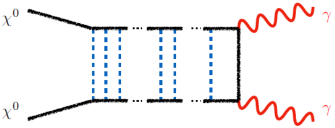
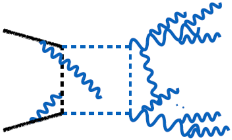
The return of the WIMP: Onium in the Sky with Loops & Logs

M. Beneke (TU München)

Higgs Centre Theory Colloquium, Edinburgh, 08 May 2026

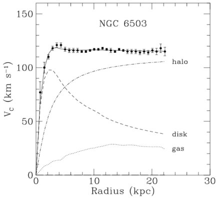


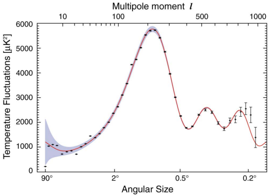


MB, **Hellmann**, **Ruiz-Femenia** [1210.7928, 1411.6924, 1411.6930]; Hellmann, Ruiz-Femenia [1303.0200] – NREFT formalism, Sommerfeld, MSSM; MB, **Bharucha**, **Hryczuk**, **Recksiegel**, Ruiz-Femenia [1611.00804] + **Dighera**, Hellmann [1601.04718] – MSSM Sommerfeld, relic density, cosmic rays; MB, Dighera, Hryczuk [1409.3049, 1607.03910] – Thermal effects in DM annihilation, IR finiteness, OPE; Sommerfeld effect, see [1601.04718]; MB, **Szafron**, **Urban** [1909.04584, 2009.00640]; Urban [2108.07285] – NLO Sommerfeld and electroweak potentials; MB, **Broggio**, **Hasner**, **Vollmann** [1805.07367] + Urban [1903.08702, 1912.02034]; MB, Urban, Vollmann [2203.01692]; MB, **Lederer**, **Peset** [2211.14341] – EW Sudakov resummation, $\gamma + X$; MB, Lederer, Urban [2209.14343], MB, **Binder**, **Garny**, **De Ros** [2403.07108] + Lederer [2411.08737]– Resonant/Quasi-bound state annihilation, unitarity violation

Some figures courtesy K. Urban, M. Vollmann

Dark matter





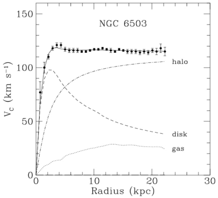
Die Rotverschiebung von extragalaktischen Nebeln

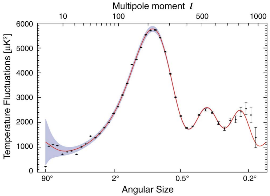
von **F. Zwicky.**

(16. II. 33.)

Um, wie beobachtet, einen mittleren Dopplereffekt von 1000 km/sek oder mehr zu erhalten, müsste also die mittlere Dichte im Comasystem mindestens 400 mal grösser sein als die auf Grund von Beobachtungen an leuchtender Materie abgeleitete[1]). Falls sich dies bewahrheiten sollte, würde sich also das überraschende Resultat ergeben, dass dunkle Materie in sehr viel grösserer Dichte vorhanden ist als leuchtende Materie.

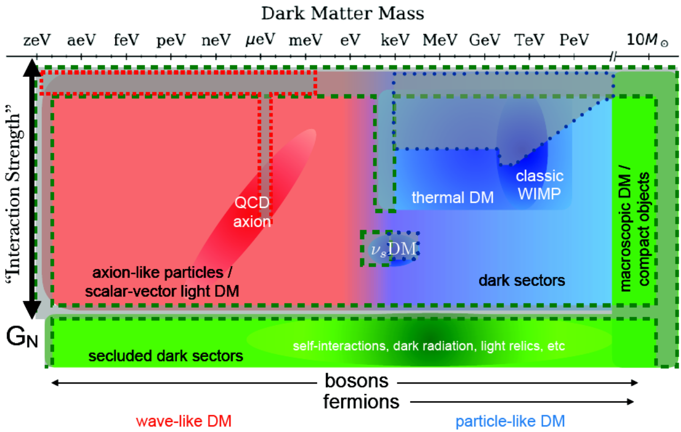
Dark matter





Unquestionable evidence of physics beyond the SM.
But only through gravitational effects.
Probably a particle.

The dark matter particle landscape



The Cold Dark Matter / WIMP paradigm

- Weak interactions with SM particles. Negligible self-interactions
- Initially in thermal equlibrium

$$\chi\chi \rightleftharpoons XX \quad (X = W^\pm, Z, q, l, \gamma, \ldots)$$

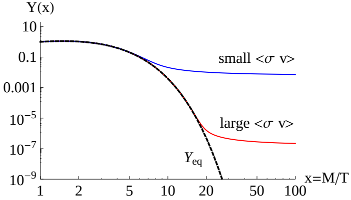
- Due to the expansion of the the co-moving density $n_\chi a^3$ (or $Y = n_\chi / s$) becomes constant at late times ("freeze-out"). Boltzmann equation

$$\frac{dn_\chi}{dt} + 3H(t) n_\chi = \langle \sigma v \rangle \, (n_\chi^{\mathrm{eq}\,2} - n_\chi^2)$$

- "Cold" – freeze-out occurs when the DM particles are **non-relativistic**.

$$\Omega_{\mathrm{DM}} h^2 = \frac{n_\chi M_\chi}{\rho_c / h^2} \approx \frac{\text{few pb}}{\langle \sigma v \rangle}$$

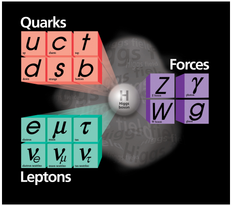
- $\Omega_{\mathrm{DM}} h^2 = 0.120$ corresponds to a typical electroweak annihilation cross section $\propto \alpha_{\mathrm{EW}}^2 / M_\chi^2$ for $M \approx 1$ TeV.



The minimal WIMP SM extension

Field	$SU(3) \times SU(2) \times U(1)_Y$
Q_i	$(3, 2, +1/6)$
U_i	$(3, 1, +2/3)$
D_i	$(3, 1, -1/3)$
L_i	$(1, 2, -1/2)$
E_i	$(1, 1, -1)$
Φ	$(1, 2, +1/2)$

Fields + symmetries + charges
(+ relativity, locality) $\Longrightarrow$ $\mathcal{L}_{\text{SM(EFT)}}$



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E_i	$(1, 1, -1)$
Φ	$(1, 2, +1/2)$
χ	$(1, N, Y)$

Fields + symmetries + charges
(+ relativity, locality) $\Longrightarrow$ $\mathcal{L}_{\mathrm{SM(EFT)}+\chi}$

Minimal extension:

- No new forces / interactions
- One extra multiplet wrt SM gauge symmetrties

Minimal models [Cirelli et al. (2007)]

SU(2) triplet – "Wino" ($Y = 0$, fermion)

$$\chi^0, \chi^\pm$$

$$\mathcal{L}_\chi = \frac{1}{2}\bar{\chi}(i\not{D} - m_\chi)\chi$$

Observed relic density for $m_\chi \approx 2.89$ TeV

Radiative mass splitting $\delta m_{0+} = 164.1$ MeV [Ibe, Matsumoto, Sato, 2012]

No free parameters

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No free parameters

SU(2) doublet – "Higgsino" ($Y = \frac{1}{2}$, fermion)

$$\chi_1^0, \chi_2^0, \chi^\pm$$

$$\mathcal{L}_\chi = \bar{\chi}(\not{D} - m_\chi)\chi + \mathcal{L}_{\mathrm{dim}-5}$$

Observed relic density for $m_\chi \approx 1.1$ TeV

Radiative mass splitting $\delta m_{0+} = 355$ MeV [Thomas, Wells, 1998]

$$\mathcal{L}_{\mathrm{dim}-5} = \frac{c}{\Lambda}(\bar{\chi}H)i\gamma_5(H^\dagger\chi)$$

needed to split the neutral states $\delta m_{12} > 200$ keV

One free parameter

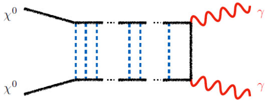
(The Minimal Supersymmetric Standard Model (MSSM) contains (a mixture) of both and a singlet ("bino"))

Large quantum (loop) effects at the TeV scale

(DM particle with SU(2) electroweak gauge interactions.)

Sommerfeld effect (non-relativistic scattering)

$$\left(\alpha_2 \frac{m_\chi}{m_W}\right)^n$$



$\mathcal{O}(1)$ changes of relic density, huge resonant $\mathcal{O}(10^3)$ effects for annihilation in the present Universe possible.

Large quantum (loop) effects at the TeV scale

(DM particle with SU(2) electroweak gauge interactions.)

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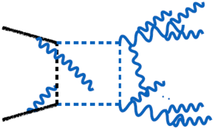
$\mathcal{O}(1)$ changes of relic density, huge resonant $\mathcal{O}(10^3)$ effects for annihilation in the present Universe possible.

EW Sudakov double logarithms (soft-collinear radiation)

$$\left(\alpha_2 \ln^2 \frac{m_\chi}{m_W}\right)^n$$



Only for exclusive final states. $\mathcal{O}(1)$ changes of the flux of cosmic rays.

Experiment pushes WIMP mass scale to **TeV**

Experiment pushes WIMP mass scale to **TeV**

↓

Electroweak Born computations break down for $m_\chi \gg m_W$

Non-perturbative effects and all-order resummations
Highly non-trivial (effective) quantum field theory

The return of the WIMP

Experiment pushes WIMP mass scale to **TeV**

↓

Electroweak Born computations break down for $m_\chi \gg m_W$

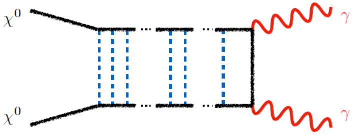
Non-perturbative effects and all-order resummations
Highly non-trivial (effective) quantum field theory

↓

Decorrelates relic density/freeze-out from annihilation in present universe, even different astrophysical objects.

$\hookrightarrow$ **new signatures / phenomenology**

Sommerfeld effect / Non-relativistic DM EFT

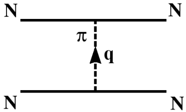


Coulomb attraction enhances small-velocity pair annihilation [Sommerfeld, 1931 (QED)]

$$S(v) = \frac{2\pi\alpha}{v}\frac{1}{1-e^{-2\pi\alpha/v}} \overset{v\to 0}{\approx} \frac{2\pi\alpha}{v}$$

Yukawa force

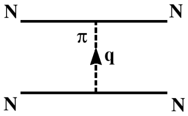
Nuclear force



$$V(r) = -\frac{g_{\pi NN}^2 e^{-m_\pi r}}{r}$$

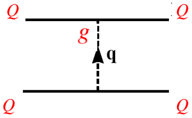
Yukawa force

Nuclear force



$$V(r) = -\frac{g_{\pi NN}^2 e^{-m_\pi r}}{r}$$

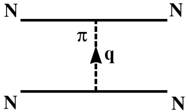
Heavy quark force



$$V(r) = -\frac{\alpha_s}{r}, \qquad \alpha_s \ll 1$$

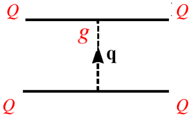
Yukawa force

Nuclear force



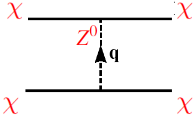
$$V(r) = -\frac{g^2_{\pi NN} e^{-m_\pi r}}{r}$$

Heavy quark force



$$V(r) = -\frac{\alpha_s}{r}, \qquad \alpha_s \ll 1$$

Dark matter force



$$V(r) = -\frac{\alpha_{\rm EW} e^{-m_{\rm EW} r}}{r}$$

(Potential) Non-relativistic dark matter EFT

$$\mathcal{L}_{\mathrm{EW}+\chi}(\mu \sim m_\chi) \quad \longrightarrow \quad \mathcal{L}_{\mathrm{PNRDMEFT}}(\mu \sim m_W)$$

- Unbroken $SU(2) \times U(1)$ gauge-theory on the left, broken on the right, non-relativistic DM fields.
- Weak coupling, but leading-order theory is not free but interacting (Yukawa-potential not a perturbation $\longrightarrow$ all-order summation in α_{EW})

(Potential) Non-relativistic dark matter EFT

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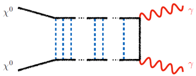
Summing ladder graphs = Solving Schrödinger equation $[x = m_\chi \alpha r,\ \epsilon_v = v/(2\alpha)]$

$$-u_\ell''(x) + \left[-\frac{e^{-\epsilon_\phi x}}{x} + \frac{\ell(\ell+1)}{x^2}\right] u_\ell(x) = \epsilon_v^2 u_\ell(x) \qquad \epsilon_\phi = m_\phi/(m_\chi \alpha)$$

for scattering states:

$$\sigma v = (\sigma v)_{\mathrm{Born}} \times |\psi_\ell(0)|^2(v)$$



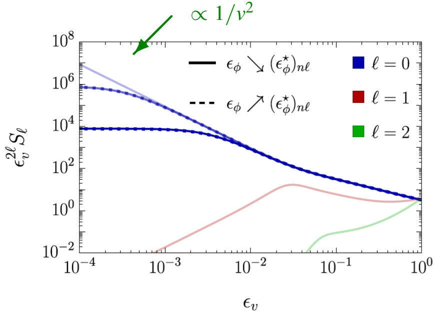
For TeV WIMP the electroweak interaction is similar to the strong interaction for heavy quarks, with the QCD confinment scale replaced by m_W.

Quarkonium	WIMP
SU(3)	broken SU(2)×U(1)
single channel/state	coupled channel problem, $I = \chi^0\chi^0, \chi^+\chi^-, \ldots$ (numerically challenging)
bound state	scattering state (primarily)
Coulomb potential ($T = 0$)	Yukawa-Coulomb
$\lvert\psi(0)\rvert^2$ depends on α_s	$\lvert\psi_{IJ}(0)\rvert^2$ a matrix, depends on $\alpha_{1,2}, m_W, v, \delta m_{IJ}$

WIMP EFT more complex than QCD but can be completely calculated from first principles (weak coupling)

Sommerfeld effect for the Yukawa force

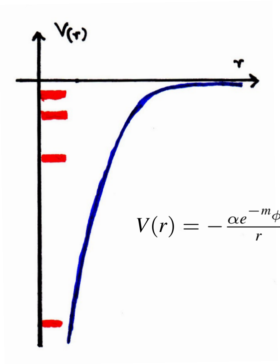
$$S_\ell = |\psi_\ell(0)|^2(v)$$



$\epsilon_v = v/(2\alpha)$

- Saturation at $v \ll m_W/m_\chi$
- Stronger $1/v^2$ growth than Coulomb $1/v$ near isolated mass values

Resonance effect for the Yukawa potential



$V(r) = -\frac{\alpha e^{-m_\phi r}}{r}$

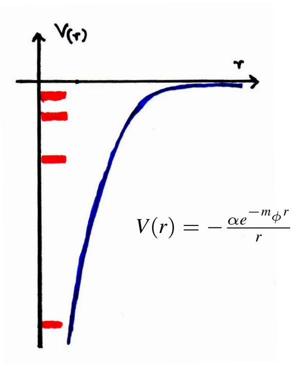
Range $r \sim 1/m_\phi$ cuts off Rydberg states $[r_{\rm Ryd} \sim n^2/(m_\chi \alpha)]$

Finite number of bound states

$$n^2 \lesssim \frac{m_\chi \alpha_2}{m_\phi}, \qquad \epsilon_\phi = m_\phi/(m_\chi \alpha)$$

Increasing m_χ adds levels from above. Zero-energy bound states for certain m_χ.

Resonance effect for the Yukawa potential



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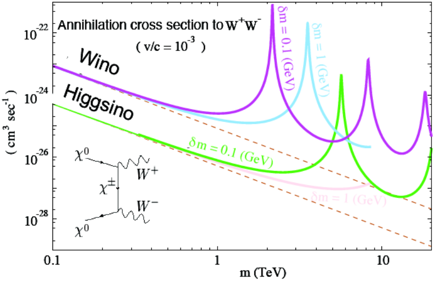
$$n^2 \lesssim \frac{m_\chi \alpha_2}{m_\phi}, \qquad \epsilon_\phi = m_\phi/(m_\chi \alpha)$$

Increasing m_χ adds levels from above. Zero-energy bound states for certain m_χ.

Non-relativistic quantum mechanics: large enhancement of scattering cross section when a zero energy bound states exists due to infinite scattering length, $S \propto\sim \frac{1}{v^2}$.

For fixed $m_\phi = m_W = 80\,\text{GeV}$ and $\alpha = \alpha_{\rm SU(2)} \approx 1/30$, resonant enhancement at certain values of dark matter mass m_χ starting in TeV range.

Wino and Higgsino [Hisano et al., 2004]



VOLUME 92, NUMBER 3 PHYSICAL REVIEW LETTERS week ending 23 JANUARY 2004

Explosive Dark Matter Annihilation

Junji Hisano,[1] Shigeki Matsumoto,[1] and Mihoko M. Nojiri[2]
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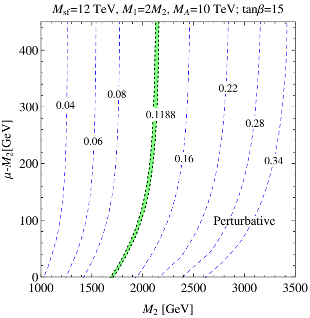
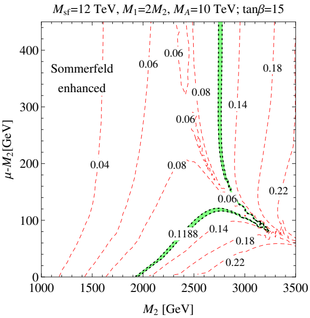
[2]*YITP, Kyoto University, Kyoto 606-8502, Japan*
(Received 25 July 2003; published 22 January 2004)

In this Letter we study pair annihilation processes of dark matter (DM) in the Universe, in the case that the DM is an electroweak gauge nonsinglet. In the current Universe, in which the DM is highly nonrelativistic, the nonperturbative effect may enhance the DM annihilation cross sections, especially for that to two photons, by several orders of magnitude. We also discuss sensitivities in future searches for anomalous γ rays from the galactic center, which originate from DM annihilation.

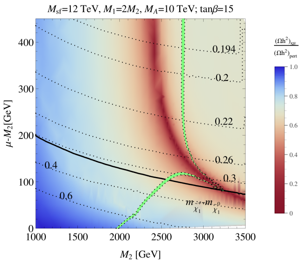
Decisive impact on DM phenomenology of WIMPs and dark sectors since 2008.

MSSM: Relic density of the mixed wino-Higgsino





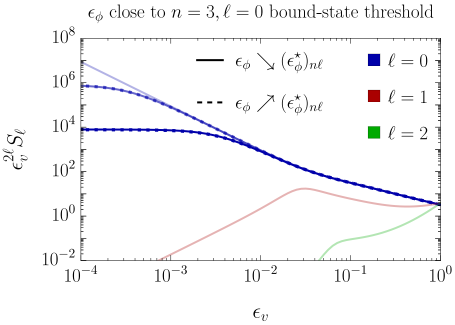
[MB, Bharucha, Dighera, Hellmann, Hryczuk, Recksiegel, Ruiz-Femenia, 1601.04718, + Lederer, in preparation]



- Correct relic density for a wide range of wino-like LSP masses 2.0 . . . 3.3 TeV. Regions around resonance are excluded by indirect non-detection. [MB, Bharucha, Hryczuk, Recksiegel, Ruiz-Femenia, 1611.00804]

Velocity spectrum peaks in higher-partial waves [2403.07108]

$$S_\ell = |\psi_\ell(0)|^2(v)$$



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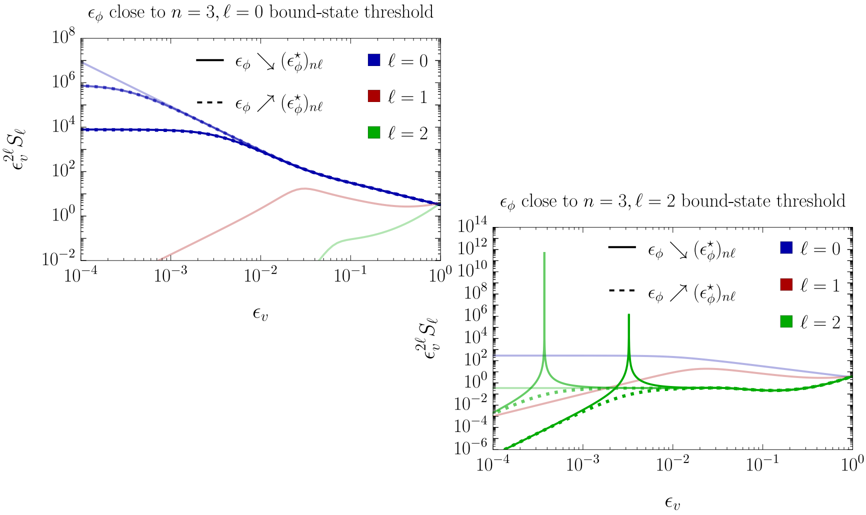
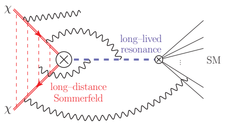
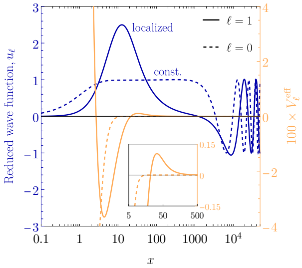


Quasi-bound/metastable state resonance effects

[MB, Lederer, Urban, 2209.14343, MB, Binder, De Ros, Garny, 2403.07108]





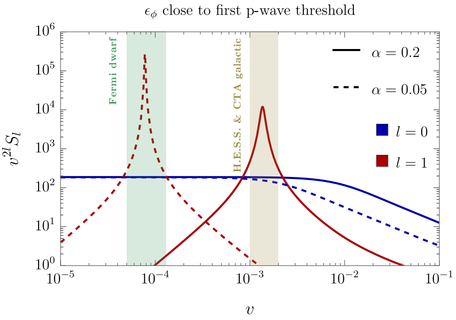
The Yukawa force can generate the resonance dynamically through a metastable $\chi\chi$ state, but not for S-wave annihilation.

For partial waves $\ell \geq 1$, the centrifugal force generates a barrier that can sustain metstable states $\rightarrow$ super-resonant enhancement: spike in velocity-dependence superimposed on the Sommerfeld enhancement.

$$S_\ell = \frac{[(2\ell+1)!!]^2\, \mathcal{P}_{n\ell}}{(2\ell+1)\epsilon_{n\ell}^{2\ell+1} T_{n\ell}} \times \frac{\gamma_{n\ell}/2}{(\epsilon_v^2 - \epsilon_{n\ell}^2)^2 + (\gamma_{n\ell}/2)^2} \qquad \text{(WKB)}$$

(For wino at $m_\chi = 11.005$ TeV.)

Quasi-bound/metastable state resonance effects



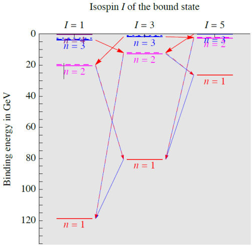
Resonance in velocity dependence when $E_{\text{kin}} = E_{\text{QBS}} > 0$.
Different astrophysical sources can exhibit very different annihilation cross sections.

Bound-state effects

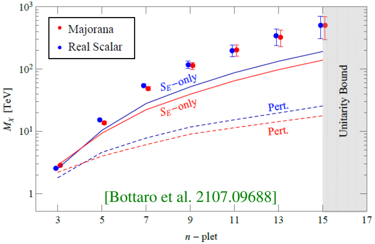
[von Harling, Petraki (2014); An, Wise, Zhang (2016); Asadi et al. (2016), Cirelli et al. (2016), Mitridate et al. (2017), Urban (2017), . . .]

Electroweak force generates a “WIMPonium” bound-state spectrum.
Annihilation via capture into bound-state

$$\chi\chi \to \mathcal{B}(\chi\chi)\ [\to \mathrm{SM}] + \gamma(W,Z)$$



[Bottaro et al. 2103.12766]



[Bottaro et al. 2107.09688]

- In minimal models importance grows with mass and EW representation.
- Not relevant for wino and Higgsino.
- Interesting effects in other DM models. Interesting signatures (radiative lines)
- Perturbative unitarity violation in radiative capture rates for Rydberg bound states in non-abelian interactions [MB, Binder, De Ros, Garny, Lederer, 2411.08737]

The quantum electroweak static potential

- "Casimir" scaling in unbroken SU(N) gauge theory up to 2-loop order
  (violated beyond [Anzai, Kiyo, Sumino, 1004.1562])

$$V(r) = -\underbrace{\frac{\alpha_{\mathrm{SU(N)}}(r)}{r}\, f(\alpha_{\mathrm{SU(N)}}(r))}_{\text{independent of rep } R \text{ of source}}\; T_R^A \otimes T_R^A \qquad \text{up to } \mathcal{O}(\alpha^3)$$

- Holds in broken SU(N) (at least at one-loop, [MB, Szafron, Urban, 1909.04584, 2009.00640; Urban, 2108.07285])

- SM broken SU(2) × U(1), generators T^A and hypercharge Y: four universal functions

$$\begin{aligned} V(r) \;&=\; V^W(r)(T^+ \otimes T^- + T^- \otimes T^+) + V^{Z,\gamma}(r)\, T^3 \otimes T^3 \\ &\quad + V^{T^3Y}(r)\left(T^3 \otimes Y + Y \otimes T^3\right) + V^{YY}(r)\, Y \otimes Y \end{aligned}$$

Analytic results in momentum space, partly numerical Fourier transformation to coordinate-space potential. Simple analytic fitting functions available.

↪ One-loop Sommerfeld effect for all "minimal" models available.

One-loop electroweak potential between static SU(2)×U(1) charges

Example: Pure wino model

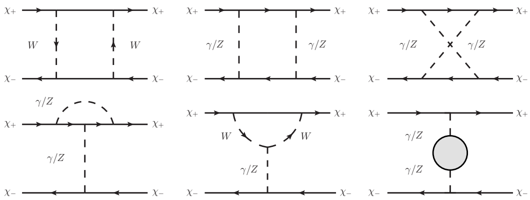
$\chi^0\chi^0 \to \chi^+\chi^-$

$$V^{(2)}_{\text{even}\, L+S}(r) = \begin{pmatrix} 0 & -\sqrt{2}\,\alpha_2\, \frac{e^{-M_W r}}{r} \\ -\sqrt{2}\,\alpha_2\, \frac{e^{-M_W r}}{r} & -\frac{\alpha_{\text{em}}}{r} - \alpha_2\, c_W^2\, \frac{e^{-M_Z r}}{r} \end{pmatrix} + \mathcal{O}(\alpha^2_{2,\text{em}})$$

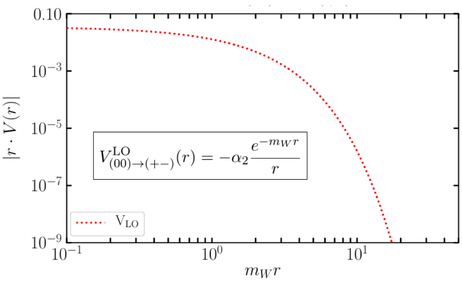
$\chi^+\chi^- \to \chi^+\chi^-$

$$V^{(2)}_{\text{odd}\, L+S}(r) = \begin{pmatrix} 0 & 0 \\ 0 & -\frac{\alpha_{\text{em}}}{r} - \alpha_2\, c_W^2\, \frac{e^{-M_Z r}}{r} \end{pmatrix} + \mathcal{O}(\alpha^2_{2,\text{em}})$$

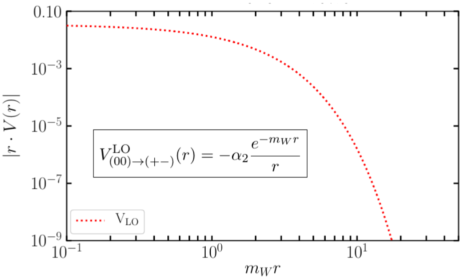
$\chi^0\chi^0$ cannot be in a state with odd L+S.

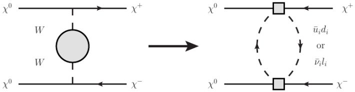


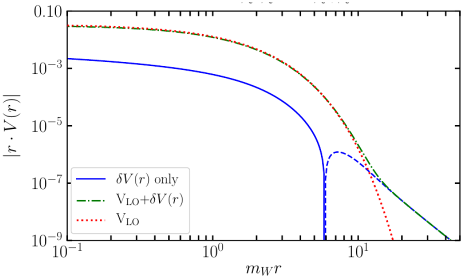
Wino potential V^W $[\chi^0\chi^0 \to \chi^+\chi^-]$



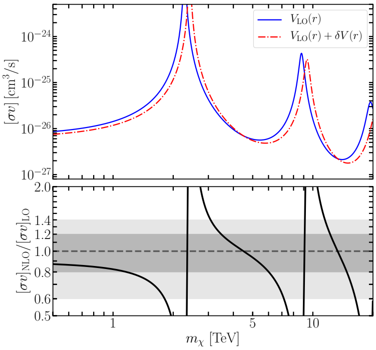
Wino potential V^W $[\chi^0\chi^0 \to \chi^+\chi^-]$



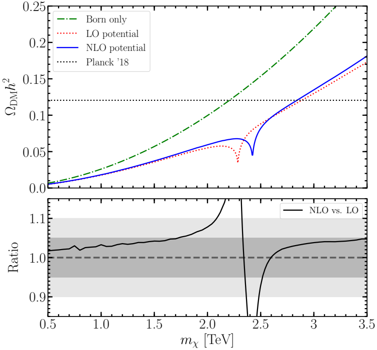




Wino annihilation to $\gamma\gamma + \frac{1}{2}\gamma Z$ and freeze-out abundance at NLO



[1909.04584, 2009.00640]



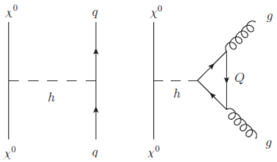
Resonances shift by 8% from 2.283 (8.773) TeV to 2.419 (9.355) TeV.

Prospects for TeV scale electro-weak WIMP detection

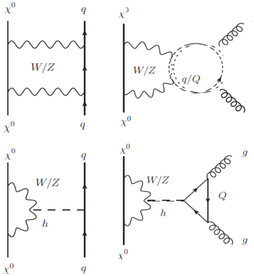
Scattering, annihilation

Production requires 100 TeV pp or multi-TeV lepton collider

Pure wino and Higgsino do not have tree-level scattering with nuclei (quarks and gluons).

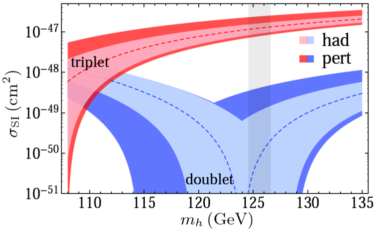


[Fig. from Hisano, DMNet 2022]



Loop-induced, independent of DM mass
EFT treatment [Hill, Solon, 1309.4092]

Cancellation of spin-0 and spin-2 contributions for Higgsino

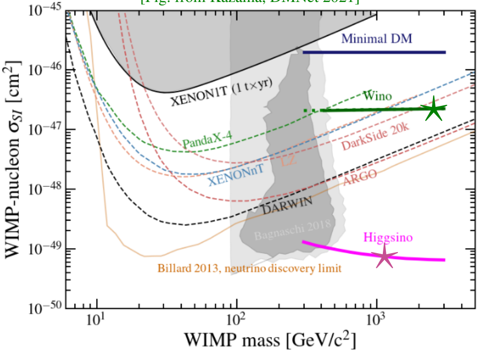


TeV WIMP direct detection

$$\sigma_{\text{wino}} = 1.3^{+1.2+0.4}_{-0.5-0.3} \times 10^{-47}\,\text{cm}^2$$

$$\sigma_{\text{Higgsino}} < 10^{-49} \ldots 10^{-48}\,\text{cm}^2$$

[Fig. from Kazama, DMNet 2021]



Thermal Wino: Best limit LZ (2025) factor 4 away, wino should certainly be seen in next-generation DD experiments (DARWIN, XLZD)
Higgsino well below neutrino "floor".

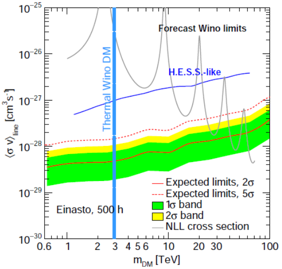
TeV WIMP indirect detection

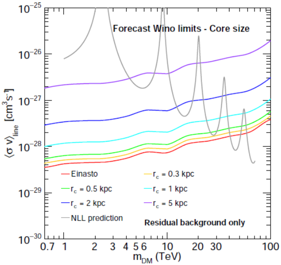
High-energy γ's from the Galactic Center and dwarf galaxies

$$\chi^0\chi^0 \to \gamma\gamma, \gamma Z, \gamma + X$$

produces an almost mono-chromatic TeV energy photon excess cosmic rays with $E_\gamma \approx m_\chi$.

CTA experiment sensitivity forecast, Galactic Center [Rinchuiso et al., 2008.00692, Rodd et al. 2405.13104]





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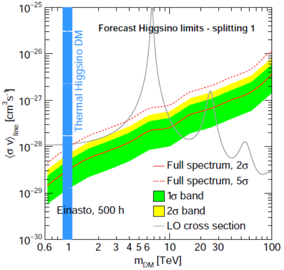
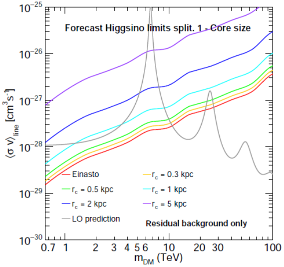
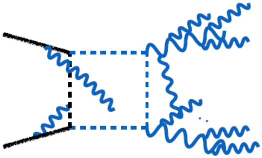




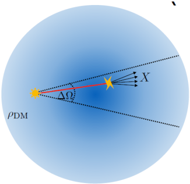
Thermal wino should be seen by CTA even under conservative assumptions on the galactic DM distribution. Higgsino observation less certain. [See, however, Safdi et al., 2506.08084, 2507.15934].

Electroweak Sudakov logarithms and the photon “line” signal



The photon "line" signal

Cosmic rays – Look for photons from DM annihilation in DM-rich targets: Galactic Center, dwarf-spheroidal satellite galaxies



$$\Phi(E_i) = \frac{1}{8\pi m_\chi^2} \int_{\Delta\Omega} d\Omega \int_{\text{l.o.s.}} ds\, \rho_{\text{DM}}^2(\mathbf{r}(s))\, \frac{d}{dE_i} [\sigma v]_{\chi\chi\to i+X}$$

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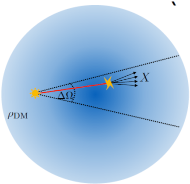
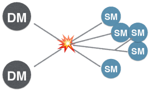
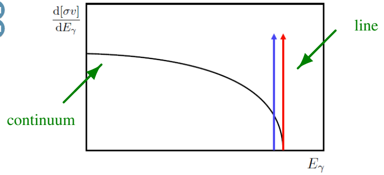


$$\Phi(E_i) = \frac{1}{8\pi m_\chi^2} \int_{\Delta\Omega} d\Omega \int_{\text{l.o.s.}} ds\, \rho_{\text{DM}}^2(\mathbf{r}(s))\, \frac{d}{dE_i} [\sigma v]_{\chi\chi\to i+X}$$

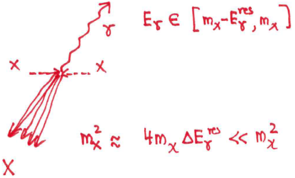
$\chi^0\chi^0 \to \gamma\gamma, \gamma Z$ produces a mono-chromatic photon excess in (here TeV energy) cosmic rays with $E_\gamma \approx m_\chi$.

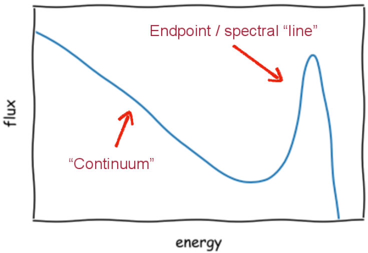




Not quite true, though: For E_γ near the endpoint, the unobserved final state is a “jet”.

The true observable is the single-inclusive photon-energy spectrum in $\chi^0\chi^0 \to \gamma + X$ with an unobserved jet-like final state X, smeared over the energy resolution $E^{\gamma}_{\rm res}$ of the instrument.

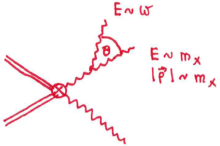




Electroweak perturbation theory develops large double logarithms $\ln \frac{m_\chi}{m_W} \times \ln \frac{m_\chi}{E^{\gamma}_{\rm res}}$.

Origin of EW double (Sudakov) logs

Analogous to QCD jets: "inhibited radiation"



- Soft and collinear radiation (incomplete cancellation of real and virtual effects)

$$I \propto \frac{g^2}{16\pi^2} \int_{E^\gamma_{\rm res}}^{m_\chi} \frac{d\omega}{\omega} \int d\cos\theta \, \frac{E(\vec{p})}{\sqrt{m_W^2 + \vec{p}^{\,2}} - |\vec{p}|\cos\theta} \propto \frac{\alpha}{4\pi} \ln\frac{m_\chi}{E^\gamma_{\rm res}} \ln\frac{m_\chi}{m_W}$$

- Resummation of large logs $L = \ln(4m_\chi^2/m_W^2)$

$$\sigma v \propto (1 + C_1\hat{\alpha}_2 + \ldots)\, \exp\left[L f_0(\hat{\alpha}_2 L) + f_1(\hat{\alpha}_2 L) + \hat{\alpha}_2 f_2(\hat{\alpha}_2 L) \ldots\right]$$

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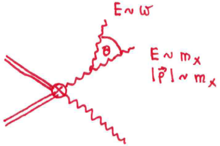

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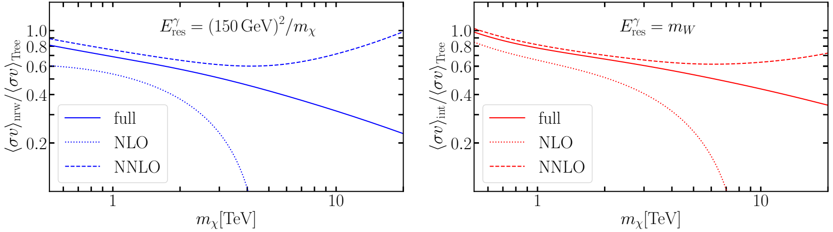
- Large logs depend on energy resolution

→ $E^\gamma_{\rm res} \sim 0$, "line", [Bauer et al. 1409.7392, Ovanesyan et al. 1409.8294, 1612.04814, NLL']
→ $E^\gamma_{\rm res} \sim m_W^2/m_\chi$, "narrow", [MB, Broggio, Hasner, Vollmann, 1805.03767, NLL']
→ $E^\gamma_{\rm res} \sim m_W$, "intermediate", [MB, Broggio, Hasner, Urban, Vollmann, 1903.08702, NLL']
→ $E^\gamma_{\rm res} \sim m_\chi(1-z) \gg m_W$, "wide", [Baumgart et al. 1712.07656 (LL), 1808.08956 (NLL), 1409.4415, 1412.8698]

Minimal triplet ("wino") model



NLO (dotted), NNLO (dashed), resummed (solid) after Sommerfeld resummation ("tree")
Left: narrow, right: intermediate resolution

Factorization (NREFT+SCET)

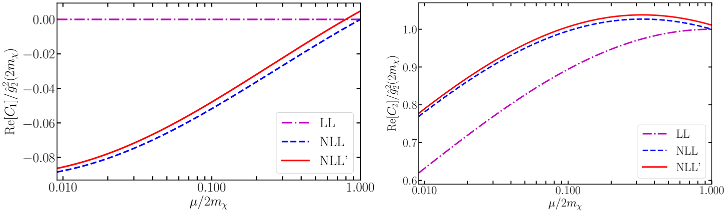
- Non-relativistic and soft-collinear effective field theory to separate the scales m_χ, $\sqrt{m_\chi m_W}$, m_W, $E^\gamma_{\rm res}$.

$$\mathcal{L}_{\rm ann} = \frac{1}{2m_\chi}\sum_i \int dsdt\, \hat{C}_i(s,t,\mu)\mathcal{O}_i$$

- Short-distance annihilation into a pair of EW gauge bosons

$$\mathcal{O}_1 = \zeta_v^{c\dagger}\Gamma^{\mu\nu}\eta_v\, \mathcal{A}^B_{\perp c,\mu}(sn_+)\mathcal{A}^B_{\perp\bar{c},\nu}(tn_-)$$
$$\mathcal{O}_2 = \frac{1}{2}\,\zeta_v^{c\dagger}\Gamma^{\mu\nu}\{T^B,T^C\}\eta_v\, \mathcal{A}^B_{\perp c,\mu}(sn_+)\mathcal{A}^C_{\perp\bar{c},\nu}(tn_-)$$

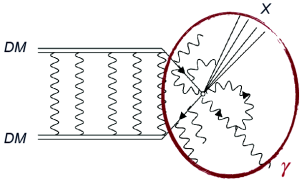
etc.



Factorization (II)

$$\frac{d(\sigma v_{\rm rel})}{dE_\gamma} = \underbrace{\sum_{I,J} S_{IJ}}_{\text{Sommerfeld}} \,\Gamma_{IJ}(E_\gamma) = \sum_{I,J} S_{IJ} \underbrace{\sum_{i,j=1,2} C_i(\mu) C_j^*(\mu)}_{\text{short distance annihilation}} \underbrace{\gamma^{ij}_{IJ}(E_\gamma,\mu)}_{\text{final state}}$$

$$\gamma^{ij}_{IJ}(E_\gamma,\mu) = \frac{1}{(\sqrt{2})^{n_{id}}}\,\frac{1}{4}\,\frac{2}{\pi m_\chi}\,\underbrace{V_{\rm int}(\mu,\nu)\,Z^{33}_\gamma(\mu,\nu)}_{(\text{anti}-)\ \text{collinear}} \times \int d\omega\, \underbrace{J_{\rm int}(4m_\chi(m_\chi - E_\gamma - \omega/2),\mu)}_{\text{hard}-\text{collinear}}\, \underbrace{W^{ij}_{IJ}(\omega,\mu,\nu)}_{\text{soft}}$$

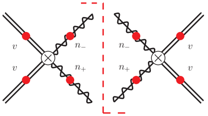


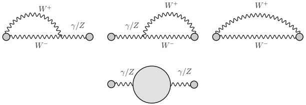
$$\gamma^{ij}_{IJ}(E_\gamma,\mu) = \frac{1}{(\sqrt{2})^{n_{id}}}\,\frac{1}{4}\,\frac{2}{\pi m_\chi}\,V_{\rm nrw}(\mu,\nu)\,Z^{33}_\gamma(\mu,\nu) \times D^i_{I,33}(\mu,\nu) D^{j\,*}_{J,33}(\mu,\nu) J^{33}_{\rm nrw}(4m_\chi(m_\chi - E_\gamma),\mu,\nu)$$

Hard-matching in the unbroken SU(2)×U(1) theory, factorization into jet and soft function requires rapidity renormalization and evolution. Soft functions defined in broken theory with non-zero SM particle masses. Resummation by solving a renormalization-group equation in μ, ν.

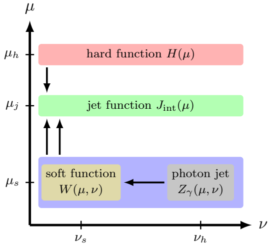
Resummation by RG equations

- NLL' accuracy
- One-loop electroweak corrections in every function

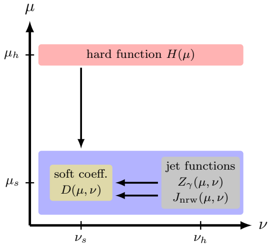




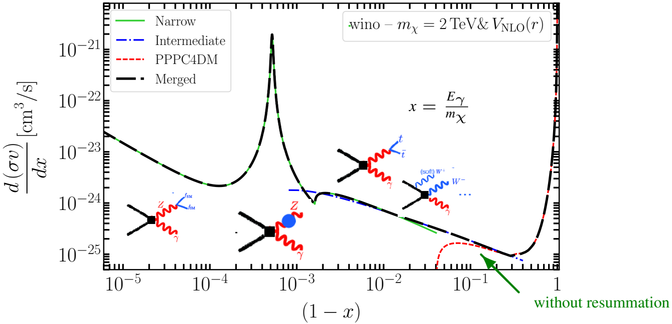
- Renormalization group evolution in virtuality μ and rapidity ν sums logarithms at NLL (one-loop anomalous dimensions + two-loop cusp)
- Convolutional matrix evolution for the soft functions (use Laplace transforms).



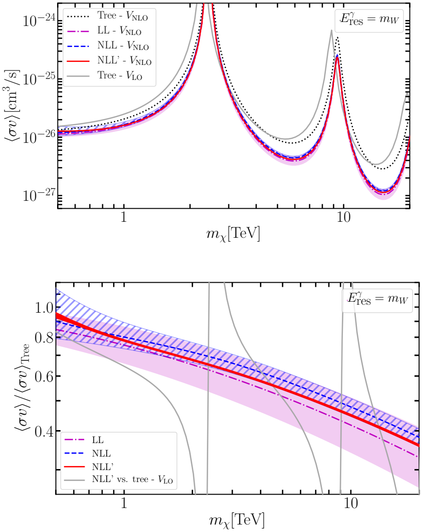
int (upper) and narrow (lower) resolution



The resummed / matched gamma energy spectrum [MB, Urban, Vollmann, 2203.01692]



Resummed endpoint bin rate (wino) [1903.08702, updated plots including NLO Sommerfeld potential from K. Urban's thesis, 2021]



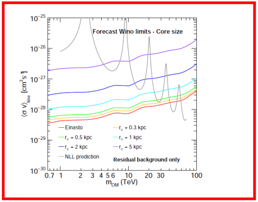
$$\langle \sigma v \rangle (E^{\gamma}_{res}) = \int_{m_\chi - E^{\gamma}_{res}}^{m_\chi} dE_\gamma \, \frac{d(\sigma v)}{dE_\gamma}$$

Factor 2 flux reduction from quantum corrections

NLL' eliminates theoretical uncertainty for all practical purposes (residual < 1%)

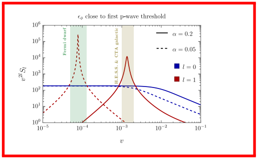
Similar results for the Higgsino and the MSSM

TeV WIMPs, especially minimal ones, are not yet excluded. Wino will be 2030-2035 (ID, DD), Higgsino maybe (ID)

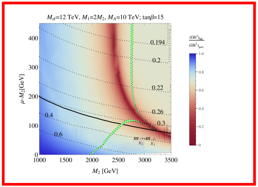


TeV mass WIMPs are theoretically rich and complex

- QCD-like non-relativistic and jet physics
- Spectacular resonance effects due to electroweak Yukawa force
- Large electroweak loop effects



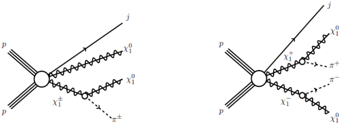
TeV mass WIMPs are phenomenologically rich – relic density, cosmic ray and direction signals are less correlated than was believed, even not for different astrophysical targets.



Back-up slides

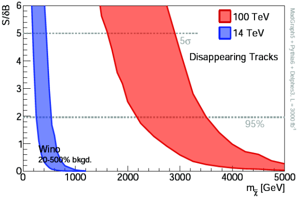
TeV WIMP at future colliders [Low, Wang, 2014, Cirelli, Sala, Taoso, 2014, ...]

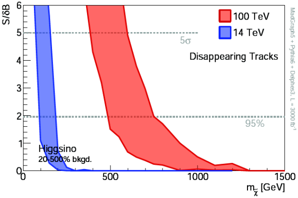
Disappearing tracks most promising: $\chi^+ \to \chi^0 + \text{soft } \pi$
Less sensitive for Higgsino (larger mass splitting, shorter lifetime)



[Fig. from Belyaev et al., 2020]

Sensitivity at LHC high-luminosity 14 TeV 3000 fb^{-1} only up to 500-600 GeV, less for Higgsino
Lepton collider: CLIC or muon collider, leaves 100 TeV pp

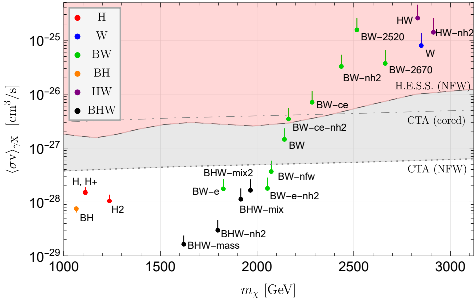




[Figs. from Low, Wang, 2014]

With more realistic track reconstruction and pile-up simulation [Saito, Sawada, Terashi, 1901.02987]: Thermal wino might be discovered at a 100 TeV hadron collider, Higgsino most likely.

$\gamma + X$ signal in the MSSM [MB, Lederer, Peset, 2211.14341]



Rule of thumb: multiply γ line-like signal by 0.6 ... 0.65 to account for electroweak resummation

model	m_{LSP} [GeV]	$m_{\chi_1^\pm}$ [GeV]	$\langle \sigma v_{rel} \rangle_{\gamma X}^{SE+NLL}$ [$cm^3 s^{-1}$]	$\frac{\langle \sigma v_{rel} \rangle_{\gamma X}^{SE+NLL}}{\langle \sigma v_{rel} \rangle_{\gamma X}^{SE}}$	$\frac{\langle \sigma v_{rel} \rangle_{tot}^{SE}}{\langle \sigma v_{rel} \rangle_{tot}^{LO}}$
pure models					
H	1111.6	1112.4	$1.50 \cdot 10^{-28}$	**0.82**	1.58
W	2849.5	2849.6	$8.04 \cdot 10^{-26}$	**0.60**	83.6
doubly mixed					
BH	1065.4	1069.8	$7.58 \cdot 10^{-29}$	**1.16**	1.27
BW	2141.7	2143.8	$1.46 \cdot 10^{-27}$	**0.64**	5.28
BW-e	1825.7	1830.5	$1.76 \cdot 10^{-28}$	**0.67**	2.25
BW-2520	2516.4	2516.9	$1.57 \cdot 10^{-25}$	**0.61**	168
BW-e-nh2	2054.0	2056.1	$1.80 \cdot 10^{-28}$	**0.64**	3.57
HW	2830.9	2831.1	$2.60 \cdot 10^{-25}$	**0.58**	322
HW-nh2	2912.2	2912.4	$1.42 \cdot 10^{-25}$	**0.55**	192
fully mixed					
BHW-mix	1916.1	1922.0	$1.14 \cdot 10^{-28}$	**0.64**	2.11
BHW-mix2	1966.0	1971.3	$1.66 \cdot 10^{-28}$	**0.64**	2.34
BHW-mass	1621.1	1632.4	$1.66 \cdot 10^{-29}$	**0.70**	1.39
BHW-nh2	1797.1	1808.5	$3.02 \cdot 10^{-29}$	**0.66**	1.64
additional					
•B	2144.9	6997.5	$4.36 \cdot 10^{-36}$	$\mathbf{2.21 \cdot 10^5}$	1.01
•BW-coan	2144.9	2147.6	$2.63 \cdot 10^{-31}$	**0.64**	1.17
•H+	1111.4	1112.2	$1.53 \cdot 10^{-28}$	**0.81**	1.58
H2	1236.6	1238.9	$1.05 \cdot 10^{-28}$	**0.78**	1.50
BH-undet	1296.1	1316.0	$3.31 \cdot 10^{-30}$	**0.86**	1.15
BW-nfw	2073.6	2075.7	$3.70 \cdot 10^{-28}$	**0.64**	4.07
BW-ce	2284.8	2285.7	$7.14 \cdot 10^{-27}$	**0.63**	15.2
BW-2670	2663.1	2664.0	$3.77 \cdot 10^{-26}$	**0.58**	54.8
BW-nh2	2436.0	2436.7	$3.29 \cdot 10^{-26}$	**0.62**	41.3
BW-ce-nh2	2162.9	2164.1	$3.49 \cdot 10^{-27}$	**0.63**	9.43