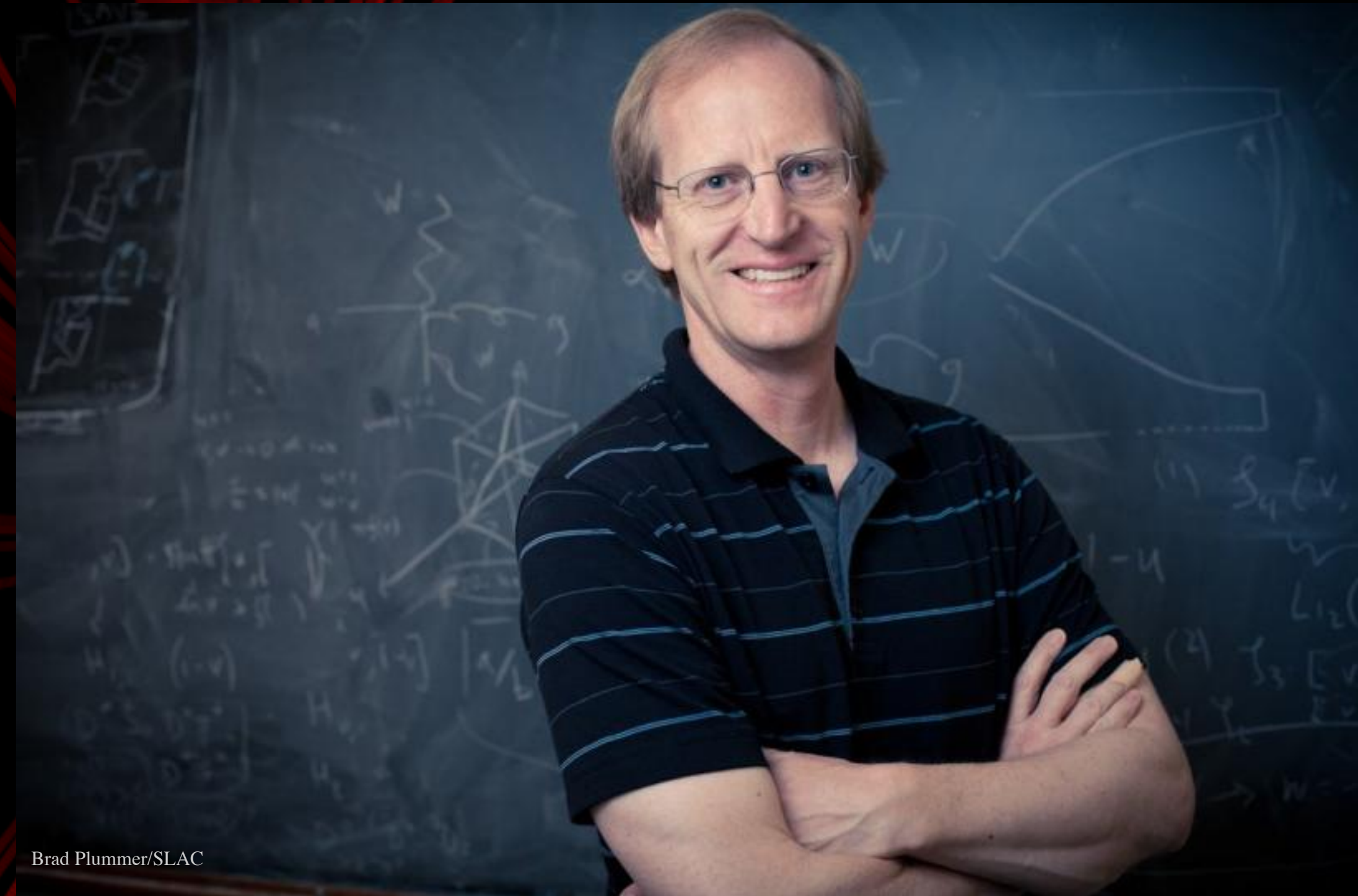


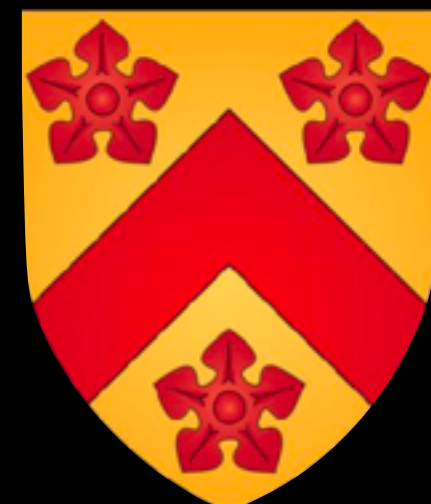
Making perturbative QFT physical

Lancefest
24 June 2026
Higgs Centre, Edinburgh



Brad Plummer/SLAC

Gavin Salam
Rudolf Peierls Centre for
Theoretical Physics
& All Souls College, Oxford



Science and
Technology
Facilities Council



The promise of perturbative QFT

Follow the rules (Feynman diagrams)

Ask a computer to work through maths

Out comes a prediction

Ask the computer to draw more diagrams, work harder, and you get more accuracy

Does it live up to the promise? [#1]

incredibly powerful, e.g. differential scattering cross-sections from first few orders of perturbative expansion in the strong coupling α_s

$$\sigma = \sigma_0 + \alpha_s \sigma_1 + \alpha_s^2 \sigma_2 + \dots$$

NLO



e.g. **B**DDK hep-ph/9403226

BDK hep-ph/9708239

BlackHat, 0803.4180

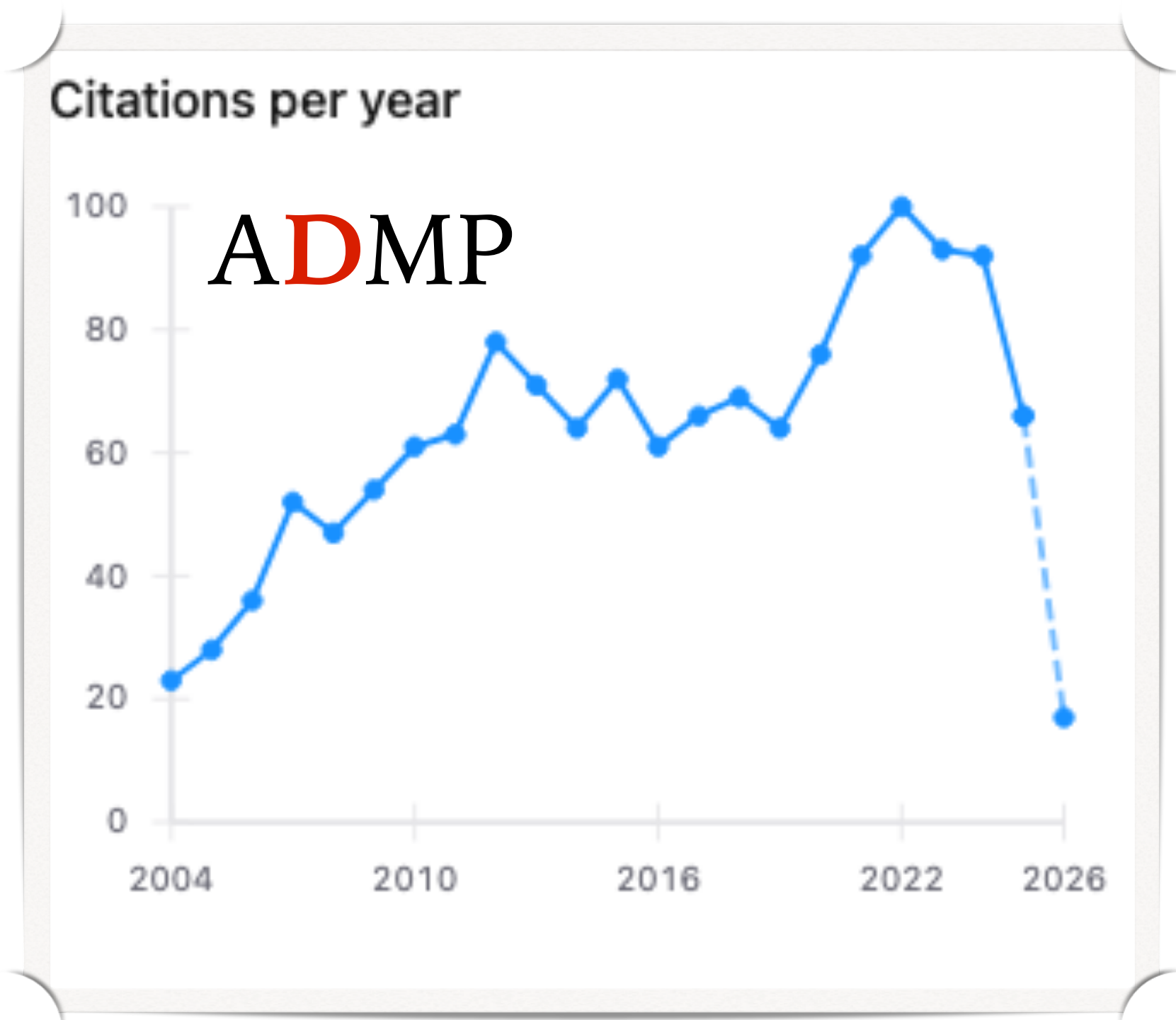


Solution of the 1-loop problem

Does it live up to the promise? [#1]

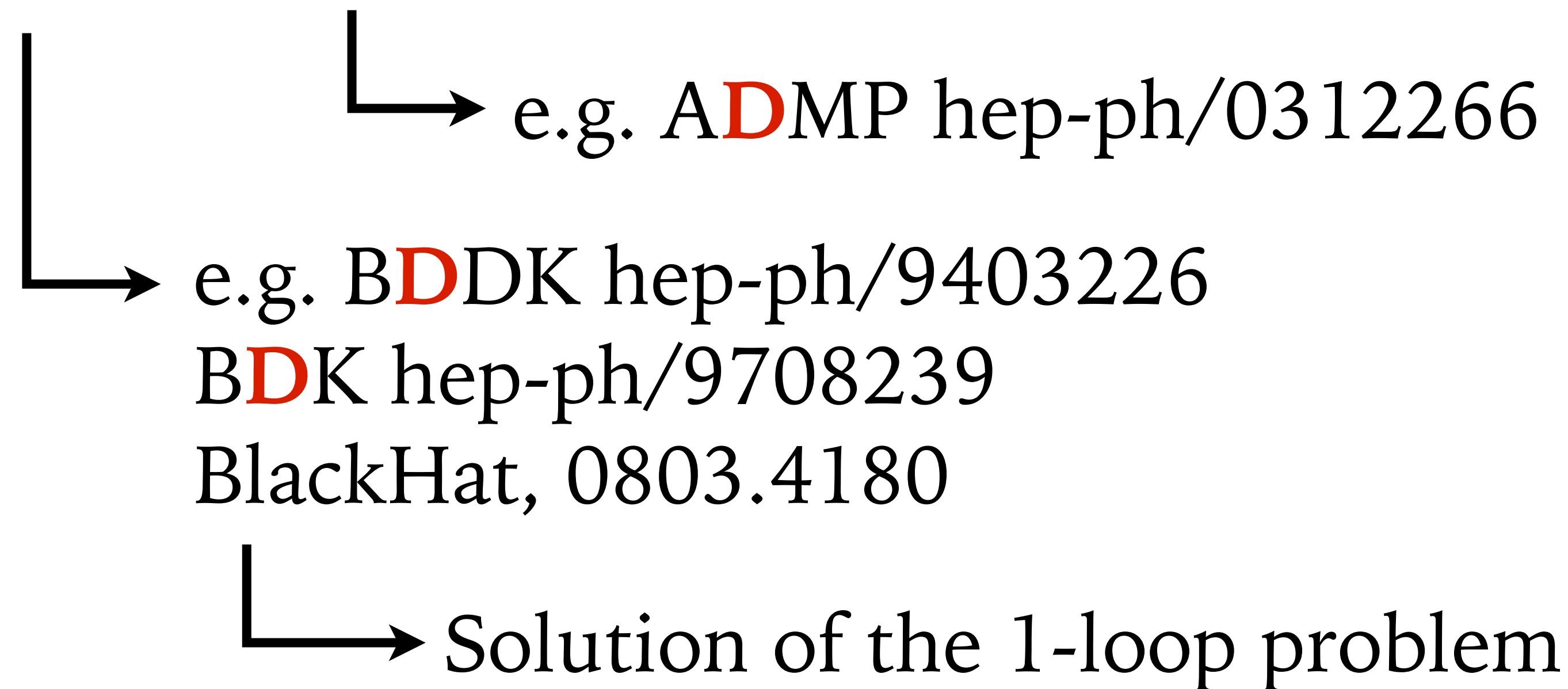
incredibly powerful, e.g. differential scattering cross-sections from first few orders of perturbative expansion in the strong coupling α_s

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NLO

NNLO



Does it live up to the promise? [#1]

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$$\sigma = \sigma_0 + \alpha_s \sigma_1 + \alpha_s^2 \sigma_2 + \dots$$

Eight loop form factors, amplitudes and patterns in planar $\mathcal{N} = 4$ super-Yang-Mills theory

Lance J. Dixon^{a,*} and Zhenjie Li^a

^aSLAC National Accelerator Laboratory, Stanford, CA 94309, USA

E-mail: lance@slac.stanford.edu, munuxilee@gmail.com

The simplest nontrivial amplitude in planar $\mathcal{N} = 4$ super-Yang-Mills theory is six-gluon scattering in the maximally-helicity-violating configuration. It has been computed to 8 loops with the help of antipodal duality, which relates it to the three-point form factor of a protected operator, the chiral stress tensor super-multiplet, represented also as $\text{tr}\phi^2$. In this talk, we describe the computation to 8 loops of another three-point form factor, for the operator $\text{tr}\phi^3$. This form factor lives in the same restricted space of polylogarithms as the $\text{tr}\phi^2$ form factor. We also report on all-order patterns for sequences of coefficients in the symbols of these polylogarithmic results, for the leading discontinuity of the $\text{tr}\phi^3$ form factor.

NLO

NNLO

↳ e.g. **ADMP** hep-ph/0312266

↳ e.g. **BDDK** hep-ph/9403226

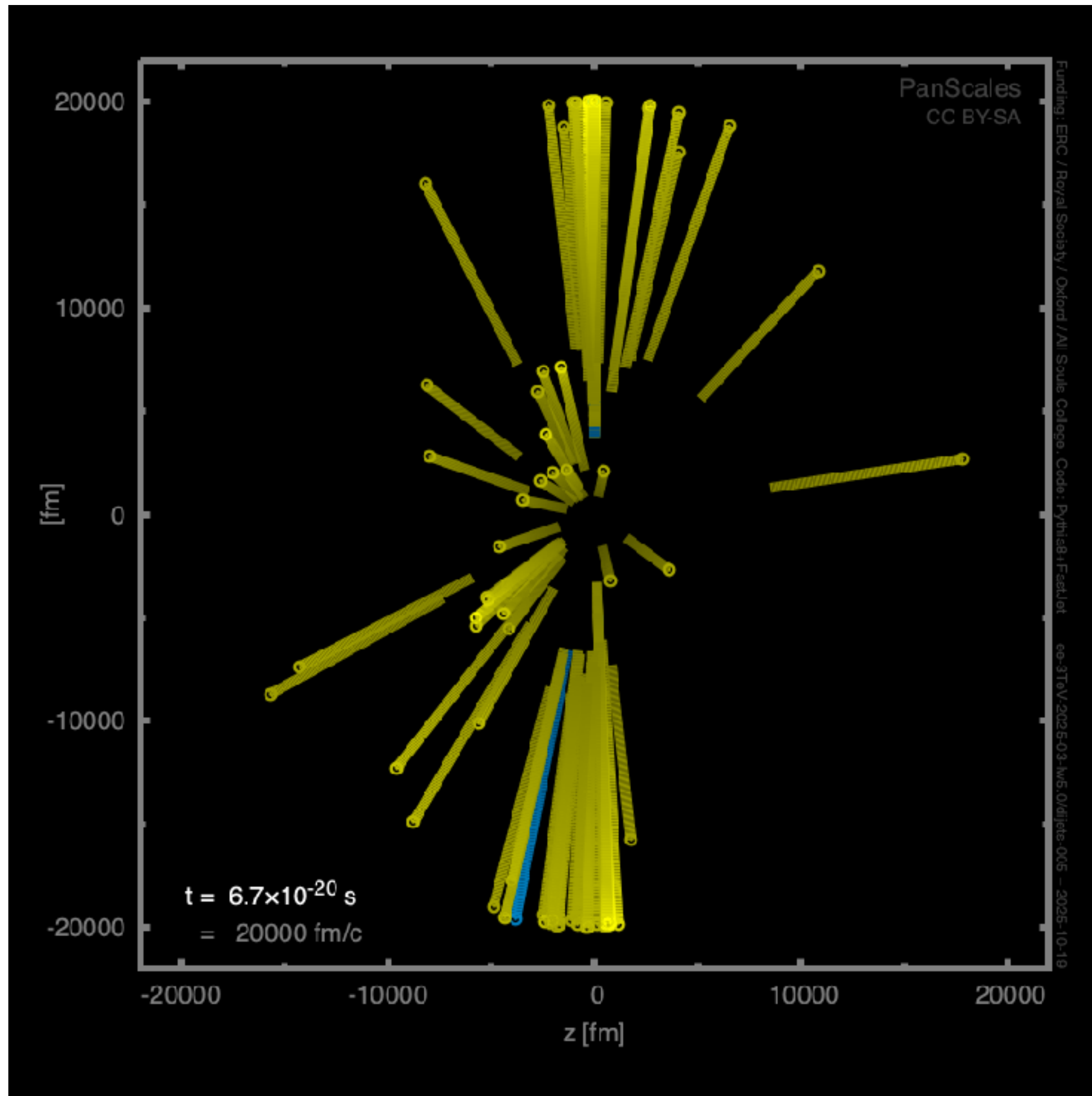
BDK hep-ph/9708239

BlackHat, 0803.4180

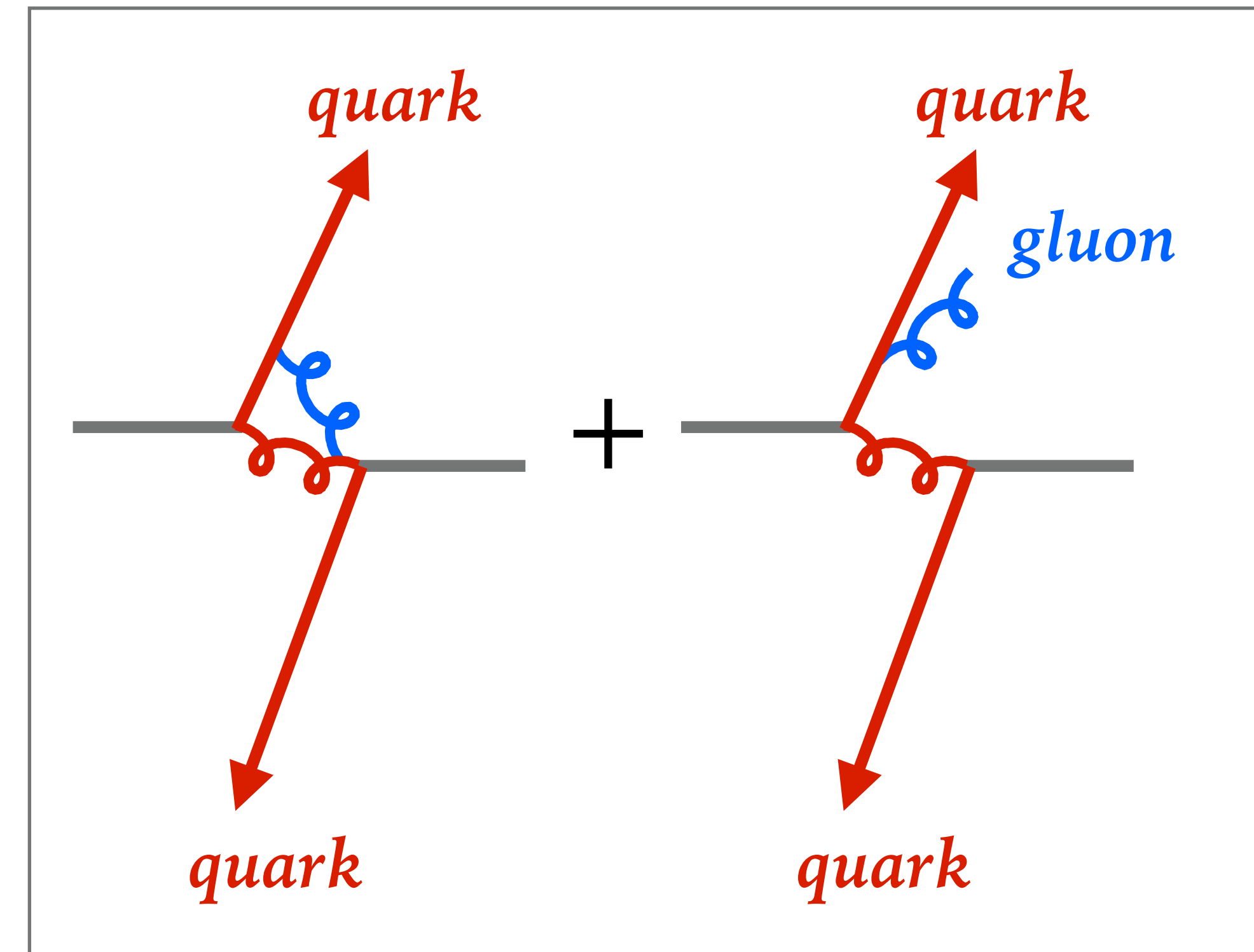
↳ Solution of the 1-loop problem

Does it live up to the promise? [#2]

The problem: **real events and power-house calculational tool look totally different**



\neq



next-to-leading order

What a NLO calculation gives you (here, Event2, $e^+e^- \rightarrow q\bar{q}$)

LO (2-particle) tree-level event

with weight 1.00000

px, py, pz, E = -1.32 -1.38 -49.96 50.00

px, py, pz, E = 1.32 1.38 49.96 50.00

What a NLO calculation gives you (here, Event2, $e^+e^- \rightarrow q\bar{q}$)

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LO event ($q\bar{q}$)

What a NLO calculation gives you (here, Event2, $e^+e^- \rightarrow q\bar{q}$)

LO (2-particle) tree-level event

with weight 1.00000

px, py, pz, E = -1.32 -1.38 -49.96 50.00

px, py, pz, E = 1.32 1.38 49.96 50.00

NLO (3-particle) tree-level event

with **weight 16.7749**

px, py, pz, E = -1.60 -1.75 -49.87 49.93

px, py, pz, E = 1.31 1.36 49.25 49.29

px, py, pz, E = 0.30 0.39 0.62 0.79

LO event ($q\bar{q}$)

NLO event, with real emission
~ LO event + extra soft gluon
and **large positive weight**

What a NLO calculation gives you (here, Event2, $e^+e^- \rightarrow q\bar{q}$)

LO (2-particle) tree-level event

with weight 1.00000

px, py, pz, E = -1.32 -1.38 -49.96 50.00

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NLO (3-particle) tree-level event

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px, py, pz, E = 1.31 1.36 49.25 49.29

px, py, pz, E = 0.30 0.39 0.62 0.79

NLO (2-particle) virtual subtraction event

with **weight -1.5868**

px, py, pz, E = -1.32 -1.38 -49.96 50.00

px, py, pz, E = 1.32 1.38 49.96 50.00

NLO (2-particle) virtual subtraction event

with **weight -15.1855**

px, py, pz, E = -1.61 -1.75 -49.94 50.00

px, py, pz, E = 1.61 1.75 49.94 50.00

NLO (2-particle) virtual finite event

with weight 0.0501, multiplying (alphas/2pi)

px, py, pz, E = -1.32 -1.38 -49.96 50.00

px, py, pz, E = 1.32 1.38 49.96 50.00

LO event ($q\bar{q}$)

NLO event, with real emission
~ LO event + extra soft gluon
and **large positive weight**

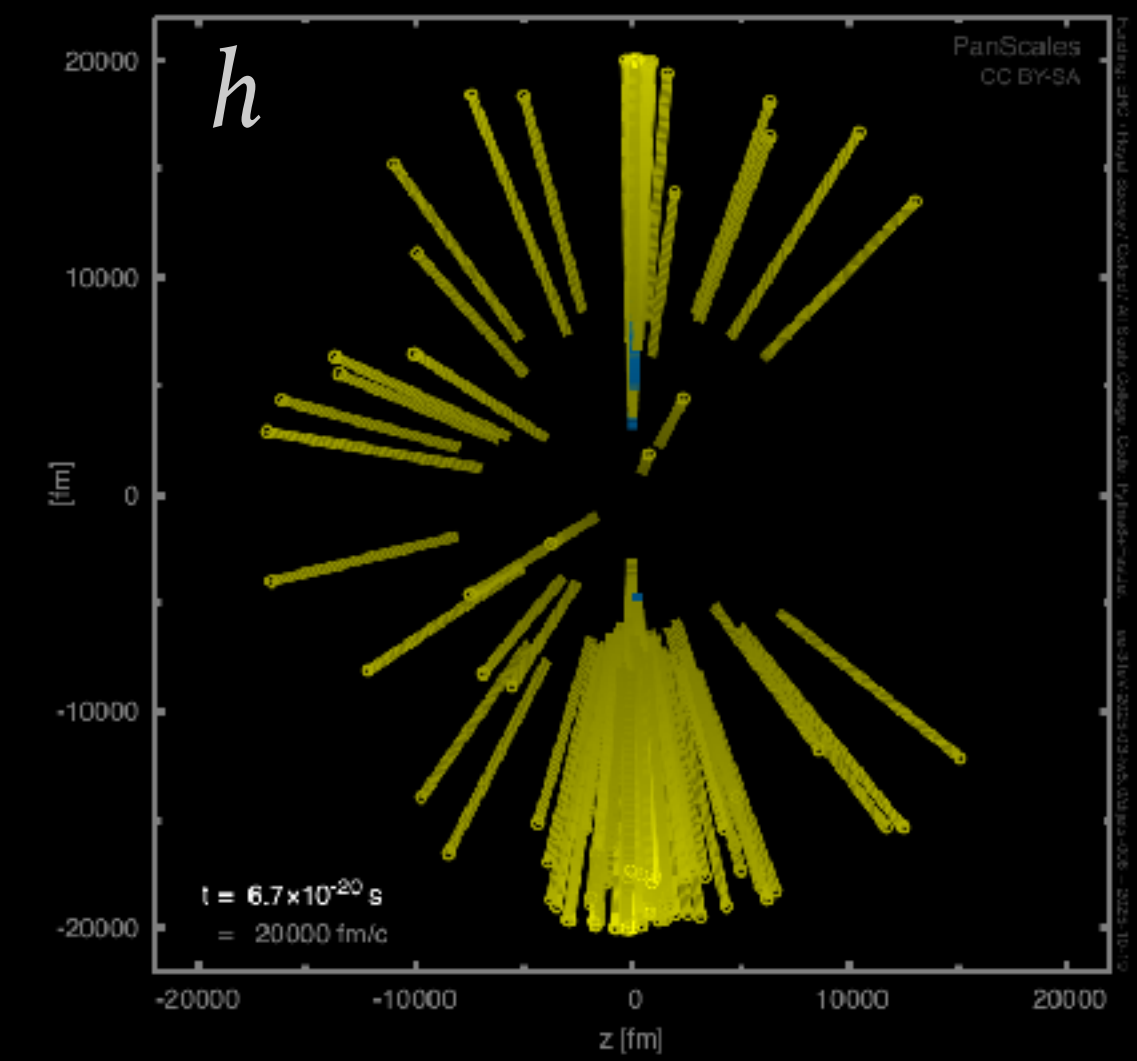
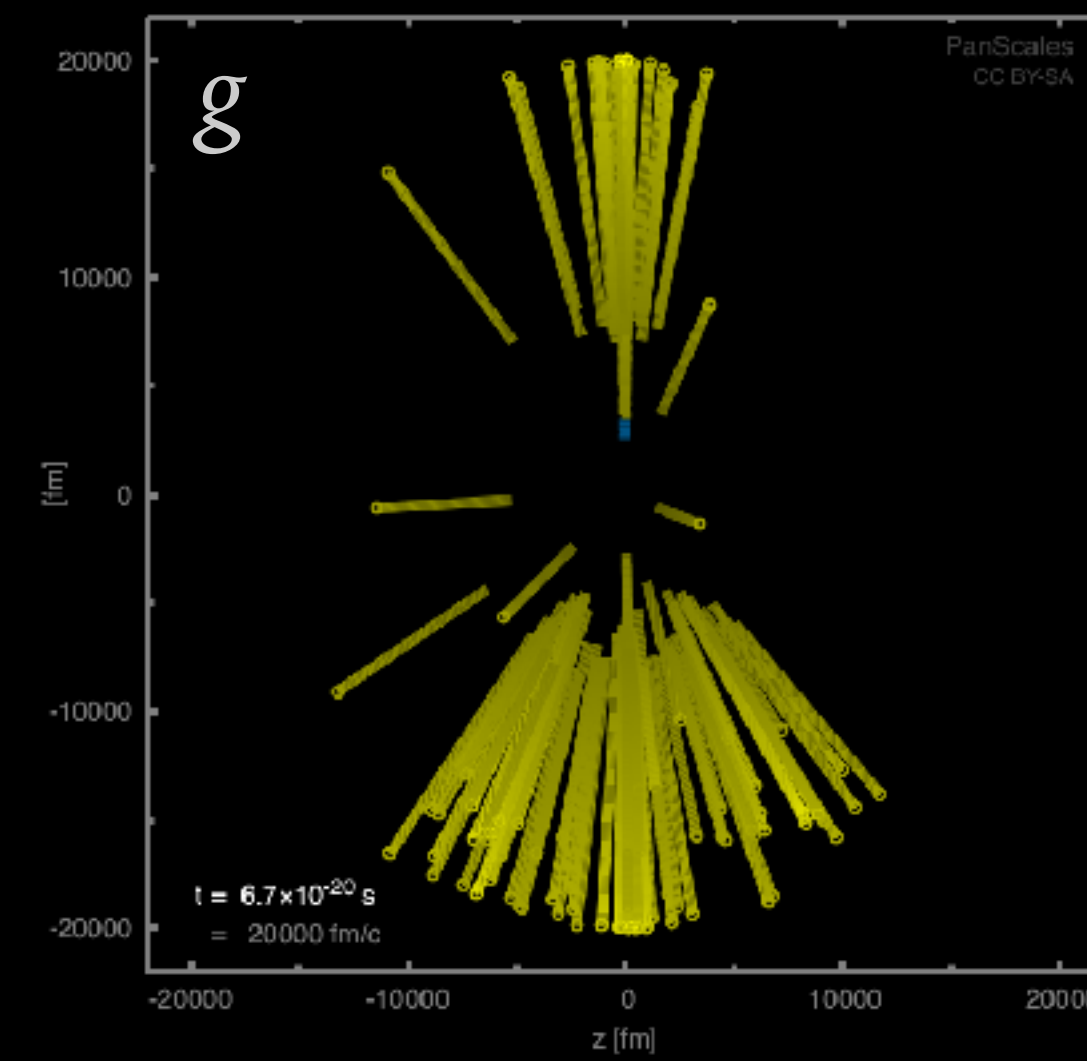
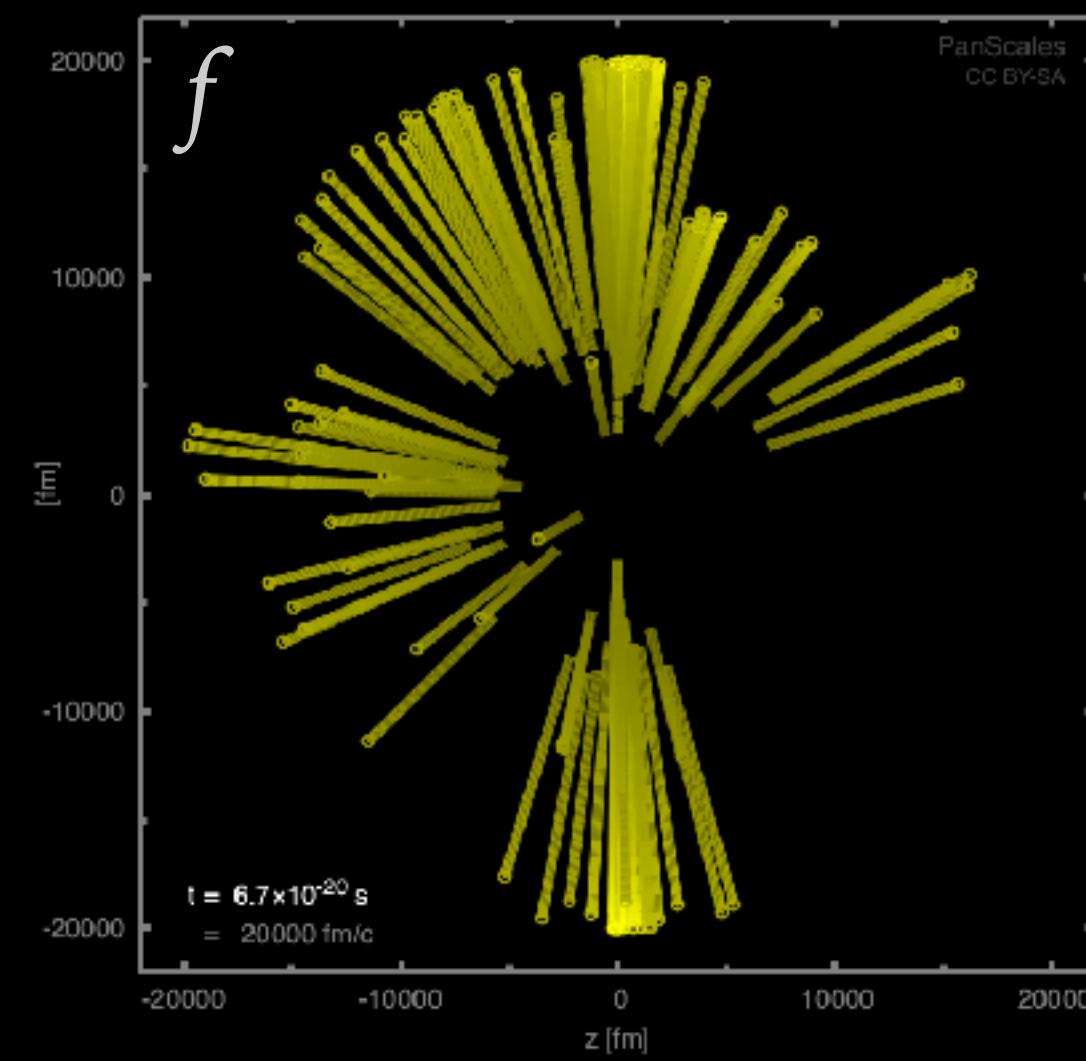
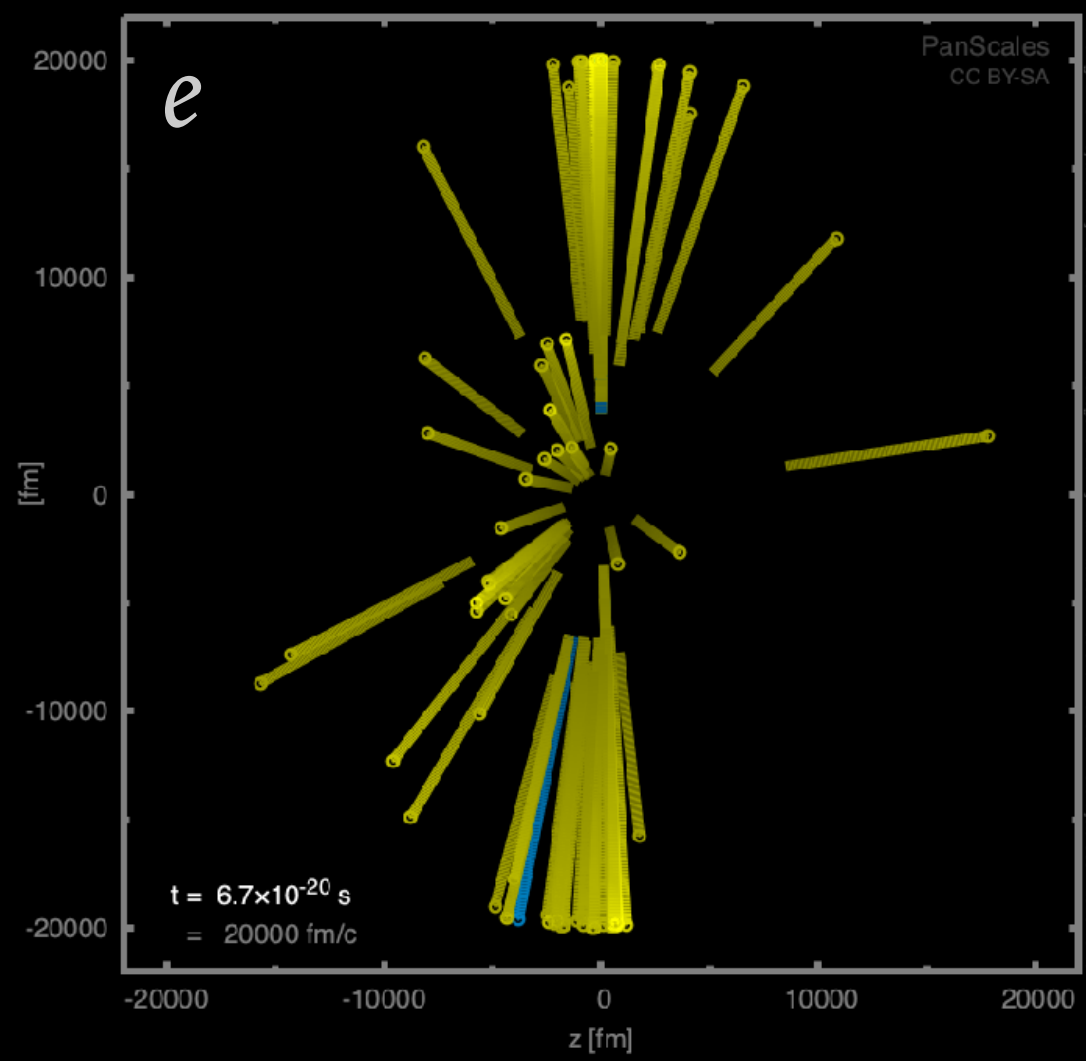
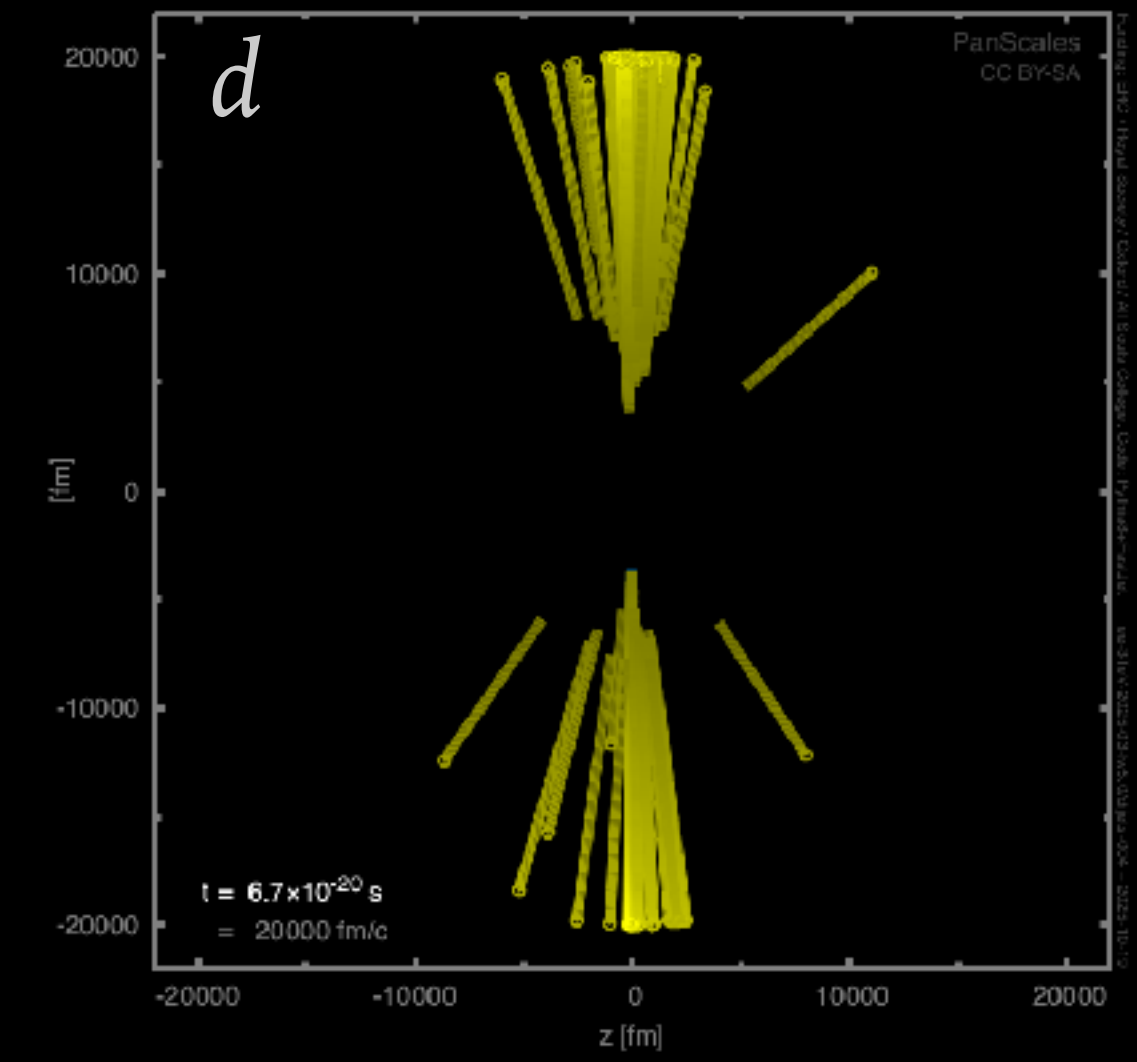
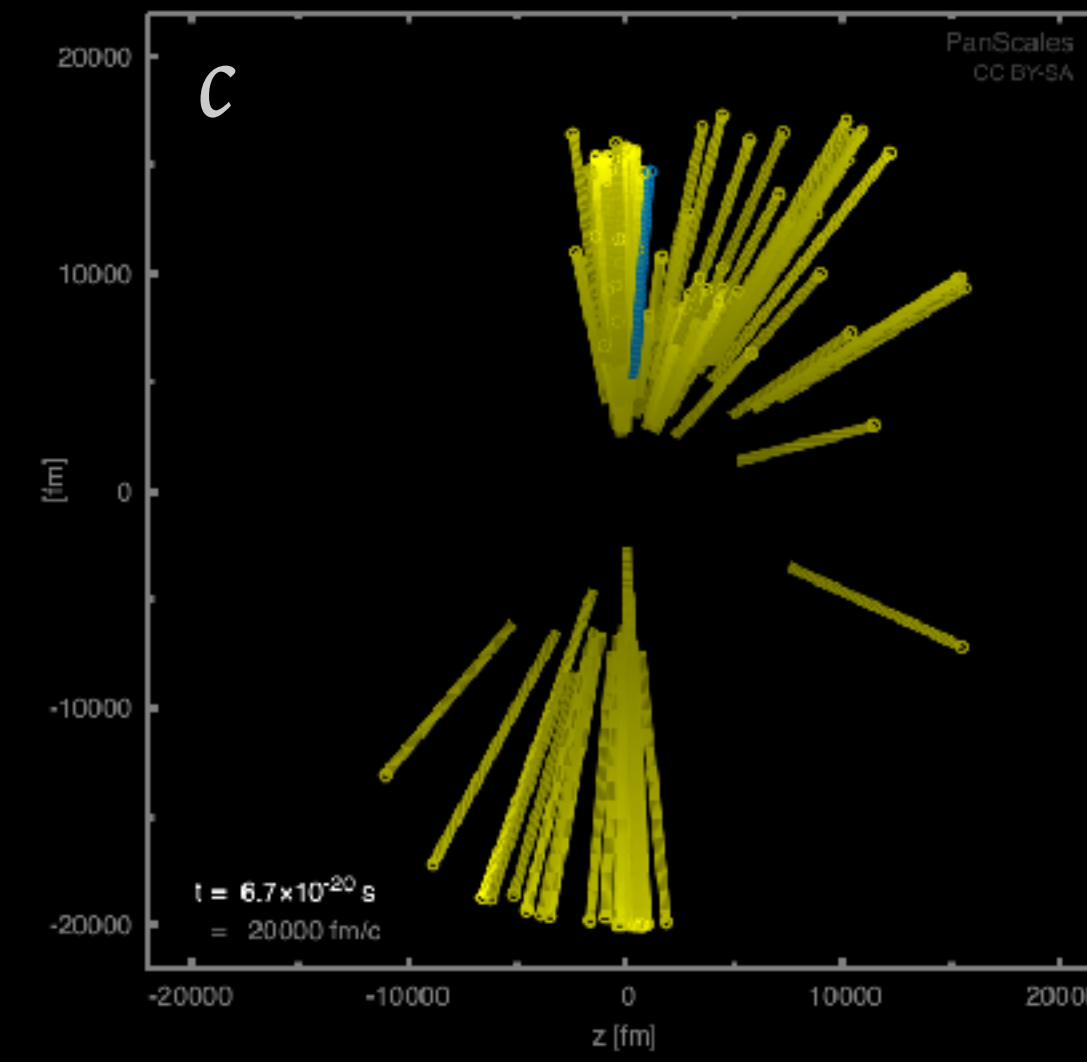
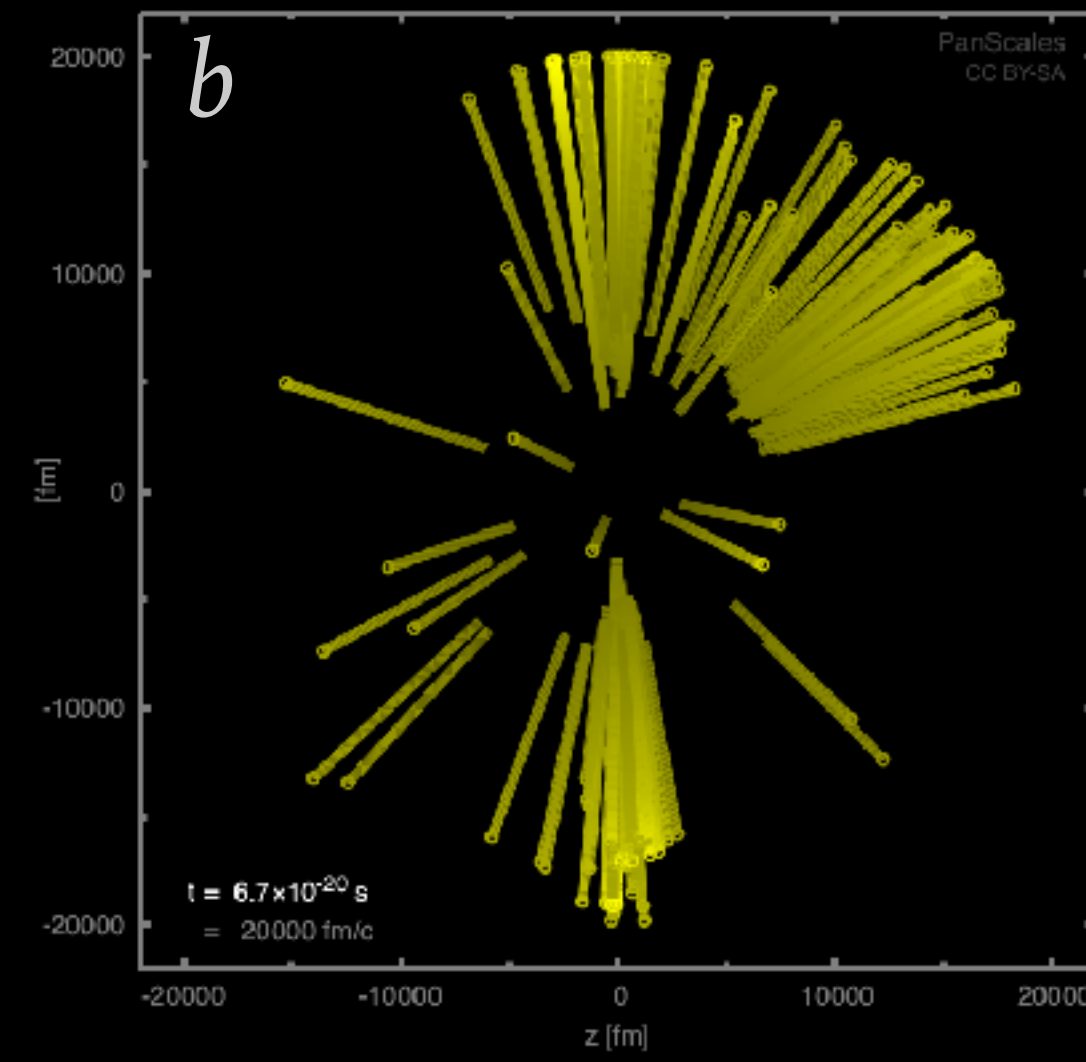
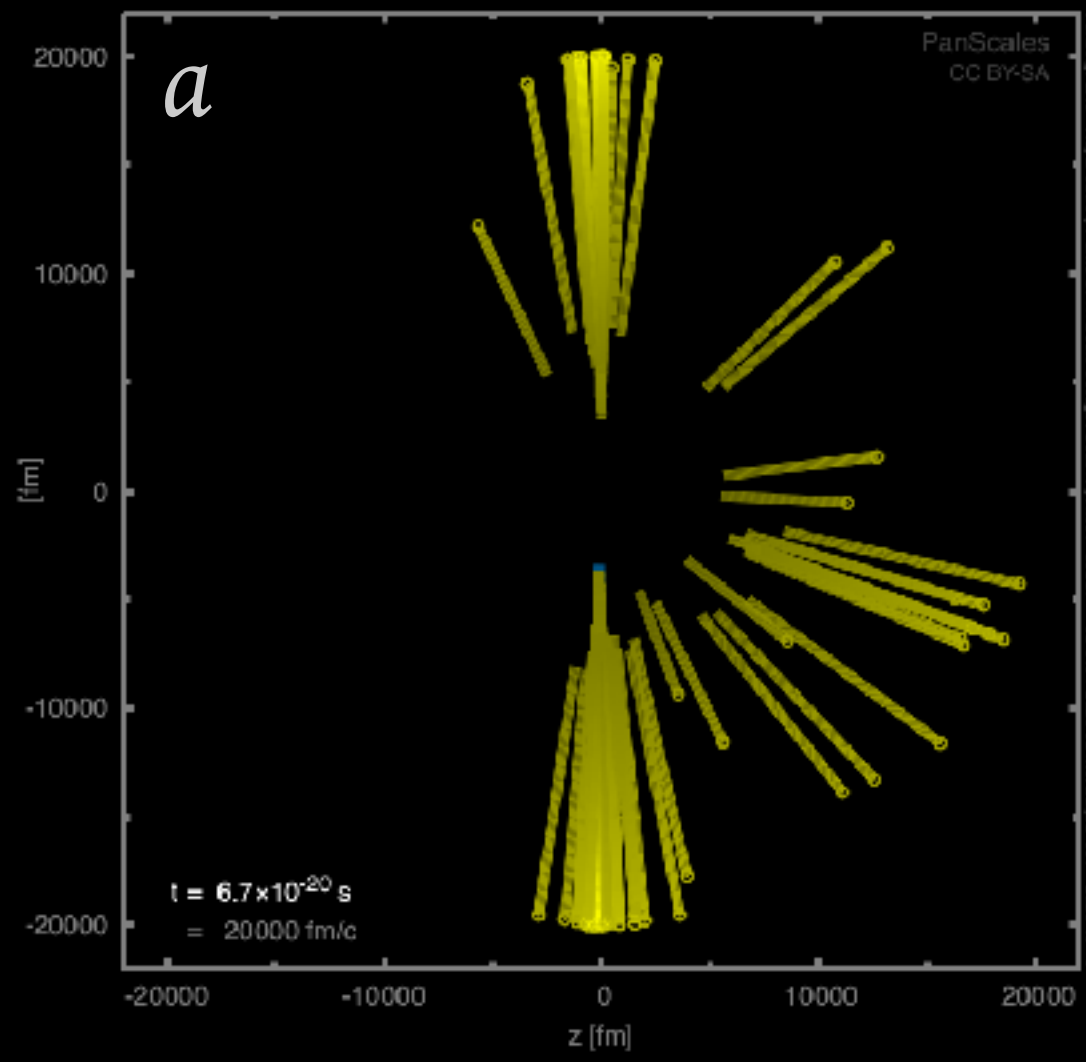
NLO event, “virtual” correction
~ LO event
and **large negative weight**

event weights are \sim probabilities

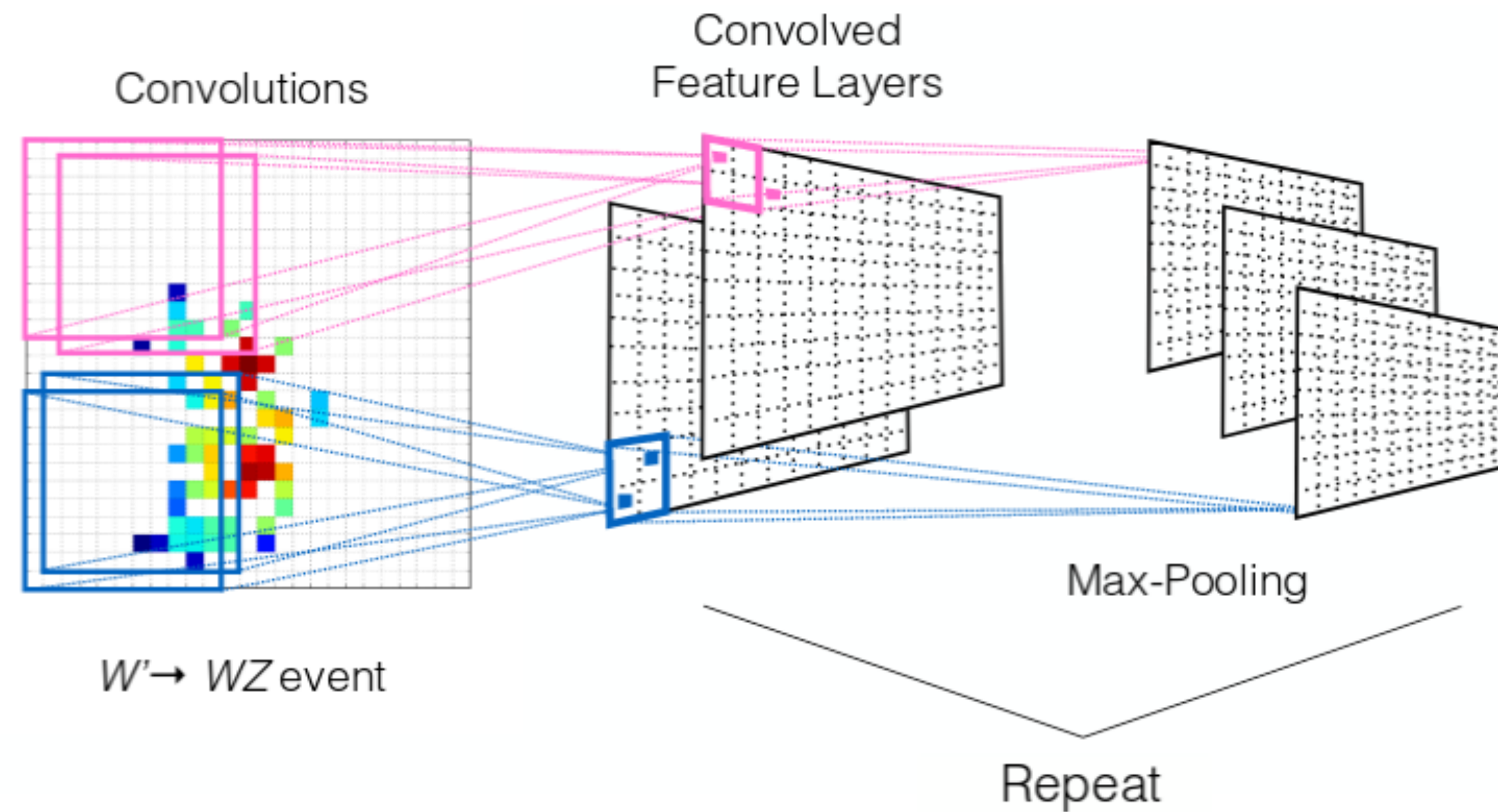
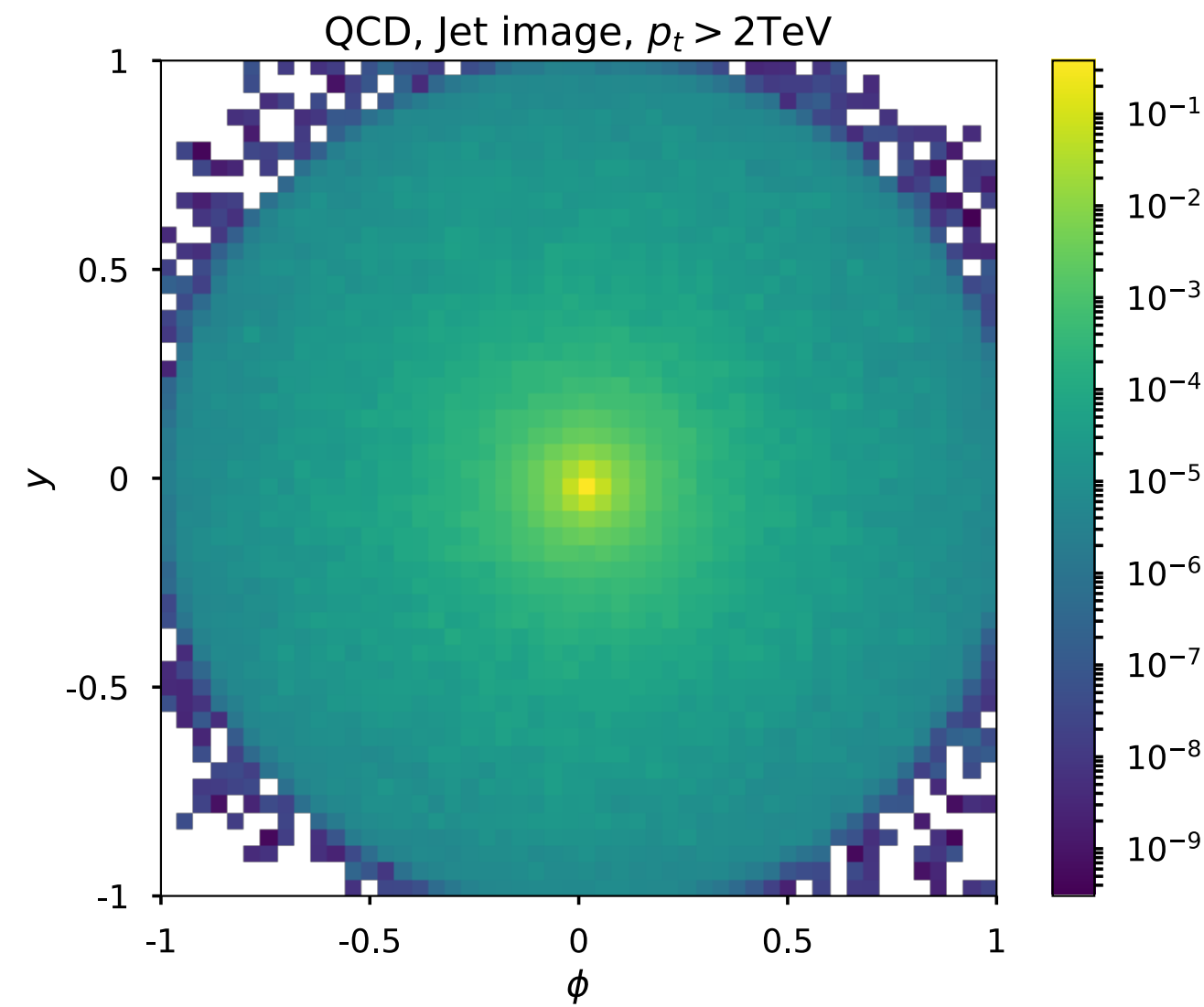
- real life doesn't have negative probabilities
- real life doesn't have (near-)divergent probabilities
- you can avoid these problems in perturbation theory if you ask very limited kinds of questions, i.e. nearly always summing real & virtual divergences (infrared safe, single momentum scale)*
- but experiments don't limit themselves to those kinds of questions

can we be flexible with the questions we ask, yet still get the benefits of perturbation theory?

* though there can still be nasty surprises, cf. Chen et al [2102.07607](#), GPS & Slade [2106.08329](#)



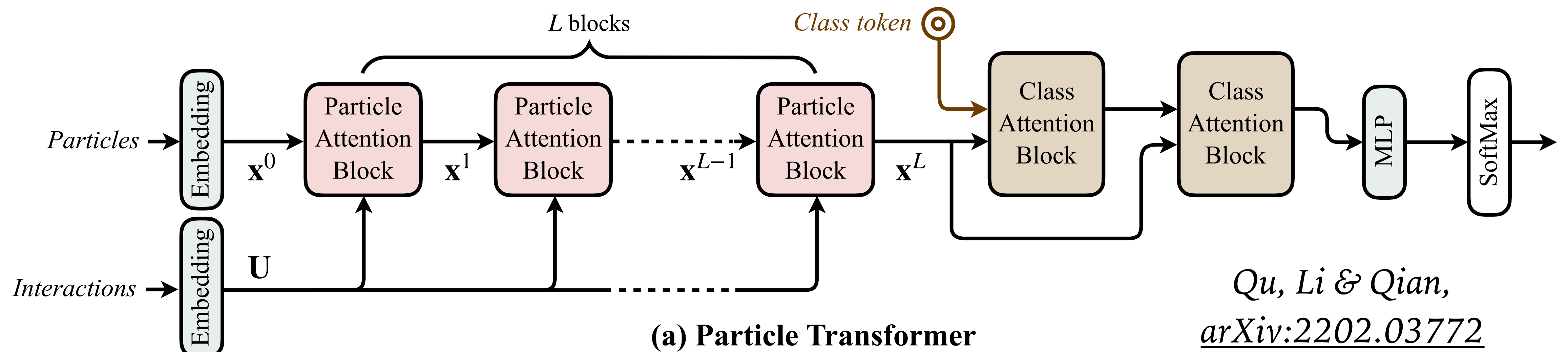
Did the jet of particles come from a quark or a W/Z/H-boson? → Use machine learning



[Cogan, Kagan, Strauss, Schwartzman [JHEP 1502 \(2015\) 118](#)]

[de Oliveira, Kagan, Mackey, Nachman, Schwartzman [JHEP 1607 \(2016\) 069](#)]

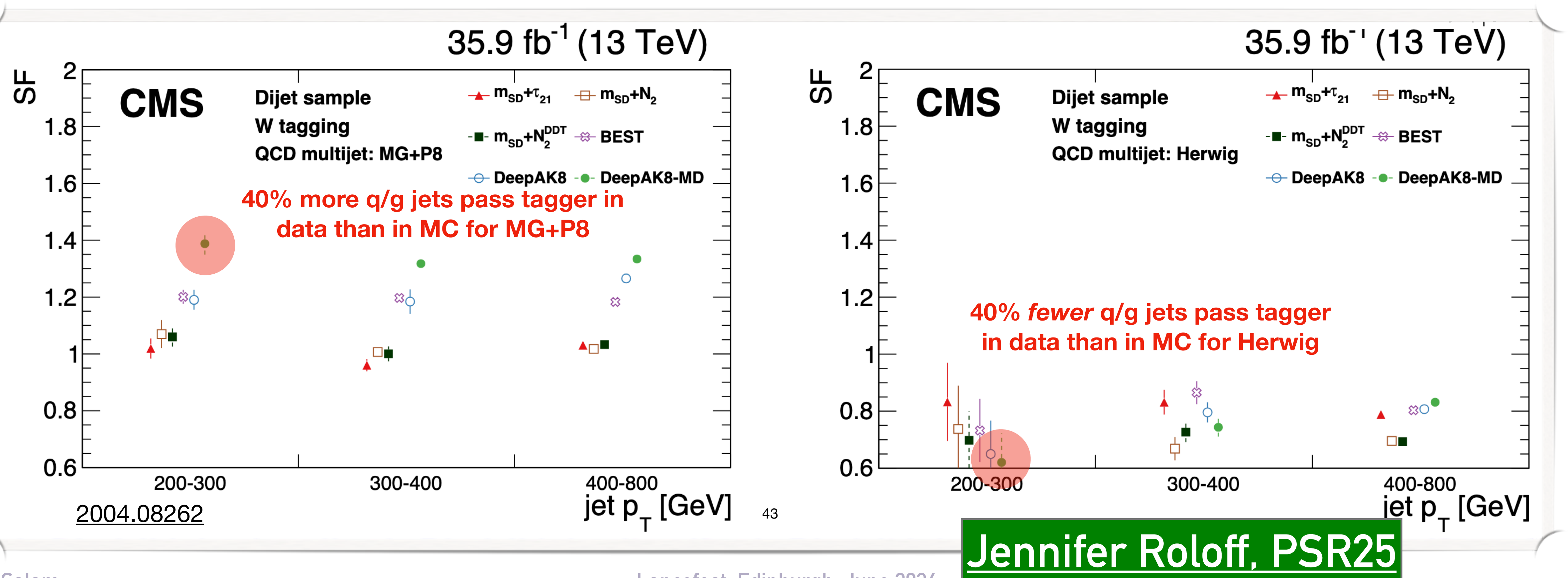
2021 Young Experimental Physicist
EPS HEPP prize



Qu, Li & Qian,
[arXiv:2202.03772](#)

We want machine learning to be accurate

- even simplest ML methods exacerbate sensitivity to imperfections in predictions
- how do we get (perturbative) QFT to be accurate even with ML?



Physical predictions for any one observable: resummation (e.g. EEC collinear limit)

In the collinear limit, $z \rightarrow 0$, the perturbative contributions to the EEC exhibits a single logarithmic series

$$\frac{d\sigma}{dz} = \sum_{L=1}^{\infty} \sum_{j=-1}^{L-1} \left(\frac{\alpha_s(\mu)}{4\pi} \right)^L c_{L,j} \mathcal{L}^j(z) + \dots, \quad (4)$$

where $\mathcal{L}^{-1}(z) = \delta(z)$ and $\mathcal{L}^j(z) = [\ln^j z/z]_+$ for $j \geq 0$ denotes a standard plus distribution. The ellipses denote

Dixon, Moult, Zhu, 1905.01310

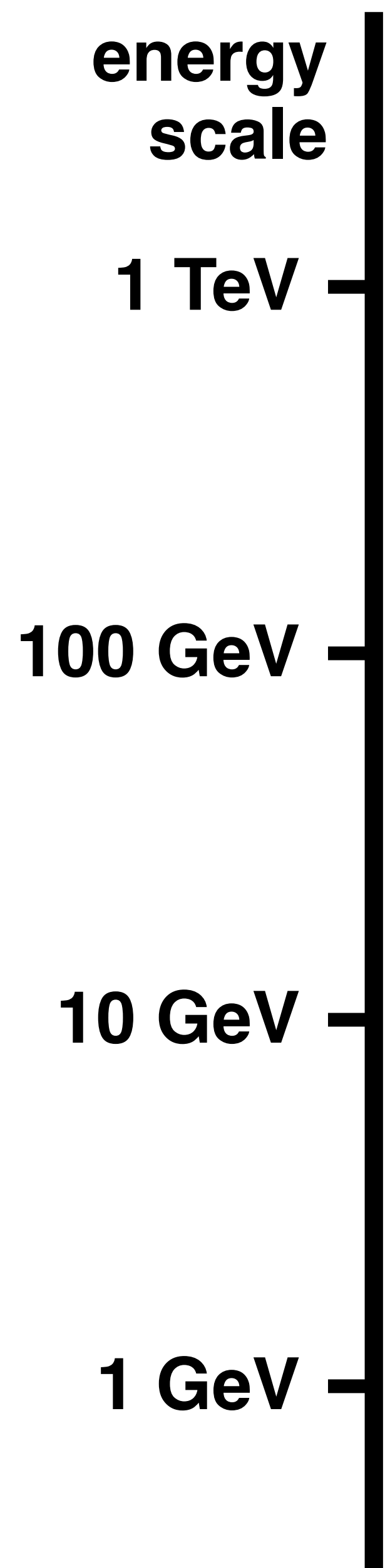
$$0 \leq z = \frac{1 - \cos \chi}{2} \leq 1$$

$$\frac{d\vec{J}_{\text{LL}}(\ln \frac{zQ^2}{\mu^2}, \mu)}{d \ln \mu^2} = \vec{J}_{\text{LL}}(\ln \frac{zQ^2}{\mu^2}, \mu) \cdot \int_0^1 dy y^2 \hat{P}_T^{(0)}(y)$$

All-order formulation often involves a differential equation in some scale

$$\vec{J}_{\text{LL}}(\ln \frac{zQ^2}{\mu^2}, \mu) = (1, 1) \cdot V \left[\left(\frac{\alpha_s(\sqrt{z}Q)}{\alpha_s(\mu)} \right)^{-\frac{\vec{\gamma}_T^{(0)}}{\beta_0}} \right]_D V^{-1},$$

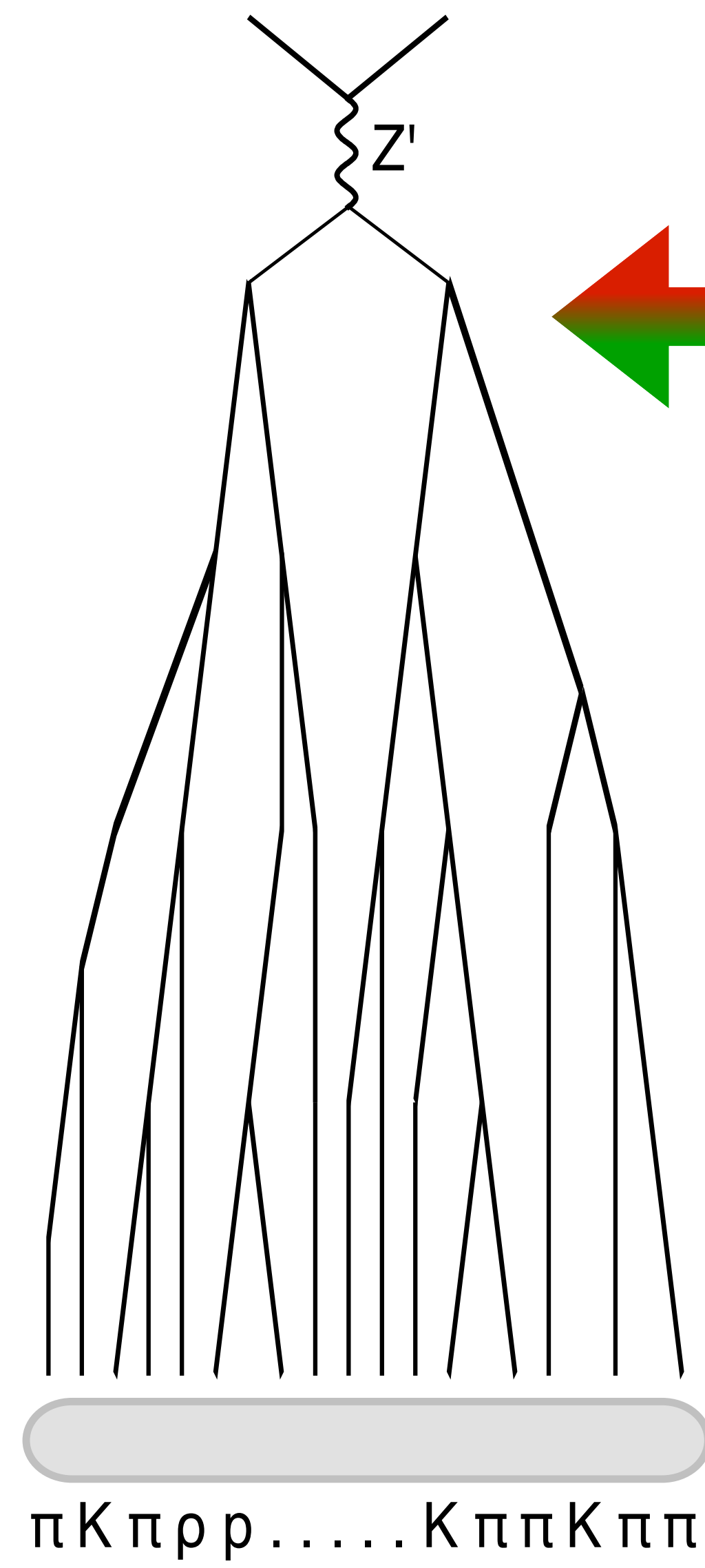
Which can be solved analytically or numerically



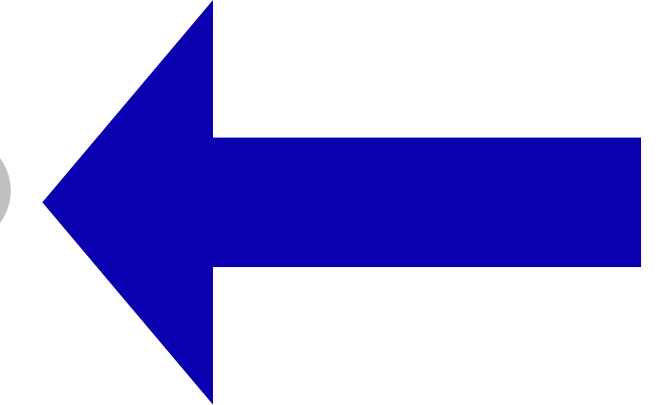
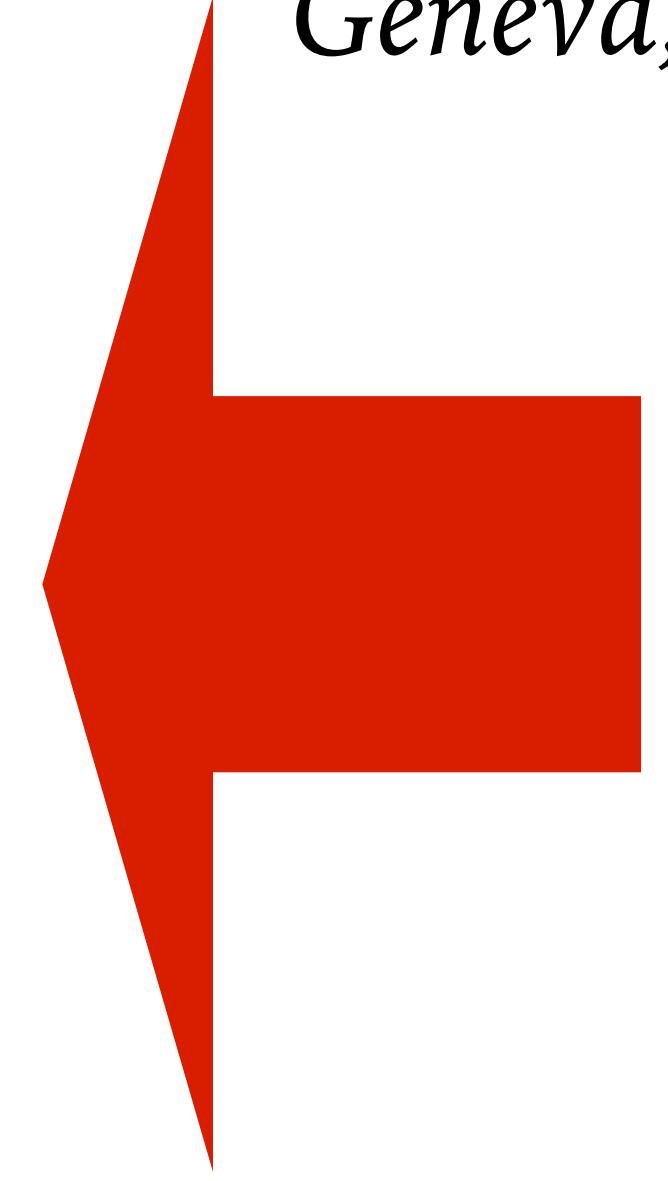
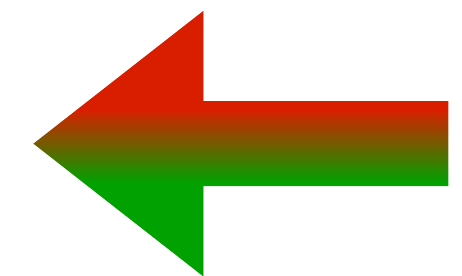
hard process

parton shower

hadronisation



*Much of past 25 years' work:
MLM, CKKW, MC@NLO,
POWHEG, MIN(N)LO, FxFx,
Geneva, UNNLOPS, Vincia, etc.*



**In standard codes,
largely based on
principles from
20-30 years ago
(Herwig, Pythia,
Sherpa)**

parton showers: the ultimate differential equation

discussion today is in the large-NC limit

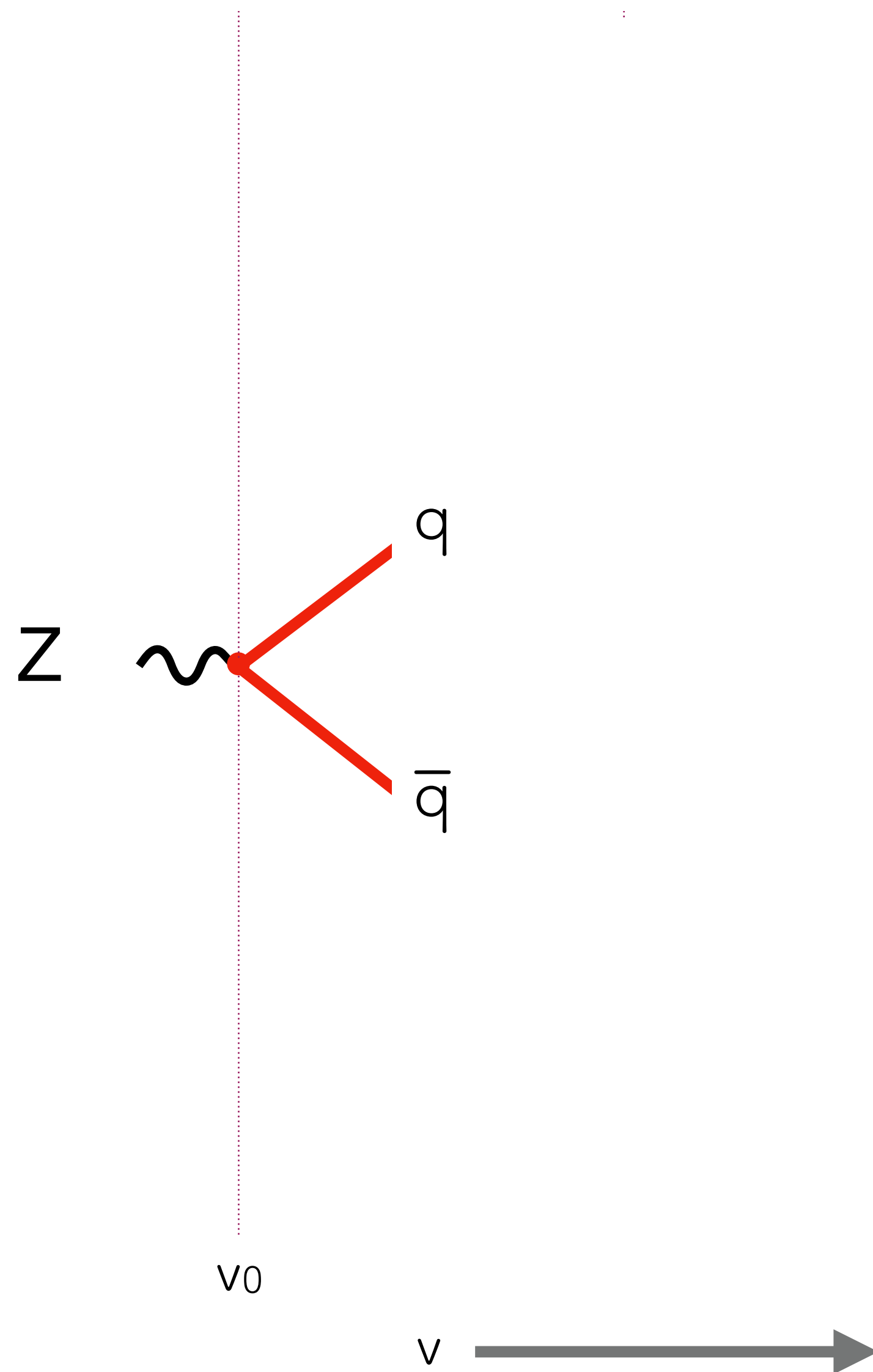
there are also various approaches for subleading-NC terms

QCD **parton shower**: an evolution equation (in **evolution scale v** , e.g. trans.mom.)

Start with quark (q) antiquark (\bar{q}) state.

Throw a random number to determine down to what scale state persists unchanged

$$\frac{dP_2(v)}{dv} = -f_{2 \rightarrow 3}^{q\bar{q}}(v) P_2(v)$$

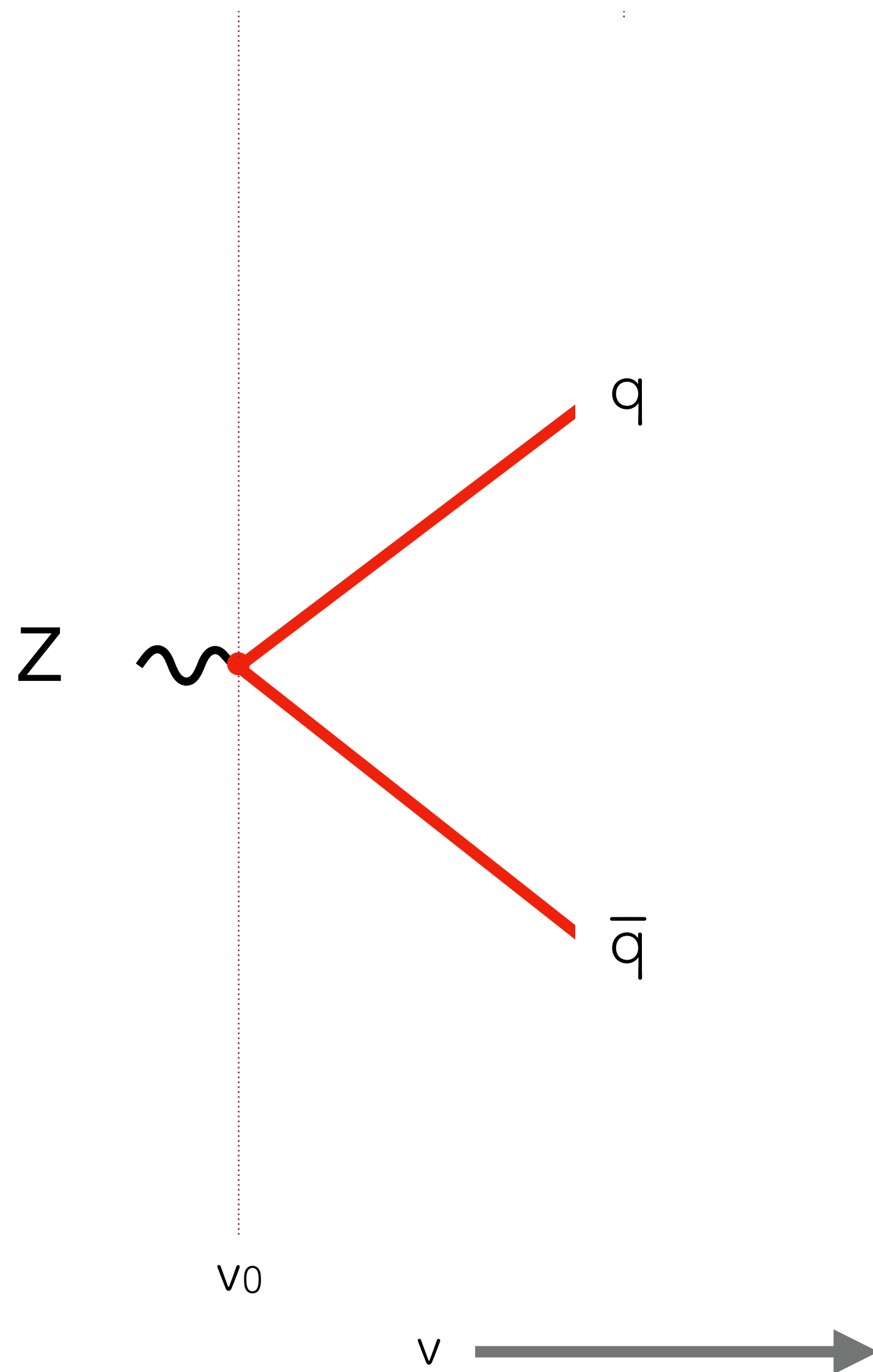


QCD **parton shower**: an evolution equation (in **evolution scale v** , e.g. trans.mom.)

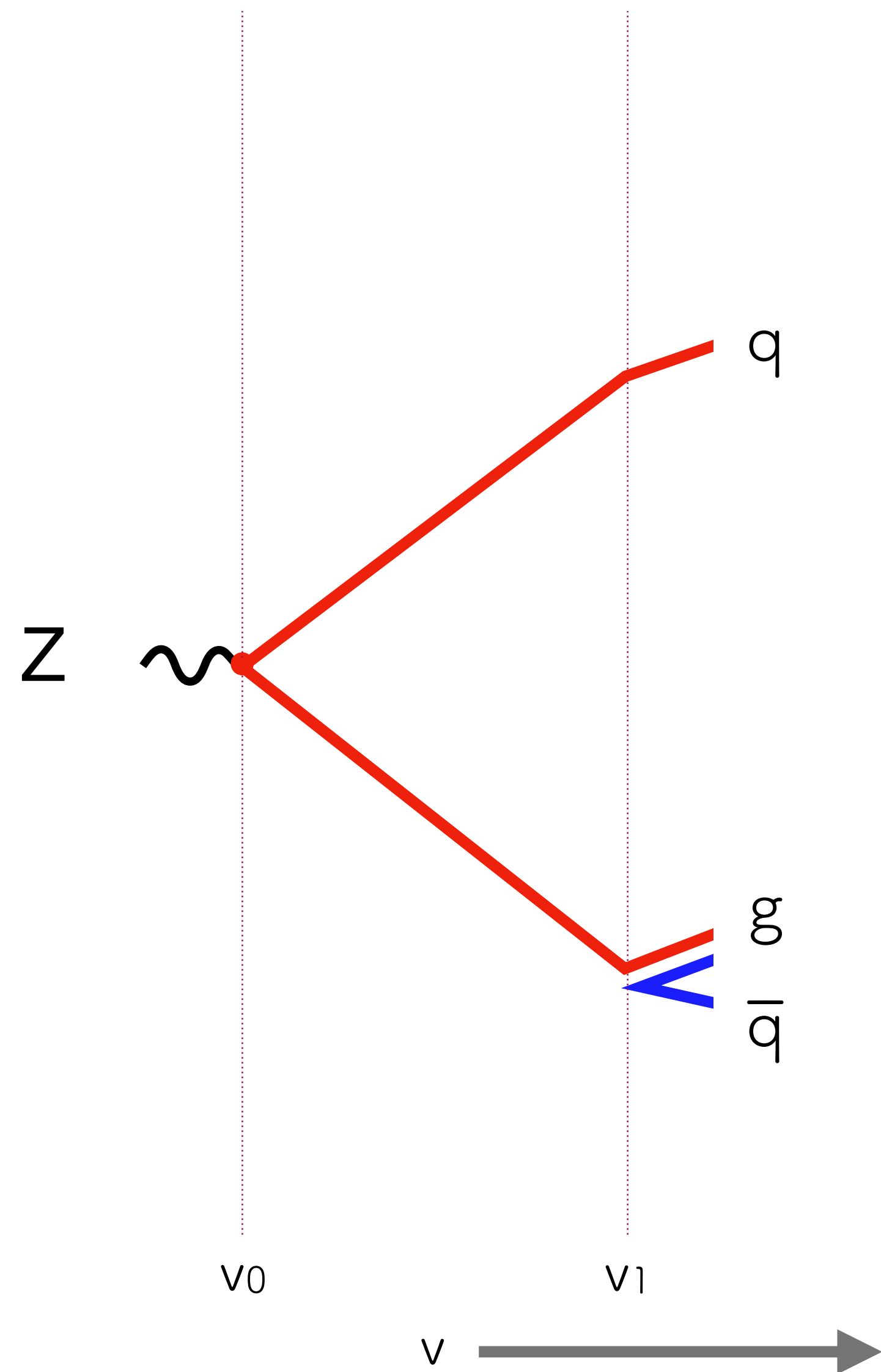
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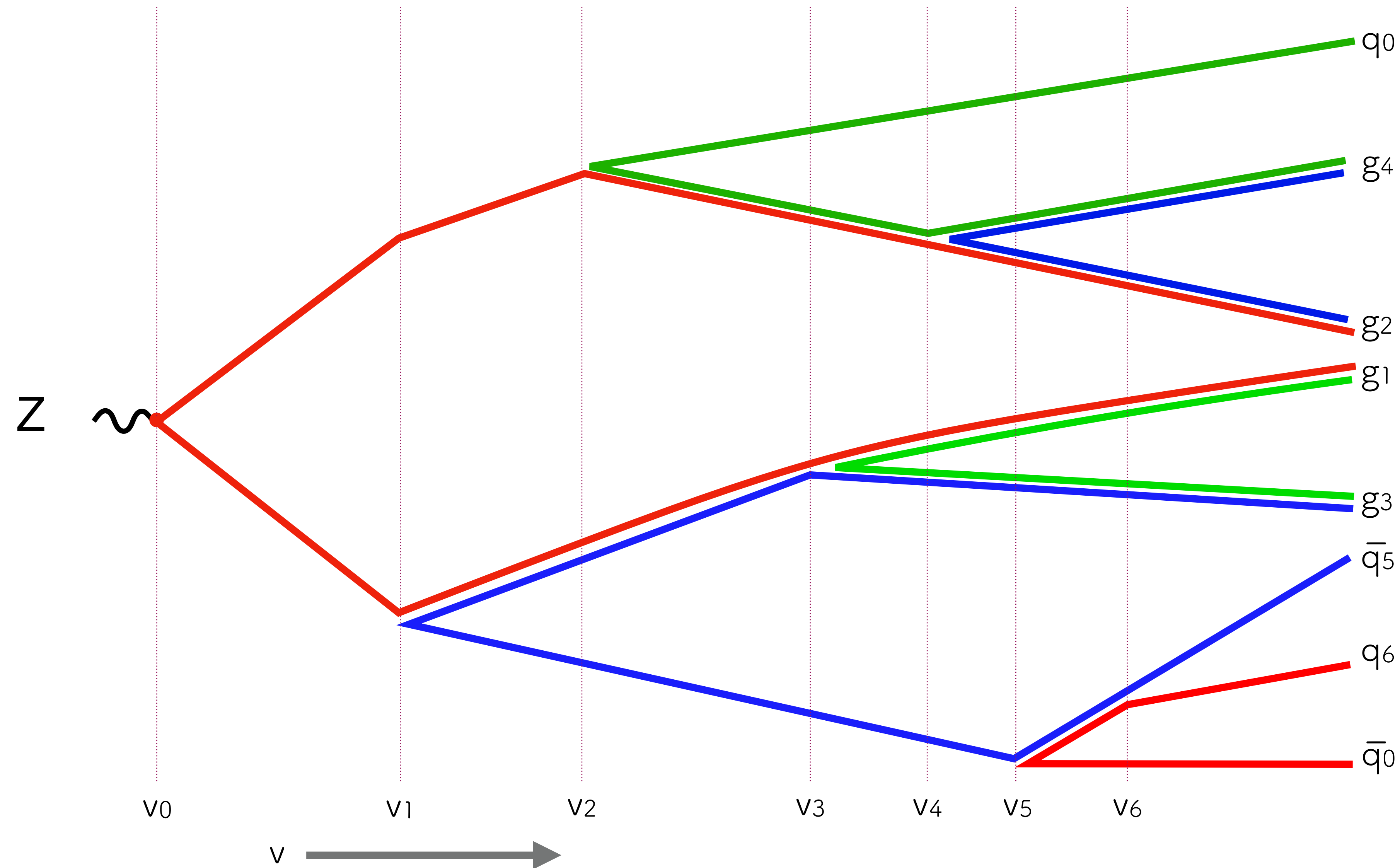
Throw a random number to determine down to what scale state persists unchanged

At some point, **state splits** ($2 \rightarrow 3$, i.e. emits gluon). Evolution equation changes

$$\frac{dP_3(\nu)}{d\nu} = - \left[f_{2 \rightarrow 3}^{qg}(\nu) + f_{2 \rightarrow 3}^{g\bar{q}}(\nu) \right] P_3(\nu)$$

gluon (g) here is drawn as two colour lines

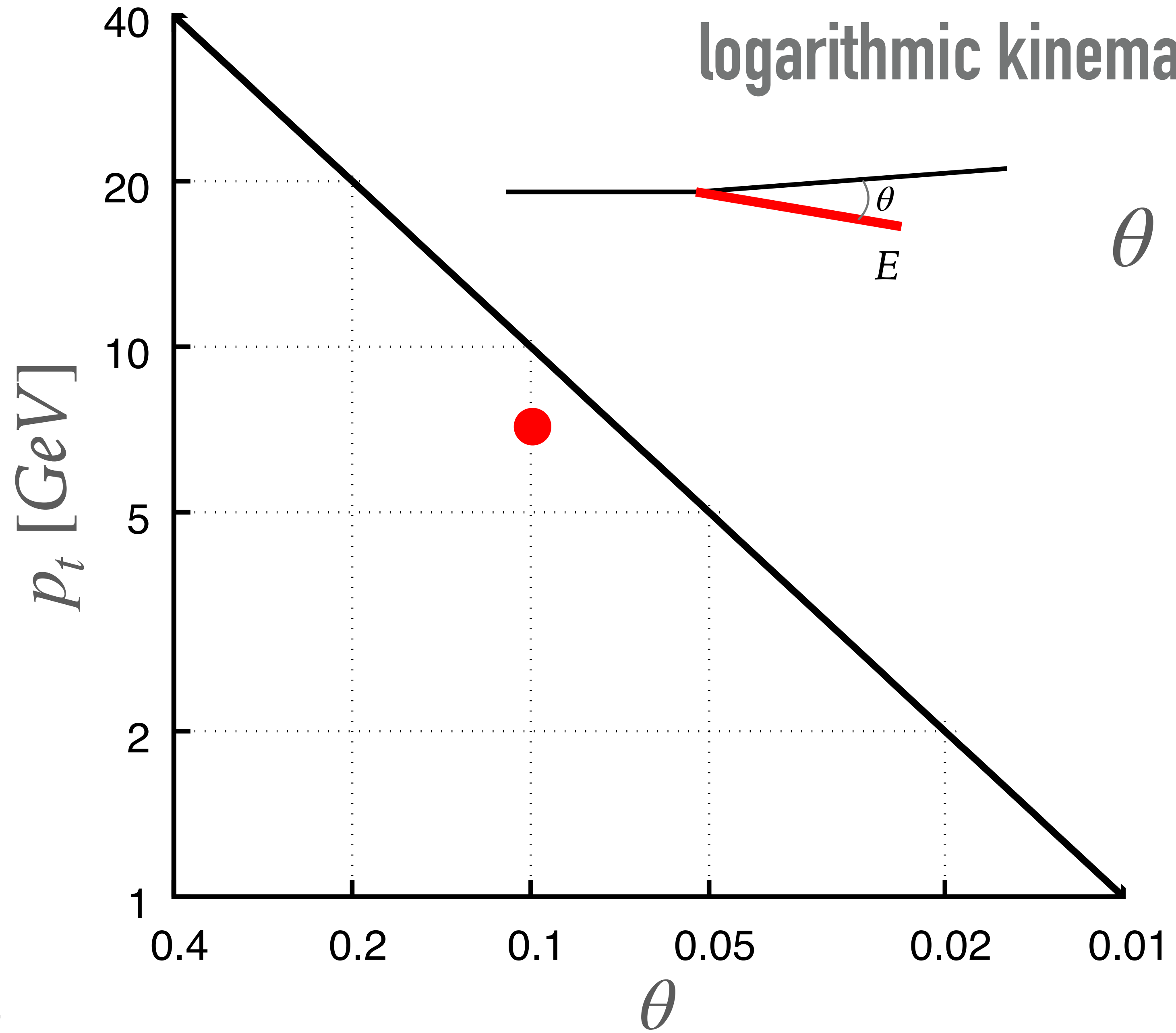
QCD **parton shower**: an evolution equation (in **evolution scale v** , e.g. trans.mom.)



self-similar
evolution
continues until it
reaches a non-
perturbative
scale

illustration by K. Hamilton

jet with $R = 0.4$, $p_t = 200 \text{ GeV}$



logarithmic kinematic plane whose two variables are

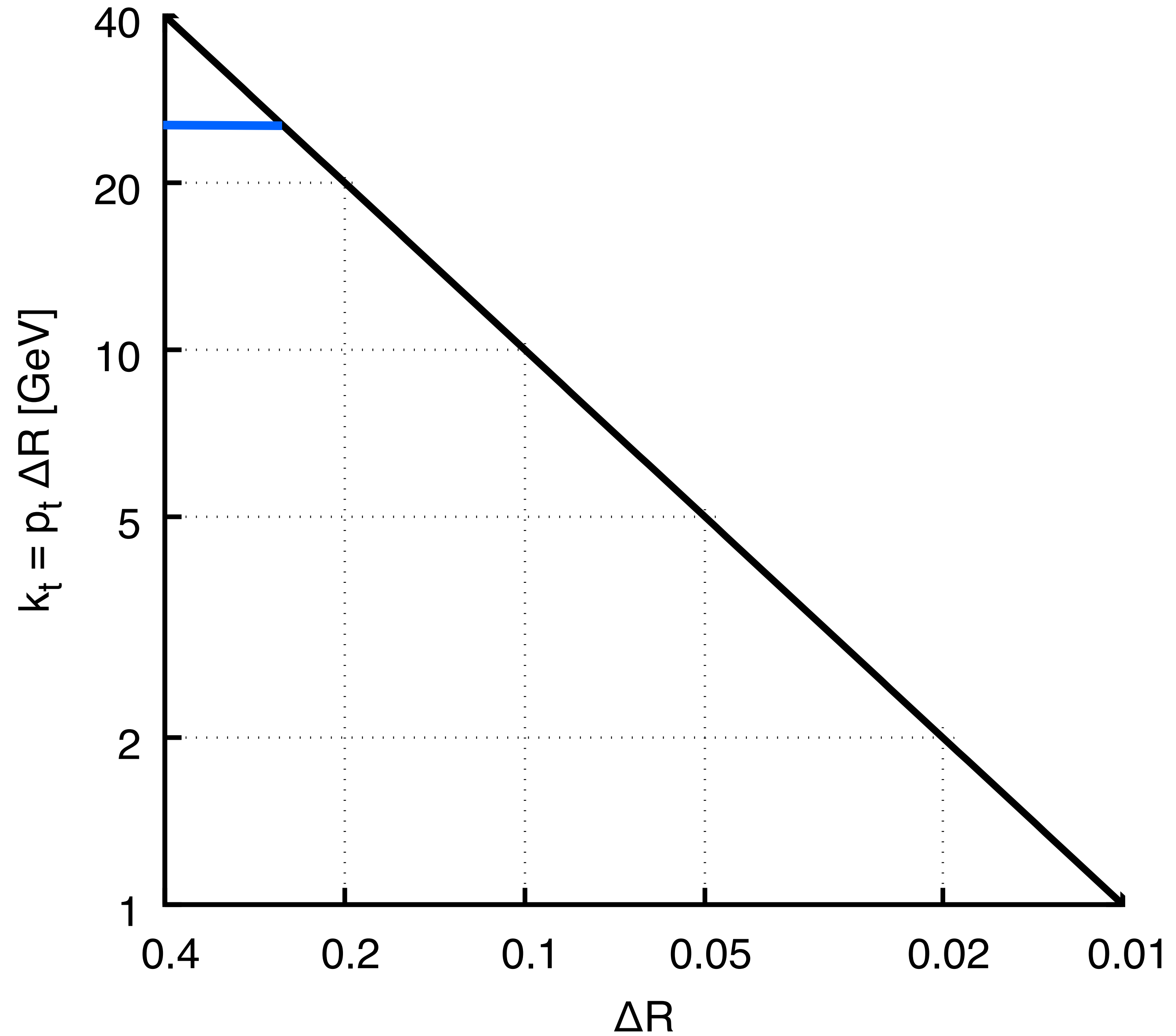
$$\theta \text{ (or } \eta = -\ln \tan \frac{\theta}{2} \text{)}$$

$$p_t = E\theta$$

Introduced for understanding Parton Shower Monte Carlos by B. Andersson, G. Gustafson L. Lonnblad and Pettersson 1989

The Lund Plane

jet with $R = 0.4$, $p_t = 200 \text{ GeV}$

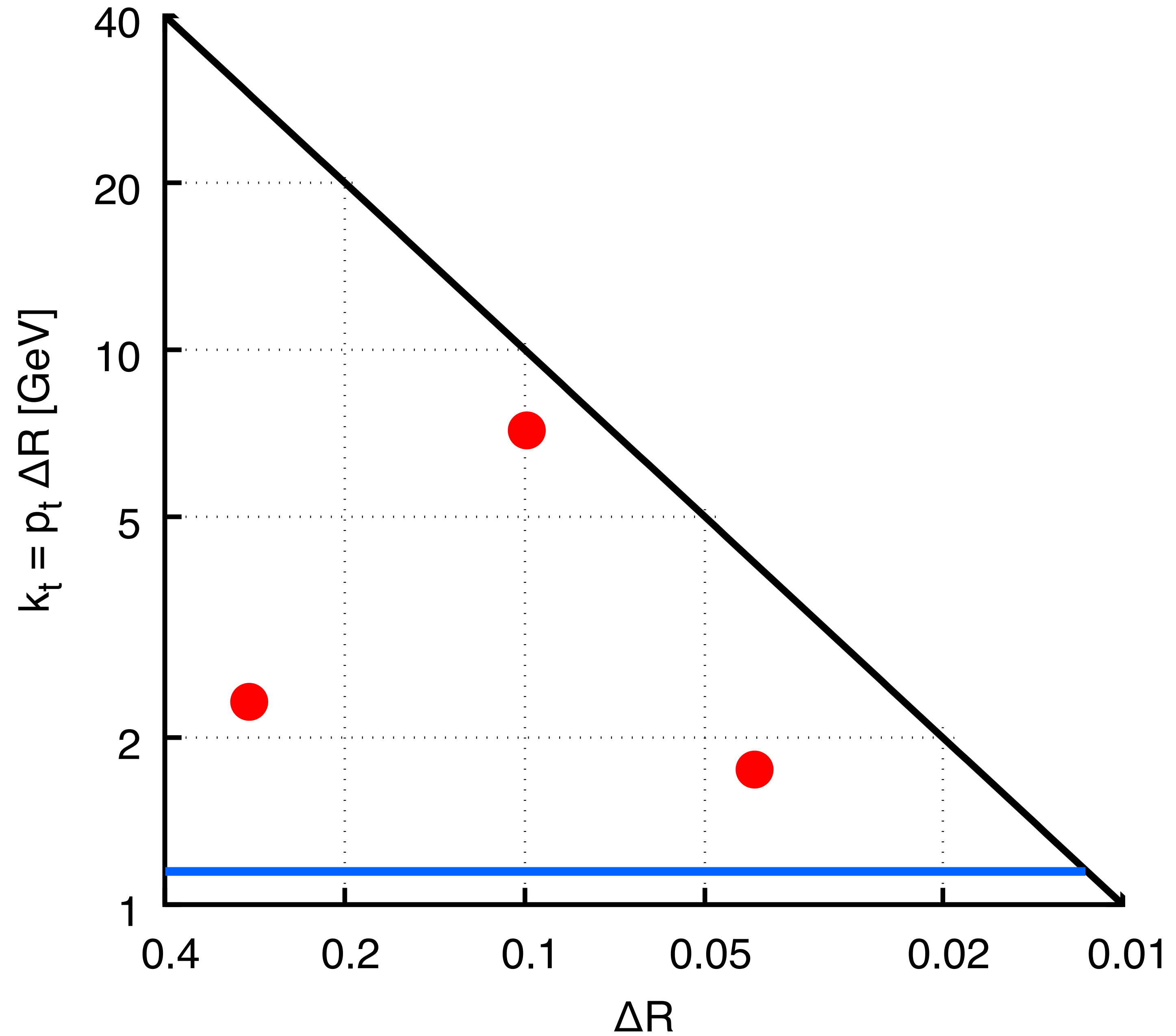


Parton shower is “differential equation” in one variable

You need to **choose** how one variable maps onto the two “logarithmic” dimensions of the Lund plane

A form of scheme choice

jet with $R = 0.4$, $p_t = 200 \text{ GeV}$

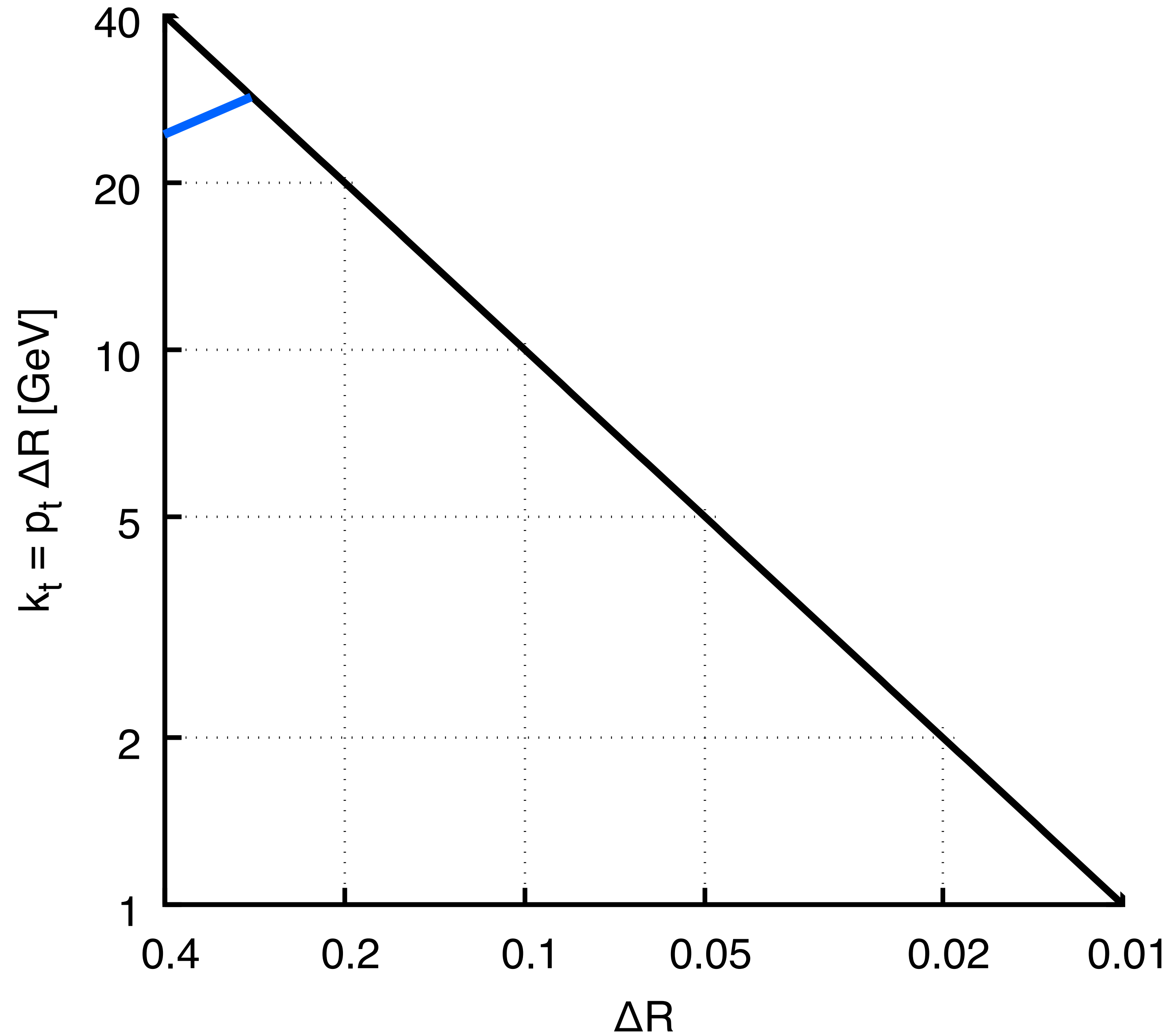


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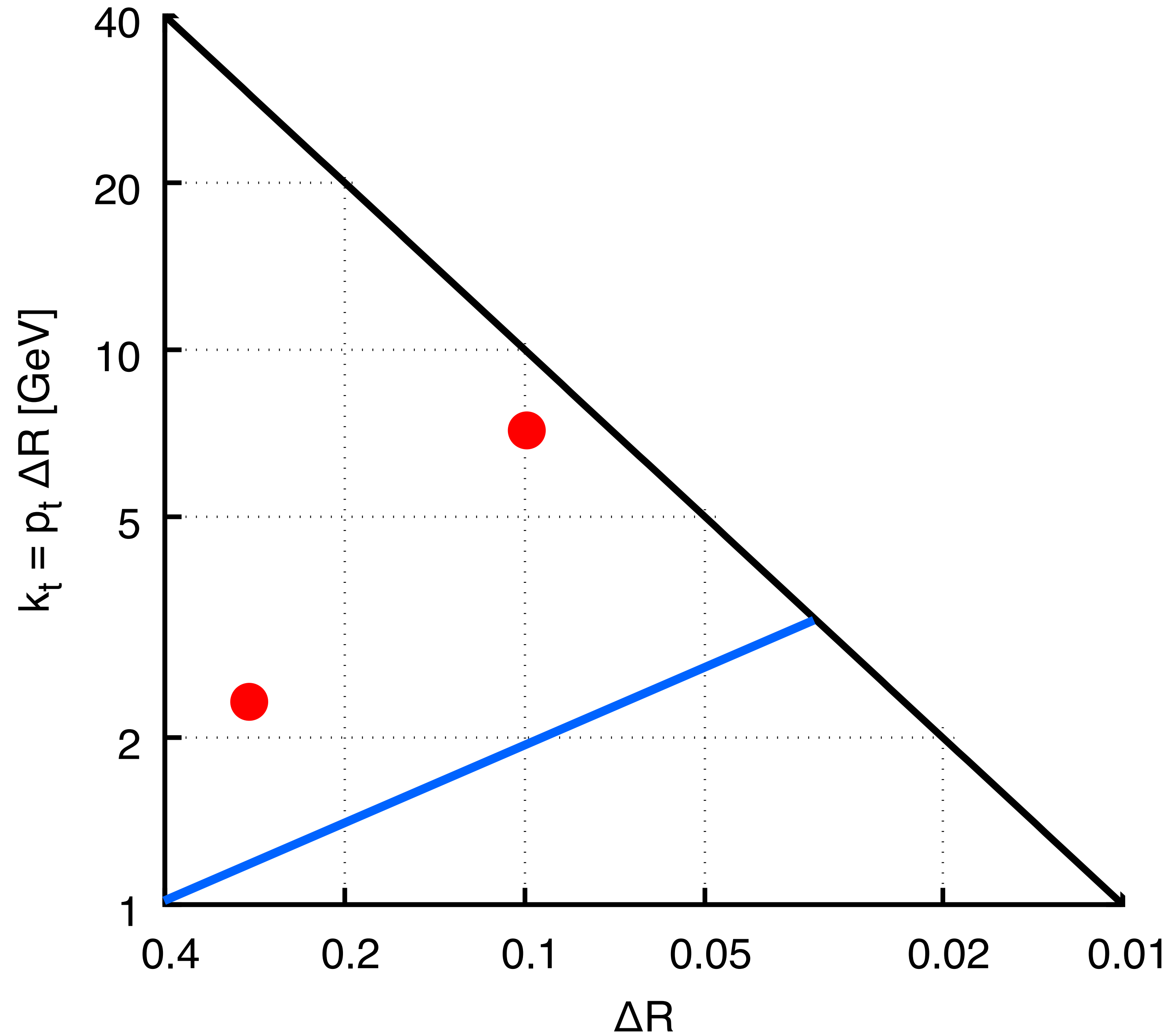


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A form of scheme choice — **up to the order you control, results should be independent of the scheme choice.**

jet with $R = 0.4$, $p_t = 200 \text{ GeV}$

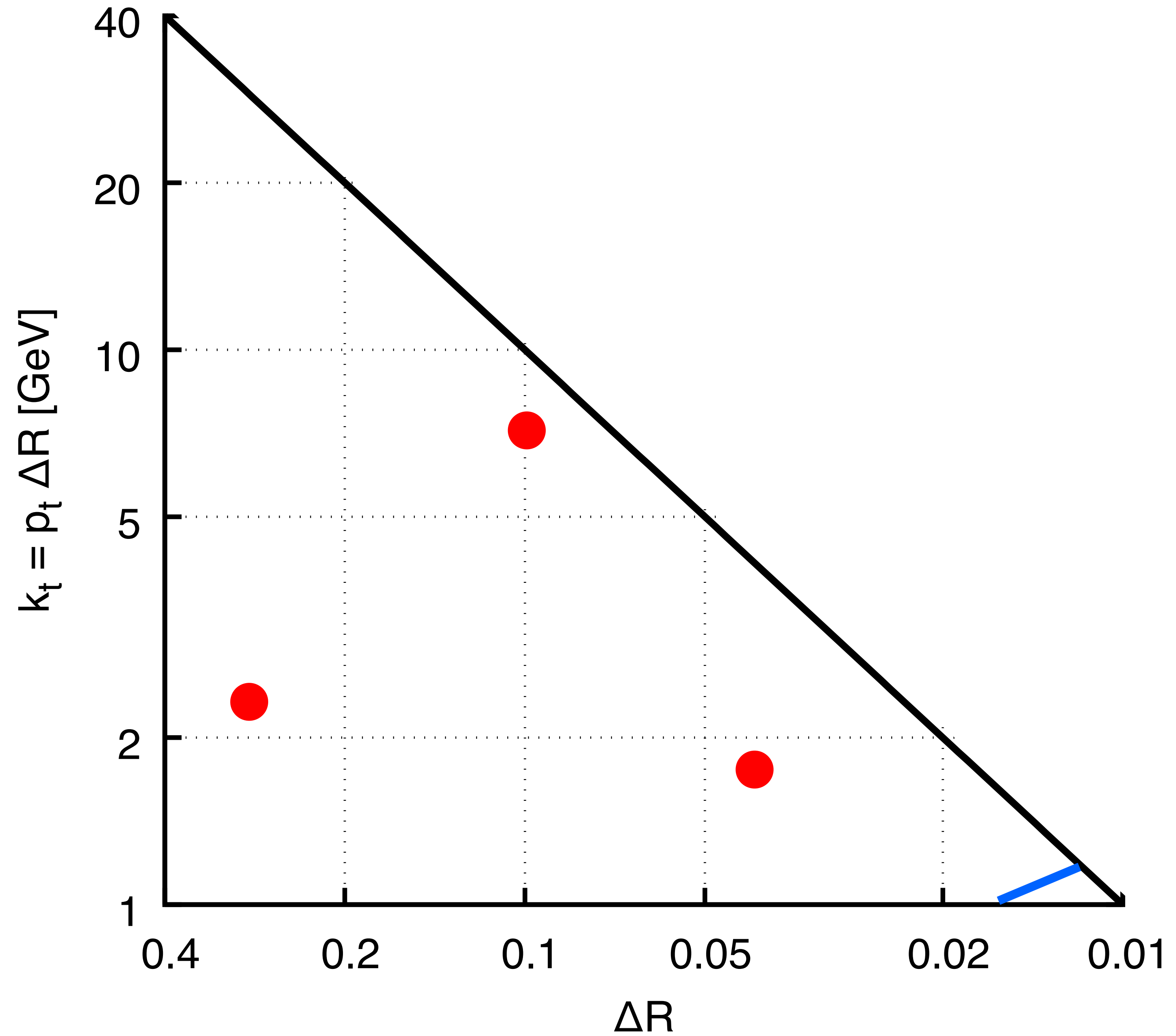


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Parton shower is “differential equation” in one variable

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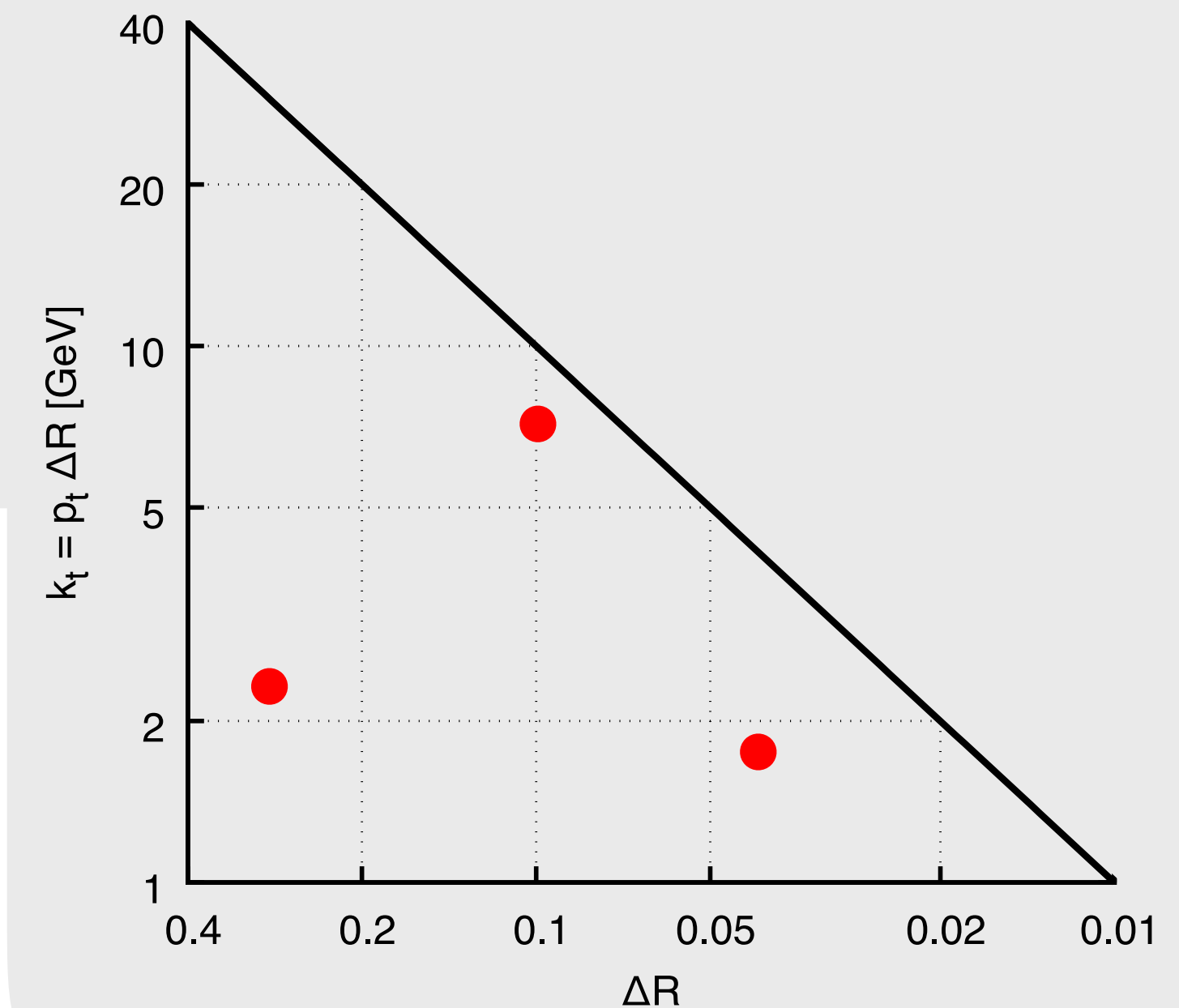
A Matrix Element condition

- correctly reproduce n -parton tree-level matrix element for arbitrary configurations, so long as all emissions well separated in the Lund diagram
(because those configurations are the most likely ones)
- supplement with unitarity, 2-loop running coupling & cusp anomalous dimension

Resummation condition:

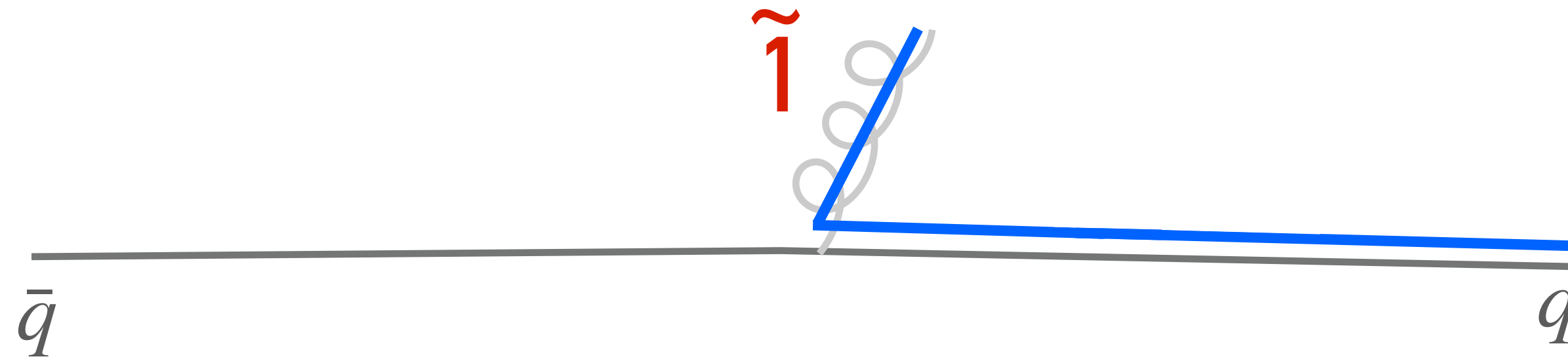
reproduce NLL ($\alpha_s^n L^n$) results for all standard resummations

- global event shapes
- non-global observables
- fragmentation functions ($\alpha_s^n L^n$ just leading log)
- multiplicities ($\alpha_s^n L^{2n-1}$)
- ...



1. Recoil: the core of any shower

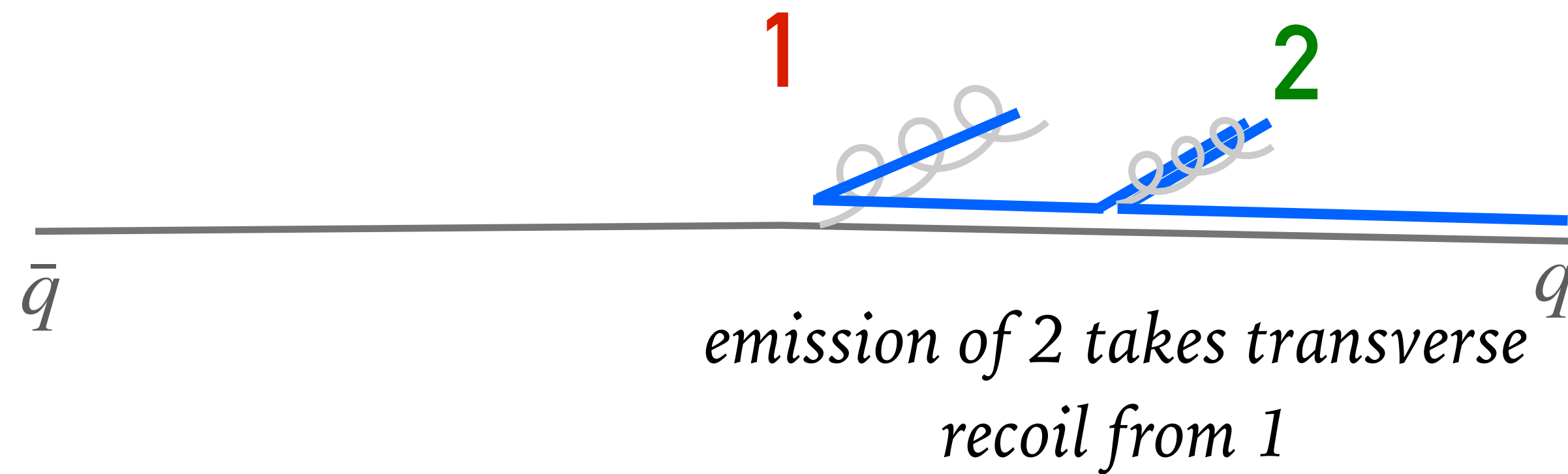
Dipole showers conserve momentum at each step. Traditional dipole-local recoil:



$$d\mathcal{P}_{\tilde{i} \rightarrow ik}^{\text{FS}} = \frac{\alpha_s(k_{\perp}^2)}{2\pi} \frac{dk_{\perp}^2}{k_{\perp}^2} \frac{dz}{z} \frac{d\varphi}{2\pi} N_{ik}^{\text{sym}} [z P_{\tilde{i} \rightarrow ik}(z)]$$

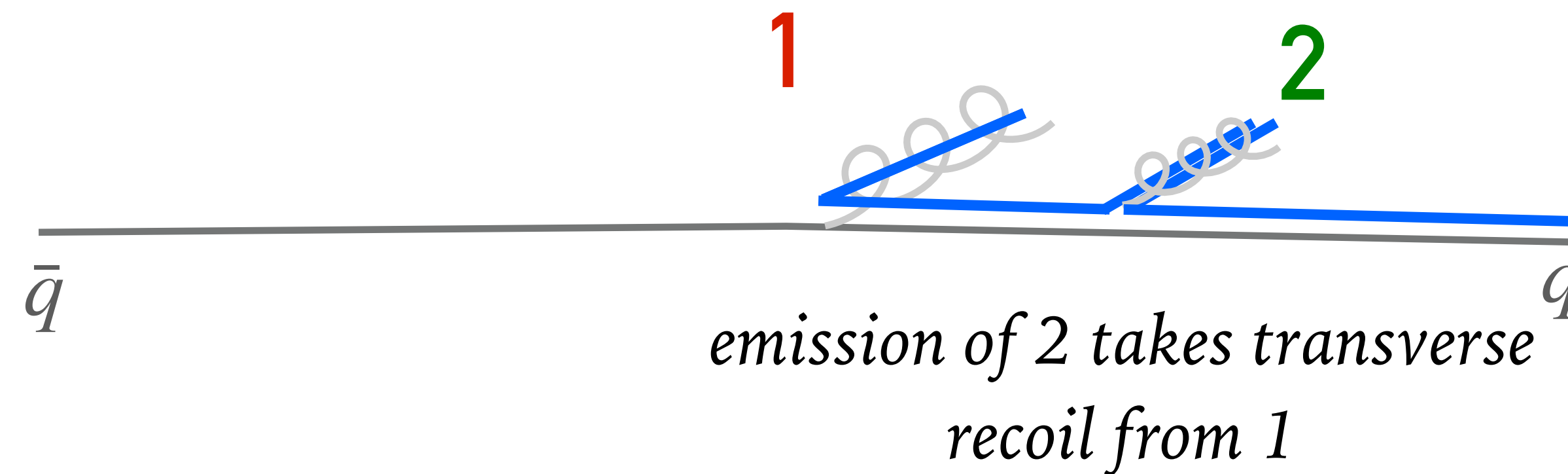
1. Recoil: the core of any shower

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Dipole showers conserve momentum at each step. Traditional dipole-local recoil:



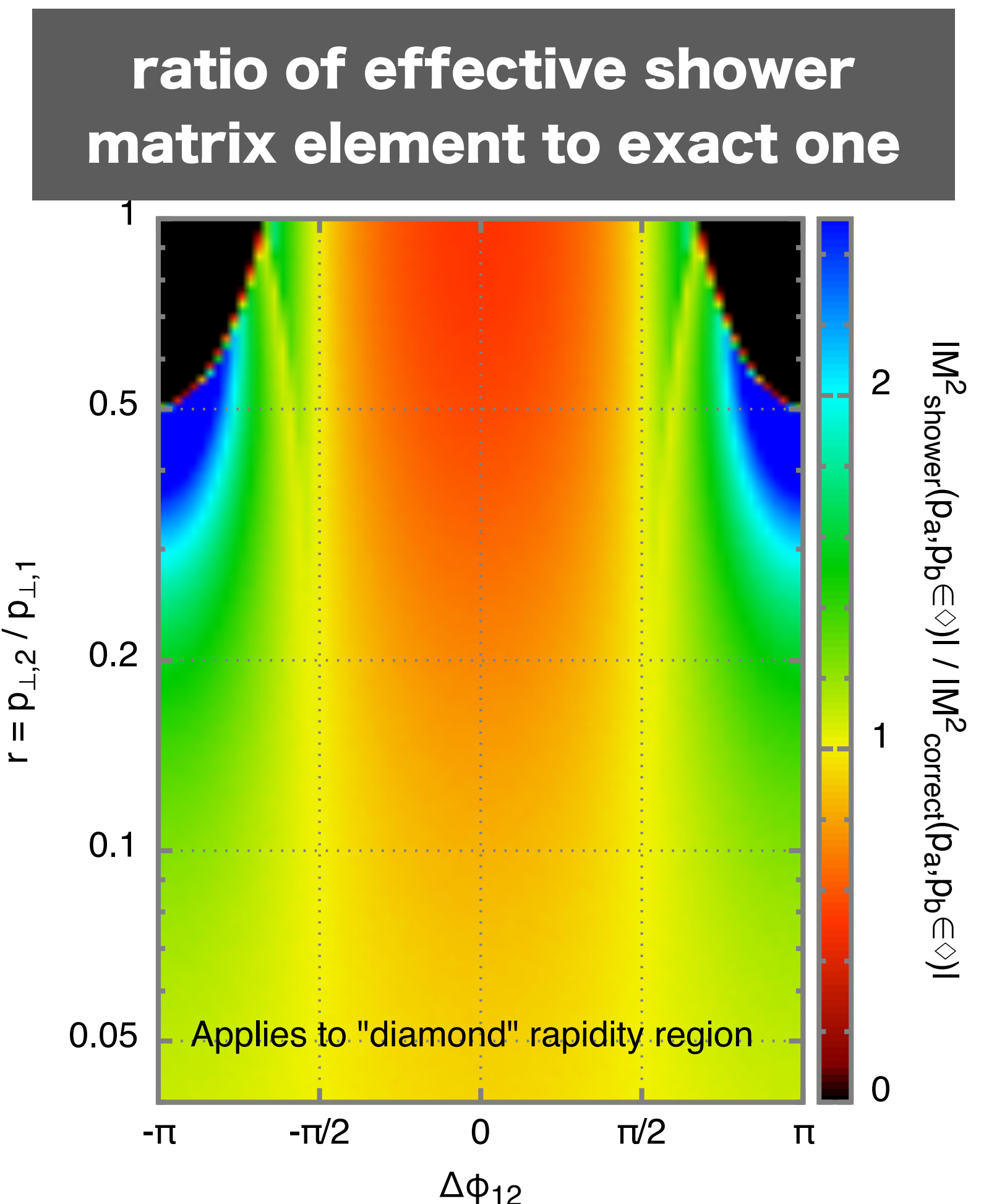
Shower initially generated matrix element for particle $\tilde{1}$, whose momentum differs (by $\sim 50\%$) from final particle 1.

Matrix element is incorrect wrt final momentum 1.

First observed: Andersson, Gustafson, Sjogren '92

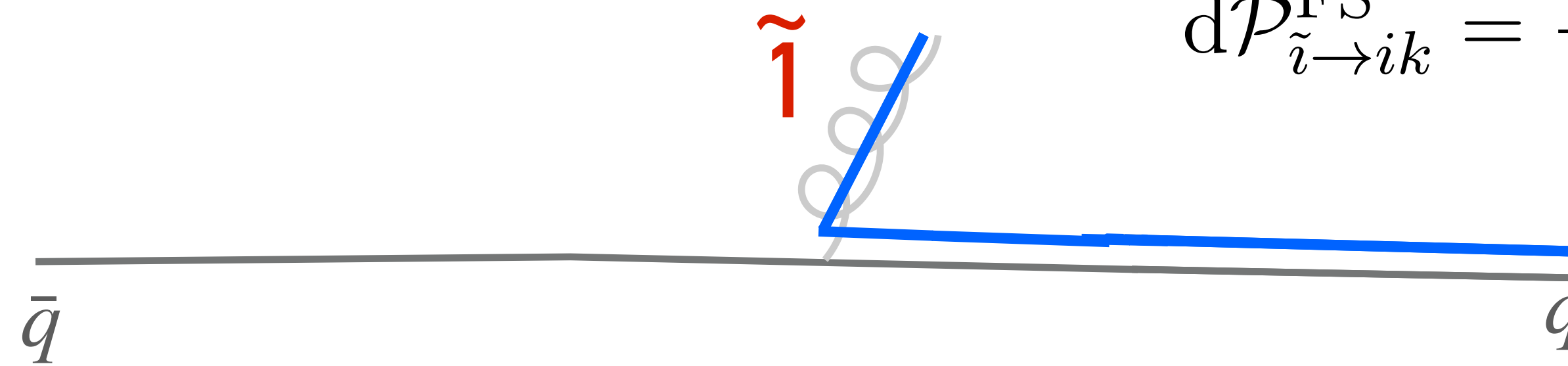
Closely related effect present for Z p_t : Nagy & Soper [0912.4534](#)

Impact on log accuracy across many observables: Dasgupta, Dreyer, Hamilton, Monni, GPS, [1805.09327](#)



1. Correct recoil rule: **no side effects on other distant emissions**

One approach

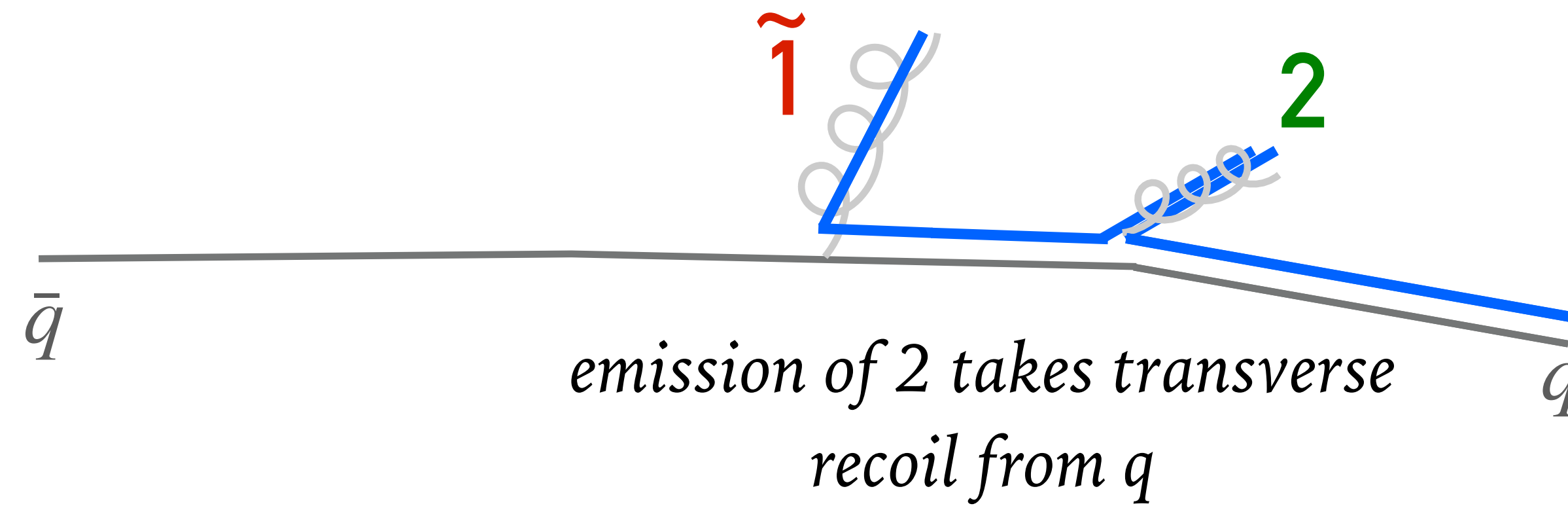


The diagram shows a horizontal line representing a quark. The left end is labeled \bar{q} and the right end is labeled q . A blue line branches off from the horizontal line, extending upwards and to the right. This blue line is labeled with a red $\tilde{1}$. A grey wavy line is attached to the blue line, representing a gluon emission.

$$d\mathcal{P}_{\tilde{i} \rightarrow ik}^{\text{FS}} = \frac{\alpha_s(k_{\perp}^2)}{2\pi} \frac{dk_{\perp}^2}{k_{\perp}^2} \frac{dz}{z} \frac{d\varphi}{2\pi} N_{ik}^{\text{sym}} [z P_{\tilde{i} \rightarrow ik}(z)]$$

1. Correct recoil rule: **no side effects on other distant emissions**

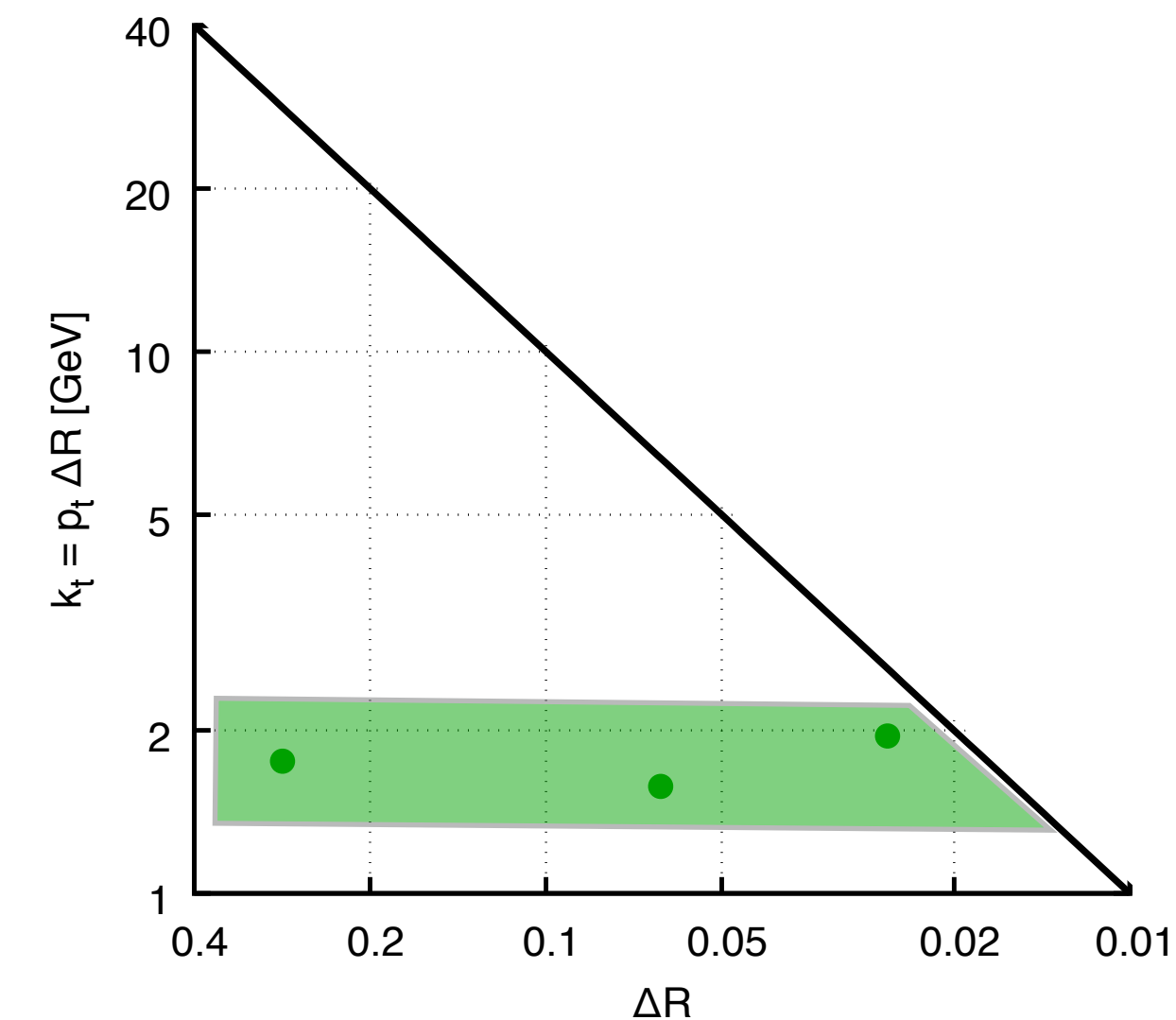
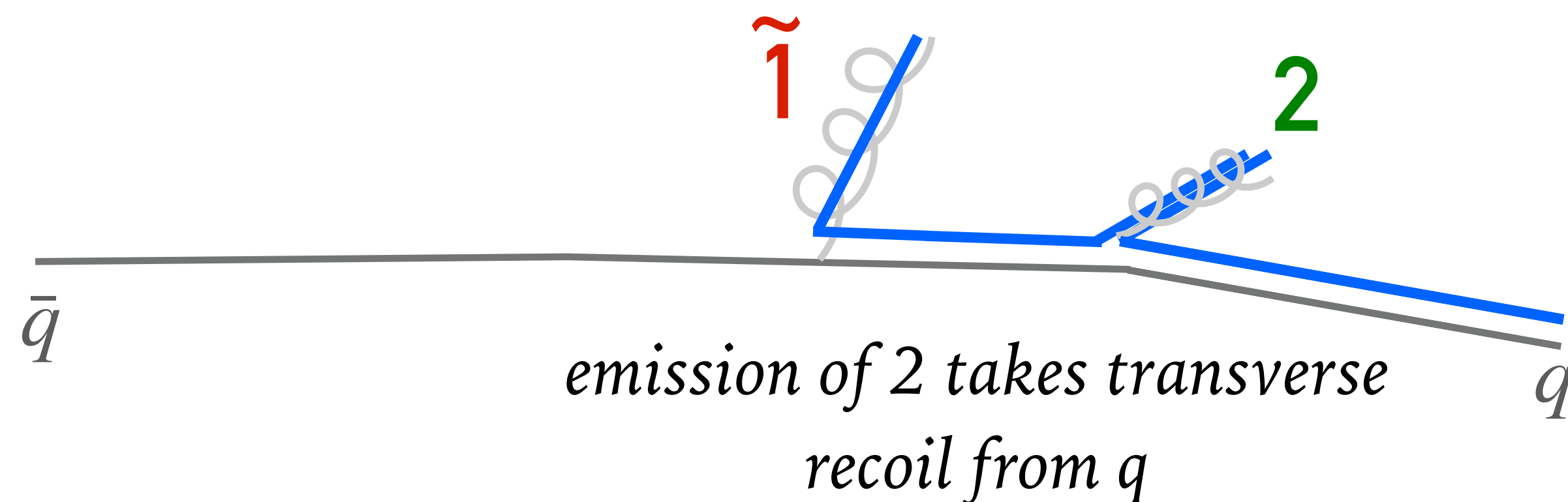
One approach



θ_{1q} left almost unchanged if \perp recoil from emission of 2 taken by (much harder) q

1. Correct recoil rule: **no side effects on other distant emissions**

One approach



θ_{1q} left almost unchanged if \perp recoil from emission of 2 taken by (much harder) q

Can be achieved in multiple ways:

- ▶ global transverse recoil
(Dasgupta et al [2002.11114](#), “PanGlobal”; Holguin Seymour & Forshaw [2003.06400](#); Alaric [2208.06057](#) + ..., Apollo, [2403.19452](#))
- ▶ local transverse recoil, with non-standard shower ordering & dipole partition
([2002.11114](#) “PanLocal”; Nagy & Soper [0912.4534](#) + ..., “Deducto”)

Alaric

Apollo

FHP

Panscales

Available for e+e-?	NLL	NLL	NLL	Partial NNLL
Available for pp?	NLL (no numerical proof beyond colour singlet)	No	No	NLL (no numerical proof beyond colour singlet)
Available for DIS?	No	No	No	NLL
Recoil distribution	Global for soft, local pT for collinear	As Alaric's soft map, different ord. variable	Global	Local (PanLocal) and global (PanGlobal) variants
Splitting functions at LO	Soft with special ref. direction, +coll remainder	Antenna functions	AP with sharp partitioning	AP with smooth partitioning
Spin correlations	Yes in e+e-	No	Yes in e+e-	Yes at NLL
Corrections beyond LC	No	No	No	Yes at NLL (NODS)
NLO matching available?	Yes in e+e-, using S-MC@NLO at exact colour	Yes in e+e- on paper, not yet implemented	Yes in e+e-	Yes for CS (dBNLO, P2B or positive weights ESME)
Multi-jet merging?	In e+e-; 5 jets (NLO), in pp; 3 jets (LO)	No	No	No
Particle masses?	Yes in e+e-	No	No	Yes in e+e-
Resonances?	Yes in e+e-	No	No	No
Publicly available?	Python codes, should become part of Sherpa	No, but should become part of Pythia8	No, but should become part of Herwig	Yes, interfaced to Pythia8

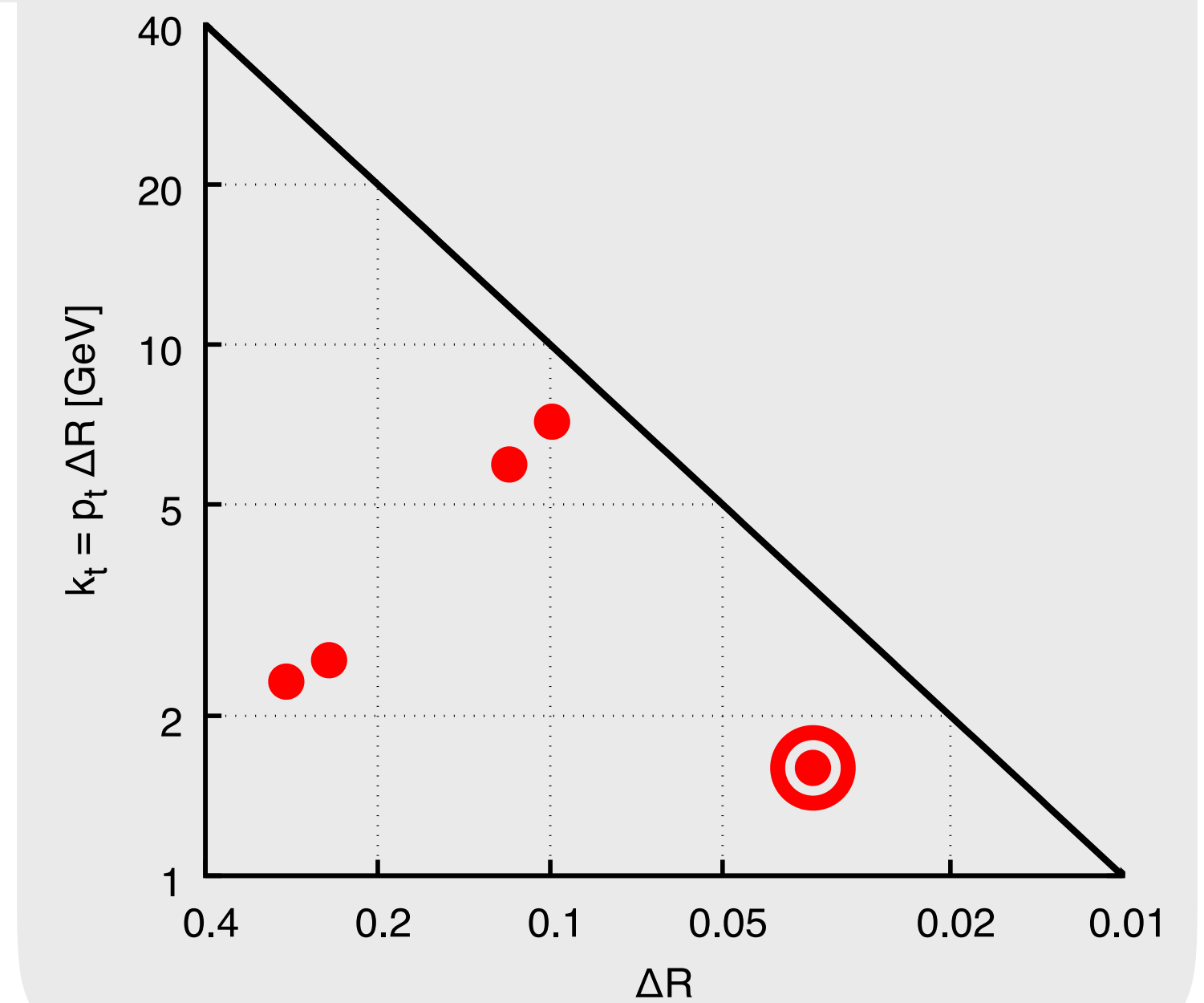
A Matrix Element condition

- correctly reproduce n -parton tree-level matrix element for arbitrary configurations, so long as all groups of up to p emissions are well separated in the Lund diagram
- For groups of k emissions, be accurate up to $p - k$ loops
- supplement with unitarity, $p + 1$ -loop running coupling & cusp anomalous dimension

Resummation condition:

reproduce N^pLL ($\alpha_s^n L^{n+1-p}$) results for all standard resummations

- global event shapes
- non-global observables
- fragmentation functions ($\alpha_s^n L^{n+1-p} \equiv N^{p-1}$ LL)
- multiplicities ($\alpha_s^n L^{2n-p}$)
- ...





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GPS
Oxford



Nicolas Schalch
Oxford



Ludovic Scyboz
Monash



Alba Soto-Ontoso
Granada



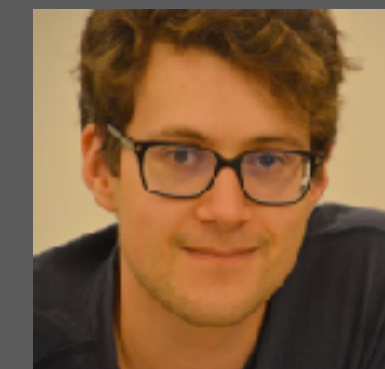
Grégory Soyez
IPhT, Saclay



Silvia Zanoli
Oxford

PanScales

A project to bring logarithmic understanding and accuracy to parton showers



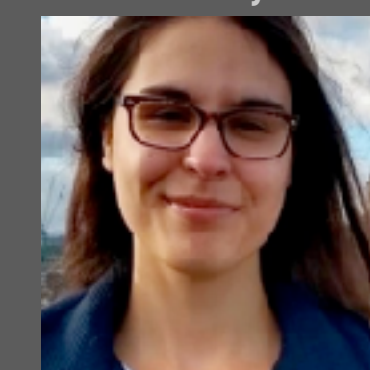
Frédéric Drever



Rob Verheyen



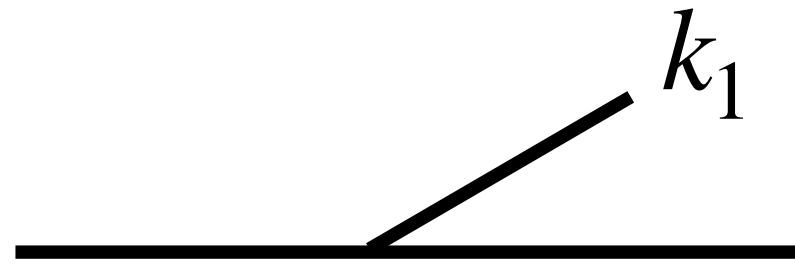
Rok Medves



Emma Slade

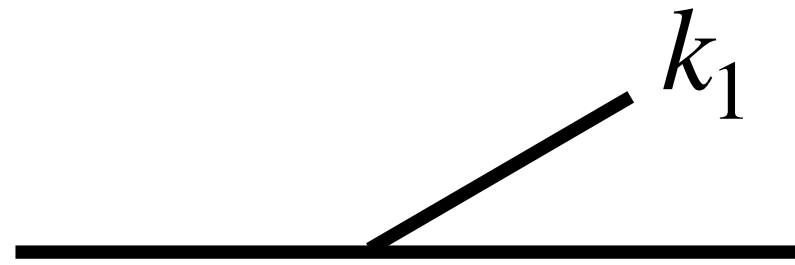
former members

Towards NLL: each new emission's distribution conditional on previous emission

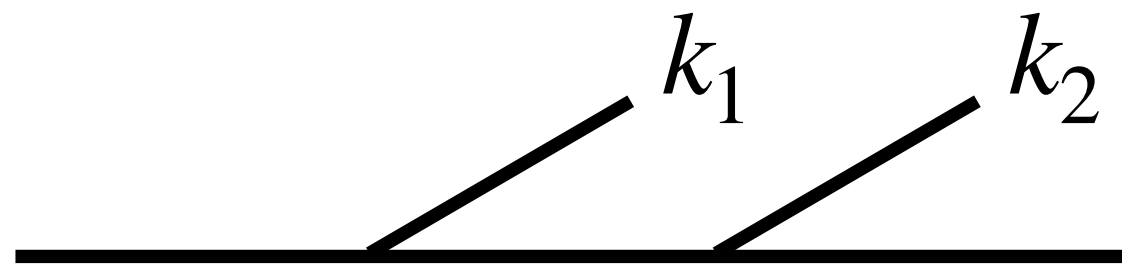


Distribute k_1 according to $M^2(k_1)$

Towards NLL: each new emission's distribution conditional on previous emission

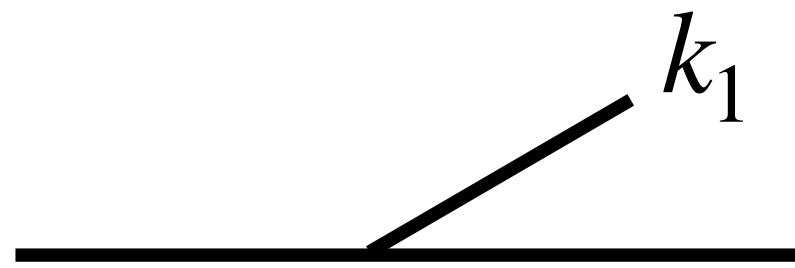


Distribute k_1 according to $M^2(k_1)$

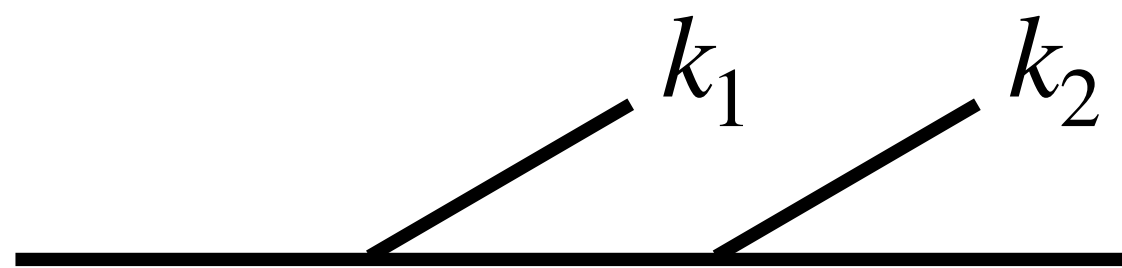


Distribute k_2 according to $M^2(k_1, k_2)/M^2(k_1)$

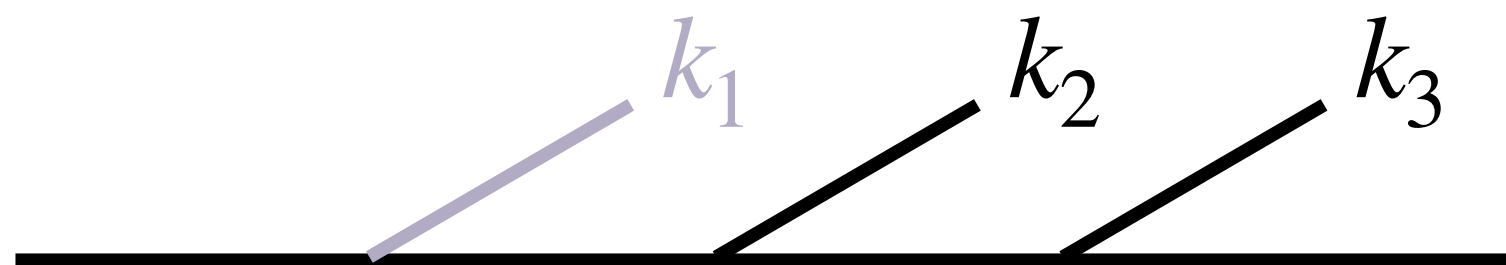
Towards NLL: each new emission's distribution conditional on previous emission



Distribute k_1 according to $M^2(k_1)$

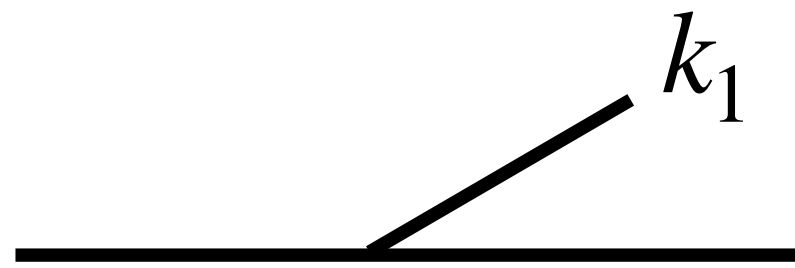


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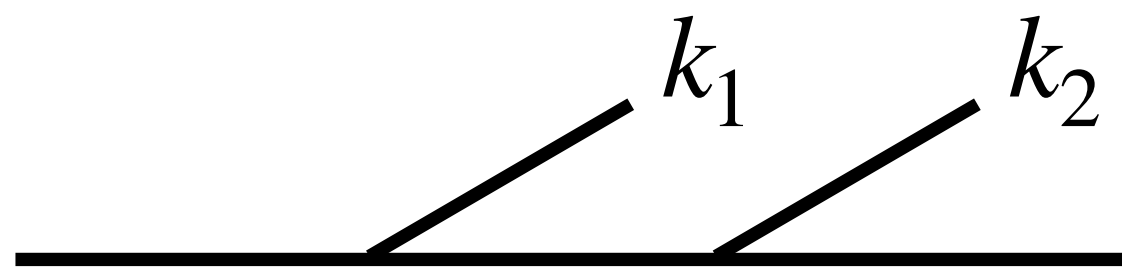


Distribute k_3 according to $M^2(k_2, k_3)/M^2(k_2)$

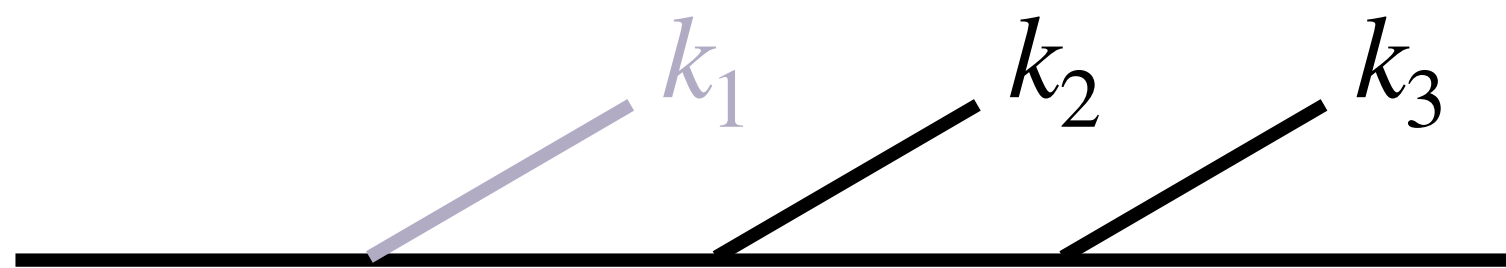
Towards NLL: each new emission's distribution conditional on previous emission



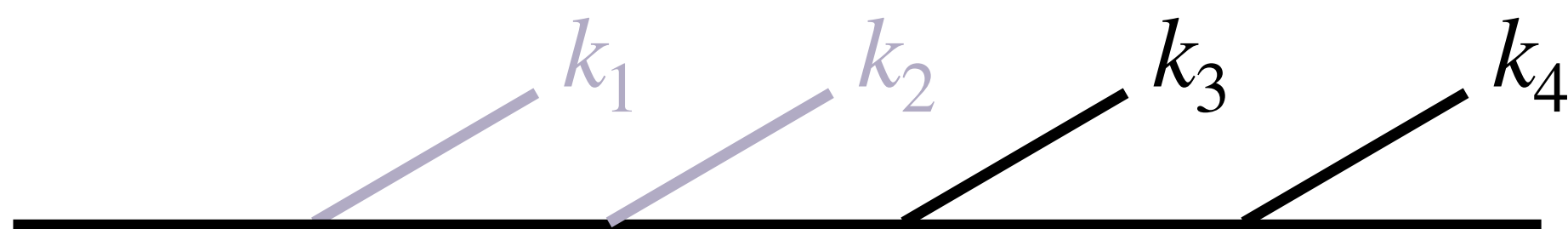
Distribute k_1 according to $M^2(k_1)$



Distribute k_2 according to $M^2(k_1, k_2)/M^2(k_1)$

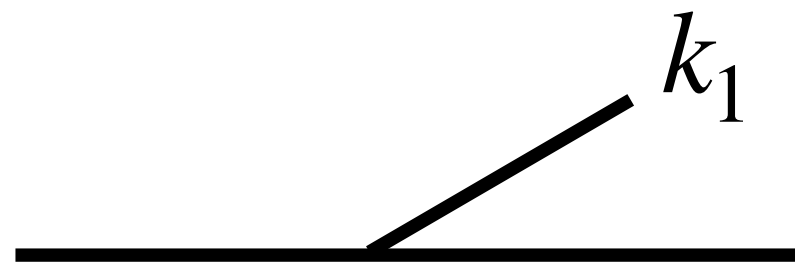


Distribute k_3 according to $M^2(k_2, k_3)/M^2(k_2)$

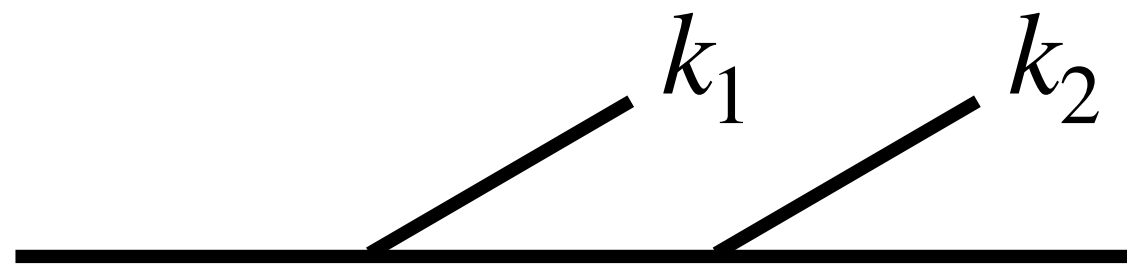


Distribute k_4 according to $M^2(k_3, k_4)/M^2(k_3)$

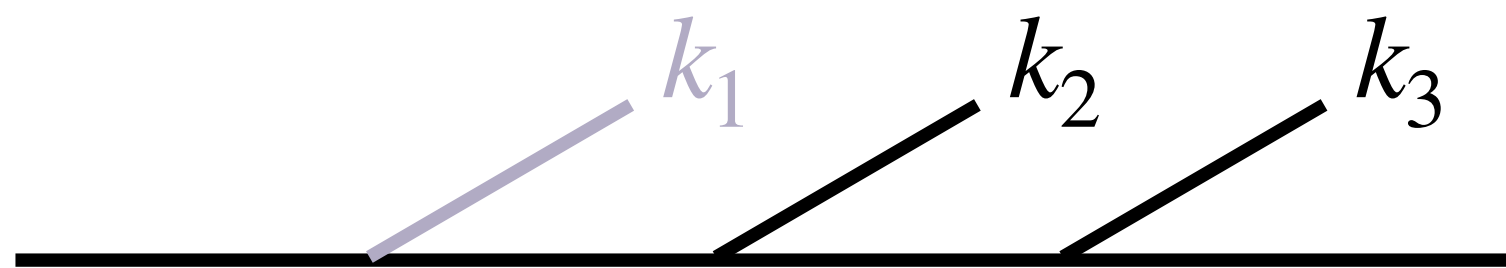
Towards NLL: each new emission's distribution conditional on previous emission



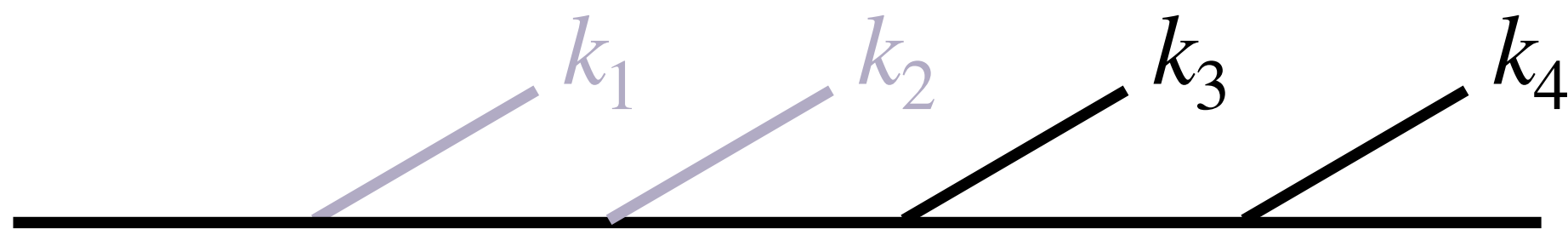
Distribute k_1 according to $M^2(k_1)$



Distribute k_2 according to $M^2(k_1, k_2)/M^2(k_1)$



Distribute k_3 according to $M^2(k_2, k_3)/M^2(k_2)$

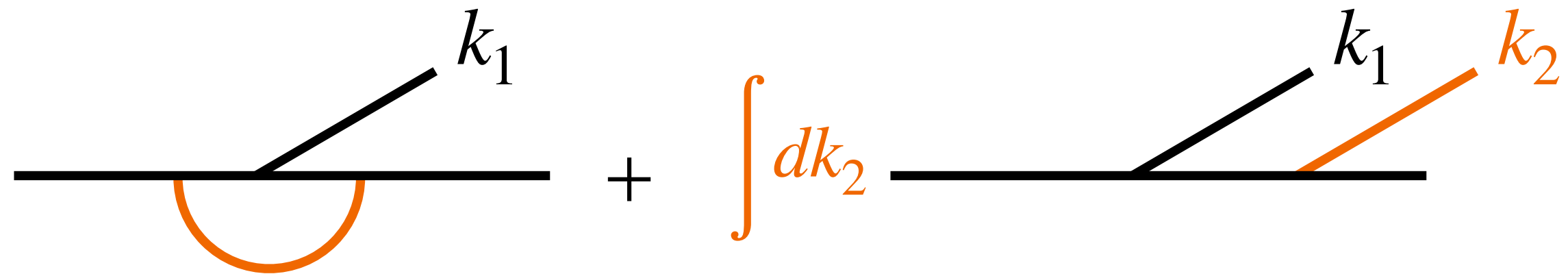


Distribute k_4 according to $M^2(k_3, k_4)/M^2(k_3)$

Relies on factorisation: e.g. $M^2(k_1, k_2, k_3, k_4)/M^2(k_1, k_2, k_3) \rightarrow M^2(k_3, k_4)/M^2(k_3)$
if 3 and 4 well separated in Lund plane from 1 and 2

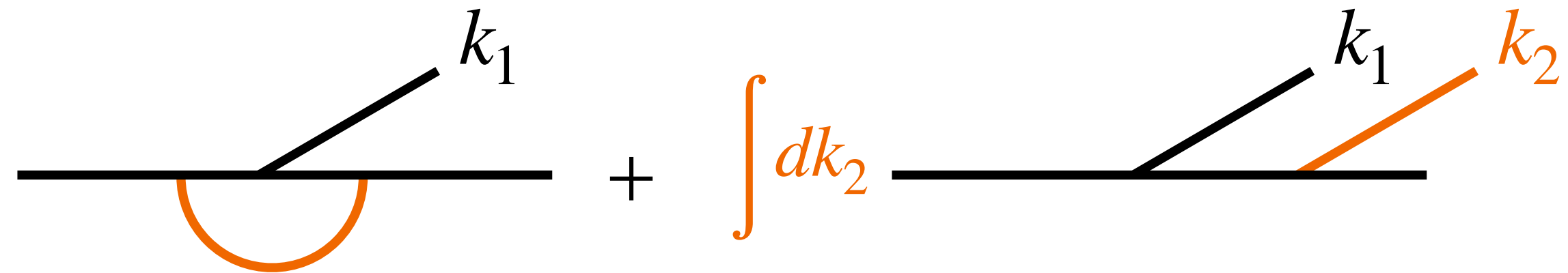
[factorised matrix elements given in Dokshitzer, Marchesini & Oriani '92, Campbell & Glover, [hep-ph/9710255](https://arxiv.org/abs/hep-ph/9710255),
Catani & Grazzini [hep-ph/9810389](https://arxiv.org/abs/hep-ph/9810389), etc.]

Account for virtual corrections associated with each emission

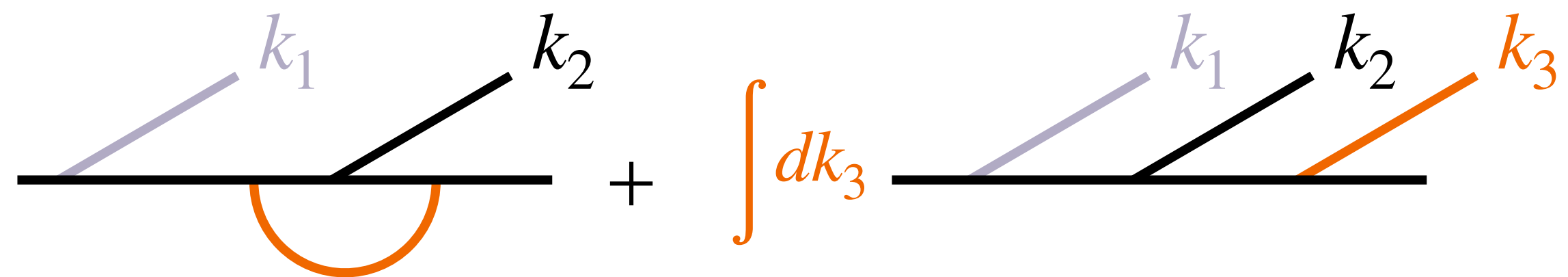


NLO correction to k_1 emission
intensity sums **loop** correction and **all possible scenarios for the next emission**

Account for virtual corrections associated with each emission



NLO correction to k_1 emission intensity sums **loop** correction and **all possible scenarios for the next emission**



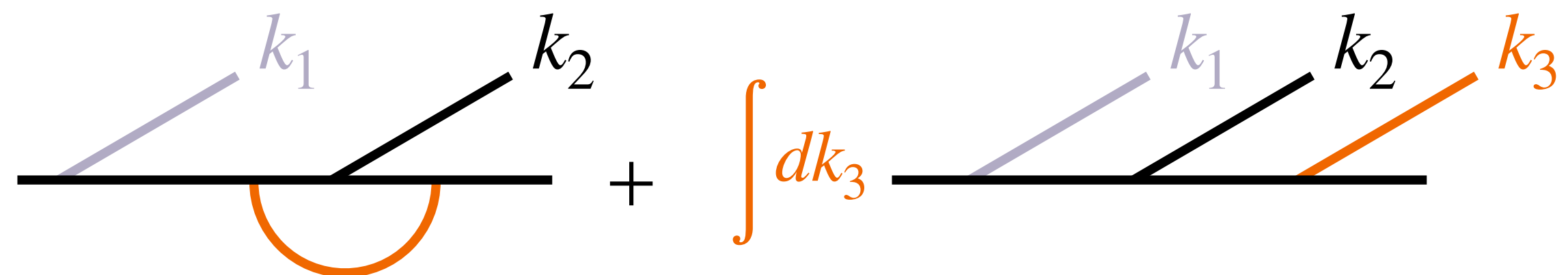
NLO correction to k_2 emission intensity sums loop correction and all possible scenarios for the next emission

etc.

Account for virtual corrections associated with each emission



NLO correction to k_1 emission intensity sums **loop** correction and **all possible scenarios for the next emission**

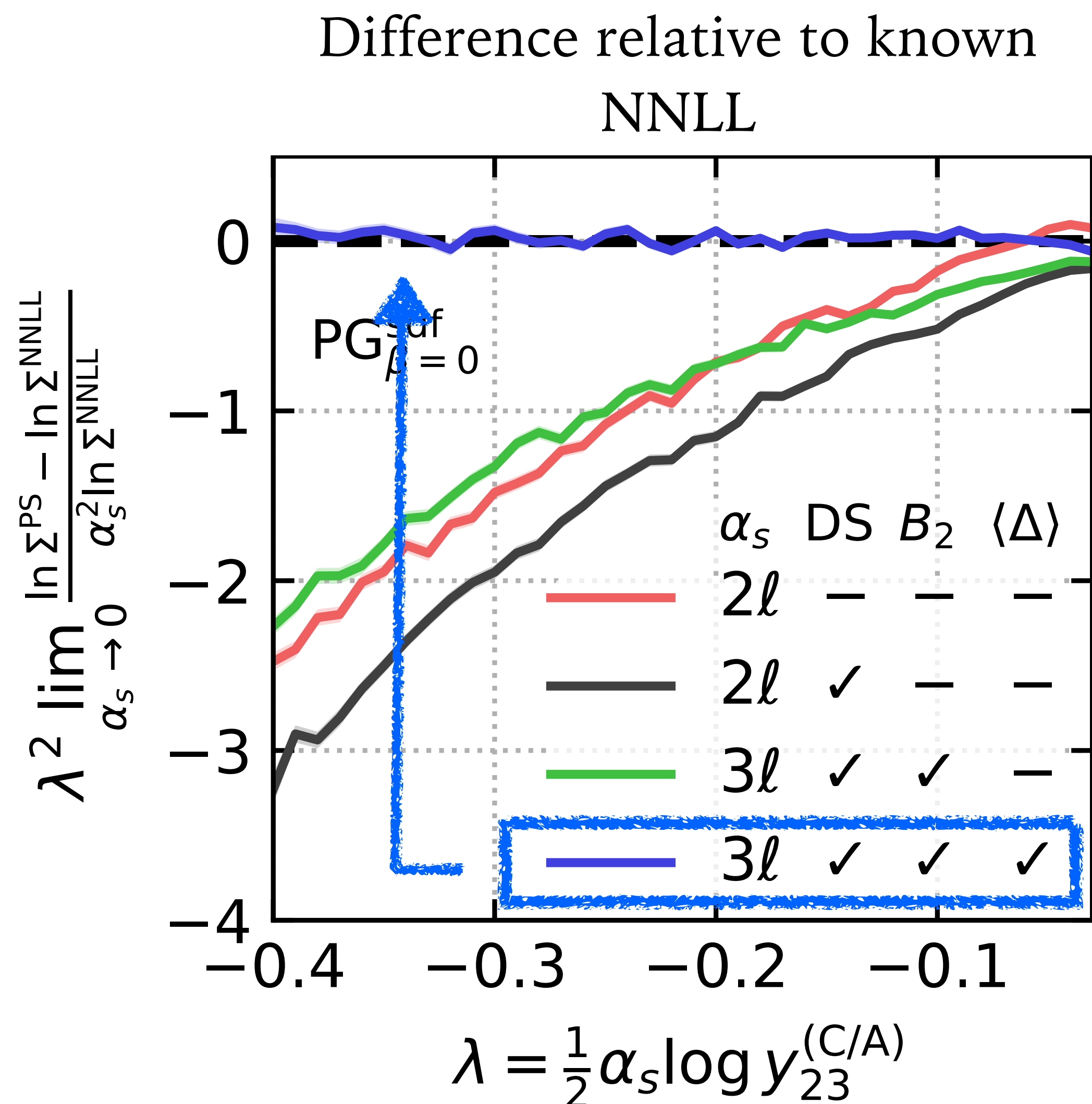


NLO correction to k_2 emission intensity sums loop correction and all possible scenarios for the next emission

etc.

Again relies on factorisation, e.g. when 1 and 2 are well separated in the Lund plane
+ careful nesting, cf. van Beekveld, Dasgupta, El-Menoufi, Helliwell, Monni, [GPS 2409.08316](#)
(see also Hartgring, Laenen & Skands, 1303.4974, Campbell et al [2108.07133](#) at fixed order)

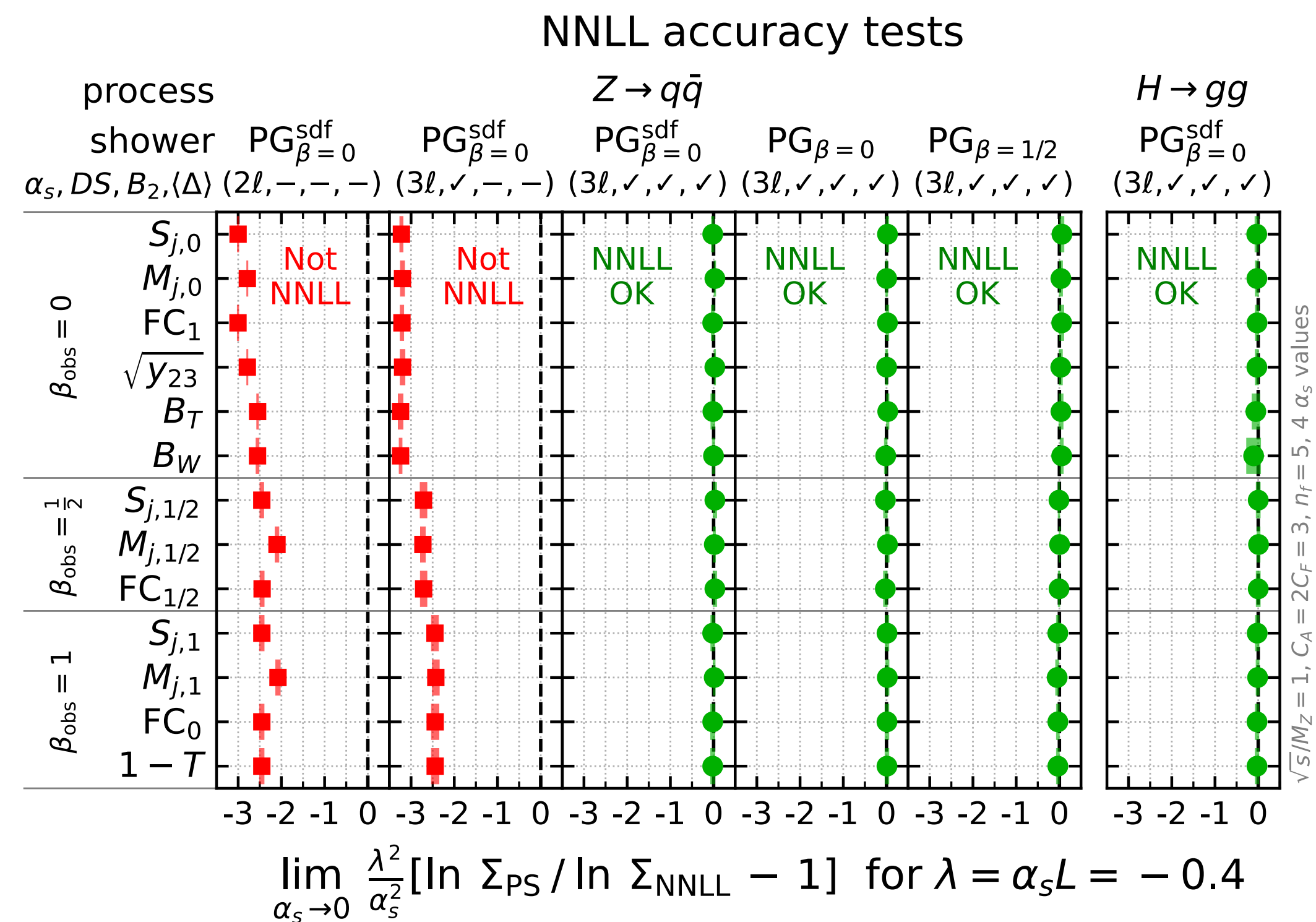
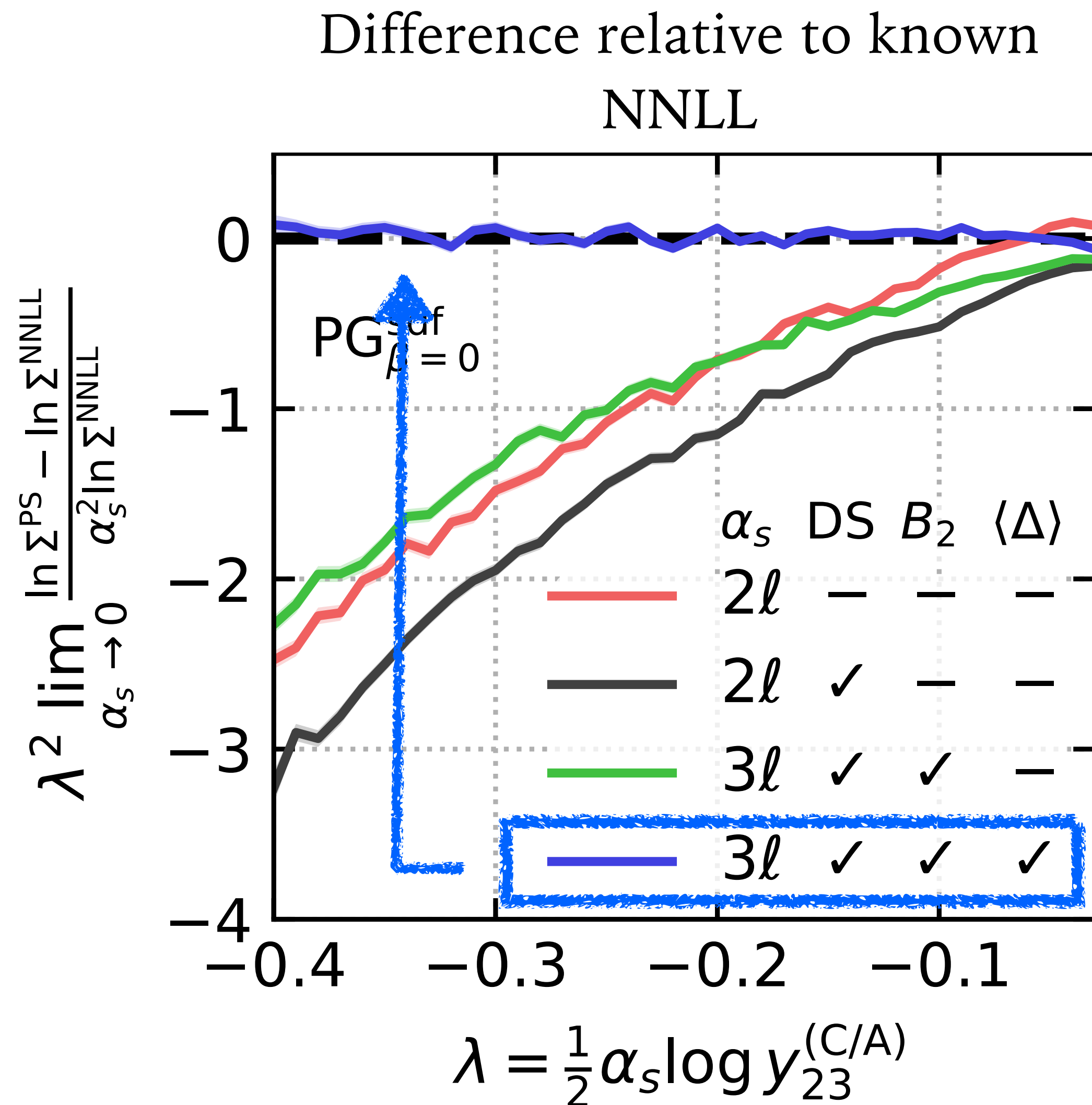
Testing NNLL for event shapes (so far only for e^+e^- collisions)



need to analyse and account for all possible sources of NNLL contribution

(some, which don't affect event shapes, are still work in progress)

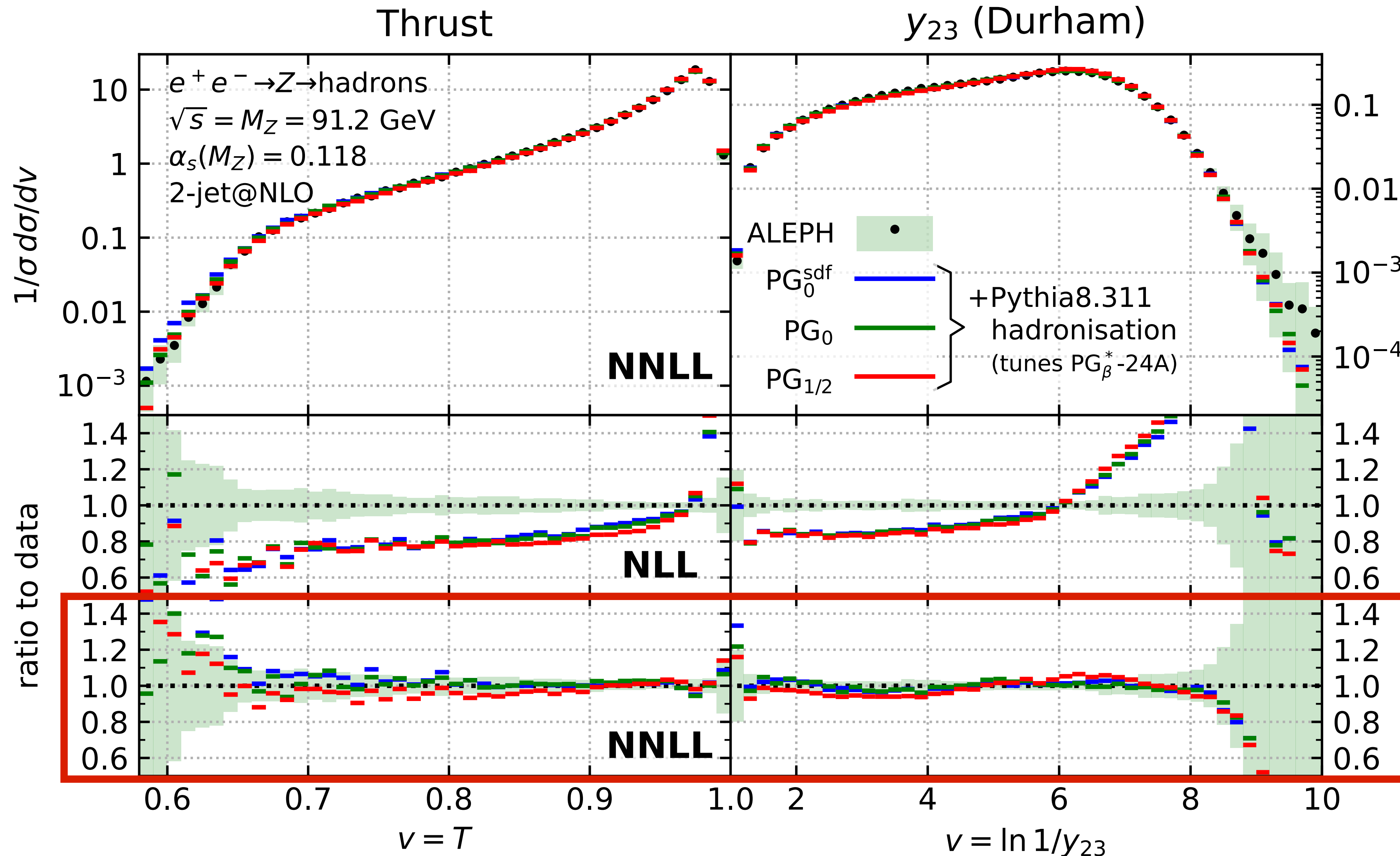
Testing NNLL for event shapes (so far only for e⁺e⁻ collisions)



need to analyse and account for all possible sources of NNLL contribution

(some, which don't affect event shapes, are still work in progress)

Comparing to LEP event-shape data



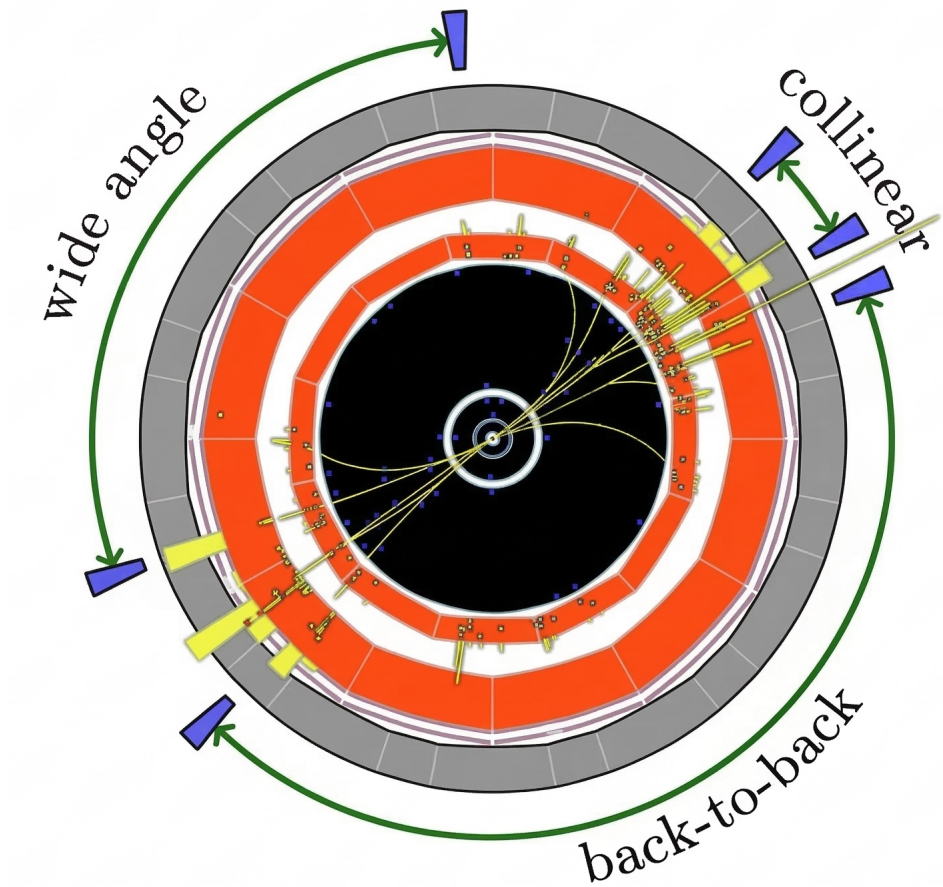
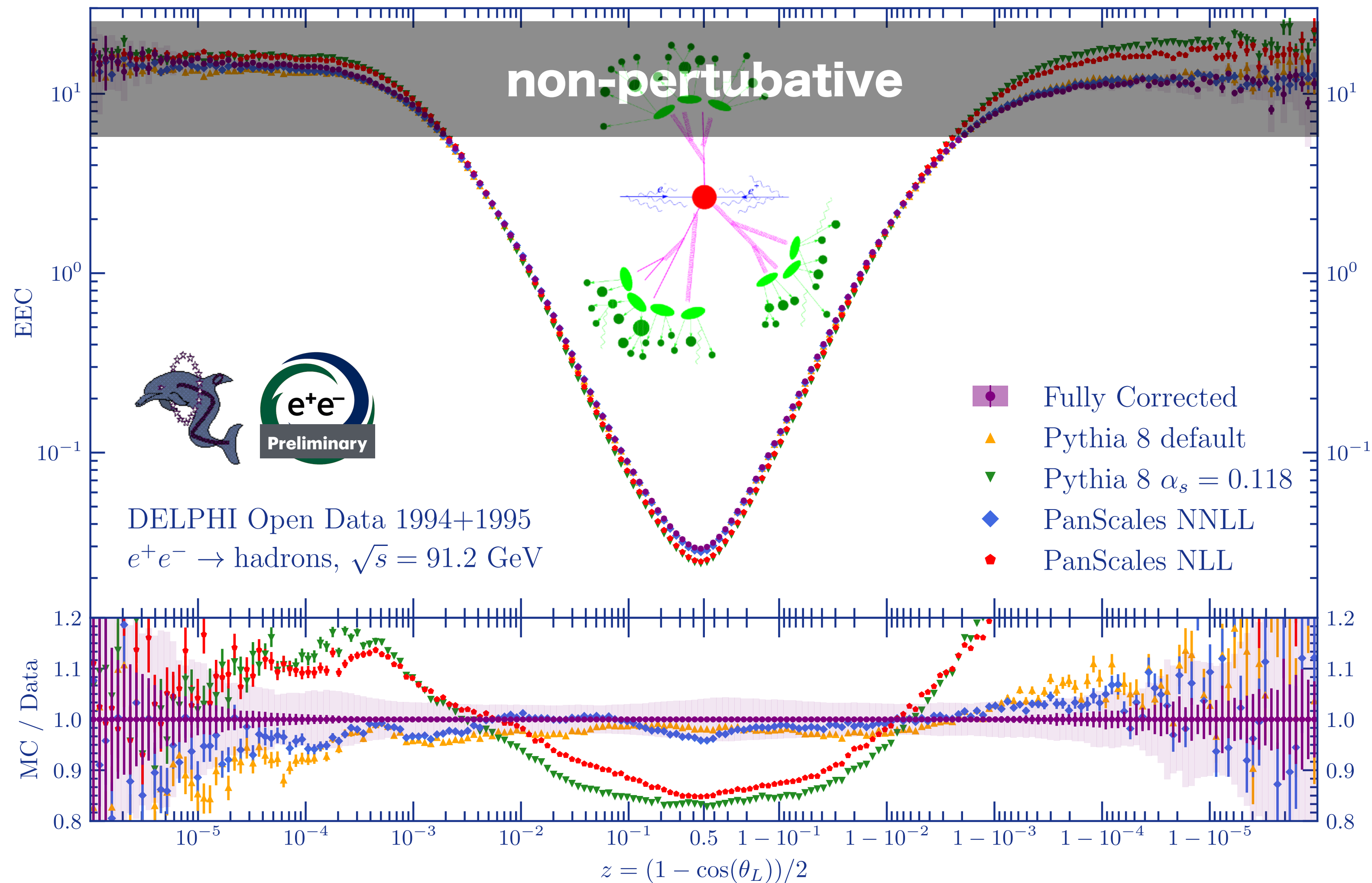
NNLL brings 20%
 effects ($\sim \alpha_s$)

Dramatically improves
 agreement with data,
 using a “normal”
 $\alpha_s = 0.118$

NB: 3-jet @ NLO still
 missing for robust
 pheno conclusions

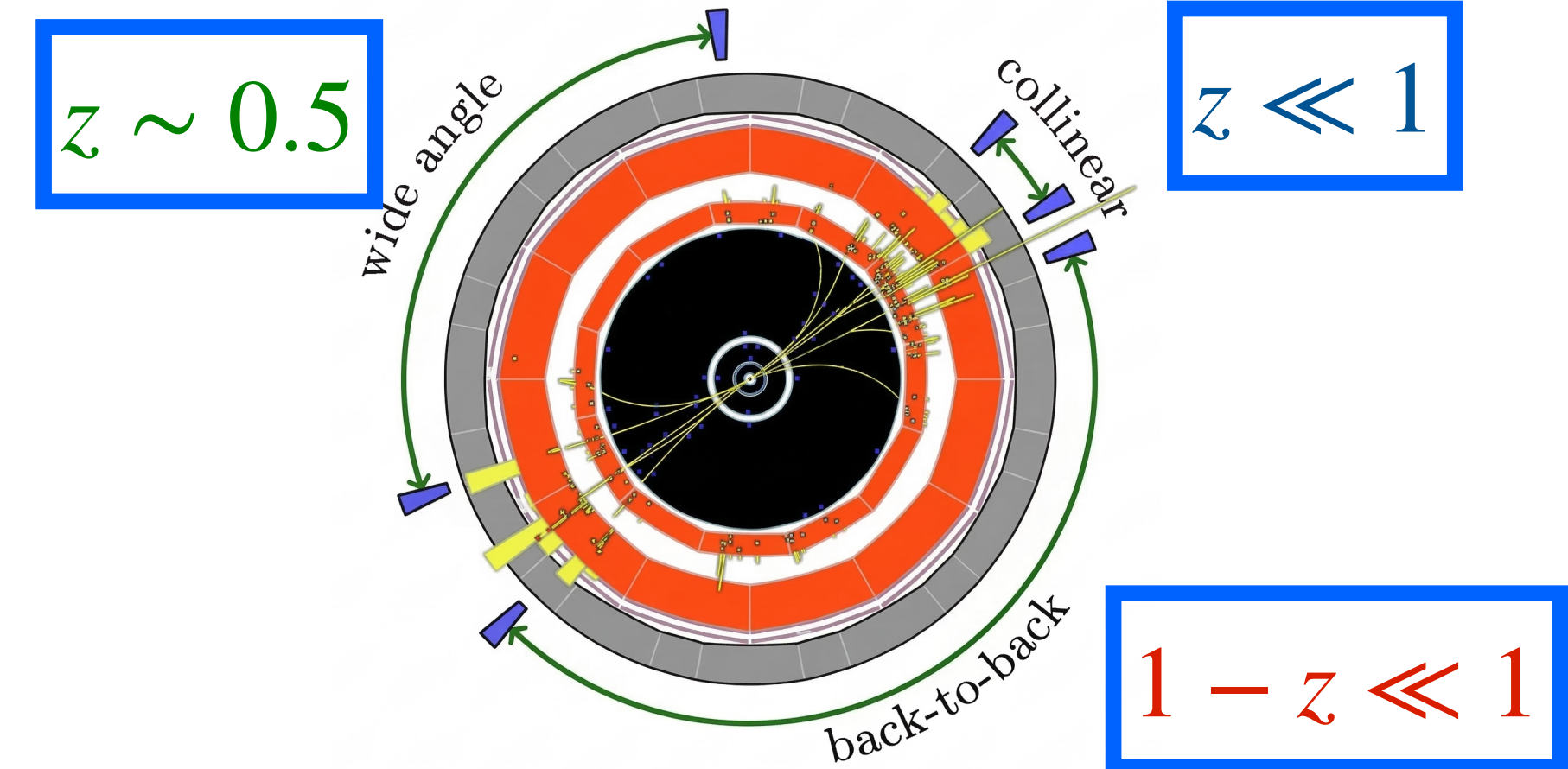
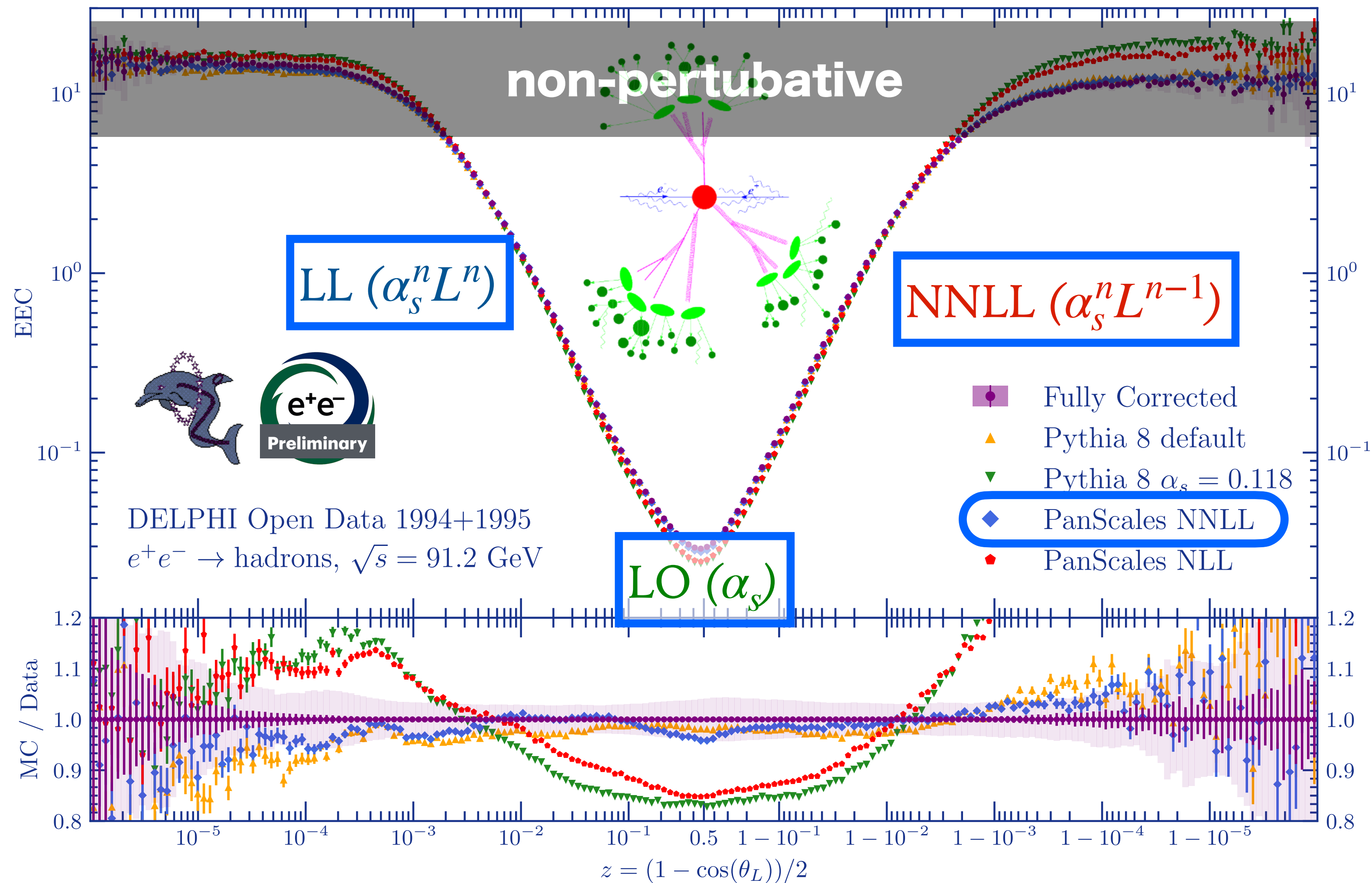
← **NNLL**
 for event shapes;
 not yet general NNLL

EEC, e^+e^- Alliance measurement on archived data



- State-of-the-art PanScales @ NNLL describe the data well without parton shower tuning
 - 3-jets in the ME
- Leading Log Pythia8 generator also describe the data with parton shower tuning

EEC, e^+e^- Alliance measurement on archived data



- State-of-the-art PanScales @ NNLL describe the data well without parton shower tuning
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- Leading Log Pythia8 generator also describe the data with parton shower tuning



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Country: US

Drafted: Undrafted, PHI, 2025

College: Toledo

Agent(s): Paul Bobbitt

The promise of perturbative QFT

Follow the rules (Feynman diagrams)

Ask a computer to work through maths

Out comes a prediction

Ask the computer to draw more diagrams, work harder, and you get more accuracy

the hope is that the technology that gets developed for “standard” pQFT calculations can be leveraged to get accuracy not just at fixed order, but across the full range of perturbative scales in simulations