A new Origin story for scattering amplitudes

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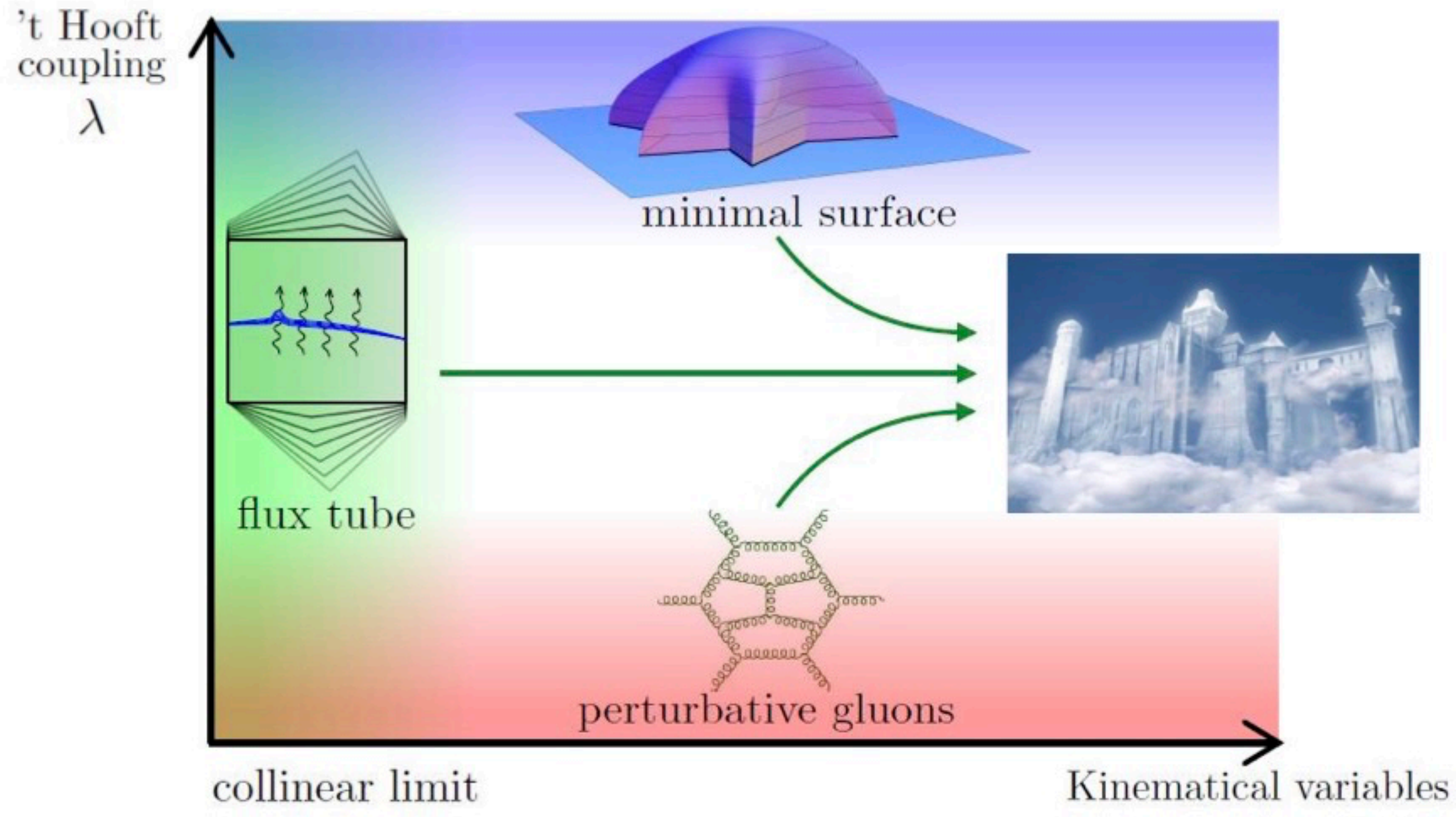
Lancefest Workshop, University of Edinburgh, 24-26 June 2026

Based on ongoing work with Thiago Fleury, Erkan Kaluç and Didina Serban…

… and 10 years of collaboration with **Lance**

Solving N = 4 SYM

- Laboratory for exploring dynamics of 4d massless gauge theories
- Symmetries open the way to new methods for computing correlation functions and amplitudes at both weak and strong coupling
- AdS/CFT and Integrability
- Amplitude/Null Polygon Wilson Loop/ Correlator duality
- #Bootstrap: Integrable Bootstrap, Amplitude/FF Function Bootstrap
- Castle in the Sky = Full solution ?



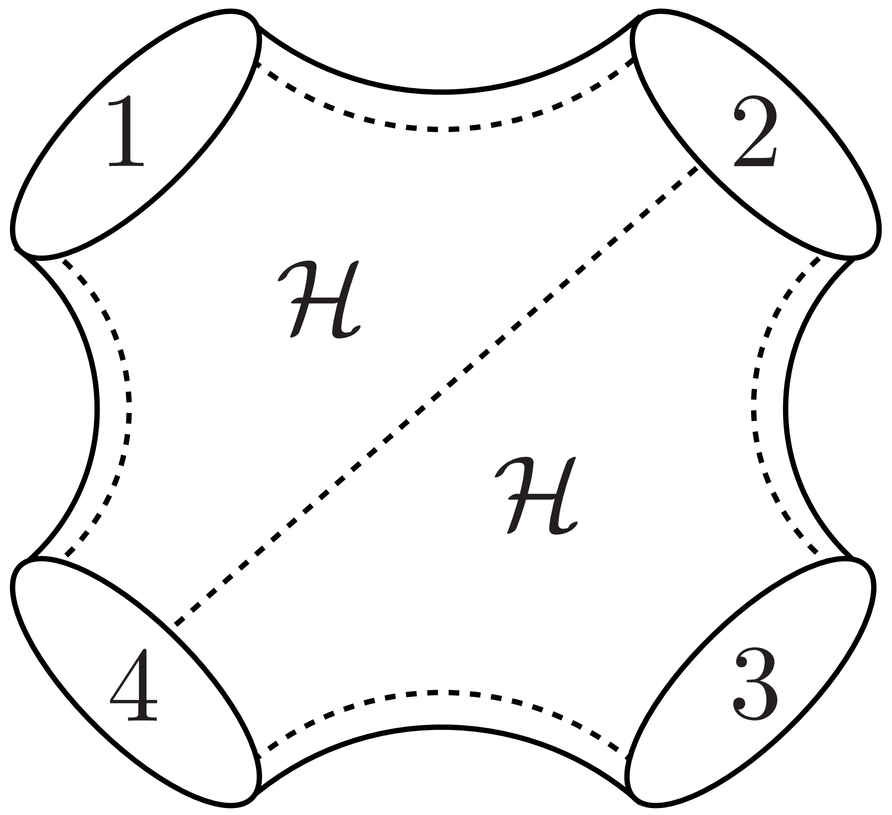
Correlators and polygons

[Fleury,Komatsu]
[Eden,Sfondrini]
[BB,Komatsu,Vieira]

New polygons from correlation functions

Planar single-trace correlators can be tessellated into hexagons (seams = Wick contractions)

Ex. 4-point function



4 hexagons are needed here
(2 in the front/back)

Generally, break n-point functions into 2(n-2) hexagons

$$F_n \quad \to \quad \mathcal{H}_1 \otimes \mathcal{H}_2 \ldots \otimes \mathcal{H}_{2(n-2)}$$

Polygon correlators

[Fleury,Komatsu]
[Bargheer,Caetano,Fleury,Komatsu,Vieira]
[Coronado]
[Caron-Huot,Coronado]
[Caron-Huot,Coronado,Muhlmann]
[Fleury,Gonçalves]
[Bercini,Fernandes,Gonçalves]
[Crisanti,Eden,Gottwald,Mastrolia,Scherdin]
[Bargheer,Bekov,Bercini,Coronado]

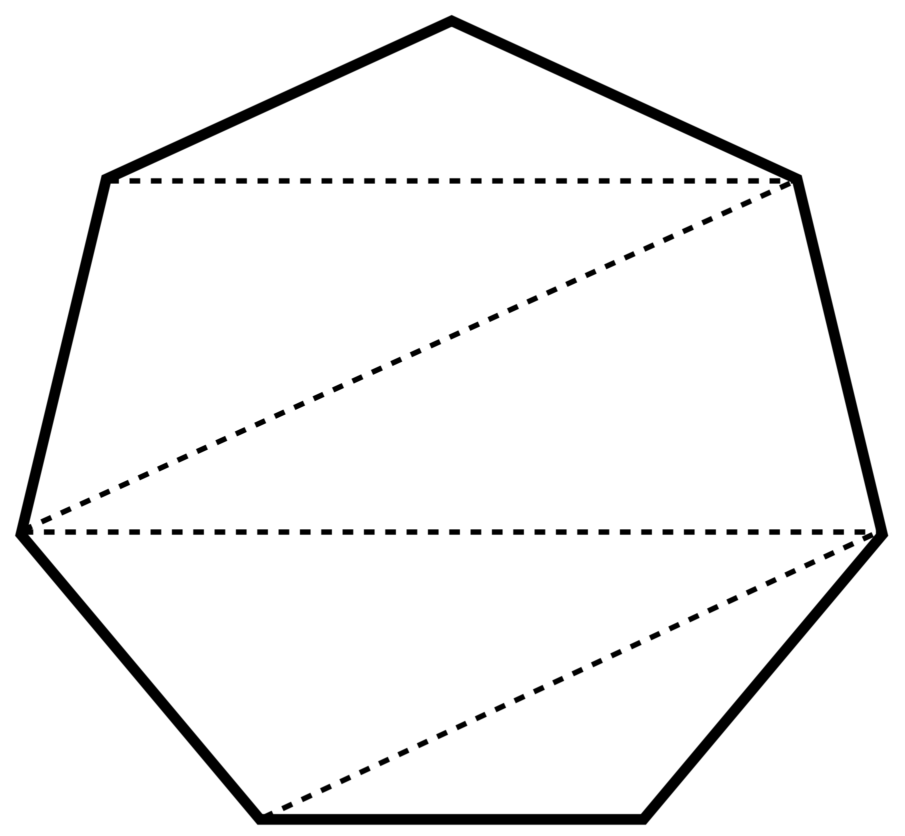
Gluing hexagons together leads to sums and integrals over magnons

The more we glue the more intricate the sums become

Simplification: large-charge correlators = **polygon correlators**

Hexagons glued together into bigger polygons

Polygons are easier to study than punctured spheres (less hexagons)



7-pt function tilling

Polygon correlators

[Fleury,Komatsu]
[Bargheer,Caetano,Fleury,Komatsu,Vieira]
[Coronado]
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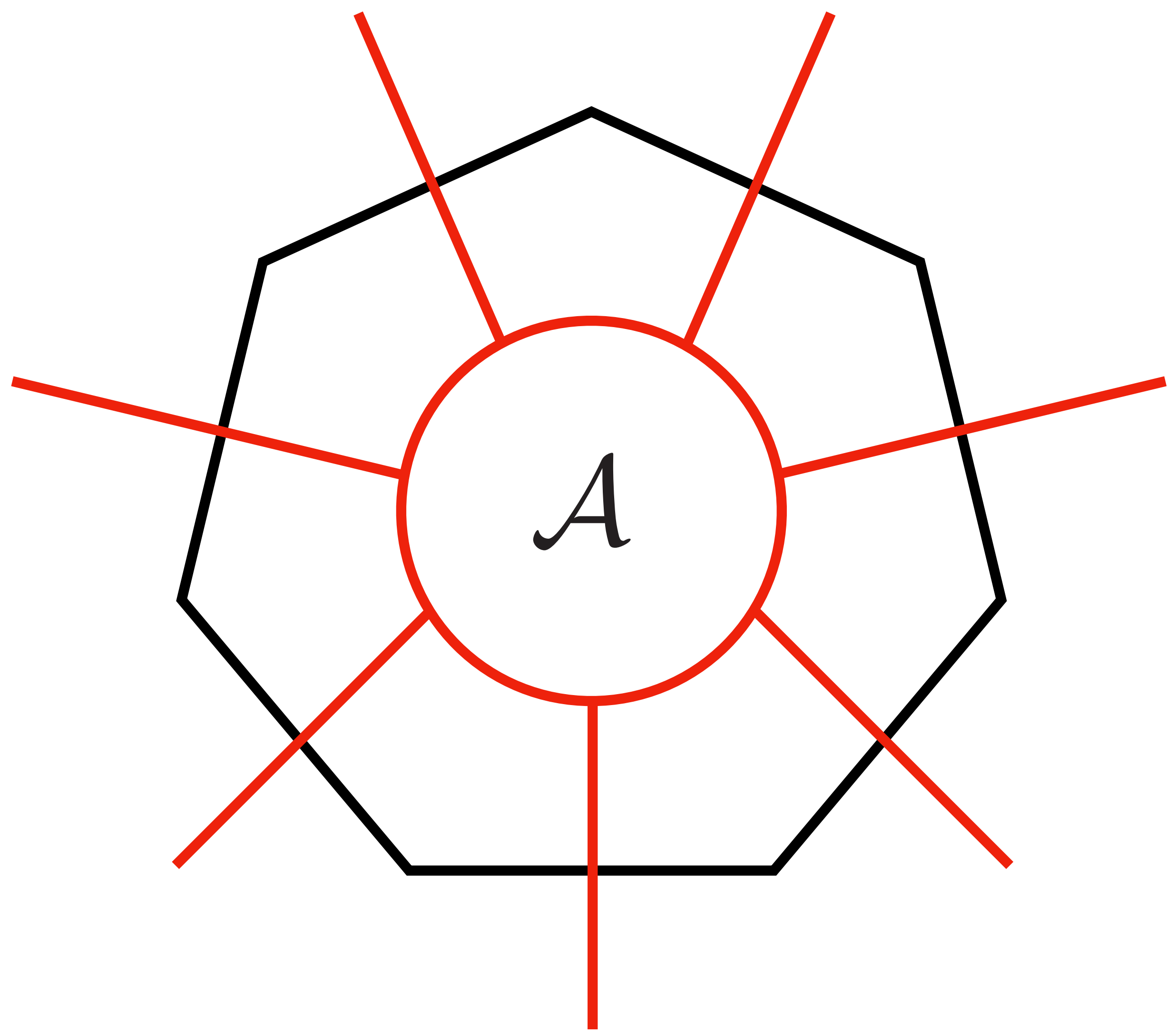


A new correlator/polygon/amplitude duality

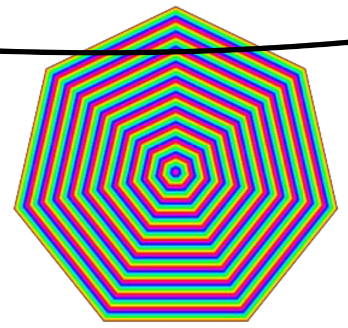
Off-shell Scattering Amplitudes or Amplitudes on Coulomb Branch (IR reg.) [Caron-Huot,Coronado]

Massive external states, but massless states propagating in loops

Origin Story

Motivation 2: Every Superhero/amplitude needs an Origin Story

?



- For $n = 6$, when all 3 cross ratios $u_i \to 0$, logarithm amplitude is remarkably simple, $\propto c(\zeta) \ln^2 u_i$ throug
  Caron-Huot, LD, Dulat, von Hippel, McLeod, Papathanasiou, 1960.
- Zeta-valued coefficients now understood to all orde
  Basso, LD, Papathanasiou, 2001.05460; hexagon origin story next
- **How about $n = 7$?**
- Analogous origin (due to Gram det): first 6 $u_i \to 0$, $u_7 \to 1$
- MHV symbol, known to 4 loops, is consistent with this, but provides no ζ-valued information
  LD, Drummond, Harrington, McLeod, Papathanasiou, Spradlin, 1612.08976

For MHV amplitude

Double-log behavior when many cross ratios vanish simultaneously / Origin limits

First observed at 6-points

Generalization using cluster algebra and tilted cusp

From very beginning: behavior was similar to behavior of Octagon correlator in null limit

Null limit for n = 4

Octagon (n=4) develops large logarithms in cross ratios in null limit $U, V \to 0$

[Coronado]
[Kostov,Petkova,Serban]
[Belitsky,Korchemsky]

Exact re-summation is possible with a simple Sudakov behavior

$$\log \mathbb{O} \approx -\frac{\Gamma_{\text{oct}}}{16} \log^2 (UV) + \dots$$

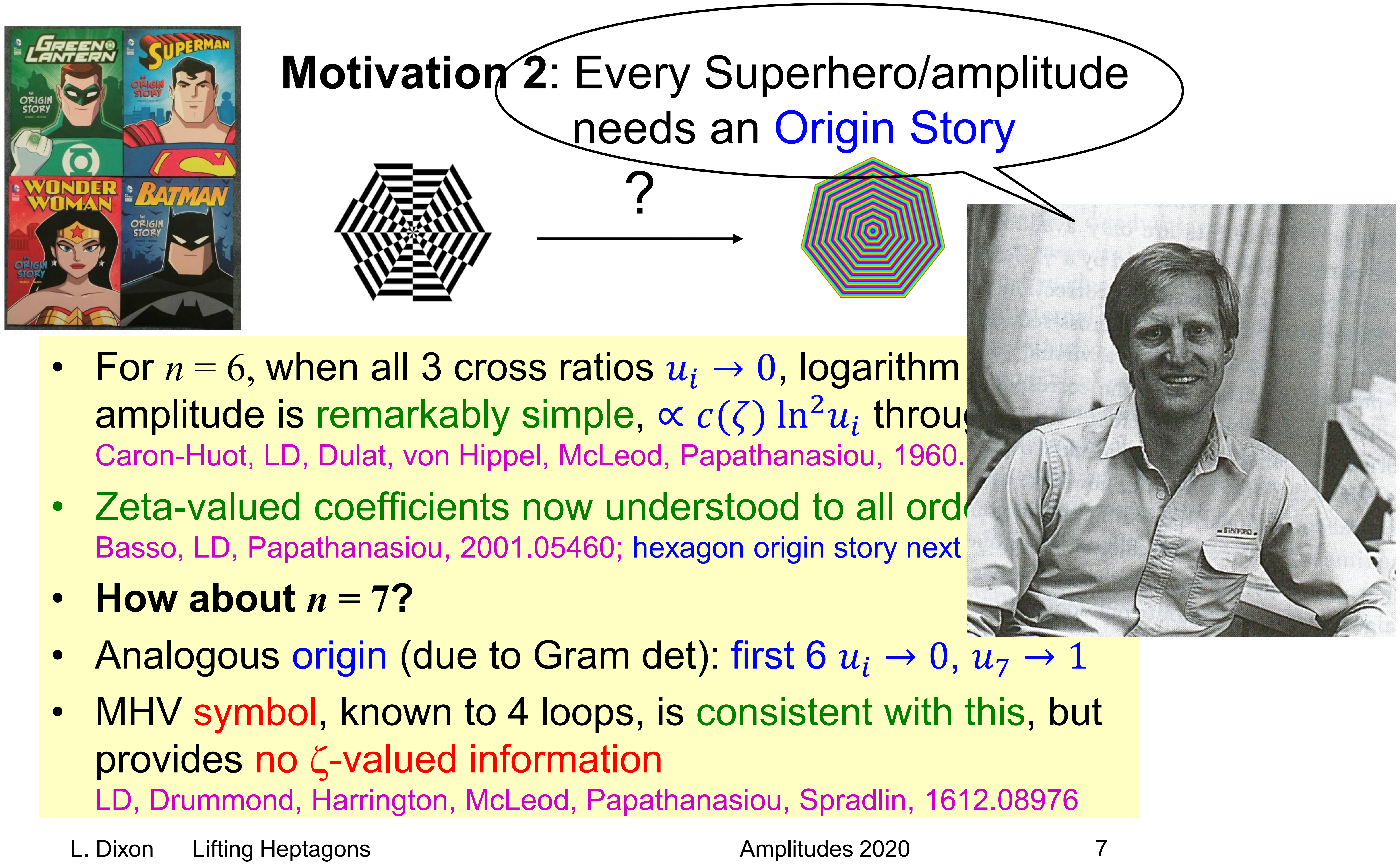
Coefficient exactly known $\Gamma_{\text{oct}} = \frac{2}{\pi^2} \log \cosh (2\pi g)$

Different from usual IR divergences controlled by cusp anomalous dimension!

Phenomenon extends to higher polygons with new anomalous dimensions

[Caron-Huot,Coronado]
[BB,Dixon,Liu,Papathanasiou]
[Bercini,Fernandes,Gonçalves]
[Bargheer,Bekov,Bercini,Coronado]
[Belitsky,Bork,Lee,Onishchenko,Smirnov]

Origin Story



For MHV amplitude

Double-log behavior when many cross ratios vanish simultaneously / Origin limits

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Generalization using cluster algebra and tilted cusp

From very beginning: behavior was similar to behavior of Octagon correlator in null limit

Origin story for polygon correlators? Not clear in general, but evidence in 2d

2d kinematics and positive region

Hard to explore Origins in full kinematics : lack “good” variables / cluster algebras

Focus on two-dimensional kinematics, with all operators lying in a plane $\mathbb{R}^{1,1}$

Configuration space = two copies of Grassmannian $\mathrm{Gr}(2,n)$

= two sets of n real spinors $\{(\lambda_i, \bar{\lambda}_i), i = 1, \ldots, n\}$

Distances: $x_{ij}^2 = (x_i - x_j)^2 \propto \langle ij \rangle \times [ij]$

where $\langle ij \rangle = \epsilon_{\alpha\beta} \lambda_i^\alpha \lambda_j^\beta$ and similarly for $[ij]$

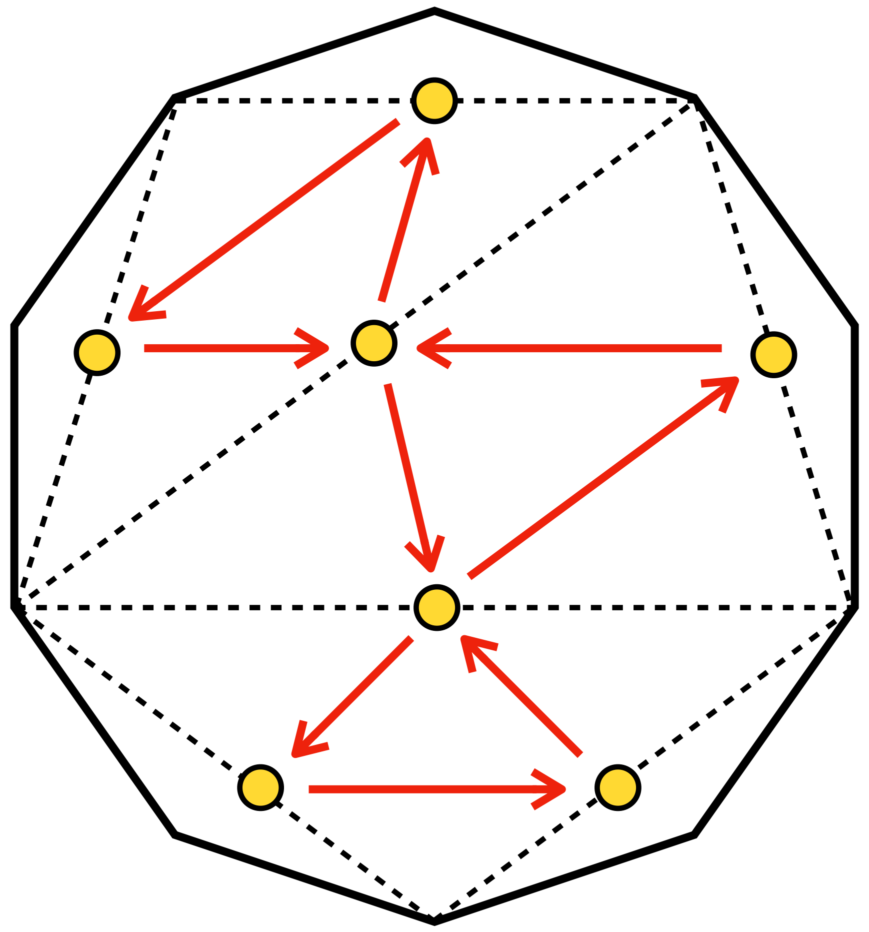
Positive region = domain defined by inequalities $\langle ij \rangle \geqslant 0 \quad 1 \leqslant i < j \leqslant n$

Boundary = where distances are going to zero = where correlators develop singularities

Facet = codim1 face where $\langle ij \rangle = 0$ for some i, j

Vertex = intersection of (n-3) facets = where singularities are in a sense “maximal”

Cluster algebra

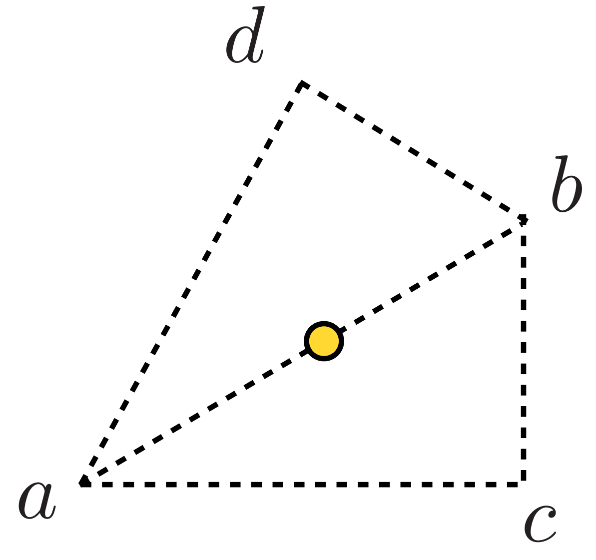


- Conformally invariant description using **cluster coordinates**

  Each **vertex** is associated with set of cluster variables / triangulation

  Cluster coordinate z is defined locally by a simple geometric rule

$$z_{ab} = \frac{\langle ad\rangle\langle cb\rangle}{\langle ac\rangle\langle bd\rangle}$$

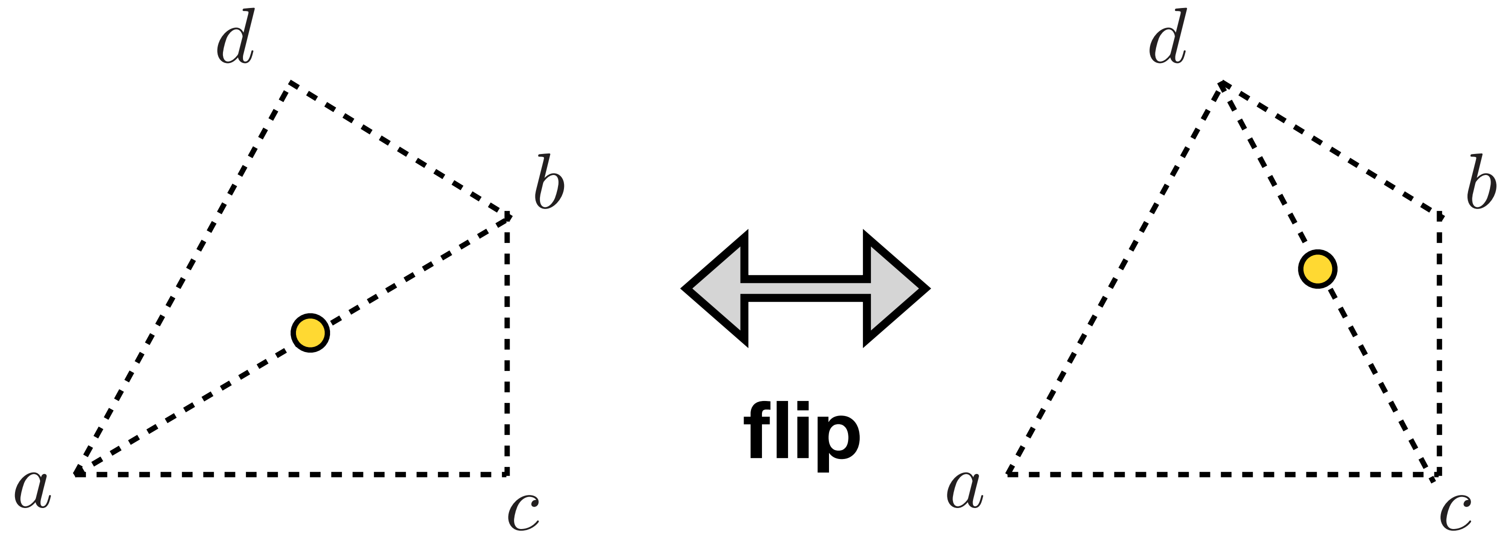


[Huge literature]

  Vertex = point where all z variables are going to zero

- Move from one **vertex** to another using cluster **mutations**

  They encode transformations of cluster coordinates associated with flips of edges in triangulation



Rules use information encoded in quiver diagram of the triangulation

$$z_i \to z_i' = 1/z_i$$
$$z_j \to z_j' = z_j\left(1 + z_i^{B_{ij}}\right)^{B_{ij}}$$

Exchange graph

Exchange graph of the n-gon cluster algebra A_{n-3}

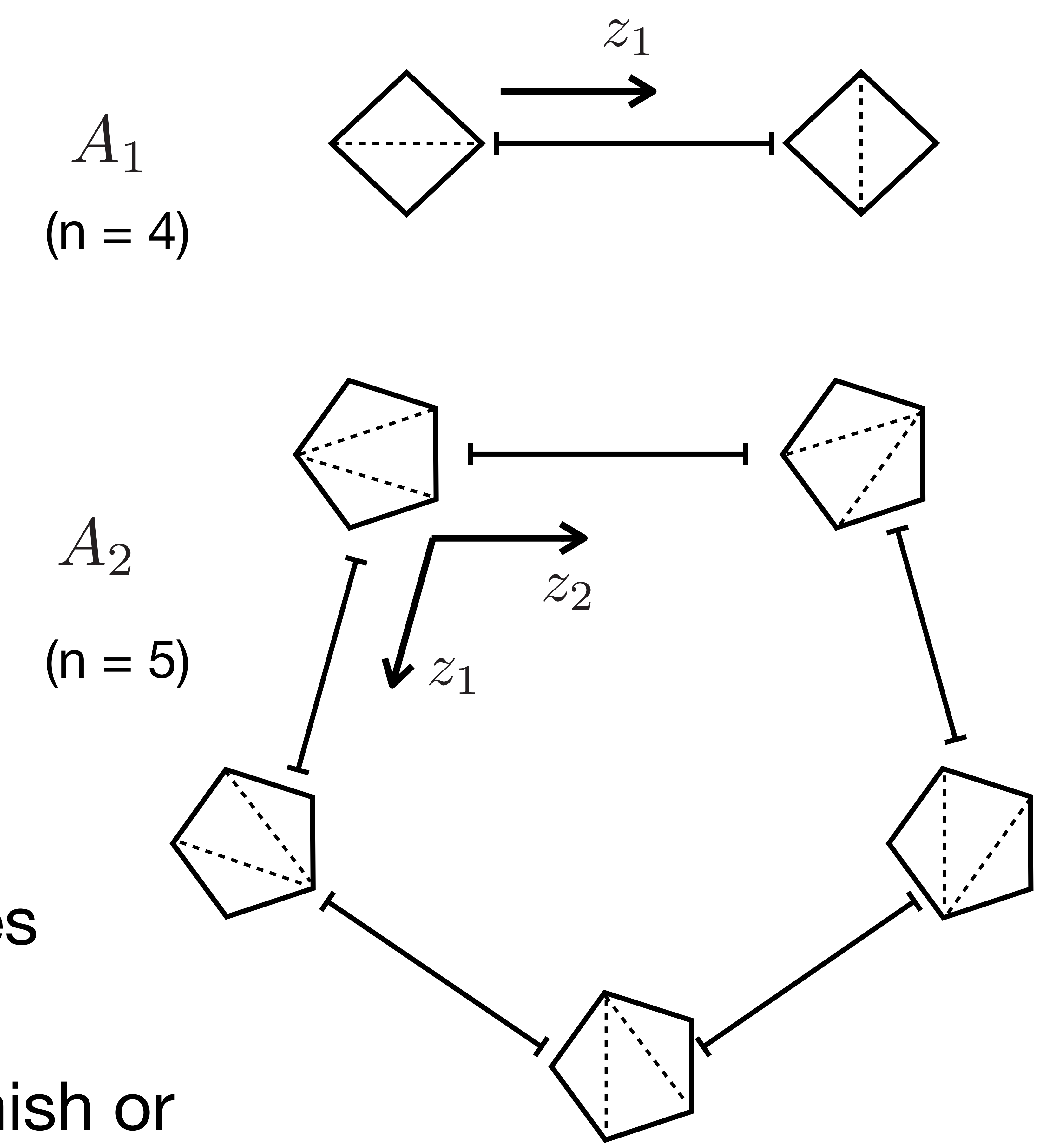
Vertices : triangulations = set of cluster coordinates

Edges : flips = mutations (bi-rational chart transformations)

Positive region

Interior of the exchange graph corresponds to positive region, defined as the set of points reached by assigning positive values to the cluster coordinates

Boundary of this region are limits where cluster coordinates vanish or become large



Origin (cluster definition) “where singularities are maximal”

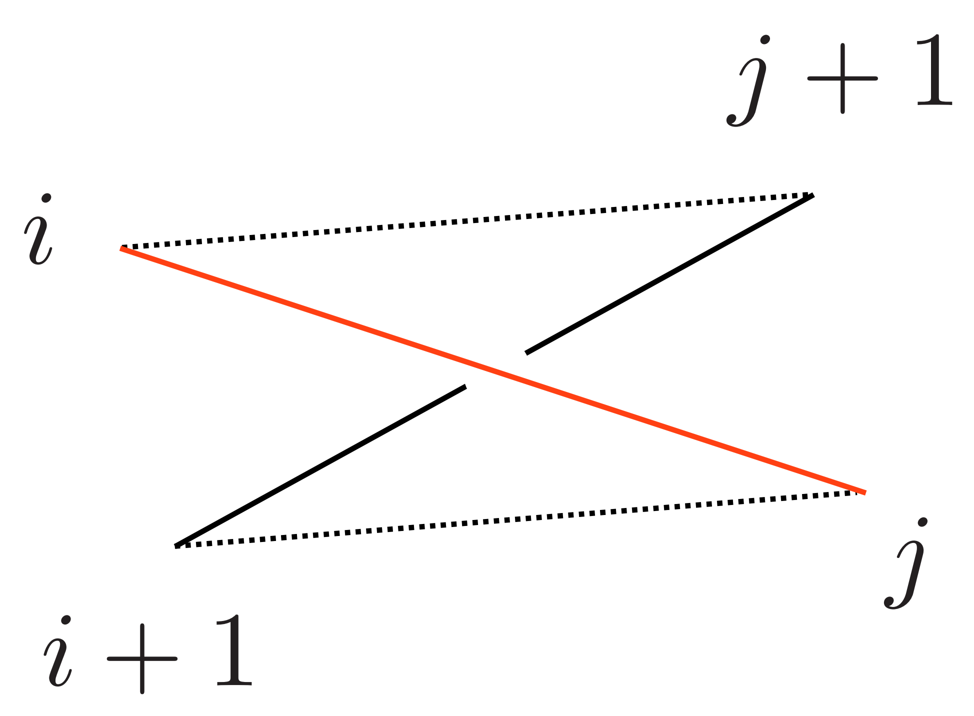
Similar to Origins in [BB,Dixon,Liu,Papathanasiou]

= Origin of a system of cluster coordinate (vertex <=> triangulation)

Binary geometry

[Arkani-Hamed,He,Lam]
[Arkani-Hamed,He,Lam,Thomas]
[Brown][DelDuca,Druc,Drummond,Duhr,
Dulat,Marzucca,Papathanasiou,Verbeek]

Attach a “u variable” (cross ratio) to each diagonal $u_{ij} = \frac{\langle i, j+1\rangle\langle i+1, j\rangle}{\langle i, j\rangle\langle i+1, j+1\rangle}$



Basis of $\frac{n(n-3)}{2}$ cross ratios in 2d (only n-3 are independent)

Relations take the elegant form

$$u_{ij} + \prod_{(kl)\,\text{crossing}\,(ij)} u_{kl} = 1$$

with product running over all chords crossing ij

Nice properties:

Variables are bounded when evaluated on positive region $0 < u_{ij} < 1$
Bounds are saturated at boundary of this region

Facet corresponds to limit where a “u” approaches 0, while crossing-related “u’s” go to 1

$$u_{ij} \to 0 \quad \Rightarrow \quad u_{kl} \to 1$$

Vertex labeled by **binary sequence** indicating which (n – 3) u’s are sent to 0 and which are set to 1

Origin limits (full definition)

We have both a left and a right polygon to parametrize

$$u_{ij}^{\text{Left}} = \frac{\langle i, j+1\rangle\langle j, i+1\rangle}{\langle i, j\rangle\langle i+1, j+1\rangle} \qquad u_{ij}^{\text{Right}} = \frac{[i, j+1][j, i+1]}{[i, j][i+1, j+1]}$$

Physical distances or cross ratios are products of left and right variables

$$\boxed{U_{ij} = u_{ij}^{\text{Left}} \times u_{ij}^{\text{Reft}}}$$

Number of vanishing cross ratios **maximized** when the sets of internal diagonals in the left and right triangulations are mutually disjoint

In that case, 2(n – 3) cross ratios approach zero, while the remaining ones approach one

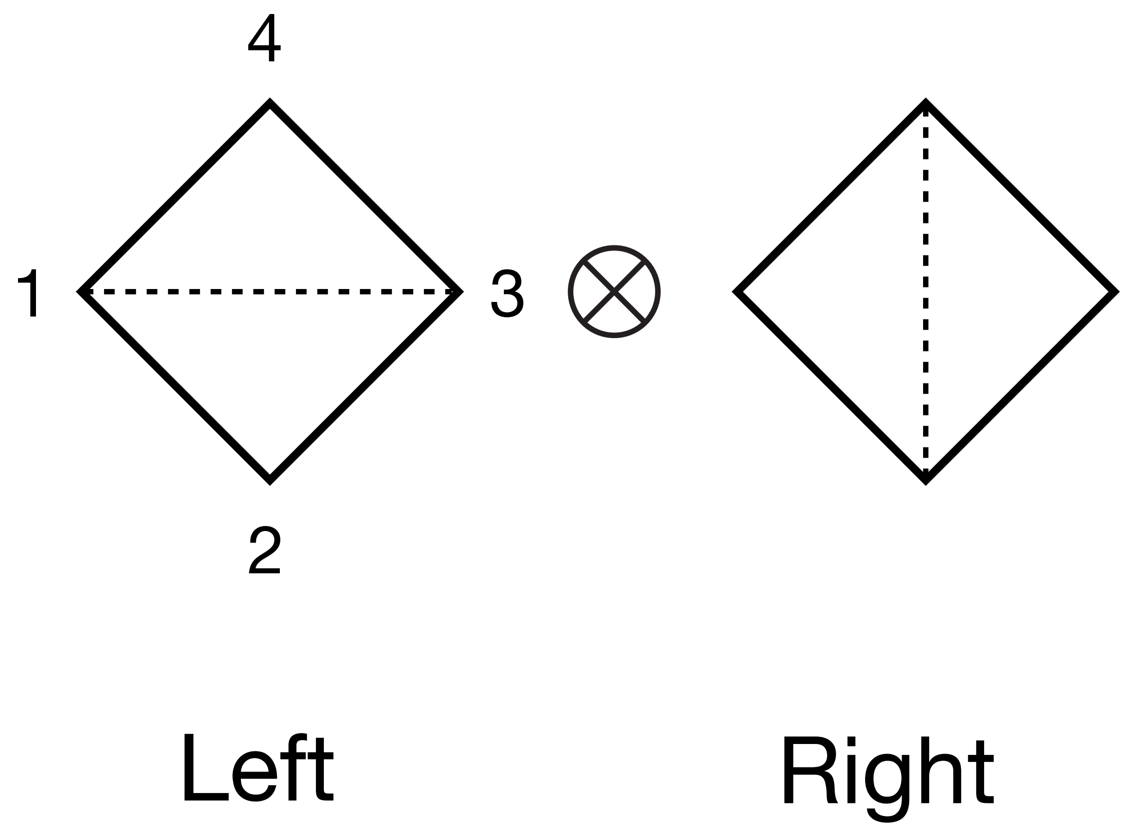
We call such configurations **Origins** or Origin limits, in analogy with scattering amplitudes

[BB,Dixon,Liu,Papathanasiou]

Examples I

Origin limits

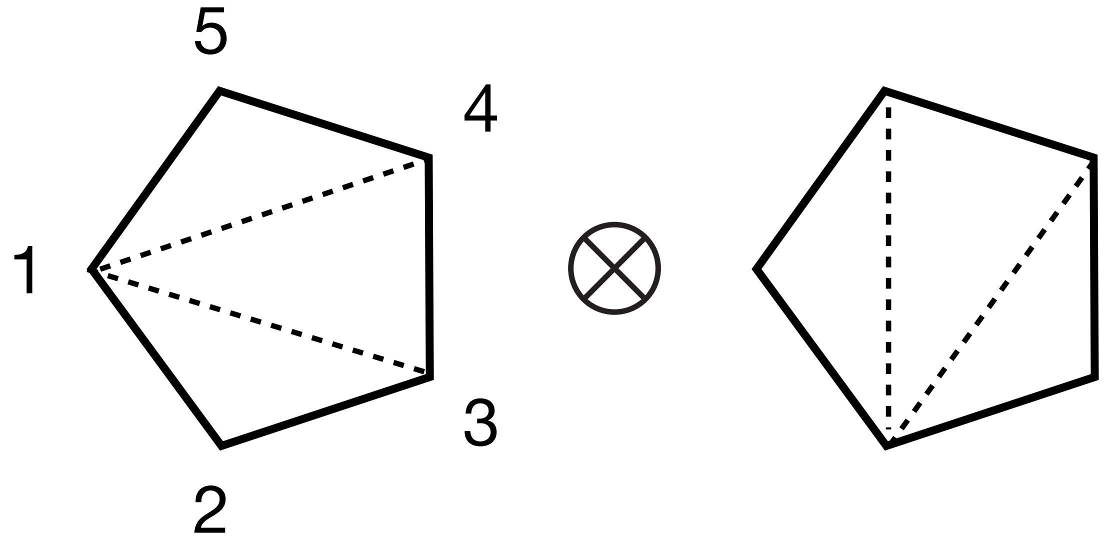
4-point function



$$U_{13}, U_{24} \to 0$$

same as null square limit

5-point function



$$U_{13}, U_{14}, U_{24}, U_{25} \to 0$$

$$U_{35} \to 1$$

Comments

1) Other choices for n=5 give cyclic images of the limit above

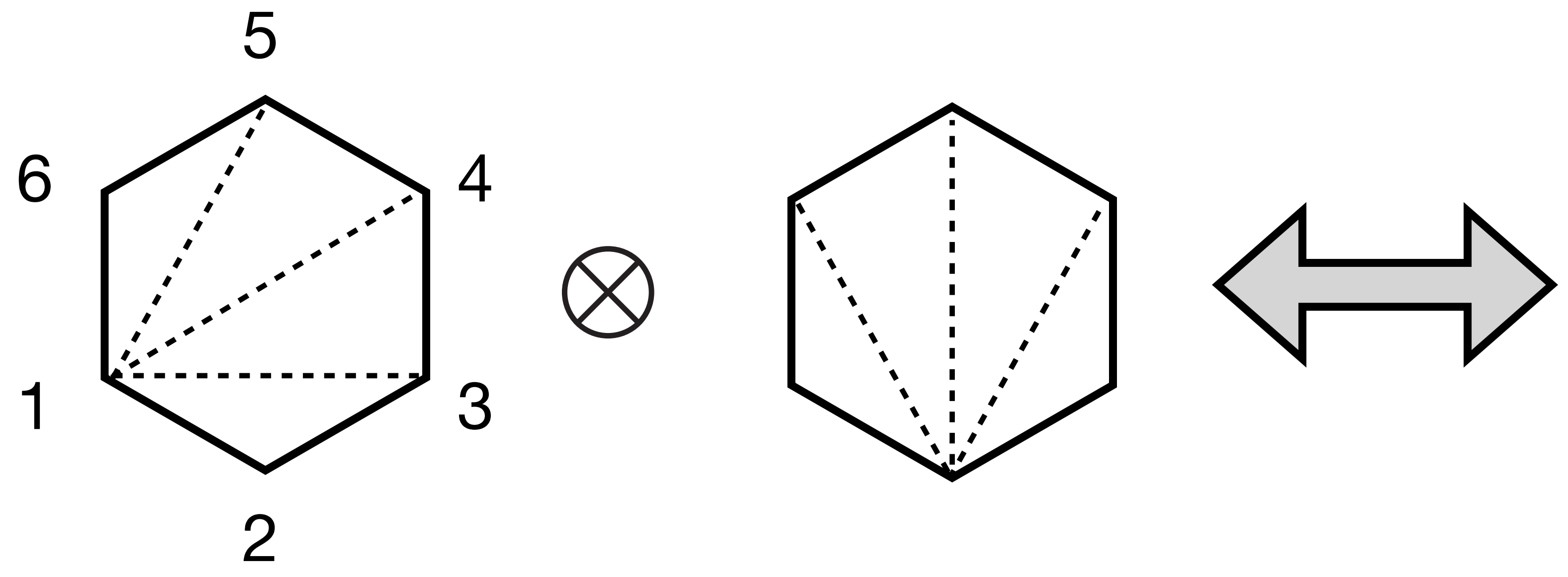
2) **No more than 4 cross ratios can approach zero simultaneously**. maximal number in 2d.

Examples II

Origin limits

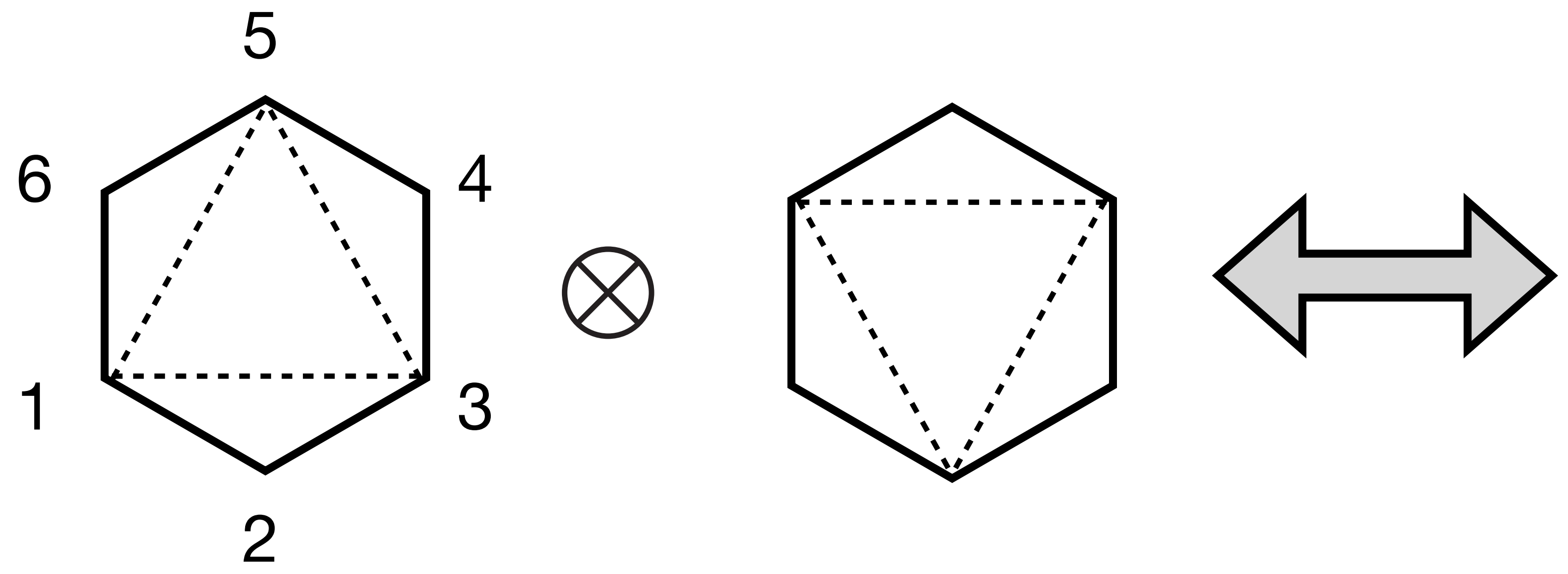
6-point function

Ex.1



$$U_{13}, U_{24}, U_{15}, U_{26}, U_{14}, U_{25} \to 0$$
$$U_{35}, U_{46}, U_{36} \to 1$$

Ex.2



$$U_{13}, U_{24}, U_{35}, U_{46}, U_{15}, U_{26} \to 0$$
$$U_{14}, U_{25}, U_{36} \to 1$$

same as null hexagon limit

...

Many Origins are dihedral images of one another. Classification of orbits?

Origin classes for hexagon

Define $(u_1, u_2, u_3, u_4, u_5, u_6; v_1, v_2, v_3) = (U_{13}, U_{24}, U_{35}, U_{46}, U_{15}, U_{26}; U_{14}, U_{25}, U_{36})$

Inequivalent classes of Origins are given by

Origin Class	u_1	u_2	u_3	u_4	u_5	u_6	v_1	v_2	v_3
O_1	0	0	1	0	0	1	0	0	1
O_2	0	1	0	0	1	0	0	0	1
O_3	0	0	1	0	1	0	0	0	1
O_4	1	0	0	0	0	1	0	0	1
O_5	0	0	1	0	0	0	0	1	1
O_6	0	0	0	0	0	0	1	1	1

Images are obtained by acting on above representatives with dihedral group

Orbits with 6,3,12,6,6,1 elements for O1 to O6 respectively

One-loop analysis

Perturbative data can be produced using hexagonalization

✓ 4pt function through all loops $U_1, U_2 \to 0$

$$\log \mathcal{C}_4 \approx -g^2 \log U_1 \log U_2 + \mathcal{O}(g^4)$$

✓ 5pt function through 5 loops at least $U_1, U_2, U_4, U_5 \to 0 \ \ (U_3 \to 1)$

$$\log \mathcal{C}_5 \approx -g^2 (\log U_1 \log U_2 + \log U_4 \log U_5 + \log U_1 \log U_5) + \mathcal{O}(g^4)$$

Higher polygons? In progress for all 6-point Origins…

Can we do better than that? **Master formula** (all-loop, all-point)

All-loop master formula

Drawing inspiration from scattering amplitudes and generalizing observations made for 4-point function, we are led to conjecture that disk correlation functions exponentiate in any Origin limit and that the exponent is a quadratic polynomial in the logarithms of the U cross ratios

[BB,Dixon,Liu,Papathanasiou]
[Coronado]
[Belitsky,Korchemsky]
[Kostov,Petkova,Serban]

Conjecture: we expect the disk n-point function to take the form

[BB,Fleury,Kaluç,Serban - in progress]

$$\log \mathcal{C}_n \approx -\frac{1}{2}\oint_{C_n} \frac{(z-1/z)dz}{2\pi i z}\mathcal{G}(z,g)\times\mathcal{S}_n(z,\{\log U_{ij}\})$$

Three main ingredients

Variable z = spectral parameter

1) $\mathcal{G}$ is a function of the coupling constant known as tilted cusp anomalous dimension

2) $\mathcal{S}_n$ is the string integrand coding the kinematics: it is rational in z and quadratic in log U's

3) C_n (contour) is such as to enclose the poles of $\mathcal{S}_n$ in the plane of z

All-loop master formula

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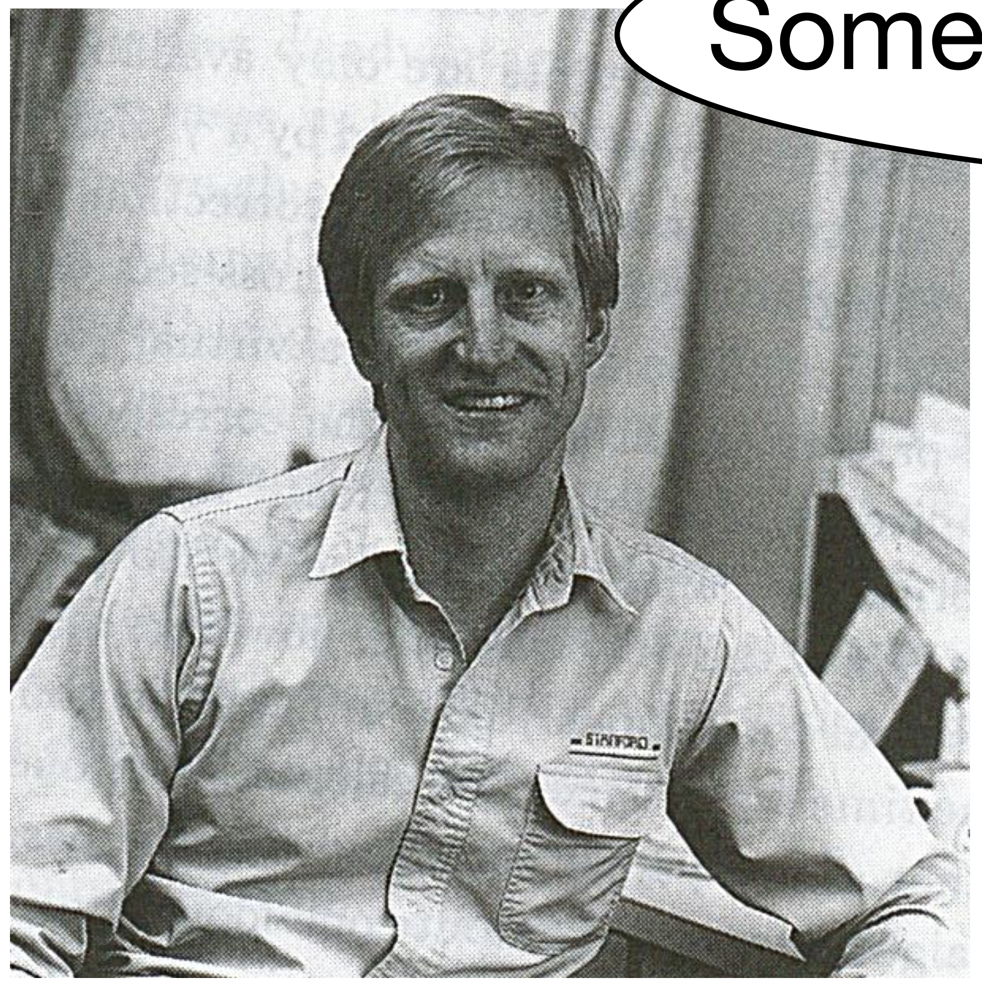
Three main ingredients

Variable z = spectral parameter

1) $\mathcal{G}$ is a function of the coupling constant known as tilted cusp anomalous dimension

$$\mathcal{G}(z,g) = \Gamma_\alpha(g) \quad \text{with} \quad z = -e^{2i\alpha}$$

Tilted cusp



Some things get better when tilted

Not just true for Pisa tower

Also for the cusp anomalous dimension!

Tilted BES equation gives a family of tilted anomalous dimensions

Tilted cusp anomalous dimension

[BB,Dixon,Liu,Papathanasiou]
[Beisert,Eden,Staudacher]

Rotation of BES kernel using tilt angle

$$\mathbb{K}(\alpha) = 2\cos\alpha \begin{bmatrix} \cos\alpha\, \mathbb{K}_{\circ\circ} & \sin\alpha\, \mathbb{K}_{\circ\bullet} \\ \sin\alpha\, \mathbb{K}_{\bullet\circ} & \cos\alpha\, \mathbb{K}_{\bullet\bullet} \end{bmatrix} \qquad \mathbb{K}_{ij} = 2j(-1)^{ij+j} \int_0^\infty \frac{dt}{t} \frac{J_i(2gt)J_j(2gt)}{e^t - 1}$$

One-parameter family of deformation of cusp anomalous dimension

$$\Gamma_\alpha(g) = 4g^2(1 + \mathbb{K}(\alpha))^{-1}_{11}$$

Same zeta-structure as for cusp but with loop coefficients dressed with trigonometric numbers

$$\Gamma_\alpha = 4g^2 - 16\zeta_2 \cos^2\alpha\, g^4 + 32\zeta_4 \cos^2\alpha(3 + 5\cos^2\alpha)\, g^6 + \ldots$$

In particular $\quad \Gamma_{\text{cusp}} = \Gamma_{\alpha=\pi/4} \qquad \Gamma_{\text{oct}} = \Gamma_{\alpha=0}$

Tilted cusp

Not just true for Pisa tower

Also for the cusp anomalous dimension!

Tilted BES equation gives a family of tilted anomalous dimensions

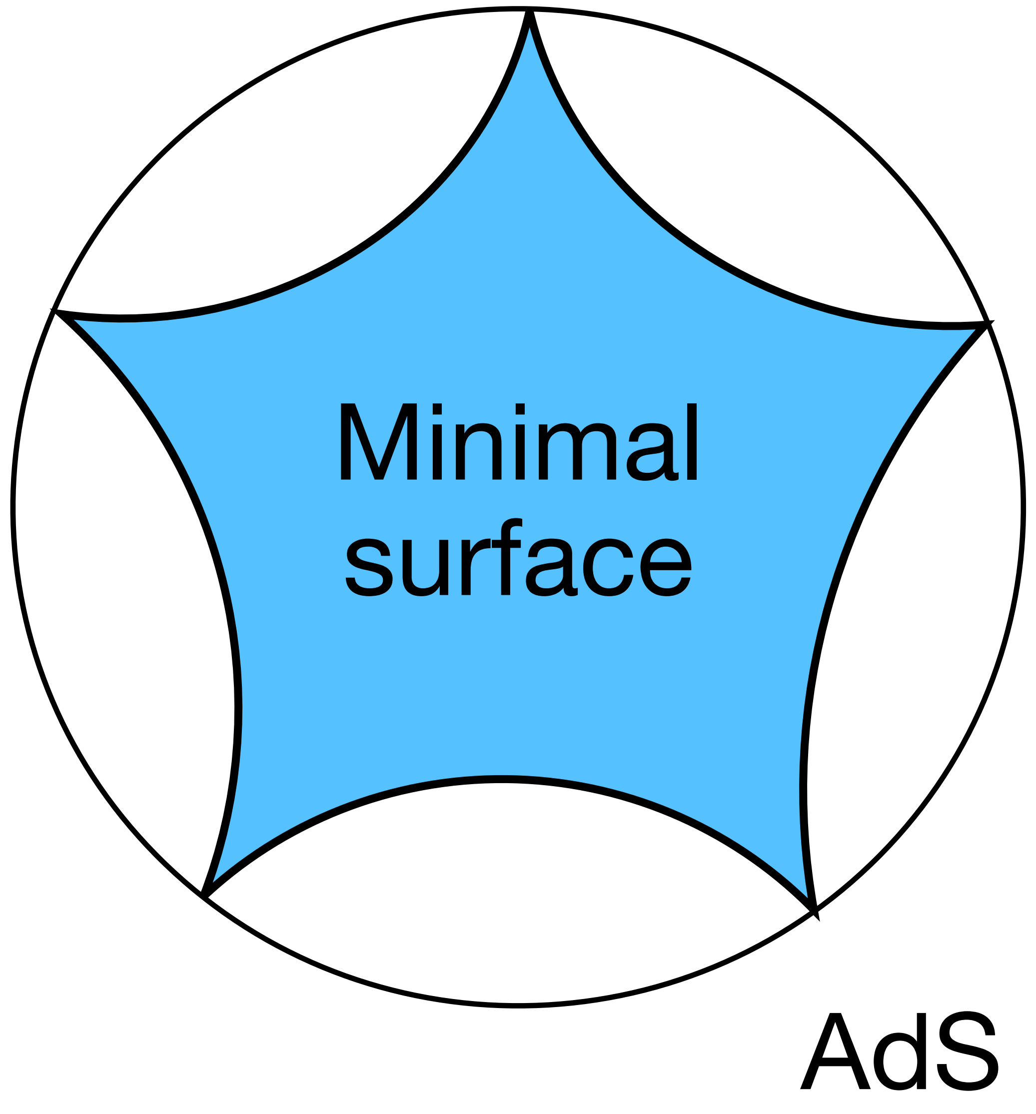
Physical meaning of tilt angle?

Not clear, but direct application to Origin limits

2) String integrand

[Caetano,Toledo]
[Bargheer,Coronado,Vieira]
[BB,Kaluç,Serban]
[Bargheer,Fleury,Lai WIP]

At strong coupling correlation functions are described by minimal surfaces in AdS



Integrability: area as the **free energy** of a system of **TBA** equations

Solutions for Y-functions simplify drastically in Origin limits: **linear** problem

$$\log Y_i(z) = \frac{1}{1 - K_{ij}(z)} \cdot I_j(z)$$

Free energy is obtained by integrating the string integrand $\mathcal{S}_n(z) = \sum_i I_i(1/z) \log Y_i(z)$

Complete classification of how and where this simplification is still lacking

3) Contour evaluation I

In **all** cases the poles are on the unit circle and in **most** cases they are simple

$$\log \mathcal{C}_n \approx \sum_{\alpha} \Gamma_\alpha(g) \times P_\alpha(\{\log U_{ij}\})$$

where the set of alpha's and associated polynomials are determined by the string integrand

Simplest example is given by the 4-point function with pole at $z^2 = 1$

$$\log \mathcal{C}_4 \approx -\frac{1}{16}\Gamma_{\alpha=0}(g)\log^2(U_1 U_2) + \frac{1}{16}\Gamma_{\alpha=\pi/2}(g)\log^2(U_1/U_2)$$

with

$\Gamma_{\alpha=0} = \Gamma_{\text{oct}} = \frac{2}{\pi^2}\log\cosh(2\pi g)$ and $\Gamma_{\alpha=\pi/2} = 4g^2$

3) Contour evaluation II

In **all** cases the poles are on the unit circle and in **most** cases they are simple

$$\log \mathcal{C}_n \approx \sum_\alpha \Gamma_\alpha(g) \times P_\alpha(\{\log U_{ij}\})$$

where the set of alpha's and associated polynomials are determined by string integrand

For 5-point function we have poles at $z^2(1-z^2)=1$ resulting in

$$\log \mathcal{C}_5 \approx \Gamma_{\pi/12}(g)\, P_1(\{\log U_{ij}\}) + \Gamma_{5\pi/12}(g)\, P_2(\{\log U_{ij}\})$$

with $P_{1,2} = \frac{1}{2} P_{\text{one-loop}} \mp \frac{1}{2\sqrt{3}}(\log^2 U_1 + \log^2 U_2 + \log^2 U_4 + \log^2 U_5 + \log U_1 \log U_4 + \log U_2 \log U_5)$

Perfect agreement with previous data through 5 loops (at least)!

Null polygon limit

Limit where consecutive operators become null separated from each other

Not an Origin limit for higher n

Yet it is cornered by Origins = double-soft limits in 2d

This hints that a similar behavior should apply $\log \mathcal{C}_n = \text{Quad-Log} + R_n$

Conjecture: sum over alpha's associated with n-th roots of unity [BB,Fleury,Kaluç,Serban]

$$\text{Quad-Log} = -\frac{1}{4n} \sum_{\alpha \in \boldsymbol{\alpha}_n} \Gamma_\alpha(g) \cos(2\alpha) \left| \sum_{k=1}^{n} e^{2ik\alpha} \log x_{k,k+1}^2 \right|^2 + \dots$$

For n = 4 it is reduces to octagon dimension but for not for higher n!

In agreement with all available perturbative data

Amplitude side: Infrared double logs depend non trivially on ratios of external masses

Remainder function

Meaning of finite part?

Does it relate to amplitudes/null Wilson loops?

Same space of functions (multiple polylogs)

Same Steinmann relations

Not enough to prove relation to WLs

But good start to bootstrap to all loops?

Need more data

Lance (March 2024):

I have discovered a truly marvelous solution to N=4 SYM, which this margin is too narrow to contain…

Happy 65 Birthday!

Thanks for your friendship and the many wonderful physics discussions!

Looking forward to many more collaborations and more keys to decoding N=4 SYM!