathscr{W}}_{\text{real}} \ni \frac{\alpha_s}{\pi}\sum_{i \nparallel j}\boldsymbol{T}_i^L \circ \boldsymbol{T}_j^R\left(\ln\frac{s_{ij}}{\mu^2} - \ln\frac{\mu^2}{\omega^2} - \ln\hat{s}_{ij}\right)$$

- Leading CVL agrees with recent result of Banfi, Forshaw, Holguin '25

$$\sigma_{\text{CVL}} \sim \left\langle \mathscr{H}_1^{\text{Born}}\, \Gamma^{\mathscr{W}}_{\text{real}}(\mu_1)\, \boldsymbol{V}^G\, \boldsymbol{V}^G\, \Gamma^{\mathscr{W}}(\mu_4) \right\rangle$$

- And the general scaling is

$$d\sigma_{\text{CVL}}/d\omega \sim \alpha_s^{2+n}\ln^{2+2n}(Q/\omega) \qquad n > 2$$

Conclusion

- Factorization at hadron colliders is much richer than in e^+e^- collisions!
- Soft-collinear factorization is broken by Glauber gluons
  - Jet and soft functions $\longrightarrow$ soft+collinear matrix elements
  - Richer mode structure: Glauber transmits scales between soft and collinear
  - Coherence violation: sensitivity to colors of individual partons
  - Interplay of different Glauber effects: Genuine, Cheshire, hard phases

Conclusion

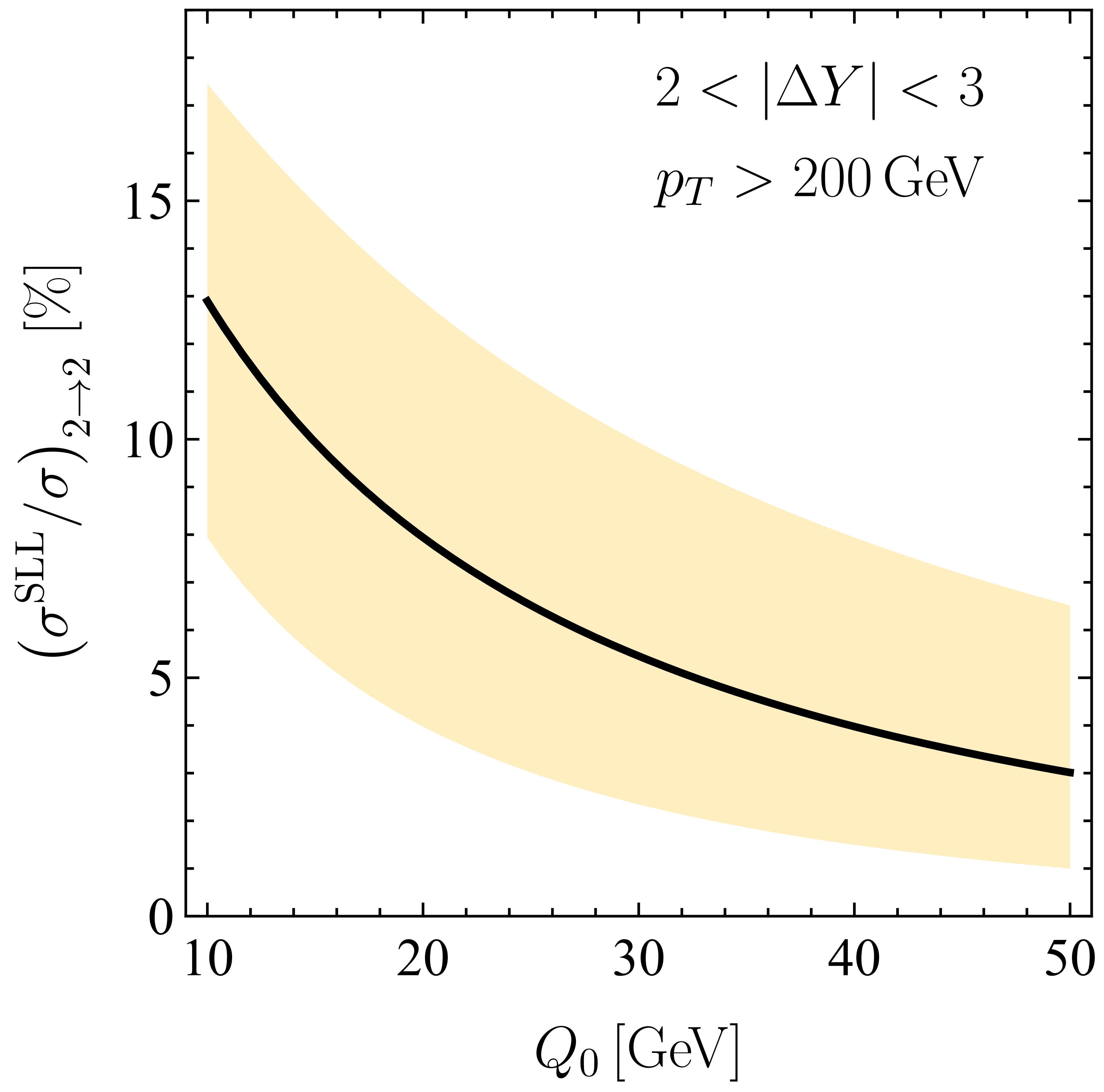
- Soft-Collinear Effective Theory provides a systematic framework for
  - factorization (= scale separation) theorems
  - resummation of CVLs and SLLs by RG methods
- Several interesting results
  - All-order resummations of leading SLLs
  - Jet cross sections: Genuine Glaubers gluons restore PDF factorization!
- Phenomenological relevance TBD
  - sizable effects for SLLs, while leading CVL in N-jettiness is small

A lot of interesting physics remains to be explored!

Extra slides

Phenomenological relevance

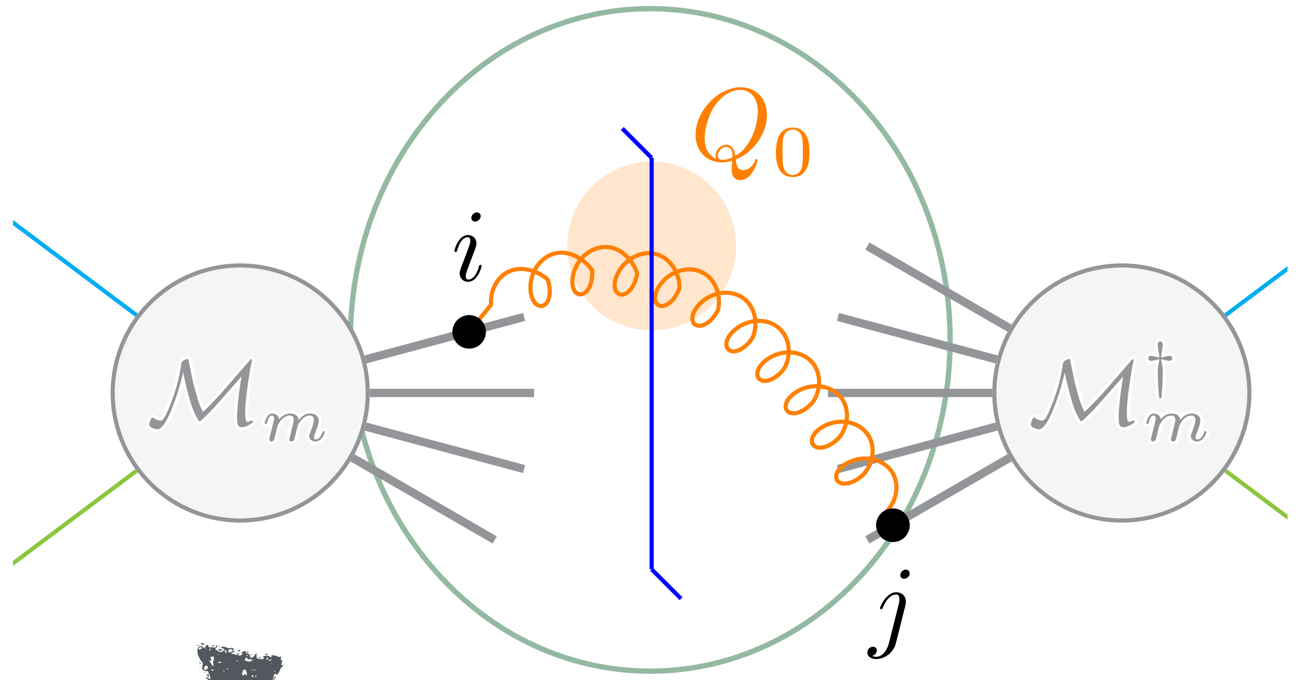
Becher, Hager, Martinelli, Neubert, DS, Stillger '25



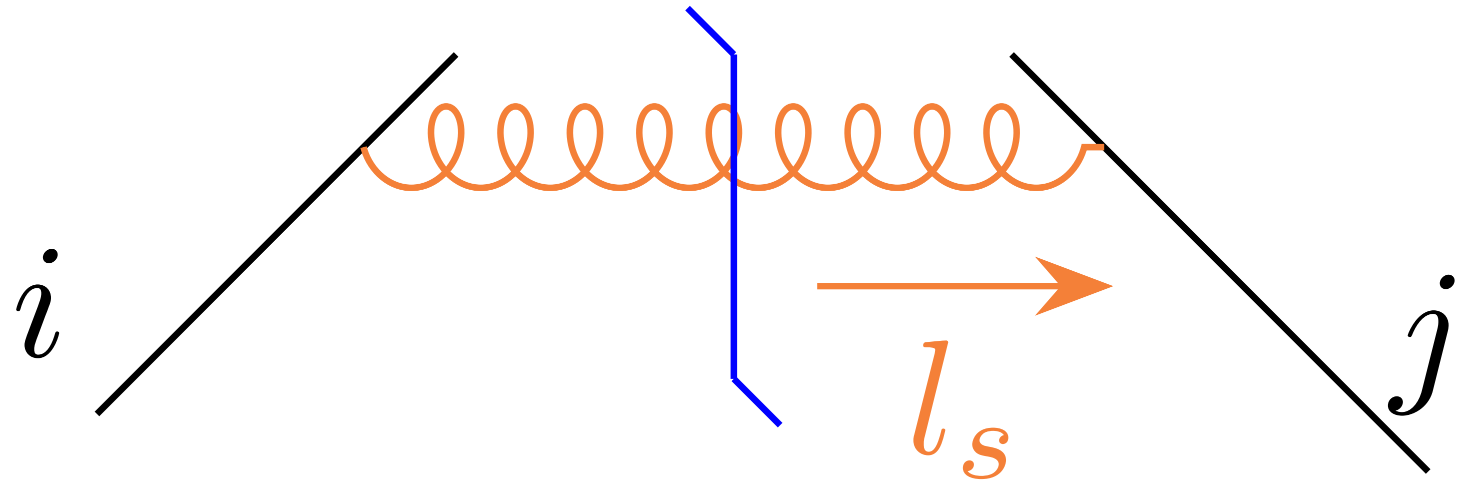
process	$\sigma_{2\to 2}$ [pb]	$\sigma^{\rm SLL}_{2\to 2}$ [pb]	process	$\sigma_{2\to 2}$ [pb]	$\sigma^{\rm SLL}_{2\to 2}$ [pb]
$qq \to qq$	231.5	12.0	$q\bar{q} \to gg$	12.4	-0.9
$qq' \to qq'$	454.4	22.2	$qg \to qg$	4104.6	403.3
$q\bar{q} \to q\bar{q}$	142.0	7.4	$gg \to q\bar{q}$	57.5	-4.4
$q\bar{q}' \to q\bar{q}'$	372.9	18.0	$gg \to gg$	2281.1	150.6
$q\bar{q} \to q'\bar{q}'$	3.6	<0.1			
$\sum$	1204.4	59.6	$\sum$	6455.6	548.6
$\sum_{\text{all channels}}$		7660.0		608.2	

- >10% for small values of Q_0
- Biggest contribution through gluonic channels
- Full cross section for the first time

RG-consistency

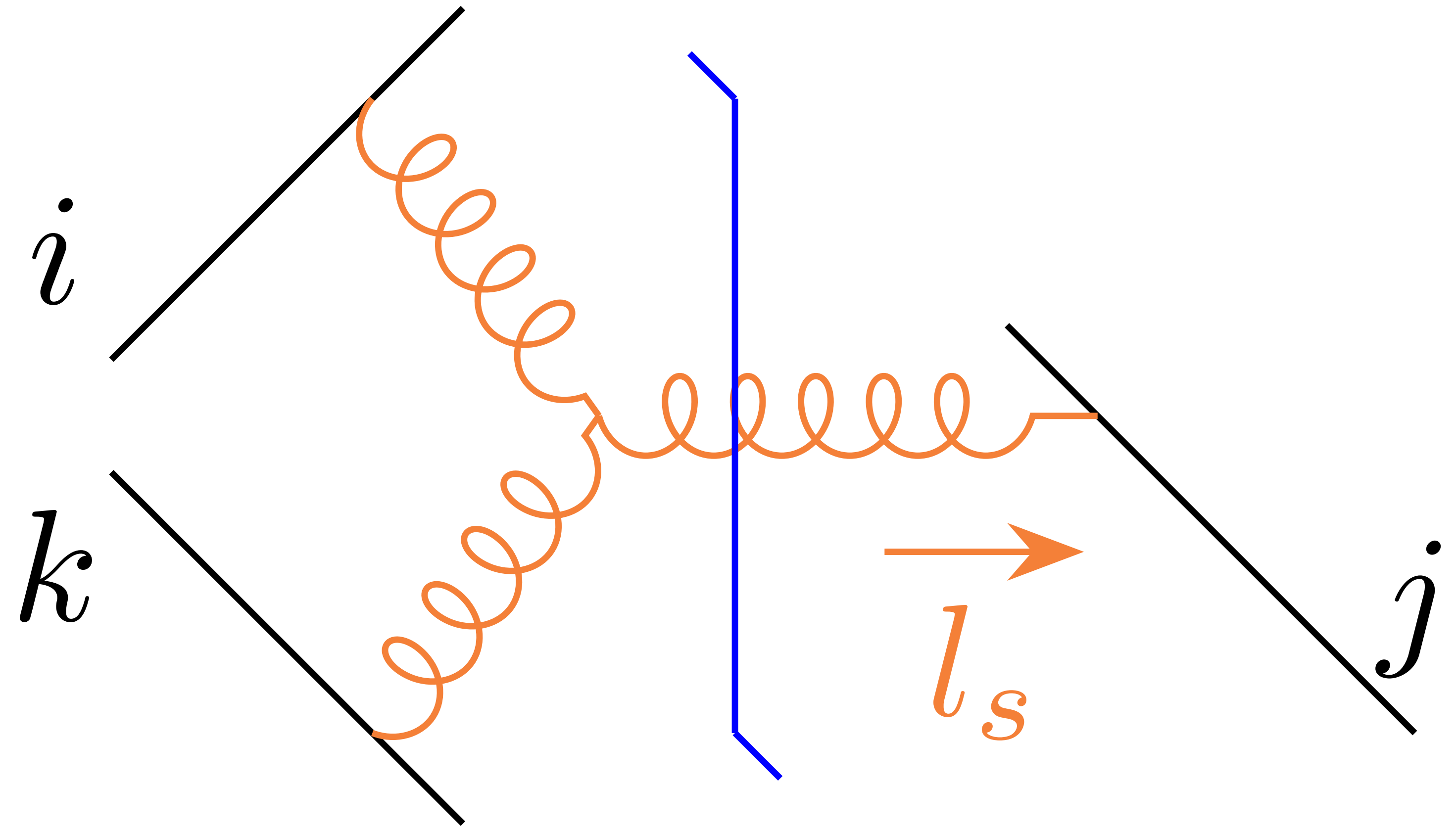


- Look at tree-level and one-loop diagrams first



eikonal factor

$$\sim \boldsymbol{J}^{\mu,a(0)} \boldsymbol{J}_\mu^{a(0)\dagger} \quad \text{with} \quad \boldsymbol{J}^{\mu,a(0)} = \sum_{i=1}^{m} \boldsymbol{T}_{iL}^a \frac{n_i^\mu}{n_i \cdot l_s}$$



one-loop soft current

Catani, Grazzini '00

- Matches structure

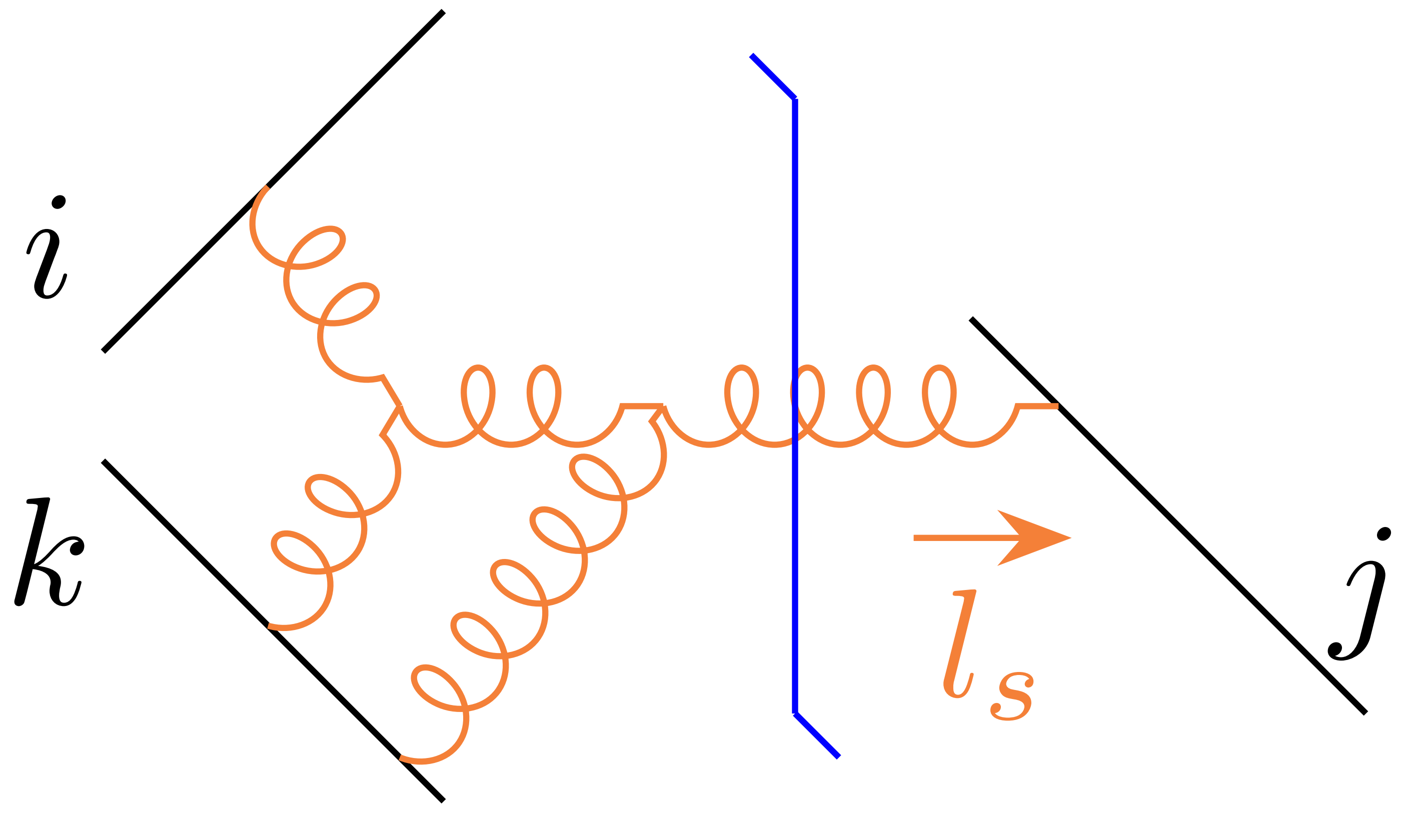
$$\mathscr{W}_m^{\text{bare}} = \mathbf{1} + \frac{\alpha_s}{4\pi} \frac{\overline{\boldsymbol{\Gamma}}}{2\varepsilon} + \left(\frac{\alpha_s}{4\pi}\right)^2 \left(\frac{\boldsymbol{V}^G \overline{\boldsymbol{\Gamma}}}{2\varepsilon^2} + \dots\right) + \left(\frac{\alpha_s}{4\pi}\right)^3 \left[\frac{\boldsymbol{\Gamma}^c \boldsymbol{V}^G \overline{\boldsymbol{\Gamma}}}{3\varepsilon^3} \left(\frac{11}{6\varepsilon} + \ln\frac{\mu_s^2}{Q^2} + \frac{9}{2} \ln\frac{\mu_s^2}{Q_0^2}\right) + \frac{\boldsymbol{V}^G \boldsymbol{V}^G \overline{\boldsymbol{\Gamma}}}{3\varepsilon^3} + \dots\right] + \mathcal{O}(\alpha_s^4)$$

✔ ✔

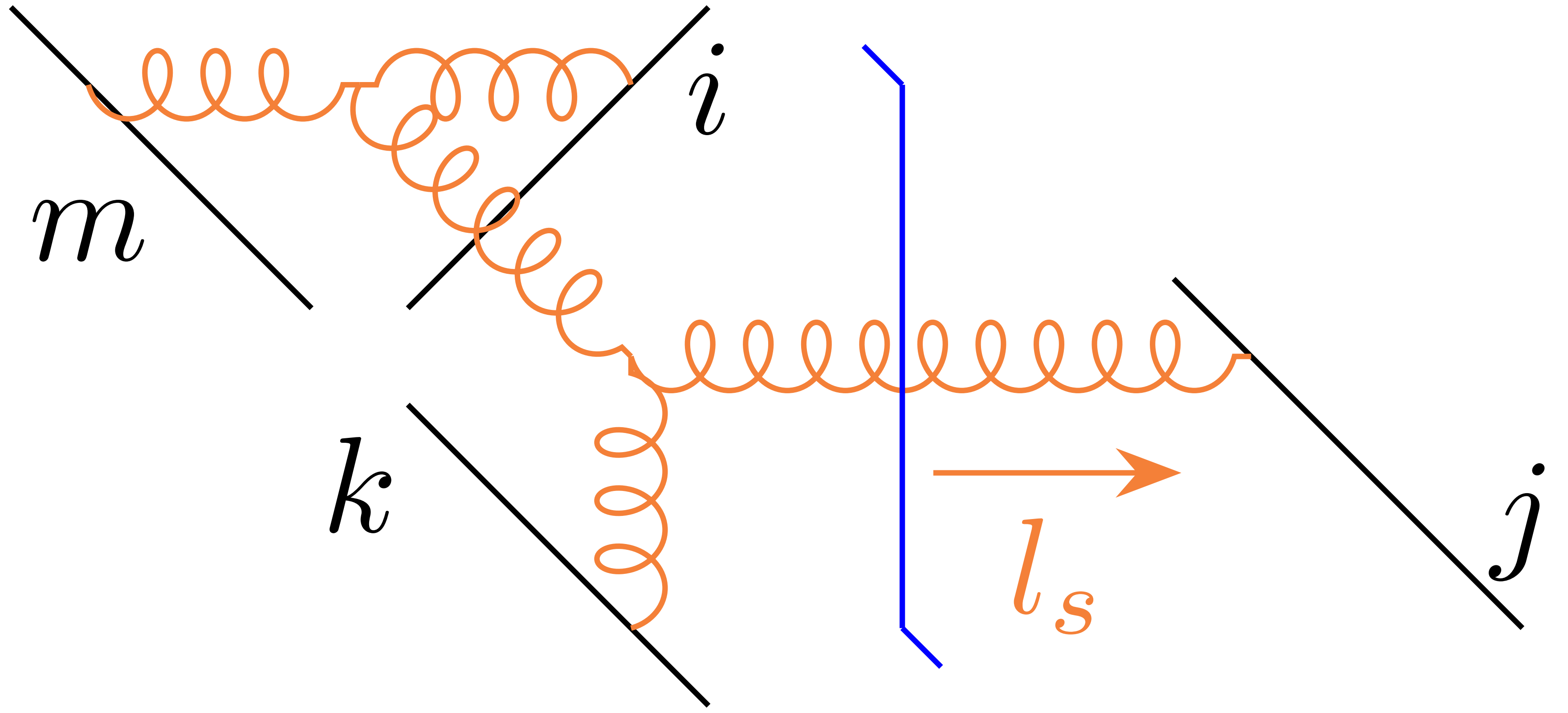
RG-consistency

- Now, go one loop further

Duhr, Gehrmann '13 / Dixon, Herrmann, Yan, Zhu '20



dipole terms



tripole terms

Large logarithm

Color-aware DGLAP

- Does not match all terms

$$\mathcal{I}_m^{\text{bare}} = \mathbf{1} + \frac{\alpha_s}{4\pi}\frac{\overline{\mathbf{\Gamma}}}{2\varepsilon} + \left(\frac{\alpha_s}{4\pi}\right)^2\left(\frac{\boldsymbol{V}^G\,\overline{\mathbf{\Gamma}}}{2\varepsilon^2} + \ldots\right) + \left(\frac{\alpha_s}{4\pi}\right)^3\left[\frac{\mathbf{\Gamma}^c\boldsymbol{V}^G\,\overline{\mathbf{\Gamma}}}{3\varepsilon^3}\left(\frac{11}{6\varepsilon} + \ln\frac{\mu_s^2}{Q^2} + \frac{9}{2}\ln\frac{\mu_s^2}{Q_0^2}\right) + \frac{\boldsymbol{V}^G\boldsymbol{V}^G\,\overline{\mathbf{\Gamma}}}{3\varepsilon^3} + \frac{\left[\mathbf{\Gamma}^C,\boldsymbol{V}^G\right]\overline{\mathbf{\Gamma}}}{12\varepsilon^3} + \ldots\right] + \mathcal{O}(\alpha_s^4)$$

✓ ✓ ✗ ✓ ✗

Glauber contribution

- In Euclidean region $s_{ij} = (p_i + p_j)^2 < 0, p_5^2 < 0$ only soft-collinear region with $k \sim (\lambda^2, \lambda, \lambda^{3/2})$
  - Cancels after q_c integration!
- Leads to a „hidden" region with $k \sim (\lambda^2, \lambda, \lambda)$ for physical scattering region
  - Couples soft and collinear sectors → collinear factorization breaking
- Perform k_+ and k_- integral via residues
  - Well-defined without additional regulators

$$I^{g} = i(4\pi)^{2-\varepsilon} \int \frac{d^d k}{(2\pi)^d} \frac{1}{-k_T^2} \frac{1}{k^+ q_c^- - k_T^2 - 2k_T \cdot q_{cT}} \times \frac{1}{\left[-k^+(p_c^- - q_c^-) - q_c^+ p_c^- - k_T^2 - 2k_T \cdot q_{cT}\right]} \times \frac{1}{\bar{p}_{\bar{c}}^+ (k^- - l_s^-)} \frac{1}{-l_s^+ k^- - k_T^2 + 2k_T \cdot l_{sT}}$$

Euclidean of-shell triangle in $d - 2\varepsilon$