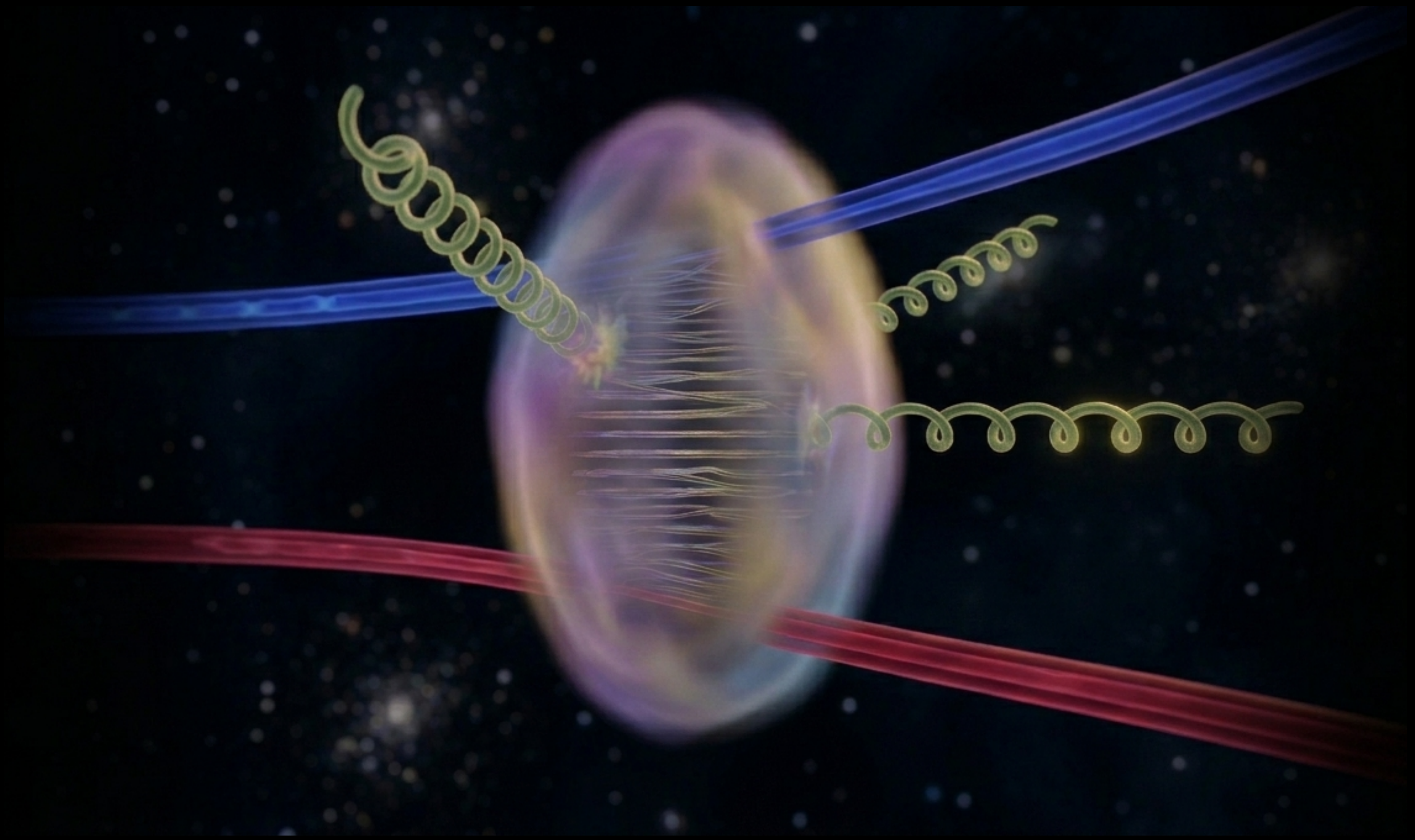
Probing High-Energy QCD through the lens of Scattering Amplitudes



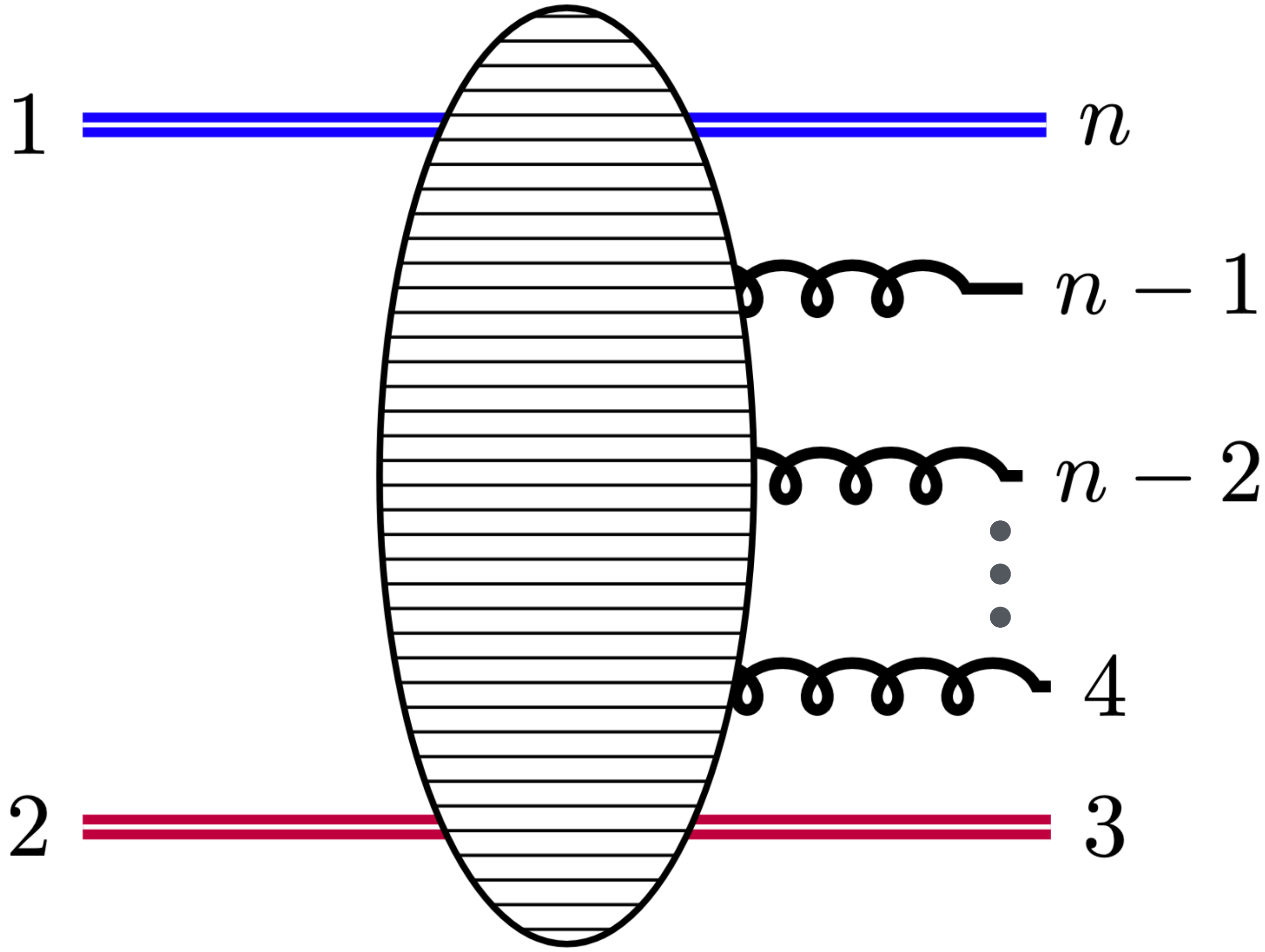
Giulio Gambuti - 2026 - Factorization in QCD and Beyond

ETH zürich

The Plan

1. **MRK limit** for 2 ➜ N processes
2. Review of **shockwave formalism**
3. **NNLL** building blocks via matching to **amplitudes**

Multi Regge Kinematics (MRK)



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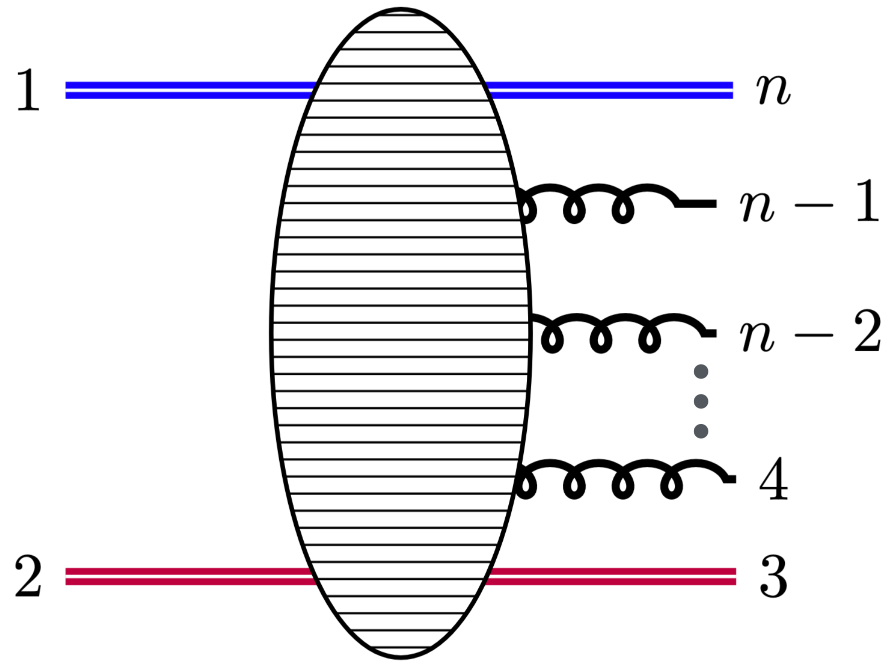

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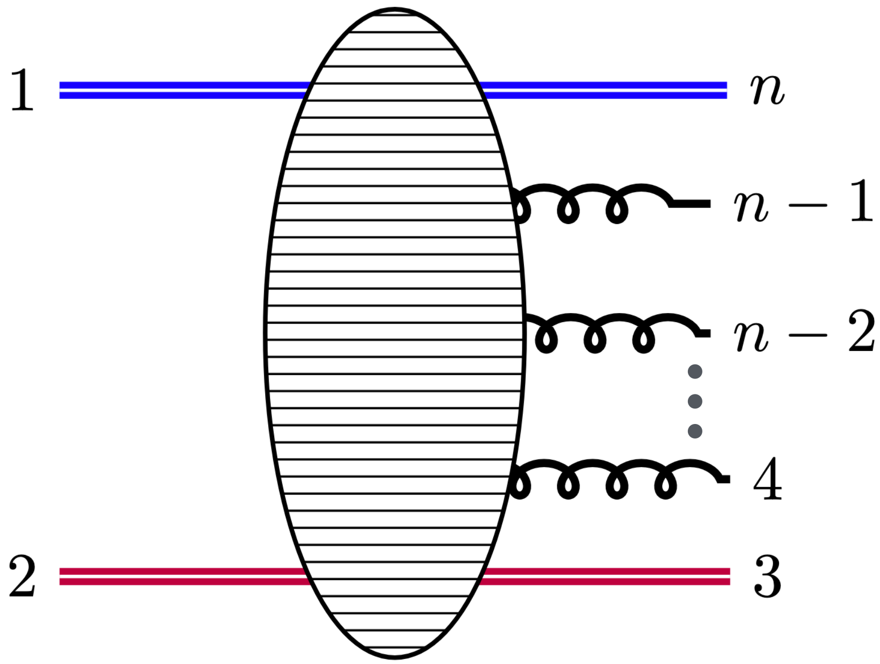

$$p^{\pm} = p^{0} \pm p^{3}$$
$$\mathbf{p} = p^{1} + i\,p^{2}$$

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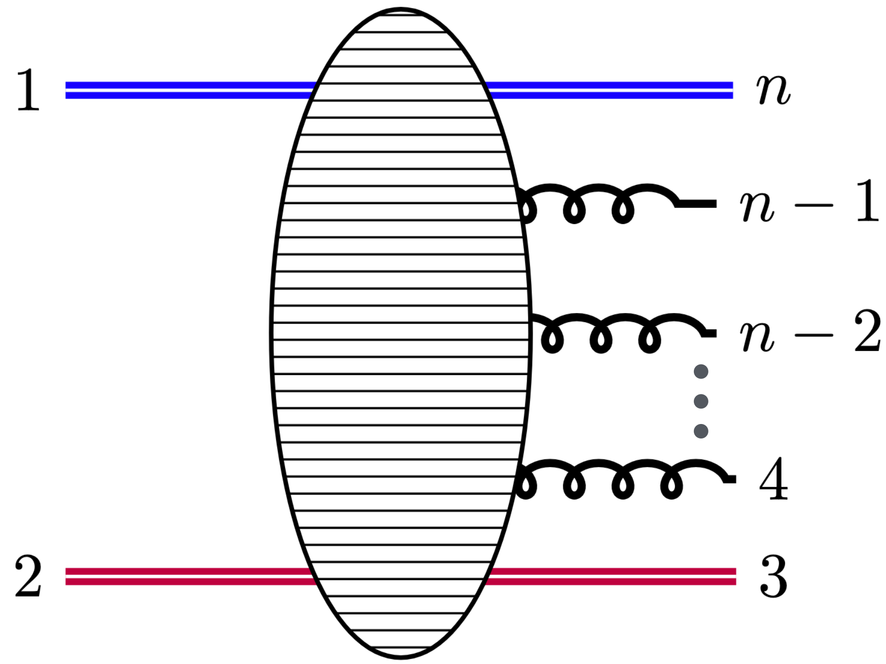

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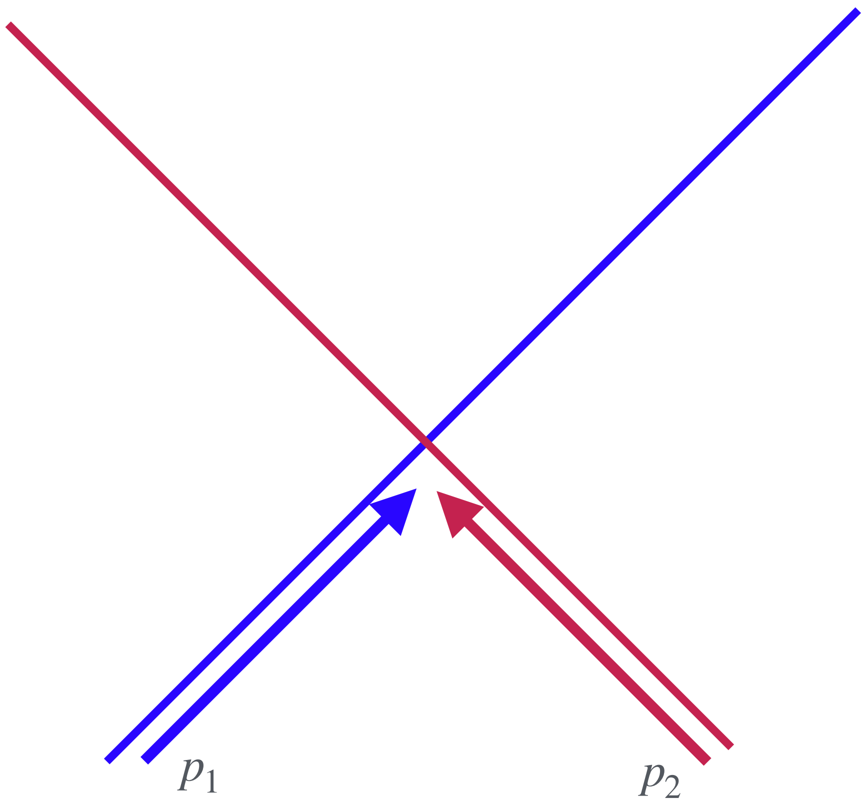

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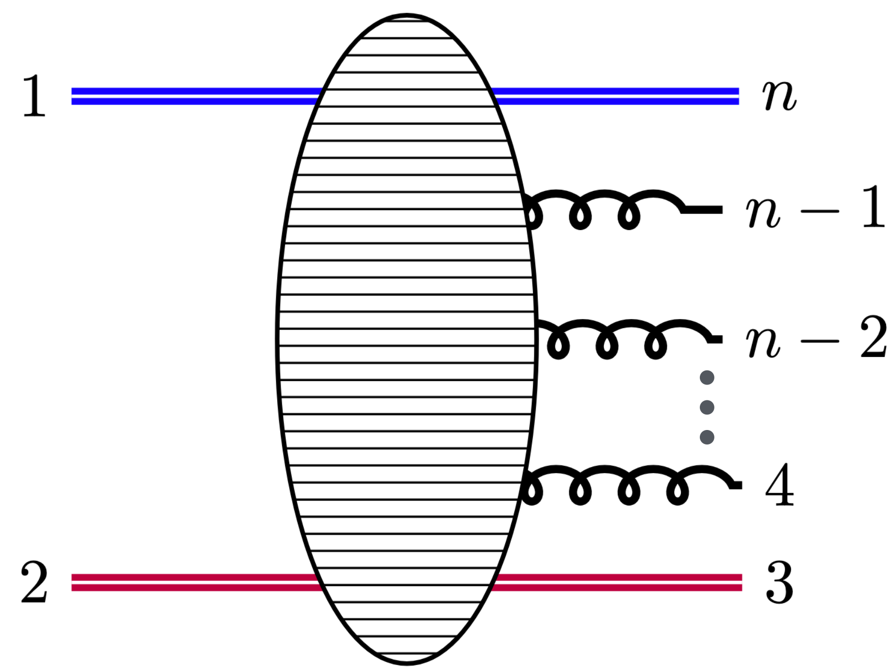

$$p^{\pm} = p^0 \pm p^3$$
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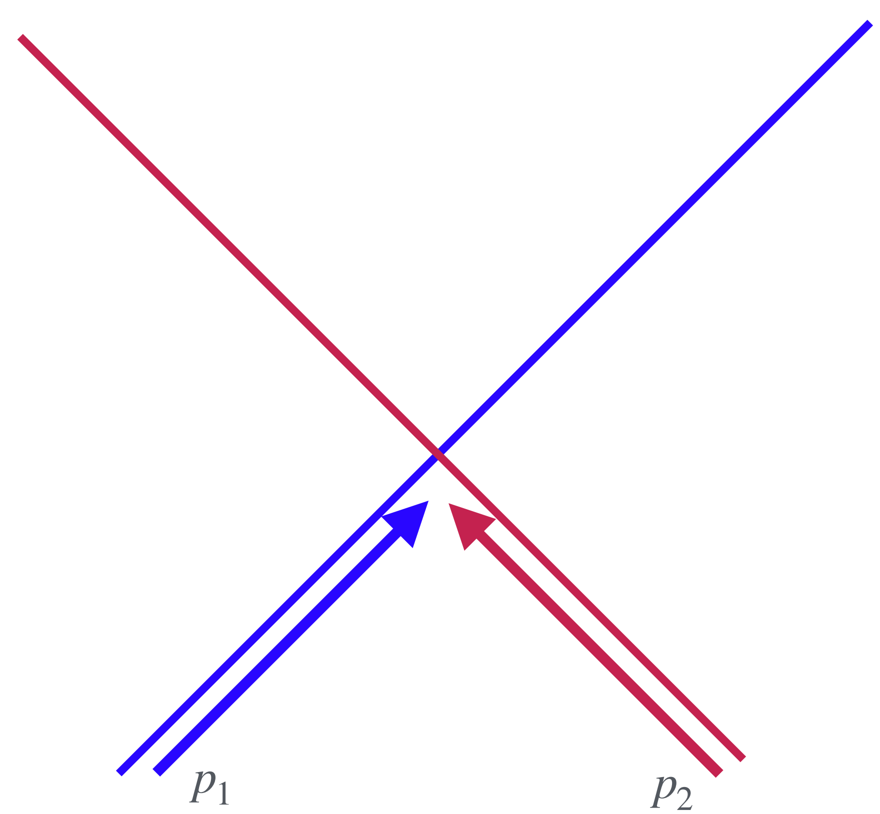
rapidity $$\eta_i = \frac{1}{2} \log \frac{p_i^+}{p_i^-}$$



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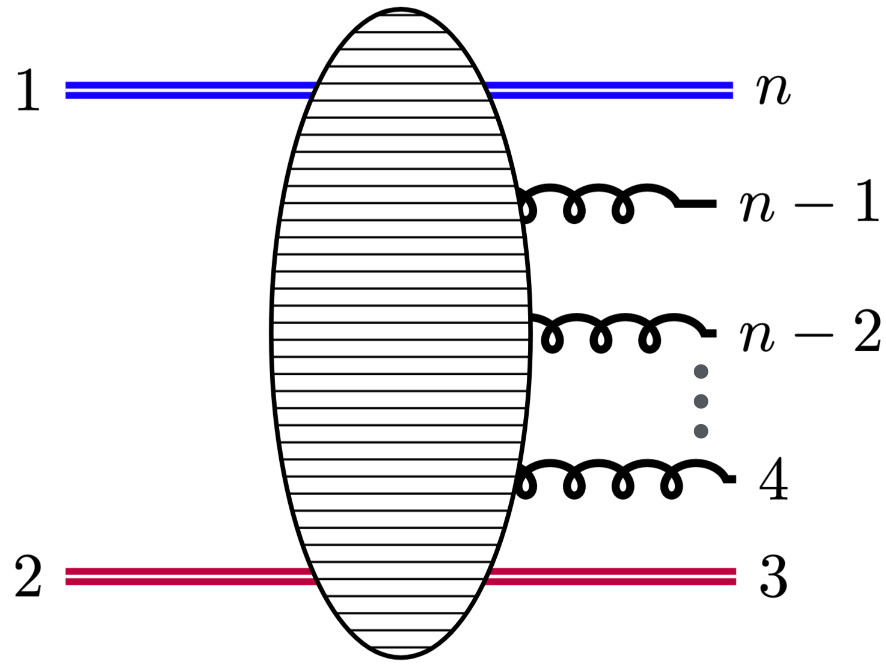

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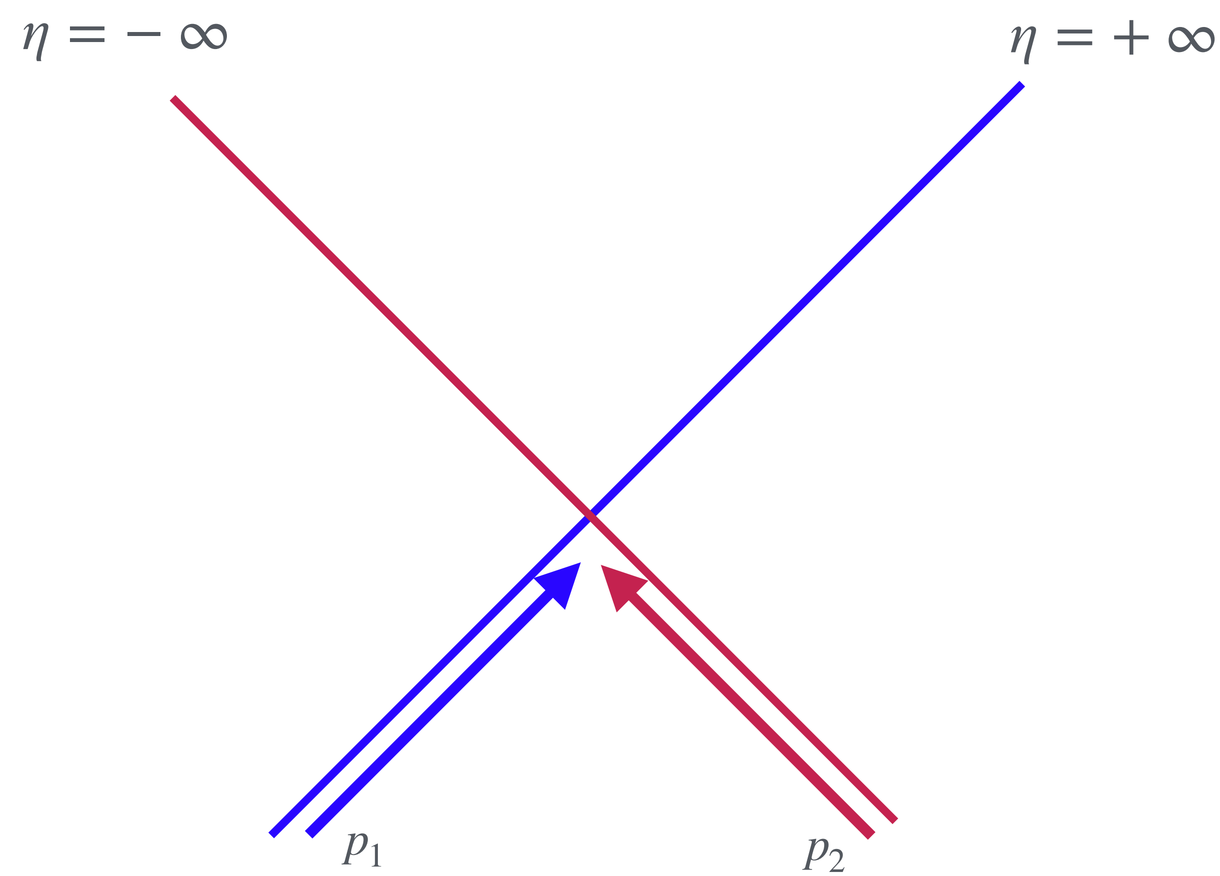

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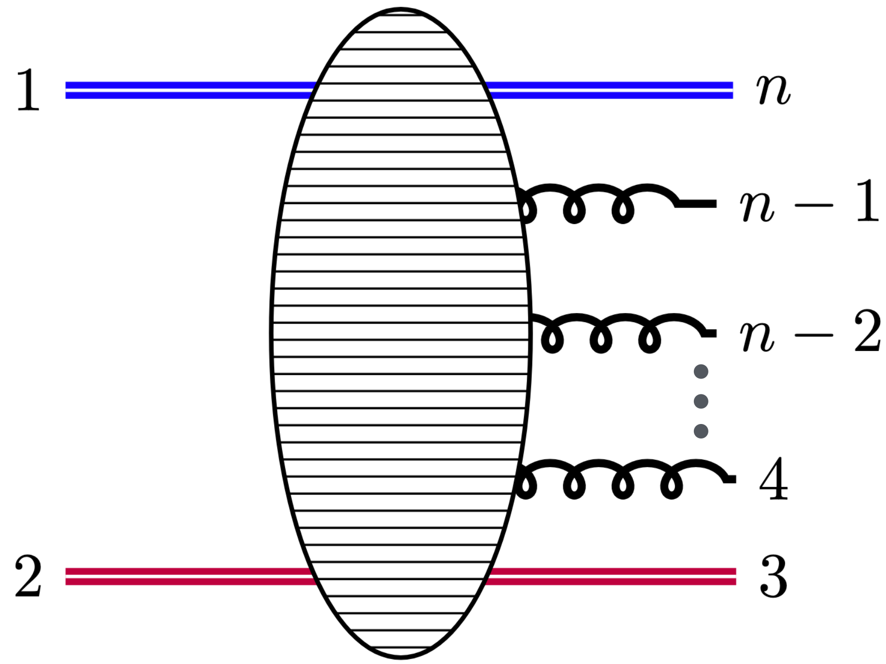

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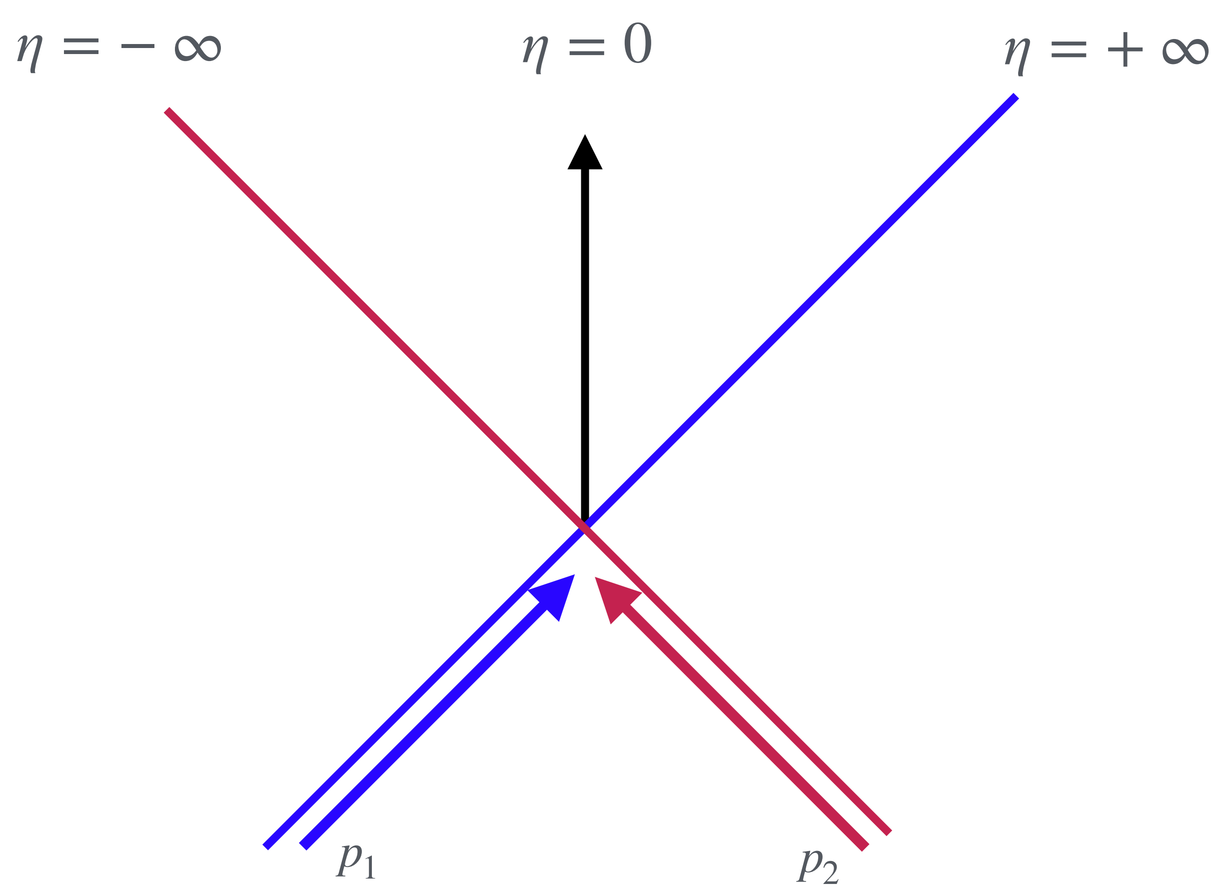

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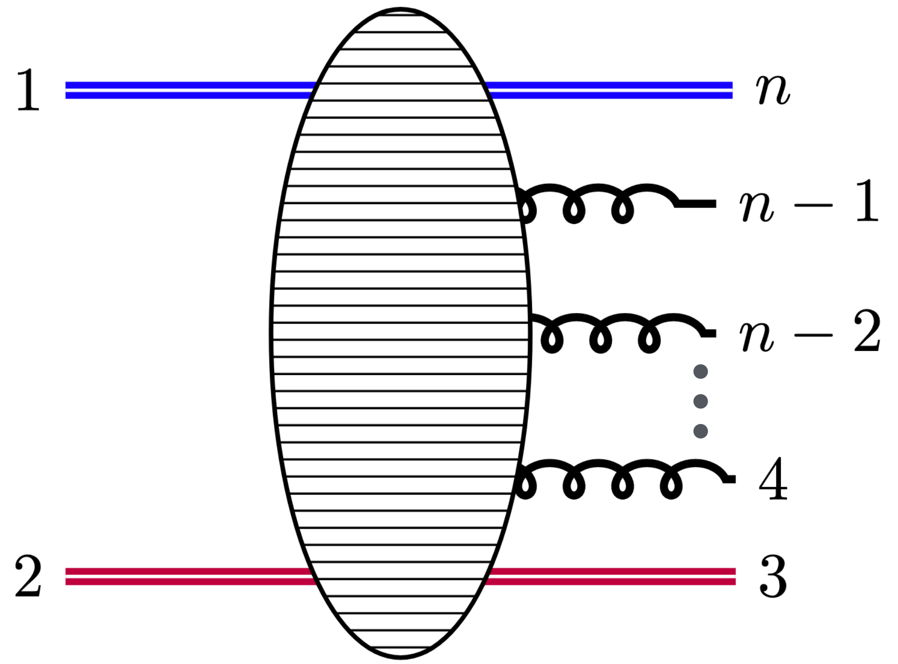

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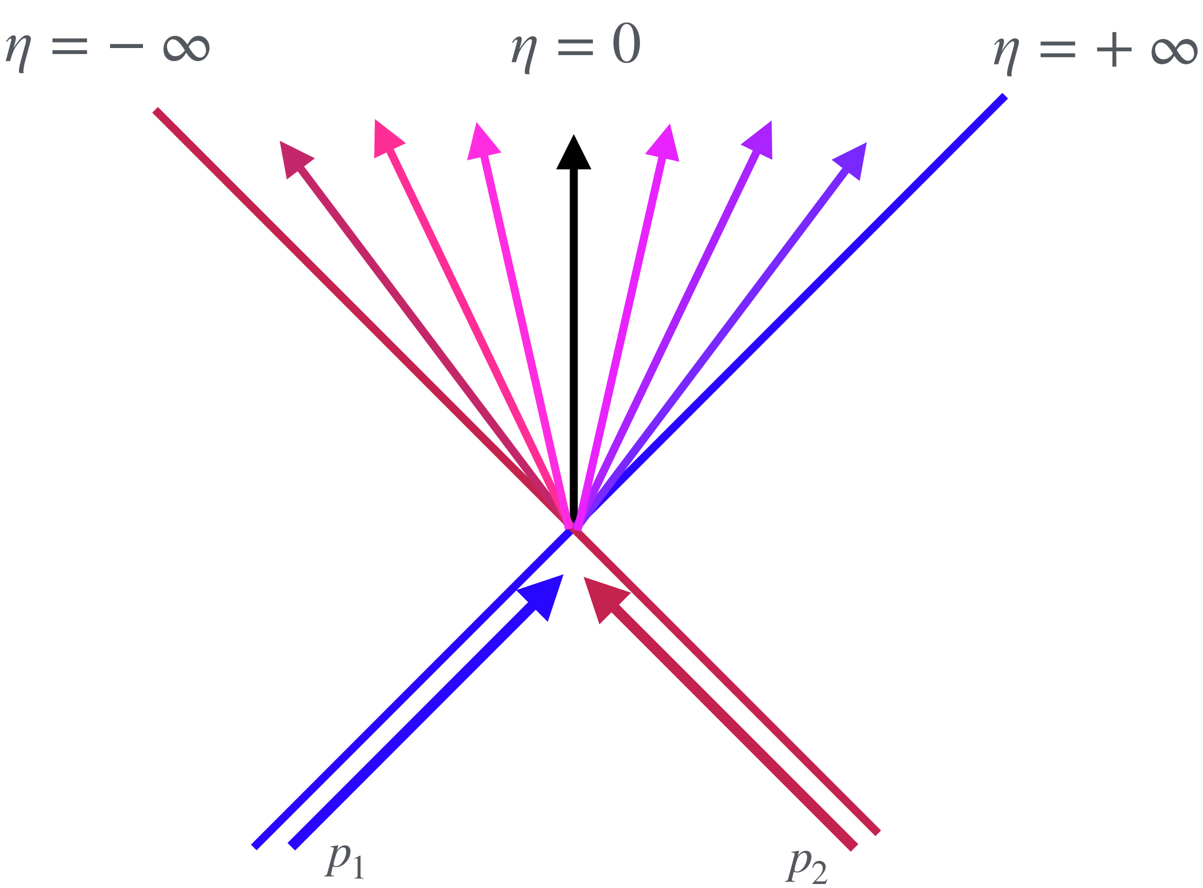
BFKL LADDER



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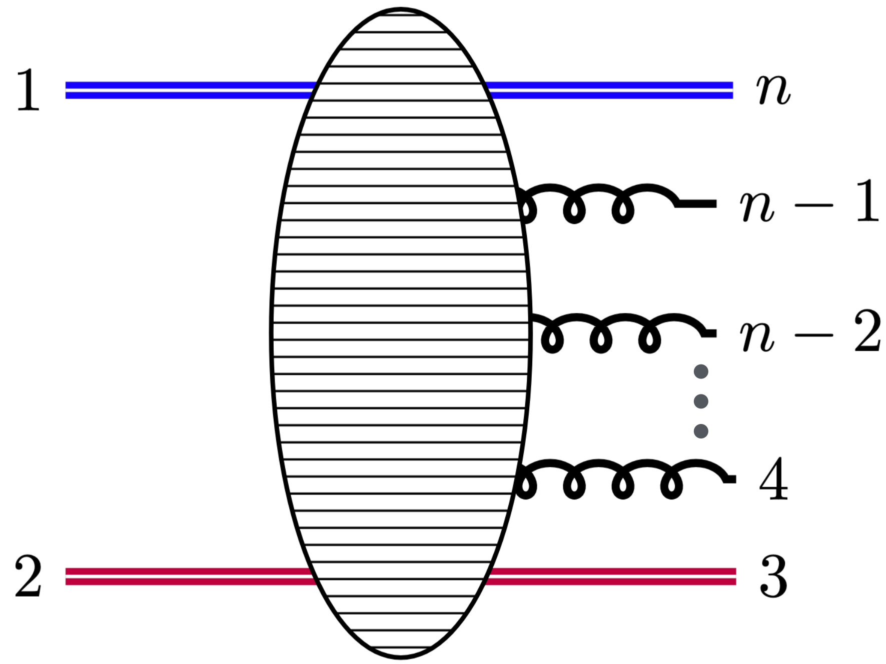

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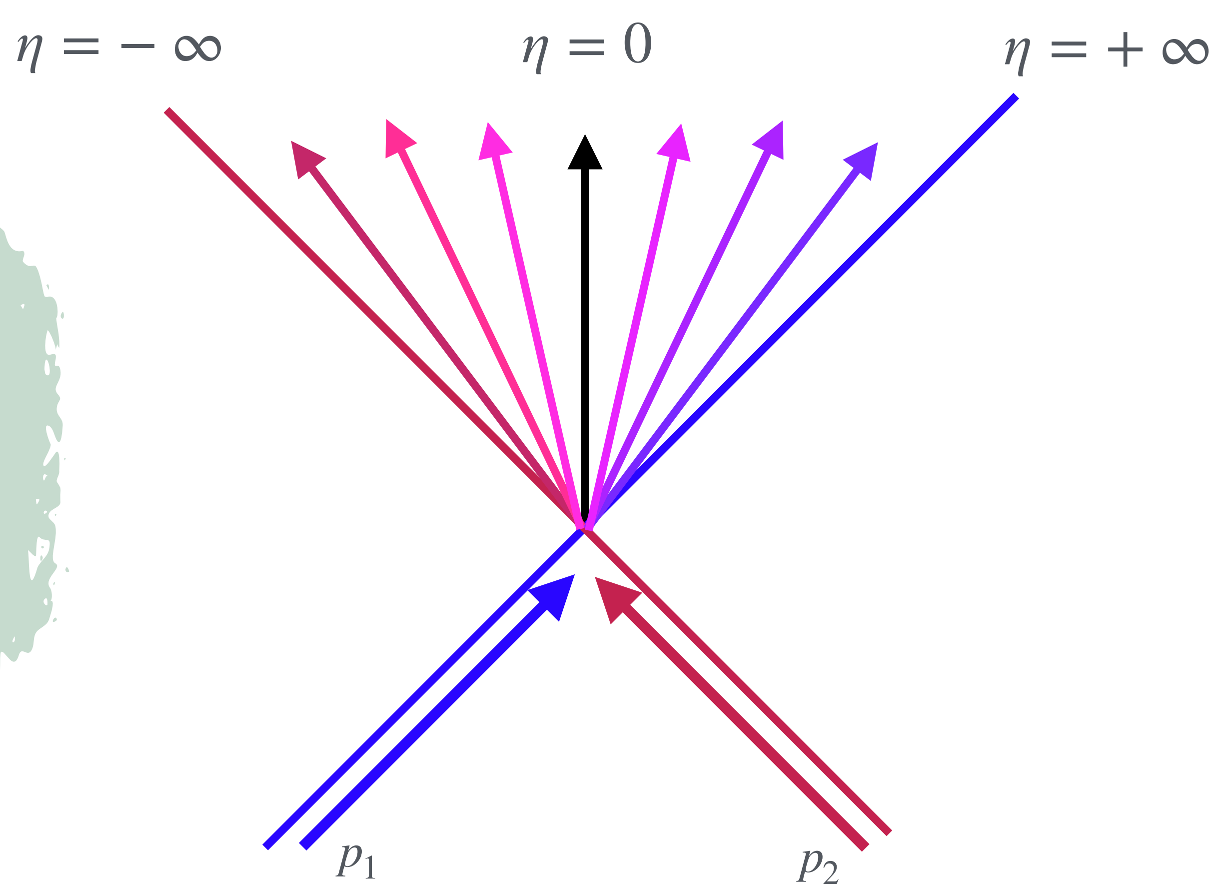
$$\eta_2 \sim \eta_3 \ll \eta_4 \ll \ldots \ll \eta_n \sim \eta_1$$

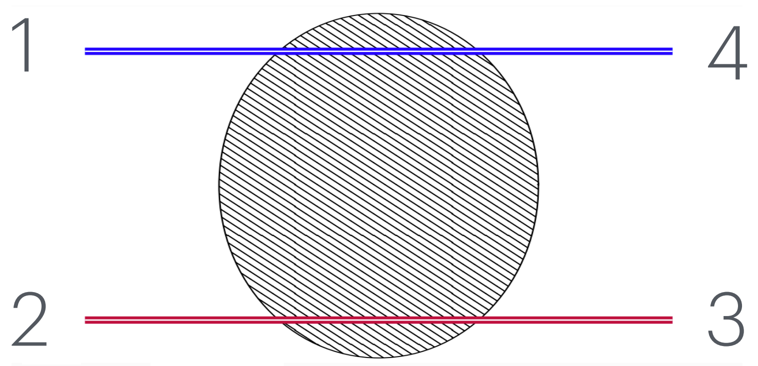
&

no transverse hierarchy $(\mathbf{p})$

BFKL LADDER



2 ➜ 2 Scattering



Regge limit $\eta_2 \sim \eta_3 \ll \eta_4 \sim \eta_1$

$$\Delta\eta_{43} = \log\left(\frac{s}{-t}\right) \to +\infty$$

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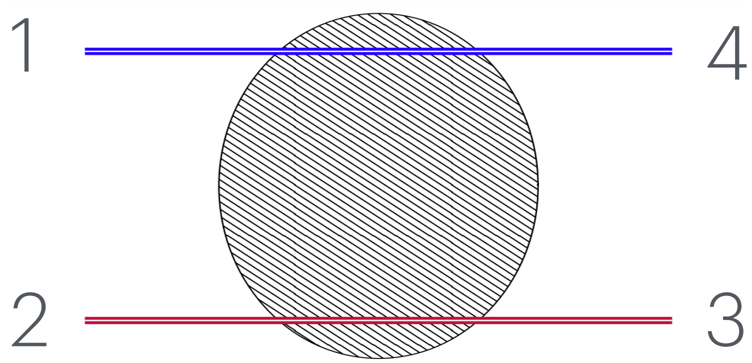

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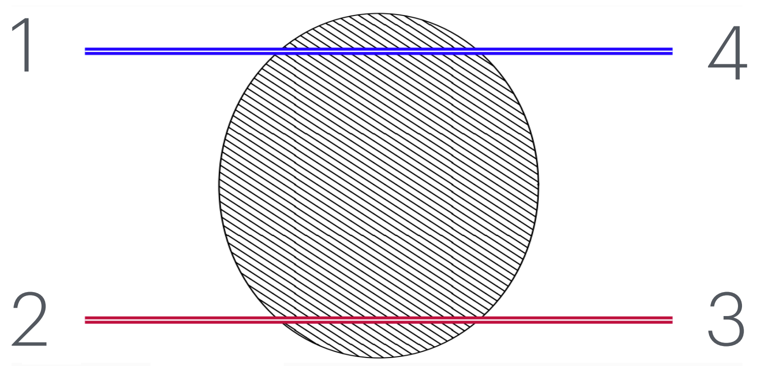

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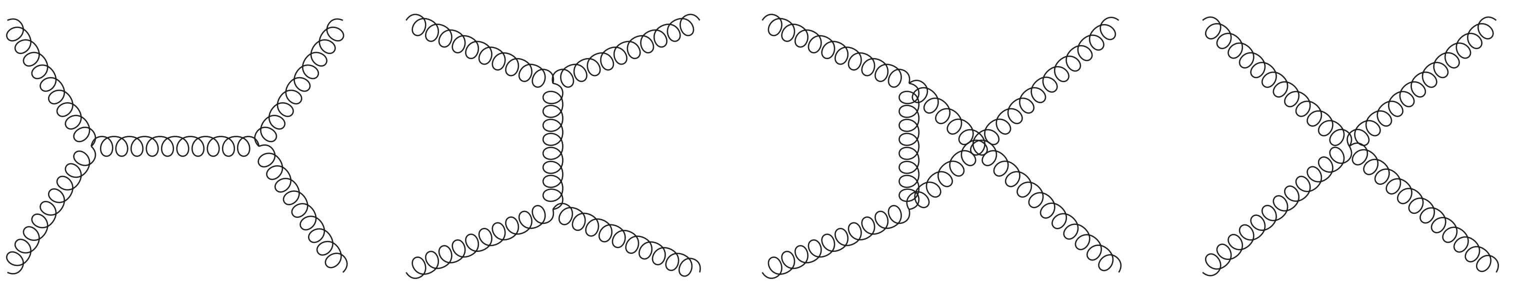
Gluon amplitude



$\sim \frac{1}{s}$ $\sim \frac{1}{t}$ $\sim \frac{1}{s+t}$ constant

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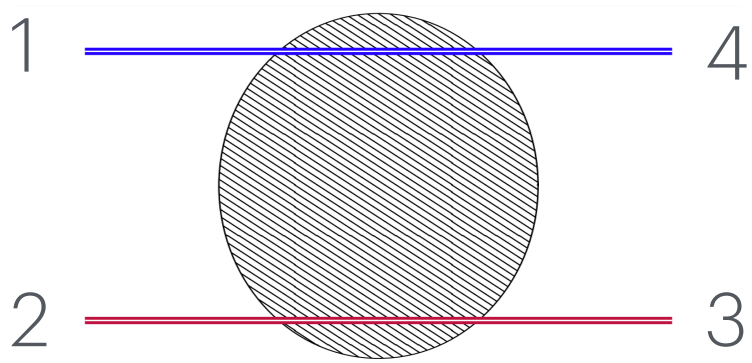

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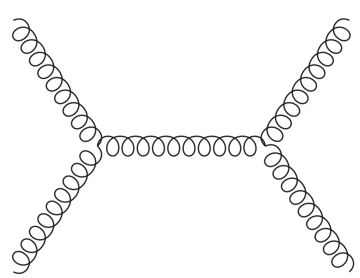
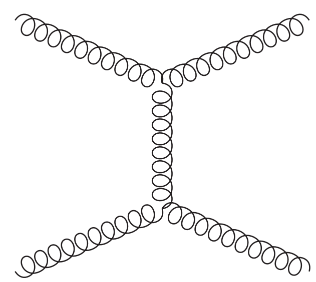
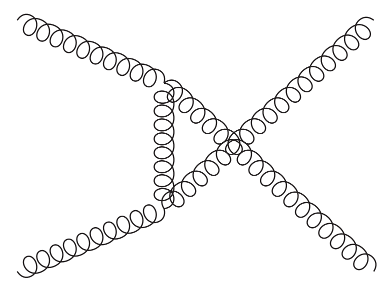


$\sim \frac{1}{s}$



$\sim \frac{1}{t}$



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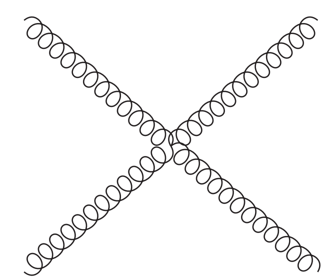

constant

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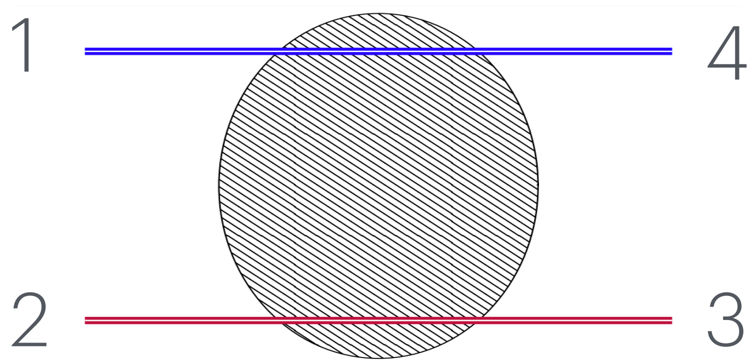

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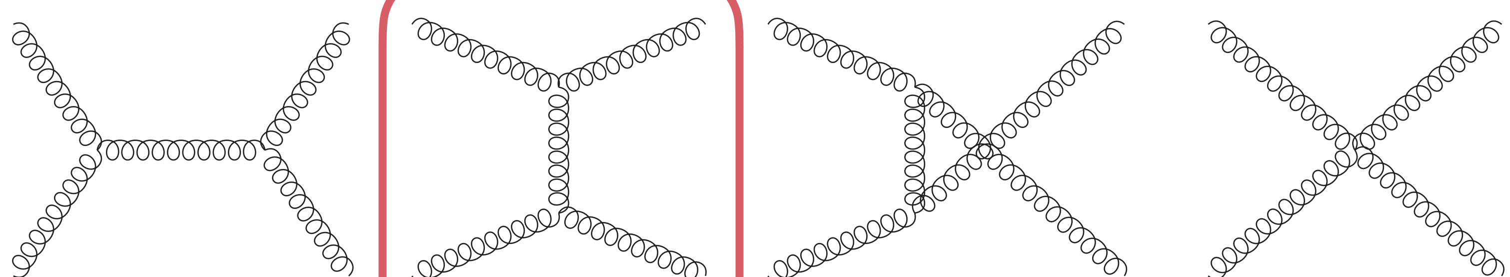

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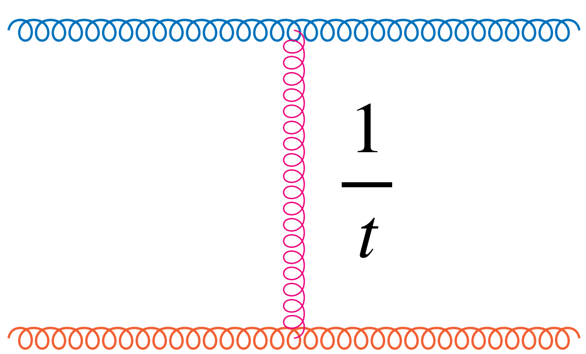

$\frac{1}{t}$

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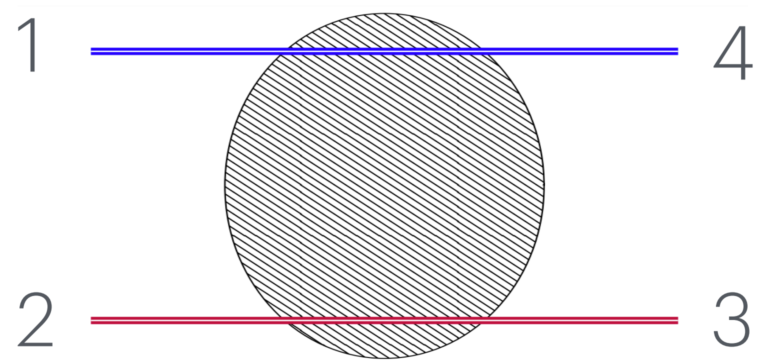

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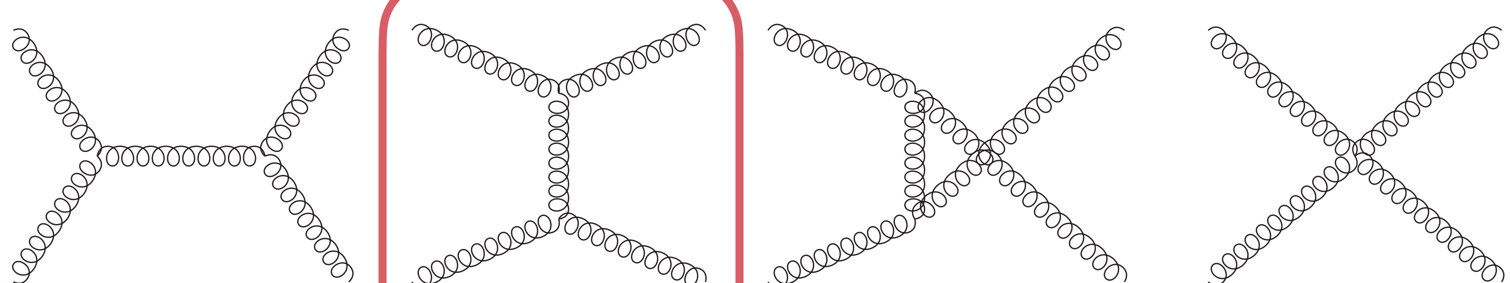

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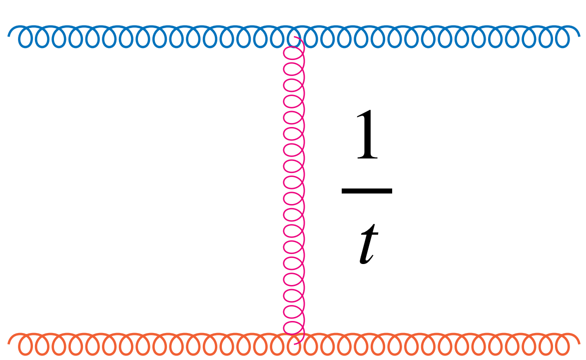
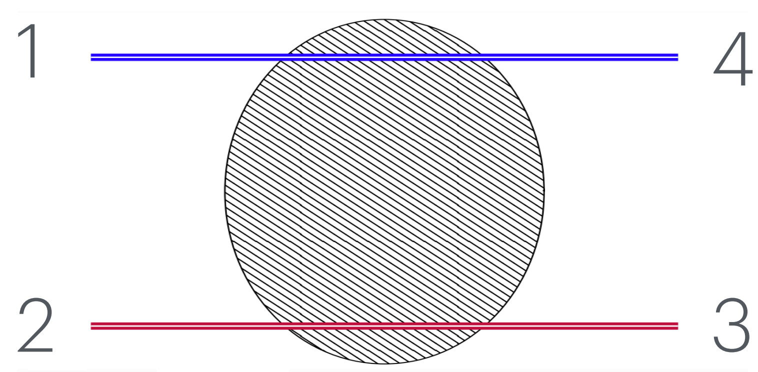


$$\mathscr{A} \propto 1 + \ldots\alpha_s L + \ldots(\alpha_s L)^2 + \ldots(\alpha_s L)^3 + \ldots$$

2 ➜ 2 Scattering



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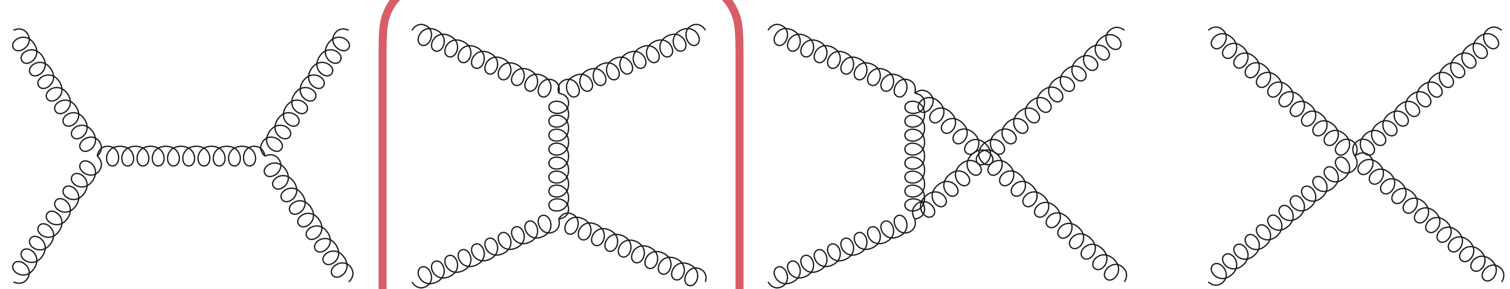

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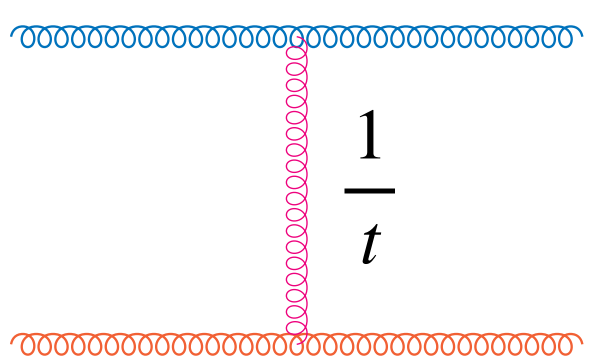
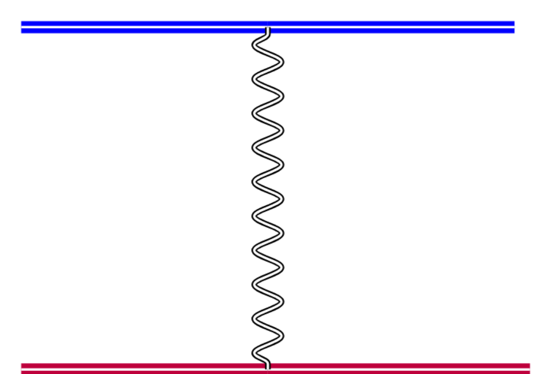


$\frac{1}{t}$

LL resummation



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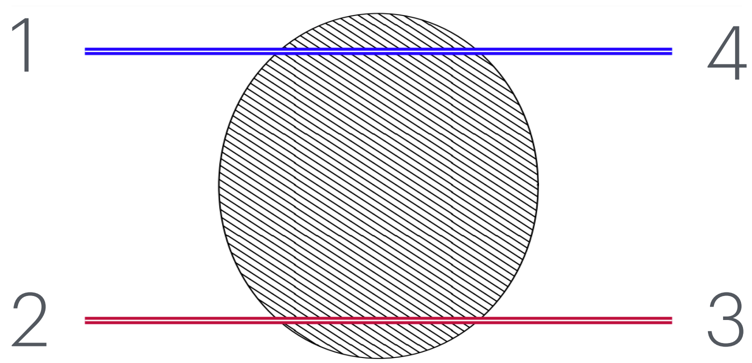

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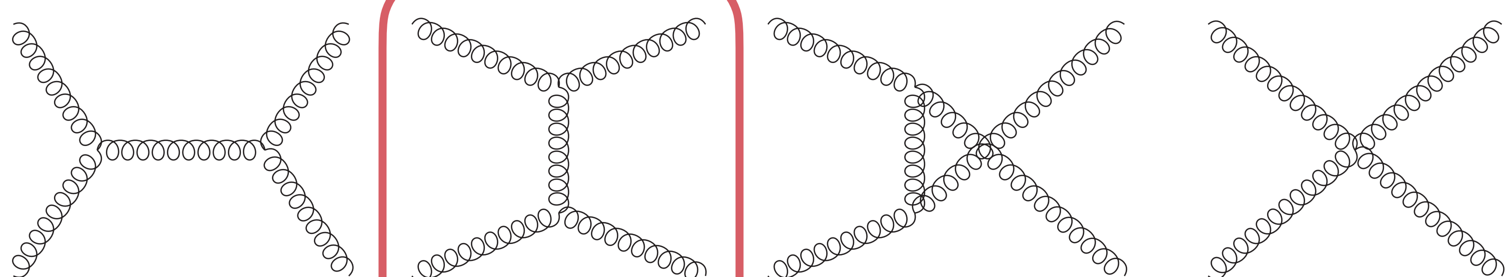

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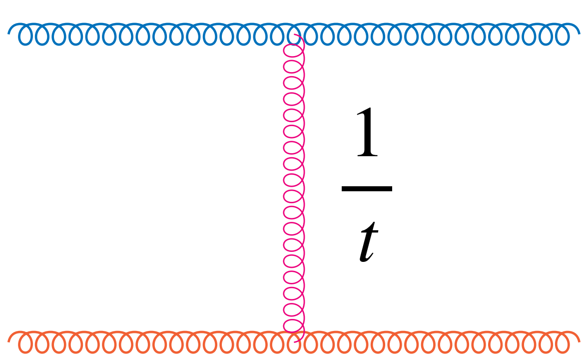
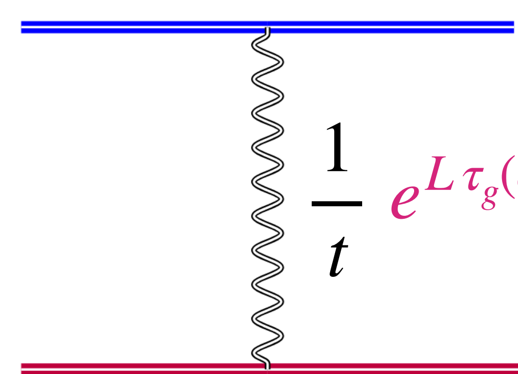


LL resummation →



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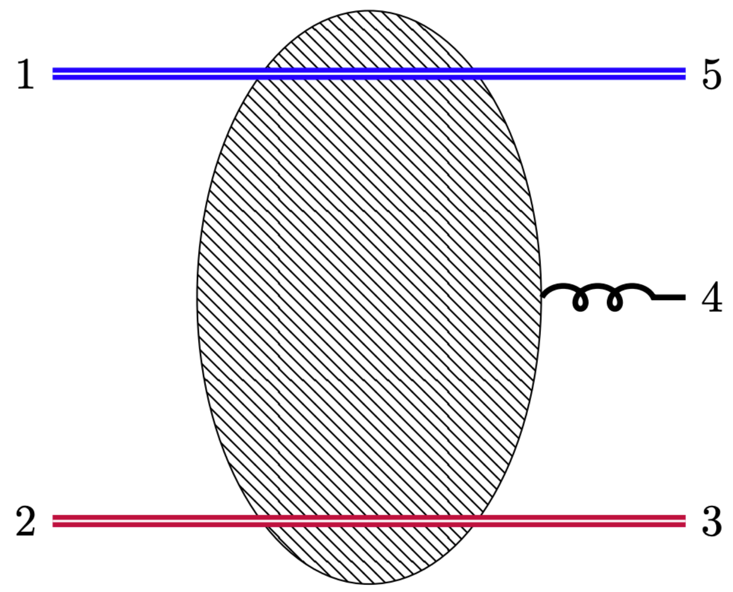
reggeised gluon

2 ➜ 3 Scattering



$$\Delta\eta_{54} \approx \log\left(\frac{s_{45}}{-s_{51}}\right)$$

$$\Delta\eta_{43} \approx \log\left(\frac{s_{34}}{-s_{23}}\right)$$

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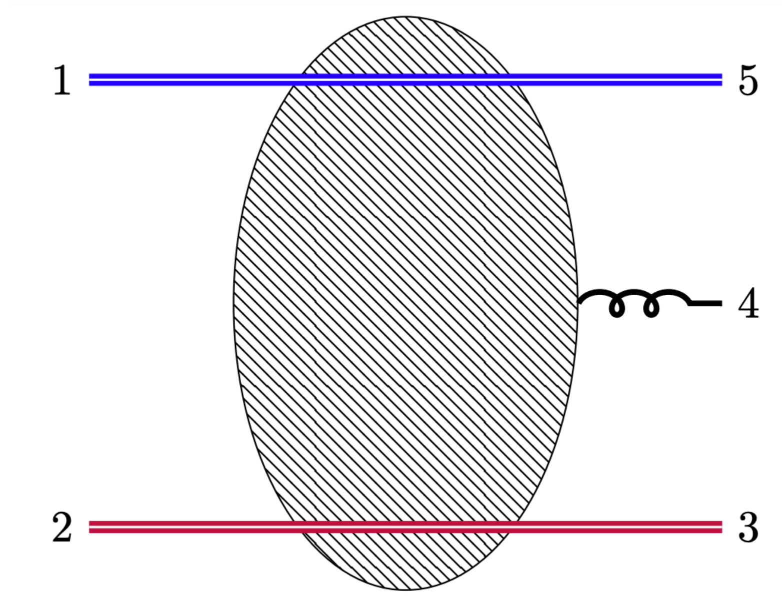

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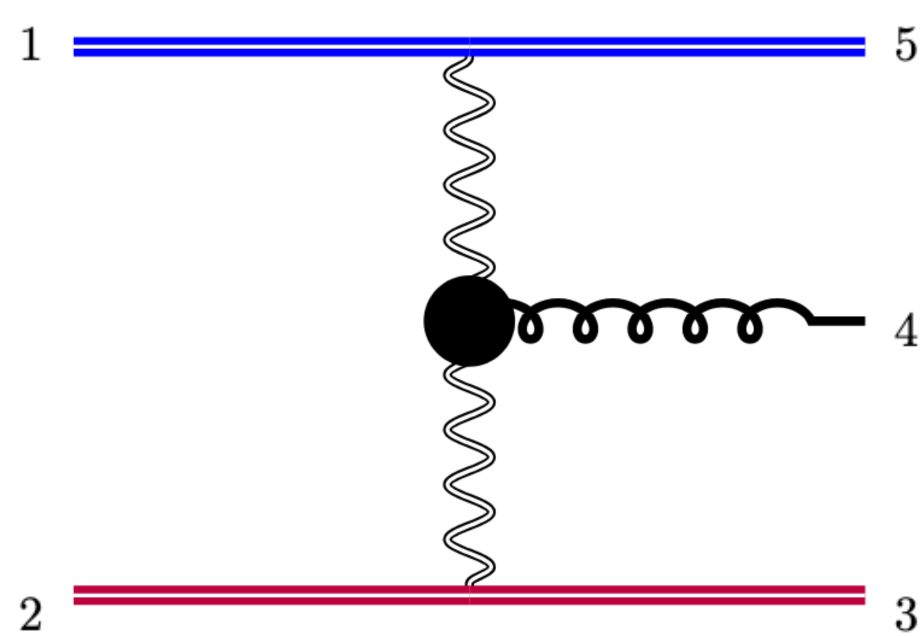

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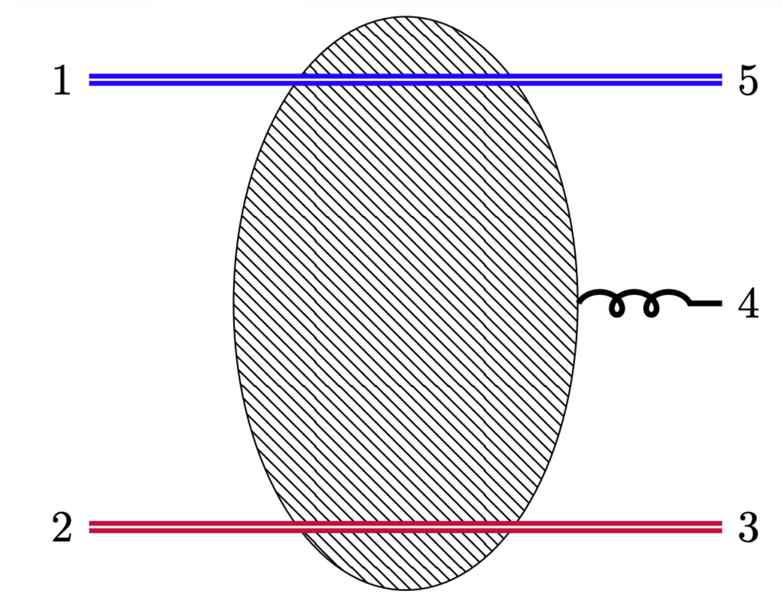

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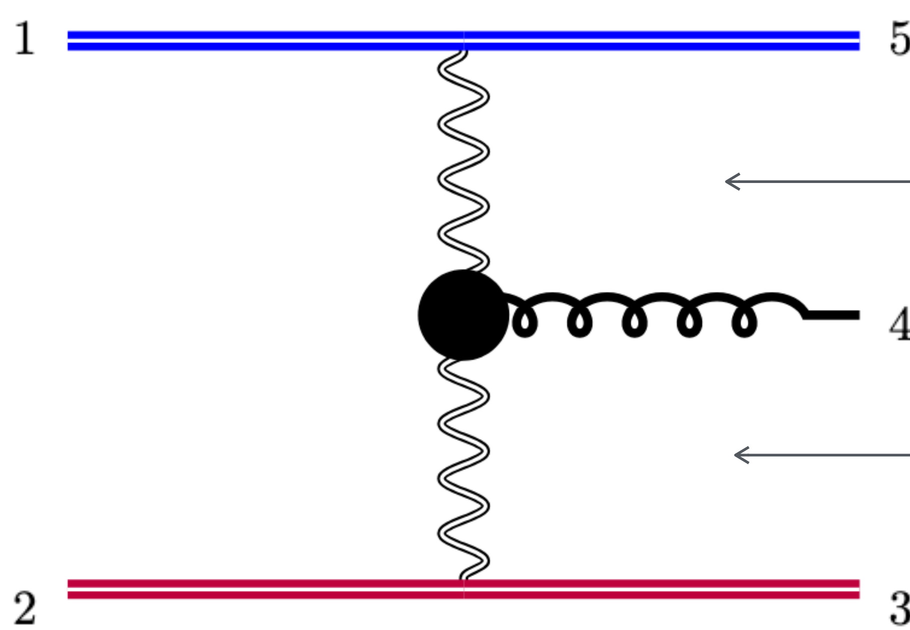

$$\frac{1}{s_{51}}\, e^{L_{45}\,\tau_g(\alpha)}$$

$$\frac{1}{s_{23}}\, e^{L_{34}\,\tau_g(\alpha)}$$

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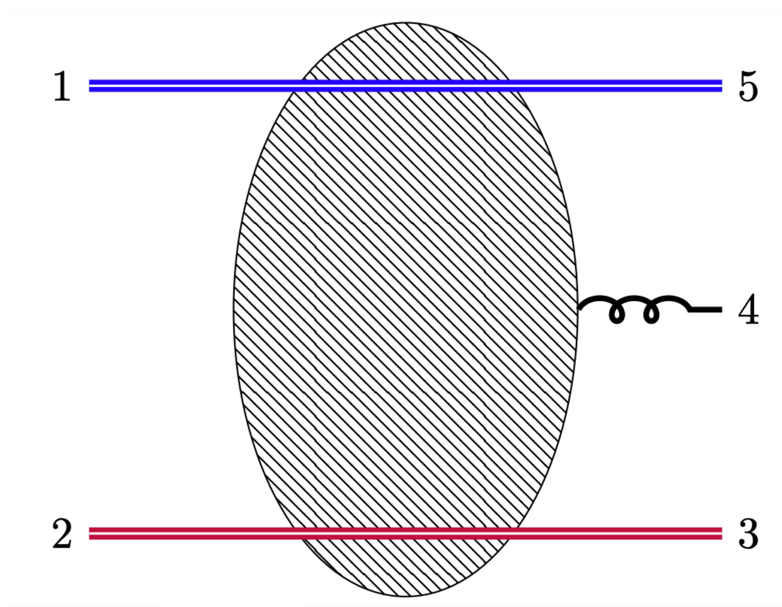

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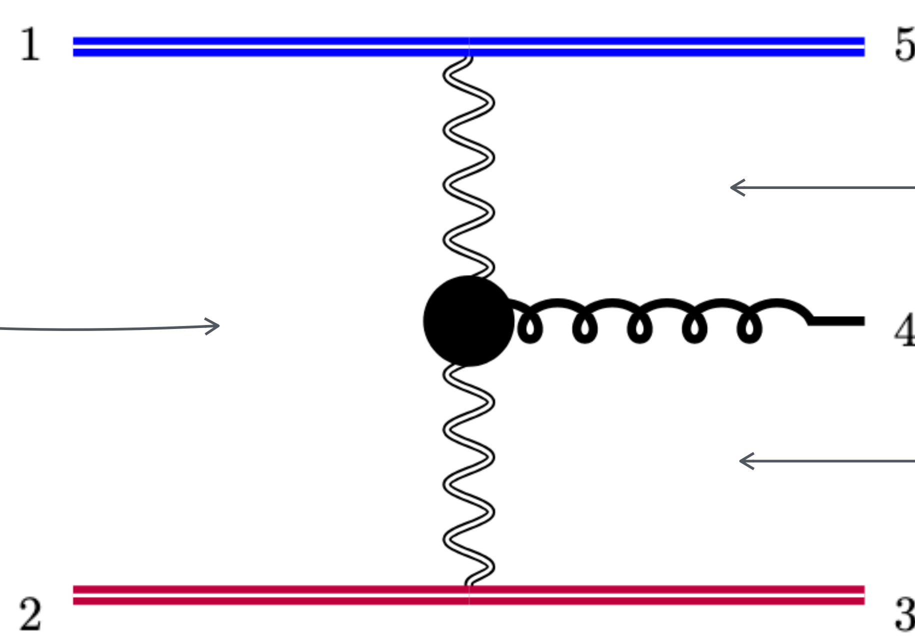

Lipatov vertex

$$\frac{1}{s_{51}} e^{L_{45}\,\tau_g(\alpha)}$$

$$\frac{1}{s_{23}} e^{L_{34}\,\tau_g(\alpha)}$$

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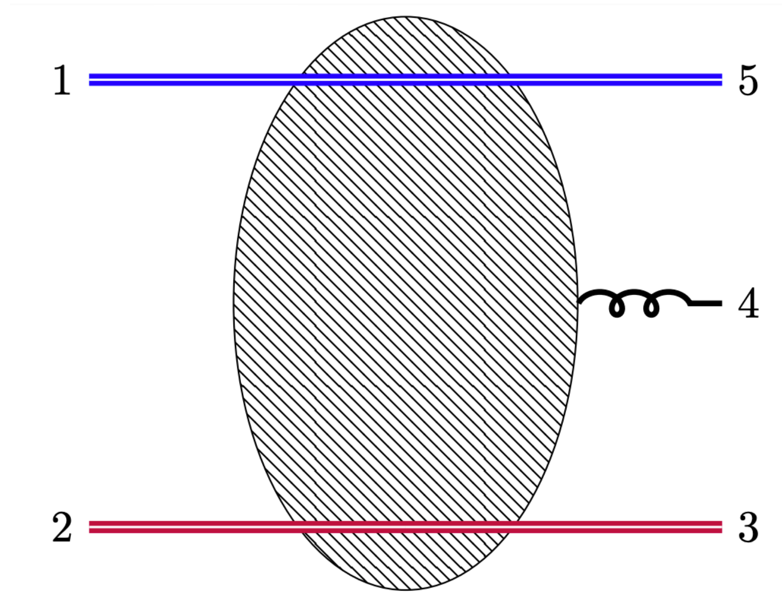

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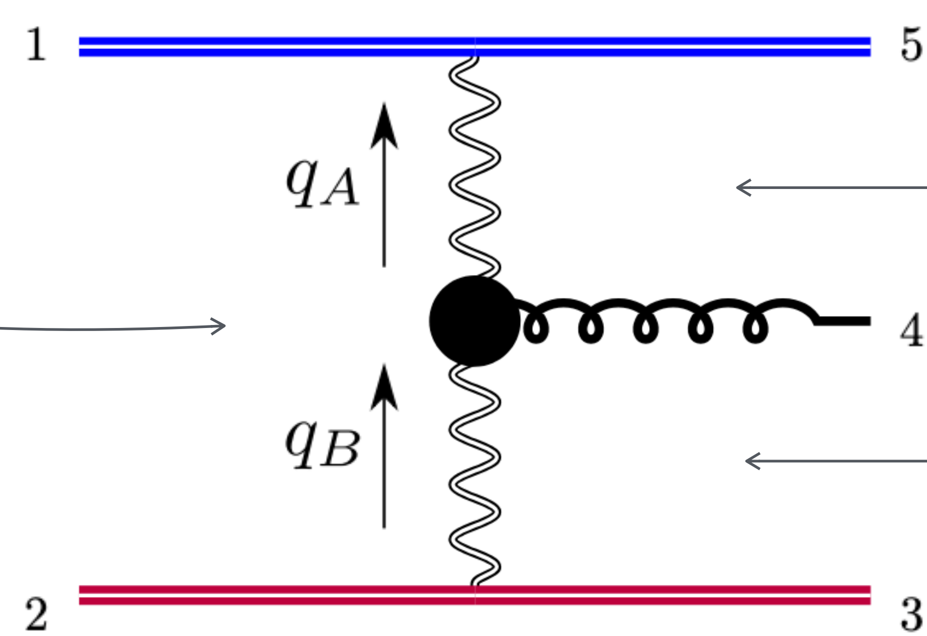


Lipatov vertex

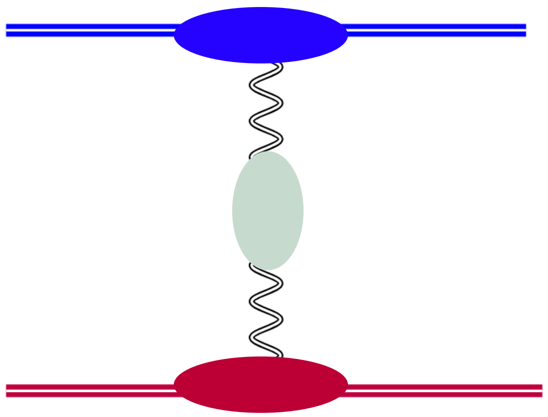
$$V_+(\mathbf{q}_A, \mathbf{p}_4) = \frac{\bar{q}_{A,\perp}\, q_{B,\perp}}{p_{4,\perp}},$$

$$V_-(\mathbf{q}_A, \mathbf{p}_4) = \frac{q_{A,\perp}\, \bar{q}_{B,\perp}}{\bar{p}_{4,\perp}}$$

$$\frac{1}{s_{51}}\, e^{L_{45}\,\tau_g(\alpha)}$$

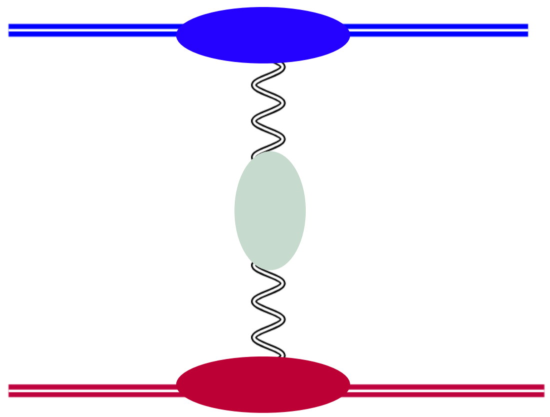
$$\frac{1}{s_{23}}\, e^{L_{34}\,\tau_g(\alpha)}$$

A universal factorised structure



$$\mathcal{I}_1 T^a \cdot \delta^{ab} \left(\frac{s}{t}\right) e^{L_{12}\tau_g} \cdot \mathcal{I}_2 T^b$$

A universal factorised structure



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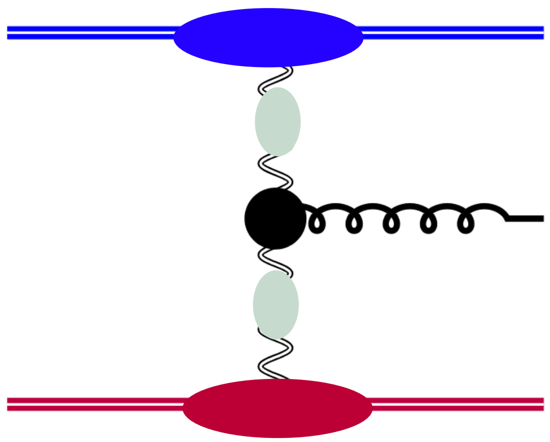
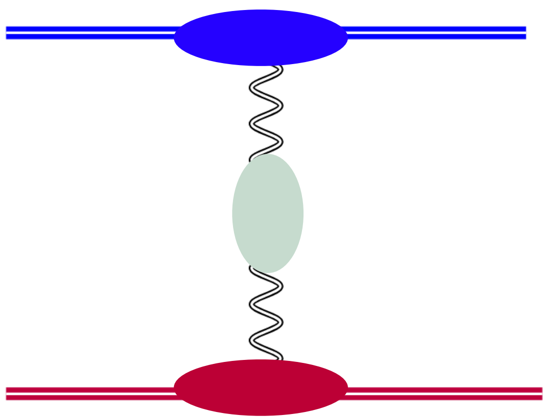


$$\mathcal{I}_1 T^a \cdot \left(\frac{s_{34}}{s_{23}}\right) e^{L_{34}\tau_g} \cdot i f^{abc} V_\lambda \cdot \left(\frac{s_{45}}{s_{51}}\right) e^{L_{45}\tau_g} \cdot \mathcal{I}_2 T^b$$

A universal factorised structure



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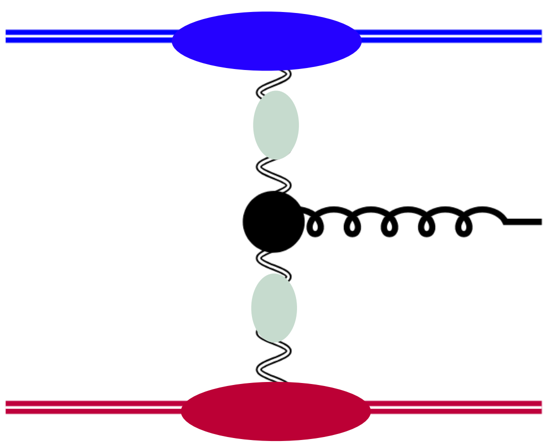
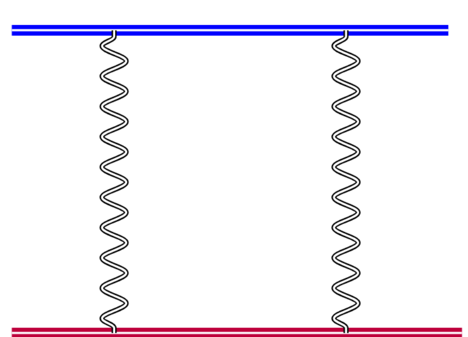
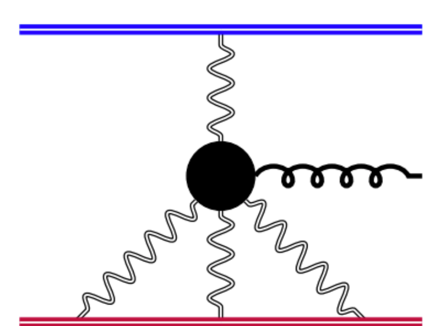


$$\mathcal{I}_1 T^a \cdot \left(\frac{s_{34}}{s_{23}}\right) e^{L_{34}\tau_g} \cdot i f^{abc} V_\lambda \cdot \left(\frac{s_{45}}{s_{51}}\right) e^{L_{45}\tau_g} \cdot \mathcal{I}_2 T^b$$

from NLL: factorisation breaking effects!





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t-channel colour flow basis

$$3 \otimes \bar{3} = (8) \quad \oplus \quad (1)$$

$$8 \otimes 8 = (8_a \oplus 10 \oplus \overline{10}) \quad \oplus \quad (0 \oplus 1 \oplus 8_s \oplus 27)$$

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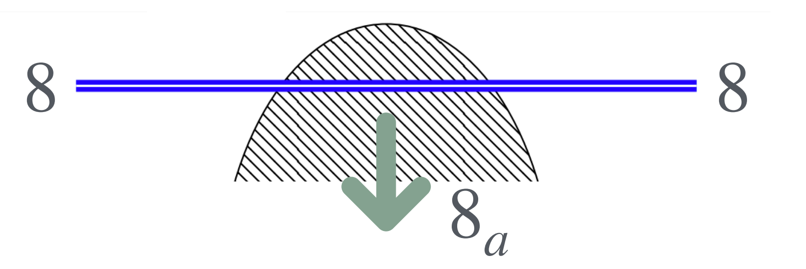

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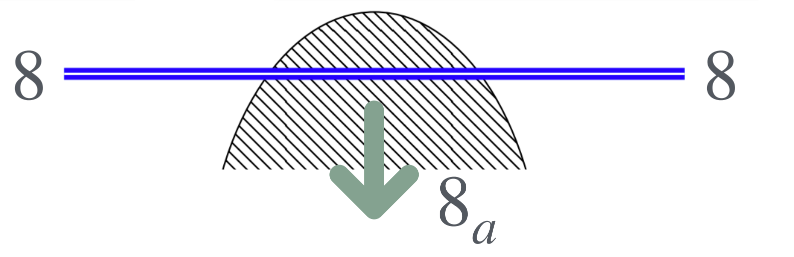

$$A = \sum C_i^{\text{even}} A_i^{\text{even}} + \sum C_i^{\text{odd}} A_i^{\text{odd}}$$

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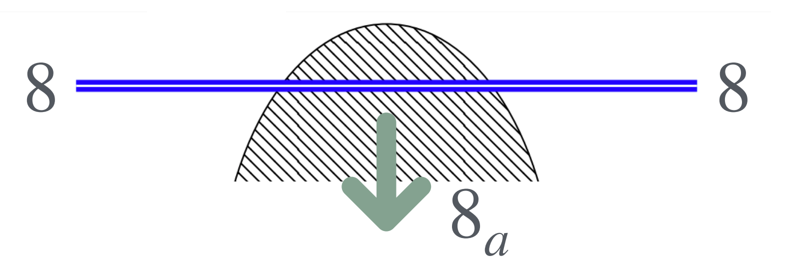

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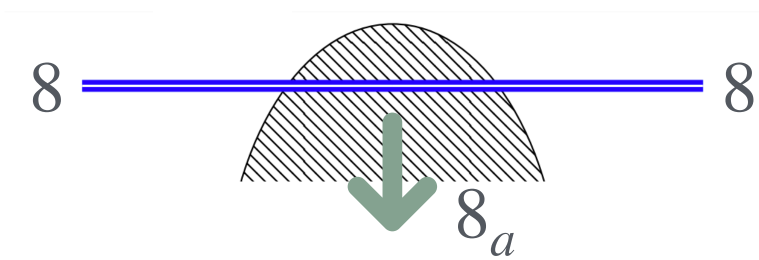

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All even amplitudes lead to logarithmically suppressed cross-section

$gg \to ggg$ MRK colour basis

(r_1, r_2)	$\mathcal{C}_i^{[gg]}$	(r_1, r_2)	$\mathcal{C}_i^{[gg]}$
$(8_a, 8_a)$	$if^{a_5ba_1}\, if^{a_3ca_2}\, if^{bca_4}$	$(10, 8_a)$	$\mathcal{T}_{10+\overline{10}}^{a_4ba_1a_5} if^{a_3ba_2}$
$(8_a, 8_s)$	$\frac{N_c^2}{N_c^2-4}\, if^{a_5ba_1}\, d^{a_3ca_2}\, d^{bca_4}$	$(8_s, 10)$	$\frac{N_c^2}{N_c^2-4}\, d^{a_5ba_1} \mathcal{T}_{10-\overline{10}}^{a_4ba_2a_3}$
$(8_s, 8_a)$	$\frac{N_c^2}{N_c^2-4}\, d^{a_5ba_1}\, if^{a_3ca_2}\, d^{bca_4}$	$(10, 8_s)$	$\frac{N_c^2}{N_c^2-4}\, \mathcal{T}_{10-\overline{10}}^{a_4ba_1a_5} d^{a_3ba_2}$
$(8_s, 8_s)$	$\frac{N_c^2}{N_c^2-4}\, d^{a_5ba_1}\, d^{a_3ca_2}\, if^{bca_4}$	$(27, 27)$	$\mathcal{T}_{27}^{bca_2a_3} \mathcal{T}_{27}^{bea_1a_5}\, if^{ca_4e}$
$(1, 8_a)$	$\frac{N_c^2}{N_c^2-1}\, \delta^{a_5a_1} if^{a_3a_4a_2}$	$(0, 0)$	$\mathcal{T}_{0}^{bca_2a_3} \mathcal{T}_{0}^{bea_1a_5}\, if^{ca_4e}$
$(8_a, 1)$	$\frac{N_c^2}{N_c^2-1}\, if^{a_5a_4a_1} \delta^{a_3a_2}$	$(10, 0)$	$\mathcal{T}_{10+\overline{10}}^{bca_1a_5} \mathcal{T}_{0}^{bea_2a_3}\, if^{ea_4c}$
$(8_a, 0)$	$if^{a_5ba_1} \mathcal{T}_0^{a_4ba_2a_3}$	$(0, 10)$	$\mathcal{T}_{0}^{bea_1a_5} \mathcal{T}_{10+\overline{10}}^{bca_2a_3}\, if^{ca_4e}$
$(0, 8_a)$	$\mathcal{T}_0^{a_4ba_1a_5} if^{a_3ba_2}$	$(10, 27)$	$\mathcal{T}_{10+\overline{10}}^{bea_1a_5} \mathcal{T}_{27}^{bca_2a_3}\, if^{ca_4e}$
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$(27, 8_a)$	$\mathcal{T}_{27}^{a_4ba_1a_5} if^{a_3ba_2}$	$(10, 10)_1$	$\mathcal{T}_{10+\overline{10}}^{bea_1a_5} \mathcal{T}_{10+\overline{10}}^{bca_2a_3}\, if^{ca_4e}$
$(8_a, 10)$	$if^{a_5ba_1} \mathcal{T}_{10+\overline{10}}^{a_4ba_2a_3}$	$(10, 10)_2$	$\mathcal{T}_{10+\overline{10}}^{bea_1a_5} \left(\mathcal{T}_{10-\overline{10}}^{bca_2a_3}\, d^{ca_4e} - \mathcal{T}_{10+\overline{10}}^{bca_2a_3}\, if^{ca_4e}/N_c\right)$

Shockwave Formalism for amplitudes

Mueller, Balitsky, Kovchegov, Jalilian-Marian,
Iancu, McLerran, Weigert, Leonidov, Kovner

When Does The Gluon Reggeize?

Simon Caron-Huot[a,b]

The main idea

1. Rapidity factorisation (OPE) + operator basis
2. Rapidity evolution: $\partial_\eta \hat{O}(\eta) = \ \ldots$
3. Equal-rapidity correlators

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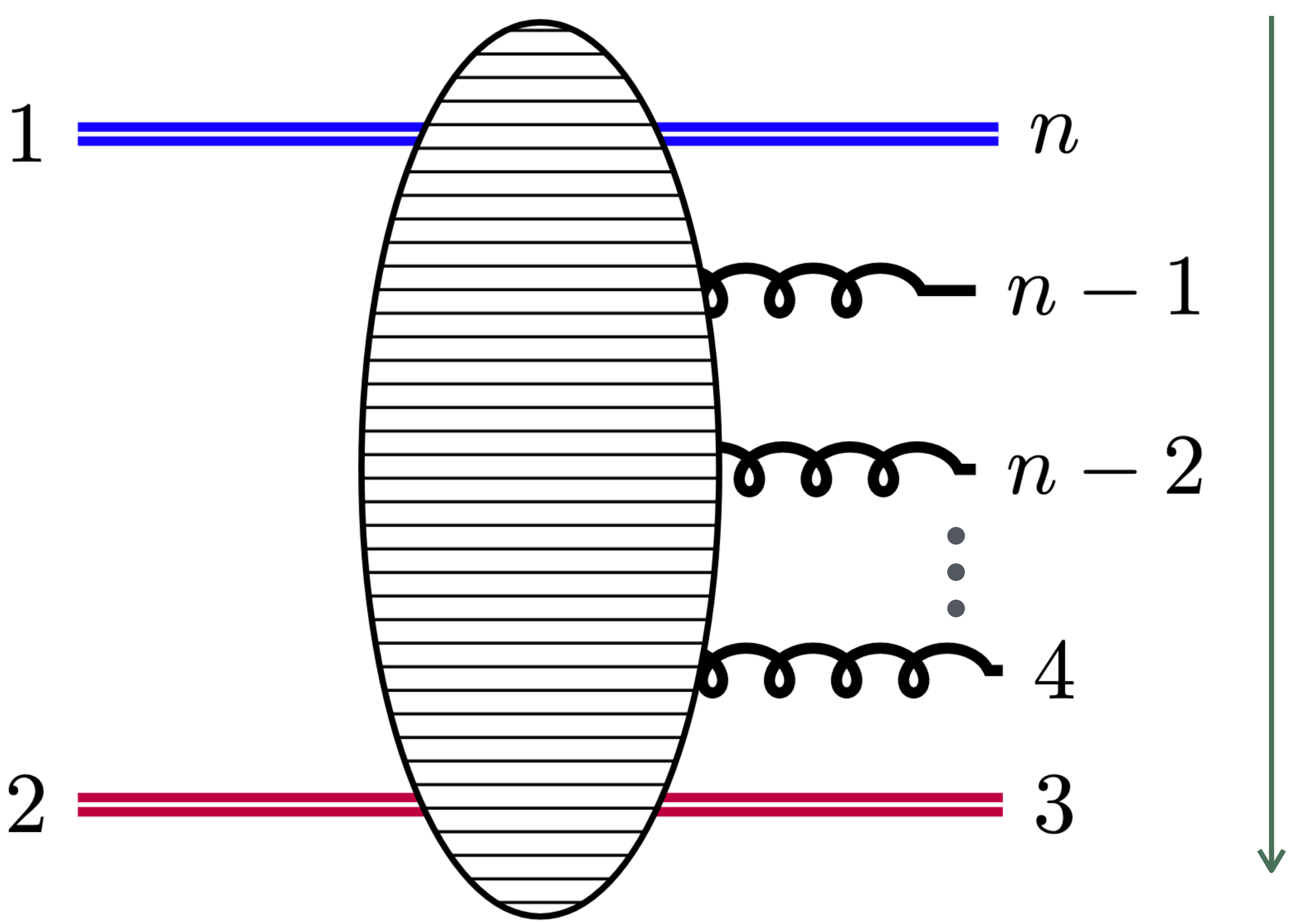

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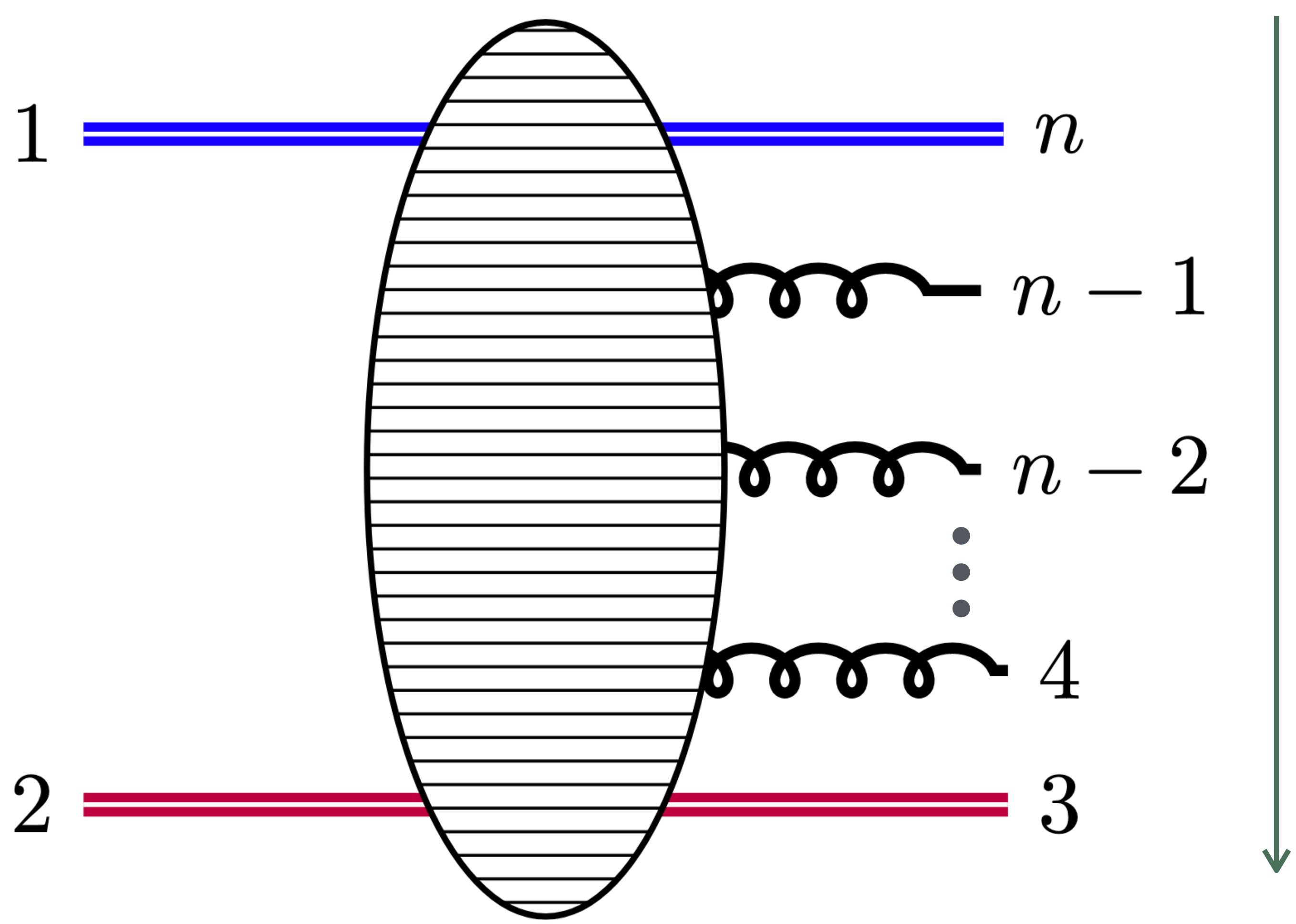

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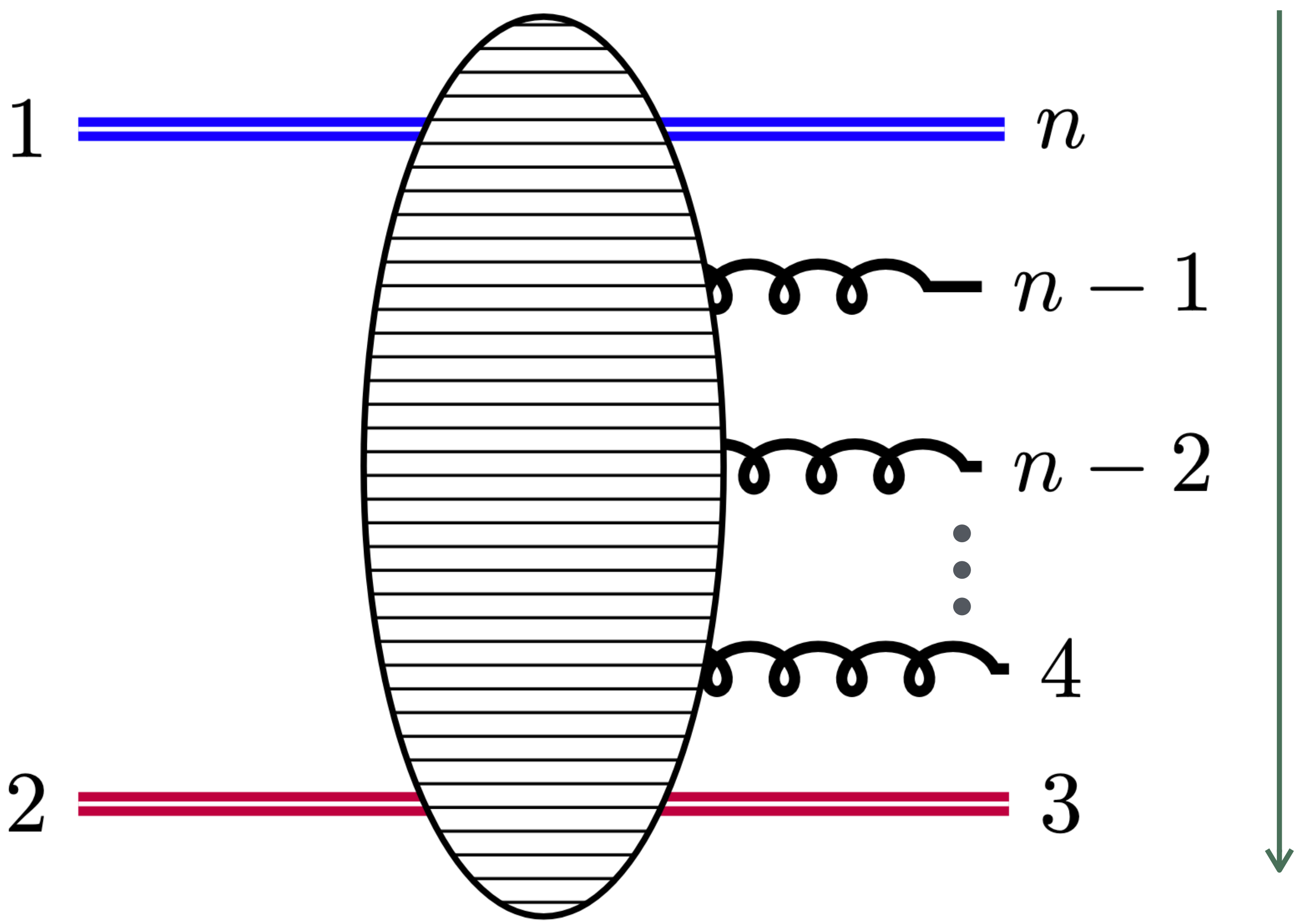

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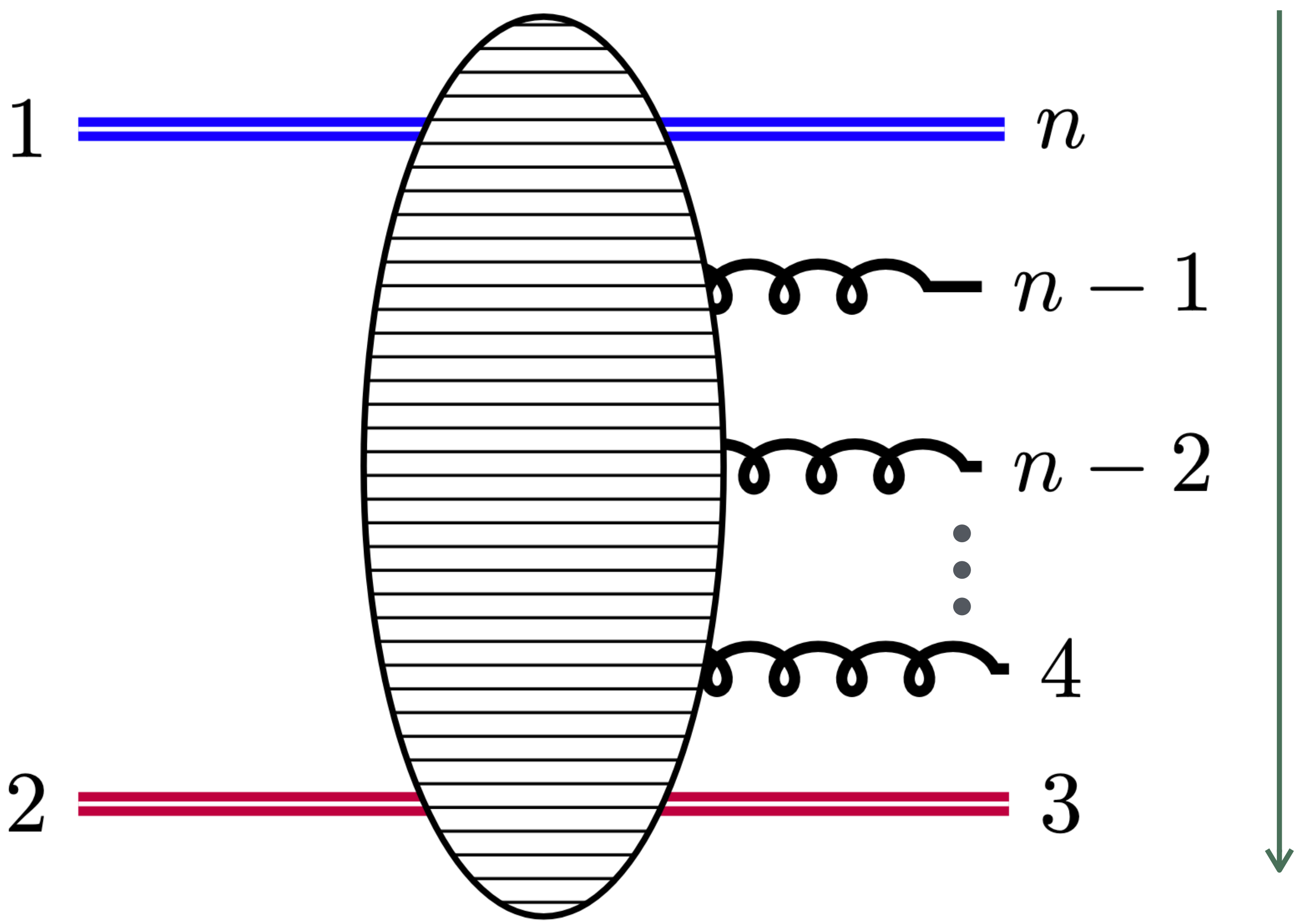

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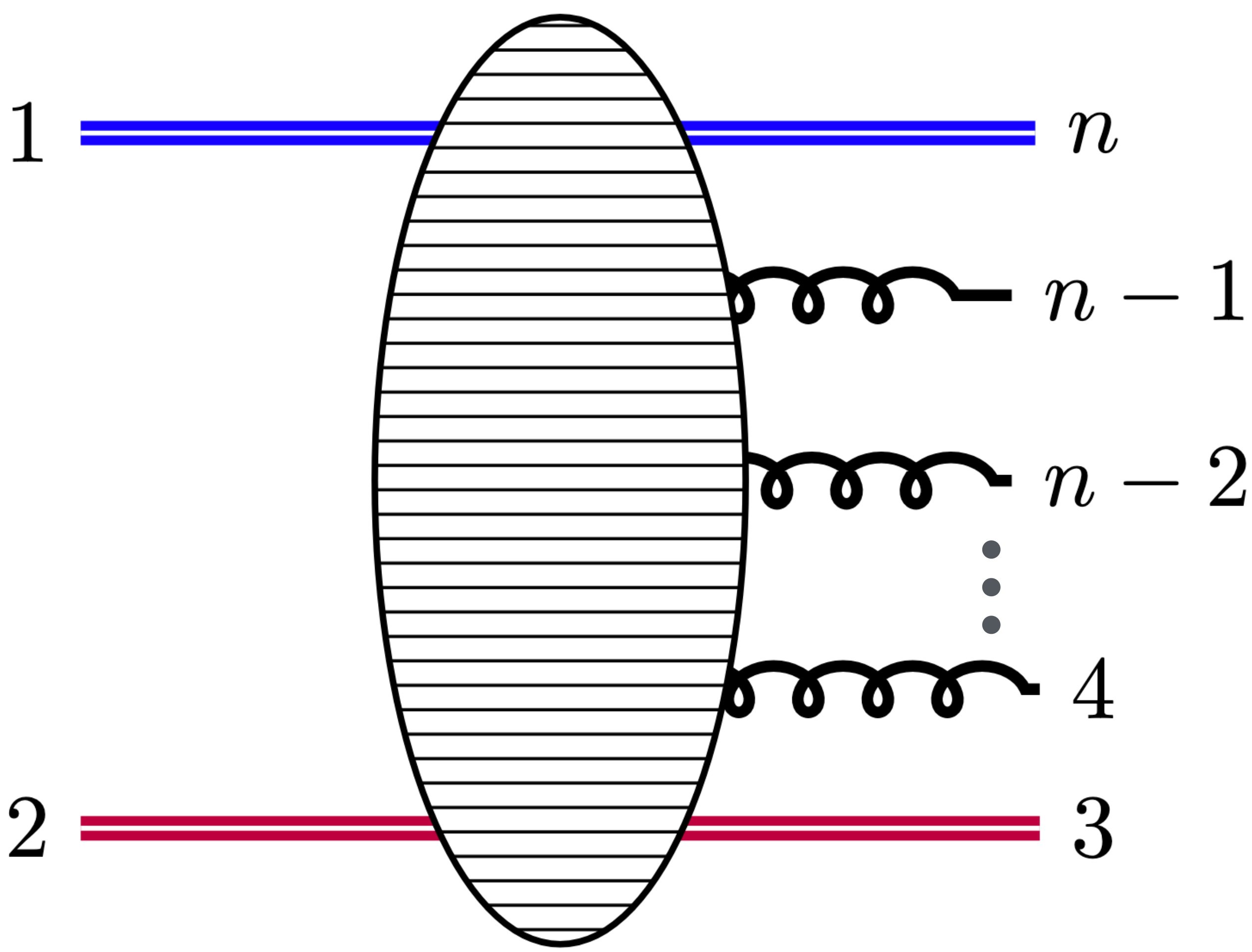

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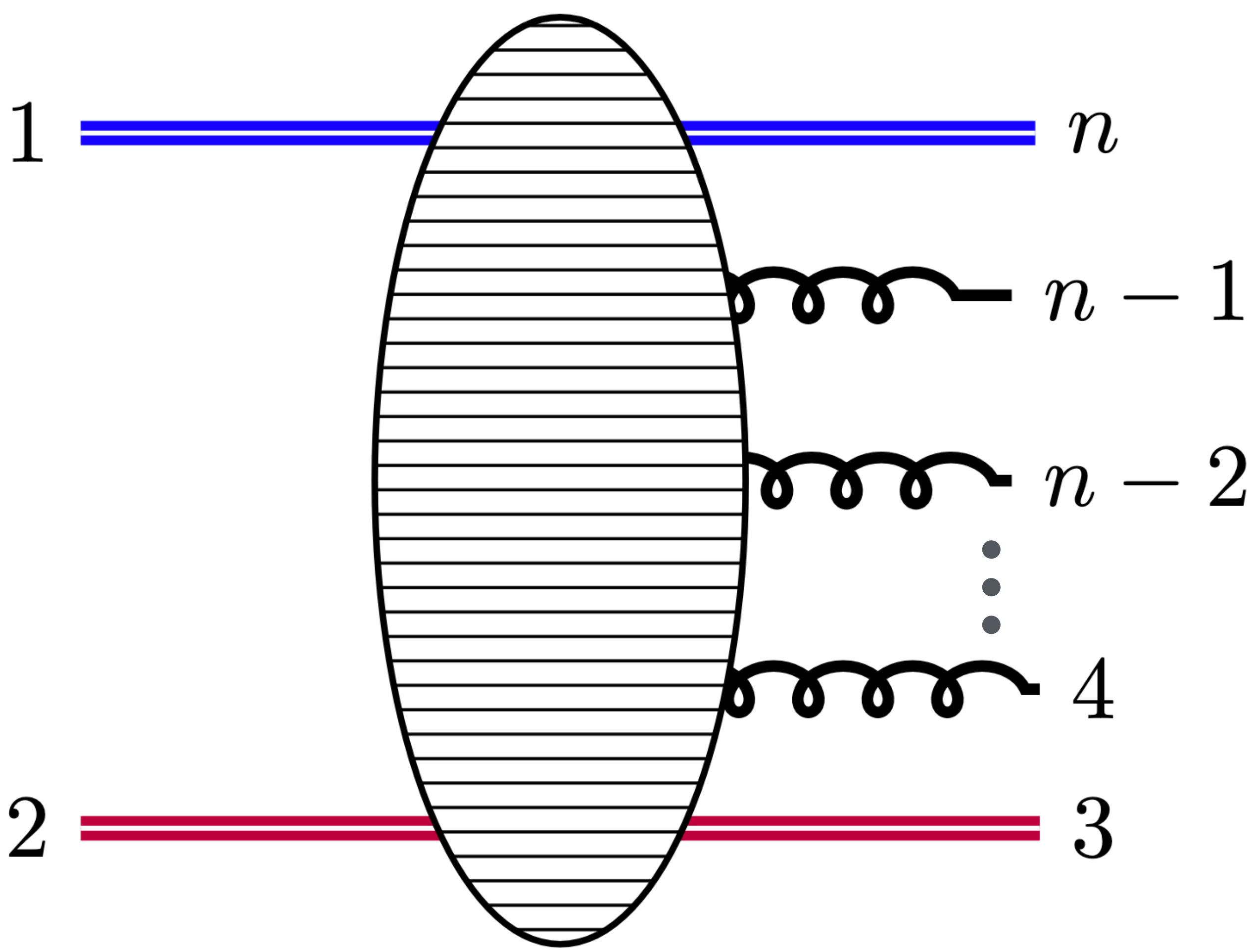

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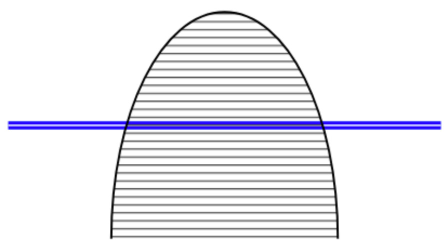
$$\langle \hat{O}_1(\eta_3, \mathbf{q_1}) \hat{O}_2(\eta_3, \mathbf{q_2}) \ldots \rangle$$

Operator basis:

straight, infinite, undecorated Wilson lines



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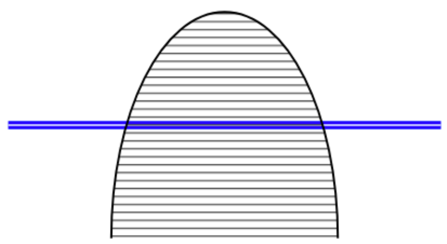

$$U_\eta(z) = \mathbb{P}\exp\left(ig\int_{-\infty}^{+\infty} dx^- A_-^a(x^+ = 0, x^-, z)T^a\right)$$

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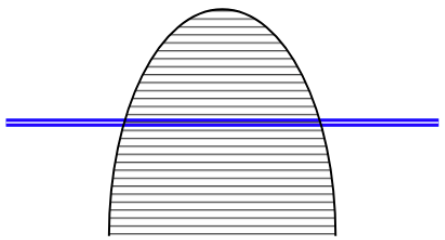

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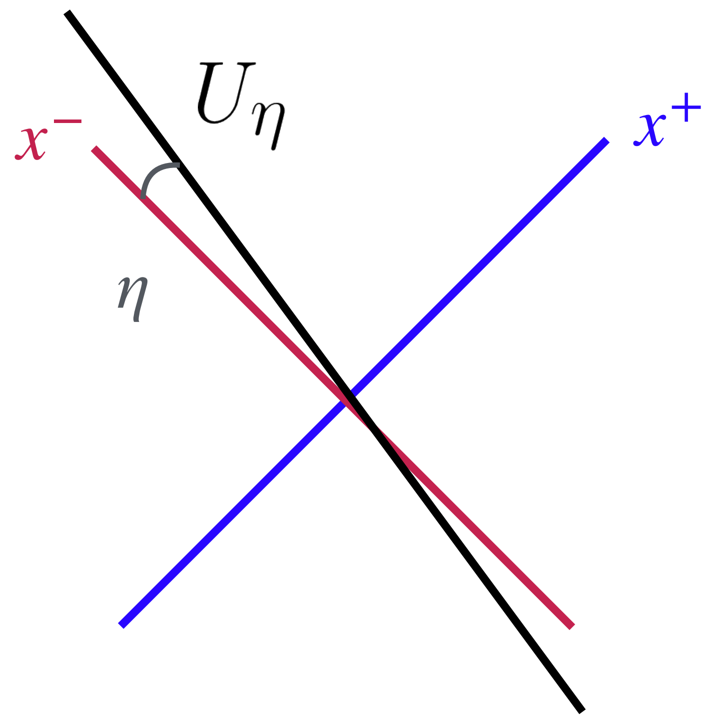

the tilt regulates the Wilson line

Rapidity evolution: $$-\frac{d}{d\eta}[U(z_1)U(z_2)\dots U(z_n)] = H\cdot[U(z_1)U(z_2)\dots U(z_n)]$$

Balitsky-JIMWLK equation

[Balitsky, Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner]

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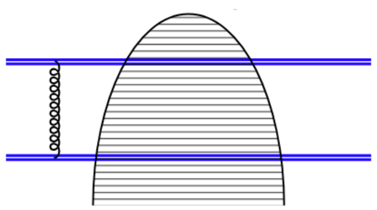
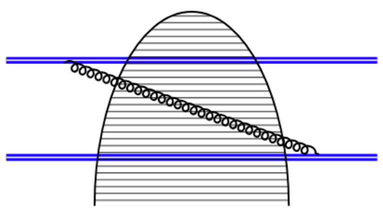
dipoles at leading order

$$H_{ij} = \frac{\alpha_s}{4\pi}\int \mathrm{d}^{2-2\epsilon} z_0 K_{ij}(z_0)\left[T^a_{i,L}T^a_{j,L} - U^{ab}_{adj.}(z_0)T^a_{i,L}T^a_{j,R} + (i\leftrightarrow j)\right]$$

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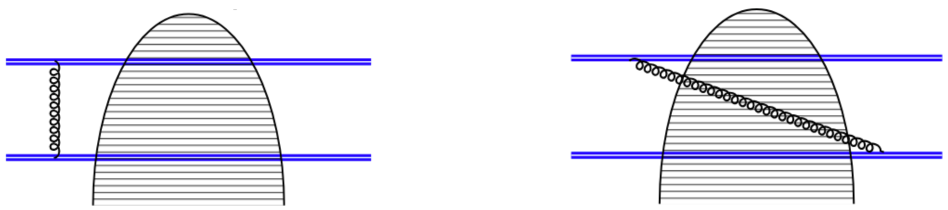

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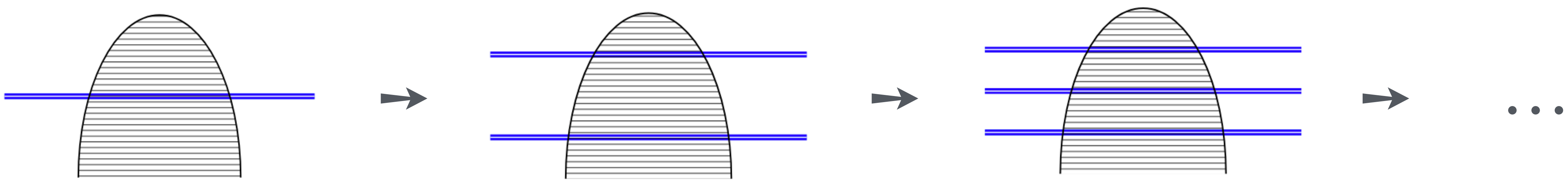

unbounded system of coupled equations!

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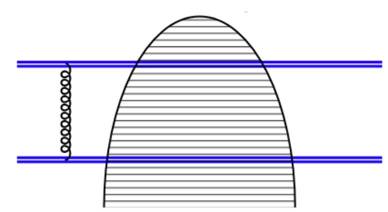

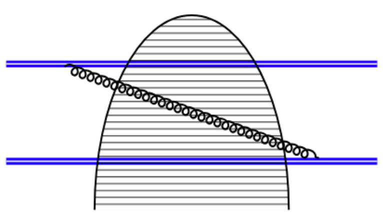

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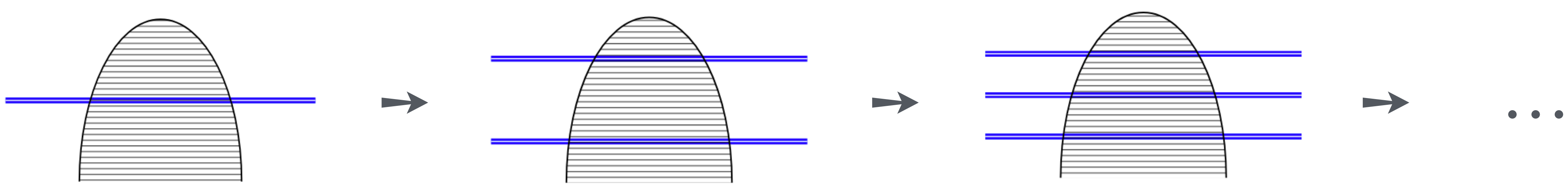

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solution: linearisation

linearisation

$$U(z) = \exp\left(ig\, W^a(z)T^a\right) \approx 1 + ig\, W^a(z)T^a + \frac{(ig)^2}{2}\, W^a(z)W^b(z)T^aT^b + \ldots$$

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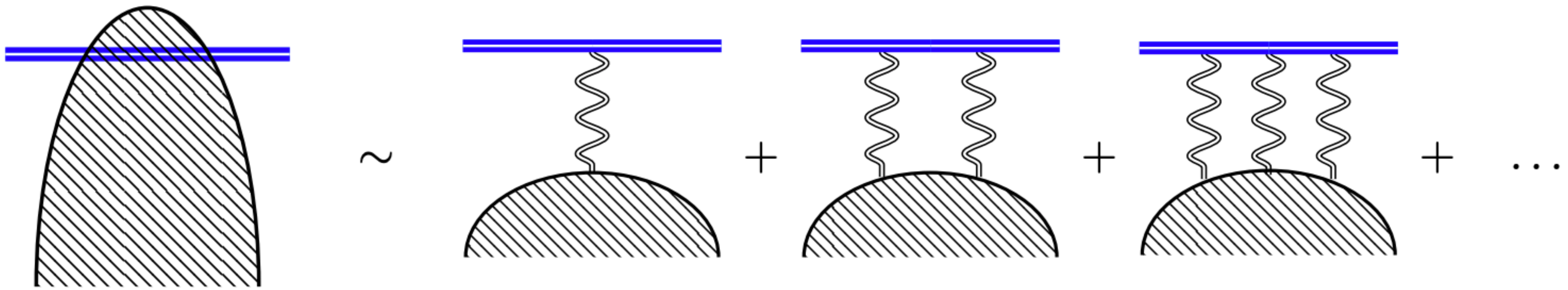

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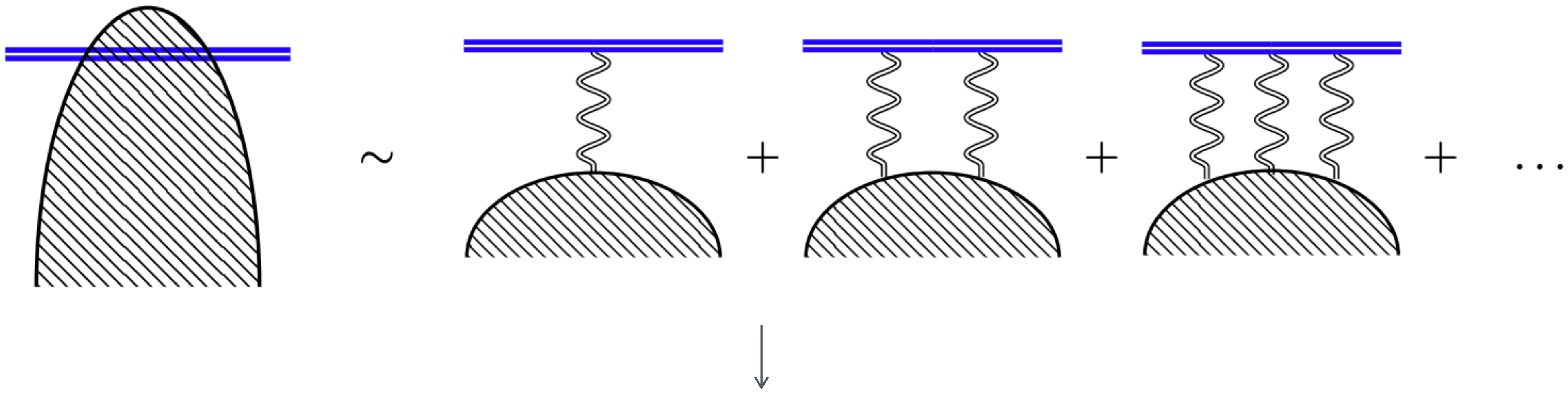

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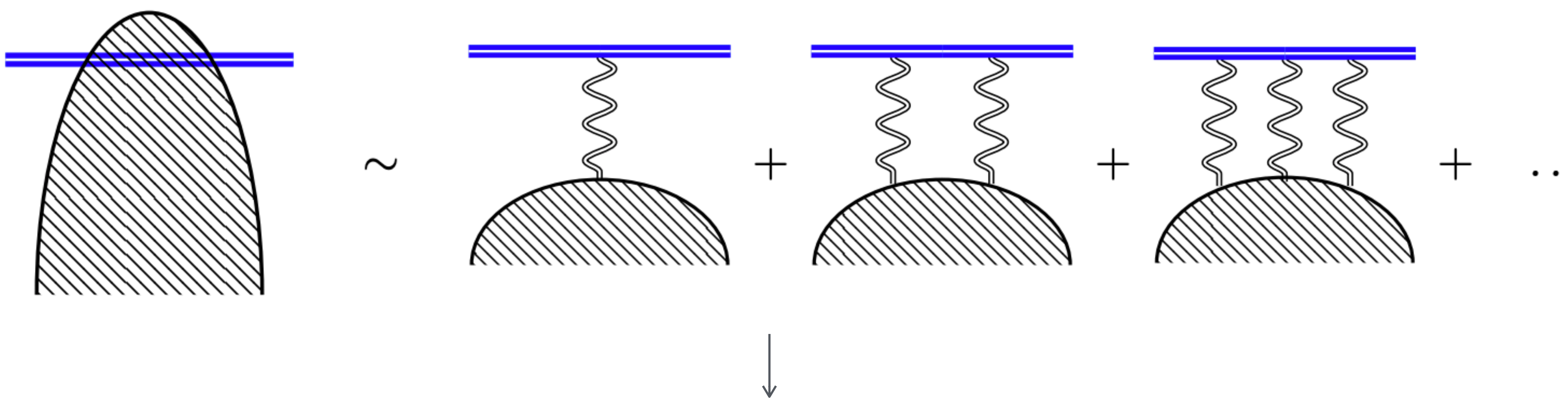

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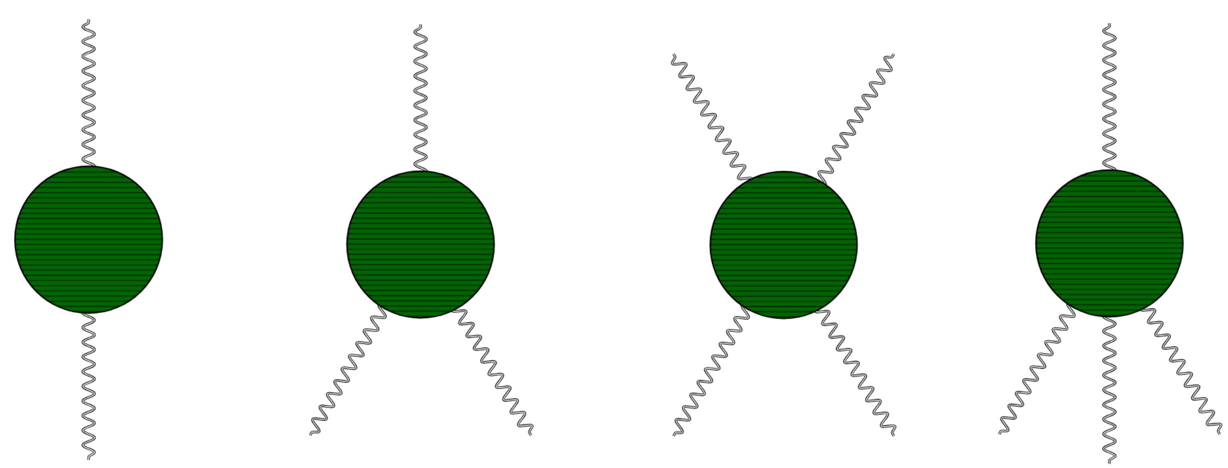

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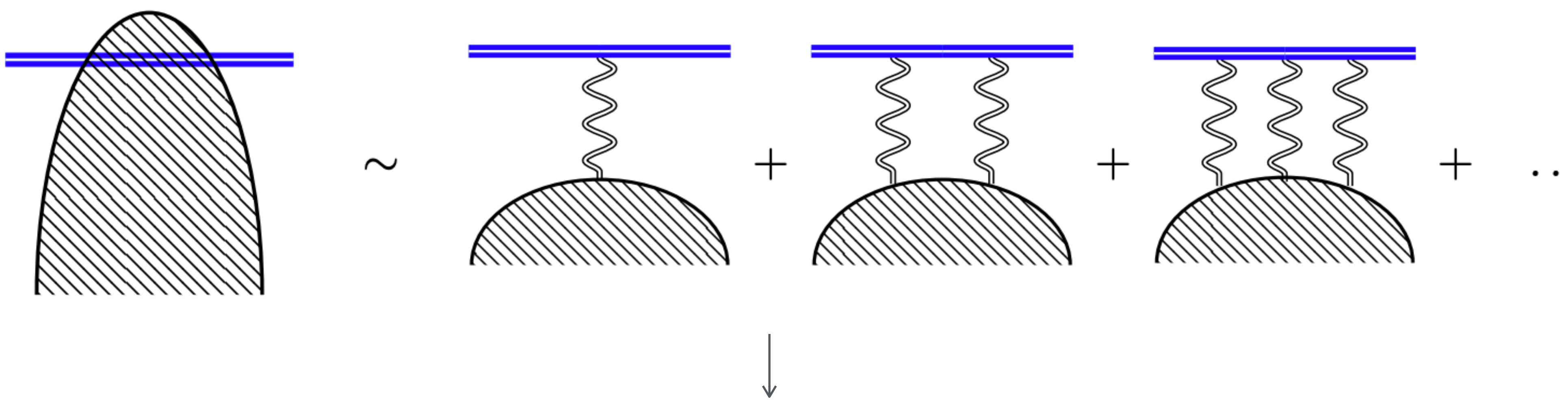

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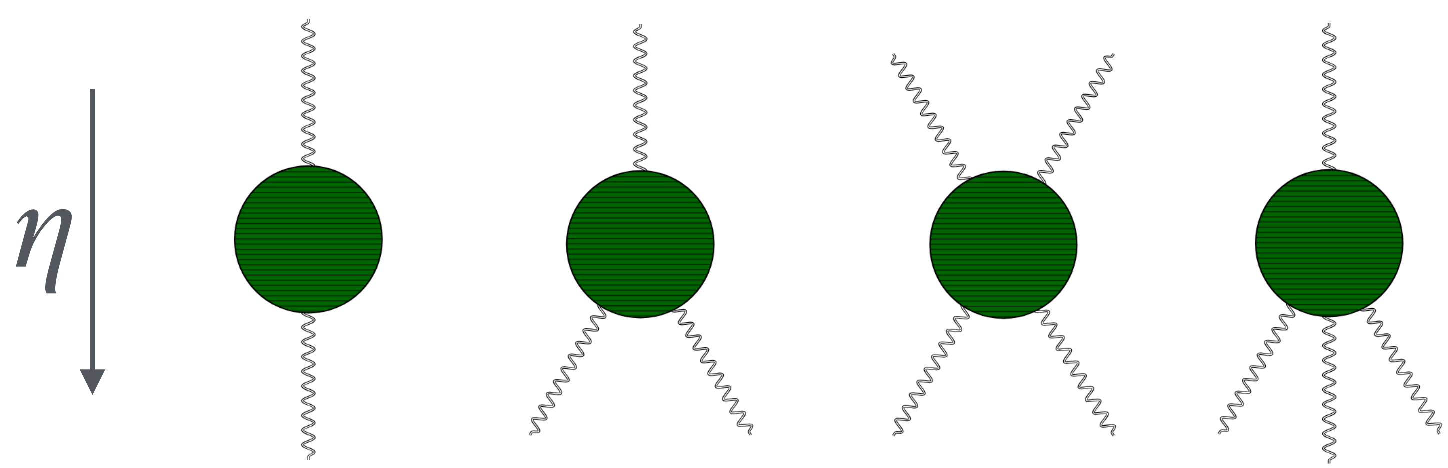

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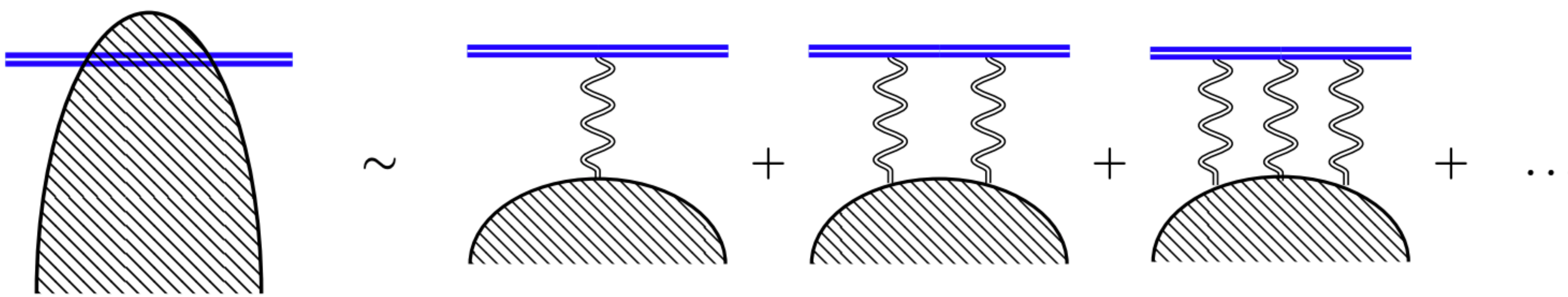

$$\downarrow$$

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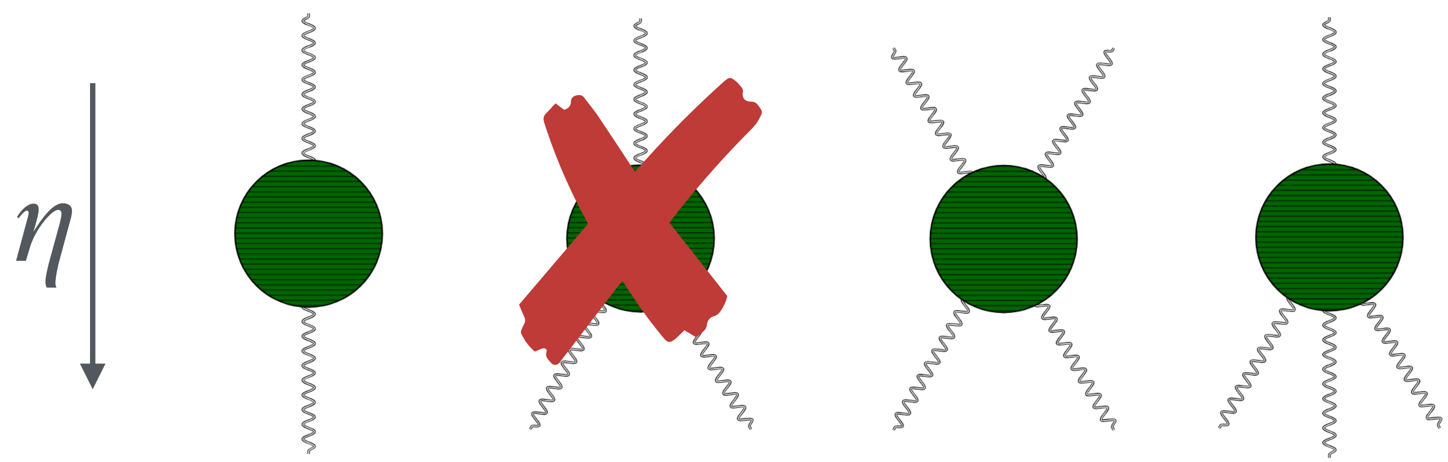

Line reversal $W^a \to -W^a$

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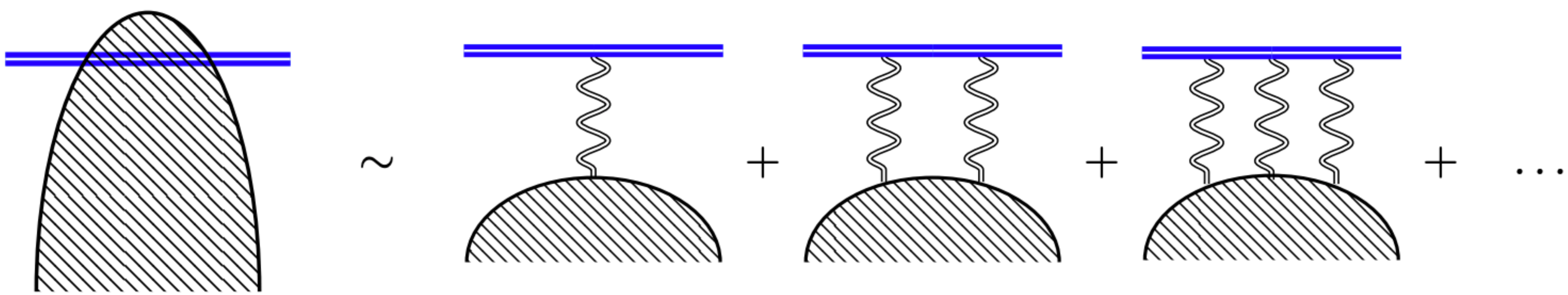
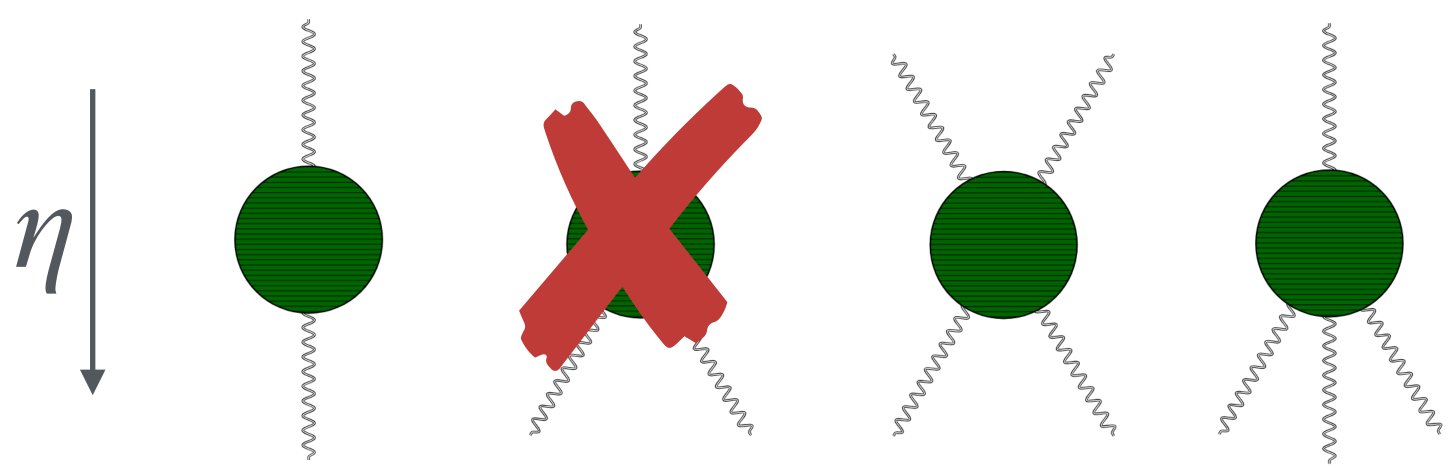


$$-\frac{d}{d\eta}[W(p_1)W(p_2)\ldots W(p_n)] = H\cdot[W(p_1)W(p_2)\ldots W(p_n)]$$

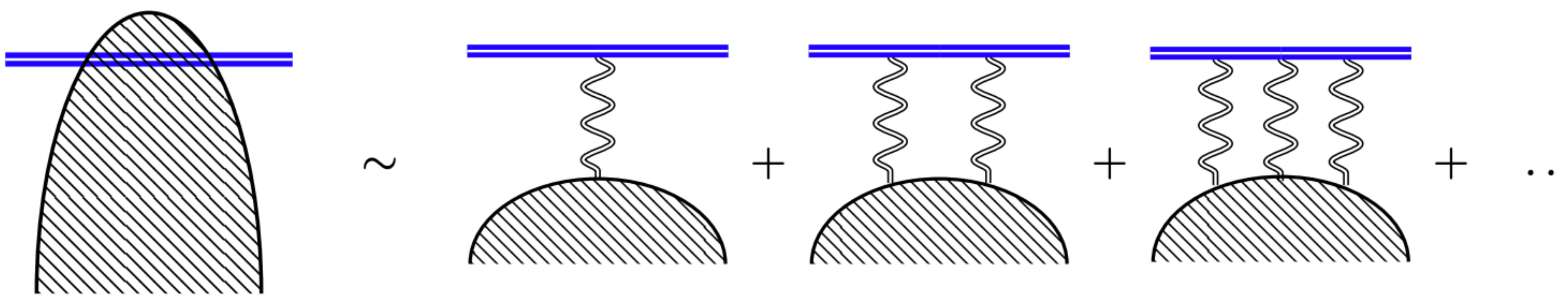


Line reversal $W^a \to -W^a$

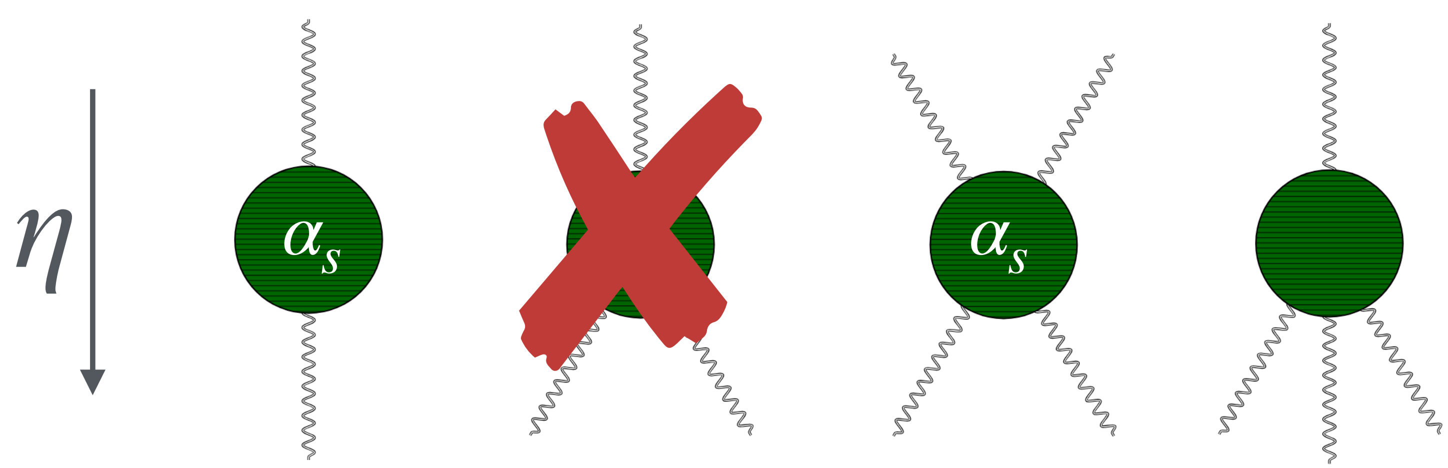
$$S(W^n \to W^{n\pm 2m}) \sim \alpha_s \alpha_s^m$$

linearisation

$$U(z) = \exp\left(ig\, W^a(z) T^a\right)$$
$$\approx 1 + ig\, W^a(z) T^a + \frac{(ig)^2}{2} W^a(z) W^b(z) T^a T^b + \dots$$



$$-\frac{d}{d\eta}[W(p_1)W(p_2)\dots W(p_n)] = H\cdot[W(p_1)W(p_2)\dots W(p_n)]$$



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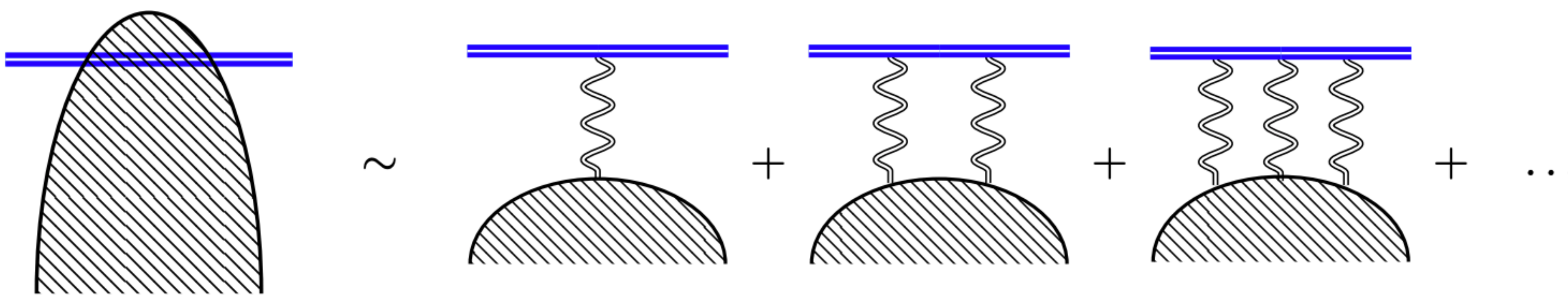
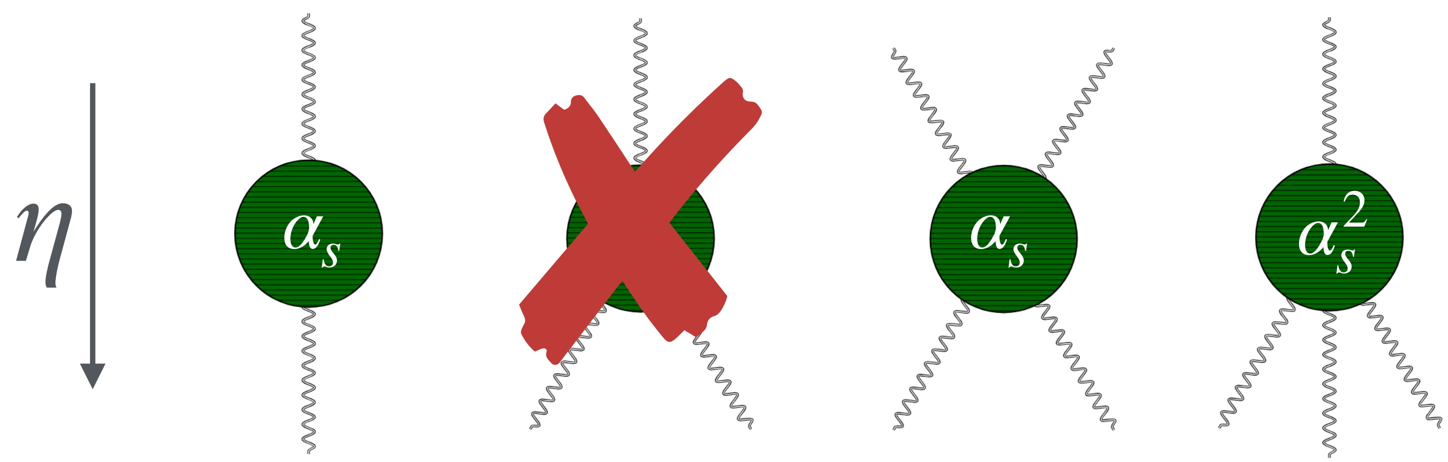


$$\downarrow$$

$$-\frac{d}{d\eta}[W(p_1)W(p_2)\dots W(p_n)] = H\cdot[W(p_1)W(p_2)\dots W(p_n)]$$



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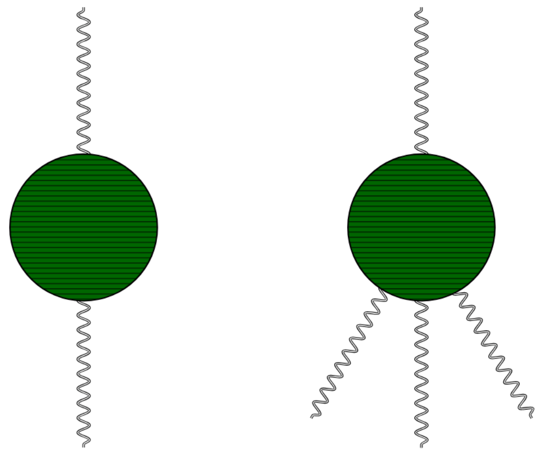
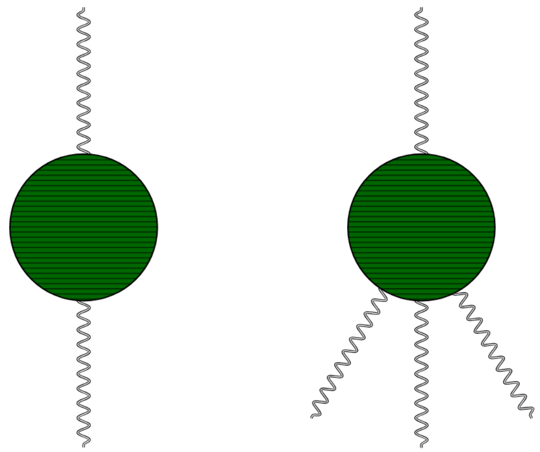


$$W_\eta = e^{(\eta' - \eta)\tau_g}\, W_{\eta'} + \mathcal{O}(\mathrm{NNLL})$$

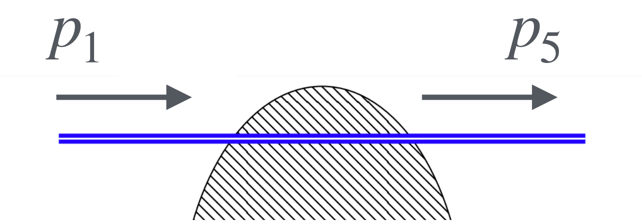


$$W_\eta = e^{(\eta' - \eta)\tau_g}\, W_{\eta'} + \mathcal{O}(\mathrm{NNLL})$$

Is W a Reggeized gluon?

$$[\![O_1 O_2 \dots O_n]\!]^{ab}_r \equiv (T_r^{c_1})_{aa_1} (T_r^{c_2})_{a_1 a_2} \dots (T_r^{c_n})_{a_{n-1} b} \, O_1^{c_1} O_2^{c_2} \dots O_n^{c_n}$$

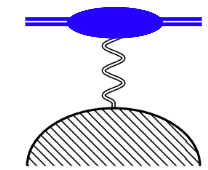
$$[\![O_1 O_2 \dots O_n]\!]^{ab} \equiv [\![O_1 O_2 \dots O_n]\!]^{ab}_{\text{adj}} \,,$$



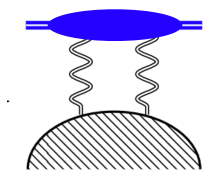
Projectile OPE

$$q_A = p_5 - p_1$$

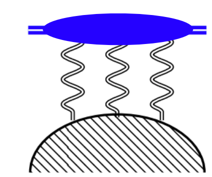
$$a^{a_1,\dagger}_{\lambda_1}(p_1) a^{a_5}_{\lambda_5}(p_5) \sim 2\pi\delta(p_1^+ - p_5^+)\delta_{\lambda_1\lambda_5} \times 2p_1^+ \times \Big\{ (ig_s)\mathcal{J}(\mathbf{q}_A)[\![W(\mathbf{q}_A)]\!]_A +$$



$$+ \frac{(ig_s)^2}{2!} \int \{\mathrm{d}\mathbf{q}\} \left[1 + \mathcal{J}'(\mathbf{q}_A, \mathbf{q})\right] \, [\![W(\mathbf{q}_A - \mathbf{q}) W(\mathbf{q})]\!]_A +$$

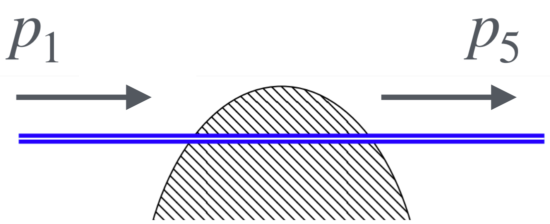


$$+ \frac{(ig_s)^3}{3!} \int \{\mathrm{d}\mathbf{q}_1\}\{\mathrm{d}\mathbf{q}_2\} \, [\![W(\mathbf{q}_A - \mathbf{q}_1) W(\mathbf{q}_1 - \mathbf{q}_2) W(\mathbf{q}_2)]\!]_A + \dots \Big\}$$



$$[\![O_1O_2\dots O_n]\!]_r^{ab} \equiv (T_r^{c_1})_{aa_1}(T_r^{c_2})_{a_1a_2}\dots(T_r^{c_n})_{a_{n-1}b}\,O_1^{c_1}O_2^{c_2}\dots O_n^{c_n}$$

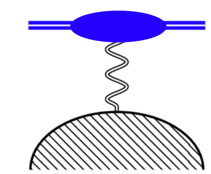
$$[\![O_1O_2\dots O_n]\!]^{ab} \equiv [\![O_1O_2\dots O_n]\!]_{\mathrm{adj}}^{ab}\,,$$



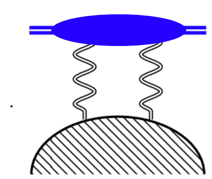
Projectile OPE

$$q_A = p_5 - p_1$$

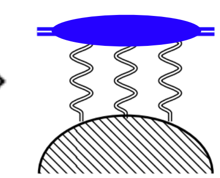
$$a_{\lambda_1}^{a_1,\dagger}(p_1)a_{\lambda_5}^{a_5}(p_5) \sim 2\pi\delta(p_1^+ - p_5^+)\delta_{\lambda_1\lambda_5} \times 2p_1^+ \times \Big\{(ig_s)\mathcal{J}(\mathbf{q}_A)[\![W(\mathbf{q}_A)]\!]_A +$$

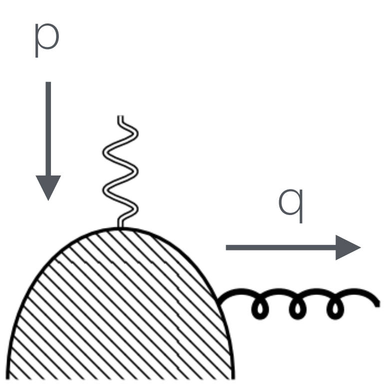


$$+\frac{(ig_s)^2}{2!}\int\{\mathrm{d}\mathbf{q}\}\,[1+\mathcal{J}'(\mathbf{q}_A,\mathbf{q})]\;[\![W(\mathbf{q}_A-\mathbf{q})W(\mathbf{q})]\!]_A +$$



$$+\frac{(ig_s)^3}{3!}\int\{\mathrm{d}\mathbf{q}_1\}\{\mathrm{d}\mathbf{q}_2\}\,[\![W(\mathbf{q}_A-\mathbf{q}_1)W(\mathbf{q}_1-\mathbf{q}_2)W(\mathbf{q}_2)]\!]_A + \dots\Big\}$$

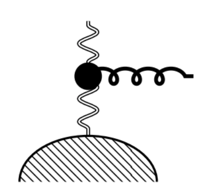




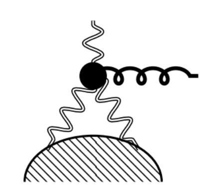
Gluon emission OPE

$$W(\mathbf{p})^b\,a_\lambda^a(q) \sim$$

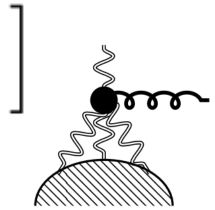
$$2g_s[\![W]\!]^{ab}(\mathbf{q}+\mathbf{p})\left[\frac{\boldsymbol{\varepsilon}_\lambda^*\cdot\mathbf{p}}{\mathbf{p}^2}+\frac{\boldsymbol{\varepsilon}_\lambda^*\cdot\mathbf{q}}{\mathbf{q}^2}\right]\mathcal{W}_\lambda(\mathbf{p},\mathbf{q})$$



$$+ig_s^2\int\{\mathrm{d}\mathbf{k}_1\}[\![W(\mathbf{q}+\mathbf{p}-\mathbf{k}_1)W(\mathbf{k}_1)]\!]^{ab}\left[\frac{\boldsymbol{\varepsilon}_\lambda^*\cdot\mathbf{p}}{\mathbf{p}^2}+\frac{\boldsymbol{\varepsilon}_\lambda^*\cdot(\mathbf{k}_1-\mathbf{p})}{(\mathbf{k}_1-\mathbf{p})^2}\right]+$$



$$+g_s^3\int\{\mathrm{d}\mathbf{k}_1\}\{\mathrm{d}\mathbf{k}_2\}[\![W(\mathbf{q}+\mathbf{p}-\mathbf{k}_1)W(\mathbf{k}_1-\mathbf{k}_2)W(\mathbf{k}_2)]\!]^{ab}\times\left[\frac{1}{6}\left(\frac{\boldsymbol{\varepsilon}_\lambda^*\cdot(\mathbf{k}_1-\mathbf{p})}{(\mathbf{k}_1-\mathbf{p})^2}\right)-\frac{1}{2}\left(\frac{\boldsymbol{\varepsilon}_\lambda^*\cdot(\mathbf{k}_2-\mathbf{p})}{(\mathbf{k}_2-\mathbf{p})^2}\right)-\frac{1}{3}\left(\frac{\boldsymbol{\varepsilon}_\lambda^*\cdot\mathbf{p}}{\mathbf{p}^2}\right)\right]$$



Equal-rapidity correlators

$$\left\langle \mathbb{T}\left[W(\mathbf{p}_1)\cdots W(\mathbf{p}_n)\right]_\eta \left[\widetilde{W}(\mathbf{q}_1)\cdots \widetilde{W}(\mathbf{q}_m)\right]_\eta \right\rangle \overset{*}{=} \delta_{nm} \sum_{\sigma\in S_n} G(\mathbf{p}_1, \mathbf{q}_{\sigma(1)}) \ldots G(\mathbf{p}_n, \mathbf{q}_{\sigma(n)}) + \mathcal{O}(\alpha_s)$$

$$G(\mathbf{p},\mathbf{q}) = \left\langle \mathbb{T}\, W^a_\eta(\mathbf{p}) \widetilde{W}^b_\eta(\mathbf{q}) \right\rangle = (2\pi)^{2-2\epsilon} \delta^{2-2\epsilon}(\mathbf{p}-\mathbf{q}) \frac{i\delta^{ab}}{\mathbf{p}^2}$$

* Two-parton scattering in the high-energy limit

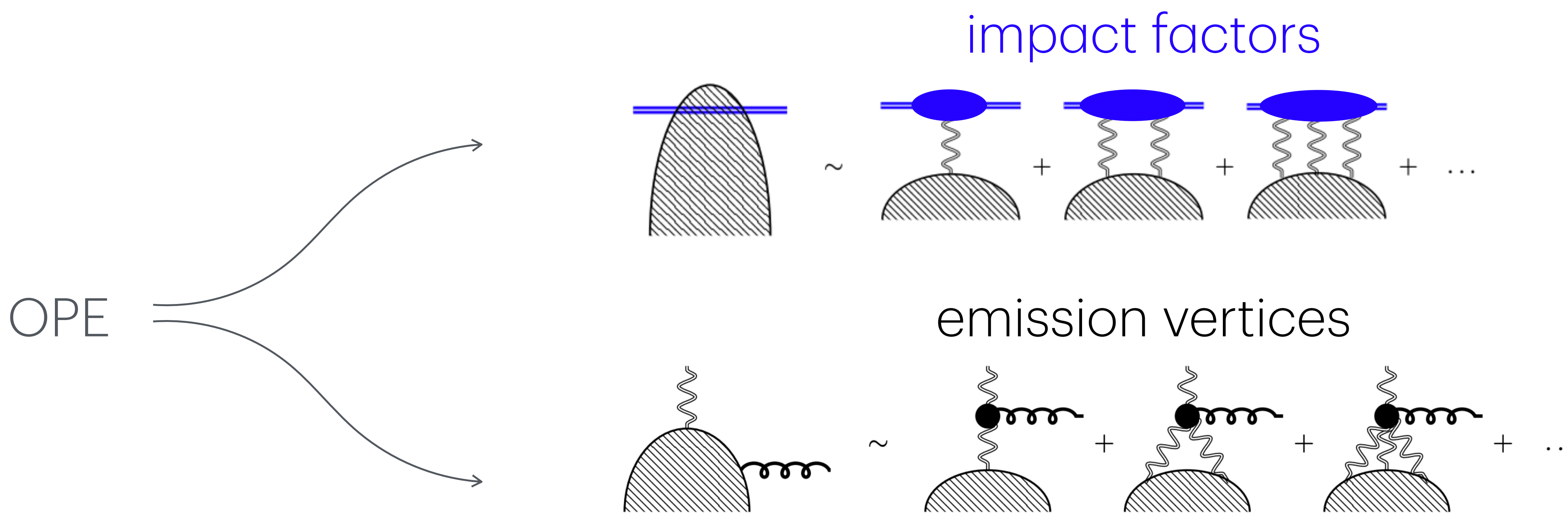
Simon Caron-Huot,[a] Einan Gardi,[b] Leonardo Vernazza[b]

Shockwave collected

Reggeon-like free fields $W^a(\mathbf{p})$

Shockwave collected

Reggeon-like free fields $\qquad W^a(\mathbf{p})$

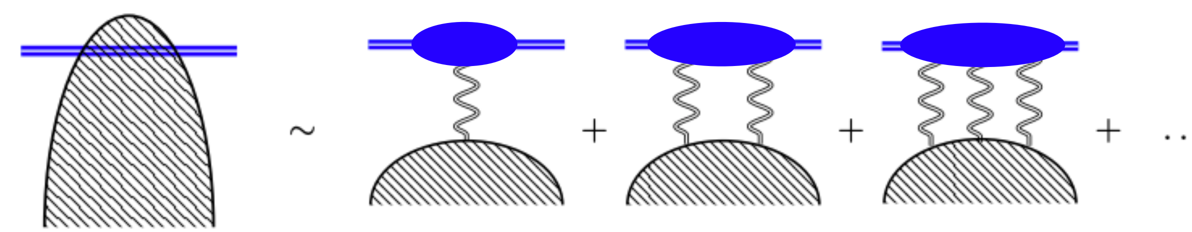


Shockwave collected

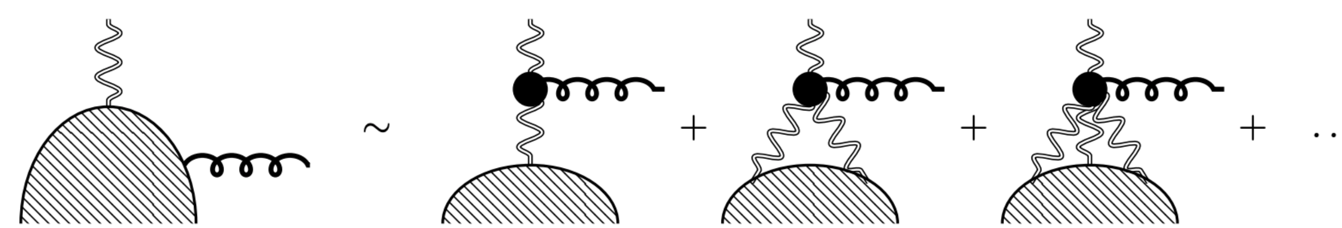
Reggeon-like free fields $W^a(\mathbf{p})$

OPE

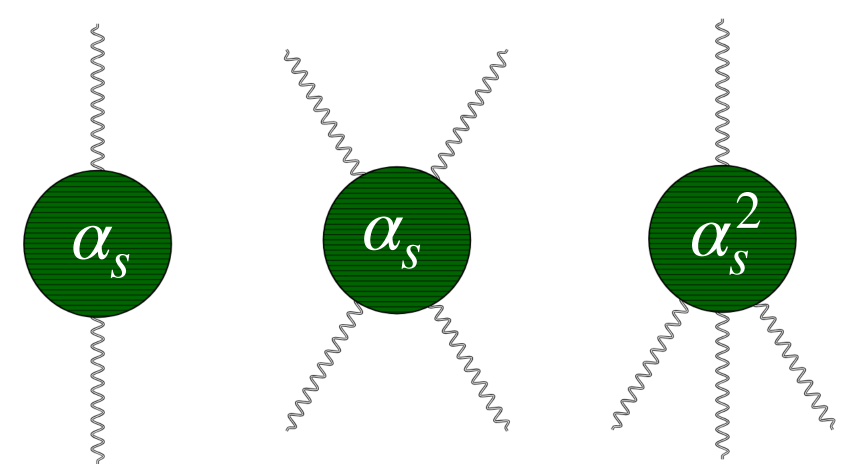
impact factors

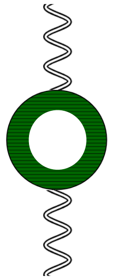


emission vertices



Rapidity evolution





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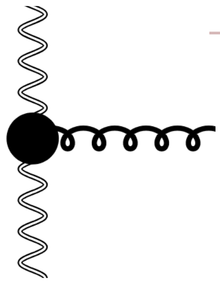

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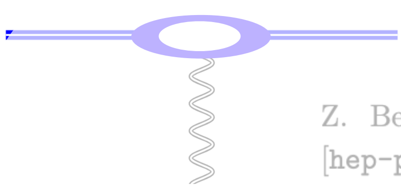

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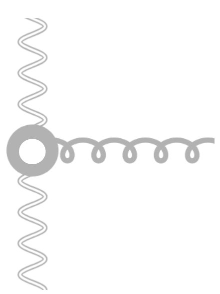

1 loop: [Del Duca, Schmidt: hep-ph/9810215] [Fadin, Fucilla, Papa: 2302.09868]

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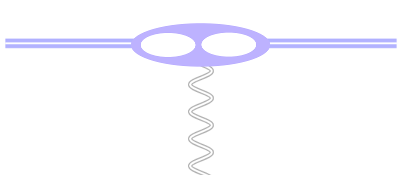

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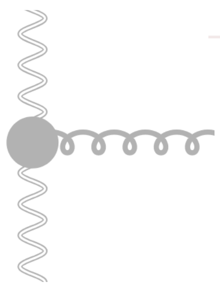

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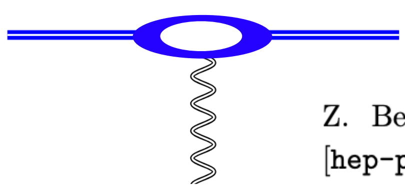

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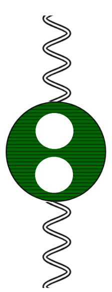

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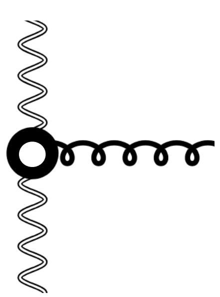

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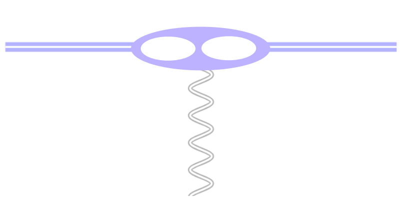

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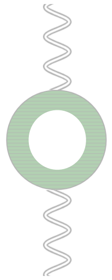

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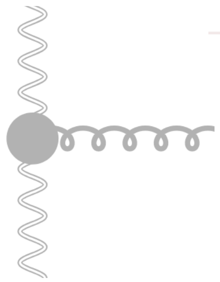

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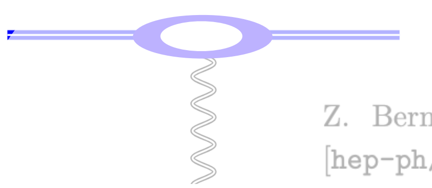

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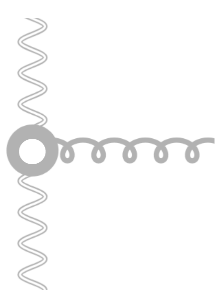
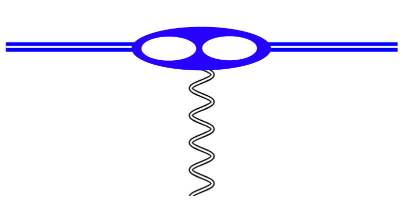
J. Blümlein, V. Ravindran and W.L. van Neerven, *Phys. Rev.* **D 58** (1998) 091502 [hep-ph/9806357].



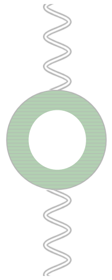
1 loop: [Del Duca, Schmidt: hep-ph/9810215] [Fadin, Fucilla, Papa: 2302.09868]

NLL



S. Caron-Huot, E. Gardi and L. Vernazza, *Two-parton scattering in the high-energy limit*, *JHEP* **06** (2017) 016 [1701.05241].

NNLL



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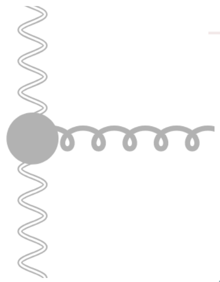
E.A. Kuraev, L.N. Lipatov and V.S. Fadin, *The Pomeranchuk Singularity in Nonabelian Gauge Theories*, *Sov. Phys. JETP* **45** (1977) 199.

I.I. Balitsky and L.N. Lipatov, *The Pomeranchuk Singularity in Quantum Chromodynamics*, *Sov. J. Nucl. Phys.* **28** (1978) 822.



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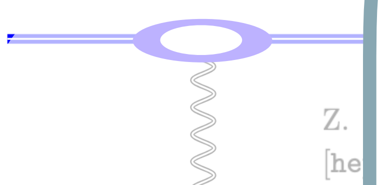
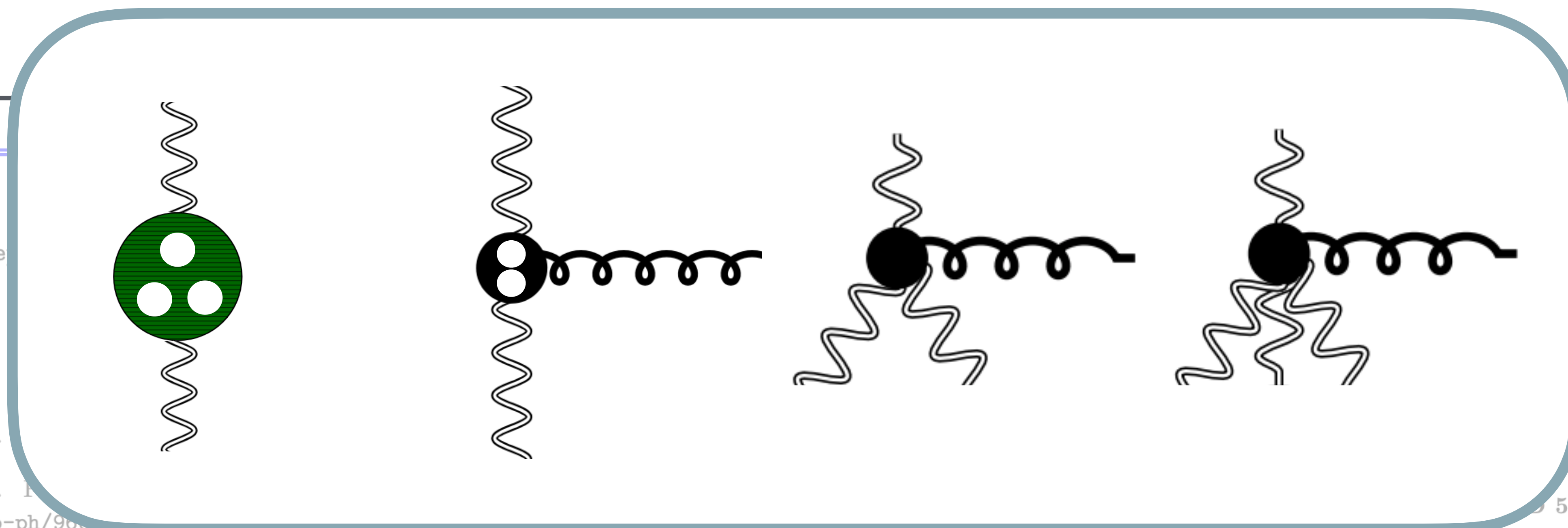
LL

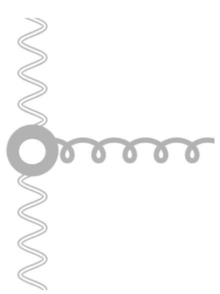


Z. ... **50** (1994) 2265
[he... p-ph/9711309].



V.S. ... (1996) 2729

V.S. F ... **58** (1998) 091502
[hep-ph/96... [hep-ph/9806357].



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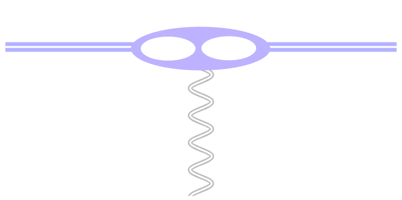
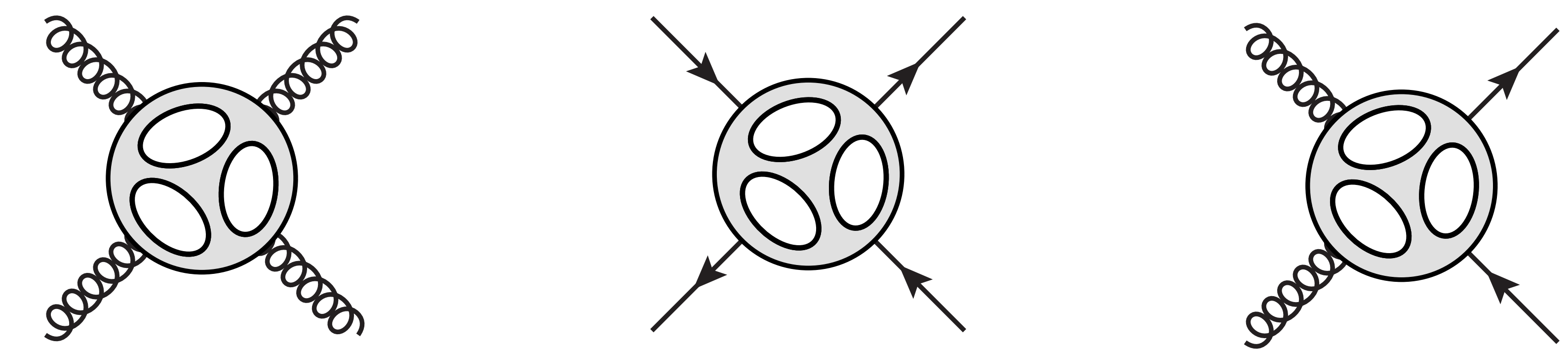
NLL



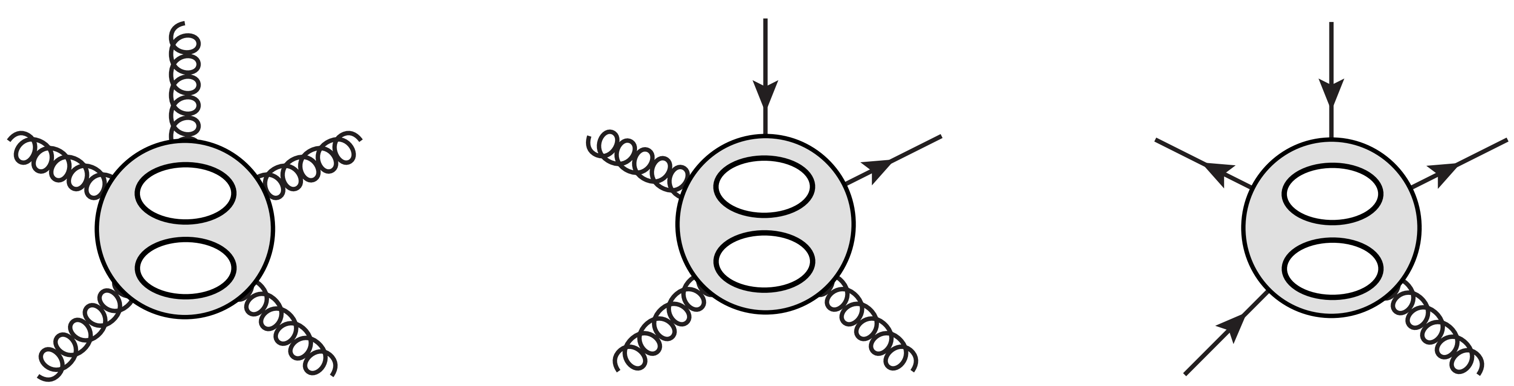
S. Caron-Huot, E. Gardi and L. Vernazza, *Two-parton scattering in the high-energy limit*, *JHEP* **06** (2017) 016 [1701.05241].

NNLL

Recent amplitude data

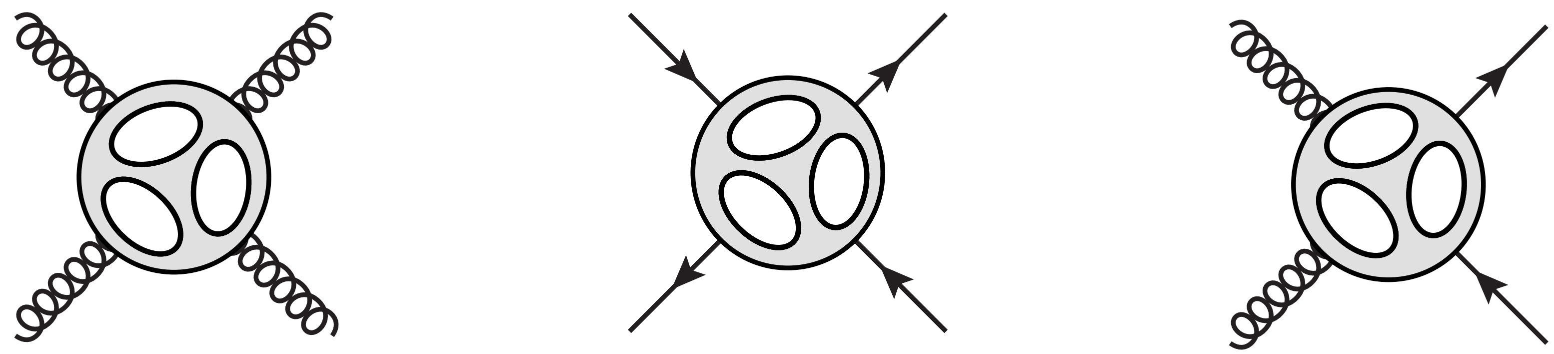


Chakraborty, Caola, **GG**, Tancredi, von Manteuffel:
2108.00055(JHEP), 2207.03503(JHEP), 2112.11097(PRL)

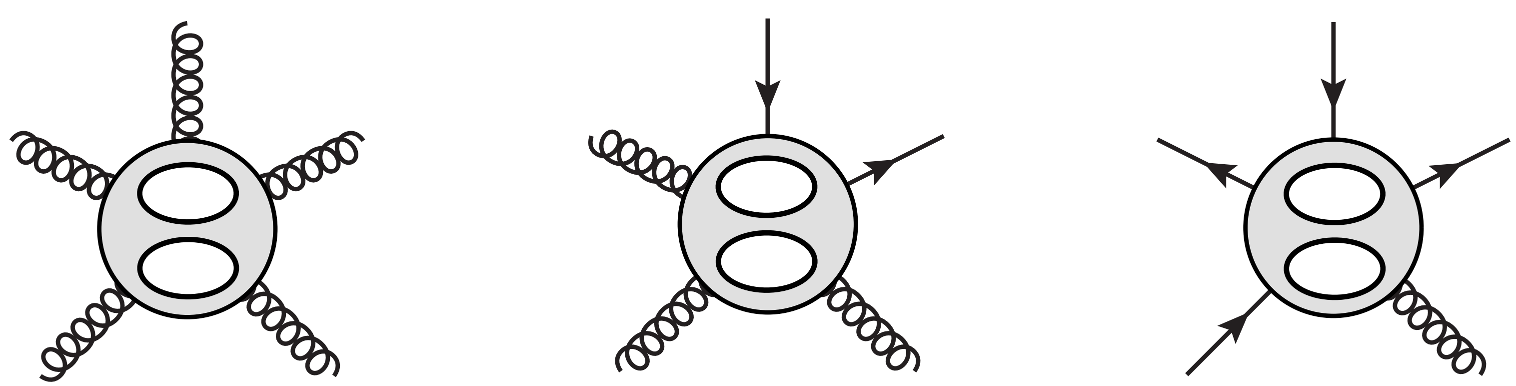


De Laurentis, Ita, Klinkert, Sotnikov: 2311.10086(PRD)
De Laurentis, Ita, Sotnikov: 2311.18752(PRD)
Agarwal, Buccioni, Caola, Devoto, **GG**, von Manteuffel,
Tancredi: 2311.16907(PRD)

Recent amplitude data



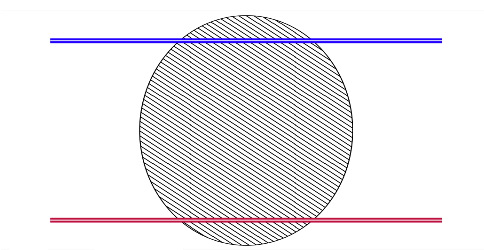
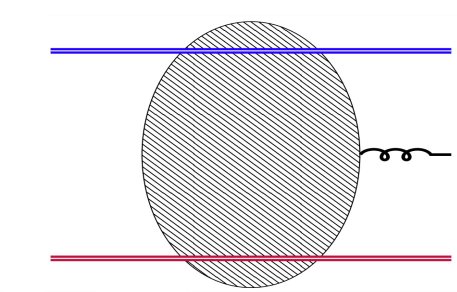
Chakraborty, Caola, **GG**, Tancredi, von Manteuffel:
2108.00055(JHEP), 2207.03503(JHEP), 2112.11097(PRL)



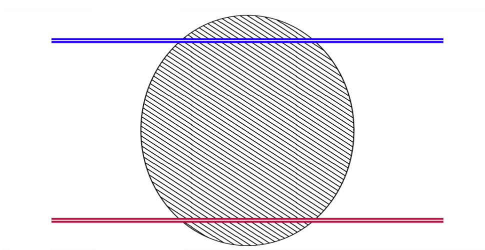
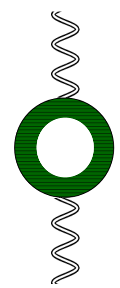
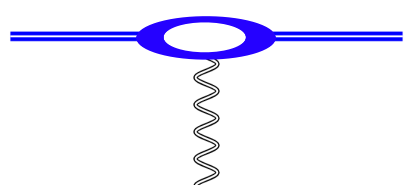
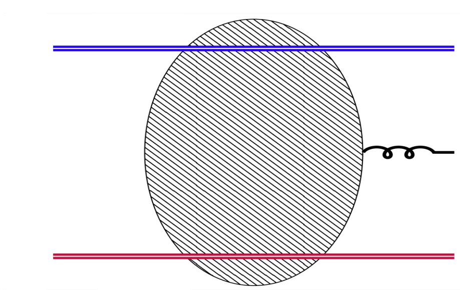
De Laurentis, Ita, Klinkert, Sotnikov: 2311.10086(PRD)
De Laurentis, Ita, Sotnikov: 2311.18752(PRD)
Agarwal, Buccioni, Caola, Devoto, **GG**, von Manteuffel, Tancredi: 2311.16907(PRD)

MRK expansion of exact amplitudes + shockwave prediction = “matching”

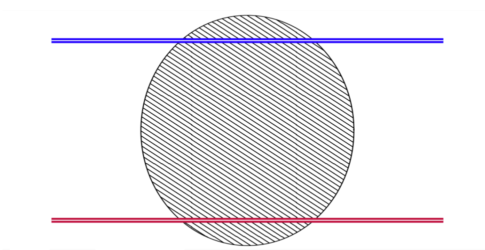
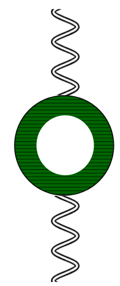
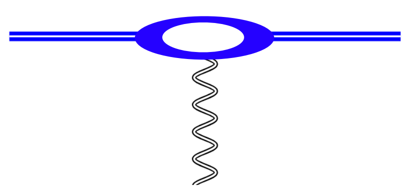
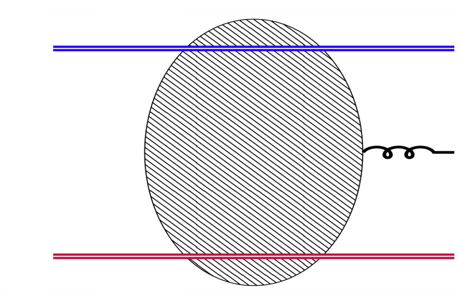
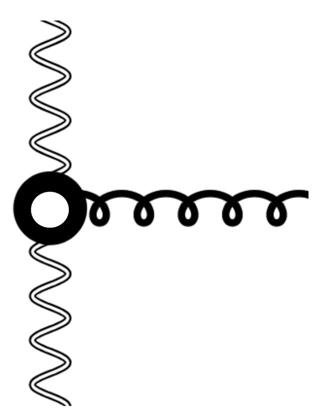
Matching to amplitudes

	1 loop	2 loop	3 loop
			
			

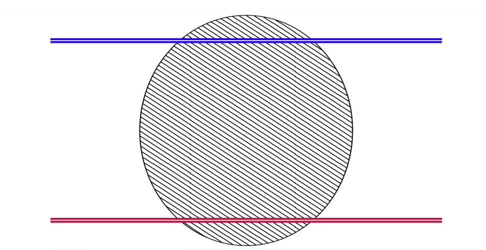
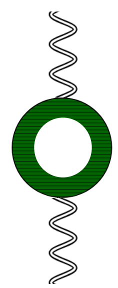
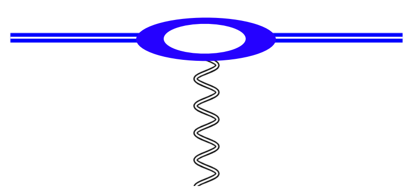
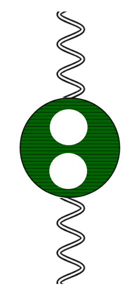
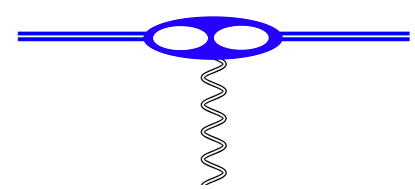
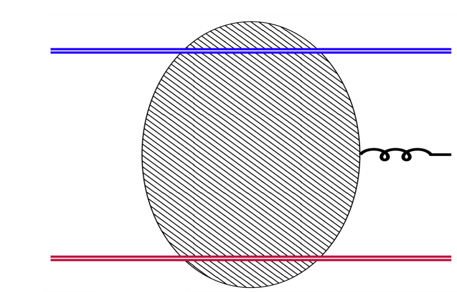
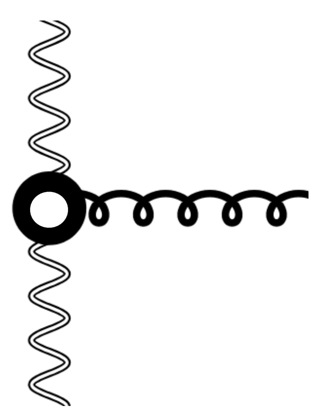
Matching to amplitudes

	1 loop	2 loop	3 loop
	**LL**  **NLL** 		
			

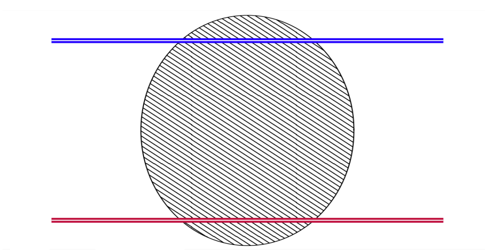
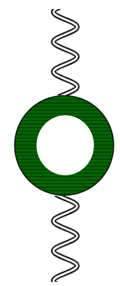
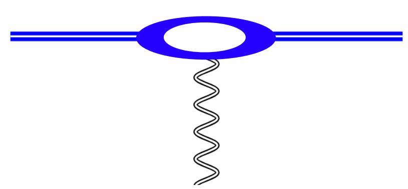
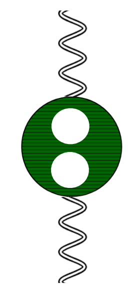
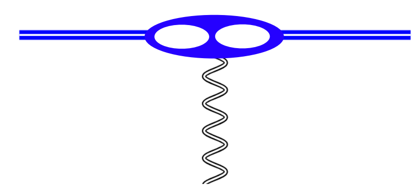
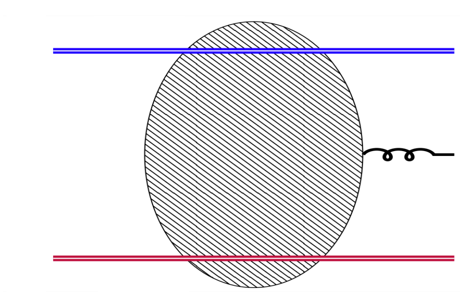
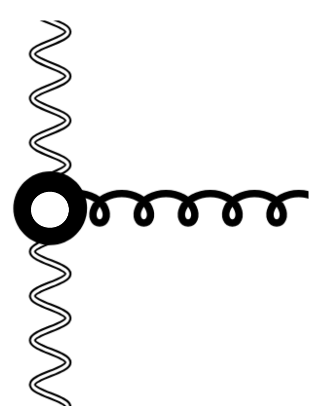
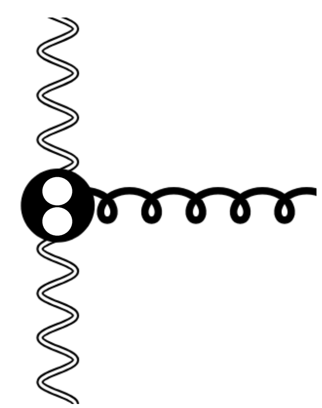
Matching to amplitudes

	1 loop	2 loop	3 loop
	LL  NLL 		
	NLL 		

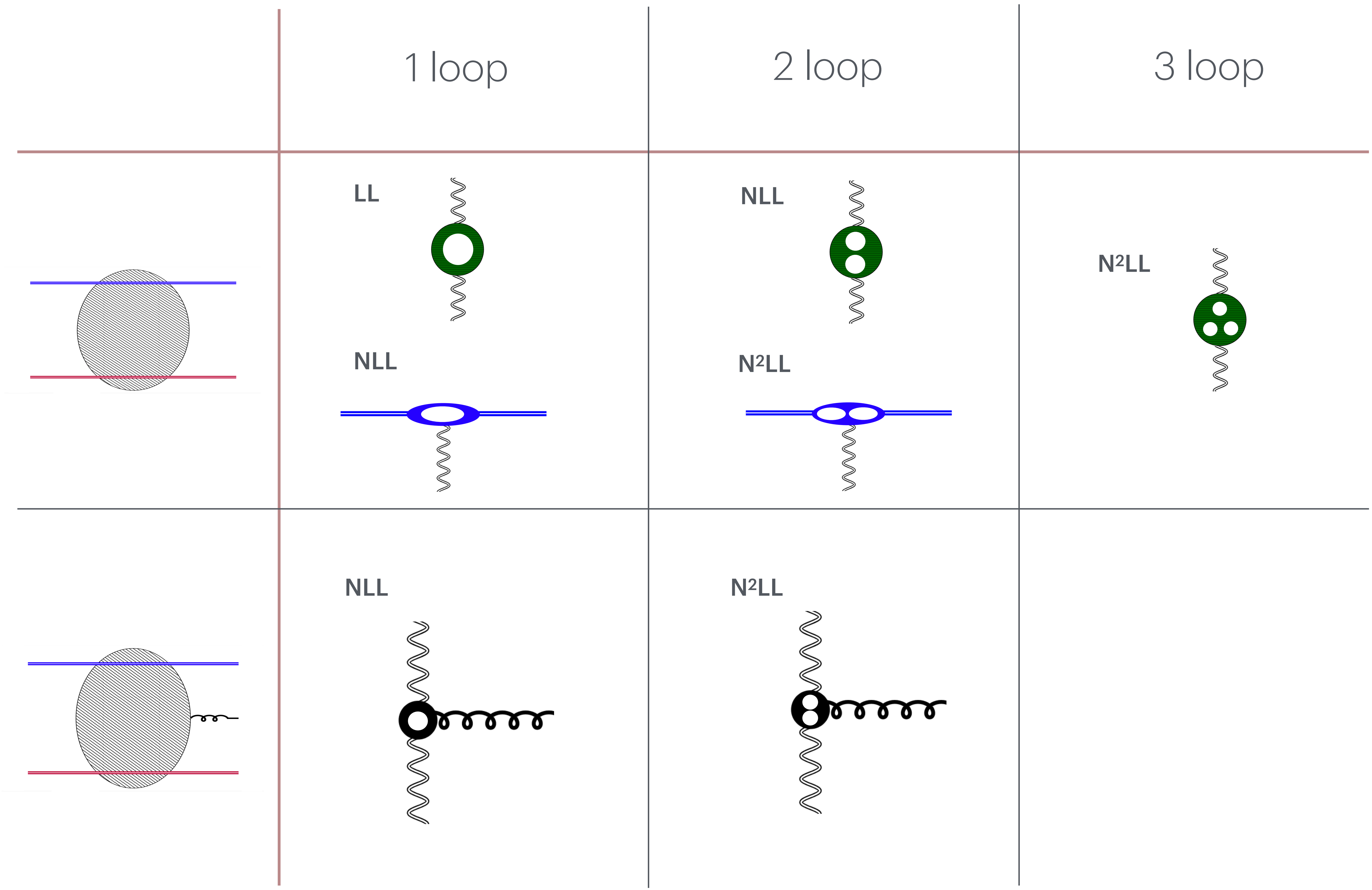
Matching to amplitudes

	1 loop	2 loop	3 loop
	**LL**  **NLL** 	**NLL**  **N^2LL** 	
	**NLL** 		

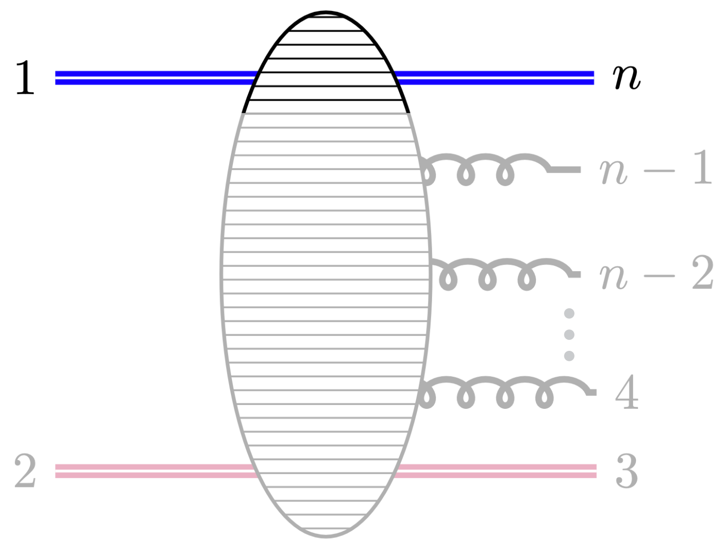
Matching to amplitudes

	1 loop	2 loop	3 loop
	**LL**  **NLL** 	**NLL**  **N²LL** 	
	**NLL** 	**N²LL** 	

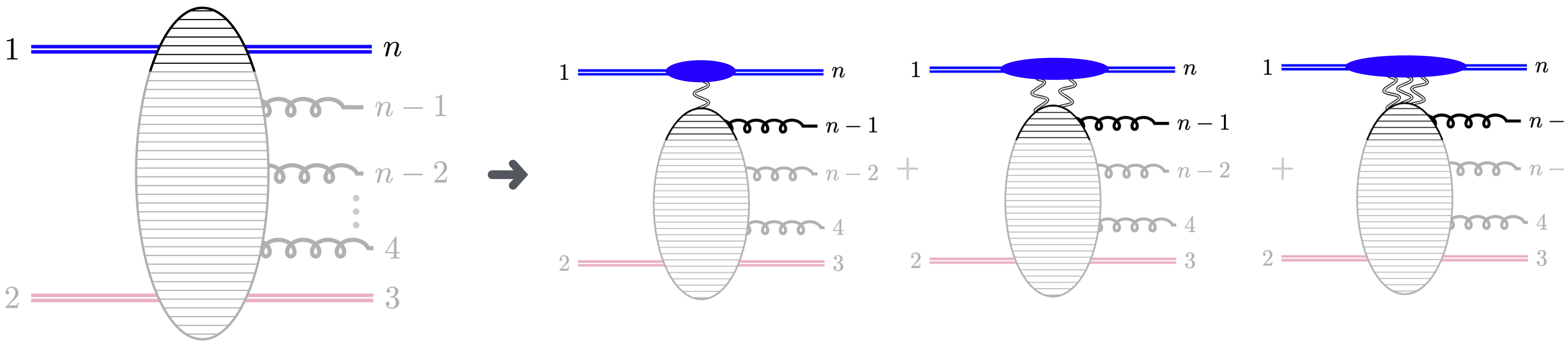
Matching to amplitudes



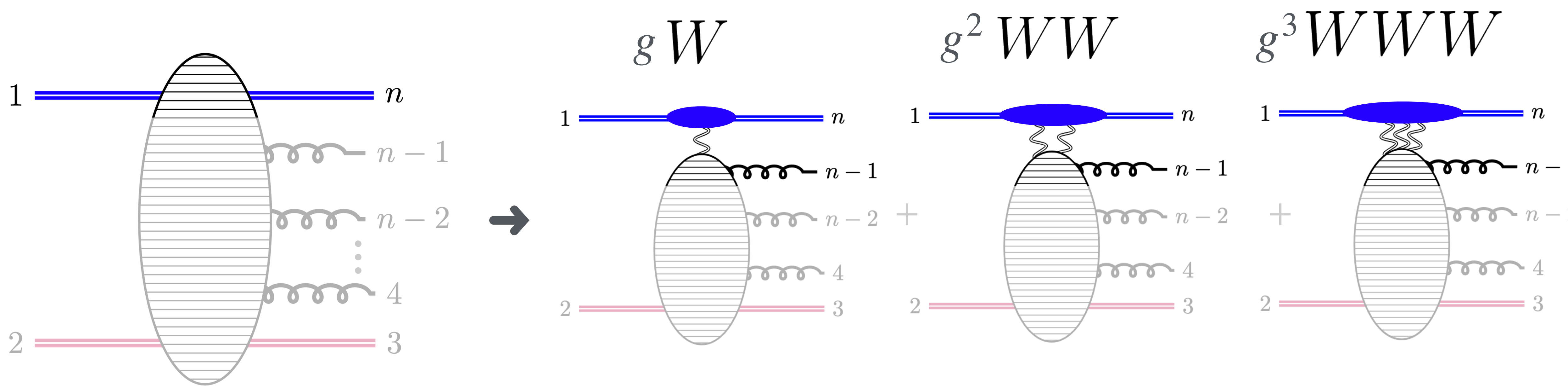
Shockwave Amplitudes



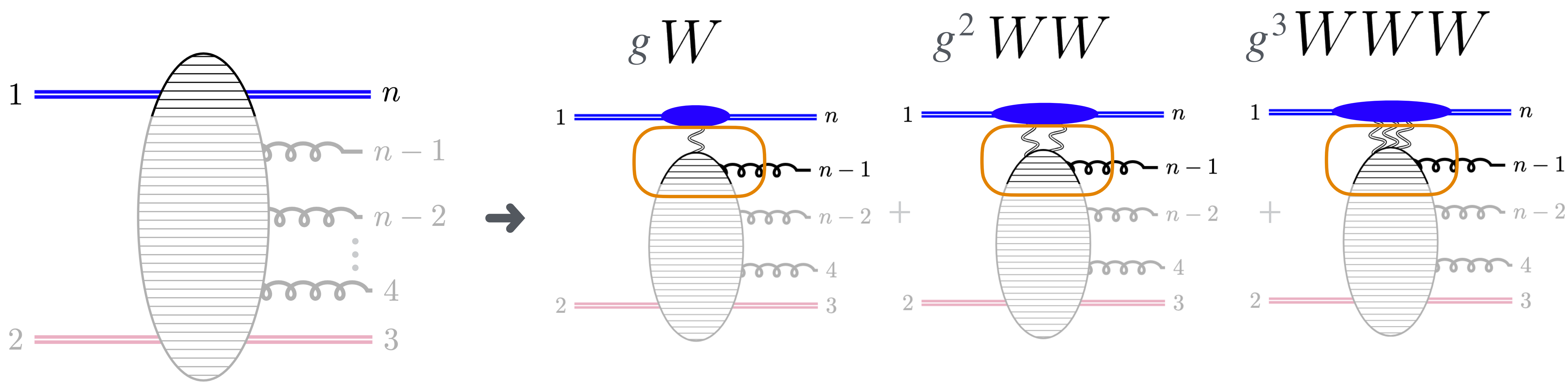
1
n
n − 1
n − 2
4
2
3



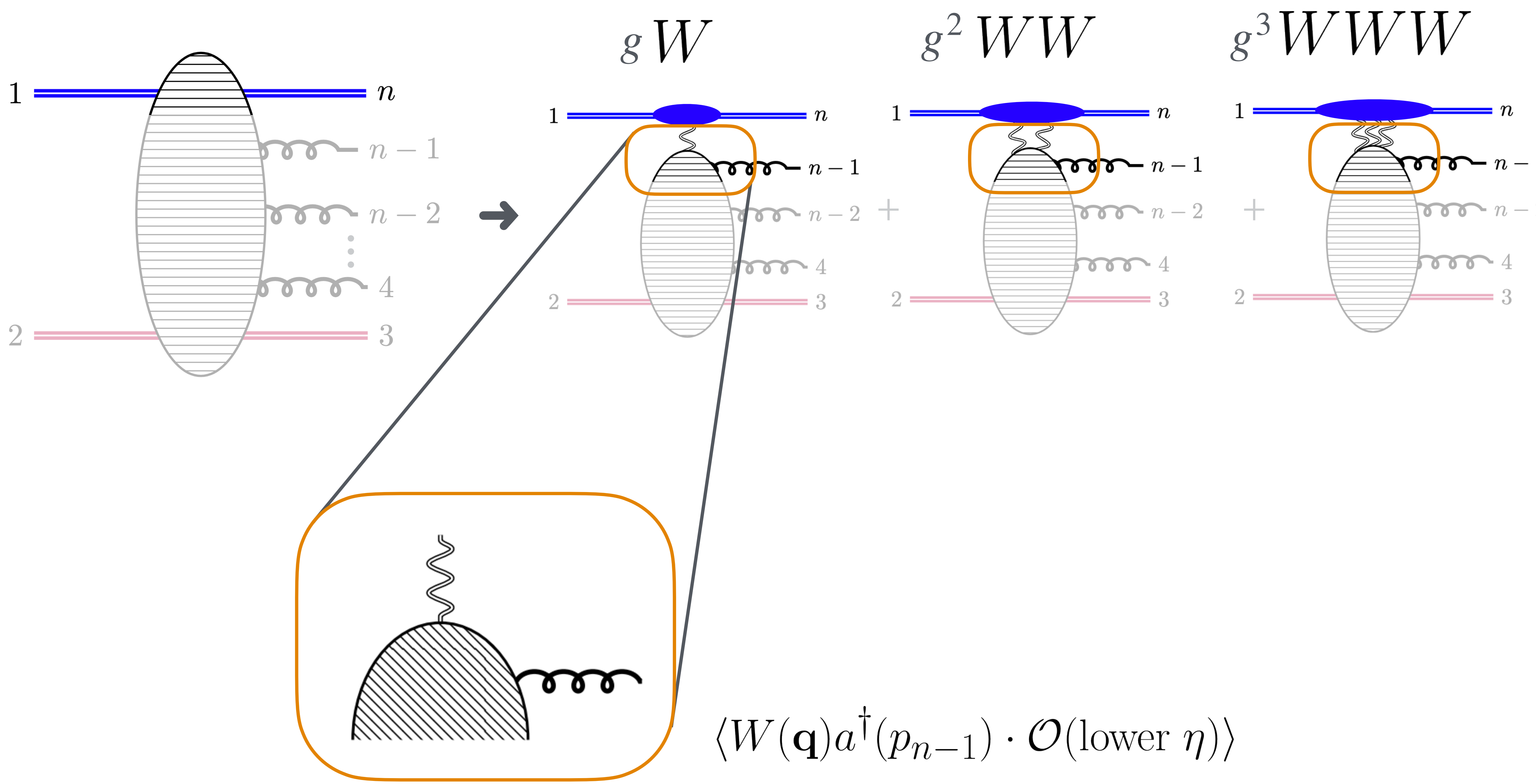
1
n
n − 1
n − 2
4
2
3
1
n
n − 1
n − 2
4
2
3
+
1
n
n − 1
n − 2
4
2
3
+
1
n
n − 1
n − 2
4
2
3



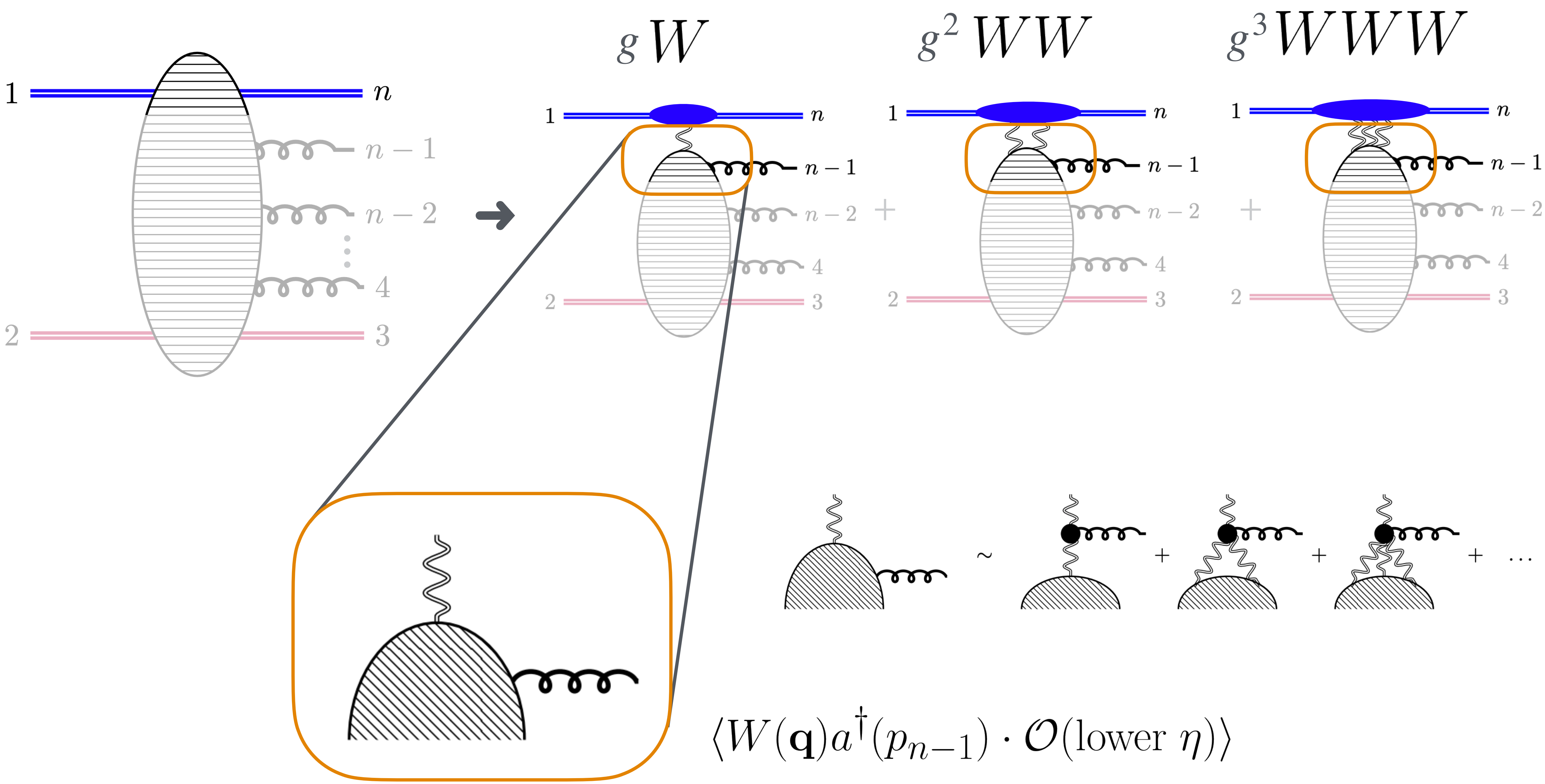
1
n
n − 1
n − 2
4
2
3
gW
g²WW
g³WWW
+
+



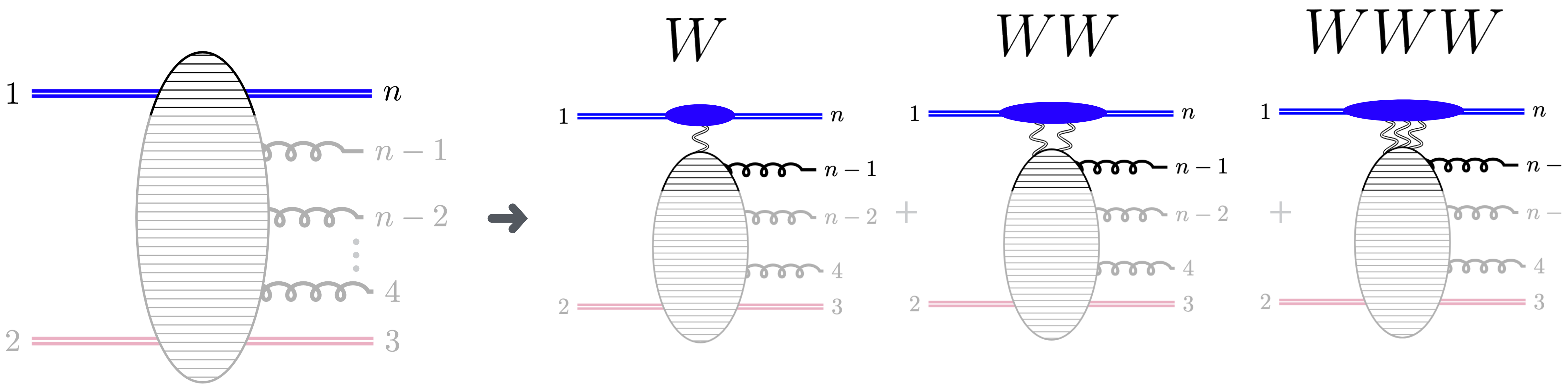
1
n
n − 1
n − 2
4
2
3
g W
g^2 WW
g^3 WWW
+
+



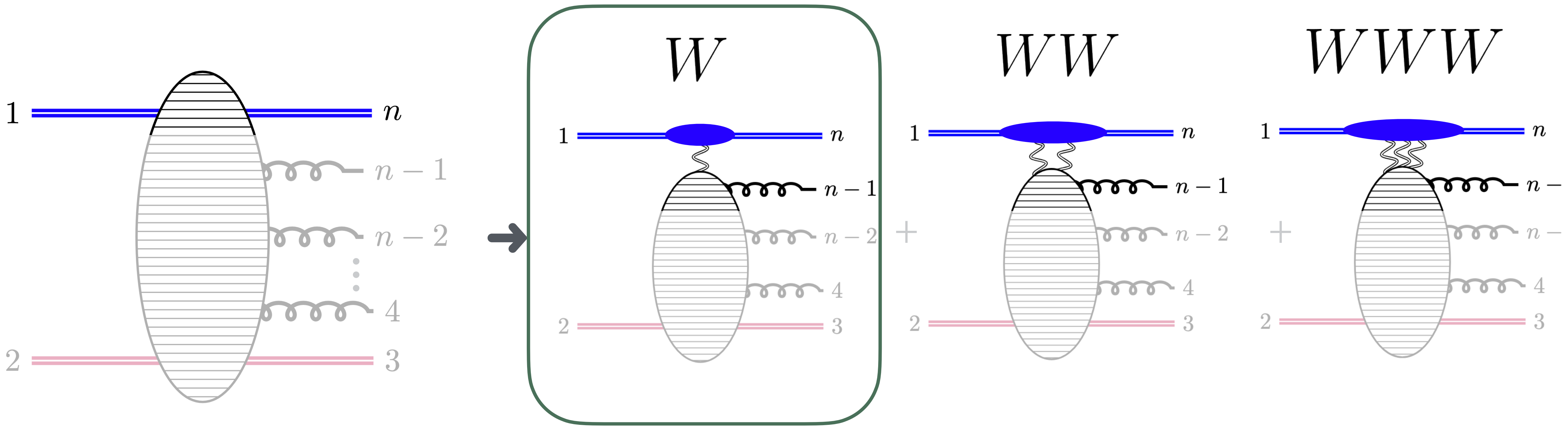
$g\,W$
$g^2\,WW$
$g^3\,WWW$
1
n
$n-1$
$n-2$
4
2
3
+
$\langle W(\mathbf{q})a^\dagger(p_{n-1}) \cdot \mathcal{O}(\text{lower } \eta)\rangle$



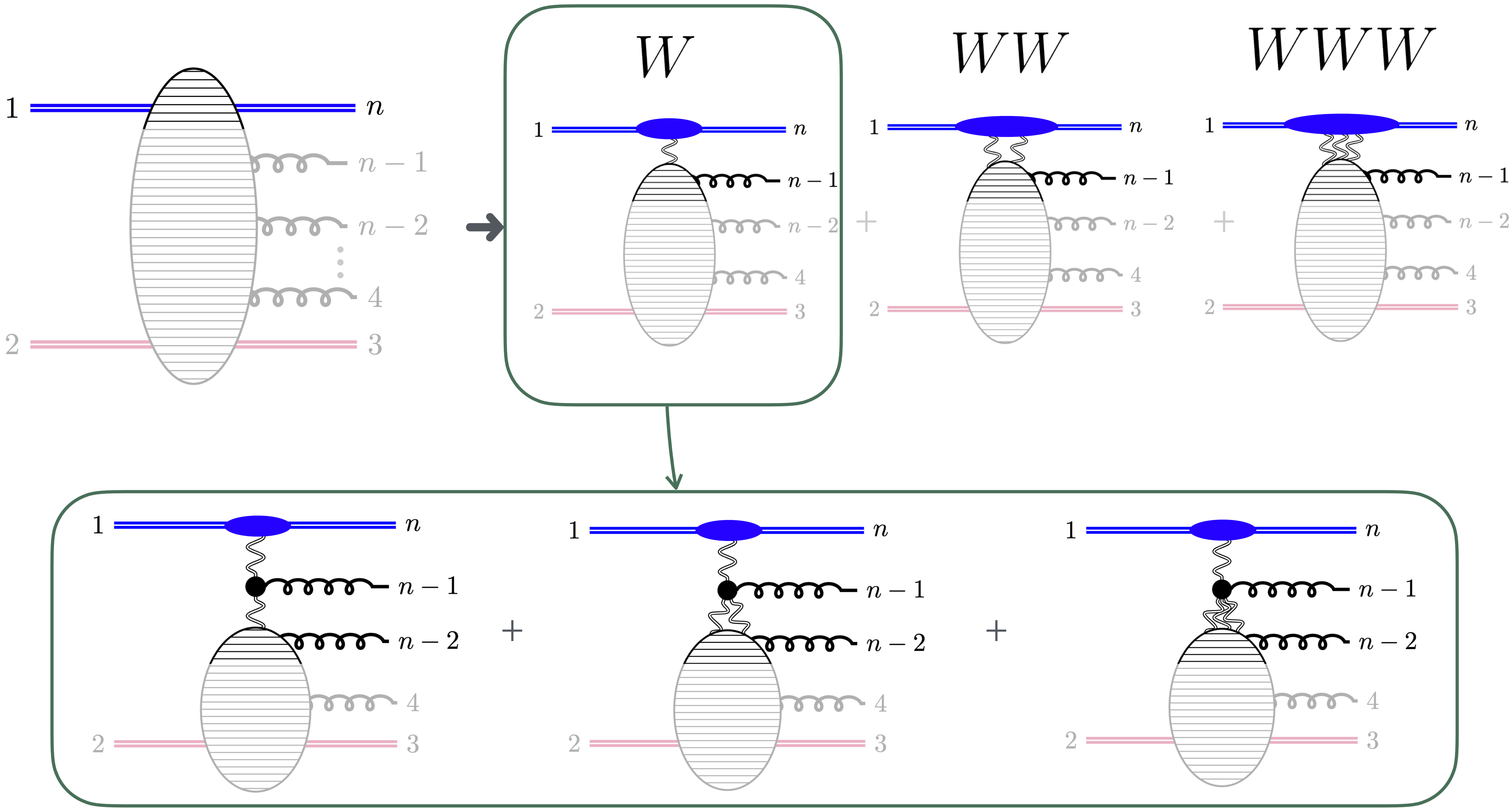
1
n
n − 1
n − 2
4
2
3
g W
g² WW
g³ WWW
+
~
+
+
+ …
⟨W(q)a†(p_{n−1}) · O(lower η)⟩



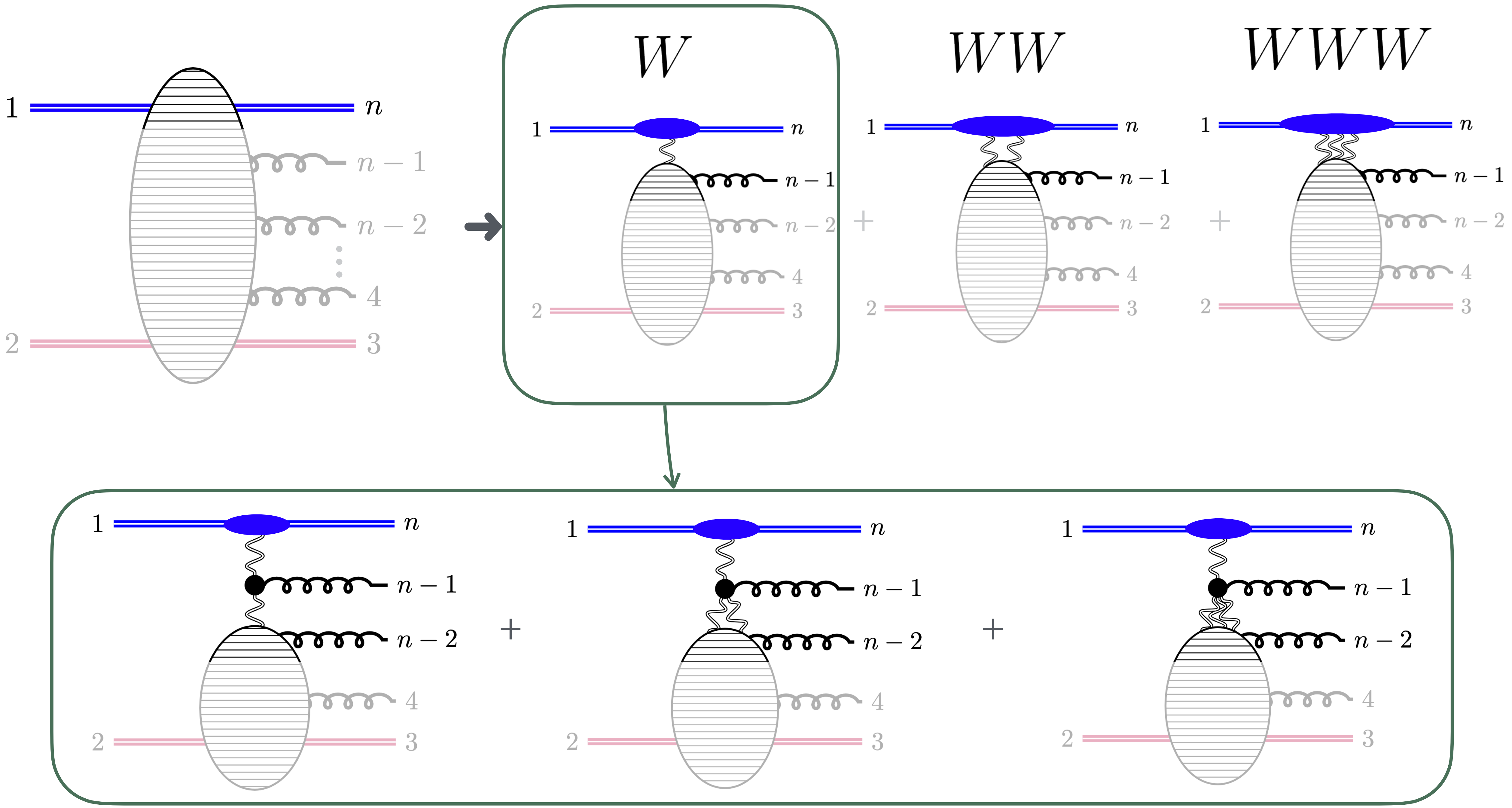
1
n
n − 1
n − 2
4
2
3
W
WW
WWW
+
+



1
n
n − 1
n − 2
4
2
3
W
1
n
n − 1
n − 2
4
2
3
+
WW
1
n
n − 1
n − 2
4
2
3
+
WWW
1
n
n − 1
n − 2
4
2
3



1
n
n − 1
n − 2
4
2
3
W
WW
WWW
+
+



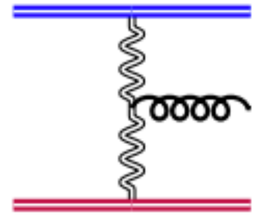
and finally...

$$\Big\langle \mathbb{T}\big[W(\mathbf{p}_1)\cdots W(\mathbf{p}_n)\big]_\eta \big[\widetilde{W}(\mathbf{q}_1)\cdots \widetilde{W}(\mathbf{q}_m)\big]_\eta \Big\rangle =$$

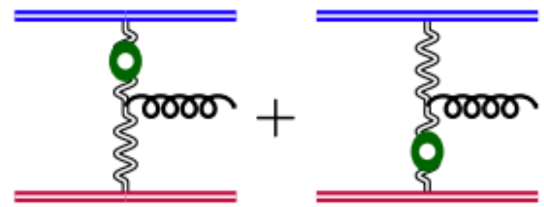
$$\delta_{nm} \sum_{\sigma \in S_n} G(\mathbf{p}_1, \mathbf{q}_{\sigma(1)}) \dots G(\mathbf{p}_n, \mathbf{q}_{\sigma(n)}) + \mathcal{O}(\alpha_s)$$

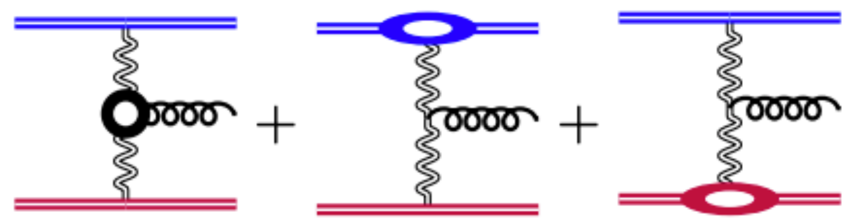
$$G(\mathbf{p}, \mathbf{q}) = \Big\langle \mathbb{T}\, W^a_\eta(\mathbf{p}) \widetilde{W}^b_\eta(\mathbf{q}) \Big\rangle = (2\pi)^{2-2\epsilon} \delta^{2-2\epsilon}(\mathbf{p} - \mathbf{q}) \frac{i\delta^{ab}}{\mathbf{p}^2} + \mathcal{O}(\alpha_s)$$

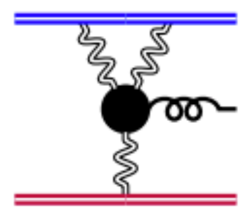
Tree

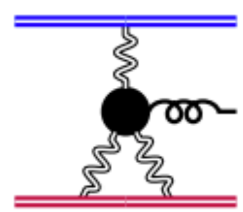
$$\mathcal{A}^{(0)}_{\rm LL} =$$

$$= \mathcal{A}^{(0)}$$

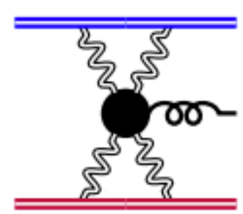
One-loop

$$\mathcal{A}^{(1)}_{\rm LL} =$$

$$= \left[L_A \tau^{(1)}_A + L_B \tau^{(1)}_B\right] \mathcal{A}^{(0)},$$

$$\mathcal{A}^{(1),(--)}_{\rm NLL} =$$

$$= \left[\bar{\mathcal{J}}^{(1)}_A + \bar{\mathcal{J}}^{(1)}_B + \overline{\mathcal{W}}^{(1)}_{\lambda_4}\right] \mathcal{A}^{(0)},$$

$$\mathcal{A}^{(1),(+-)}_{\rm NLL} =$$

$$= i\pi B^{(1)}_{-+} \boldsymbol{\mathcal{T}}_{-+} \mathcal{A}^{(0)},$$

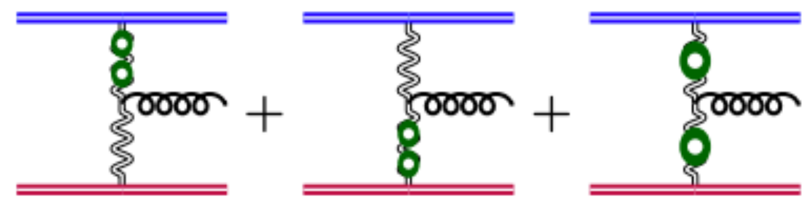
$$\mathcal{A}^{(1),(-+)}_{\rm NLL} =$$

$$= i\pi B^{(1)}_{+-} \boldsymbol{\mathcal{T}}_{+-} \mathcal{A}^{(0)},$$

$$\mathcal{A}^{(1),(++)}_{\rm NLL} =$$

$$= i\pi B^{(1)}_{--} \boldsymbol{\mathcal{T}}_{--} \mathcal{A}^{(0)},$$

Two-loop

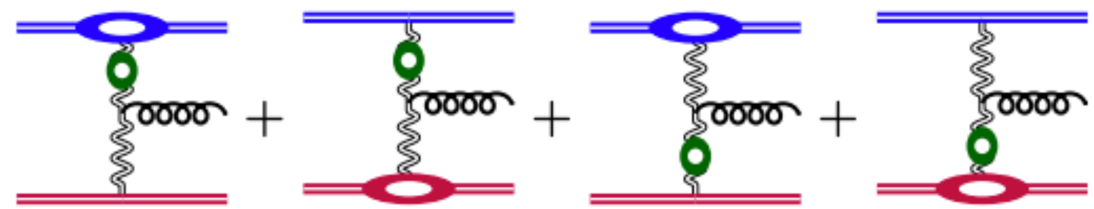
(odd,odd)

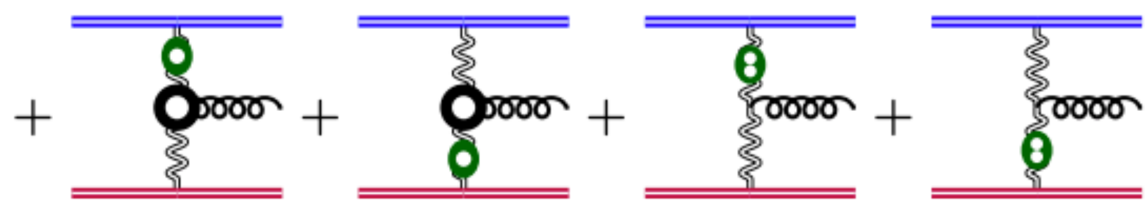
$$\mathcal{A}^{(2)}_{\mathrm{LL}} =$$



$$= \frac{1}{2}(L_A \tau_A^{(1)} + L_B \tau_B^{(1)})^2 \mathcal{A}^{(0)},$$

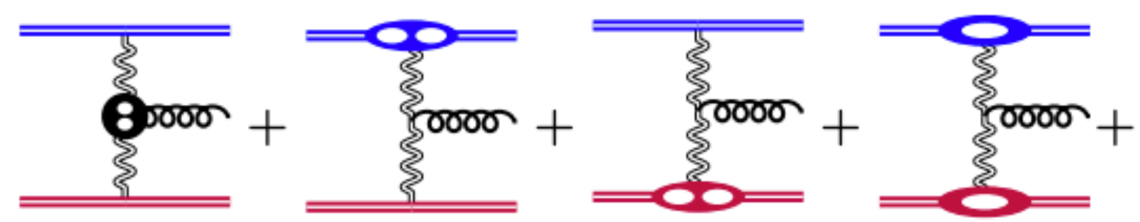
$$\mathcal{A}^{(2),(--)}_{\mathrm{NLL}} =$$

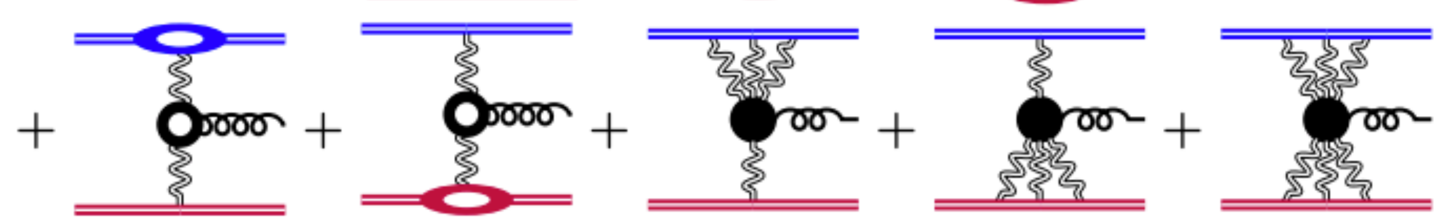




$$= \Big[(L_A \tau_A^{(2)} + L_B \tau_B^{(2)}) + (L_A \tau_A^{(1)} + L_B \tau_B^{(1)})(\bar{\mathcal{J}}_A^{(1)} + \bar{\mathcal{J}}_B^{(1)} + \overline{\mathcal{W}}^{(1)}_{\lambda_4})\Big] \mathcal{A}^{(0)},$$

$$\mathcal{A}^{(2),(--)}_{\mathrm{NNLL}} =$$





$$= \Big[\overline{\mathcal{W}}^{(2)}_{\lambda_4} + \bar{\mathcal{J}}_A^{(2)} + \bar{\mathcal{J}}_B^{(2)} + \bar{\mathcal{J}}_A^{(1)} \bar{\mathcal{J}}_B^{(1)} + \overline{\mathcal{W}}^{(1)}_{\lambda_4}(\bar{\mathcal{J}}_A^{(1)} + \bar{\mathcal{J}}_B^{(1)})$$

$$+ (i\pi)^2 \Big(B^{(2)}_{+-} \boldsymbol{\mathcal{T}}^2_{+-} + B^{(2)}_{--} \boldsymbol{\mathcal{T}}^2_{--} + B^{(2)}_{-+} \boldsymbol{\mathcal{T}}^2_{-+} - B^{(2)}_{d} \frac{N_c^2}{4}\Big)\Big] \mathcal{A}^{(0)}$$

Extracting Data from Amplitudes

MRK
helicity amplitudes

$$\mathcal{H} = \sum_{i,c} R_{ic}\, F_i\, \mathcal{C}_c$$

MRK
helicity amplitudes

$$\mathcal{H} = \sum_{i,c} R_{ic}\, F_i\, \mathcal{C}_c$$

rational

MRK
helicity amplitudes

$$\mathcal{H} = \sum_{i,c} R_{ic}\, F_i\, \mathcal{C}_c$$

rational transcendental

MRK
helicity amplitudes

$$\mathcal{H} = \sum_{i,c} R_{ic} \, F_i \, \mathcal{C}_c$$

rational transcendental colour

MRK
helicity amplitudes

$$\mathcal{H} = \sum_{i,c} R_{ic}\, F_i\, \mathcal{C}_c$$

rational — transcendental — colour

MRK variable
$x \to 0$

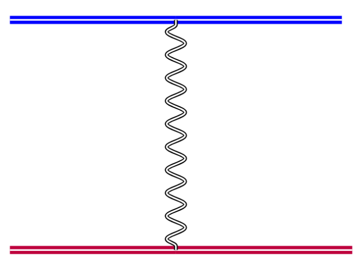
MRK helicity amplitudes

$$\mathcal{H} = \sum_{i,c} R_{ic}\, F_i\, \mathcal{C}_c$$

rational transcendental colour

MRK variable

$x \to 0$



$1/x$

$$\{s_{12}, s_{23}\} \to \left\{\frac{s}{x}, s_{23}\right\}$$

HPLs (trivial expansion)

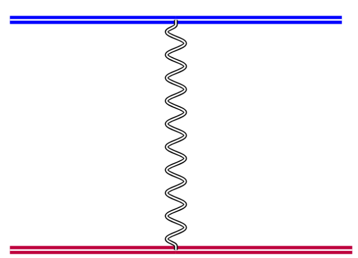
MRK helicity amplitudes

$$\mathcal{H} = \sum_{i,c} R_{ic}\, F_i\, \mathcal{C}_c$$

rational transcendental colour

MRK variable

$x \to 0$



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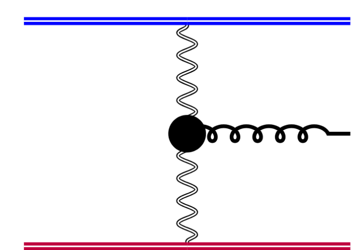

$1/x^2$

$$\{s_{12}, s_{23}, s_{34}, s_{45}, s_{51}\} \to \left\{\frac{s}{x^2}, s_{23}, \frac{s_1}{x}, \frac{s_2}{x}, s_{51}\right\}$$

Pentagon functions,
non-trivial expansion

[same strategy as Federico's talk]

MRK expansion

Multi-Regge limit of the two-loop five-point amplitudes in $\mathcal{N} = 4$ super Yang-Mills and $\mathcal{N} = 8$ supergravity

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Simon Caron-Huot,[a] Dmitry Chicherin,[b] Johannes Henn,[b] Yang Zhang[c,d] and Simone Zoia[b]

MRK expansion

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$$\mathcal{H} = \frac{1}{x^{\#}} \sum_{\ell,k,c} \alpha_s^{\ell} \log^{k}(x)\, R'_{\ell,k,i}\, F'_i\, \mathcal{C}_c$$

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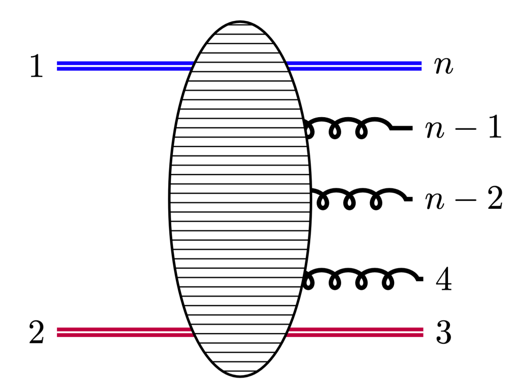
compare w/ shockwave prediction



MRK expansion

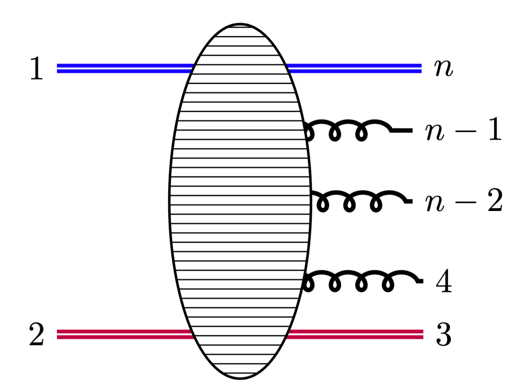
Multi-Regge limit of the two-loop five-point amplitudes in $\mathcal{N}=4$ super Yang-Mills and $\mathcal{N}=8$ supergravity

Simon Caron-Huot,[a] Dmitry Chicherin,[b] Johannes Henn,[b] Yang Zhang[c,d] and Simone Zoia[b]

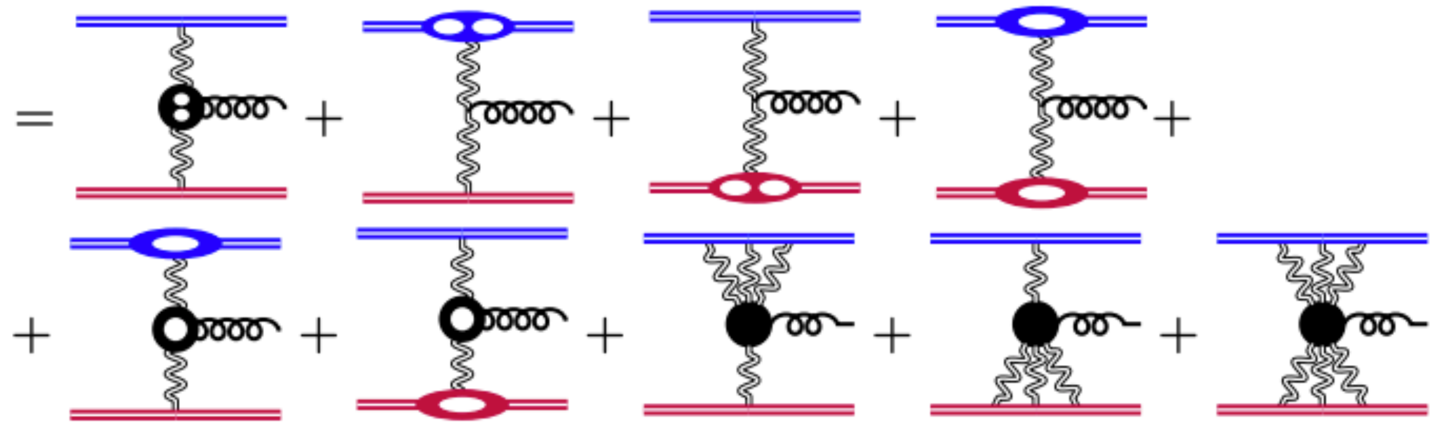
$$\mathcal{H} = \frac{1}{x^{\#}} \sum_{\ell,k,c} \alpha_s^{\ell} \log^{k}(x)\, R'_{\ell,k,i}\, F'_i\, \mathcal{C}_c + \mathcal{O}\left(\frac{1}{x^{\#-1}}\right)$$

deflection effects

compare w/ shockwave prediction



$$\mathcal{A}^{(2),(--)}_{\mathrm{NNLL}} =$$



$$= \Big[\overline{\mathcal{W}}^{(2)}_{\lambda_4} + \bar{\mathcal{J}}^{(2)}_A + \bar{\mathcal{J}}^{(2)}_B + \bar{\mathcal{J}}^{(1)}_A \bar{\mathcal{J}}^{(1)}_B + \overline{\mathcal{W}}^{(1)}_{\lambda_4} (\bar{\mathcal{J}}^{(1)}_A + \bar{\mathcal{J}}^{(1)}_B)$$

$$+ (i\pi)^2 \Big(B^{(2)}_{+-} \boldsymbol{\mathcal{T}}^2_{+-} + B^{(2)}_{--} \boldsymbol{\mathcal{T}}^2_{--} + B^{(2)}_{-+} \boldsymbol{\mathcal{T}}^2_{-+} - B^{(2)}_d \frac{N_c^2}{4} \Big) \Big] \mathcal{A}^{(0)}$$

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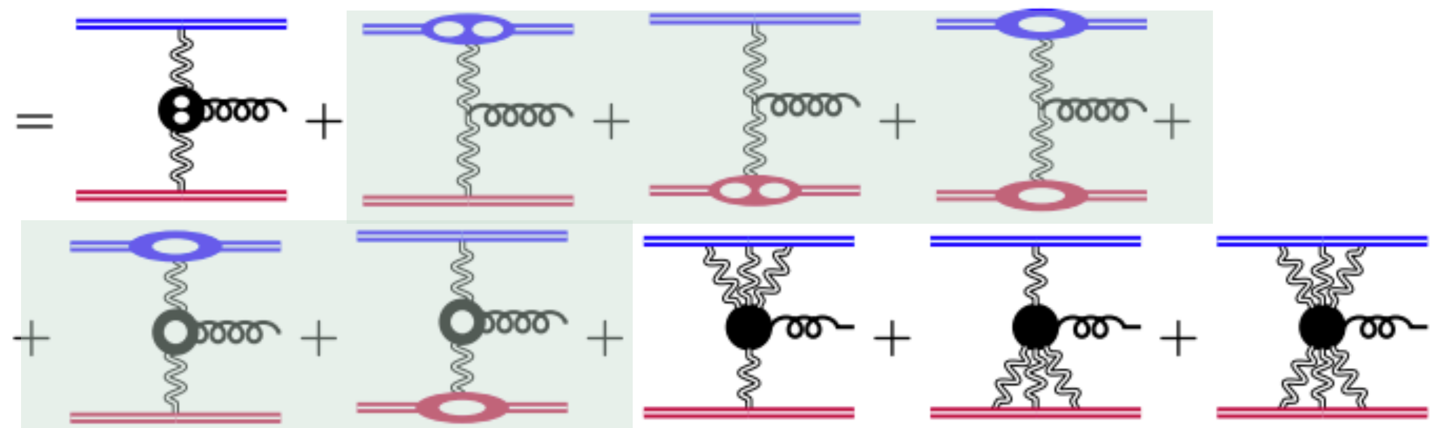

$$= \Big[\overline{\mathcal{W}}^{(2)}_{\lambda_4} + \bar{\mathcal{J}}^{(2)}_{A} + \bar{\mathcal{J}}^{(2)}_{B} + \bar{\mathcal{J}}^{(1)}_{A}\bar{\mathcal{J}}^{(1)}_{B} + \overline{\mathcal{W}}^{(1)}_{\lambda_4}(\bar{\mathcal{J}}^{(1)}_{A} + \bar{\mathcal{J}}^{(1)}_{B})$$

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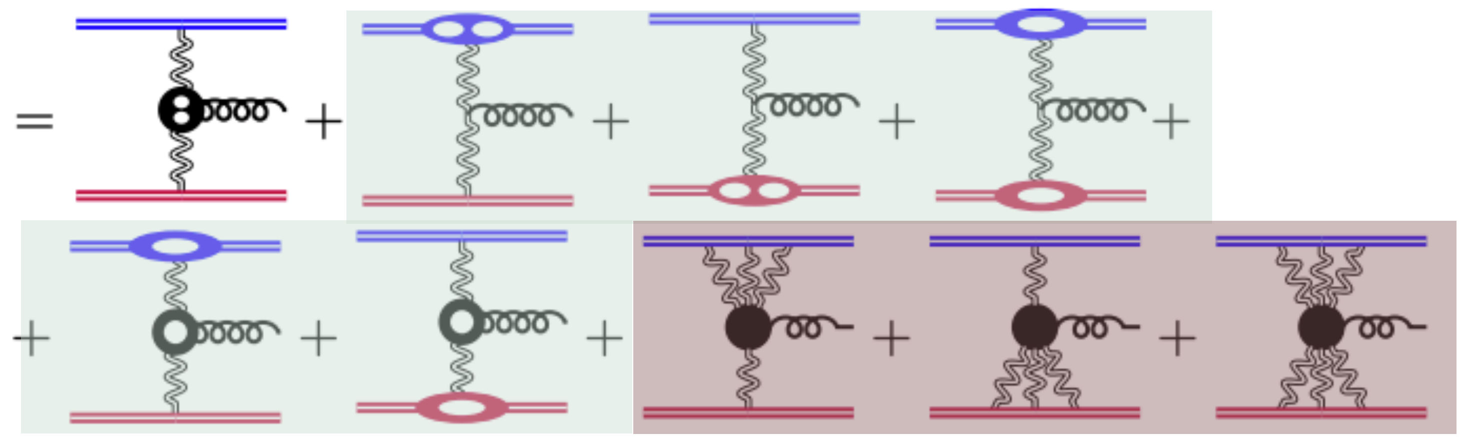

$$= \Bigg[\overline{\mathcal{W}}^{(2)}_{\lambda_4} + \bar{\mathcal{J}}^{(2)}_{A} + \bar{\mathcal{J}}^{(2)}_{B} + \bar{\mathcal{J}}^{(1)}_{A}\bar{\mathcal{J}}^{(1)}_{B} + \overline{\mathcal{W}}^{(1)}_{\lambda_4}\left(\bar{\mathcal{J}}^{(1)}_{A} + \bar{\mathcal{J}}^{(1)}_{B}\right)$$

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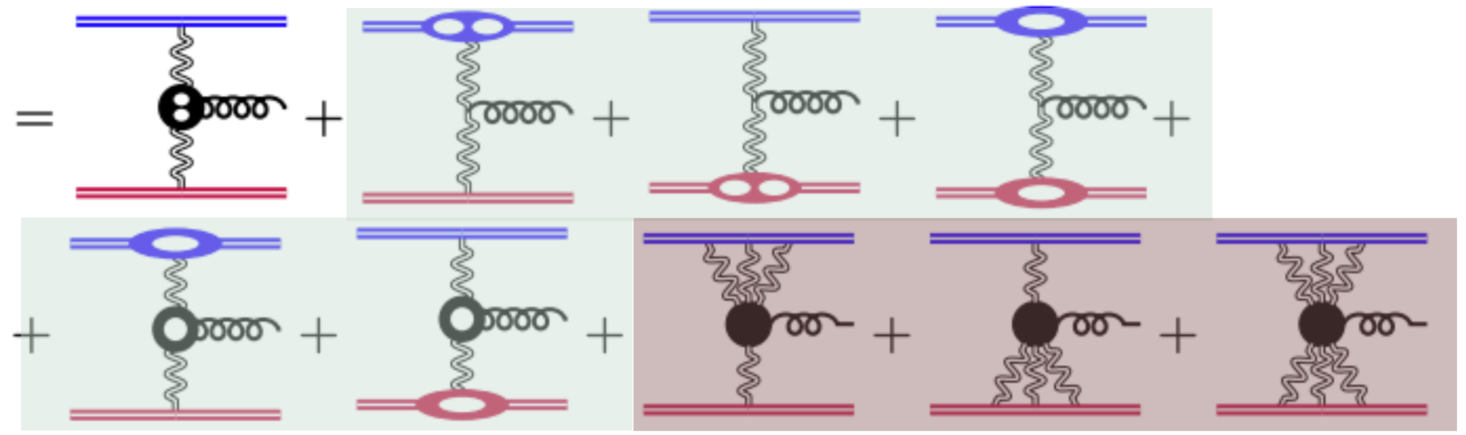

$$= \Big[\overline{\mathcal{W}}^{(2)}_{\lambda_4} + \bar{\mathcal{J}}^{(2)}_A + \bar{\mathcal{J}}^{(2)}_B + \bar{\mathcal{J}}^{(1)}_A \bar{\mathcal{J}}^{(1)}_B + \overline{\mathcal{W}}^{(1)}_{\lambda_4}\left(\bar{\mathcal{J}}^{(1)}_A + \bar{\mathcal{J}}^{(1)}_B\right)$$

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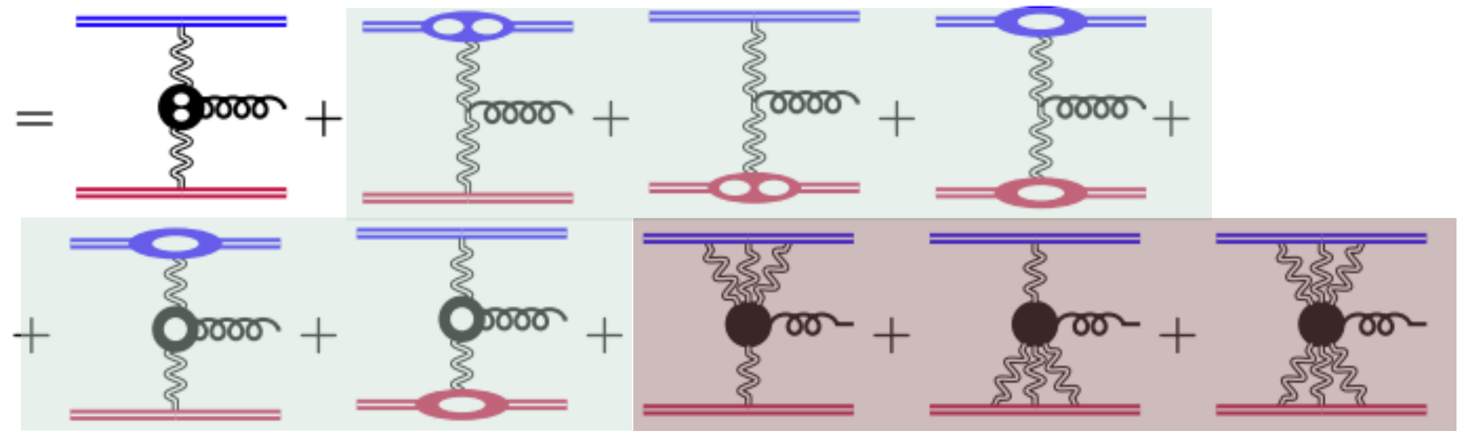
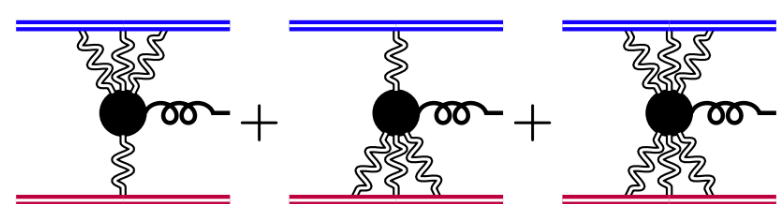
additional universal (factorised) contributions!

$$\mathcal{A}^{(2),(--)}_{\mathrm{NNLL}} =$$



$$= \left[\overline{\mathcal{W}}^{(2)}_{\lambda_4} + \bar{\mathcal{J}}^{(2)}_A + \bar{\mathcal{J}}^{(2)}_B + \bar{\mathcal{J}}^{(1)}_A \bar{\mathcal{J}}^{(1)}_B + \overline{\mathcal{W}}^{(1)}_{\lambda_4}\left(\bar{\mathcal{J}}^{(1)}_A + \bar{\mathcal{J}}^{(1)}_B\right) + (i\pi)^2 \left(B^{(2)}_{+-}\boldsymbol{\mathcal{T}}^2_{+-} + B^{(2)}_{--}\boldsymbol{\mathcal{T}}^2_{--} + B^{(2)}_{-+}\boldsymbol{\mathcal{T}}^2_{-+} - B^{(2)}_d \frac{N_c^2}{4}\right)\right]\mathcal{A}^{(0)}$$

additional universal (factorised) contributions!

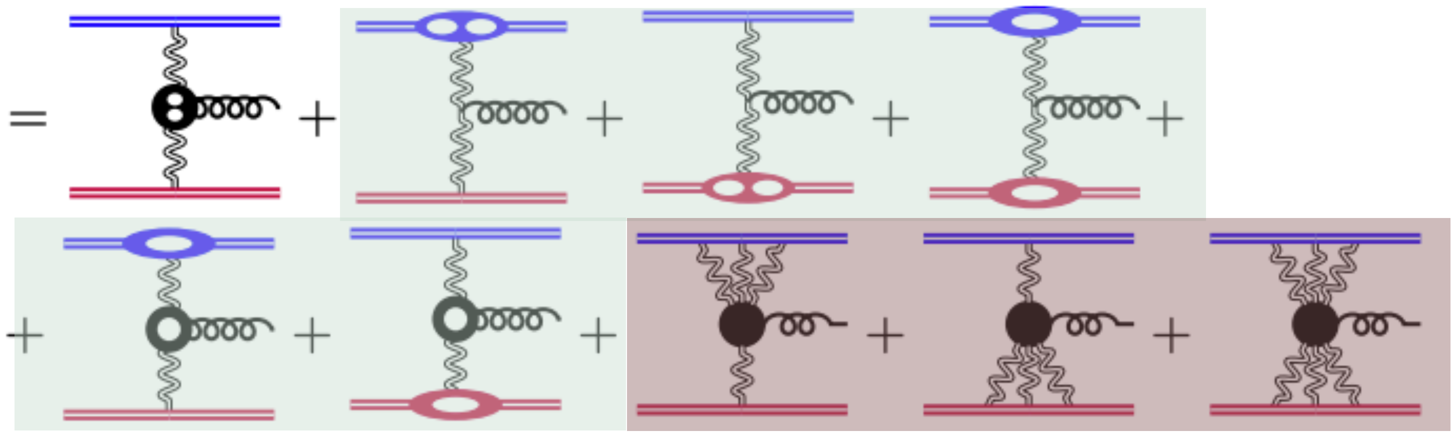


$$\approx (i\pi)^2 \frac{N_c^2}{4}\left(B^{(2)}_{+-} + B^{(2)}_{--} + B^{(2)}_{-+} - B^{(2)}_d\right)\mathcal{A}^{(0)}$$

Disentangling the Regge cut and Regge pole in perturbative QCD

Giulio Falcioni,[1,*] Einan Gardi,[1,†] Niamh Maher,[1,‡] Calum Milloy,[2,§] and Leonardo Vernazza[2,3,¶]

$\mathcal{A}^{(2),(--)}_{\rm NNLL} =$



$$= \Big[\, \overline{\mathcal{W}}^{(2)}_{\lambda_4} + \bar{\mathcal{J}}^{(2)}_A + \bar{\mathcal{J}}^{(2)}_B + \bar{\mathcal{J}}^{(1)}_A \bar{\mathcal{J}}^{(1)}_B + \overline{\mathcal{W}}^{(1)}_{\lambda_4}\big(\bar{\mathcal{J}}^{(1)}_A + \bar{\mathcal{J}}^{(1)}_B\big)$$

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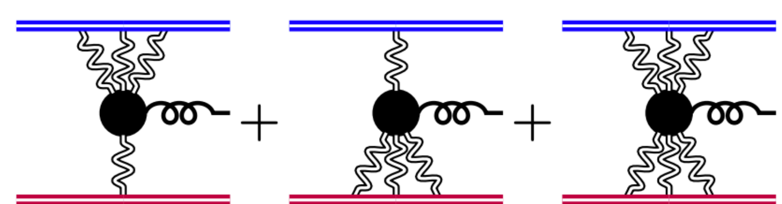
additional universal (factorised) contributions!



$$\approx (i\pi)^2 \frac{N_c^2}{4} \Big(B^{(2)}_{+-} + B^{(2)}_{--} + B^{(2)}_{-+} - B^{(2)}_{d} \Big) \mathcal{A}^{(0)}$$

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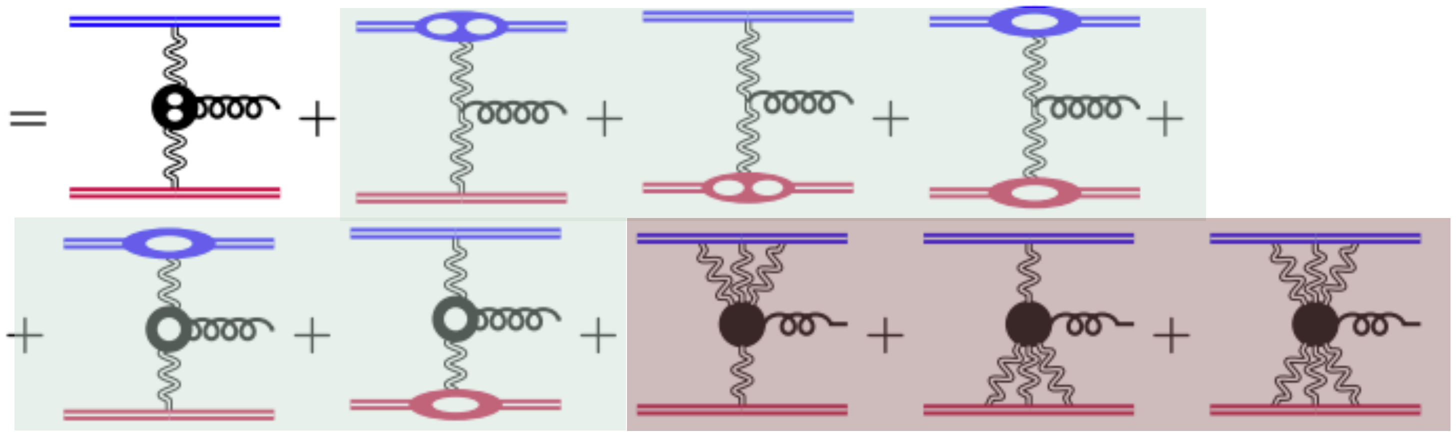
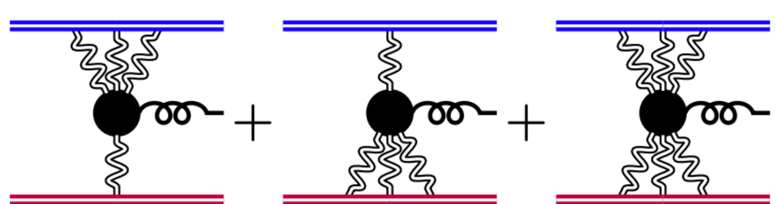


$$= \Big[\overline{\mathcal{W}}^{(2)}_{\lambda_4} + \bar{\mathcal{J}}^{(2)}_A + \bar{\mathcal{J}}^{(2)}_B + \bar{\mathcal{J}}^{(1)}_A\bar{\mathcal{J}}^{(1)}_B + \overline{\mathcal{W}}^{(1)}_{\lambda_4}\big(\bar{\mathcal{J}}^{(1)}_A + \bar{\mathcal{J}}^{(1)}_B\big)$$
$$+ (i\pi)^2\Big(B^{(2)}_{+-}\mathbf{\mathcal{T}}^2_{+-} + B^{(2)}_{--}\mathbf{\mathcal{T}}^2_{--} + B^{(2)}_{-+}\mathbf{\mathcal{T}}^2_{-+} - B^{(2)}_d\frac{N_c^2}{4}\Big)\Big]\mathcal{A}^{(0)}$$

additional universal (factorised) contributions!

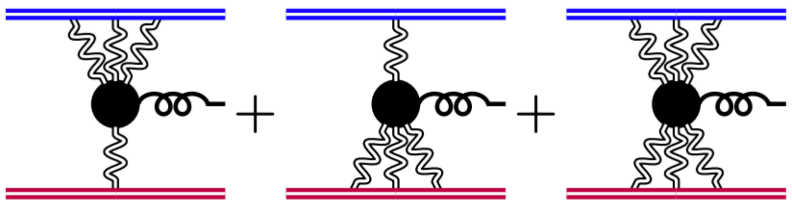


$$\approx (i\pi)^2\frac{N_c^2}{4}\Big(B^{(2)}_{+-} + B^{(2)}_{--} + B^{(2)}_{-+} - B^{(2)}_d\Big)\mathcal{A}^{(0)}$$

mismatch between W and reggeized gluon at NNLL!

an open question:

Can one redefine the W field so that



do not contribute

to the Regge pole?

an open question:

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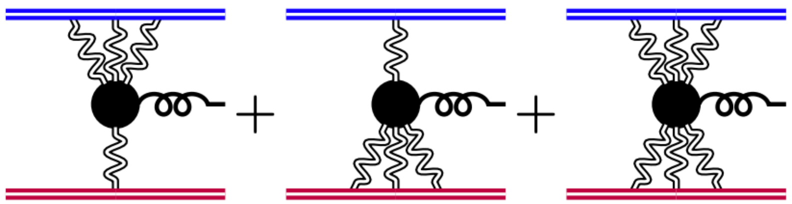


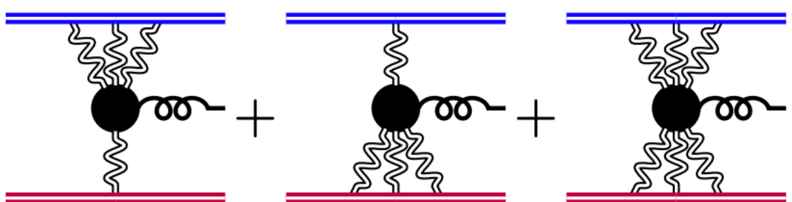
do not contribute

to the Regge pole?

Or: in terms of W

an open question:

Can one redefine the W field so that

 do not contribute

to the Regge pole?

Or: in terms of WTF is a reggeon?

Tullio Regge

Infrared Structure

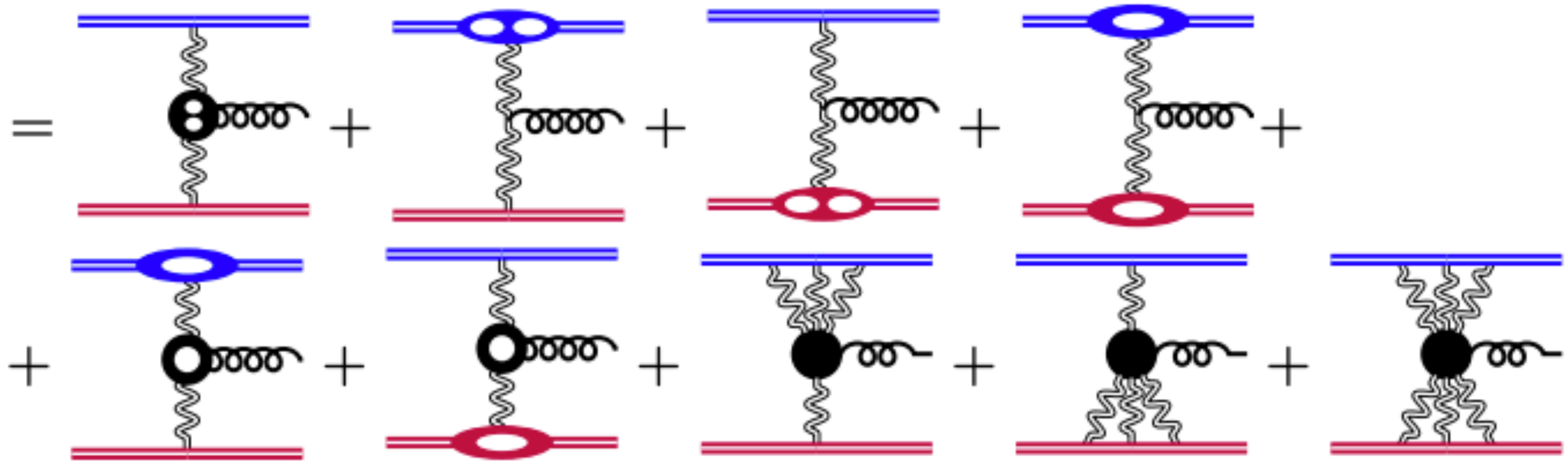
$$\boldsymbol{\mathcal{H}}^{[AB]} = \lim_{\epsilon \to 0} \mathbf{Z}_{IR}^{-1} \boldsymbol{\mathcal{B}}^{[AB]}$$

$$\mathbf{Z}_{IR}(\epsilon, \{p\}, \mu) = \exp\left[\int_{\mu}^{\infty} \frac{\mathrm{d}\mu'}{\mu'} \boldsymbol{\Gamma}_{IR}(\{p\}, \mu')\right]$$

$$\begin{aligned}
\boldsymbol{\Gamma}_{IR} = {} & \gamma_K \mathcal{C}_A \ln\frac{-s_{51}}{\mu^2} - \frac{\gamma_K}{2} \ln\frac{-s_{51}}{\rho^2} (\mathbf{T}_{+}^{15})^2 + 2\gamma_A \\
& + \gamma_K \mathcal{C}_B \ln\frac{-s_{23}}{\mu^2} - \frac{\gamma_K}{2} \ln\frac{-s_{23}}{\rho^2} (\mathbf{T}_{+}^{23})^2 + 2\gamma_B \\
& + \gamma_K L_A (\mathbf{T}_{+}^{15})^2 + \gamma_K L_B (\mathbf{T}_{+}^{23})^2 \\
& + \frac{\gamma_K}{2} \left(-\mathcal{C}_4 \ln\frac{\mu^2}{\mathbf{p}_4^2} + \ln\frac{\rho^2}{\mathbf{p}_4^2} (\mathbf{T}_{+}^{15})^2 + \ln\frac{\rho^2}{\mathbf{p}_4^2} (\mathbf{T}_{+}^{23})^2 - i\pi \boldsymbol{\mathcal{T}}_{++} \right) + \gamma_4 \\
& + \frac{\gamma_K}{2} \times i\pi \left(\boldsymbol{\mathcal{T}}_{+-} + \boldsymbol{\mathcal{T}}_{--} + \boldsymbol{\mathcal{T}}_{-+} \right).
\end{aligned}$$

The vertex finite remainder

$$\mathcal{A}^{(2),(--)}_{\text{NNLL}} =$$



$$= \Bigg[\overline{\mathcal{W}}^{(2)}_{\lambda_4} + \bar{\mathcal{J}}^{(2)}_{A} + \bar{\mathcal{J}}^{(2)}_{B} + \bar{\mathcal{J}}^{(1)}_{A}\,\bar{\mathcal{J}}^{(1)}_{B} + \overline{\mathcal{W}}^{(1)}_{\lambda_4}\left(\bar{\mathcal{J}}^{(1)}_{A} + \bar{\mathcal{J}}^{(1)}_{B}\right)$$

$$+ (i\pi)^2 \left(B^{(2)}_{+-}\boldsymbol{\mathcal{T}}^2_{+-} + B^{(2)}_{--}\boldsymbol{\mathcal{T}}^2_{--} + B^{(2)}_{-+}\boldsymbol{\mathcal{T}}^2_{-+} - B^{(2)}_{d}\frac{N_c^2}{4} \right) \Bigg] \mathcal{A}^{(0)}$$

The vertex finite remainder

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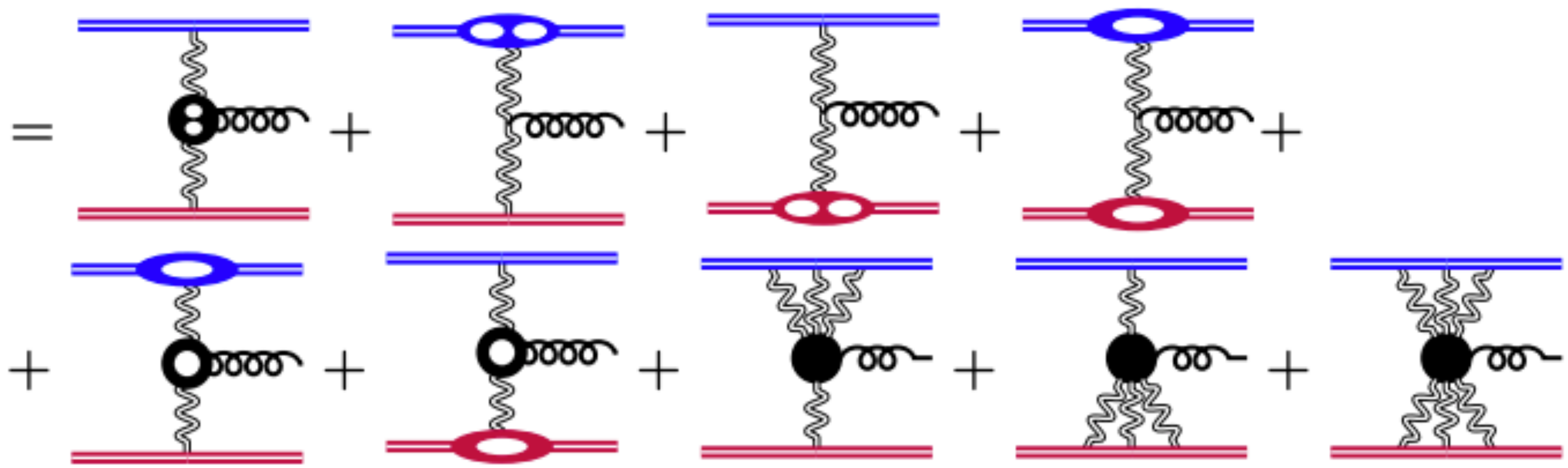


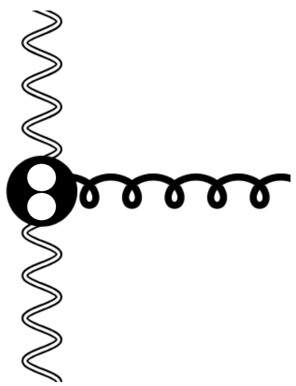
$$= \Big[\overline{\mathcal{W}}^{(2)}_{\lambda_4} + \bar{\mathcal{J}}^{(2)}_A + \bar{\mathcal{J}}^{(2)}_B + \bar{\mathcal{J}}^{(1)}_A \bar{\mathcal{J}}^{(1)}_B + \overline{\mathcal{W}}^{(1)}_{\lambda_4}(\bar{\mathcal{J}}^{(1)}_A + \bar{\mathcal{J}}^{(1)}_B)$$

$$+ (i\pi)^2 \left(B^{(2)}_{+-} \boldsymbol{\mathcal{T}}^2_{+-} + B^{(2)}_{--} \boldsymbol{\mathcal{T}}^2_{--} + B^{(2)}_{-+} \boldsymbol{\mathcal{T}}^2_{-+} - B^{(2)}_d \frac{N_c^2}{4} \right) \Big] \mathcal{A}^{(0)}$$

$$\mathcal{F}^{(2),(--)}_{\rm NNLL} = \Big\{ \hat{\mathcal{U}}^{(2)}_{\lambda_4} + \hat{\mathcal{I}}_A{}^{(2)} + \hat{\mathcal{I}}_B{}^{(2)} + \hat{\mathcal{I}}_A{}^{(1)} \hat{\mathcal{I}}_B{}^{(1)} + \hat{\mathcal{U}}^{(1)}_{\lambda_4} (\hat{\mathcal{I}}_A{}^{(1)} + \hat{\mathcal{I}}_B{}^{(1)})$$

$$+ (i\pi)^2 \left[\hat{B}^{(2)}_{+-} \left(\boldsymbol{\mathcal{T}}^2_{+-} - \frac{N_c^2}{4} \right) + \hat{B}^{(2)}_{--} \left(\boldsymbol{\mathcal{T}}^2_{--} - \frac{N_c^2}{4} \right) + \hat{B}^{(2)}_{-+} \left(\boldsymbol{\mathcal{T}}^2_{-+} - \frac{N_c^2}{4} \right) \right] \Big\} \mathcal{A}^{(0)}$$

QCD vertex



$h_{w,i}$

Transcendental

(weight w)

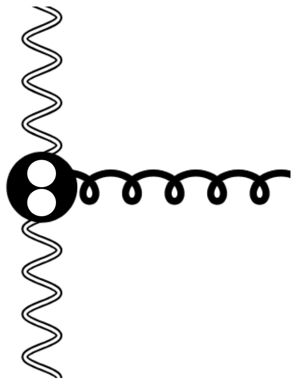
r_j

Rational

$$\hat{\mathcal{U}}^{(1)}_{+,QCD} = \frac{N_c}{2}\left(5\zeta_2 - h_{1,2}\left(h_{1,2}+3r_3\right) - i\pi h_{1,1}\right) - \frac{N_c-N_f}{3}\left(r_1 h_{1,2}+r_2\right),$$

$$\begin{aligned}
\hat{\mathcal{U}}^{(2)}_{+,QCD} = N_c^2\Bigg[&\frac{1}{144}i\pi\Big(-72\zeta_3 + h_{1,1}\left(-36\zeta_2 + 9h_{1,2}\left(3r_3+4h_{1,2}\right) - 456\right) + 464 \\
&- 27r_3\left(h_{1,3}h_{1,4} + 8h_{2,2} - 8h_{2,3}\right)\Big) + \frac{1}{432}\Big(216r_2 - 1809\zeta_4 + 216r_1h_{1,2} - 2872 \\
&+ 36\zeta_2\left(-18h_{1,1}^2 + 3\left(9r_3 - 7h_{1,2}\right)h_{1,2} + 209\right) - 9\big(-6h_{1,2}^4 + 98h_{1,2}^2 + 9r_3\big(2h_{1,2}^3 \\
&+ 3\left(\left(h_{1,1}-4\right)h_{1,1}+24\right)h_{1,2} + h_{1,1}\left(h_{1,3}h_{1,4}+8h_{2,2}-8h_{2,3}\right) + 64h_{3,6}\big)\big)\Big)\Bigg] \\
+ N_c\left(N_c-N_f\right)\Bigg[&\frac{1}{216}i\pi\Big(36r_4 + 36r_2\left(h_{1,1}-1\right) + 108r_3h_{1,2} + 3h_{1,1}\left(3r_1h_{1,2}-40\right) \\
&- 9r_1\left(12h_{1,2} + h_{1,3}h_{1,4} + 8h_{2,2} - 8h_{2,3}\right) + 112\Big) + \frac{1}{648}\Big(36\zeta_2\left(9r_1h_{1,2}+55\right) \\
&+ 36\left(3\left(5r_3+r_6\right) - 113r_1\right)h_{1,2} + 36r_2\left(3h_{1,2}^2 - 15\zeta_2 + 6h_{1,1} - 137\right) - 9\big(9r_1h_{1,2}h_{1,1}^2 \\
&- 3\big(4r_4 - 12r_3h_{1,2} + r_1\left(36h_{1,2} - h_{1,3}h_{1,4} - 8h_{2,2} + 8h_{2,3}\right) - 4\big)h_{1,1} + 2\big(3r_1h_{1,2}^3 \\
&+ \left(6r_5+2\right)h_{1,2}^2 - 18\left(r_1-r_3\right)\left(h_{1,3}h_{1,4} + 8h_{2,2} - 8h_{2,3}\right) + 96r_1h_{3,6}\big)\big) - 260\Big)\Bigg] \\
+ N_c\beta^{(0)}\Bigg[&\frac{1}{8}i\pi\left(h_{1,1}^2 + 2h_{1,2}^2 + 4\zeta_2 - 8h_{2,1}\right) + \frac{1}{48}\Big(-h_{1,4}^3 - 3h_{1,1}^2h_{1,4} + 3h_{1,2}^2h_{1,4} \\
&- 9h_{1,2}\left(h_{1,3}h_{1,4} + 8h_{2,2} - 8h_{2,3}\right) - 48\left(2\zeta_2h_{1,4} - 2h_{3,4} + 2h_{3,5} + h_{3,7}\right) \\
&+ 3h_{1,3}^2h_{1,4} + 232\zeta_3 + 3h_{1,1}\left(5h_{1,2}^2 + 2h_{1,3}h_{1,2} - 16h_{2,1}\right)\Big)\Bigg] \\
+ \frac{\left(N_c-N_f\right)^2}{54}\Big[&\left(r_2+r_1h_{1,2}\right)\left(6h_{1,1}-20\right) + 3h_{1,2}h_{1,2}\Big] + \frac{N_f}{2N_c}\Big[r_2 + \left(r_1-2r_3\right)h_{1,2}\Big]
\end{aligned}$$

N=4 vertex



$h_{w,i}$

Transcendental

(weight w)

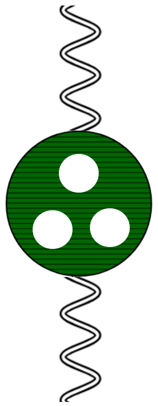
r_j

Rational

$$\hat{\mathcal{U}}^{(1)}_{\mathcal{N}=4} = \frac{N_c}{2}\left(-h_{1,2}^2 - i\pi h_{1,1} + 5\zeta_2\right),$$

$$\hat{\mathcal{U}}^{(2)}_{\mathcal{N}=4} = -\frac{N_c^2}{4}\left(h_{1,2}^2\left(7\zeta_2 - i\pi h_{1,1}\right) + \zeta_2 h_{1,1}\left(6h_{1,1} + i\pi\right) - \frac{h_{1,2}^4}{2} + 2i\pi\zeta_3 + \frac{67}{4}\zeta_4\right)$$

QCD gluon Regge trajectory



$$K(\alpha_s(\mu)) = -\frac{1}{4}\int_{\infty}^{\mu^2} \frac{d\lambda^2}{\lambda^2}\gamma^{\mathrm{K}}\left(\alpha_s(\lambda^2)\right)$$

$$\begin{aligned}
\tau_1 &= K_1 + \mathcal{O}(\epsilon),\\
\tau_2 &= K_2 - \frac{56 n_f}{27} + N_c\left(\frac{404}{27} - 2\zeta_3\right) + \mathcal{O}(\epsilon),\\
\tau_3 &= K_3 + N_c^2\left(16\zeta_5 + \frac{40\zeta_2\zeta_3}{3} - \frac{77\zeta_4}{3} - \frac{6664\zeta_3}{27}\right.\\
&\quad \left. - \frac{3196\zeta_2}{81} + \frac{297029}{1458}\right) + \frac{n_f}{N_c}\left(-4\zeta_4 - \frac{76\zeta_3}{9} + \frac{1711}{108}\right)\\
&\quad + N_c n_f\left(\frac{412\zeta_2}{81} + \frac{2\zeta_4}{3} + \frac{632\zeta_3}{9} - \frac{171449}{2916}\right)\\
&\quad + n_f^2\left(\frac{928}{729} - \frac{128\zeta_3}{27}\right) + \mathcal{O}(\epsilon),
\end{aligned}$$

Disentangling the Regge cut and Regge pole in perturbative QCD

Giulio Falcioni,[1,*] Einan Gardi,[1,†] Niamh Maher,[1,‡] Calum Milloy,[2,§] and Leonardo Vernazza[2,3,¶]

The Two-Loop Lipatov Vertex in QCD

Samuel Abreu,[a,b] Giuseppe De Laurentis,[a] Giulio Falcioni,[c,d] Einan Gardi,[a] Calum Milloy,[e] Leonardo Vernazza[e]

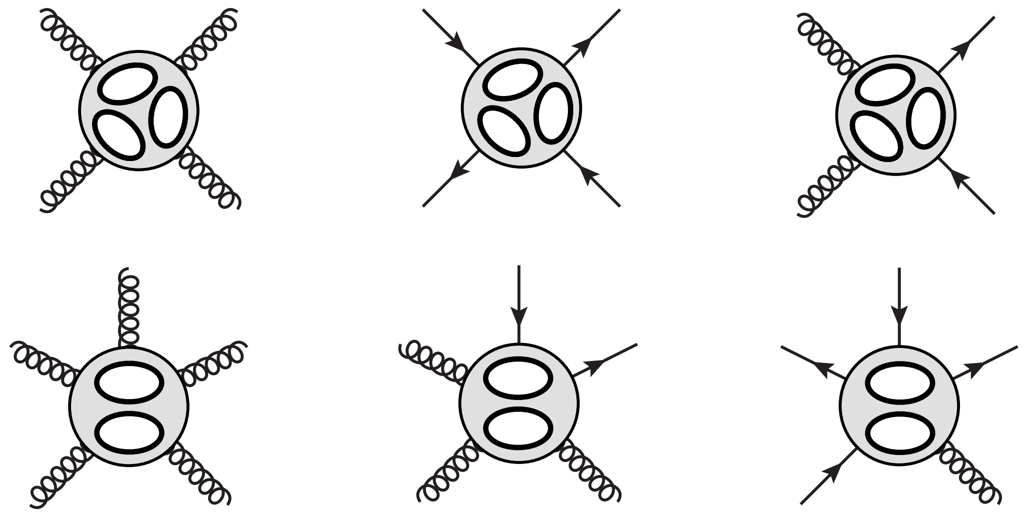
Three-loop gluon scattering in QCD and the gluon Regge trajectory

Fabrizio Caola,[1,2,*] Amlan Chakraborty,[3,†] Giulio Gambuti,[1,4,‡]
Andreas von Manteuffel,[3,§] and Lorenzo Tancredi[5,6,¶]

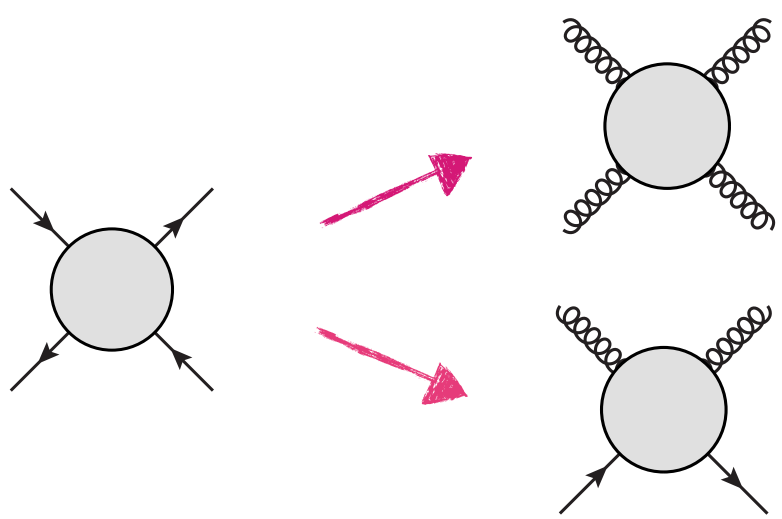
Investigating the universality of five-point QCD scattering amplitudes at high energy

Federico Buccioni,[a] Fabrizio Caola,[b,c] Federica Devoto,[b,d] Giulio Gambuti[b]

redundant amplitude information



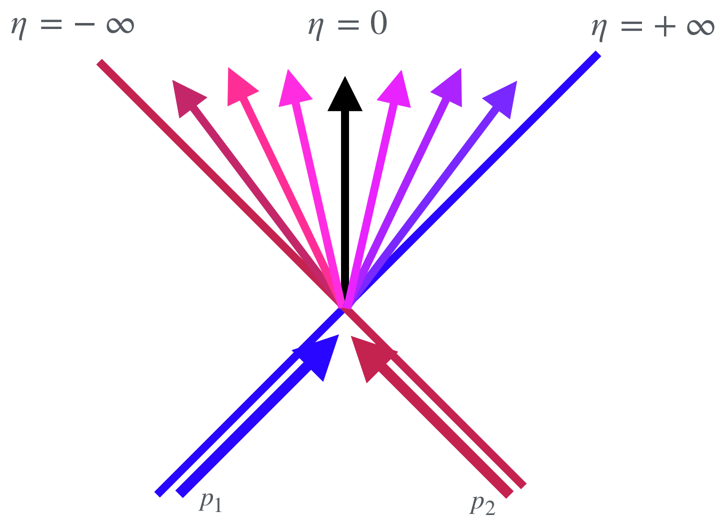
extraction + check of universality and factorisation !



Recap

Recap

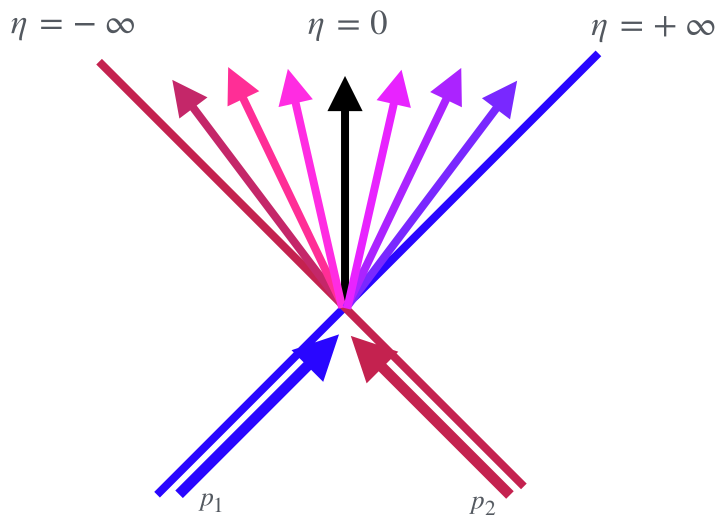
large rapidity gaps



$$\eta_2 \sim \eta_3 \ll \eta_4 \ll \ldots \ll \eta_n \sim \eta_1$$

Recap

large rapidity gaps



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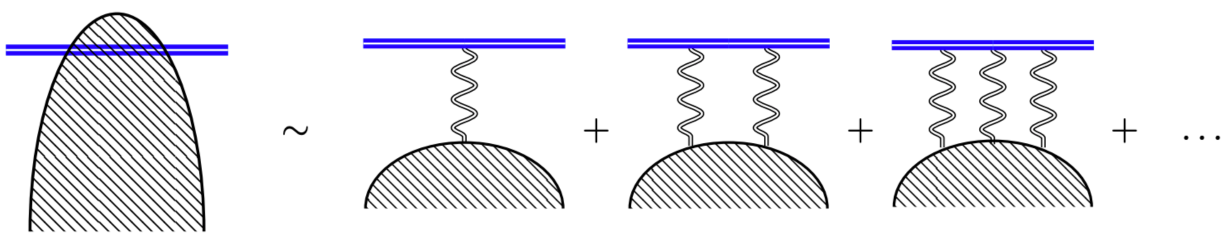
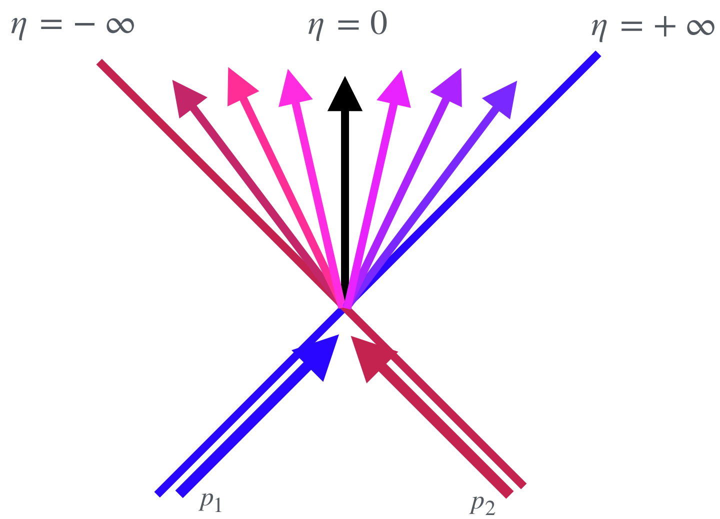
W-field expansion and RRGE



Balitsky-JIMWLK evolution

Recap

large rapidity gaps



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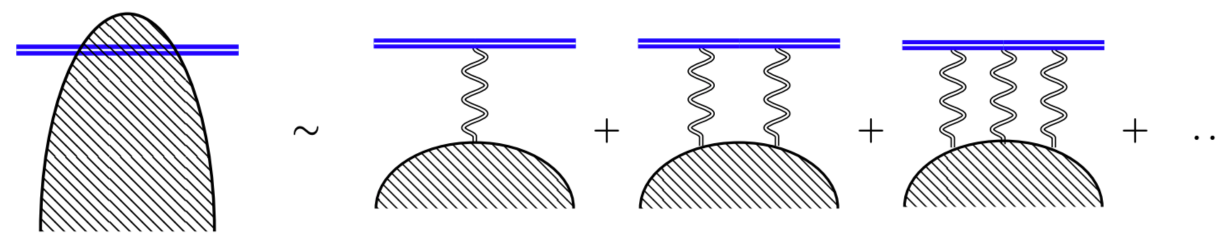
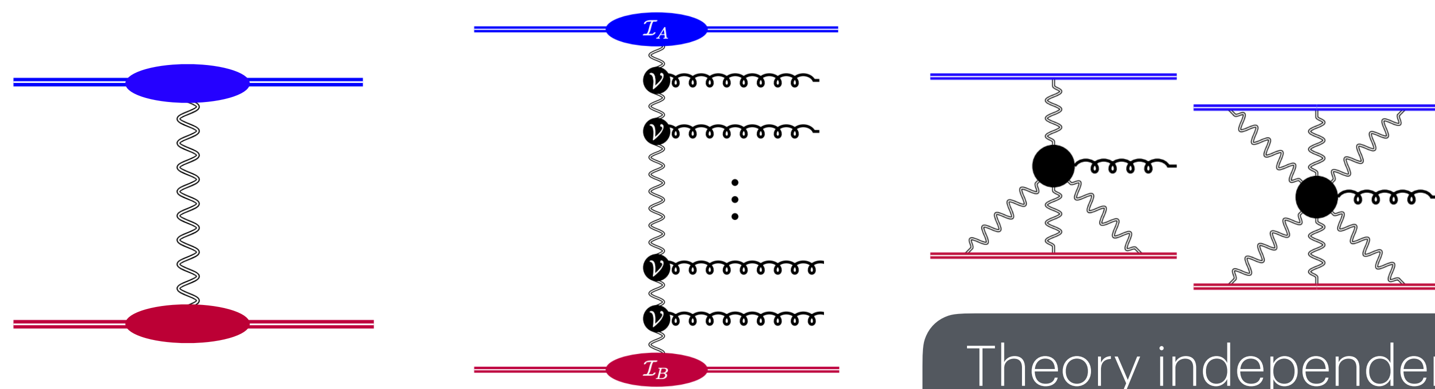


Balitsky-JIMWLK evolution

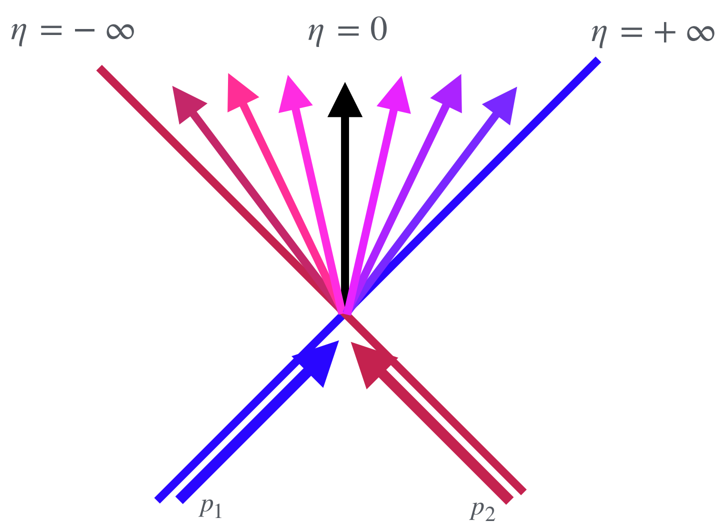
a diagrammatic approach



Theory independent at NNLL

Recap

large rapidity gaps



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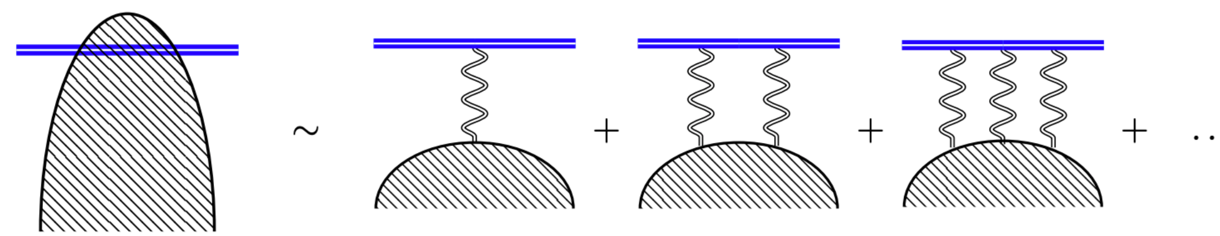
W-field expansion and RRGE



Balitsky-JIMWLK evolution

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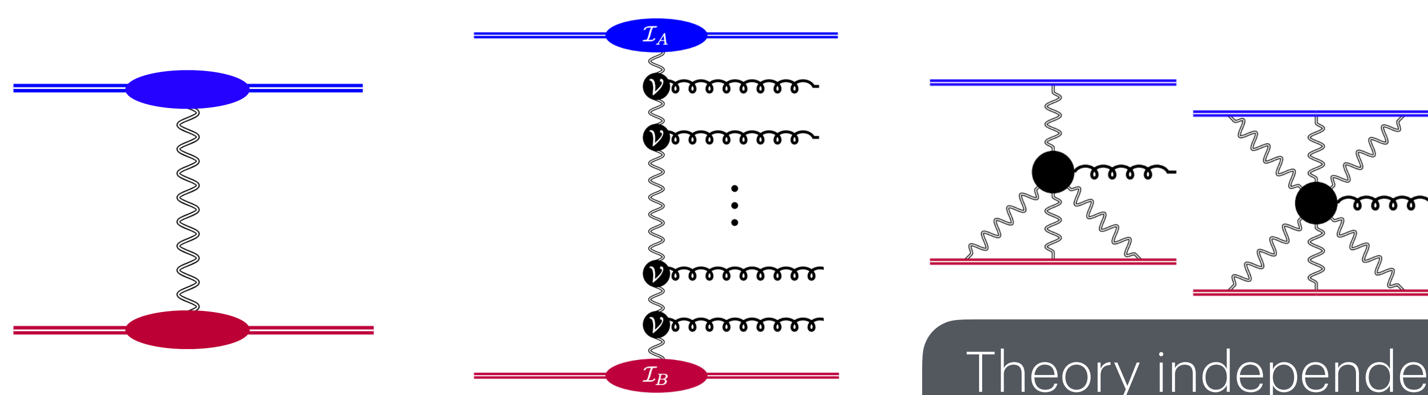
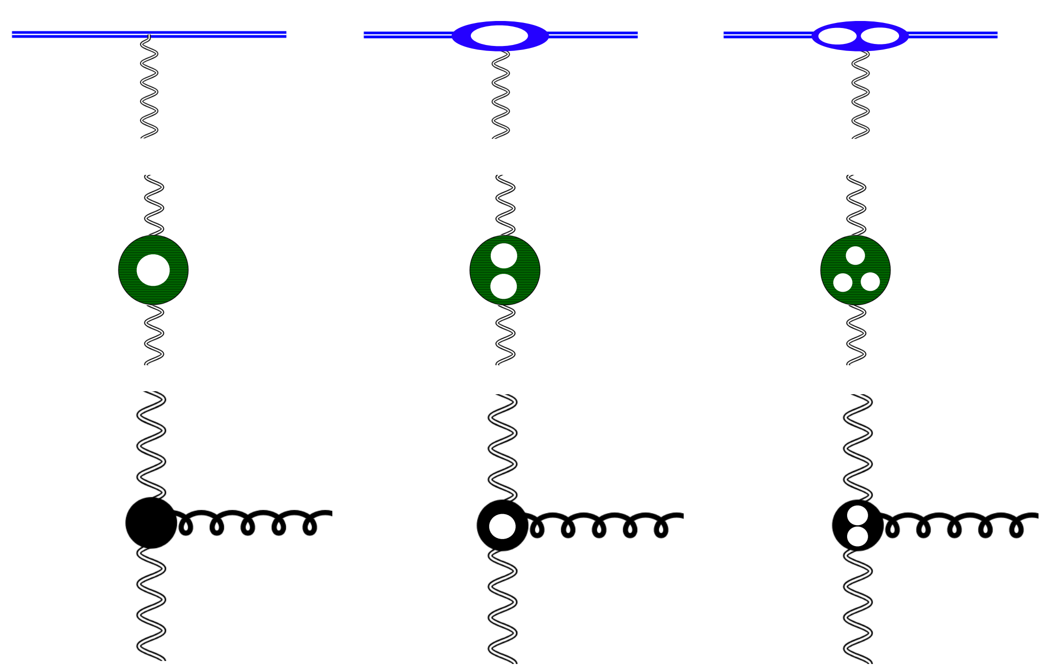
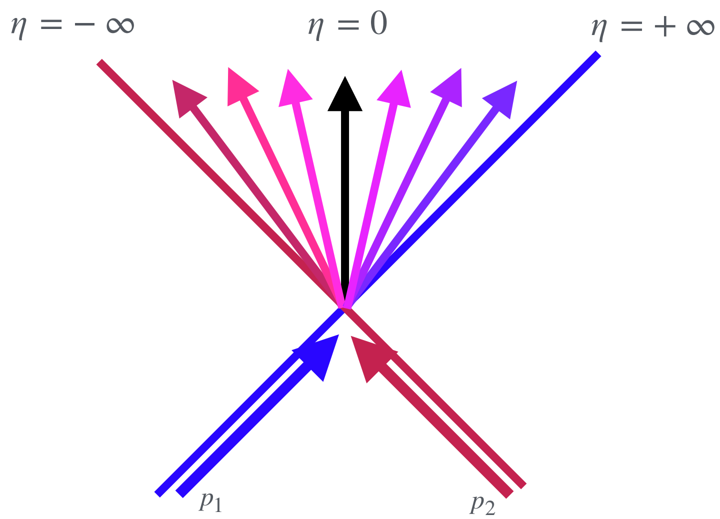


Theory independent at NNLL



Recap

large rapidity gaps



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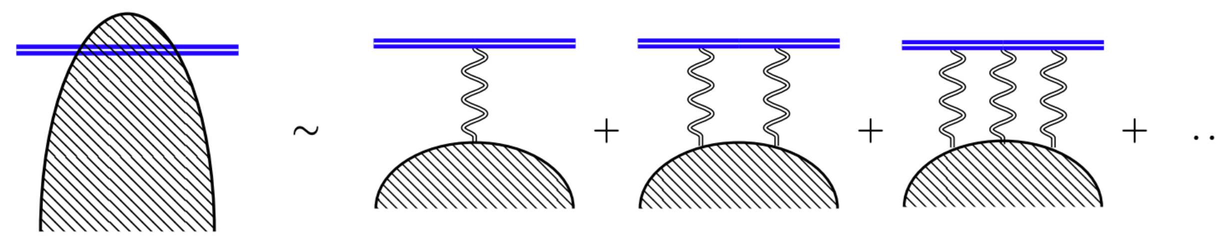

Balitsky-JIMWLK evolution

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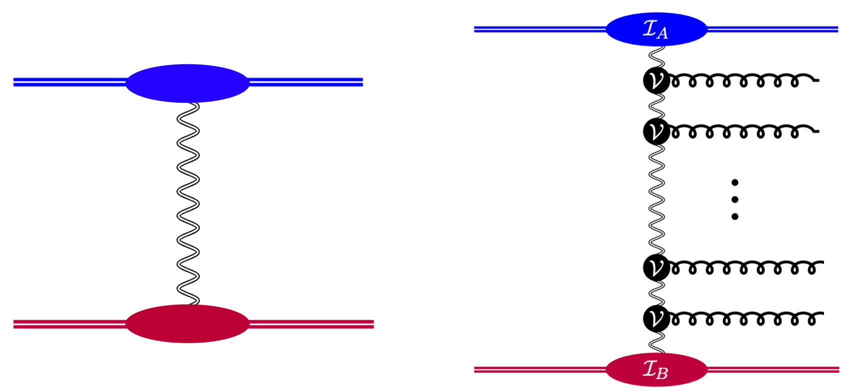
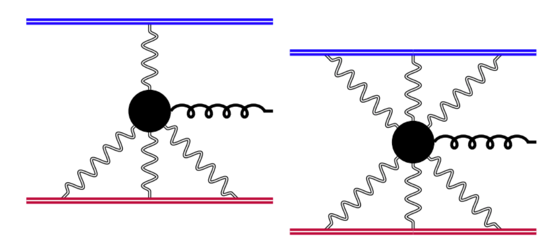
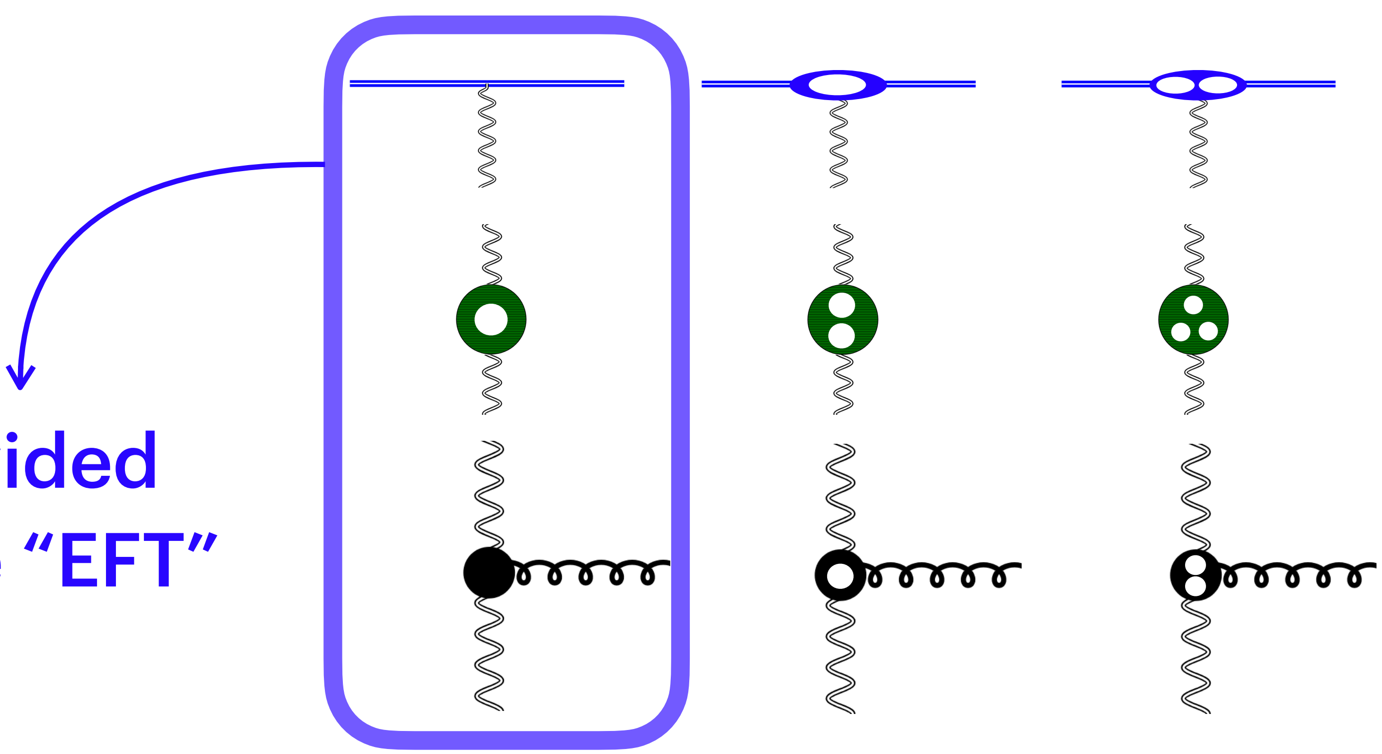




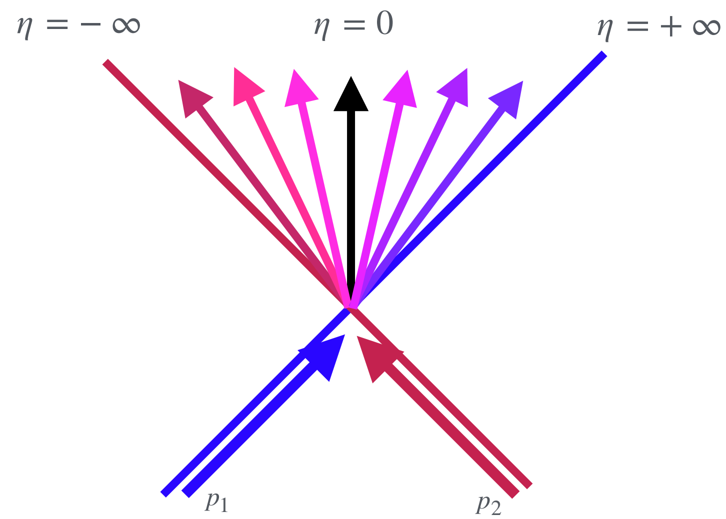
Theory independent at NNLL



Provided by the “EFT”

Recap

large rapidity gaps



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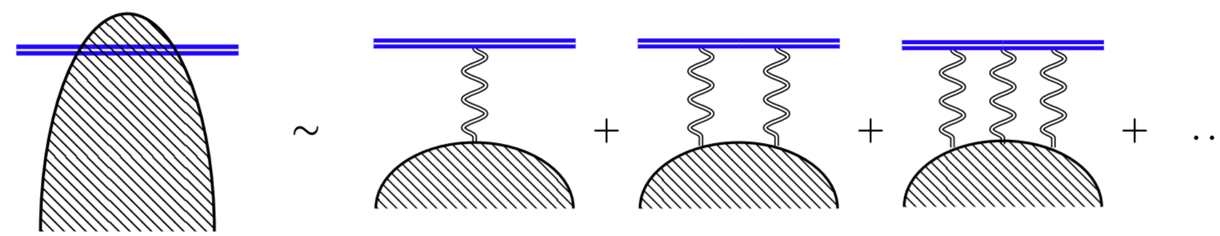
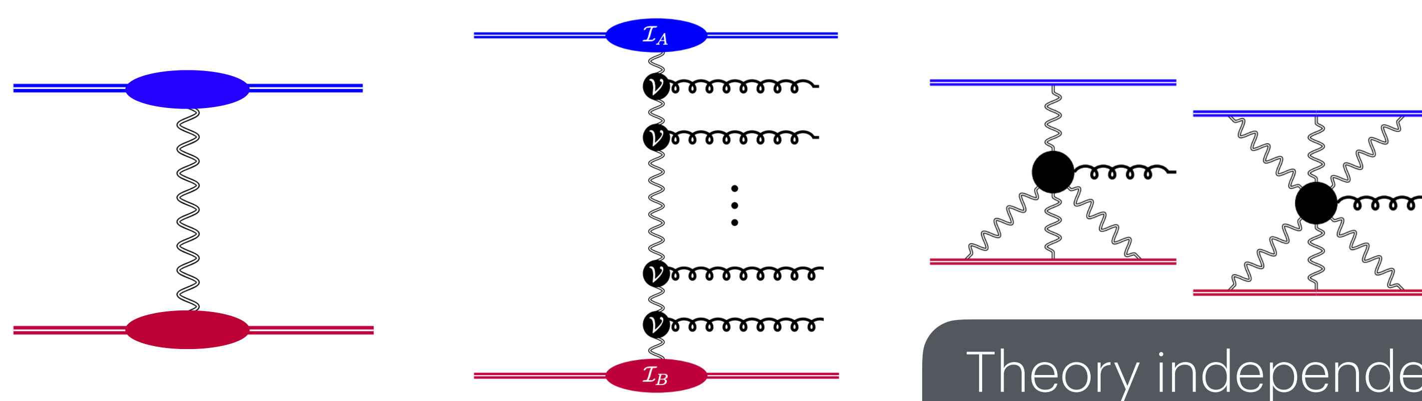
W-field expansion and RRGE

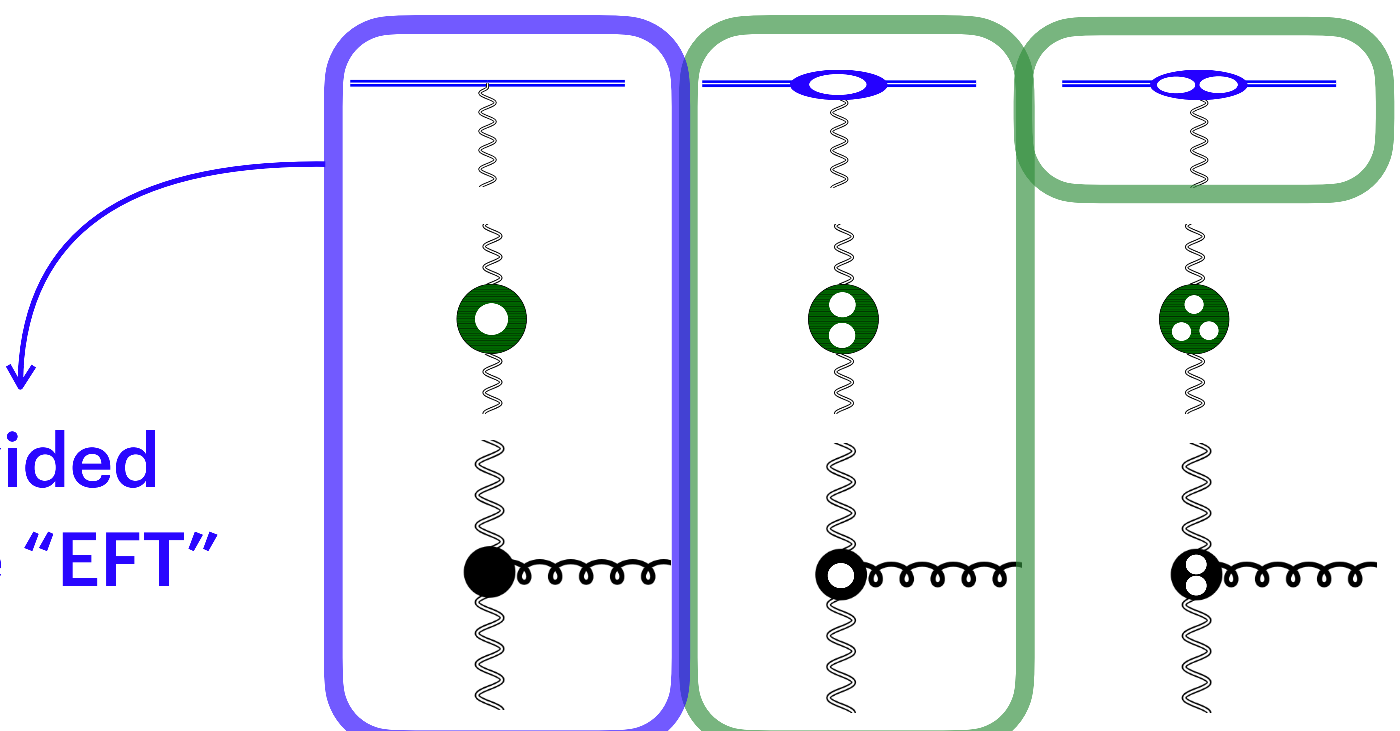


Balitsky-JIMWLK evolution

a diagrammatic approach



Theory independent at NNLL



Provided by the "EFT"

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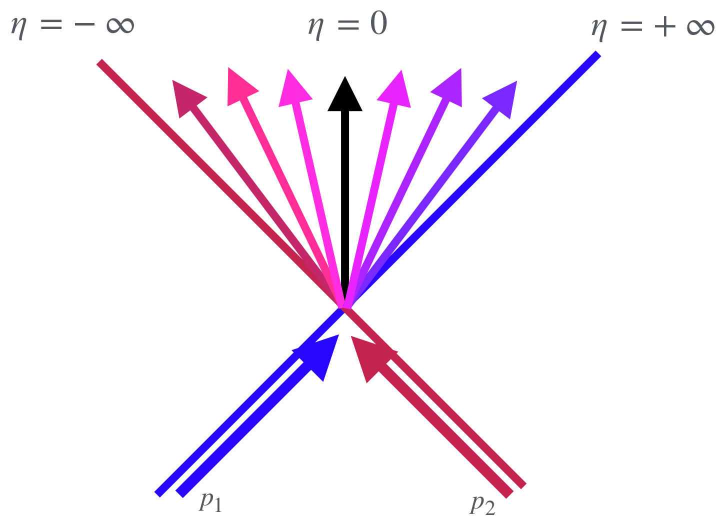

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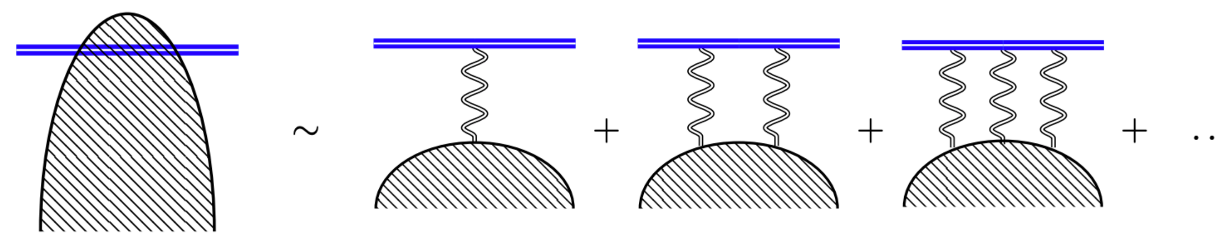

Balitsky-JIMWLK evolution

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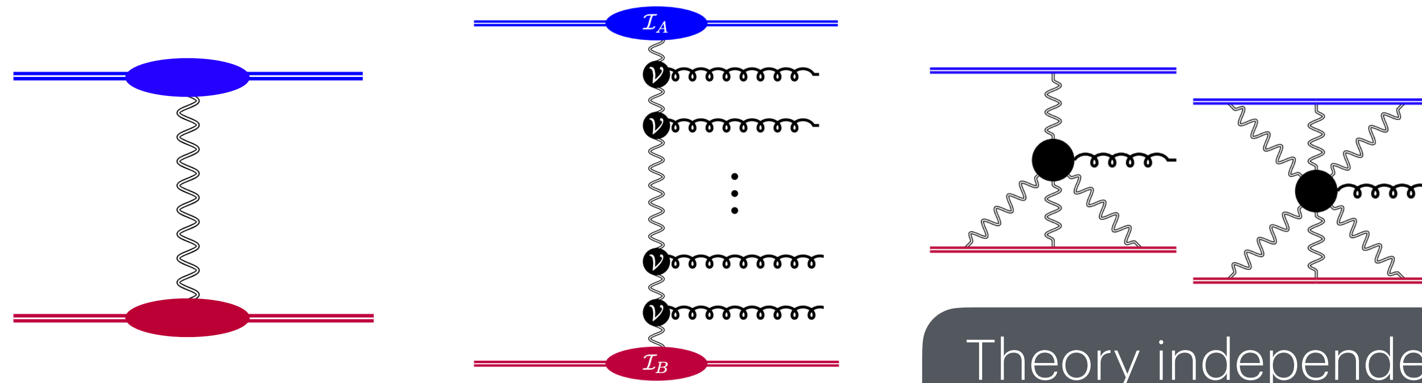
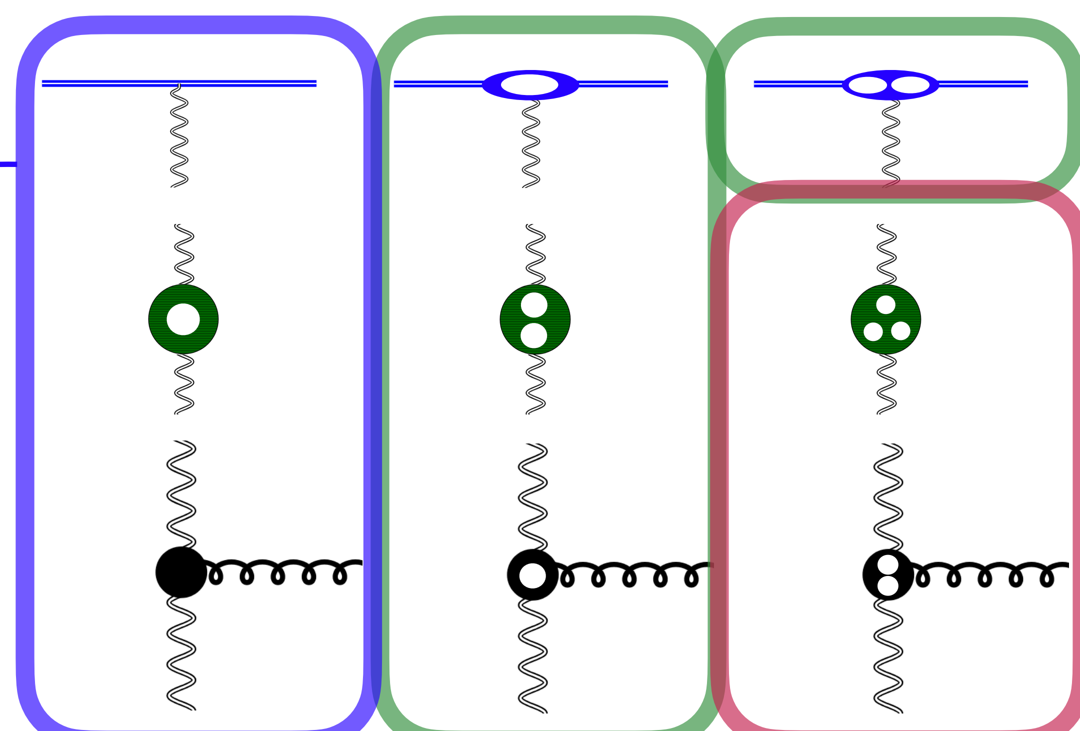


Theory independent at NNLL



Provided by the "EFT"

NEW!

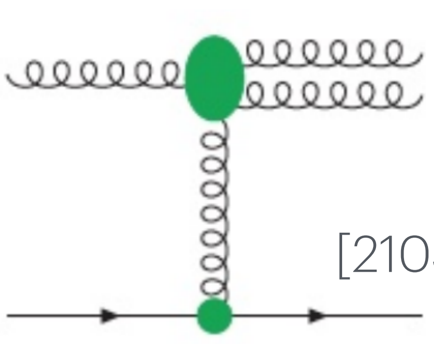
Future Directions

W⇔ reggeised gluon @ NNLL

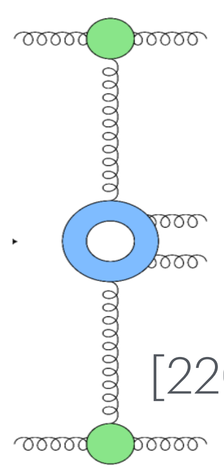
&

BFKL evolution

NMRK limits

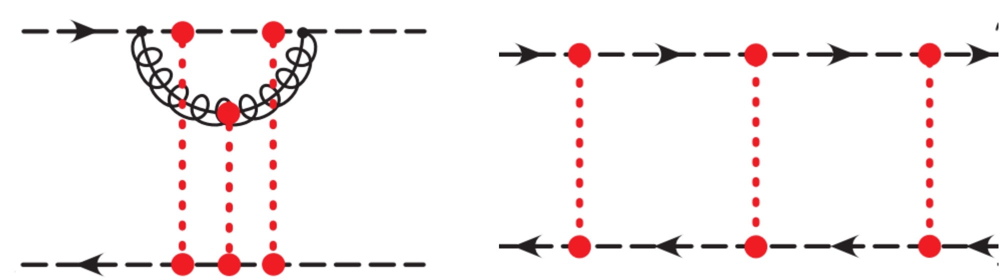


[2103.16593]



[2204.12459]

precise comparison w/ Glauber SCET



Backup

Single-valued transcendental functions

Multi-Regge limit of the two-loop five-point amplitudes in $\mathcal{N}=4$ super Yang-Mills and $\mathcal{N}=8$ supergravity

Simon Caron-Huot,[a] Dmitry Chicherin,[b] Johannes Henn,[b] Yang Zhang[c,d] and Simone Zoia[b]

$$g_{1,4}=\ln(z\bar{z}),\quad g_{1,5}=\ln\left((1-z)(1-\bar{z})\right),$$

$$g_{1,6}=\ln(z)-\ln(\bar{z}),\quad g_{1,7}=\ln(1-z)-\ln(1-\bar{z}),$$

$$g_{2,1}=D_2(z,\bar{z}),$$

$$g_{2,2}=\mathrm{Li}_2(z)+\mathrm{Li}_2(\bar{z}),$$

$$g_{2,3}=\mathrm{Li}_2\left(\frac{z}{1-\bar{z}}\right)+\mathrm{Li}_2\left(\frac{\bar{z}}{1-z}\right)+(g_{1,4}-g_{1,5})\ln(|1-z-\bar{z}|)$$
$$+\,i\pi\,(g_{1,6}+g_{1,7})\,\mathrm{sgn}[\mathrm{Im}(z)]\Theta\left(\mathrm{Re}(z)-\frac{1}{2}\right),$$

$$g_{3,1}=D_3(z,\bar{z}),$$

$$g_{3,2}=D_3(1-z,1-\bar{z}),$$

$$g_{3,3}=\mathrm{Li}_3(z)-\mathrm{Li}_3(\bar{z}),$$

$$g_{3,4}=\mathrm{Li}_3(1-z)-\mathrm{Li}_3(1-\bar{z})$$

$$g_{3,5}=\mathrm{Li}_3\left(\frac{z\bar{z}}{(1-z)(1-\bar{z})}\right)+\frac{1}{2}\ln(1-z-\bar{z})\ln^2\left(\frac{z\bar{z}}{(1-z)(1-\bar{z})}\right),$$

$$g_{3,6}=2\,\mathrm{Li}_3\left(\frac{z}{1-\bar{z}}\right)-2\mathrm{Li}_3\left(\frac{\bar{z}}{1-z}\right)-\ln\left(\frac{z\bar{z}}{(1-z)(1-\bar{z})}\right)D_2\left(\frac{z}{1-\bar{z}},\frac{\bar{z}}{1-z}\right)$$
$$+\,\frac{i\pi}{2}\left[(g_{1,4}-g_{1,5})^2+(g_{1,6}+g_{1,7})^2\right]\mathrm{sgn}[\mathrm{Im}(z)]\,\Theta\left(\mathrm{Re}(z)-\frac{1}{2}\right),$$

$$g_{3,9}=\mathrm{Li}_3\left(\frac{1-z-\bar{z}}{(1-z)(1-\bar{z})}\right),$$

$$D_2(z,\bar{z})=\mathrm{Li}_2(z)-\mathrm{Li}_2(\bar{z})+\frac{\ln(z\bar{z})}{2}\left(\ln(1-z)-\ln(1-\bar{z})\right),$$

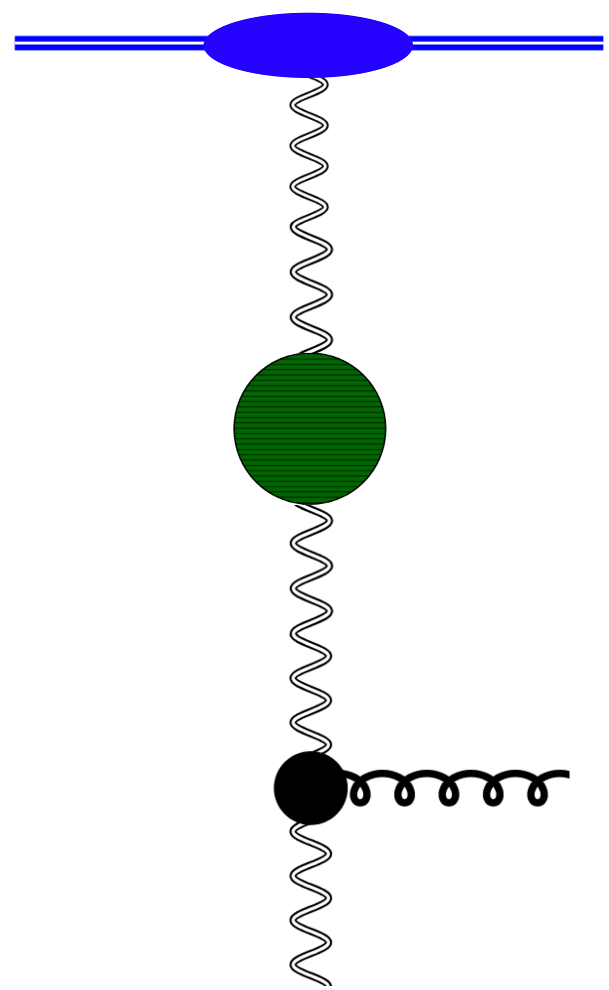
$$D_3(z,\bar{z})=\mathrm{Li}_3(z)+\mathrm{Li}_3(\bar{z})-\frac{1}{2}\ln(z\bar{z})\left(\mathrm{Li}_2(z)+\mathrm{Li}_2(\bar{z})\right)-\frac{1}{4}\ln^2(z\bar{z})\ln\left((1-z)(1-\bar{z})\right)$$

Rational Functions

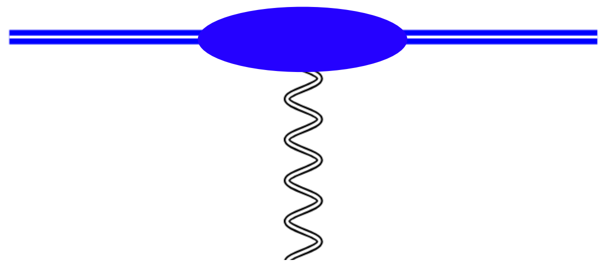
$$r_1 = \frac{z^3 + (1-\bar{z})^3}{(1-z-\bar{z})^3}, \quad r_2 = \frac{z(1-\bar{z})}{(1-z-\bar{z})^2}\left(\frac{1}{1-z} + \frac{1}{\bar{z}}\right), \quad r_3 = \frac{1+z-\bar{z}}{1-z-\bar{z}},$$

$$r_4 = \frac{z(1-\bar{z})}{(1-z)\bar{z}}, \quad r_5 = \frac{z(1-\bar{z})}{(1-z-\bar{z})^2}, \quad r_6 = \frac{z(1-\bar{z})(z-\bar{z})}{\bar{z}(1-z)(1-z-\bar{z})},$$

(anti-)symmetric under $z \to 1-\bar{z}$

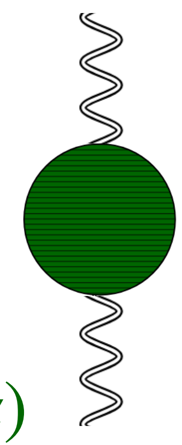


Impact Factors



$$\mathcal{J} = 1 + \alpha_s J_1 + \alpha_s^2 J_2 + \ldots$$

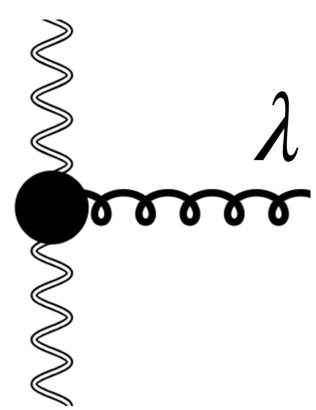
Regge Trajectory



$e^{L\,\tau_g(\alpha)}$

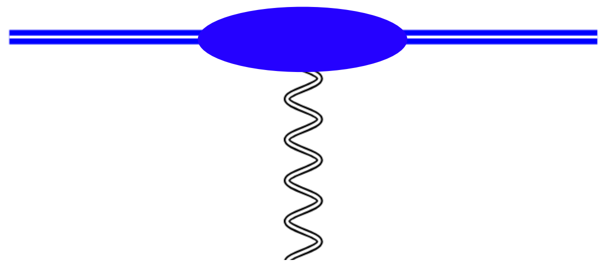
$$\tau_g(\alpha) = \alpha_s \tau_1 + \alpha_s^2 \tau_2 + \alpha_s^3 \tau_3 + \ldots$$

Lipatov Vertex



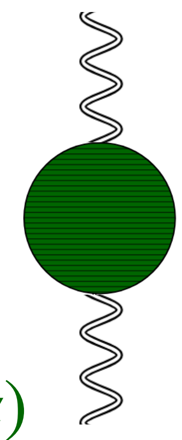
$$V^\lambda = V_0^\lambda + \alpha_s V_1^\lambda + \alpha_s^2 V_2^\lambda$$

Impact Factors



$$\mathcal{J} = 1 + \alpha_s J_1 + \alpha_s^2 J_2 + \ldots$$

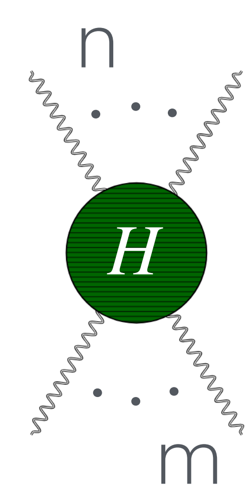
Regge Trajectory



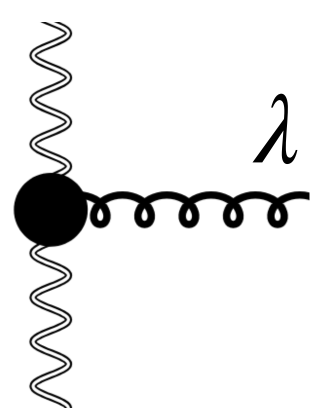
$e^{L\,\tau_g(\alpha)}$

$$\tau_g(\alpha) = \alpha_s \tau_1 + \alpha_s^2 \tau_2 + \alpha_s^3 \tau_3 + \ldots$$

and similarly for all

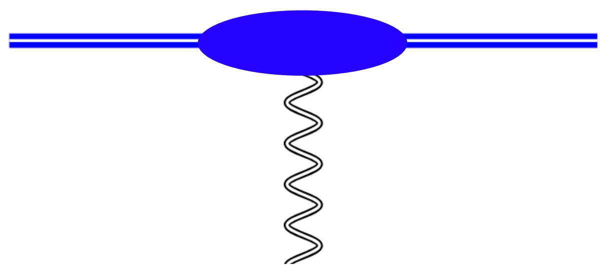


Lipatov Vertex



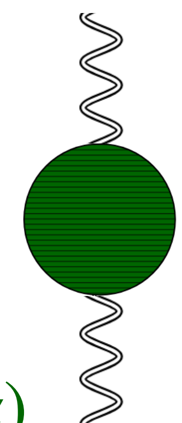
$$V^\lambda = V_0^\lambda + \alpha_s V_1^\lambda + \alpha_s^2 V_2^\lambda$$

Impact
Factors



$$\mathcal{J} = 1 + \alpha_s J_1 + \alpha_s^2 J_2 + \ldots$$

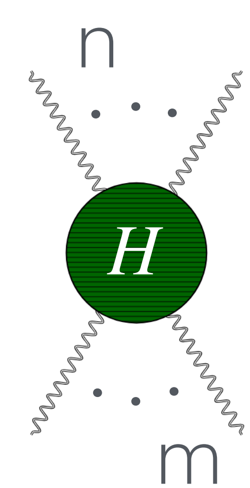
Regge
Trajectory



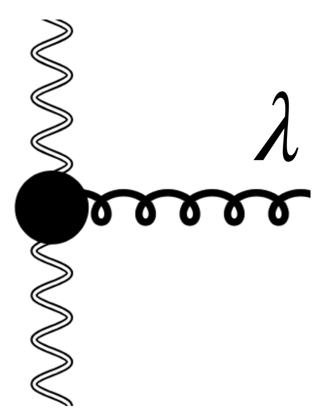
$e^{L\,\tau_g(\alpha)}$

$$\tau_g(\alpha) = \alpha_s \tau_1 + \alpha_s^2 \tau_2 + \alpha_s^3 \tau_3 + \ldots$$

and similarly for all

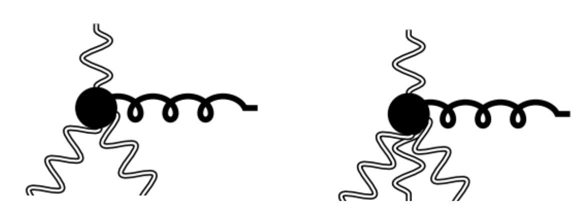


Lipatov
Vertex



$$V^\lambda = V_0^\lambda + \alpha_s V_1^\lambda + \alpha_s^2 V_2^\lambda$$

and similarly for



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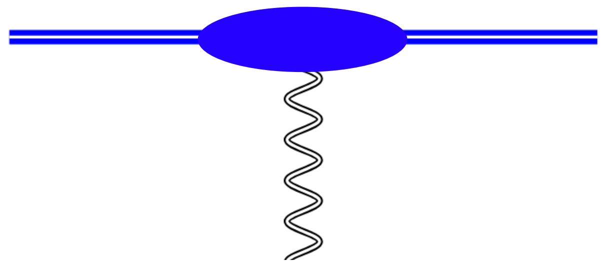
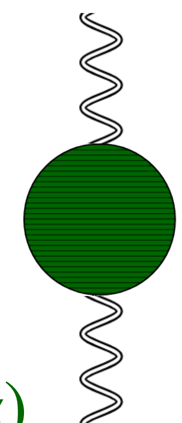
LL

Impact Factors



$$\mathcal{J} = 1 + \alpha_s J_1 + \alpha_s^2 J_2 + \ldots$$

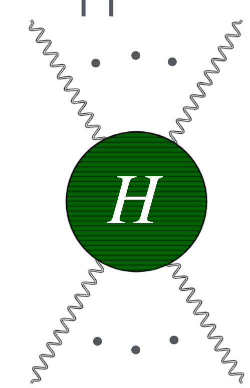
Regge Trajectory



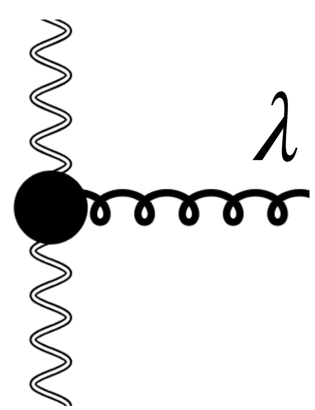
$e^{L\,\tau_g(\alpha)}$

$$\tau_g(\alpha) = \alpha_s \tau_1 + \alpha_s^2 \tau_2 + \alpha_s^3 \tau_3 + \ldots$$

and similarly for all

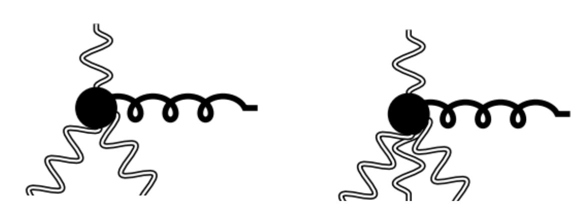


Lipatov Vertex



$$V^\lambda = V_0^\lambda + \alpha_s V_1^\lambda + \alpha_s^2 V_2^\lambda$$

and similarly for



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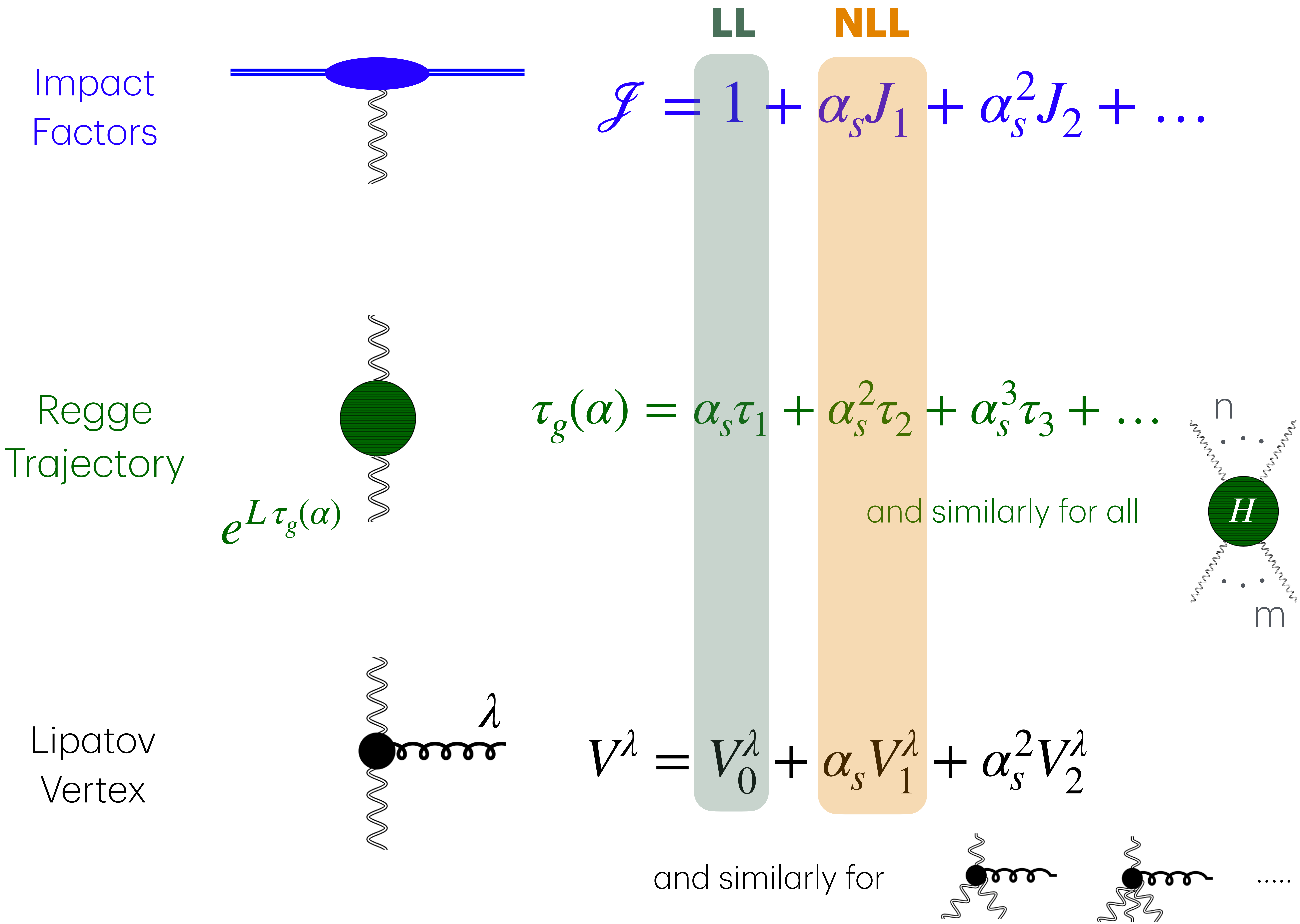
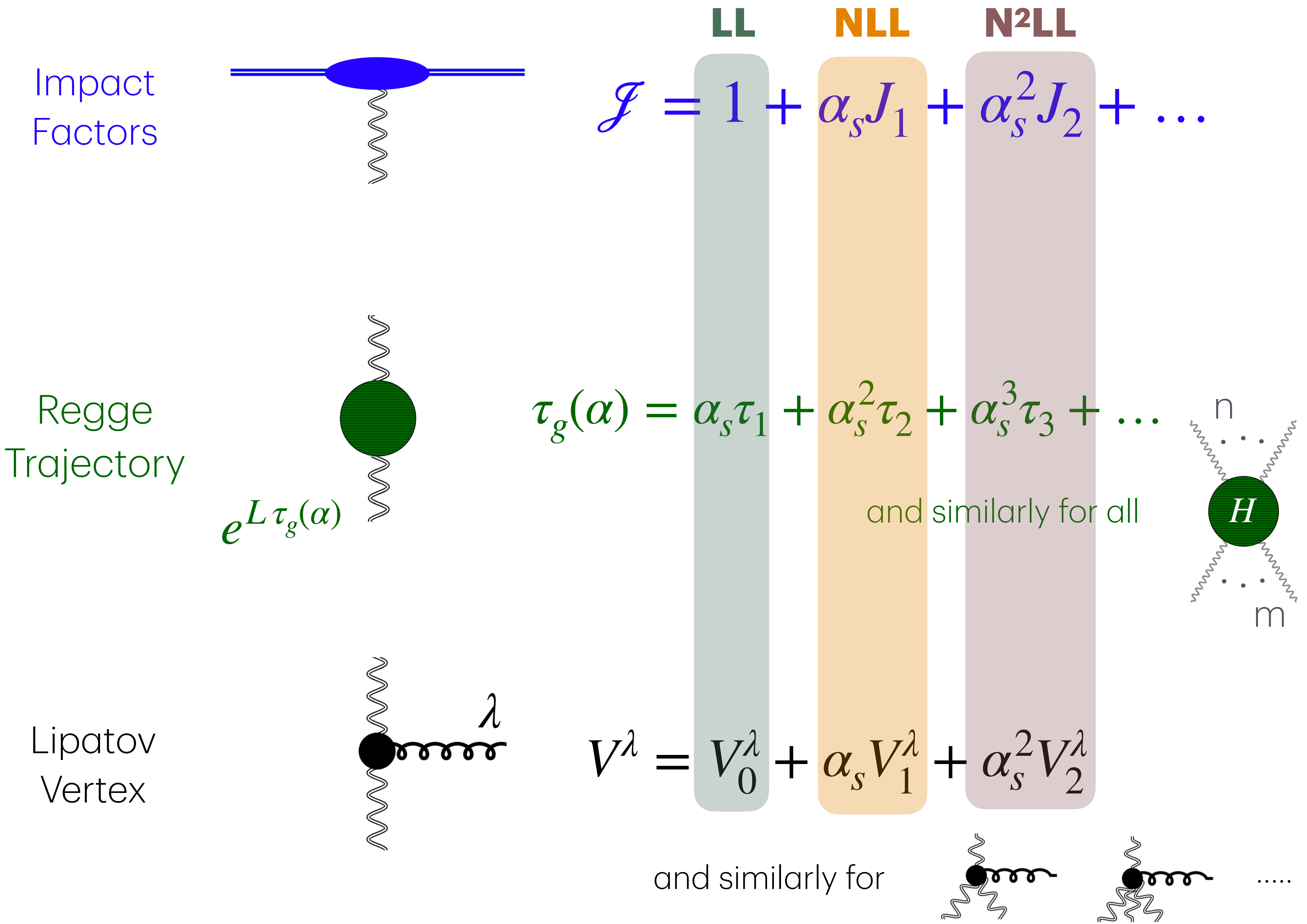

LL
NLL
Impact
Factors
$\mathcal{J} = 1 + \alpha_s J_1 + \alpha_s^2 J_2 + \ldots$
Regge
Trajectory
$e^{L\,\tau_g(\alpha)}$
$\tau_g(\alpha) = \alpha_s \tau_1 + \alpha_s^2 \tau_2 + \alpha_s^3 \tau_3 + \ldots$
n
and similarly for all
H
m
Lipatov
Vertex
λ
$V^\lambda = V_0^\lambda + \alpha_s V_1^\lambda + \alpha_s^2 V_2^\lambda$
and similarly for
.....


LL
NLL
N²LL
Impact Factors
$\mathcal{J} = 1 + \alpha_s J_1 + \alpha_s^2 J_2 + \ldots$
Regge Trajectory
$e^{L\,\tau_g(\alpha)}$
$\tau_g(\alpha) = \alpha_s \tau_1 + \alpha_s^2 \tau_2 + \alpha_s^3 \tau_3 + \ldots$
and similarly for all
n
H
m
Lipatov Vertex
λ
$V^\lambda = V_0^\lambda + \alpha_s V_1^\lambda + \alpha_s^2 V_2^\lambda$
and similarly for
.....