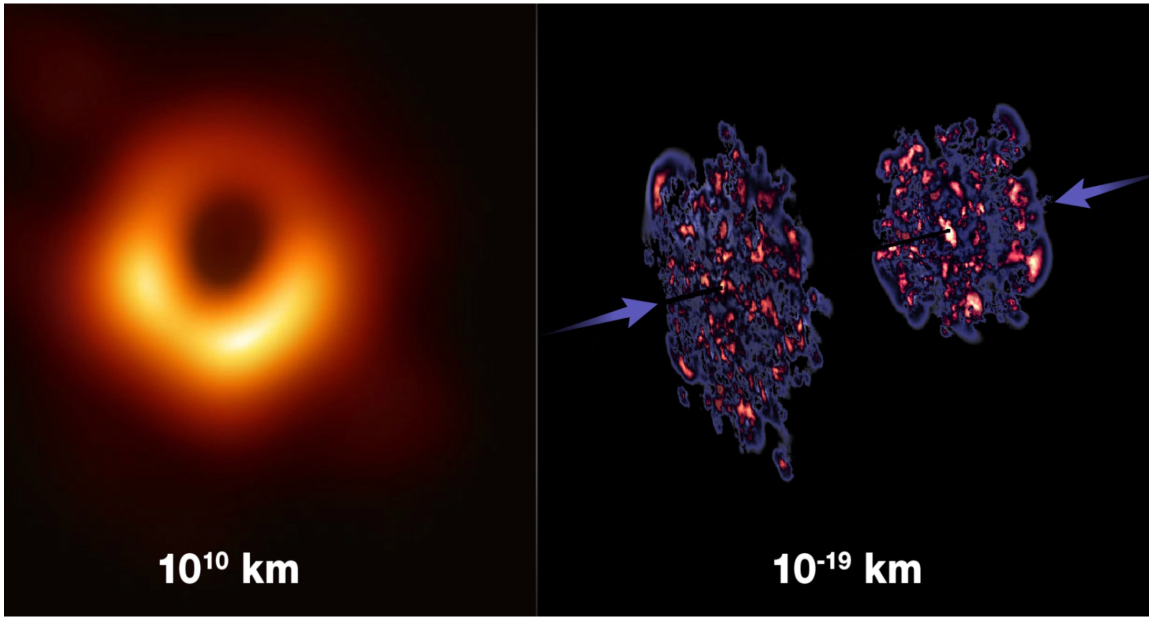
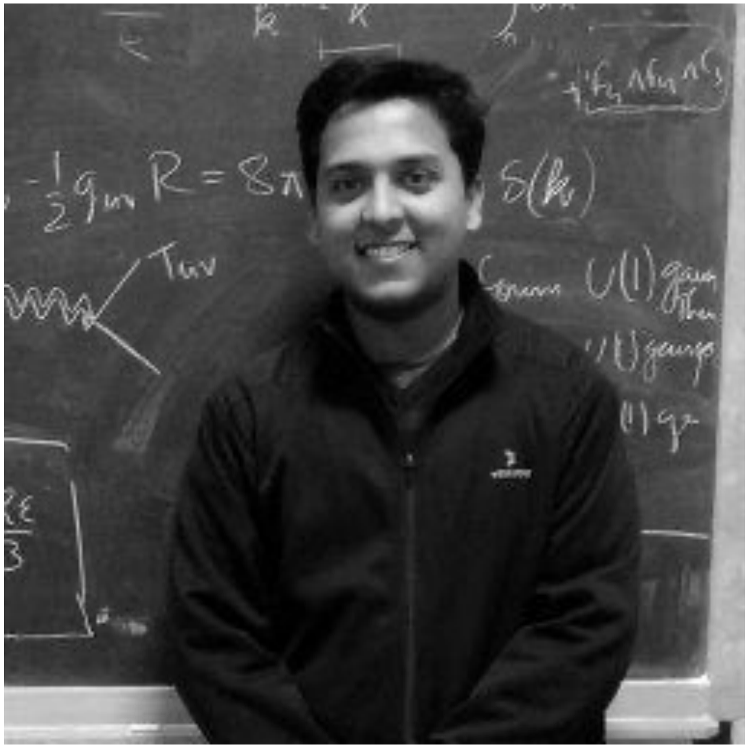
Shockwave double copy of radiation in QCD and gravity



Raju Venugopalan

Brookhaven National Lab, CFNS, Stony Brook, and Higgs Center, Edinburgh

Workshop on factorization in QCD and beyond, May 6-8, 2026

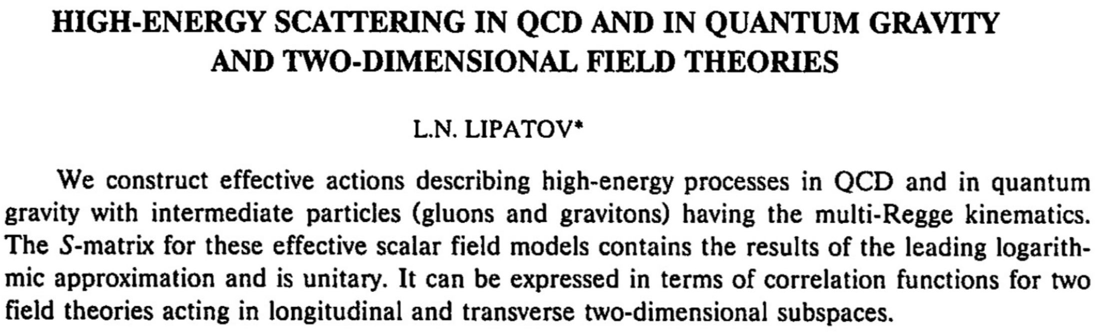


Based in part on work

(arXiv:2311.03463, 2312.03507, 2312.11652, long review **2507.21252,** 2605.03038, 2606.XXXXX)

with Himanshu Raj (Simons Confinement+ QCD Strings Collaboration Fellow at Stony Brook)

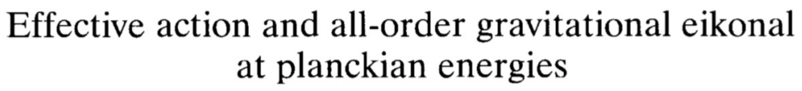
$2 \rightarrow N + 2$ amplitudes in trans-Planckian gravitation scattering: from wee partons to Black Holes



HIGH-ENERGY SCATTERING IN QCD AND IN QUANTUM GRAVITY AND TWO-DIMENSIONAL FIELD THEORIES

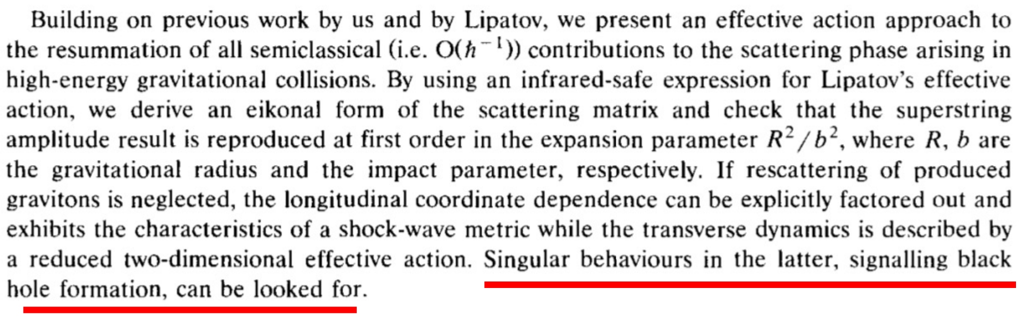
L.N. LIPATOV*

We construct effective actions describing high-energy processes in QCD and in quantum gravity with intermediate particles (gluons and gravitons) having the multi-Regge kinematics. The S-matrix for these effective scalar field models contains the results of the leading logarithmic approximation and is unitary. It can be expressed in terms of correlation functions for two field theories acting in longitudinal and transverse two-dimensional subspaces.

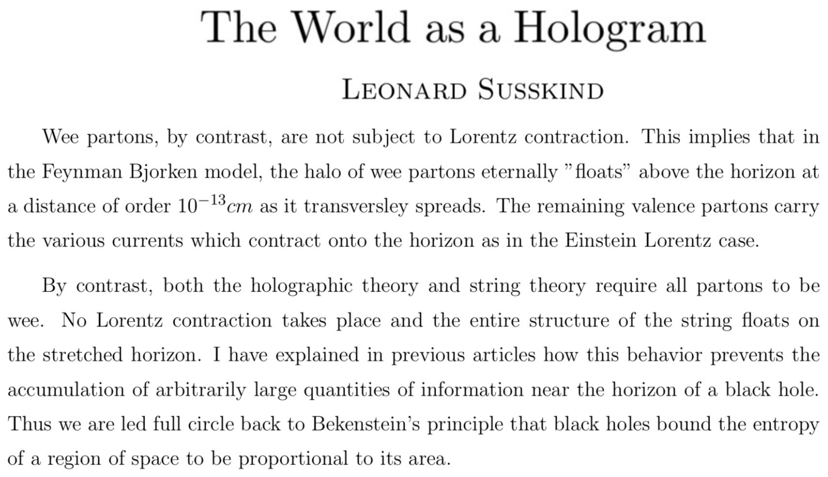


Effective action and all-order gravitational eikonal at planckian energies

AMATI,CIAFALONI,VENEZIANO **NPB403 (1993)707**



Building on previous work by us and by Lipatov, we present an effective action approach to the resummation of all semiclassical (i.e. $O(\hbar^{-1})$) contributions to the scattering phase arising in high-energy gravitational collisions. By using an infrared-safe expression for Lipatov's effective action, we derive an eikonal form of the scattering matrix and check that the superstring amplitude result is reproduced at first order in the expansion parameter R^2/b^2, where R, b are the gravitational radius and the impact parameter, respectively. If rescattering of produced gravitons is neglected, the longitudinal coordinate dependence can be explicitly factored out and exhibits the characteristics of a shock-wave metric while the transverse dynamics is described by a reduced two-dimensional effective action. Singular behaviours in the latter, signalling black hole formation, can be looked for.



The World as a Hologram

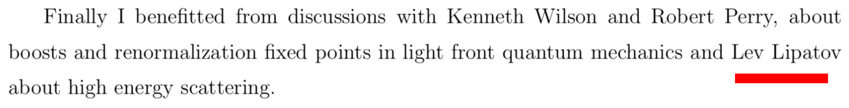
LEONARD SUSSKIND

Wee partons, by contrast, are not subject to Lorentz contraction. This implies that in the Feynman Bjorken model, the halo of wee partons eternally "floats" above the horizon at a distance of order $10^{-13}cm$ as it transversley spreads. The remaining valence partons carry the various currents which contract onto the horizon as in the Einstein Lorentz case.

By contrast, both the holographic theory and string theory require all partons to be wee. No Lorentz contraction takes place and the entire structure of the string floats on the stretched horizon. I have explained in previous articles how this behavior prevents the accumulation of arbitrarily large quantities of information near the horizon of a black hole. Thus we are led full circle back to Bekenstein's principle that black holes bound the entropy of a region of space to be proportional to its area.

***J.Math.Phys.* 36 (1995) 6377; > 4 K cites !**

In Acknowledgements:

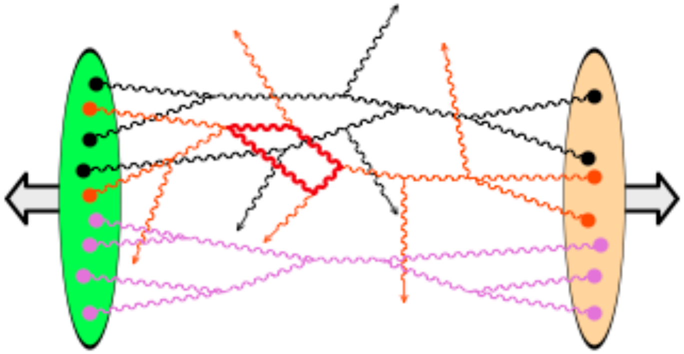


Finally I benefitted from discussions with Kenneth Wilson and Robert Perry, about boosts and renormalization fixed points in light front quantum mechanics and Lev Lipatov about high energy scattering.

30+ years of work by ACV et al. exploring gravitational shockwave collisions in this 2-D EFT

DeVecchia, Heissenberg, Russo, Veneziano, Phys. Rept. (2024)

Shockwaves in QCD?



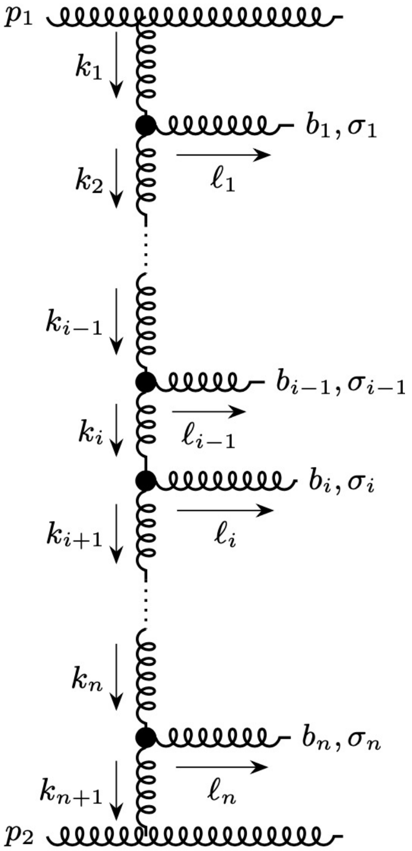
Dynamics in QCD is fundamentally quantum – is it meaningful to talk about semi-classical shockwaves and shockwave collisions?

We will argue that it is a meaningful (and indeed, highly efficient) paradigm to understand the Regge regime of 2→ n scattering (Q^2=fixed $\gg \Lambda_{QCD}^2$, $s \to \infty, x_{Bj} \to 0$, in DIS kinematics)

This framework is far more robust for large nuclei ($A \gg 1$) than for protons; the large number of color sources in the former already reside predominantly in classical configurations at modest boosts

It allows us to address interesting problems that are otherwise intractable using amplitude methods – such as, explore the matter produced in ultrarelativistic heavy-ion collisions or very high multiplicity p+p and p+A collisions

2→ N QCD amplitudes in Regge asymptotics: BFKL paradigm



Multiparticle production in multi-Regge kinematics of QCD:

$$y_0^+ \gg y_1^+ \gg y_2^+ \gg \cdots \gg y_N^+ \gg y_{N+1}^+ \qquad \text{with} \qquad \boldsymbol{k}_i \simeq \boldsymbol{k}$$

y_i=Ln(x_i/x_{i+1})

Building blocks: Lipatov vertices and reggeized propagators

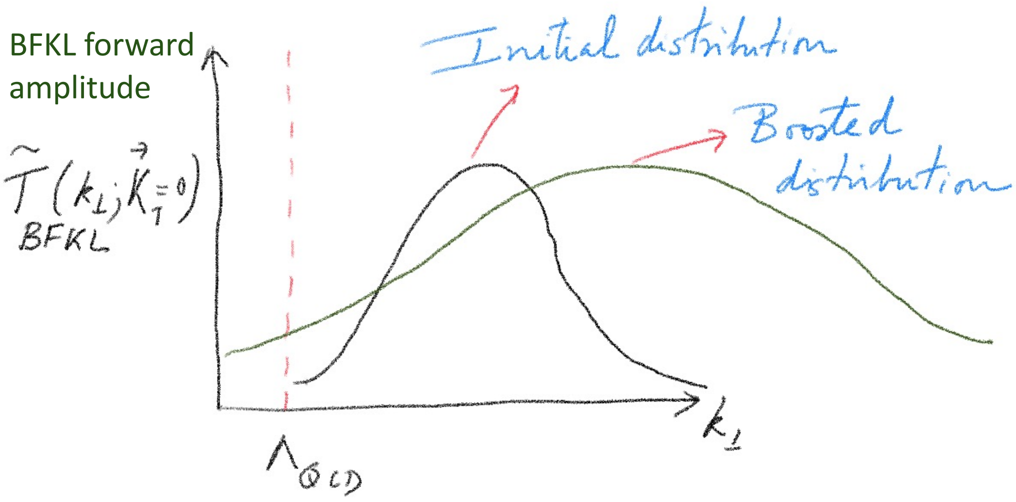
BFKL ladder is ordered in rapidity . Produced partons are wee in longitudinal momentum(` `slow”) but hard in transverse momentum – weak coupling Regge regime of QCD

RG description rapidity of evolution given by the BFKL Hamiltonian
Rapid growth of the amplitude with energy

Reviews: Del Duca, hep-ph/9503226
Forshaw & Ross, Cambridge Univ. Press (1998)
Del Duca, Dixon, arXiv:2203.13026
Raj, Venugopalan, arXiv:2507.21252

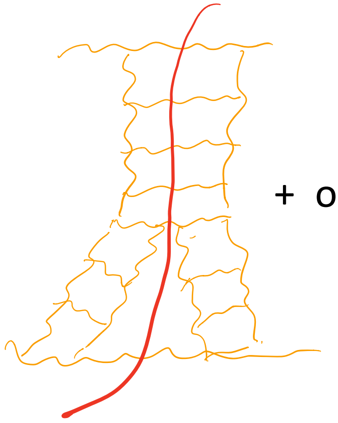
BFKL: Balitsky-Fadin-Kuraev-Lipatov (1976-1978)

Breakdown of BFKL paradigm: infrared diffusion and gluon saturation



NLLx BFKL does not cure infrared diffusion

Gluon saturation cures infrared diffusion



+ other higher twist cuts become O(1) for gluon occupancies $N \equiv \frac{xG_A(x,Q_S^2)}{2(N_c^2-1)\pi R_A^2 Q_S^2} = \frac{1}{\alpha_S(Q_S)}$

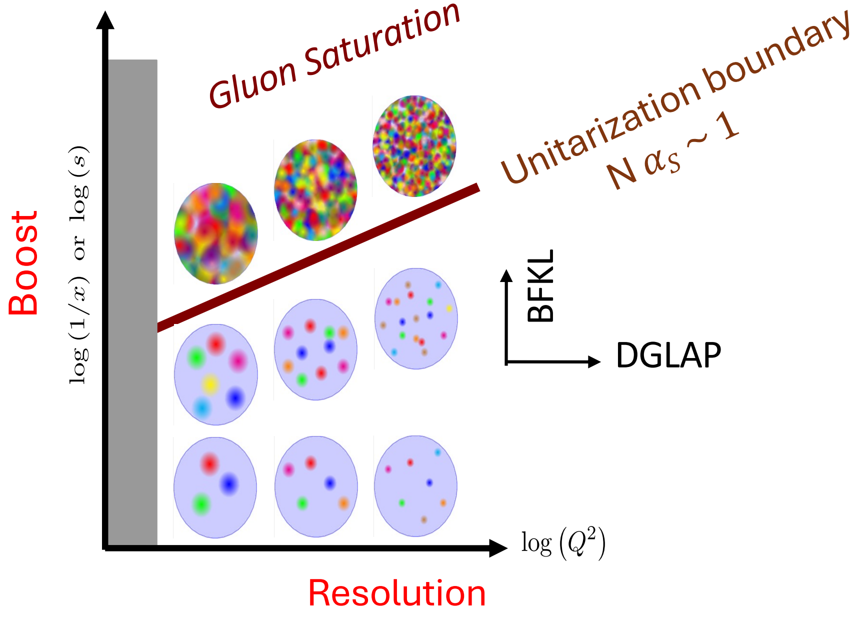
Classicalization when $\alpha_S(Q_S) \ll 1$ for saturation scale $Q_S \gg \Lambda_{QCD}$

Gluon saturation: classicalization and unitarization of cross-sections

For a fixed large Q^2 there is an $x_0(Q^2)$ such that below x_0 the OPE breaks down…

significant nonperturbative corrections in the leading twist coefficient and anomalous dimension functions due to diffusion of gluons to small values of transverse momentum.

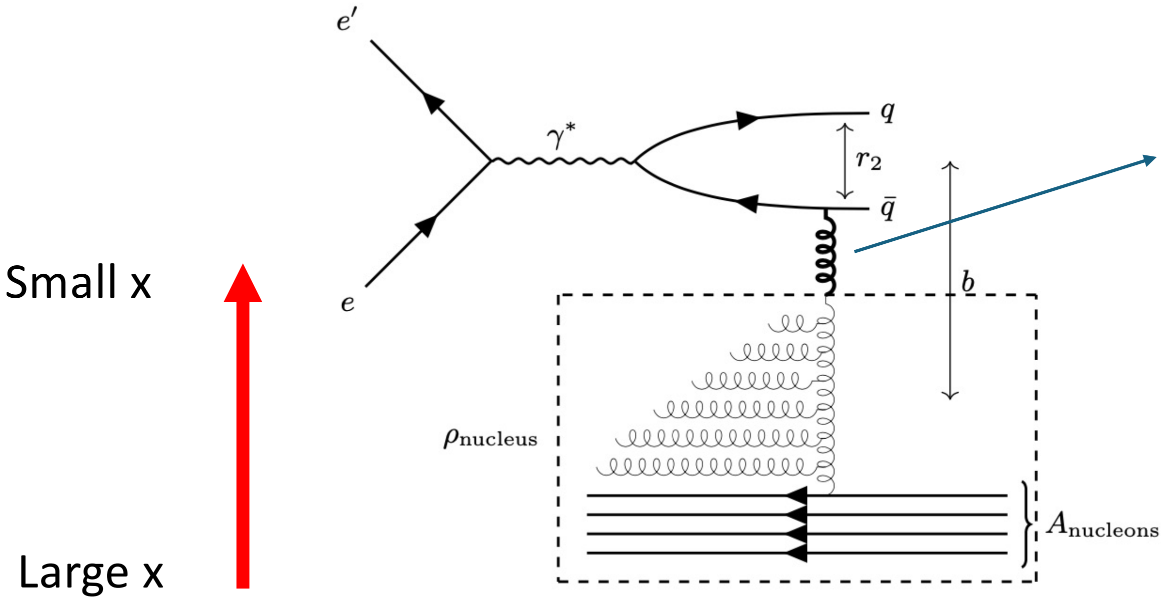
A. H. Mueller, PLB 396 (1997) 251



Gluon saturation: nontrivial fixed point defines emergent close packing scale Q_S^2(x)

This intuition is simply realized in the BK equation

An EFT of saturated glue: The Color Glass Condensate



Reggeized gluon as field sourced by blob's color charge density ρ

Dense close-packed ($1/Q_S$) classical blob

gluon "shockwave"

McLerran,RV (1993-1994)
Balitsky (1996)

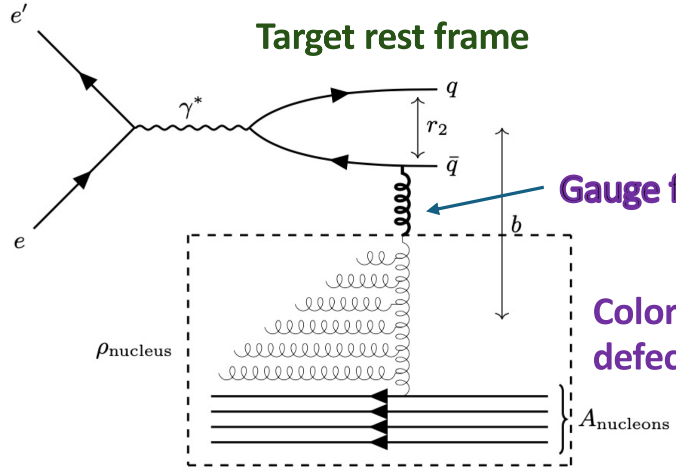
The high occupancy state typically corresponds to a higher dimensional (large charge) classical representation

The "glassy" behavior is because the large x modes are static random sources (long lived "impurities") on interaction time scales – small x modes are dynamical

Color Glass Condensate review: Gelis,Iancu,Jalilian-Marian, RV:arXiv 1002.0333

CGC EFT: Born-Oppenheimer separation of slow and fast partons

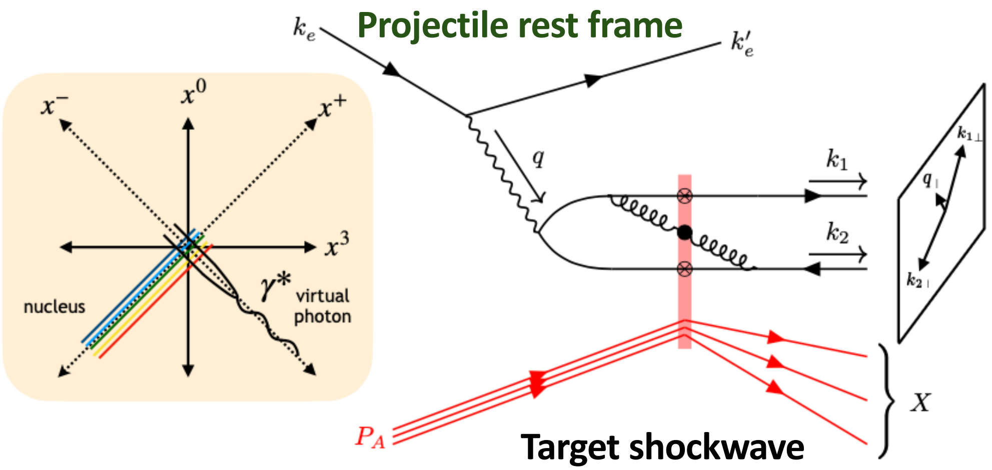
Target rest frame



Gauge field in bulk

Color sources/ defects

Projectile rest frame



Target shockwave

Born-Oppenheimer separation of static light cone sources and dynamical fields

Generating functional

$$\mathcal{Z}[j] = \int [d\rho] W_{\Lambda^+}[\rho] \left\{ \frac{\int^{\Lambda^+} [dA] \delta(A^+) e^{iS_{\Lambda^+}[A,\rho] - \int j \cdot A}}{\int^{\Lambda^+} [dA] \delta(A^+) e^{iS_{\Lambda^+}[A,\rho]}} \right\}$$

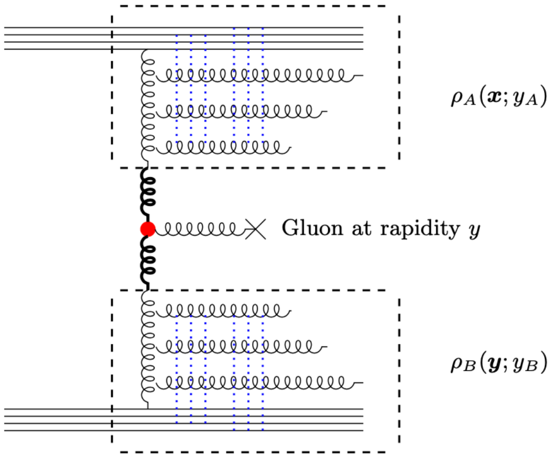
Gauge invariant stochastic distribution of color sources/impurities

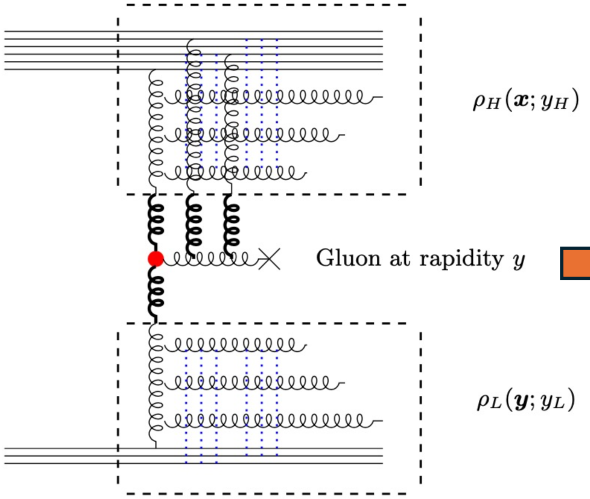
Shockwave RG: BK, JIMWLK

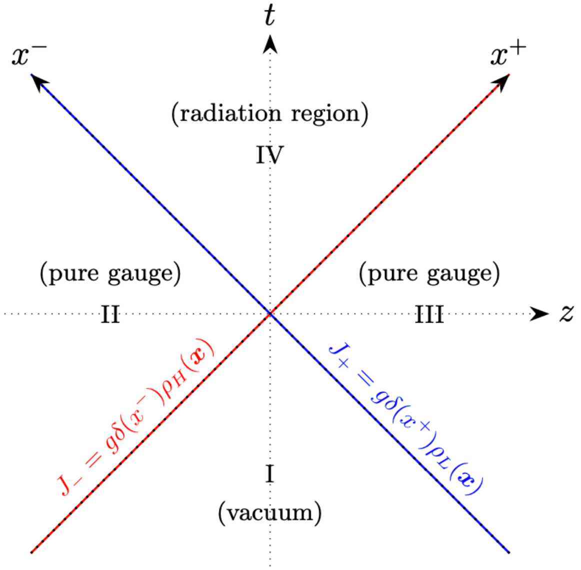
QCD path integral of wee fields coupled to sources

Λ^+ is scale separating sources and fields

Gluon shockwave collisions: Lipatov vertex and reggeization







Weizsäcker-Williams gluon radiation field in light cone gauge

$$a_i(k) = -\frac{2ig}{k^2+i\epsilon}\int\frac{d^2\boldsymbol{q}_2}{(2\pi)^2}\left(q_{2i} - k_i\frac{\boldsymbol{q}_2^2}{\boldsymbol{k}^2}\right)\underbrace{\frac{\rho_L(\boldsymbol{q}_2)}{\boldsymbol{q}_2^2}\left(U(\boldsymbol{k}+\boldsymbol{q}_2) - (2\pi)^2\delta^2(\boldsymbol{k}+\boldsymbol{q}_2)\right)}$$

$$U(x^-,\boldsymbol{x})\delta\left(x^+\right) = \exp\left(ig\int_{-\infty}^{x^-} dz^-\bar{A}_-(z^-,\boldsymbol{x})\cdot T\right)$$

$$\bar{A}_\mu(x^-,\boldsymbol{x}) = -g\delta_{\mu-}\delta\left(x^-\right)\frac{\rho_H(\boldsymbol{x})}{\Box_\perp}$$

Lipatov vertex
in $A^- = 0$ gauge

reggeized gluons from
semi-classical source dists.

Ln (U) → *reggeized gluon*

Blaizot,Gelis,RV (2004)
Gelis-Mehtar-Tani (2005)

Jalilian-Marian,Jeon,RV (2000); Caron-Huot (2013)

Probability of producing n particles in theory with sources

Lehmann-Symanzik-Zimmerman (LSZ)

$$\langle p_1 \cdots p_{n\text{out}} | 0_{\text{in}} \rangle = \frac{1}{Z^{n/2}} \int \left[\prod_{i=1}^{n} d^4x_i e^{ip_i \cdot x_i} \, (\partial^2_{x_i} + m^2) \frac{\delta}{i\delta\rho(x_i)} \right] e^{i\mathcal{V}[\rho]}$$

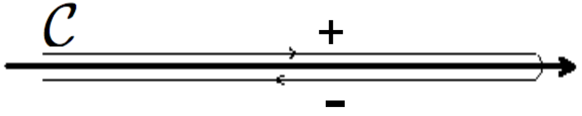
n-particle probability $P_n = \frac{1}{n!} \prod_{i=1}^{n} \int \frac{d^3p_i}{2E_{p_i}} |\langle p_1 \cdots p_n \text{out} | 0_{\text{in}} \rangle|^2$

$$P_n = \frac{1}{n!} \mathcal{D}^n[j_+, j_-] \exp\left(iV[j_+] - iV^\star[j_-]\right)|_{j_+ = j_- = j}$$

$$D[j_+, j_-] \equiv \frac{1}{Z} \int_{x,y} G^0_{+-}(x,y)(\partial_x^2 + m^2)(\partial_y^2 + m^2) \frac{\delta}{\delta j_+(x)} \frac{\delta}{\delta j_-(y)}$$

$$\int \frac{d^3\vec{p}}{(2\pi)^3 2E_p} e^{ip\cdot(x-y)}$$

Schwinger-Keldysh contour



$$G^0_{++} = \frac{i}{p^2 - m^2 + i\epsilon}$$

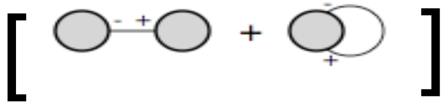
$$G^0_{--} = \frac{-i}{p^2 - m^2 - i\epsilon}$$

$$G^0_{+-} = 2\pi\theta(-p^0)\delta(p^2 - m^2)$$

N-particle distributions: inclusive multiplicity

$$\langle n\rangle = \sum_n n\,P_n \equiv D\,\underbrace{\left[e^{D} e^{iV} e^{-iV}\right]}_{e^{iV_{\rm SK}}}$$

$$\langle n\rangle = \int_{x,y} Z\,G^0_{+-}(x,y)\,\left[\Gamma_+(x)\,\Gamma_-(y)+\Gamma_{+-}(x,y)\right]$$



$$\Gamma_\pm(x) = \frac{\partial_x^2+m^2}{Z}\,\frac{\delta iV_{\sf SK}}{\delta j_\pm(x)}\Big|_{j_+=j_-=j}$$

One-point function in the background field

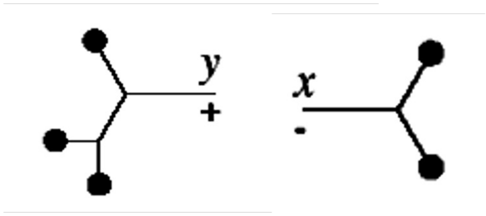
$$\Gamma_{+-}(x,y) = \frac{(\partial_x^2+m^2)(\partial_y^2+m^2)}{Z}\,\frac{\delta^2 iV_{\sf SK}}{\delta j_\pm(x)\delta j_-(y)}\Big|_{j_+=j_-=j}$$

Two-point function in background field

Gelis, RV ; NPA776 (2006)135
NPA 779 (2006)177

Inclusive multiplicity to LO in strong fields: O (1/g^2)

$$\langle n \rangle_{\mathrm{LO}} =$$



$$\text{but } (g\rho)^{\infty}$$

arbitrary number of insertions of sources ρ

In the Schwinger Keldysh formalism, each node of a tree includes a sum over $\pm$

$$G^0_{i+}(x,y) - G^0_{i-}(x,y) = G^0_R(x,y) \; ; \; i = \pm$$

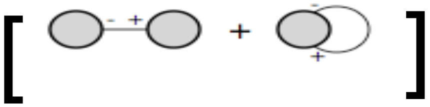
Recursive use of this identity shows that sum of all tree diagrams is the

retarded solution of classical equations of motion with $\lim_{x^0 \to -\infty} \phi_c(x) = 0$

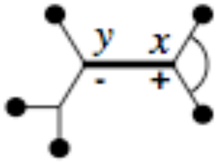
Leading order result for the inclusive multiplicity in the strong fields in a heavy-ion collision is given by solutions of the QCD Yang-Mills equations

Inclusive multiplicity at NLO in strong fields: O (g^0)

$$\langle n \rangle = \int_{x,y} Z\, G^0_{+-}(x,y)\,[\Gamma_+(x)\,\Gamma_-(y) + \Gamma_{+-}(x,y)]$$



$$\langle n \rangle_{\mathrm{NLO}} =$$



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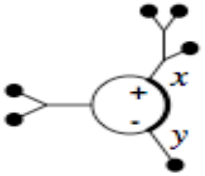


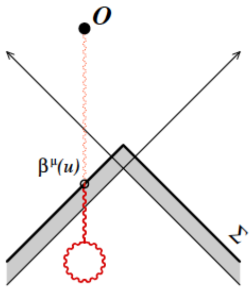
$$\phi_c^{(0)}\,\phi_c^{(1)}$$

$$\langle \delta\phi_{\mathrm{quant.}}\delta\phi_{\mathrm{quant.}} \rangle$$

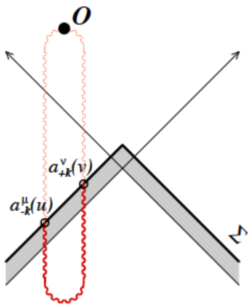
Product of classical field and
1-loop correction to classical field

Small fluctuation propagator
in classical background field

QCD factorization of wee gluon distributions of the nuclei



$\beta(\vec{u})$



$\mathcal{G}(\vec{u},\vec{v})$

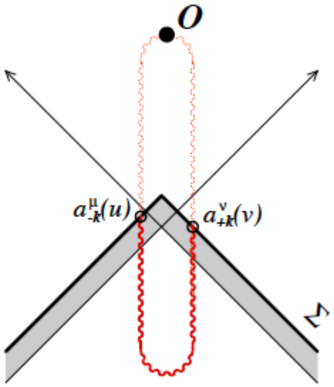
$$\mathcal{O}_{\rm NLO} = \left[\frac{1}{2}\int_{\vec{u},\vec{v}} \mathcal{G}(\vec{u},\vec{v})\ \mathcal{T}_u\mathcal{T}_v + \int_{\vec{u}} \beta(\vec{u})\ \mathcal{T}_u\right]\mathcal{O}_{\rm LO}$$

$$\mathcal{T}_u = \frac{\delta}{\delta A(\vec{u})}$$

linear operator of source
on initial “Cauchy” surface

These 1-and 2-point have logs
- are resummed to all orders by
the JIMWLK Hamiltonian

$$\mathcal{O}_{\rm NLO} = \left[\ln\left(\frac{\Lambda^+}{p^+}\right)\mathcal{H}_1 + \ln\left(\frac{\Lambda^-}{p^-}\right)\mathcal{H}_2\right]\mathcal{O}_{\rm LO}$$



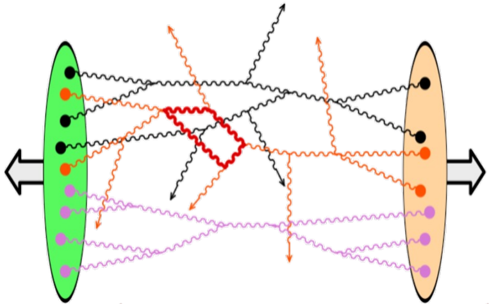
Quantum fluctuations that cross-talk between
nuclei before collision are suppressed at
leading log accuracy

➡ QCD factorization

Gelis,Lappi,RV (2008)

Factorization + temporal evolution in the Glasma

$$T^{\mu\nu}_{\mathrm{LO}} = \frac{1}{4} g^{\mu\nu} F^{\lambda\delta} F_{\lambda\delta} - F^{\mu\lambda} F^{\nu}_{\lambda}$$



$\rho_1 \sim \frac{1}{g}$ $\rho_2 \sim \frac{1}{g}$

$$\mathrm{O}\left(\frac{Q_S^4}{g^2}\right)$$

ε=20-40 GeV/fm^3 for τ=0.3 fm @ RHIC
scale set by Q_S in the nuclei

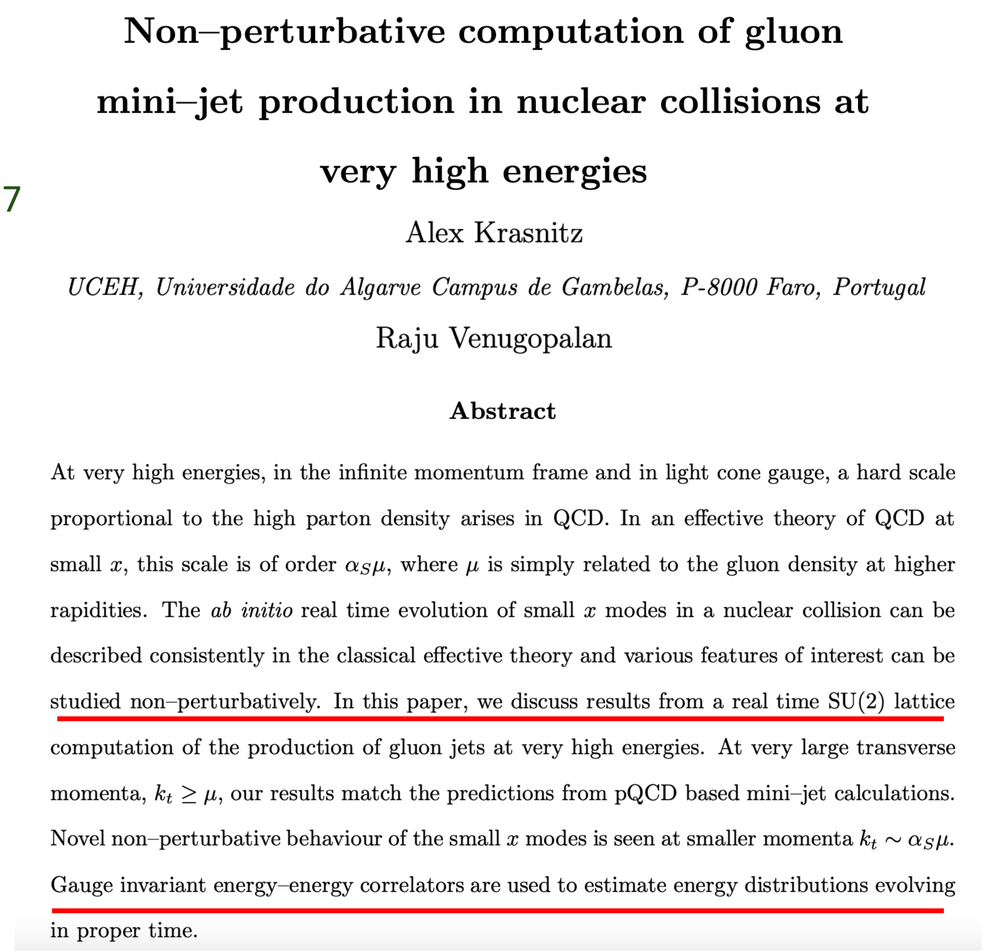
$$\langle T^{\mu\nu}(\tau, \underline{\eta}, x_\perp)\rangle_{\mathrm{LLog}} = \int [D\rho_1 d\rho_2]\, W_{Y_1}[\rho_1]\, W_{Y_2}[\rho_2]\, T^{\mu\nu}_{\mathrm{LO}}(\tau, x_\perp)$$

$$Y_1 = Y_{\mathrm{beam}} - \eta\,;\, Y_2 = Y_{\mathrm{beam}} + \eta$$

Glasma factorization => universal “density matrices W” ⊗ “matrix element”

Gluon radiation in shockwave collisions

Nucl. Phys. B 557 (1999) 237



Non–perturbative computation of gluon mini–jet production in nuclear collisions at very high energies

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Raju Venugopalan

Abstract

At very high energies, in the infinite momentum frame and in light cone gauge, a hard scale proportional to the high parton density arises in QCD. In an effective theory of QCD at small x, this scale is of order $\alpha_S\mu$, where μ is simply related to the gluon density at higher rapidities. The *ab initio* real time evolution of small x modes in a nuclear collision can be described consistently in the classical effective theory and various features of interest can be studied non–perturbatively. In this paper, we discuss results from a real time SU(2) lattice computation of the production of gluon jets at very high energies. At very large transverse momenta, $k_t \geq \mu$, our results match the predictions from pQCD based mini–jet calculations. Novel non–perturbative behaviour of the small x modes is seen at smaller momenta $k_t \sim \alpha_S\mu$. Gauge invariant energy–energy correlators are used to estimate energy distributions evolving in proper time.

We computed energy-energy correlators but the
jet community was not
interested at that time

The Glasma at NLO: explosive growth of time-dependent fluctuations

$$\langle n \rangle_{\mathrm{NLO}} =$$

Evolution of small fluctuations from -∞ in the shockwave backgrounds

Gluon pair production contribution

One loop corrections to classical field

In our previous discussion, we proved the factorization of only static modes in each nucleus (for which the initial Cauchy surface was the backward "Milne" wedge in spacetime)

– these correspond to p_η=0 (LC modes corresponding to BFKL/JIMWLK evolution of sources)

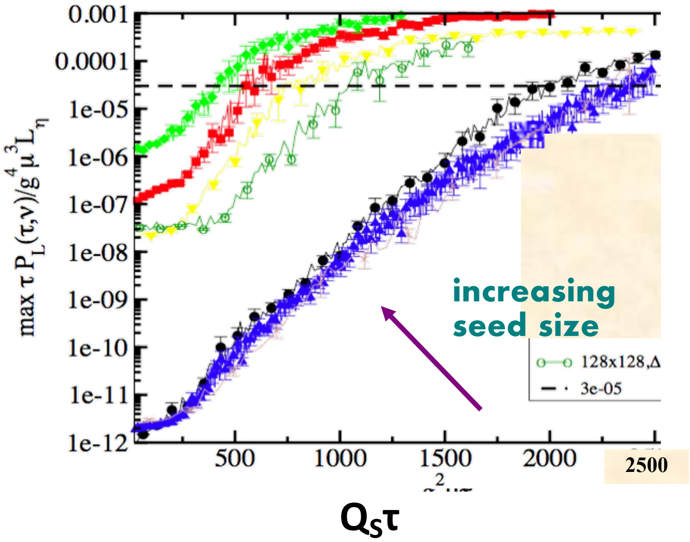
But for $p_\eta \neq 0$ modes, $\mathbf{T}_u \mathcal{A}(x) \sim \dfrac{\delta \mathcal{A}(x)}{\delta \mathcal{A}(0,y)} \sim \exp\left(\sqrt{Q_s \tau}\right)$

The Glasma at NLO: plasma instabilities

At LO: boost invariant gauge fields $A_{cl}^{\mu,a}(x_T,\tau) \sim 1/g$

NLO: $A^{\mu,a}(x_T,\tau,\eta) = A_{cl}^{\mu,a}(x_T,\tau) + a^{\mu,a}(\eta)$

$a^{\mu,a}(\eta) = O(1)$



- Small fluctuations grow exponentially as

$$e^{\sqrt{Q_S\tau}}$$

- Same order of classical field at

$$\tau = \frac{1}{Q_S}\ln^2\frac{1}{\alpha_S}$$

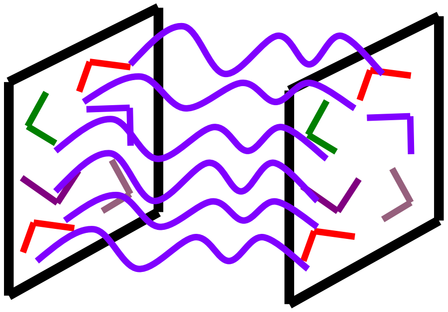
- Resum such contributions to all orders

$$(g\, e^{\sqrt{Q_S\tau}})^n$$

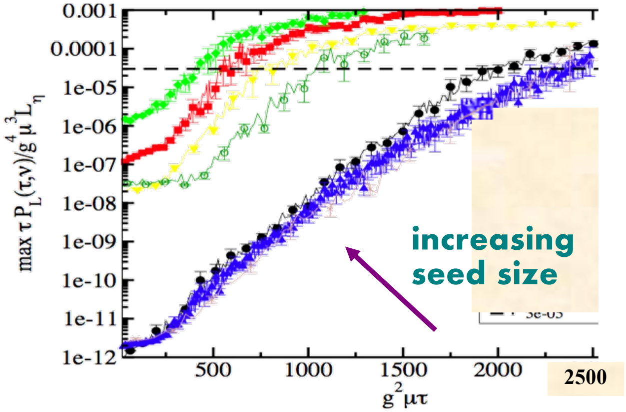
$$T^{\mu\nu}_{\rm resum} = \int_{\tau=0^+}[da]\; F_{\rm init.}[a]\; T_{\rm LO}[A_{\rm cl}+a]$$

Romatschke,Venugopalan (2005)
Dusling,Gelis,Venugopalan (2011)
Gelis, Epelbaum (2013)

From Glasma to Plasma



Quant. fluctuations grow exponentially after collision



increasing seed size

$$\langle\langle T^{\mu\nu}\rangle\rangle_{\rm LLx+Linst.} = \int [D\rho_1][D\rho_2]\, W_{\rm Y_{beam}-Y}[\rho_1]\, W_{\rm Y_{beam}+Y}[\rho_2]$$

$$\times \int [da(u)]\; F_{\rm init}[a]\; T^{\mu\nu}_{\rm LO}[A_{\rm cl}(\rho_1,\rho_2)+a]$$

Path integral over multiple initializations of classical trajectories in one event can lead to quasi-ergodic “eigenstate thermalization”

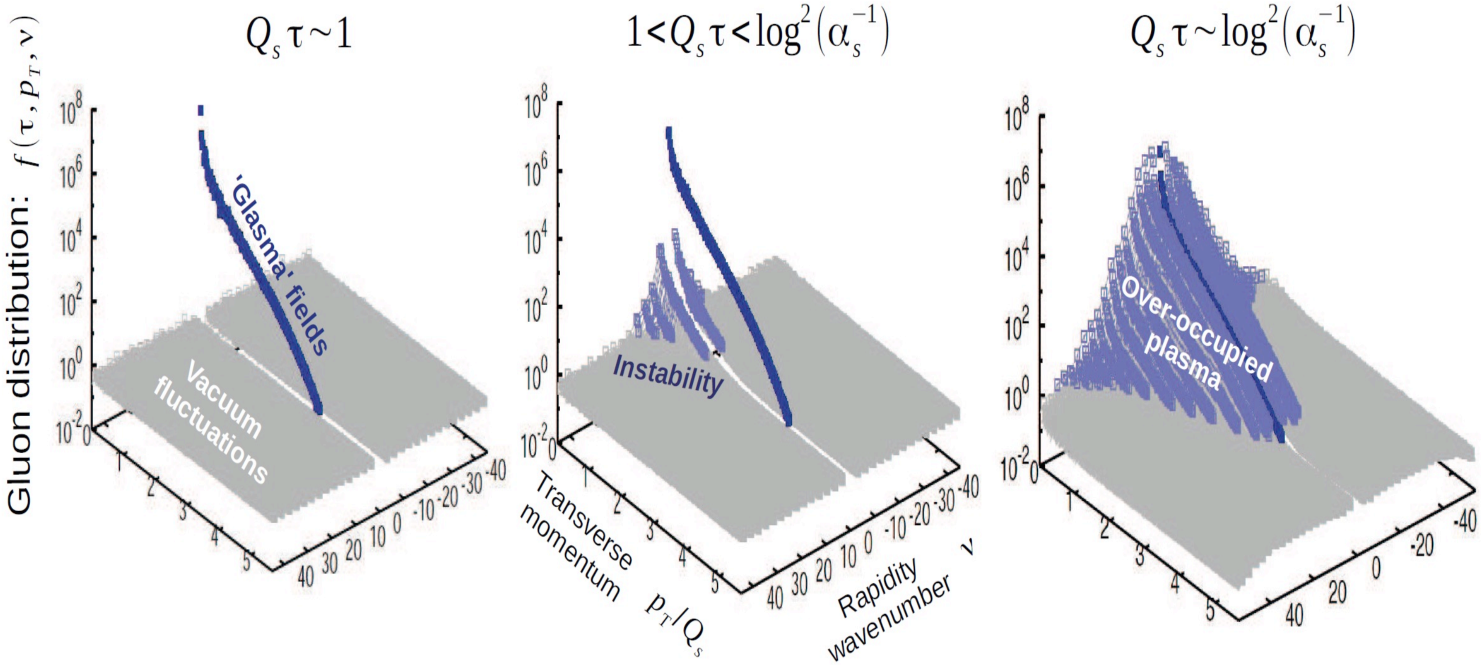
Berry; Srednicki; Rigol et al.; …

This *scrambling of information* is seen in many systems in nature and can be understood to lead to decoherence of the primordial classical fields

From Glasma to Quark Gluon Plasma

Glasma fields produced in the shock wave collision are unstable to quantum fluctuations…

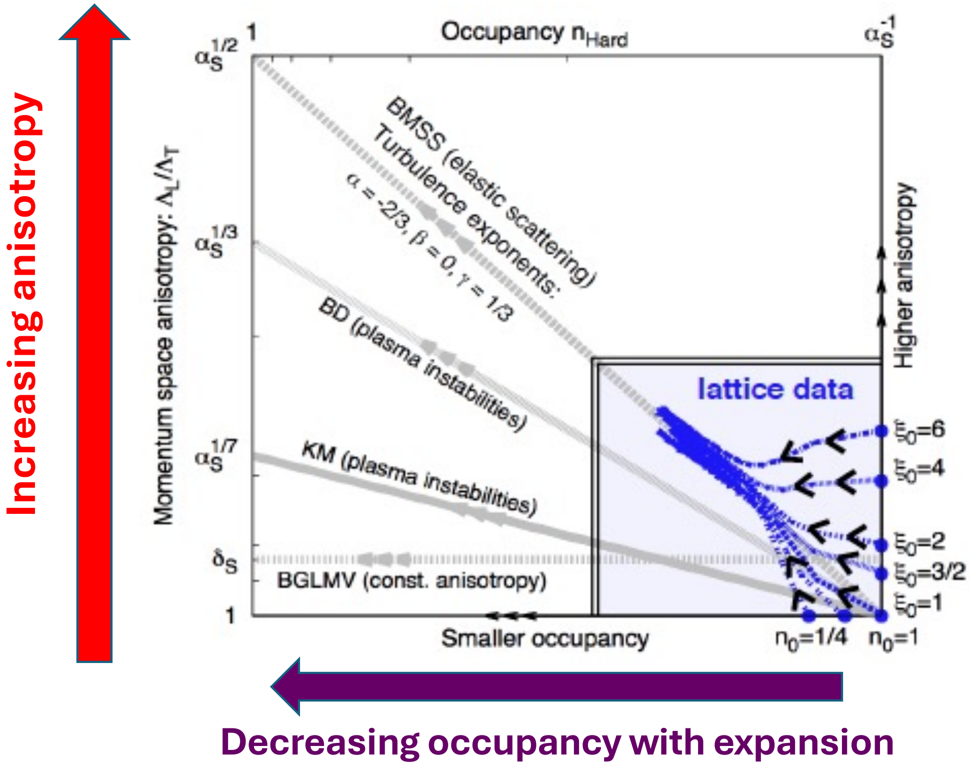
This instability leads to rapid overpopulation of all momentum modes

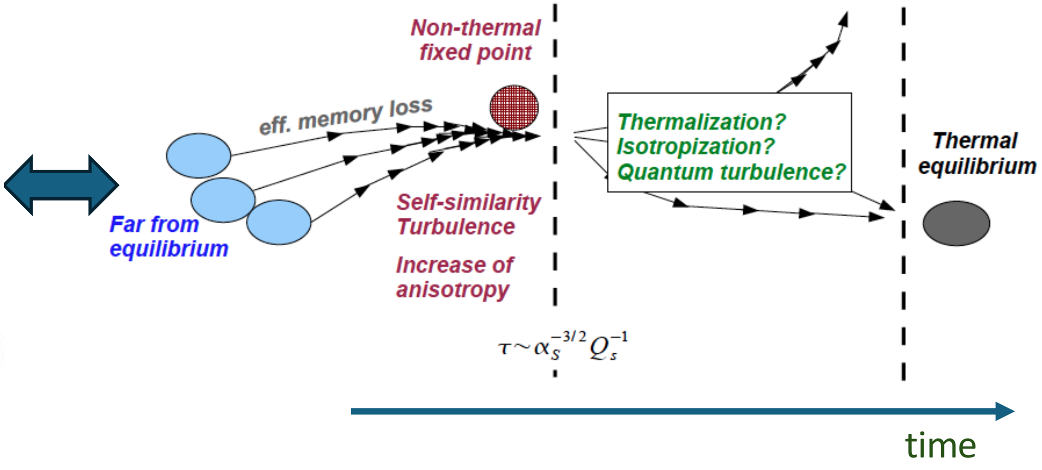


Classical-statistical QFT numerical lattice simulations of gluon fields exploding into the vacuum

Berges,Schenke,Schlichting,RV, NPA 931 (2014) 348

Heavy-ion collisions in the shockwave formalism

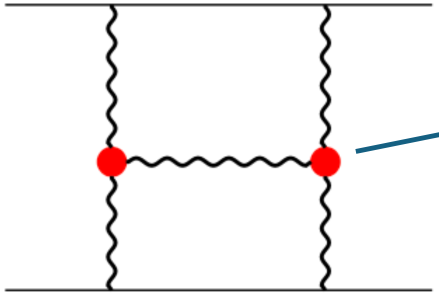




QCD thermalization: Ab initio approaches and interdisciplinary connections
Jürgen Berges, Michal P. Heller, Aleksas Mazeliauskas, and RV
Rev. Mod. Phys. **93**, 035003 (2021)

QCD-gravity double copy in Regge asymptotics

From QCD to gravity in Regge asymptotics: Lipatov vertex



H-diagram of Amati, Ciafaloni, Veneziano

Gravitation Lipatov vertex:

$$\Gamma_{\mu\nu}(\boldsymbol{q}_1,\boldsymbol{q}_2) \equiv \underbrace{\frac{1}{2}C_\mu(\boldsymbol{q}_1,\boldsymbol{q}_2)C_\nu(\boldsymbol{q}_1,\boldsymbol{q}_2)}_{\text{Double copy of QCD Lipatov vertex}} - \underbrace{\frac{1}{2}N_\mu(\boldsymbol{q}_1,\boldsymbol{q}_2)N_\nu(\boldsymbol{q}_1,\boldsymbol{q}_2)}_{\text{Double copy of QED Bremsstrahlung vertex}}$$

$$N_\mu(\boldsymbol{q}_1,\boldsymbol{q}_2) = \sqrt{\boldsymbol{q}_1^2\boldsymbol{q}_2^2}\left(\frac{p_{1\mu}}{p_1\cdot k} - \frac{p_{2\mu}}{p_2\cdot k}\right)$$

S-matrix power counting a la ACV:

$$S = e^{2i(\delta_0+\delta_1+\delta_2+\cdots)} \qquad \delta_0 = Gs\log\left(\frac{L}{b}\right), \qquad \delta_1 = \frac{6G^2s}{\pi b^2}\log s\,, \qquad \delta_2 = \frac{2G^3s^2}{b^2}\left[1+\frac{i}{\pi}\log s\left(\log\frac{L^2}{b^2}+2\right)\right]$$

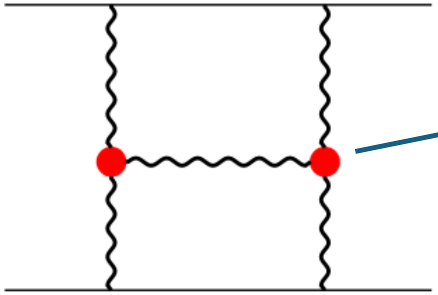
Leading Eikonal term (real)

Sub-leading quantum gravity correction $\sim \frac{l_P^2}{b^2}$

Sub-leading loop contribution $\sim \frac{R_S^2}{b^2}$
- includes absorptive piece

$$\boxed{\delta_2 \gg \delta_1 \text{ for } R_S \gg l_P}$$

From QCD to gravity in Regge asymptotics: Lipatov vertex



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$$N_\mu(\boldsymbol{q}_1,\boldsymbol{q}_2) = \sqrt{\boldsymbol{q}_1^2\boldsymbol{q}_2^2}\left(\frac{p_{1\mu}}{p_1\cdot k} - \frac{p_{2\mu}}{p_2\cdot k}\right)$$

Lipatov (1982)

In amplitudes language, the extra terms in the double copy are imposed by so-called Steinmann relations - required by unitarity to cancel spurious poles of energy variables ($s_1= (k+p_1)^2$ and $s_2=(k+p_2)^2$) in overlapping channels

Lipatov vertex from shockwave collisions

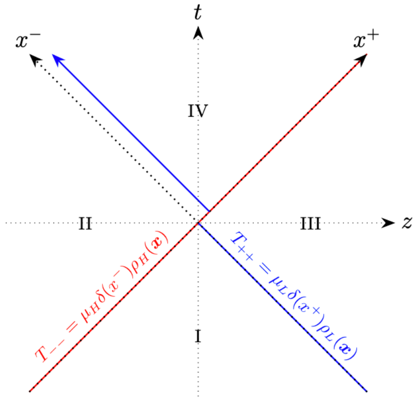
We will now sketch how the Lipatov vertex is recovered in shockwave collisions

Aichelburg-Sexl shockwave metric

$$ds^2 = 2dx^+dx^- - \delta_{ij}dx^i dx^j + f(x^-, \boldsymbol{x})\,(dx^-)^2$$

with $f(x^-, \boldsymbol{x}) = 2\kappa^2 \mu_H \delta(x^-) \dfrac{\rho_H(\boldsymbol{x})}{\Box_\perp} = \dfrac{\kappa^2}{\pi} \mu_H \delta(x^-) \displaystyle\int d^2\boldsymbol{y}\ \ln \Lambda|\boldsymbol{x} - \boldsymbol{y}| \rho_H(\boldsymbol{y})$

Soln of Einstein's eqns sourced by the EM tensor $T_{\mu\nu} = \delta_{\mu -}\delta_{\nu -}\mu_H \delta(x^-)\rho_H(\boldsymbol{x})$



$\mu_H = m_H\, \gamma$ = fixed for $\gamma \to \infty$

$\kappa^2 = 8\, \pi\, G$

Shockwave collisions: single shock background

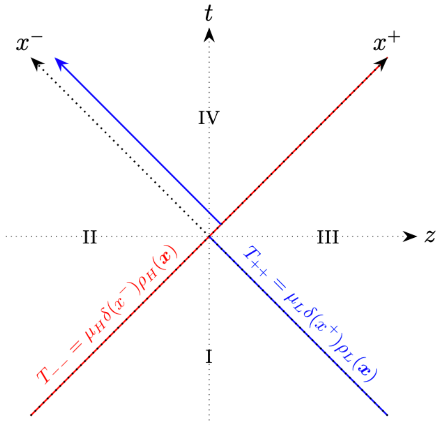
Linearizing around the metric $g_{\mu\nu} = \bar{g}_{\mu\nu} + \kappa\, h_{\mu\nu}$

fixing light cone gauge $h_{\mu +}$=0, find

$$h_{ij}(x^+, x^-, \boldsymbol{x}) = V(x^-, \boldsymbol{x}) h_{ij}(x^+, x^- = x_0^-, \boldsymbol{x})$$

with the gravitational Wilson line $V(x^-, \boldsymbol{x}) \equiv \exp\left(\frac{1}{2}\int_{x_0^-}^{x^-} dz^- \bar{g}_{--}(z^-, \boldsymbol{x})\, \partial_+\right)$

Exactly analogous to the QCD case
with $A_- \rightarrow\ g_{--}$ and $T^a \rightarrow \partial_+$



$\mu_H = m_H\, \gamma$ = fixed for $\gamma \rightarrow \infty$

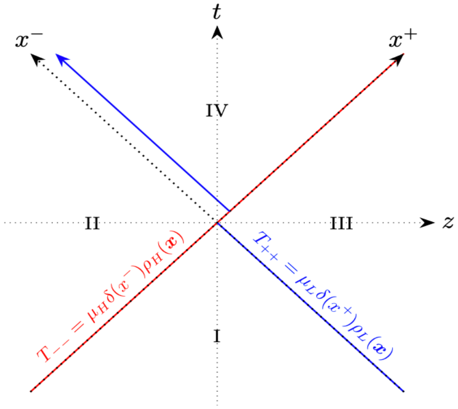
κ^2=8 π G

Melville,Nachulich,Schnitzer,White,
arXiv:1306.6019

See also Saotome and Akhoury, arXiv:1210.8111

Shockwave collisions: “dilute-dense” approximation

Now consider the interaction of the “dilute” source ρ_L with the dense ρ_H shockwave:



$\mu_H = m_H\,\gamma$ = fixed for $\gamma \to \infty$

$\kappa^2 = 8\,\pi\,G$

$$T_{\mu\nu} = \delta_{\mu-}\delta_{\nu-}\mu_H\delta\left(x^-\right)\rho_H(\boldsymbol{x}) + \delta_{\mu+}\delta_{\nu+}\mu_L\delta\left(x^+\right)\rho_L(\boldsymbol{x})$$

Solve for metric in region IV – forward lightcone

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu} \qquad \bar{g}_{--} = 2\kappa\mu_H\delta\left(x^-\right)\frac{\rho_H(\boldsymbol{x})}{\Box_\perp}$$

We decompose the perturbation $h_{\mu\nu}$ into a term linear in ρ_L and one bi-linear in $\rho_L\rho_H$ (dilute-dilute limit)

Linearized Einstein’s equations in light-cone gauge ($h_{+\mu}$=0) take the form

$$\bar{g}_{--}\partial_+^2\tilde{h}_{ij} - \Box\tilde{h}_{ij} = \kappa^2\left[\left(2\partial_i\partial_j - \Box_\perp\delta_{ij}\right)\frac{1}{\partial_+^2}T_{++} + 2T_{ij} - \delta_{ij}T - \frac{2}{\partial_+}\left(\partial_i T_{+j} + \partial_j T_{+i} - \delta_{ij}\partial_k T_{+k}\right)\right]$$

$$\tilde{h}_{ij} \equiv h_{ij} - \tfrac{1}{2}\delta_{ij}h \text{ where } h = \delta_{ij}h_{ij}$$

Raj, RV, arXiv:2311.03463, arXiv:2406.10483, arXiv:2507.21252

Shockwave collisions in general relativity: geodesics

Unlike QCD case, sub-eikonal contributions T_{+i}, T_{ij} are required for consistency of equations of motion

These are not uniquely fixed by energy-momentum conservation, the dynamics of the sources is needed to fix this. In the point particle approximation,

$$T^{\mu\nu}(x) = \frac{\mu_L}{\sqrt{-g}} \int_{-\infty}^{\infty} d\lambda \; \dot{X}^\mu \dot{X}^\nu \; \delta^{(4)}(x - X(\lambda))$$

Solution of the corresponding null geodesic equations $\ddot{X}^\mu + \Gamma^\mu_{\nu\rho} \dot{X}^\nu \dot{X}^\rho = 0 \, , \qquad g_{\nu\rho} \dot{X}^\nu \dot{X}^\rho = 0$

in shockwave background given by $X^- = \lambda \, , \quad X^i = b^i - \kappa^2 \mu_H X^- \Theta(X^-) \frac{\partial_i \rho_H(\boldsymbol{b})}{\Box_\perp}$

$$X^+ = -\kappa^2 \mu_H \Theta(X^-) \frac{\rho_H(\boldsymbol{b})}{\Box_\perp} + \frac{\kappa^4 \mu_H^2}{2} X^- \Theta(X^-) \left(\frac{\partial_i \rho_H(\boldsymbol{b})}{\Box_\perp} \right)^2$$

These geodesic solutions allow us to reconstruct the required components of the stress-energy tensor

Shockwave collisions in general relativity: Lipatov vertex

Solving eqns of motion, taking the Fourier transform, and putting the graviton momenta on-shell, one obtains

Gravitational
radiational field

$$\tilde{h}^{(2)}_{ij}(k) = \frac{2\kappa^3 \mu_H \mu_L}{k^2 + i\epsilon k^-} \int \frac{d^2\boldsymbol{q}_2}{(2\pi)^2} \Gamma_{ij}(\boldsymbol{q}_1, \boldsymbol{q}_2) \frac{\rho_H}{\boldsymbol{q}_1^2} \frac{\rho_L}{\boldsymbol{q}_2^2}$$

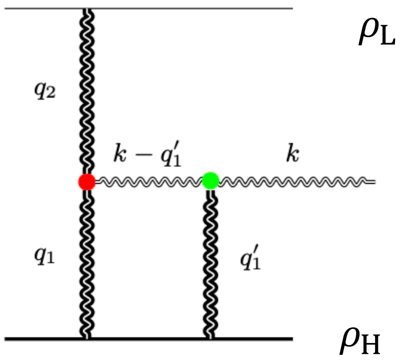
Gravitational Lipatov vertex

likewise for other components, recovering $\Gamma_{\mu\nu}(\boldsymbol{q}_1, \boldsymbol{q}_2) \equiv \frac{1}{2} C_\mu(\boldsymbol{q}_1, \boldsymbol{q}_2) C_\nu(\boldsymbol{q}_1, \boldsymbol{q}_2) - \frac{1}{2} N_\mu(\boldsymbol{q}_1, \boldsymbol{q}_2) N_\nu(\boldsymbol{q}_1, \boldsymbol{q}_2)$

Compare to gauge theory
radiation field

$$a_i(k) = \frac{g^3}{k^2 + i\epsilon k^-} \int \frac{d^2\boldsymbol{q}_2}{(2\pi)^2} C_i(\boldsymbol{q}_1, \boldsymbol{q}_2) \frac{\rho_H \cdot T}{\boldsymbol{q}_1^2} \frac{\rho_L}{\boldsymbol{q}_2^2}$$

Shockwave collisions: first dilute-dense correction



M. Fite, H. Raj, RV, in preparation

The shockwave computation can be extended straightforwardly to O($\rho_{\rm L}\,\rho_{\rm H}^2$)

$$\tilde{h}^{(1,2)}_{ij}(k) = \frac{2\mu_L\mu_H^2\kappa^6(-ik^-)}{k^2+i\epsilon k^-}\int \frac{d^2\boldsymbol{q}_1 d^2\boldsymbol{q}'_1 d^2\boldsymbol{q}_2}{(2\pi)^4}\delta^{(2)}(\boldsymbol{k}-\boldsymbol{q}_1-\boldsymbol{q}'_1-\boldsymbol{q}_2)\frac{\tilde{\rho}_H(\boldsymbol{q}_1)}{\boldsymbol{q}_1^2}\,\frac{\tilde{\rho}_H(\boldsymbol{q}'_1)}{\boldsymbol{q}'^2_1}\,\frac{\tilde{\rho}_L(\boldsymbol{q}_2)}{\boldsymbol{q}_2^2}V_{ij}(\boldsymbol{q}_1,\boldsymbol{q}'_1,\boldsymbol{q}_2)$$

$$\frac{\boldsymbol{q}_2^2}{\boldsymbol{k}^2}\,\tilde{S}_{ij}(\boldsymbol{q}_1,\boldsymbol{q}'_1,\boldsymbol{q}_2) - \Gamma_{ij}(\boldsymbol{q}_1,\boldsymbol{q}_2)$$

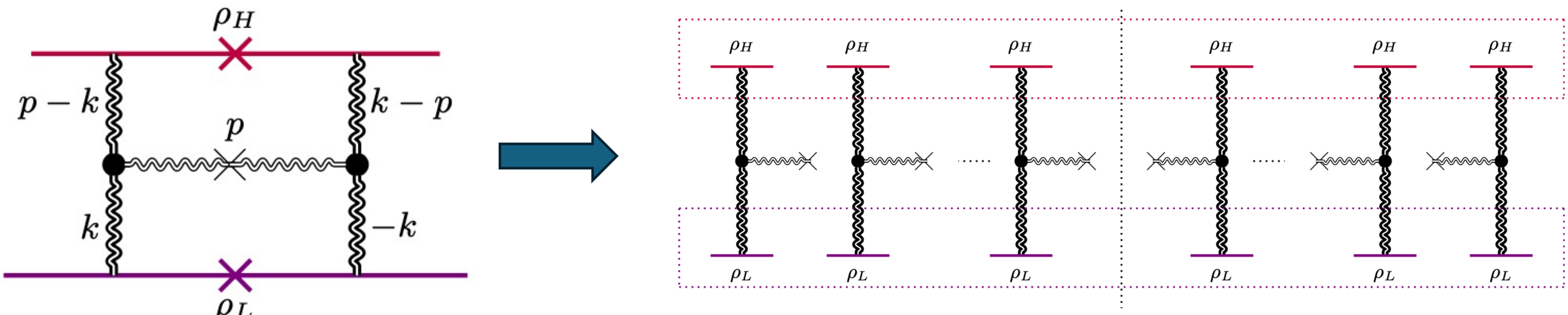
$$\alpha\left(-2k_ik_j + \boldsymbol{k}^2\,\delta_{ij}\right) + \left(q_{1i}\,q'_{1j} + q'_{1i}\,q_{1j} - \delta_{ij}\,\boldsymbol{q}_1\!\cdot\!\boldsymbol{q}'_1\right) + \beta\left(k_i\,s_j + k_j\,s_i - \delta_{ij}\,\boldsymbol{k}\!\cdot\!\boldsymbol{s}\right)$$

$$\alpha = \frac{1}{2} + \frac{\boldsymbol{s}\!\cdot\!\boldsymbol{q}_2 - \boldsymbol{q}_1\!\cdot\!\boldsymbol{q}'_1}{\boldsymbol{k}^2} + \frac{4(\boldsymbol{q}_1\!\cdot\!\boldsymbol{q}_2)(\boldsymbol{q}'_1\!\cdot\!\boldsymbol{q}_2)}{\boldsymbol{k}^4} \qquad \beta = 1 + \frac{2\,\boldsymbol{s}\!\cdot\!\boldsymbol{q}_2}{\boldsymbol{k}^2} \qquad \boldsymbol{s} \equiv \boldsymbol{q}_1 + \boldsymbol{q}'_1 = \boldsymbol{k} - \boldsymbol{q}_2\,.$$

Multi-graviton Lipatov radiation a la CGC EFT in GR?

In QCD, multi-gluon radiation in shockwave scattering (to LLx accuracy) is given by

$$\left\langle \frac{d^n N_n}{d^3\boldsymbol{p}_1\cdots d^3\boldsymbol{p}_n}\right\rangle_{\rm LLog} = \int [D\rho_1]\,[D\rho_2]\; W_{Y_1}[\rho_1]\; W_{Y_2}[\rho_2]\; \left.\frac{dN}{d^3\boldsymbol{p}_1}\right|_{\rm LO} \cdots \left.\frac{dN}{d^3\boldsymbol{p}_n}\right|_{\rm LO}$$



Gelis, Lappi, RV, arXiv 0807.1306

Corresponding "n-particle" distribution is a negative binomial distribution (NBD) $P_{n;r} = \frac{\Gamma(n+r)}{\Gamma(r)\Gamma(n+1)}\frac{\bar{n}^n r^r}{(\bar{n}+r)^{n+r}}$

with $r = \zeta\frac{(N_c^2-1)S_\perp Q_S^2}{2\pi}$

Can a similar t-channel fractionation occur in GR in the strong field regime, as b→ R_S ?

Gelis,Lappi,McLerran, arXiv: 0905.3234

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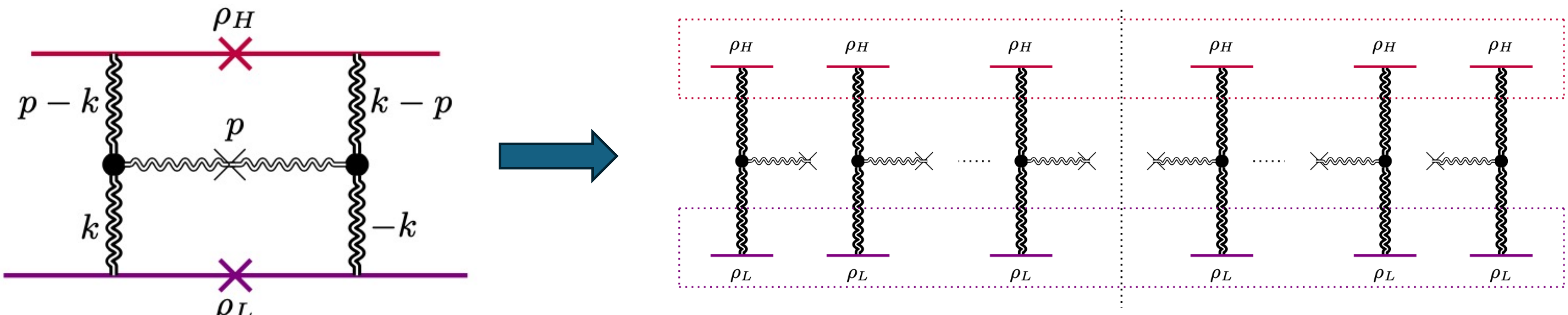

Gelis, Lappi, RV, arXiv 0807.1306

As we will argue, such radiation can be understood as a particular squeezed coherent state

This is interesting because it has been argued that such states can enhance quantum effects that are naturally only accessible at Planck scale resolution.

T. Guerrero, Class. Quant. Grav. 37 (2020)
Parikh, Wilczek, Zaharaide, PRL 127 (2021)

Multi-graviton radiation: generalized Susskind-Glogower squeezed state?

Stasto, Raj, RV: arXiv 2605.03038

If GR radiation is a NBD, this distribution corresponds to a squeezed state

$$|z;r\rangle = (1-|z|^2)^{r/2} e^{za^\dagger\sqrt{\hat{N}+r}}|0\rangle$$ where $|z^2| = p = \frac{\bar{n}}{\bar{n}+r}$ and $\hat{N} = a^\dagger a$

which is an eigenstate of the (generalized) Susskind-Glogower operator (gSG) $\hat{A} = a\sqrt{\frac{1}{\hat{N}+r-1}}$

$$\hat{A}|z;r\rangle = z\,|z;r\rangle$$

For NBD parameter r=1, usual Susskind-Glogower-Barnett-Pegg phase operator $\hat{A}_{r=1}|\phi\rangle = e^{i\phi}|\phi\rangle$ with $|\phi\rangle = \frac{1}{\sqrt{2\pi}}\sum_{n=0}^{\infty} e^{in\phi}|n\rangle$.

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For a pure single-mode squeezed state, $\Delta X^2\,\Delta P^2 = \frac{\hbar^2}{4}$

In the gSG case of interest to us, $(\Delta X)^2 = \left(\delta + \frac{1}{2}\right) e^{-2\xi}$ and $(\Delta P)^2 = \left(\delta + \frac{1}{2}\right) e^{+2\xi}$ with $\sqrt{(\Delta X)^2(\Delta P)^2} = \frac{1}{2} + \delta$

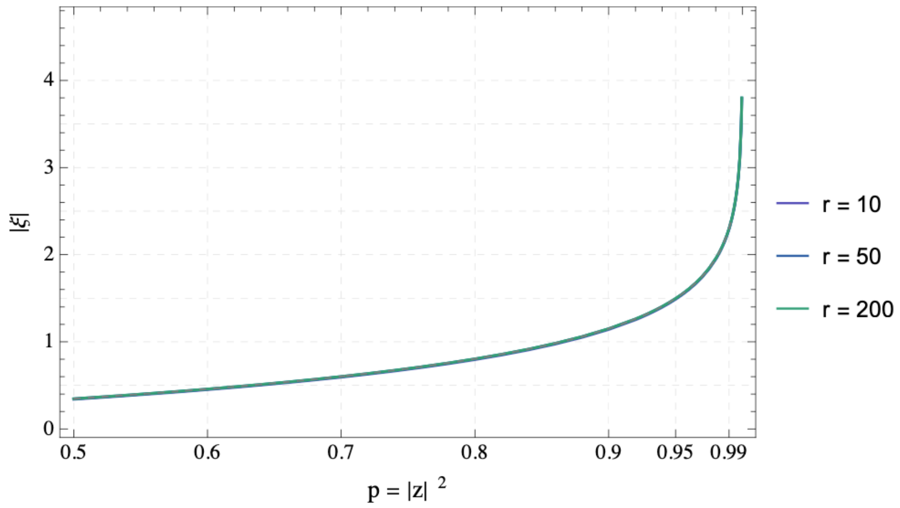
$$\xi = \frac{1}{4}\ln\frac{(\Delta P)^2}{(\Delta X)^2}$$

$$(\Delta X)^2 = \frac{1}{2} + \frac{rp}{1-p} + (1-p)^r p\,A_2(p) - 2(1-p)^{2r} p\,A_1(p)^2$$
$$(\Delta P)^2 = \frac{1}{2} + \frac{rp}{1-p} - (1-p)^r p\,A_2(p)$$

A_1 and A_2 are infinite sums in powers of p with binomial prefactors

Multi-graviton radiation: generalized Susskind-Glogower squeezed state?

Stasto, Raj, RV: arXiv 2605.03038



Wide parameter space for NBD parameter r > 1
where δ is small (close to minimum uncertainty)
but squeezing parameter $\xi \gg 1$

For r large, the squeezing parameter $\xi = Ln(\bar{n})$
For a gravitational wave measured by LIGO, $\bar{n} \approx 4.10^{36}$, so very large squeezings are possible

Specifically, quatum fluctuations on the Planck scale (10^{-35} m) can be enhanced to detectable levels at current and future gravitational wave observatories

Caveat: The statistics are "super-Poisson" which means that quantum effects can also be mimicked by classical sources

D. Carney, arXiv:2408.00094

Double Copy: gluon → gravitational radiation in shockwave collisions

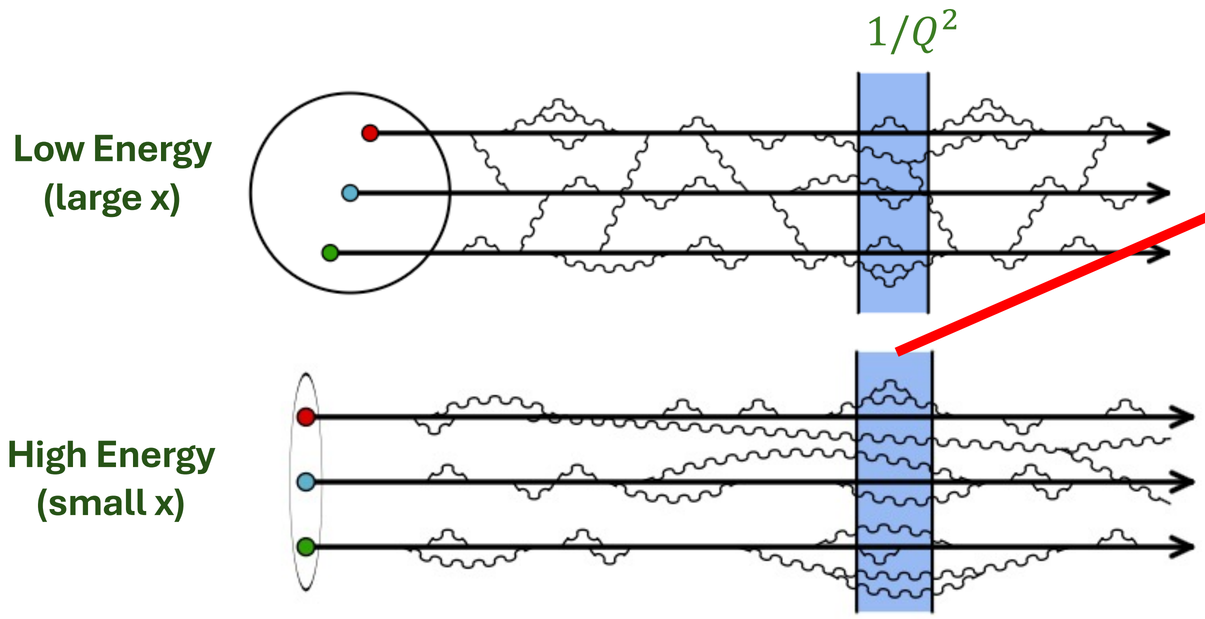
$$\begin{tikzcd}
\text{QCD at high occupancy} \arrow[d, "\text{Strong field semi-classical double copy}"'] & \text{perturbative QCD} \arrow[l, "\text{IR}"'] \arrow[d, "\text{BCJ double copy}"] \\
\text{Gravity at high occupancy} & \text{perturbative gravity} \arrow[l, "\text{UV}"']
\end{tikzcd}$$

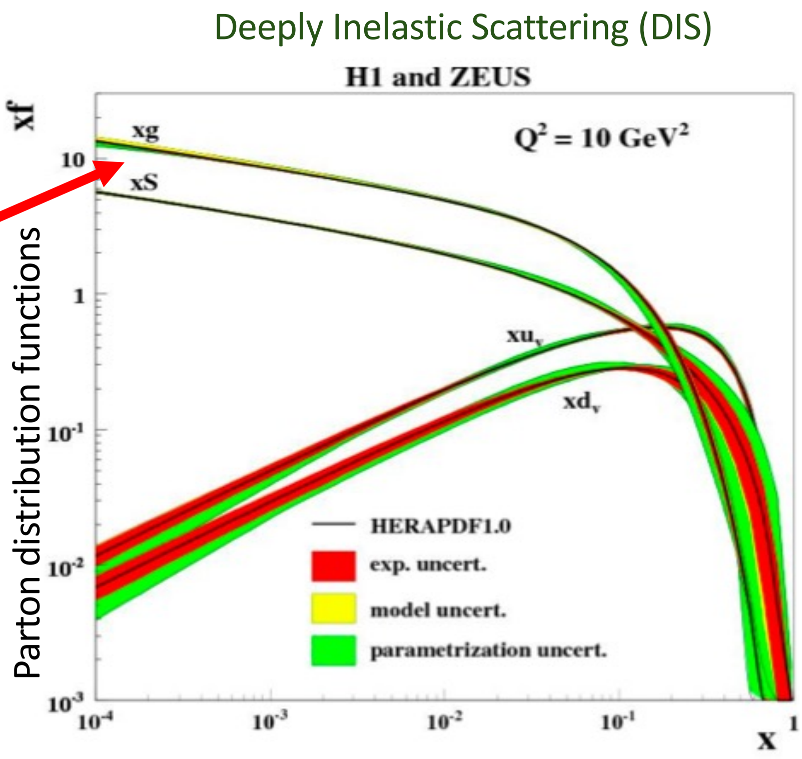
Bern, Carrasco, Johannson,
arXiv: 1004.0476

Monteiro,O’Connell,White, arXIv:1410.0239
Goldberger, Ridgeway, arXiv:1611.03493

The BCJ double copy has been exploited to perform highly precise computations of BH inspiral dynamics leading to an explosion of interest in the topic

Spacetime picture of wee partons in a hadron/nucleus





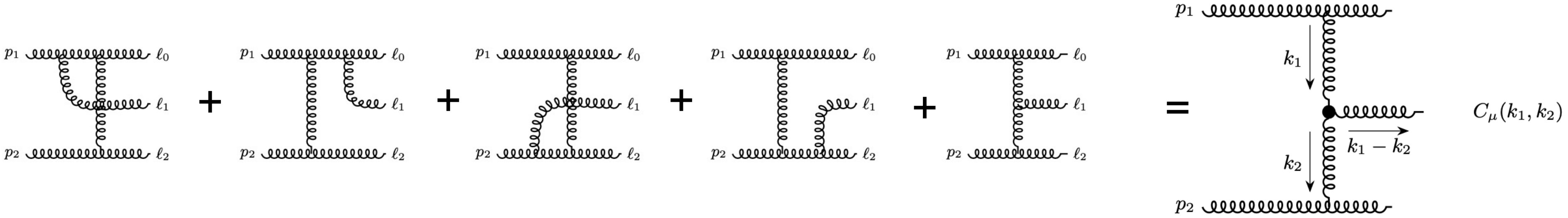
This is further enhanced by radiation with increasing boost

In the probe rest frame, “wee parton” (Bjorken/Feynman x << 1) fluctuations in the hadron live longer

Suppression in coupling compensated by large phase space for soft glue: $\alpha_S\, Ln\left(\frac{1}{x}\right) \sim 1$

BFKL: Building blocks

Lipatov effective vertex:



$$C_\mu(\boldsymbol{q}_1, \boldsymbol{q}_2) \simeq -\boldsymbol{q}_{1\mu} + \boldsymbol{q}_{2\mu} + p_{1\mu}\left(\frac{p_2 \cdot k}{p_1 \cdot p_2} - \frac{\boldsymbol{q}_1^2}{p_1 \cdot k}\right) - p_{2\mu}\left(\frac{p_1 \cdot k}{p_1 \cdot p_2} - \frac{\boldsymbol{q}_2^2}{p_2 \cdot k}\right)$$

Gauge covariant, satisfies $k_\mu \, C^\mu$ =0

BFKL: Building blocks

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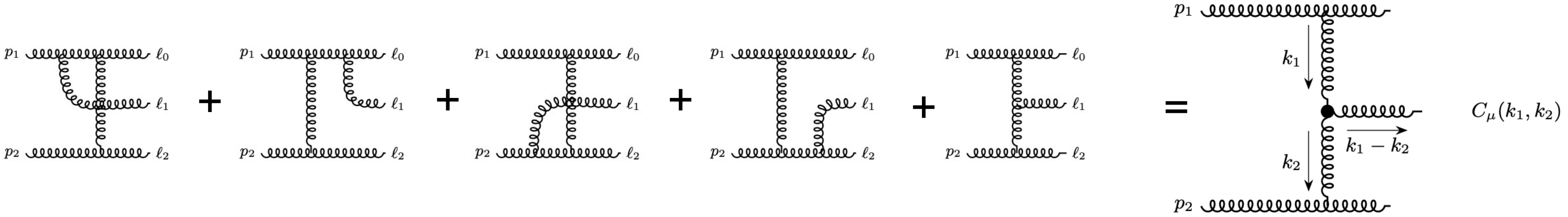

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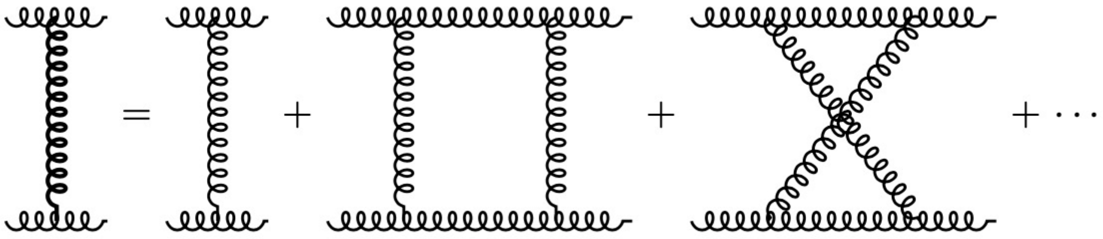
Gauge covariant, satisfies $k_\mu\, C^\mu$ =0

Reggeized gluon:

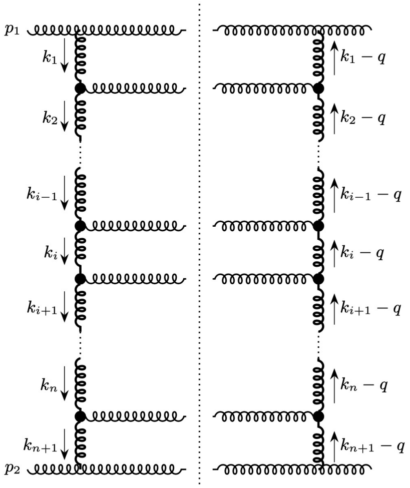


$$\frac{1}{t_i} \to \frac{1}{t_i} e^{\alpha(t_i)(y_{i-1} - y_i)}$$

$$\alpha(t) = \alpha_s N_c\, t \int \frac{d\boldsymbol{k}}{(2\pi)^2} \frac{1}{\boldsymbol{k}^2(\boldsymbol{q} - \boldsymbol{k})^2}\ , \qquad t = -\boldsymbol{q}^2$$

2 → N + 2 amplitude in the Regge limit: the BFKL equation

BFKL Pomeron: compound color singlet state of two reggeized gluons



The imaginary part of this 2 → N + 2 amplitude simplifies greatly in Mellin space

$$\mathcal{M}_\ell(\boldsymbol{q}^2) \equiv \int_1^\infty d\left(\frac{s}{\boldsymbol{k}^2}\right) \frac{\mathrm{Im}\, \mathcal{A}^{\mu\mu'\nu\nu'}_{2\to 2}(s,t)}{\mathcal{A}_0^{\mu\mu'\nu\nu'}(s,t)} \left(\frac{s}{\boldsymbol{k}^2}\right)^{-\ell-1}$$

with $$\mathcal{M}_\ell(\boldsymbol{q}^2) = 2\pi \boldsymbol{q}^2 \alpha_s N_c^2 (N_c^2 - 1) \int \frac{d^2\boldsymbol{k}}{(2\pi)^2} \frac{1}{\boldsymbol{k}^2 (\boldsymbol{q}-\boldsymbol{k})^2} f_\ell(\boldsymbol{k},\boldsymbol{q})$$

where $f_l(k,q)$ satisfies the BFKL integral equation

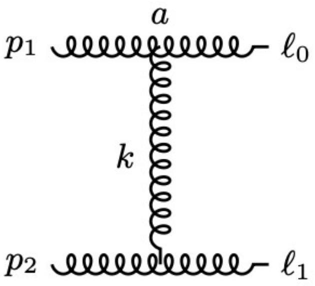
$$(\ell - \alpha(\boldsymbol{k}^2) - \alpha((\boldsymbol{q}-\boldsymbol{k})^2)) f_\ell(\boldsymbol{k},\boldsymbol{q}) = 1 - 2\alpha_s N_c \int \frac{d^2\boldsymbol{k}'}{(2\pi)^2} \frac{f_\ell(\boldsymbol{k}',\boldsymbol{q})}{\boldsymbol{k}'^2 (\boldsymbol{q}-\boldsymbol{k}')^2} \left(\boldsymbol{q}^2 - \frac{\boldsymbol{k}^2 (\boldsymbol{q}-\boldsymbol{k}')^2 + \boldsymbol{k}'^2 (\boldsymbol{q}-\boldsymbol{k})^2}{(\boldsymbol{k}-\boldsymbol{k}')^2} \right)$$

(Can be reexpressed as an RG evolution eqution for parton dists. in rapidity governed by a BFKL Hamiltonian)

Some features of the BFKL equation and its solution

Key to the construction of the BFKL ladder are multi-Regge asymptotics and dispersive techniques

Building block is the 2 → 2 Born amplitude and three gluon vertex



$$\mathcal{A}^{\alpha\alpha'\beta\beta'}_{2\to2,p_1+p_2\to\ell_0+\ell_1} = \Gamma^{\alpha\alpha' c}_{p_1\ell_0}\frac{s}{t}\Gamma^{\beta\beta' c}_{p_2\ell_1}$$

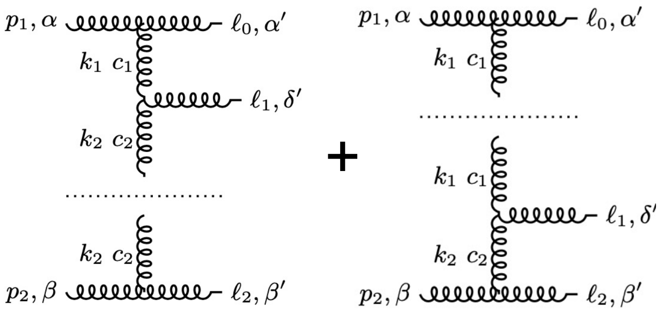
3g vertex

Construct 2→ 3 amplitude from simultaneous residues of cut diagrams

$$P_{k_2^2}\mathcal{A}^{\alpha\alpha'\beta\beta'\delta'}_{2\to2+1} = \mathcal{A}^{\alpha\alpha' c_2\delta'}_{p_1+(-k_2)\to\ell_0+\ell_1}\frac{s}{k_2^2}\Gamma^{\beta\beta' c_2}_{p_2\ell_2}$$

$$\mathcal{A}^{\alpha\alpha'\beta\beta'\delta'}_{2\to2+1} = ig\frac{s}{k_1^2 k_2^2}\Gamma^{\alpha\alpha' c_1}_{p_1\ell_0}\Gamma^{\beta\beta' c_2}_{p_2\ell_2} f^{c_1\delta' c_2} C_\nu(k_1,k_2)\epsilon^\nu(\ell_1)$$

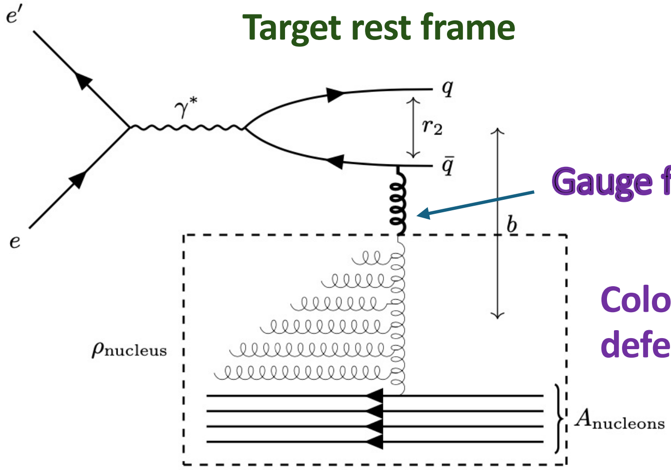
Lipatov vertex



Cut diagrams contributing to pole reconstruction of 2→ 3 ampltude

Iterate to all orders to construct the 2→ n BFKL ladder
– the same procedure applies in gravity

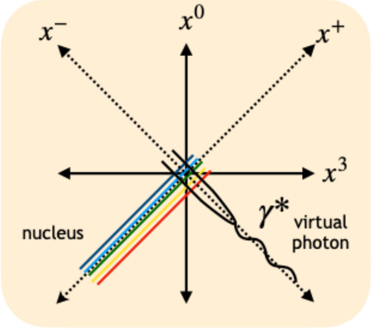
The CGC EFT

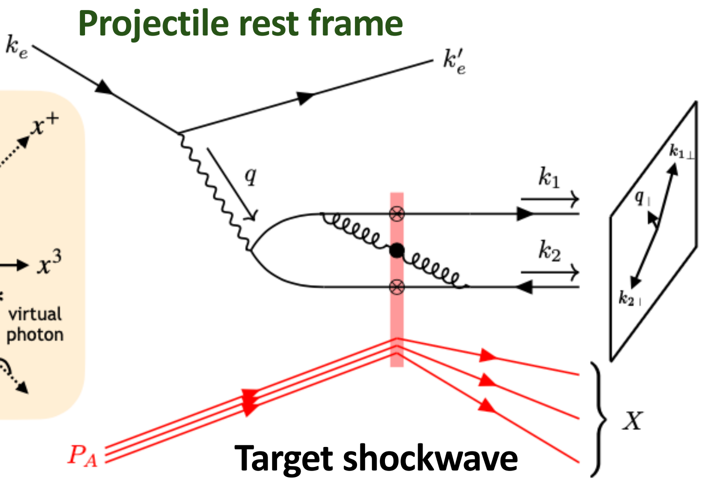


Target rest frame

Gauge field in bulk

Color sources/ defects





Projectile rest frame

Target shockwave

$$\langle \mathcal{O} \rangle = \int [D\rho] W_{\Lambda^+}[\rho] \int_{\Lambda^+} [DA] \mathcal{O}[A] \exp\left[-\frac{i}{4} \int d^4x\, F^a_{\mu\nu} F^{\mu\nu,a} - \frac{1}{N_c} \int d^2x_\perp \mathrm{Tr}\left(\rho(x_\perp) \ln K(x_\perp)\right) \right]$$

$$K(x_\perp) = P_{x^-} \exp\left(i \int dx^+ A^-_a(x_\perp, 0, x^+)\, T_a \right)$$

Invariance of $\langle \mathcal{O} \rangle$ with Λ^+ leads to RG in rapidity (Ln(Λ^+)) – B-JIMWLK hierarchy

$$\frac{\partial}{\partial Y} \langle \mathcal{O}[\rho] \rangle_Y = \frac{1}{2} \langle \int_{x,y} \frac{\partial}{\partial \rho^a(x)} \chi^{ab}_{x,y} \frac{\partial}{\partial \rho^b(y)} \mathcal{O}[\rho] \rangle_Y$$

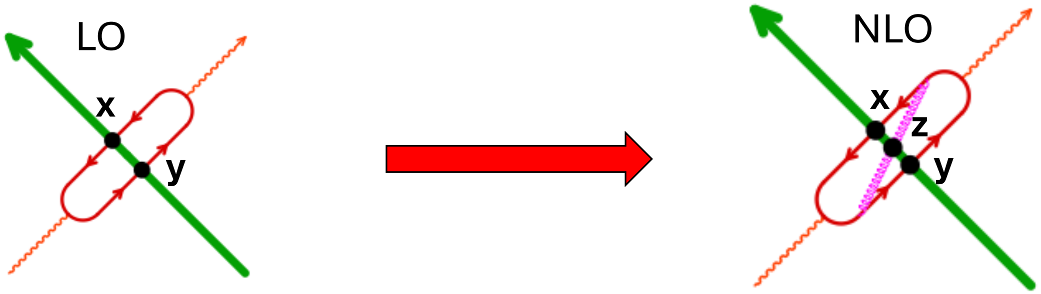
Balitsky, hep-ph/9509348
Jalilian-Marian,Kovner, Leonidov, Weigert, hep-ph/9706377
Iancu, Leonidov, McLerran, hep-ph/0011241

$\Downarrow$

"diffusion coefficient": retarded Green function in shockwave background

DIS: dipole evolution in gluon shockwave background

Example: 2-point “dipole” correlator:



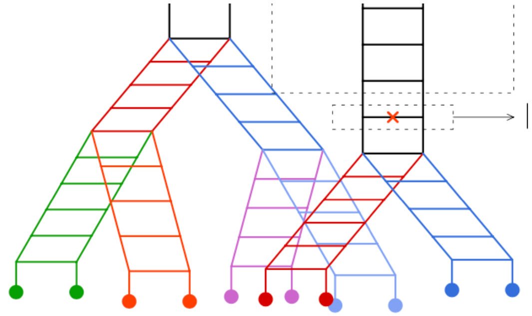
Cross-section free rapidity divergence → RG equation for Wilson line correlators sourced by shockwave

$$\frac{\partial}{\partial Y}\langle \mathrm{Tr}(V_x V_y^\dagger)\rangle_Y = -\frac{\alpha_S N_c}{2\pi^2}\int_{z_\perp} \underbrace{\frac{(x_\perp - y_\perp)^2}{(x_\perp - z_\perp)^2(z_\perp - y_\perp)^2}}_{\text{BFKL kernel}} \langle \mathrm{Tr}(V_x V_y^\dagger) - \frac{1}{N_c}\mathrm{Tr}(V_x V_z^\dagger)\ \mathrm{Tr}(V_z V_y^\dagger)\rangle_Y$$

Y = Ln(1/x)

Closed form expression (A >>1, Nc→ ∞):
non-linear Balitsky-Kovchegov (BK) eqn.

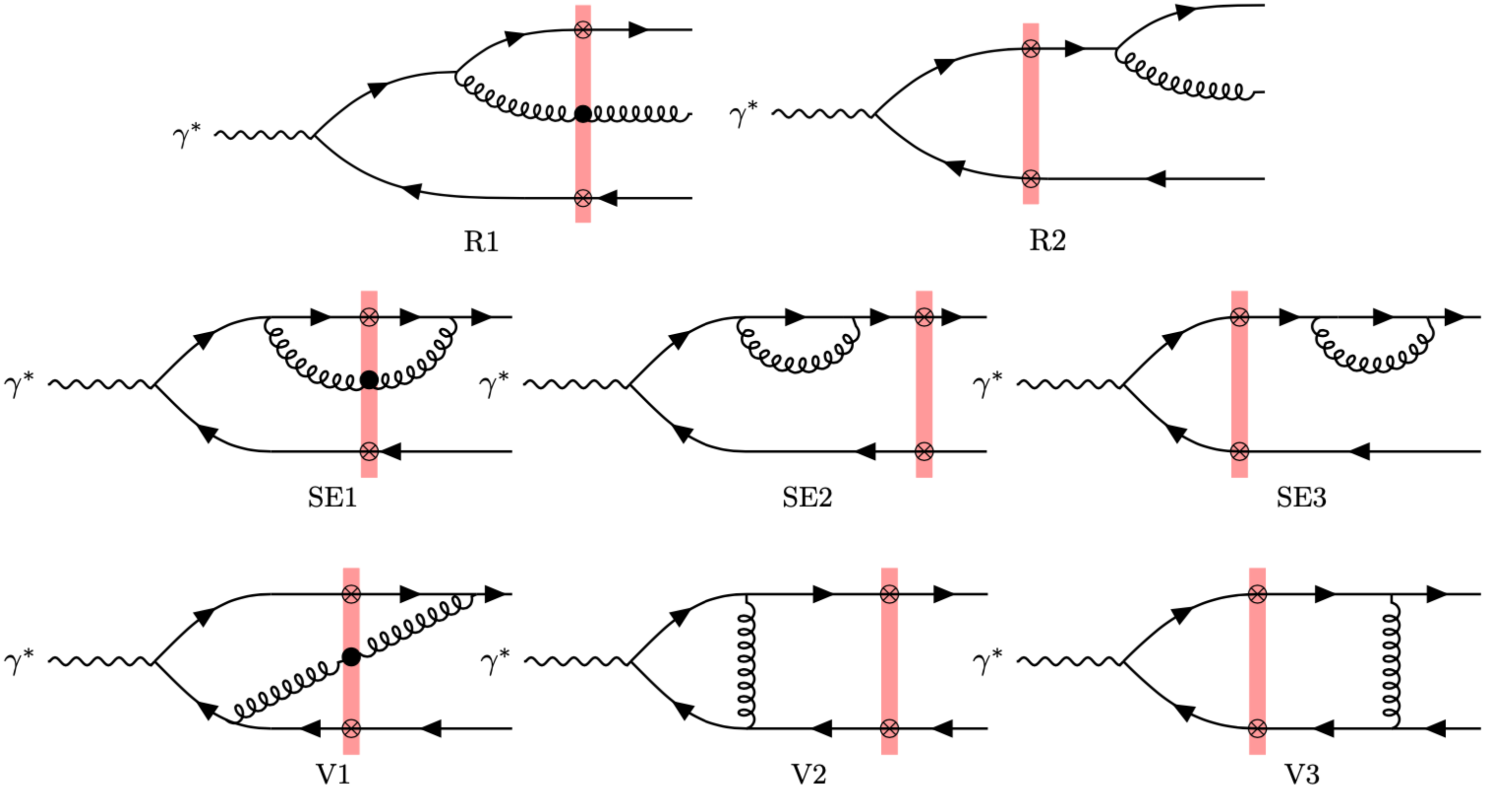
Evolved shockwave scattering off dipole probe contains all-twist multi-pomeron ”fan” diagrams



BFKL obtained as the leading twist result…

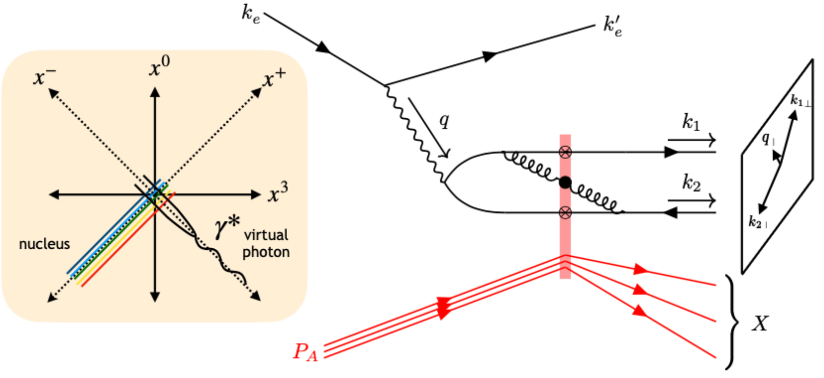
NLO impact factor for inclusive di-jet production in e+A

Caucal,Salazar,RV, JHEP 11 (2021) 222



Quark and gluon shockwave propagators ∝ F.T. of Wilson lines of the color sources ρ

Gluon Weizsäcker-Williams distribution: complete NLO results



Back-to-back di-jets in DIS

Factorization of small-x TMDs to NLO accuracy

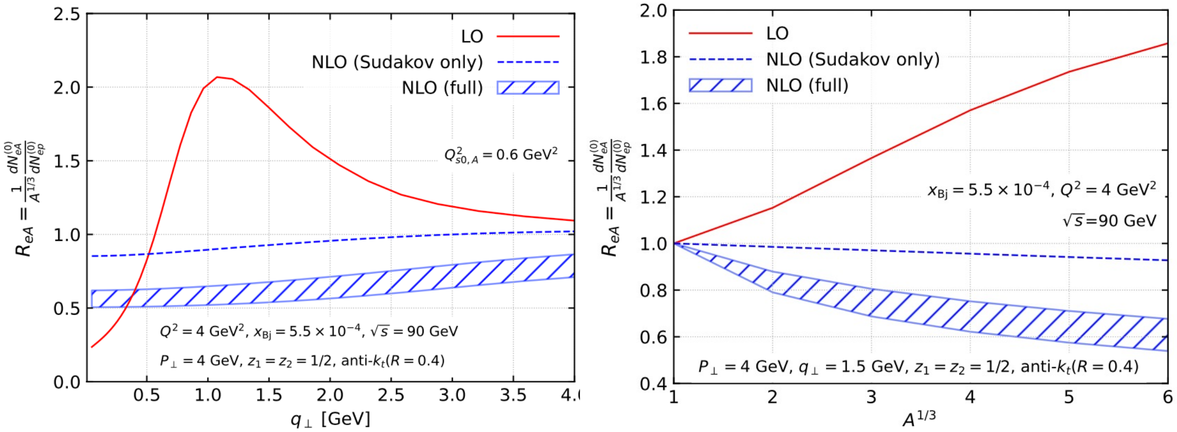
$$\begin{aligned}
\mathrm{d}\sigma^{(0),\lambda=\mathrm{T}} &= \mathcal{H}_{\mathrm{LO}}^{0,\lambda=\mathrm{T}} \int \frac{\mathrm{d}^2\boldsymbol{B}_\perp}{(2\pi)^2} \int \frac{\mathrm{d}^2\boldsymbol{r}_{bb'}}{(2\pi)^2} e^{-i\boldsymbol{q}_\perp\cdot\boldsymbol{r}_{bb'}} \boxed{\hat{G}^0_{\eta_c}(\boldsymbol{r}_{bb'},\mu_0)\mathcal{S}(\boldsymbol{P}_\perp^2,\mu_0^2)} \\
&\times \left\{1 + \frac{\alpha_s(\mu_R)N_c}{2\pi} f_1^{\lambda=\mathrm{T}}(\chi,z_1,R) + \frac{\alpha_s(\mu_R)}{2\pi N_c} f_2^{\lambda=\mathrm{T}}(\chi,z_1,R) + \alpha_s(\mu_R)\beta_0 \ln\left(\frac{\mu_R^2}{\boldsymbol{P}_\perp^2}\right)\right\} \\
&+ \mathcal{H}_{\mathrm{LO}}^{0,\lambda=\mathrm{T}} \int \frac{\mathrm{d}^2\boldsymbol{B}_\perp}{(2\pi)^2} \int \frac{\mathrm{d}^2\boldsymbol{r}_{bb'}}{(2\pi)^2} e^{-i\boldsymbol{q}_\perp\cdot\boldsymbol{r}_{bb'}} \boxed{\hat{h}^0_{\eta_c}(\boldsymbol{r}_{bb'},\mu_0)\mathcal{S}(\boldsymbol{P}_\perp^2,\mu_0^2)} \\
&\times \frac{-2\chi^2}{1+\chi^4}\left\{\frac{\alpha_s(\mu_R)N_c}{2\pi}\left[1+\ln(R^2)\right] + \frac{\alpha_s(\mu_R)}{2\pi N_c}\left[-\ln(z_1 z_2 R^2)\right]\right\} + \mathcal{O}\left(\frac{q_\perp}{P_\perp}, \frac{Q_s}{P_\perp}, \alpha_s R^2, \alpha_s^2\right)
\end{aligned}$$

$\hat{G}^0$ and $\hat{h}^0$ respectively are unpolarized and linearly polarized WW distributions,

$\mathcal{S}$ the Sudakov soft factor resumming double+single logs in P_T/q_T

f_1 and f_2 are finite pure $O(\alpha_S)$ contributions

Caucal,Salazar,Schenke,Stebel,RV (PRL 2024)

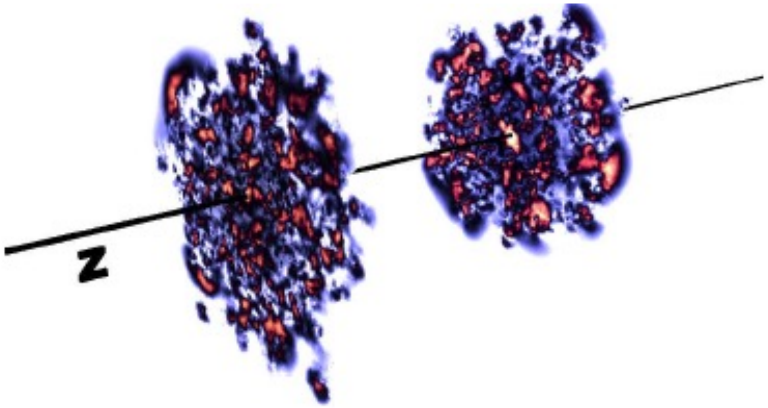


Global analyses to extract "universal" TMDs from p+A collisions at the LHC and e+A collisions from the EIC

The lumpy Glasma at LO: Yang-Mills equations

Collisions of lumpy gluon ``shock" waves

Non-equil. computations on lattice:
Krasnitz, RV (1998)
Krasnitz, Nara, RV (2001)
Lappi (2003)



Leading order solution: Solution of QCD Yang-Mills eqns

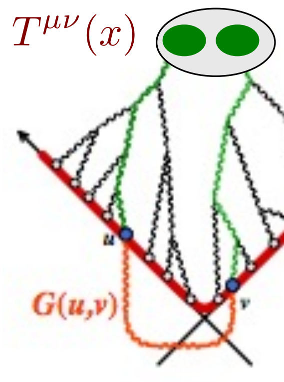
$$D_\mu F^{\mu\nu,a} = \delta^{\nu +}\rho^a_A(x_\perp)\delta(x^-) + \delta^{\nu -}\rho^a_B(x_\perp)\delta(x^+)$$

$$x^\pm = t \pm z$$

$$F^{\mu\nu,a} = \partial_\mu A^{\nu,a} - \partial_\nu A^{\nu,a} + g f^{abc} A^{\mu,b} A^{\nu,c}$$

Spectrum of initial fluctuations in the little bang

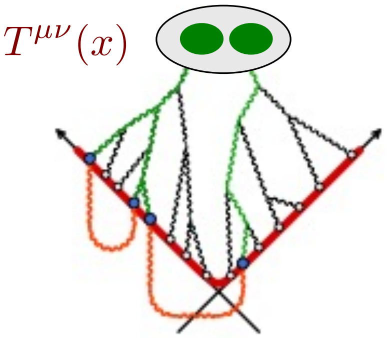
Dusling,Gelis,RV (2011)
Gelis,Epelbaum (2013)



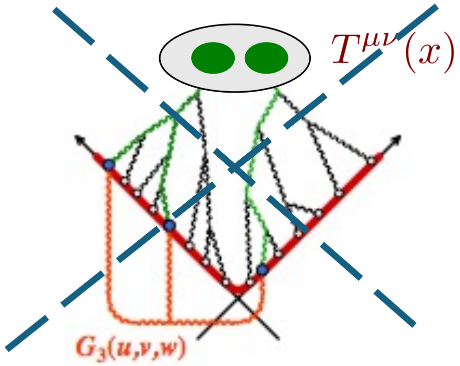
$$\mathcal{G}^{\mu\nu} = \int \frac{d^3k}{(2\pi)^3 2E_k} a^{\mu}_{-k}(\vec{u})\; a^{\nu}_{+k}(\vec{v})$$

$$\left[\frac{\delta^2 S_{\rm YM}}{\delta A^\mu A^\nu}\right]_{\rm A=A_{cl}} a^{\nu}_{\pm k} = 0 \qquad \lim_{x^0\to-\infty} a^{\mu}_{\pm k,\lambda a}(x) = \epsilon^\mu(k)\; T^a\; e^{\pm ik\cdot x}$$

Higher orders:



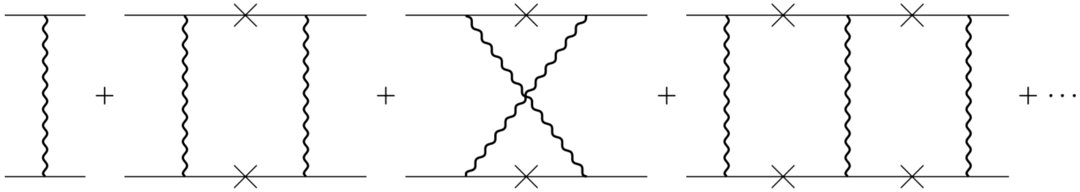
$$(g\exp(\sqrt{Q_S\tau}))^4 \sim O(1)$$



$$g(g\exp(\sqrt{Q_S\tau}))^3 \sim O(g)$$

From QCD to gravity in Regge asymptotics: reggeization

In Einstein gravity, at large impact parameters, the dominant contribution is eikonal scattering

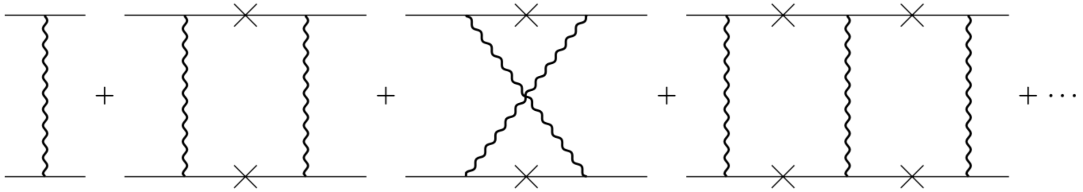


$$i\mathcal{M}_{\text{Eik}} = 2s \int d^2\boldsymbol{b}\; e^{-i\mathbf{q}\cdot\mathbf{b}} \left(e^{i\chi(\boldsymbol{b},s)} - 1\right)$$

with $\chi(\boldsymbol{b}, s) = \frac{\kappa^2 s}{2} \int \frac{d^2\boldsymbol{k}}{(2\pi)^2} \frac{1}{\boldsymbol{k}^2} e^{i\boldsymbol{b}\cdot\boldsymbol{k}}$

From QCD to gravity in Regge asymptotics: reggeization

In Einstein gravity, at large impact parameters, dominant contribution is eikonal scattering

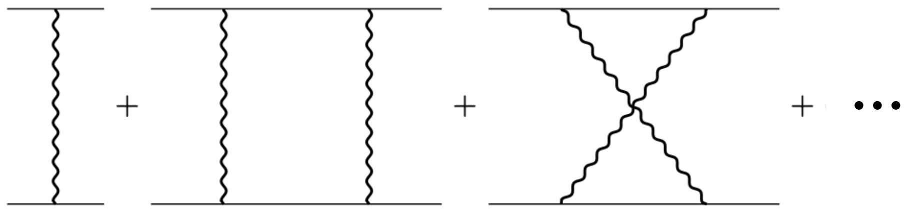


$$i\mathcal{M}_{\text{Eik}} = 2s \int d^2\boldsymbol{b}\; e^{-i\mathbf{q}\cdot\mathbf{b}} \left(e^{i\chi(\boldsymbol{b},s)} - 1\right) \qquad \text{with} \quad \chi(\boldsymbol{b},s) = \frac{\kappa^2 s}{2} \int \frac{d^2\boldsymbol{k}}{(2\pi)^2} \frac{1}{\boldsymbol{k}^2} e^{i\boldsymbol{b}\cdot\boldsymbol{k}}$$

Genuine loop contributions formally suppressed by R_S^2/b^2

$$\mathcal{M}^{(1)} \sim \frac{\kappa^2}{8\pi^2}\left(-i\pi s \log\left(\frac{-t}{\Lambda^2}\right) + t \log\left(\frac{s}{-t}\right) \log\left(\frac{-t}{\Lambda^2}\right)\right)$$

Eikonal Loop



Bartels,Lipatov,Sabio-Vera,arXiv:1208.3423
Melville,Naculich,Schnitzer,White, arXiv:1306.6019

Graviton Regge trajectory $\alpha(t) = -\kappa^2 t \int \frac{d^2\boldsymbol{k}}{(2\pi)^2} \frac{1}{\boldsymbol{k}^2\,(\boldsymbol{q}-\boldsymbol{k})^2}\left[(\boldsymbol{k}\cdot(\boldsymbol{q}-\boldsymbol{k}))^2 \left(\frac{1}{\boldsymbol{k}^2} + \frac{1}{(\boldsymbol{q}-\boldsymbol{k})^2}\right) - \boldsymbol{q}^2\right], \qquad \boldsymbol{q}^2 = -t$

The IR virtual divergence cancels in the inclusive cross-section

Lipatov, PLB 116B (1982); JETP 82 (1982)

GR Born amplitude

$$\mathcal{M}^{\text{Born}}_{\mu\nu\mu'\nu'\alpha\beta\alpha'\beta'}(s,t) = \left(i\kappa P_{\mu\nu,\mu'\nu'} p_1^\rho p_1^\sigma\right) \frac{P^{\rho\sigma\rho'\sigma'}}{q^2} \left(i\kappa P_{\alpha\beta\alpha'\beta'} p_2^{\rho'} p_2^{\sigma'}\right) \Longrightarrow \; = \frac{1}{4}\frac{\kappa^2 s^2}{t} P_{\mu\nu\mu'\nu'} P_{\alpha\beta\alpha'\beta'}$$

$$\Downarrow$$

Propagator in De Donder gauge

As in the QCD case, contracting this with polarization tensors gives

$$\boxed{\mathcal{M}^{\text{Born}}_{p_1+p_2\to\ell_0+\ell_1} = V_{p_1,\ell_0} \frac{\kappa^2 s^2}{t} V_{p_2,\ell_1}}$$

where $$V_{p_1,\ell_0} = 4\,\epsilon_{\mu\nu}(p_1)\epsilon_{\mu'\nu'}(\ell_0)\;\frac{1}{2}\left[\Gamma^{\mu\mu'}_{p_1,\ell_0}\Gamma^{\nu\nu'}_{p_1,\ell_0} + \Gamma^{\mu\nu'}_{p_1,\ell_0}\Gamma^{\nu\mu'}_{p_1,\ell_0} - \left(\delta^{\mu\nu} - \frac{p_1^\mu p_2^\nu + p_1^\nu p_2^\mu}{p_1\cdot p_2}\right)\left(\delta^{\mu'\nu'} - \frac{\ell_0^{\mu'} p_2^{\nu'} + \ell_0^{\nu'} p_2^{\mu'}}{\ell_0\cdot p_2}\right)\right]$$

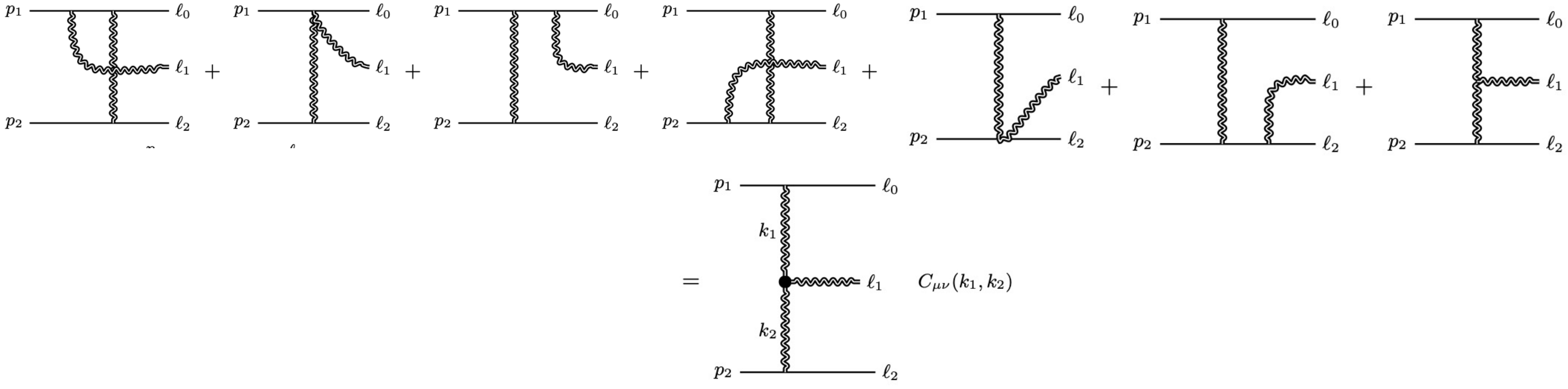
$$\Uparrow$$

$$\left(-g_{\mu\mu'} + \frac{p_{2\mu}p_{1\mu'} + p_{2\mu'}\ell_{0\mu}}{p_2\cdot p_1} + (p_1-\ell_0)^2 \frac{p_{2\mu}p_{2\mu'}}{2(p_2\cdot p_1)^2}\right)$$

A double copy of the vertex that that appeared in the QCD Born amplitude

The Born amplitude in this form is a projection of the 2→ 2 graviton scattering amplitude on the physical 2-D subspace spanned by the gravitational polarization vectors

GR Lipatov vertex



The BFKL equation in Einstein gravity

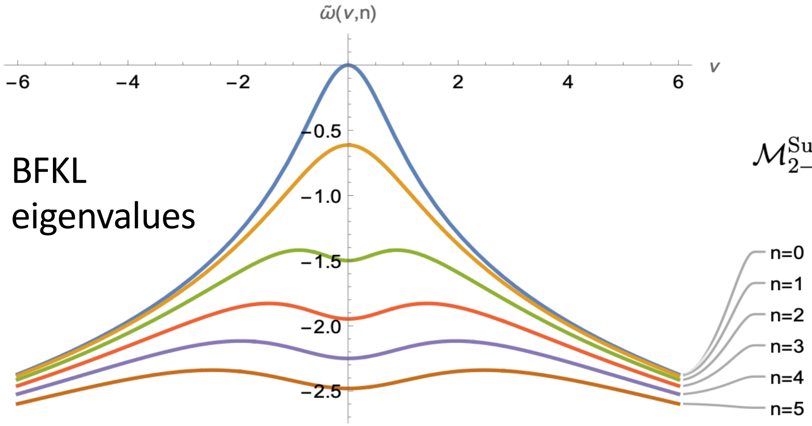
I. Rothstein, M. Saavedra, arXiv:2412.04428
H. Raj, RV, arXiv:2507.21252

The BFKL construction follows identically as in QCD…

Integral equation derived by Lipatov for the Mellin amplitude: $\mathcal{M}_\ell(t) = \frac{t}{16}\int \frac{d^2\boldsymbol{k}}{(2\pi)^2}\frac{1}{\boldsymbol{k}^2(\boldsymbol{q}-\boldsymbol{k})^2} f_\ell(\boldsymbol{k},\boldsymbol{q})$

$$(\ell - \alpha(\boldsymbol{k}^2) - \alpha((\boldsymbol{q}-\boldsymbol{k})^2)) f_\ell(\boldsymbol{k},\boldsymbol{q}) = 1 + \frac{\kappa^2}{4\pi}\int \frac{d^2\boldsymbol{k}'}{(2\pi)^2}\frac{f_\ell(\boldsymbol{k}',\boldsymbol{q})}{\boldsymbol{k}'^2(\boldsymbol{q}-\boldsymbol{k}')^2}\mathcal{K}_G(\boldsymbol{k},\boldsymbol{k}')$$

$$C^{\mu_i\nu_i}(k_i, k_{i+1})C_{\mu_i\nu_i}(q-k_i, q-k_{i+1})$$

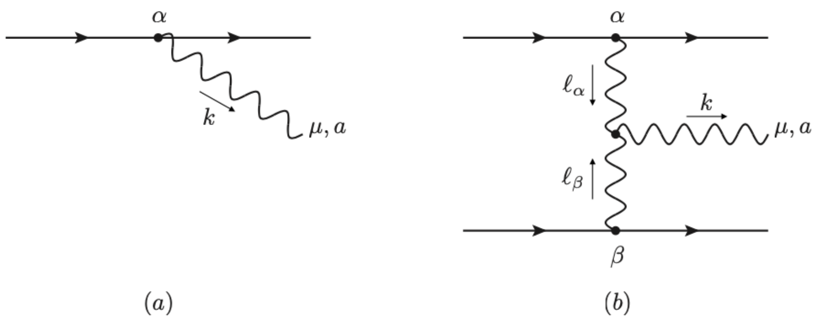


$$\mathcal{M}_{2\to2}^{\text{Sudakov}+L^2} \sim \mathcal{M}^{\text{Born}}\left(-\frac{s}{t}\right)^{\kappa^2 t\log(-t/\Lambda_{\text{IR}})/8\pi^2}\frac{1}{3}\left[1+\left(-\frac{s}{t}\right)^{\sqrt{-3\kappa^2 t/8\pi^2}}+\left(-\frac{s}{t}\right)^{-\sqrt{-3\kappa^2 t/8\pi^2}}\right]$$

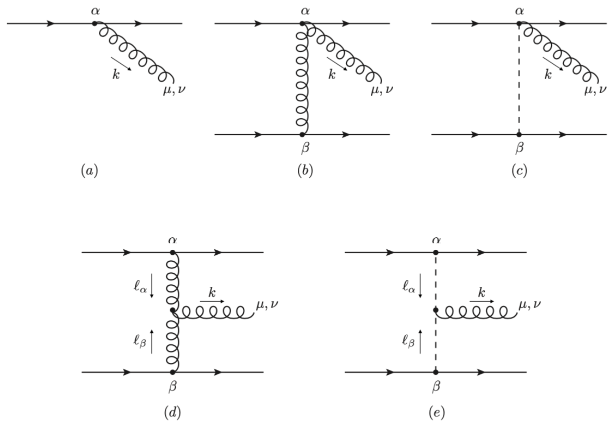
Growth with energy is slower than the Born amplitude ∝ s^2

- *Very interestingly, the soft limit of the Lipatov vertex gives the ultrarelativistic limit of Weinberg's radiative amplitude for soft graviton emission*

Classical color-kinematic duality



From Goldberger, Ridgwa
arXiv:1611.03493



A color-kinematic duality does exist but it requires that one include sub-eikonal corrections to the Lipatov vertex

For this, require a detailed theory of sources: Yang-Mills+Wong equations for classical color sources c^a :

$$D_\mu F_a^{\mu\nu} = g J_a^\nu \qquad J_a^\mu(x) = \sum_{\alpha=1,2} \int d\tau c_\alpha^a(\tau) v_\alpha^\mu(\tau) \delta^d\left(x - x_\alpha(\tau)\right)$$

$$\frac{dc^a}{d\tau} = g f^{abc} v^\mu A_\mu^b(x(\tau)) c^c(\tau) \qquad \frac{dp^\mu}{d\tau} = g c^a F^\mu_{a\nu} v^\nu$$

Classical color-kinematic duality

Ultrarelativistic limit of Goldberger-Ridgway solution

$$A^{\mu,a}(k) = -\frac{g^3}{k^2}\int \frac{d^2\boldsymbol{q}_2}{(2\pi)^2}\frac{e^{-i\boldsymbol{q}_1\cdot\boldsymbol{b}_1}}{\boldsymbol{q}_1^2}\frac{e^{-i\boldsymbol{q}_2\cdot\boldsymbol{b}_2}}{\boldsymbol{q}_2^2}\Bigg[if^{abc}c_1^b c_2^c\left(-\boldsymbol{q}_1^\mu + \boldsymbol{q}_2^\mu + p_1^\mu\left(\frac{p_2\cdot k}{p_1\cdot p_2} - \frac{\boldsymbol{q}_1^2}{p_1\cdot k}\right) - p_2^\mu\left(\frac{p_1\cdot k}{p_1\cdot p_2} - \frac{\boldsymbol{q}_2^2}{p_2\cdot k}\right)\right)$$

QCD Lipatov vertex

$$+ c_1\cdot c_2\left\{\frac{q_1^2 c_1^a}{p_1\cdot k}\left(-q_2^\mu + \frac{k\cdot q_2}{k\cdot p_1}p_1^\mu + \frac{k\cdot p_1}{p_1\cdot p_2}p_2^\mu - \frac{k\cdot p_2}{p_1\cdot p_2}p_1^\mu\right) + \frac{q_2^2 c_2^a}{p_2\cdot k}\left(-q_1^\mu + \frac{k\cdot q_1}{k\cdot p_2}p_2^\mu + \frac{k\cdot p_2}{p_1\cdot p_2}p_1^\mu - \frac{k\cdot p_1}{p_1\cdot p_2}p_2^\mu\right)\right\}\Bigg]$$

sub-eikonal correction

↑ $1/p_1^+$

↑ $1/p_2^-$

Sub-Eikonal contributions are not universal – for instance, they depend on the spin of the particles

Classical color-kinematic replacement rule:

$$c_\alpha^a \to p_\alpha^\mu\ ,$$

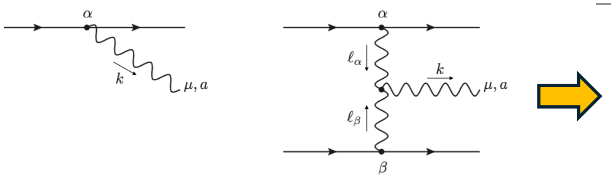
$$if^{a_1a_2a_3} \to \Gamma^{\nu_1\nu_2\nu_3}(q_1,q_2,q_3) = -\frac{1}{2}\left(\eta^{\nu_1\nu_3}(q_1-q_3)^{\nu_2} + \eta^{\nu_1\nu_2}(q_2-q_1)^{\nu_3} + \eta^{\nu_2\nu_3}(q_3-q_2)^{\nu_1}\right)$$

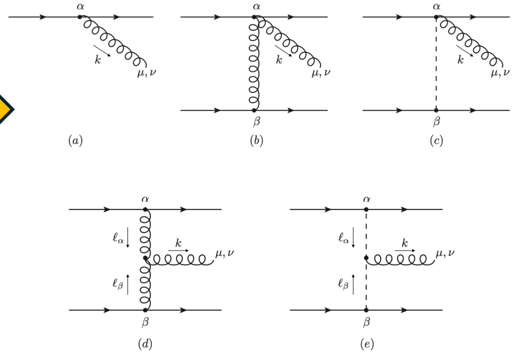
$$g \to \kappa\ ,$$

Gluon 3-pt vertex with f^{abc} stripped off

Lipatov vertex from classical color-kinematic duality

Consider pert. solutions of Yang-Mills radiation field in collision of colored charges c_α^a (Wong equations)





Goldberger, Ridgway, arXiv:1611.03493

Taking ultrarelativistic limit
(keeping sub-eikonal terms, beyond leading QCD Lipatov vertex term)
and making replacements

$$c_\alpha^a \to p_\alpha^\mu$$

$$if^{a_1 a_2 a_3} \to \Gamma^{\nu_1\nu_2\nu_3}(q_1, q_2, q_3) = -\frac{1}{2}\left(\eta^{\nu_1\nu_3}(q_1 - q_3)^{\nu_2} + \eta^{\nu_1\nu_2}(q_2 - q_1)^{\nu_3} + \eta^{\nu_2\nu_3}(q_3 - q_2)^{\nu_1}\right)$$

$$g \to \kappa$$

recovers our previous result for the radiation field in terms of the gravitational Lipatov vertex

Raj,RV, arXiv:2312.03507

We can also show that the soft limit of the gravitational Lipatov vertex
gives the ultrarelativistic limit of the Weinberg soft graviton emission vertex

This is consistent with the observation that the soft limit of the classical double copy recovers the Weinberg emission vertex

P.V. Athira,A. Manu, arXiv:1907.10021

Classical color-kinematic duality

Ultrarelativistic limit of Goldberger-Ridgway solution

$$A^{\mu,a}(k) = -\frac{g^3}{k^2}\int \frac{d^2\boldsymbol{q}_2}{(2\pi)^2}\frac{e^{-i\boldsymbol{q}_1\cdot\boldsymbol{b}_1}}{\boldsymbol{q}_1^2}\frac{e^{-i\boldsymbol{q}_2\cdot\boldsymbol{b}_2}}{\boldsymbol{q}_2^2}\Bigg[if^{abc}c_1^bc_2^c\left(-\boldsymbol{q}_1^\mu+\boldsymbol{q}_2^\mu+p_1^\mu\left(\frac{p_2\cdot k}{p_1\cdot p_2}-\frac{\boldsymbol{q}_1^2}{p_1\cdot k}\right)-p_2^\mu\left(\frac{p_1\cdot k}{p_1\cdot p_2}-\frac{\boldsymbol{q}_2^2}{p_2\cdot k}\right)\right)$$

QCD Lipatov vertex

$$+\,c_1\cdot c_2\left\{\frac{q_1^2c_1^a}{p_1\cdot k}\left(-q_2^\mu+\frac{k\cdot q_2}{k\cdot p_1}p_1^\mu+\frac{k\cdot p_1}{p_1\cdot p_2}p_2^\mu-\frac{k\cdot p_2}{p_1\cdot p_2}p_1^\mu\right)+\frac{q_2^2c_2^a}{p_2\cdot k}\left(-q_1^\mu+\frac{k\cdot q_1}{k\cdot p_2}p_2^\mu+\frac{k\cdot p_2}{p_1\cdot p_2}p_1^\mu-\frac{k\cdot p_1}{p_1\cdot p_2}p_2^\mu\right)\right\}\Bigg]$$

sub-eikonal correction

$\uparrow$ $1/p_1^+$ $\qquad$ $\uparrow$ $1/p_2^-$

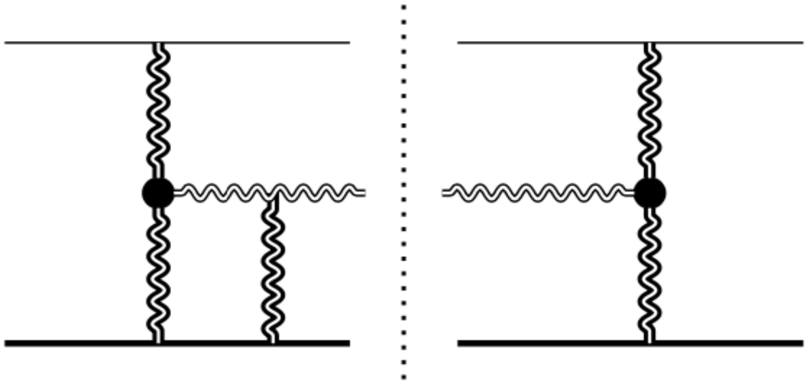
Sub-Eikonal contributions are not universal – for instance, they depend on the spin of the particles

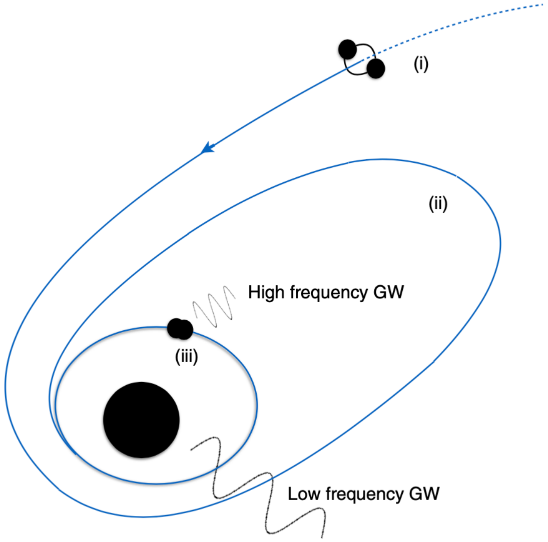
Performing the substitution, one finds the result we obtained by direct computation!

$$A^{\mu\nu}(k) = \frac{\kappa^3 s}{2\,k^2}\int\frac{d^2\boldsymbol{q}_2}{(2\pi)^2}\frac{e^{-i\boldsymbol{q}_1\cdot\boldsymbol{b}_1}}{\boldsymbol{q}_1^2}\frac{e^{-i\boldsymbol{q}_2\cdot\boldsymbol{b}_2}}{\boldsymbol{q}_2^2}\frac{1}{2}\left[C^\mu C^\nu - N^\mu N^\nu + k^\mu\left(\cancel{\frac{p_1^\nu}{p_1\cdot k}\boldsymbol{q}_1^2}+\frac{p_2^\nu}{p_2\cdot k}\boldsymbol{q}_2^2\right)\right]$$

Unphysical – drops out when contracted with the gravitational polarization tensor

RG description in gravity a la CGC EFT in QCD?





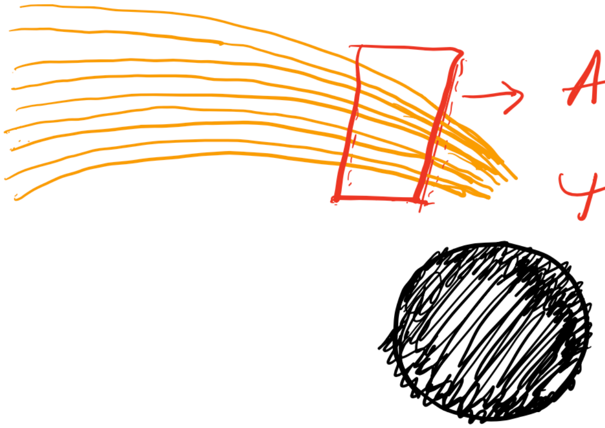
The “dilute-dense” shockwave framework allows us to compute tidal effects

May be particularly relevant for radiation in Extreme Mass Ratio Black Hole Inspirals

Isabelle Fite, Himanshu Raj and RV, in prep.

Geodesic congruence: the geometry of quantum information

The Raychaudhuri equation
- key in Hawking-Penrose singularity theorems :



A congruence of the geodesics of "wee" gravitons

Volume change of geodesic convergence

$$\begin{aligned}\dot{\theta} &= -\Omega^{i}{}_{j}\Omega^{j}{}_{i} + K^{i}{}_{i} \\ &= -\frac{1}{3}\theta^2 - \sigma_{ij}\sigma^{ij} + \omega_{ij}\omega^{ij} + K^{i}{}_{i}.\end{aligned}$$

Bulk scalar Shear tensor Rotation tensor Includes Ricci curvature + stochastic graviton noise

H.-T. Cho and B.-L Hu, arxiv:2301.06325
M. Parikh, F. Wilczek, G. Zaharaide, PRL (2021)

Remarkably, the Raychaudhuri equation can be rephrased as a Bishop-Gromov upper bound on the “complexity volume in D-1 dimensions“ of gate complexity - in quantum information theory?

A. R. Brown, arXiv:2112.05724

Pure speculation: Can this complexity picture provide further insight into the RG fixed point of BH formation?

Dvali, RV, arXiv:2106.11989