Collinear factorisation breaking in $2 \to 3$ exclusive photoproduction processes

Factorisation Workshop Edinburgh

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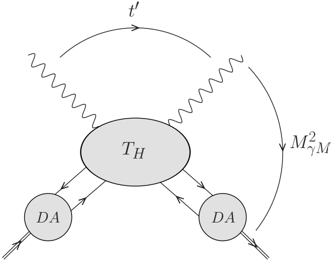
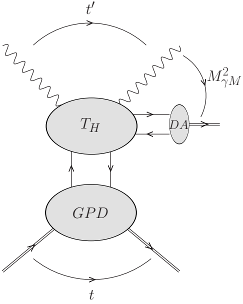
Arguments for justifying collinear factorisation

$\gamma N \to \gamma N' M$ example



large angle factorisation
à la Brodsky Lepage

$\Longrightarrow$



Collinear factorisation of the amplitude justified at large $M^2_{\gamma M}$, t', u', and small t.

$$\mathcal{M} = \int dx \int dz \, T_H(x,z) \, GPD(x) \, DA(z)$$

$GPD(x)$: Generalised parton distribution

What has been achieved so far

Work on this since early 2010s:

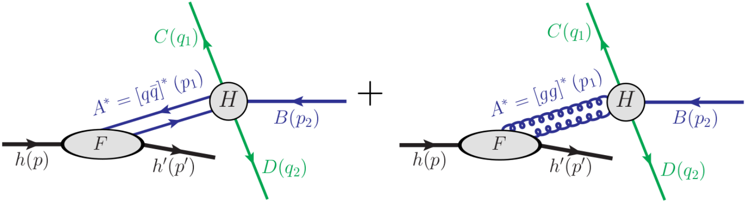
- $\gamma p \to n \pi^+ \rho^0_T$ [El Beiyad, Pire, Segond, Szymanowski, Wallon; 1001.4491]
  Target: Probing chiral-odd GPDs at leading twist
- $\gamma N \to \gamma N' M$ [Boussarie, Crnkovic, Duplancic, **S.N.**, Passek-Kumericki, Pire, Szymanowski, Wallon; 1609.03830, 1809.08104, 2212.00655, 2302.12026, 2511.19720]
  Calculation at NLO ongoing!
- $\gamma N \to \gamma \gamma N$ [Grocholski, Pedrak, Pire, Sznajder, Szymanowski, Wagner; 1708.01043, 2110.00048, 2204.00396] Calculated at NLO!
- $\gamma N \to N' M_1 M_2$ [2605.03880; **S.N.**, Perez, Szymanowski, Wallon]

All processes above chosen such that they are sensitive to quark GPDs only

Proof of factorisation

Collinear factorsiation proved for a wide range of $2 \to 3$ exclusive processes in [Qiu, Yu; 2205.07846, 2210.07995], including *all* previously mentioned photoproduction processes.

Proof relies on *large relative transverse momentum* of produced outgoing states:



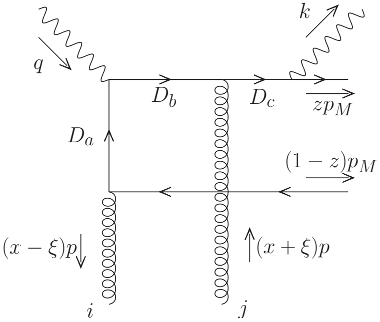
Seems natural to extend to processes sensitive to gluon channels, e.g. $\pi^0\gamma$ exclusive photoproduction...

Naive calculation for exclusive $\pi^0\gamma$ photoproduction

Let us naively calculate the amplitude *at leading order* and *leading twist*. Consider diagram on the right.

Work in kinematics where p and p_M define the $+$ and $-$ lightcone directions respectively.

q and k have large (equal) transverse momenta.



$$T_H \sim \frac{\mathrm{Tr}\left[\not{p}_M \gamma^5 \not{\epsilon}_k \left(\not{k} + z\not{p}_M\right) \gamma^j \left(\not{q} - (x-\xi)\not{p} - \bar{z}\not{p}_M\right) \not{\epsilon}_q \left(-(x-\xi)\not{p} - \bar{z}\not{p}_M\right) \gamma^i\right]}{\left[2z\, kp_M\right]\left[-2\,(x-\xi)\,qp - 2\bar{z}\,qp_M + 2\bar{z}\,(x-\xi)\,pp_M + i\epsilon\right]\left[2\bar{z}\,(x-\xi)\,pp_M + i\epsilon\right]}$$

$$\xrightarrow{x\to\xi,\bar{z}\to 0} \propto \frac{x-\xi}{\left[(x-\xi) + A\bar{z} - i\epsilon\right]\left[\bar{z}\,(x-\xi) + i\epsilon\right]}\,, \qquad A \equiv \frac{qp_M}{qp} > 0.$$

Naive calculation for exclusive $\pi^0\gamma$ photoproduction

Including the gluon GPD with projector factors $\left(\frac{H_g(x)}{(x-\xi+i\epsilon)(x+\xi-i\epsilon)}\right)$, and DA $(z\bar{z})$. This gives

$$\text{Integrand of } \mathcal{M} \sim \frac{\bar{z}\,(x-\xi)\,H_g(x)}{(x-\xi+i\epsilon)\,[(x-\xi)+A\bar{z}-i\epsilon]\,[\bar{z}\,(x-\xi)+i\epsilon]}$$

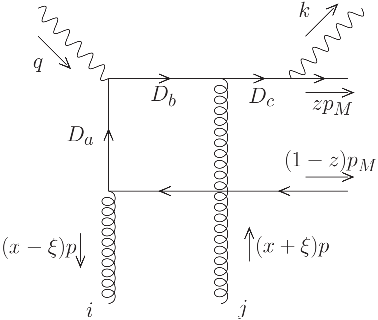
$$\longrightarrow \frac{H_g(x)}{[(x-\xi)+A\bar{z}-i\epsilon]\,[x-\xi+i\epsilon]}$$

The integral over z and x diverges if the GPD $H_g(x)$ is non-vanishing at $x=\xi$:

$$\mathcal{M} \sim \int_{-1}^{1} dx \int_{0}^{1} dz \frac{1}{[(x-\xi)+A\bar{z}-i\epsilon]\,[x-\xi+i\epsilon]}$$

$$\supset \int_{-1}^{1} dx \frac{\ln(x-\xi-i\epsilon)}{[x-\xi+i\epsilon]} \qquad \Longrightarrow \text{ divergent imaginary part!}$$

Naive calculation for exclusive $\pi^0\gamma$ photoproduction



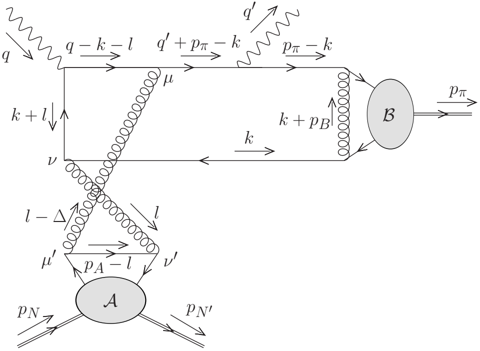
$$\mathcal{M} \sim \int_{-1}^{1} dx \int_{0}^{1} dz \frac{1}{[(x-\xi)+A\bar{z}-i\epsilon]\,[x-\xi+i\epsilon]}$$

$\implies$ The "*pinching*" is caused by propagators D_a and D_b.

Divergence also present in $\gamma N \to N' M_1 M_2$ which can exchange two gluons in the t-channel [2605.03880; **S.N.**, Perez, Szymanowski, Wallon]

Region analysis: Glauber pinch

[2311.09146, 2409.16067; **S.N.**, Schoenleber, Szymanowski, Wallon]



$$\lambda = \frac{\{m_M,\, m_N,\, \sqrt{t}\}}{Q}$$

$$p_N,\, p_{N'},\, p_A,\, \Delta \sim Q\left(1,\lambda^2,\lambda\right),$$

$$\Delta^+ < 0.$$

$$p_\pi,\, p_B \sim Q\left(\lambda^2,1,\lambda\right)$$

$$q,\, q' \sim Q\left(1,1,1\right),$$

$$q^2,\, q'^2 \sim \lambda^2 Q^2$$

$$[\text{Loop}]\ l \sim Q\left(\lambda,\lambda,\lambda\right)$$

$$[\text{Loop}]\ k \sim Q\left(\lambda,\lambda,\lambda\right)$$

Start from loop momenta with soft scaling, as this is necessarily a pinched configuration.

Collinear pinch, i.e. $l \sim Q(1,\lambda^2,\lambda)$ and $k \sim Q(\lambda^2,1,\lambda)$ can be quite easily demonstrated, and shown to be of leading power.

Region analysis: Glauber pinch

l^- *pinch*:

$$(l-\Delta)^2 + i0 = -2\Delta^+ l^- + \mathcal{O}(\lambda^2) + i0$$
$$\implies l^- = \mathcal{O}(\lambda^2) - i0\,.$$

$$(p_A - l)^2 + i0 = -2p_A^+ l^- + \mathcal{O}(\lambda^2) + i0$$
$$\implies l^- = \mathcal{O}(\lambda^2) + \mathrm{sgn}(p_A^+)\, i0\,.$$

$p_A^+ > 0$: *DGLAP* region of GPD

l^+ *pinch*:

$$(q-k-l)^2 + i0$$
$$= -2q^+k^- - 2q^-l^+ + \mathcal{O}(\lambda) + i0$$
$$\implies l^+ = \mathcal{O}(\lambda) + i0\,.$$

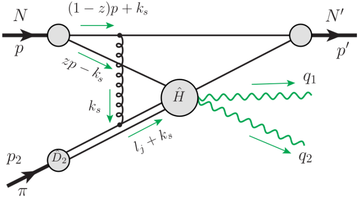
$$(k+l)^2 + i0 = 2l^+k^- + \mathcal{O}(\lambda^2) + i0$$
$$\implies l^+ = \mathcal{O}(\lambda) - \mathrm{sgn}(k^-) i0\,.$$

$l \sim Q(\lambda, \lambda^2, \lambda)$ pinched to be in the *soft-to-collinear* Glauber region.

Power-counting shows that the Glauber pinch is of leading power. (Backup)

Deformable vs non-deformable Glauber pinches

Deformable *collinear-to-collinear* Glauber momentum $k_s \sim Q(\lambda^2, \lambda^2, \lambda)$ identified in $\pi^0 N \to \gamma\gamma N$ [Qiu, Yu; 2205.07846]



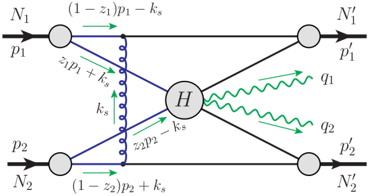
Related to $\gamma N \to \gamma N M$ through crossing.

$p \sim Q(1, \lambda^2, \lambda)$

$\implies k_s^-$ is pinched to $\mathcal{O}(\lambda^2)$

BUT k_s^+ not pinched! Deform all the way to collinear region!

Non-deformable Glauber momentum



Both k_s^+ and k_s^- pinched to be $\mathcal{O}(\lambda^2)$

Connection to superleading logs (!)

Factorisation breaking diagram in $\gamma N \to \gamma N \pi^0$ [2311.09146, 2409.16067; **S.N.**, Schoenleber, Szymanowski, Wallon]

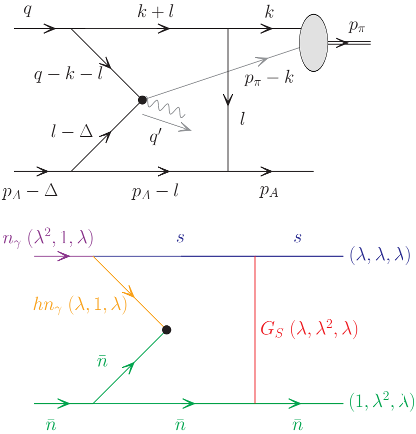
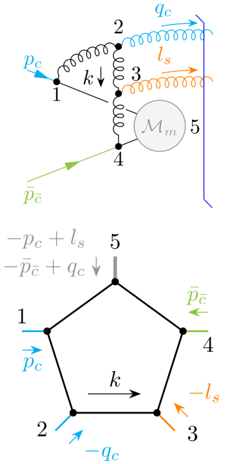


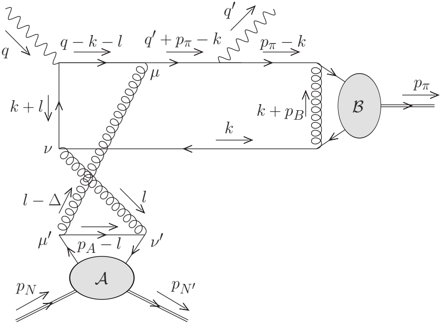
Diagram identified in [2408.10308; Becher, Hager, Jaskiewicz, Neubert, Schwienbacher]



Backup

Power counting

Consider [omitting the $i\epsilon$ prescriptions]



$$\mathcal{M} = \int F_A^{\mu'\nu'} g_{\mu\mu'} g_{\nu\nu'} \operatorname{tr}\left[F_H^{\mu\nu} F_B\right]$$

$$F_A^{\mu'\nu'} = dl^- d^2 l_\perp \left(\frac{\operatorname{tr}\left[\not{\mathcal{A}}\gamma^{\nu'}\left(\not{p}_A - \not{l}\right)\gamma^{\mu'}\right]}{l^2\,(l-\Delta)^2\,(p_A-l)^2} \right)$$

$$F_B = dk^+ d^2 k_\perp \left(\frac{\not{k}\not{\mathcal{B}}\left(\not{p}_\pi - \not{k}\right)}{k^2\,(p_\pi-k)^2\,(p_B-k)^2} \right)$$

$$F_H^{\mu\nu} = dk^- dl^+ \frac{\not{\epsilon}^*_{q'}\left(\not{q}' + \not{p}_\pi - \not{k}\right)\gamma^\mu\left(\not{q} - \not{k} - \not{l}\right)\not{\epsilon}_q\left(\not{k} + \not{l}\right)\gamma^\nu}{(q'+p_\pi-k)^2\,(q-k-l)^2\,(k+l)^2}$$

For the purposes of the power counting, $\not{\mathcal{A}}$ and $\not{\mathcal{B}}$ can be taken to be $\mathcal{A}^\mu\gamma_\mu$ and $\mathcal{B}^\mu\gamma_\mu$, with $\mathcal{A}^\mu \sim (1,\lambda^2,\lambda)$ and $\mathcal{B}^\mu \sim (\lambda^3,\lambda,\lambda^2)$.

Backup

Power counting in the collinear region

For the collinear pinch, $k \sim (\lambda^2, 1, \lambda)$ and $l \sim (1, \lambda^2, \lambda)$

Next, project onto the *transverse* polarizations of the gluons, i.e. pick the indices μ, μ', ν, ν' to be transverse, by virtue of the WIs.

$$F_A^{\mu'_\perp \nu'_\perp} \sim \lambda^4 \frac{\lambda^2}{\lambda^6} = \lambda^0 \,, \qquad F_B \sim \lambda^4 \frac{\lambda^3}{\lambda^6} = \lambda^1 \,, \qquad F_H^{\mu_\perp \nu_\perp} \sim \lambda^0 \frac{\lambda^0}{\lambda^0} = \lambda^0$$

$\implies \mathcal{M}_{\text{coll}} \sim \lambda$.

This exactly corresponds to the leading power scaling found through *Libby-Sterman power counting rules* for the collinear pinch configuration.

Backup

Power counting in the Glauber region

For the Glauber pinch, $k \sim (\lambda^2, \lambda, \lambda)$ and $l \sim (\lambda, \lambda, \lambda)$

Here, as before, we take the $\mu = \mu' = \perp$ indices to be transverse for the collinear momentum. For the Glauber momentum l, the leading power contribution is obtained using $\nu = \perp$ and $\nu' = +$. This implies that

$$F_A^{\mu'_\perp +} \sim \lambda^4 \frac{\lambda^1}{\lambda^6} = \lambda^{-1}\,, \quad F_B \sim \lambda^3 \frac{\lambda^3}{\lambda^4} = \lambda^2\,, \quad F_H^{\mu_\perp \nu_\perp} \sim \lambda^2 \frac{\lambda}{\lambda^3} = \lambda^0\,.$$

$\implies \mathcal{M}_{\text{glau}} \sim \lambda$

$\implies$ same as the collinear pinch.