Hopf algebra - inspired by Lance

Lancefest Workshop
Higgs Centre for Theoretical Physics
June 24, 2026

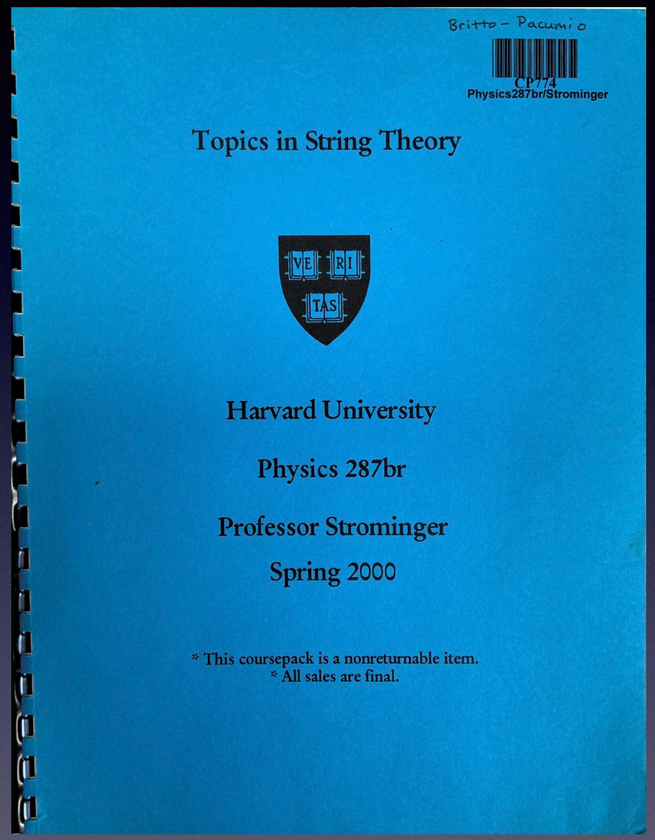
Ruth Britto
Trinity College Dublin

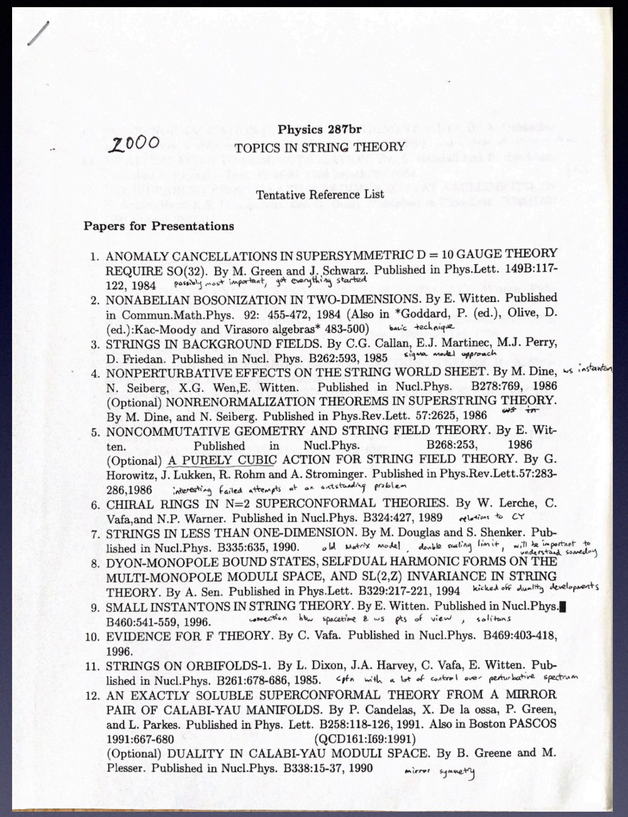
Trinity College Dublin
Coláiste na Tríonóide, Baile Átha Cliath
The University of Dublin

Funded by
the European Union

European Research Council
Established by the European Commission

Starting from strings





2000

Physics 287br
TOPICS IN STRING THEORY

Tentative Reference List

Papers for Presentations

1. ANOMALY CANCELLATIONS IN SUPERSYMMETRIC D = 10 GAUGE THEORY REQUIRE SO(32). By M. Green and J. Schwarz. Published in Phys.Lett. 149B:117-122, 1984 possibly most important, got everything started
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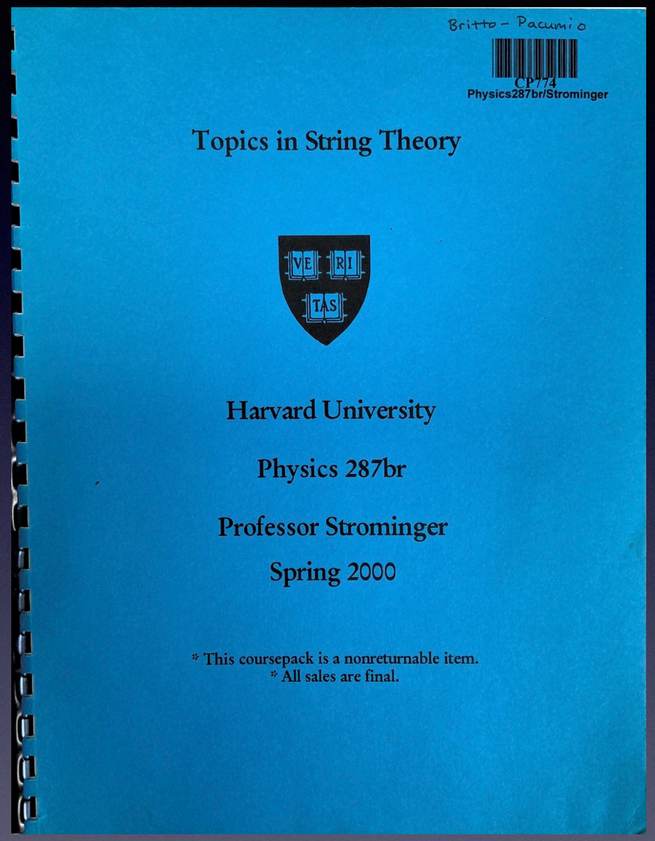
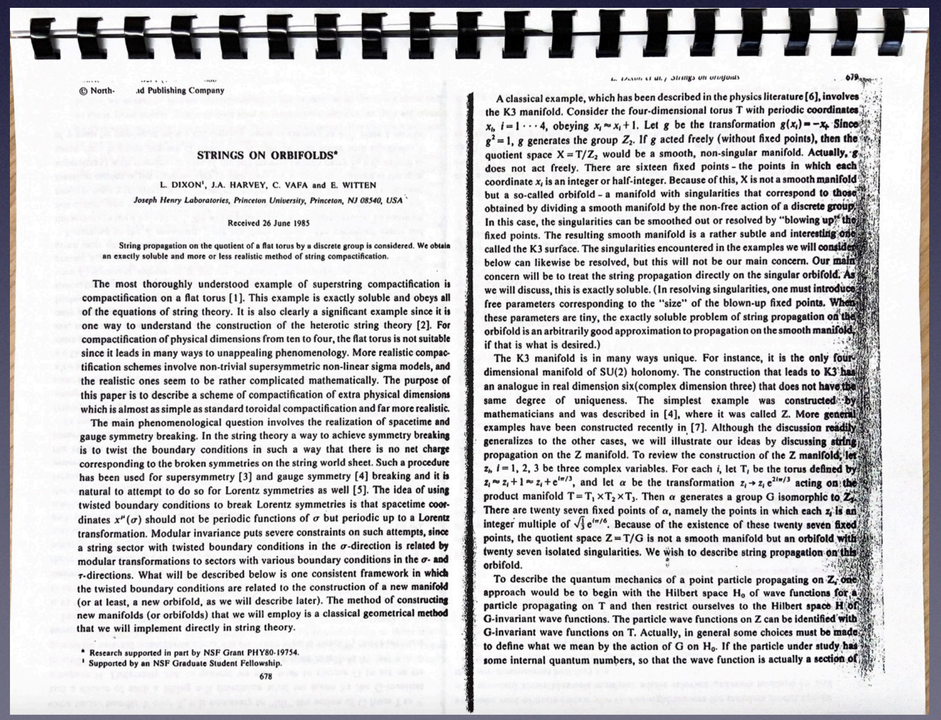
Starting from strings





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L. DIXON et al. / Strings on orbifolds 679

STRINGS ON ORBIFOLDS*

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Received 26 June 1985

String propagation on the quotient of a flat torus by a discrete group is considered. We obtain an exactly soluble and more or less realistic method of string compactification.

The most thoroughly understood example of superstring compactification is compactification on a flat torus [1]. This example is exactly soluble and obeys all of the equations of string theory. It is also clearly a significant example since it is one way to understand the construction of the heterotic string theory [2]. For compactification of physical dimensions from ten to four, the flat torus is not suitable since it leads in many ways to unappealing phenomenology. More realistic compactification schemes involve non-trivial supersymmetric non-linear sigma models, and the realistic ones seem to be rather complicated mathematically. The purpose of this paper is to describe a scheme of compactification of extra physical dimensions which is almost as simple as standard toroidal compactification and far more realistic.

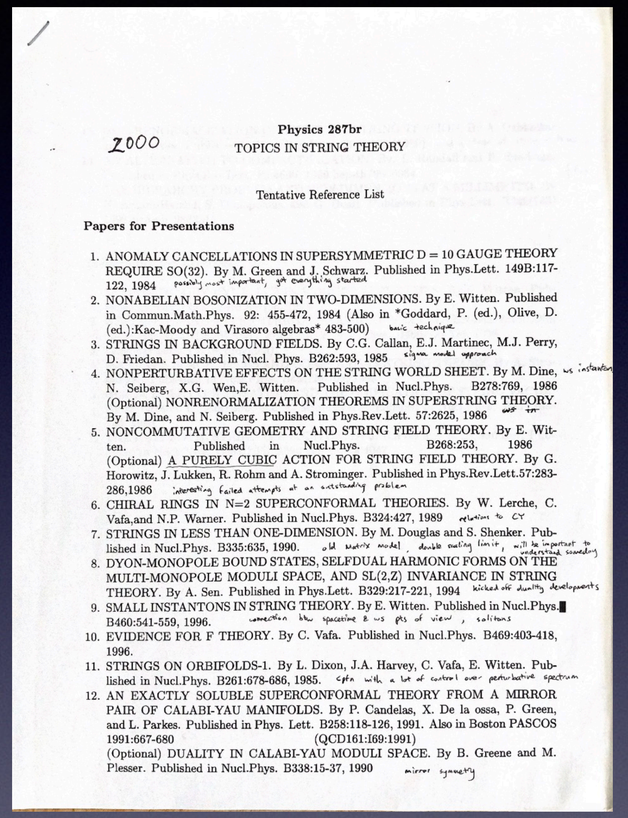
The main phenomenological question involves the realization of spacetime and gauge symmetry breaking. In the string theory a way to achieve symmetry breaking is to twist the boundary conditions in such a way that there is no net charge corresponding to the broken symmetries on the string world sheet. Such a procedure has been used for supersymmetry [3] and gauge symmetry [4] breaking and it is natural to attempt to do so for Lorentz symmetries as well [5]. The idea of using twisted boundary conditions to break Lorentz symmetries is that spacetime coordinates $x^\mu(\sigma)$ should not be periodic functions of σ but periodic up to a Lorentz transformation. Modular invariance puts severe constraints on such attempts, since a string sector with twisted boundary conditions in the σ-direction is related by modular transformations to sectors with various boundary conditions in the σ- and τ-directions. What will be described below is one consistent framework in which the twisted boundary conditions are related to the construction of a new manifold (or at least, a new orbifold, as we will describe later). The method of constructing new manifolds (or orbifolds) that we will employ is a classical geometrical method that we will implement directly in string theory.

* Research supported in part by NSF Grant PHY80-19754.
† Supported by an NSF Graduate Student Fellowship.

A classical example, which has been described in the physics literature [6], involves the K3 manifold. Consider the four-dimensional torus T with periodic coordinates x_i, $i = 1 \cdots 4$, obeying $x_i \approx x_i + 1$. Let g be the transformation $g(x_i) = -x_i$. Since $g^2 = 1$, g generates the group Z_2. If g acted freely (without fixed points), then the quotient space $X = T/Z_2$ would be a smooth, non-singular manifold. Actually, g does not act freely. There are sixteen fixed points – the points in which each coordinate x_i is an integer or half-integer. Because of this, X is not a smooth manifold but a so-called orbifold – a manifold with singularities that correspond to those obtained by dividing a smooth manifold by the non-free action of a discrete group. In this case, the singularities can be smoothed out or resolved by "blowing up" the fixed points. The resulting smooth manifold is a rather subtle and interesting one called the K3 surface. The singularities encountered in the examples we will consider below can likewise be resolved, but this will not be our main concern. Our main concern will be to treat the string propagation directly on the singular orbifold. As we will discuss, this is exactly soluble. (In resolving singularities, one must introduce free parameters corresponding to the "size" of the blown-up fixed points. When these parameters are tiny, the exactly soluble problem of string propagation on the orbifold is an arbitrarily good approximation to propagation on the smooth manifold, if that is what is desired.)

The K3 manifold is in many ways unique. For instance, it is the only four-dimensional manifold of SU(2) holonomy. The construction that leads to K3 has an analogue in real dimension six (complex dimension three) that does not have the same degree of uniqueness. The simplest example was constructed by mathematicians and was described in [4], where it was called Z. More general examples have been constructed recently in [7]. Although the discussion readily generalizes to the other cases, we will illustrate our ideas by discussing string propagation on the Z manifold. To review the construction of the Z manifold, let z_i, $i = 1, 2, 3$ be three complex variables. For each i, let T_i be the torus defined by $z_i \approx z_i + 1 \approx z_i + e^{i\pi/3}$, and let α be the transformation $z_i \to z_i e^{2i\pi/3}$ acting on the product manifold $T = T_1 \times T_2 \times T_3$. Then α generates a group G isomorphic to Z_3. There are twenty seven fixed points of α, namely the points in which each z_i is an integer multiple of $\sqrt{3}\, e^{i\pi/6}$. Because of the existence of these twenty seven fixed points, the quotient space $Z = T/G$ is not a smooth manifold but an orbifold with twenty seven isolated singularities. We wish to describe string propagation on this orbifold.

To describe the quantum mechanics of a point particle propagating on Z, one approach would be to begin with the Hilbert space H_0 of wave functions for a particle propagating on T and then restrict ourselves to the Hilbert space H of G-invariant wave functions. The particle wave functions on Z can be identified with G-invariant wave functions on T. Actually, in general some choices must be made to define what we mean by the action of G on H_0. If the particle under study has some internal quantum numbers, so that the wave function is actually a section of



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2004: from twistor strings to SYM amplitudes

arXiv:hep-th/0312171 v2 6 Oct 2004

hep-th/0312171

Perturbative Gauge Theory As A String Theory In Twistor Space

Edward Witten

Institute For Advanced Study, Princeton NJ 08540 USA

Perturbative scattering amplitudes in Yang-Mills theory have many unexpected properties, such as holomorphy of the maximally helicity violating amplitudes. To interpret these results, we Fourier transform the scattering amplitudes from momentum space to twistor space, and argue that the transformed amplitudes are supported on certain holomorphic curves. This in turn is apparently a consequence of an equivalence between the perturbative expansion of $\mathcal{N} = 4$ super Yang-Mills theory and the D-instanton expansion of a certain string theory, namely the topological B model whose target space is the Calabi-Yau supermanifold $\mathbb{CP}^{3|4}$.

2004: from twistor strings to SYM amplitudes

arXiv:hep-th/0312171 v2 6 Oct 2004

SLAC–PUB–7106
hep-ph/9601359
January, 1996

CALCULATING SCATTERING AMPLITUDES EFFICIENTLY*

LANCE DIXON

Stanford Linear Accelerator Center
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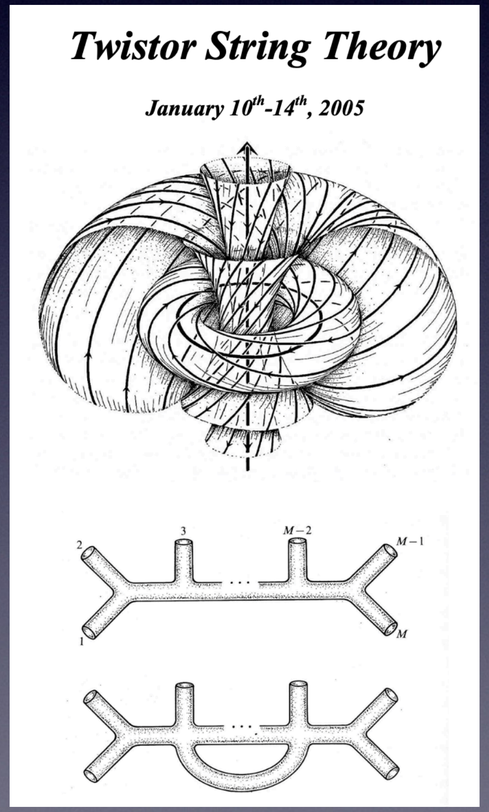
ABSTRACT

We review techniques for more efficient computation of perturbative scattering amplitudes in gauge theory, in particular tree and one-loop multi-parton amplitudes in QCD. We emphasize the advantages of (1) using color and helicity information to decompose amplitudes into smaller gauge-invariant pieces, and (2) exploiting the analytic properties of these pieces, namely their cuts and poles. Other useful tools include recursion relations, special gauges and supersymmetric rearrangements.

Invited lectures presented at the Theoretical Advanced Study Institute in Elementary Particle Physics (TASI 95): QCD and Beyond
Boulder, CO, June 4-30, 1995

2005



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people.maths.ox.ac.uk/lmason/Tws/

Twistor String Theory

[Conference Poster]

[Link to program and transparencies]

[Link to `From Twistors to Amplitudes,' a QMUL workshop.]

THE MATHEMATICAL INSTITUTE
University of Oxford

London Mathematical Society Workshop

10-14 January 2005

Abstract

This meeting was organised to take stock of the rapid progress being made on twistor-string theory and to encourage further cross-fertilization between string-theory, twistor theory and perturbative gauge theory. Twistor string theory was introduced by Witten in hep-th/0312171 as a string theory in twistor space that makes contact with N=4 super Yang-Mills theory on space-time via a generalization of the Penrose-Ward transform augmented by certain D-instanton corrections. It promises to combine many of the most attractive features of string theory and twistor theory and has implications not only for Yang-Mills but also for (conformal) gravity. It has in particular led to major advances in the calculations of Yang-Mills scattering amplitudes with applications to collider physics.

Z. Bern, P. Candelas, X. de la Ossa, S. Huggett, L. Mason

2006

CERNCOURIER

HIGGS AND ELECTROWEAK | FEATURE

Zurich workshop faces the LHC's precision challenge

27 March 2007

With Switzerland's reputation for exactitude, Zurich was the ideal place for scientists to discuss high precision for hard processes at the LHC in last year's HP² workshop.

Workshop participants in front of the ETH building in Zurich.

With the imminent start-up of the LHC, particle physics is about to enter a new regime, which should provide solutions to puzzles such as the origin of electroweak symmetry-breaking and the existence of supersymmetry. The LHC will produce head-on collisions between protons or heavy-ions, but at the fundamental level these come down to interactions between partons, that is, quarks and gluons. For this reason, all the interesting new reactions will be initiated essentially by quantum chromodynamic (QCD) hard-scattering, and any claim for new physics will require a precise understanding of known, Standard Model processes.

To prepare for the "precision challenge" at the LHC, the particle theory groups of ETH Zurich and Zurich University organized a workshop on High Precision for Hard Processes (HP²). The three-day workshop took place on the ETH campus in early September 2006, involving about 65 participants. These included 15 diploma and doctoral students, indicating that precision calculations for the LHC is a lively field that attracts many young researchers.

HP² addressed the precision challenge with reviews of results from Fermilab's Tevatron, expectations at the LHC and measurements of parton distributions. A few benchmark reactions, such as single-inclusive jet-production and vector-boson production, will already be accessible with very low luminosity at the LHC. These can provide precise constraints on the proton structure, which is relevant to all hadron-collider reactions.

Left to right: Darren Forde, Ruth Britto, David Dunbar and Lance Dixon relaxing with other participants at the reception, following an intense day of talks at the HP² workshop.

Likewise, precision studies, such as investigating the properties of the top quark, demand a better description of the full characteristics of an event. These will include improved jet-algorithms and a better understanding of soft physics in hard interactions. Searches will often involve multiparticle final states, calling for a precise description of high-multiplicity processes.

arXiv:hep-ph/9306240 v2 18 May 1998

hep-ph/9306240

SLAC–PUB–5947
SPhT/92–048
UCLA–92–043
May, 1993
(T)

Dimensionally Regulated Pentagon Integrals*

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Abstract

We present methods for evaluating the Feynman parameter integrals associated with the pentagon diagram in $4-2\epsilon$ dimensions, along with explicit results for the integrals with all masses vanishing or with one non-vanishing external mass. The scalar pentagon integral can be expressed as a linear combination of box integrals, up to $\mathcal{O}(\epsilon)$ corrections, a result which is the dimensionally-regulated version of a $D=4$ result of Melrose, and of van Neerven and Vermaseren. We obtain and solve differential equations for various dimensionally-regulated box integrals with massless internal lines, which appear in one-loop n-point calculations in QCD. We give a procedure for constructing the tensor pentagon integrals needed in gauge theory, again through $\mathcal{O}(\epsilon^0)$.

*Research supported in part by the Department of Energy under contract DE-AC03-76SF00515 (SLAC), by the Texas National Research Laboratory Commission under grant FCFY9202 (Z.B.), and by the *Direction des Sciences de la Matière* of the *Commissariat à l'Energie Atomique* of France (Saclay).

arXiv:hep-ph/9212308v1 24 Dec 1992

hep-ph/9212308

SLAC–PUB–6001 (T)
CERN-TH.6756/92
UCLA/92/42
December, 1992

Dimensionally Regulated One-Loop Integrals

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Abstract

We describe methods for evaluating one-loop integrals in $4-2\epsilon$ dimensions. We give a recursion relation that expresses the scalar n-point integral as a cyclicly symmetric combination of $(n-1)$-point integrals. The computation of such integrals thus reduces to the calculation of box diagrams ($n=4$). The tensor integrals required in gauge theory may be obtained by differentiating the scalar integral with respect to certain combinations of the kinematic variables. Such relations also lead to differential equations for scalar integrals. For box integrals with massless internal lines these differential equations are easy to solve.

Submitted to Physics Letters B

[b]Research supported by the Texas National Research Laboratory Commission grant FCFY9202.

*Research supported by the Department of Energy under grant DE-AC03-76SF00515.

[†]Laboratory of the *Direction des Sciences de la Matière* of the *Commissariat à l'Energie Atomique* of France.

arXiv:hep-ph/9403226 v2 16 Mar 1994

hep-ph/9403226

SLAC-PUB-6415
Saclay/SPhT–T94/20
UCLA/TEP/94/4
SWAT-94-17

One-Loop n-Point Gauge Theory Amplitudes, Unitarity and Collinear Limits

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ABSTRACT

We present a technique which utilizes unitarity and collinear limits to construct ansätze for one-loop amplitudes in gauge theory. As an example, we obtain the one-loop contribution to amplitudes for n gluon scattering in $N=4$ supersymmetric Yang-Mills theory with the helicity configuration of the Parke-Taylor tree amplitudes. We prove that our $N=4$ ansatz is correct using general properties of the relevant one-loop n-point integrals. We also give the "splitting amplitudes" which govern the collinear behavior of one-loop helicity amplitudes in gauge theories.

[‡]Research supported in part by the US Department of Energy under grant DE-FG03-91ER40662 and in part by the Alfred P. Sloan Foundation under grant APBR-3222.

*Research supported by the Department of Energy under grant DE-AC03-76SF00515.

[†]Address after Sept. 1, 1994: University of Swansea, Swansea, SA2 8PP, UK.

[b]Research supported in part by the NSF under grant PHY-9218990 and in part by the Department of Energy under grant DE-FG03-91ER40662.

[‡]Laboratory of the *Direction des Sciences de la Matière* of the *Commissariat à l'Energie Atomique* of France.

http://arxiv.org/abs/hep-ph/9409265v1

arXiv:hep-ph/9409265v1 9 Sep 1994

SLAC-PUB-6563
Saclay/SPhT–T94/95
UCLA/TEP/94/29
SWAT-94-36

Fusing Gauge Theory Tree Amplitudes Into Loop Amplitudes

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ABSTRACT

We identify a large class of one-loop amplitudes for massless particles that can be constructed via unitarity from tree amplitudes, without any ambiguities. One-loop amplitudes for massless supersymmetric gauge theories fall into this class; in addition, many non-supersymmetric amplitudes can be rearranged to take advantage of the result. As applications, we construct the one-loop amplitudes for n-gluon scattering in $N=1$ supersymmetric theories with the helicity configuration of the Parke-Taylor tree amplitudes, and for six-gluon scattering in $N=4$ super-Yang-Mills theory for all helicity configurations.

[‡]Research supported in part by the US Department of Energy under grant DE-FG03-91ER40662 and in part by the Alfred P. Sloan Foundation under grant BR-3222.

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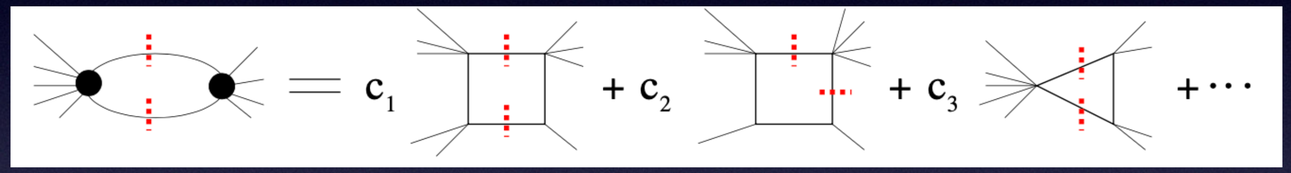
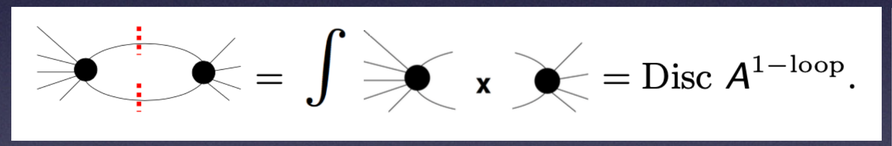
[‡]Laboratory of the *Direction des Sciences de la Matière* of the *Commissariat à l'Energie Atomique* of France.

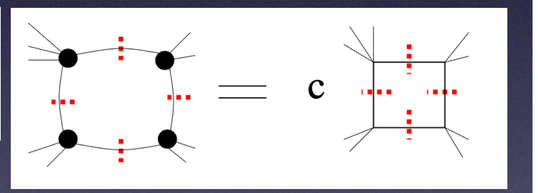
Unitarity, master integrals, dimensional regularization…

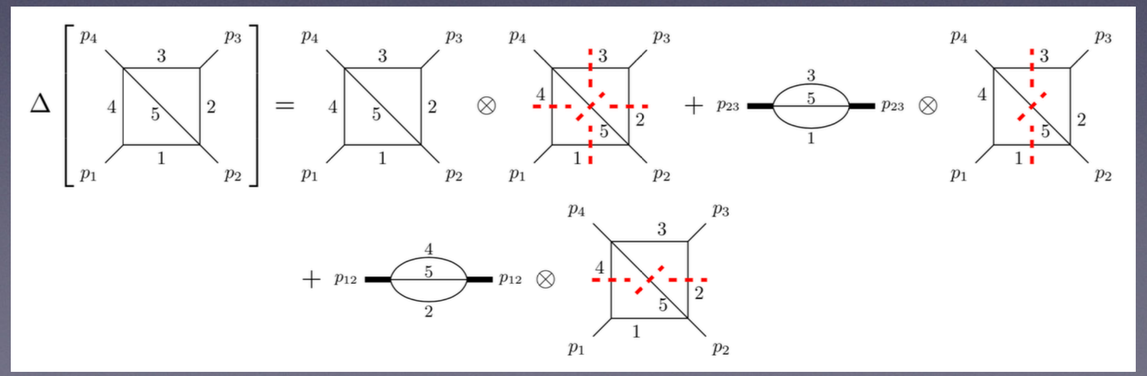
Cuts: the theme of my career

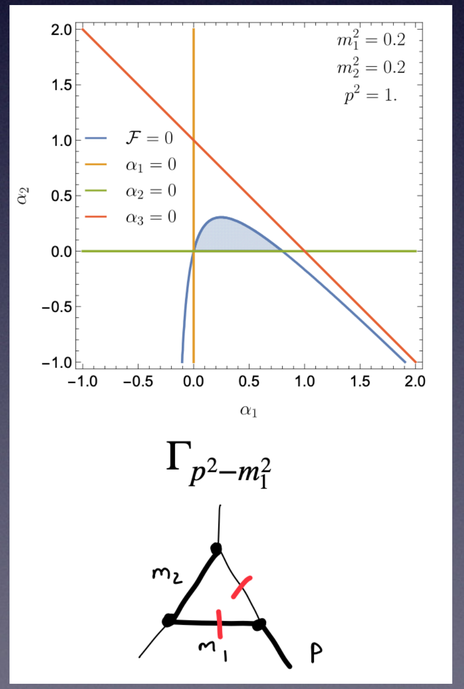
Unitarity and on-shell methods











Amplitudes 2019

Trinity College Dublin

1-5 July 2019

Organizers:

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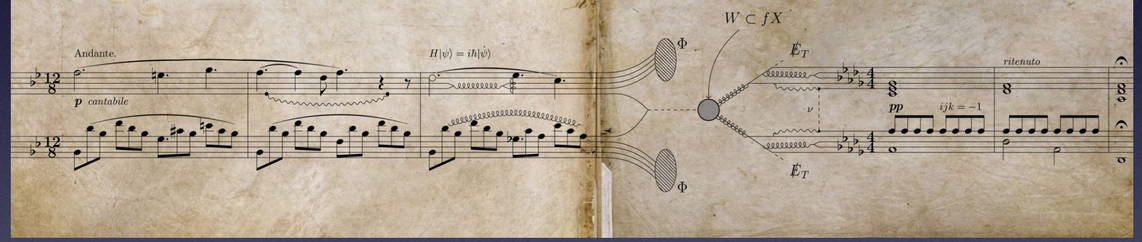
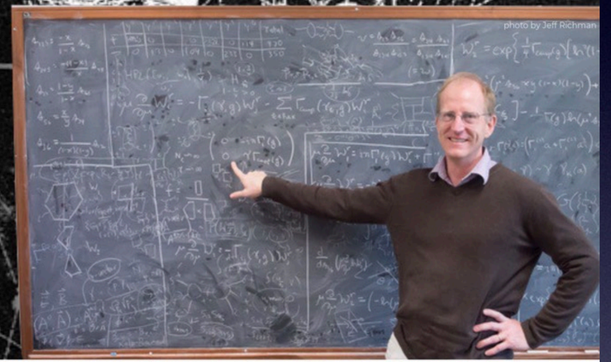
Donal O'Connell
Dmytro Volin

Prof. Lance J. Dixon (SLAC)
presents

Journey to the Heart of Matter

a public lecture

Thursday July 4th, 8pm
Tercentenary Hall
Trinity College Dublin



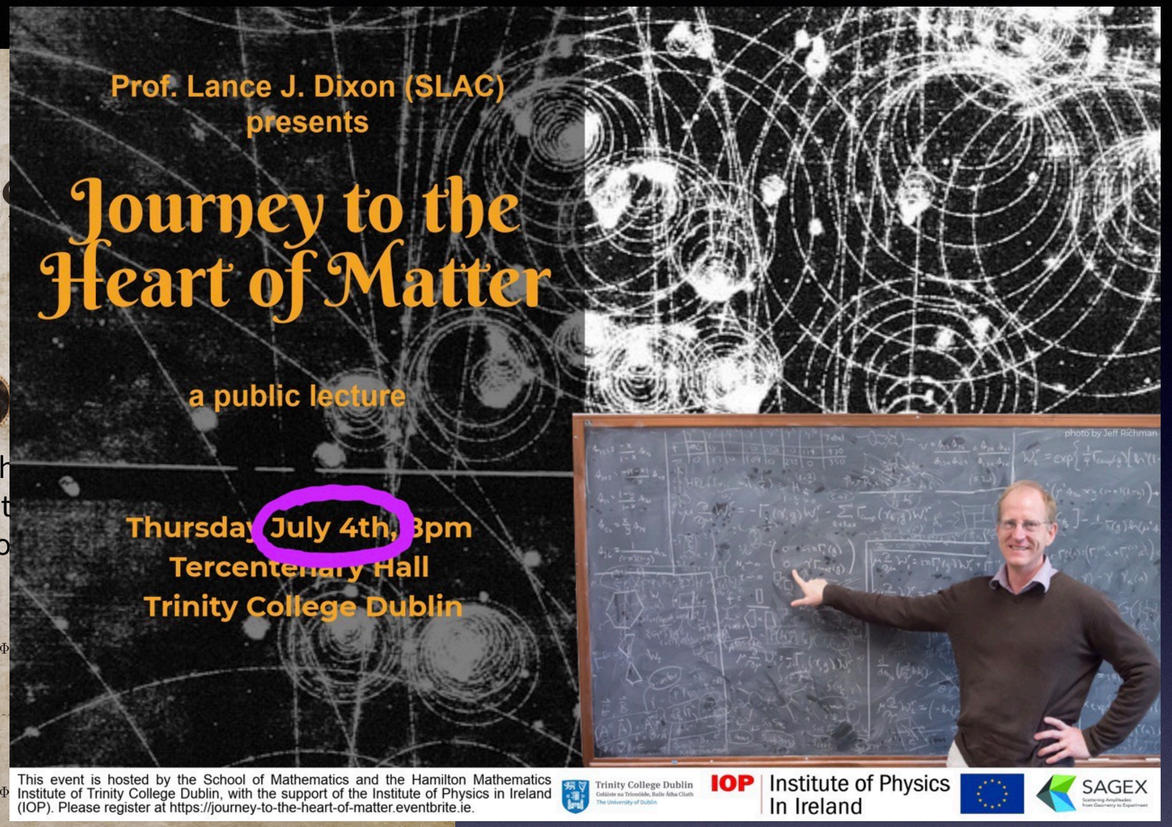
This event is hosted by the School of Mathematics and the Hamilton Mathematics Institute of Trinity College Dublin, with the support of the Institute of Physics in Ireland (IOP). Please register at https://journey-to-the-heart-of-matter.eventbrite.ie.

Trinity College Dublin
Coláiste na Tríonóide, Baile Átha Cliath
The University of Dublin

IOP Institute of Physics In Ireland

SAGEX
Scattering Amplitudes: From Geometry to Experiment

mplitudes 201
Trinity College Dublin
1-5 July 2019
Andante.
p cantabile


Prof. Lance J. Dixon (SLAC)
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Journey to the Heart of Matter
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Thursday July 4th, 3pm
Tercentenary Hall
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photo by Jeff Richman
This event is hosted by the School of Mathematics and the Hamilton Mathematics Institute of Trinity College Dublin, with the support of the Institute of Physics in Ireland (IOP). Please register at https://journey-to-the-heart-of-matter.eventbrite.ie.
Trinity College Dublin
IOP
Institute of Physics In Ireland
SAGEX

Amplitudes 2018
Jun 18 – 22, 2018
SLAC National Accelerator Laboratory or https://stanford.zoom.us/j/490970988
US/Pacific timezone

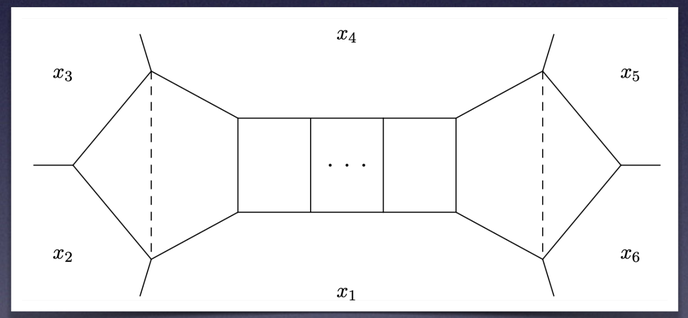
2018 at GGI

I presented “Reduction of Hypergeometric Integrals” - hypergeometric coaction work with Abreu, Duhr, Gardi, Matthew

$$\begin{aligned}\Delta_\epsilon\Big({}_2F_1(\alpha,\beta;\gamma;x)\Big) &= {}_2F_1(1+a\epsilon,b\epsilon;1+c\epsilon;x)\otimes {}_2F_1(\alpha,\beta;\gamma;x)\\ &\quad -\frac{b\epsilon}{1+c\epsilon}\,{}_2F_1(1+a\epsilon,1+b\epsilon;2+c\epsilon;x) \qquad (5.14)\\ &\quad \otimes \frac{\Gamma(1-\beta)\Gamma(\gamma)}{\Gamma(1-\beta+\alpha)\Gamma(\gamma-\alpha)}x^{1-\alpha}{}_2F_1\left(\alpha,1+\alpha-\gamma;1-\beta+\alpha;\frac{1}{x}\right),\end{aligned}$$

2018 at GGI

- Lance asked if we could write the coaction of ${}_2F_1(ig, -ig; 1; x)$ that featured in his paper with Caron-Huot, von Hippel, McLeod, Papathanasiou on double pentaladders to all orders.
- It is an interesting function, and different from typical Feynman integrals.

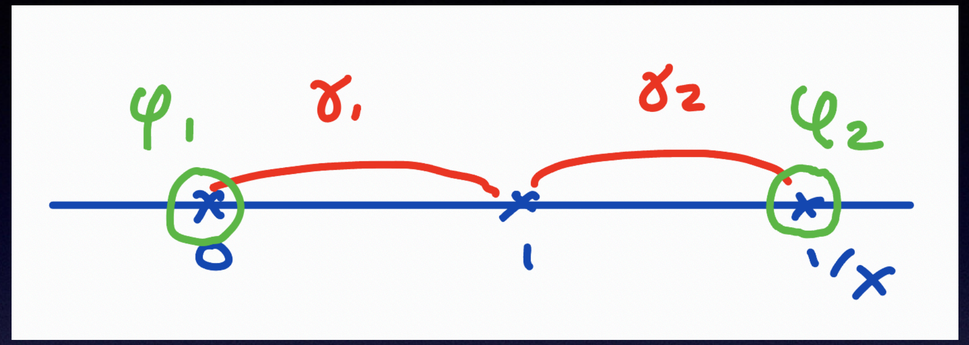


$$\Omega(u,v,w,g^2) = -\sum_{k=1}^{\infty}[(-\sqrt{xyz})^k + (-\sqrt{xy/z})^k]\hat{F}^j_{-ik}(x)\hat{F}^j_{-ik}(y) - \hat{F}_0^{2ig}(x)\hat{F}_0^{2ig}(y)\,. \qquad (4.5)$$

$$\hat{F}_0^{2ig}(x) = \frac{\pi g}{\sinh \pi g}\, {}_2F_1(ig, -ig, 1, x)$$

$$\hat{F}_0^{2ig}(x) = \frac{\pi g}{\sinh \pi g}\left(1 + g^2 \sum_{n=1}^{\infty}\frac{x^n}{n^2}\sum_{j,k=0}^{\infty}(-1)^j (ig)^{j+k} Z_{\underbrace{1,\ldots,1}_{j}}(n-1) Z_{\underbrace{1,\ldots,1}_{k}}(n-1)\right)$$

Here is the coaction.



$$\Delta\left[{}_2F_1(ig,-ig,1,x)\right] = {}_2F_1(ig,-ig,1,x)\otimes {}_2F_1(ig,-ig,1,x) - igx\,{}_2F_1(1+ig,1-ig,2,x)\otimes ig(1-x)\,{}_2F_1(1+ig,1-ig,2,1-x)\,.$$

Well, so what?

I don’t know what we can do with that.

But all the elements are special…

$$\mathcal{S}\left[\begin{pmatrix} {}_2F_1(ig,-ig,1,x)^{\partial r} & igx\,{}_2F_1(1+ig,1-ig,2,x)^{\partial r} \\ ig(x-1)\,{}_2F_1(1+ig,1-ig,2,1-x)^{\partial r} & {}_2F_1(ig,-ig,1,1-x)^{\partial r} \end{pmatrix}\right]$$

$$= \begin{pmatrix} 1+(ig)^2(1-x)\otimes x+(ig)^4(1-x)\otimes x\otimes(1-x)\otimes x+\cdots & -(ig)(\otimes(1-x))-(ig)^3(1-x)\otimes x\otimes(1-x)-\cdots \\ (ig)(\otimes x)+(ig)^3 x\otimes(1-x)\otimes x+\cdots & 1+(ig)^2 x\otimes(1-x)+(ig)^4 x\otimes(1-x)\otimes x\otimes(1-x)+\cdots \end{pmatrix}$$

We see antipodal duals.

The antipode map S

1. arXiv:2502.00862 [pdf, other] hep-th hep-ph doi 10.1103/PhysRevD.111.L101901

   Antipodal self-duality of square fishnet graphs

   Authors: Lance J. Dixon, Claude Duhr

   Abstract: In strongly-deformed planar $\mathcal{N} = 4$ super-Yang-Mills theory, or fishnet theory, a point-split single-trace correlation function of four dimension-m scalar operators is given by a single Feynman integral, which involves integrating over locations of a $m \times m$ grid of points. We show that for any integer m this square fishnet graph is invariant under the combined action of a kinematic m... ▽ More

   Submitted 15 May, 2025; **v1** submitted 2 February, 2025; **originally announced** February 2025.

   Comments: 9 pages, 1 figure. v2: minor corrections to match published version

   Report number: BONN-TH-2025-02

   Journal ref: Phys. Rev. D 111, L101901 (2025)

2. arXiv:2308.08199 [pdf, other] hep-th hep-ph

   An Eight Loop Amplitude via Antipodal Duality

   Authors: Lance J. Dixon, Yu-Ting Liu

   Abstract: We compute the six-particle maximally-helicity-violating (MHV) amplitude in planar $\mathcal{N} = 4$ super-Yang-Mills theory at eight loops, using antipodal duality and the recently computed eight-loop three-point form factor for the chiral stress energy tensor multiplet. Antipodal duality maps the form factor symbol to the amplitude symbol on a two-dimensional parity-preserving surface in the three-di... ▽ More

   Submitted 16 August, 2023; **originally announced** August 2023.

   Comments: 50 pages, 3 figures, 6 tables, 8 loops

   Report number: SLAC-PUB-17693

3. arXiv:2212.02410 [pdf, other] hep-th hep-ph doi 10.1103/PhysRevLett.130.111601

   Antipodal Self-Duality for a Four-Particle Form Factor

   Authors: Lance J. Dixon, Ömer Gürdoğan, Yu-Ting Liu, Andrew J. McLeod, Matthias Wilhelm

   Abstract: We bootstrap the symbol of the maximal-helicity-violating four-particle form factor for the chiral part of the stress-tensor supermultiplet in planar $\mathcal{N} = 4$ super-Yang-Mills theory at two loops. When minimally normalized, this symbol involves only 34 letters and obeys the extended Steinmann relations in all partially-overlapping three-particle momentum channels. In addition, the remainder... ▽ More

   Submitted 30 November, 2024; **v1** submitted 5 December, 2022; **originally announced** December 2022.

   Comments: 9 pages, 1 figure and 1 table. v2: minor clarifications, references added, journal version. v3: ancillary file containing 3 loop symbol added

   Report number: CERN-TH-2022-190, SLAC-PUB-17711

4. arXiv:2112.06243 [pdf, other] hep-th hep-ph doi 10.1103/PhysRevLett.128.111602

   Folding Amplitudes into Form Factors: An Antipodal Duality

   Authors: Lance J. Dixon, Omer Gurdogan, Andrew J. McLeod, Matthias Wilhelm

   Abstract: We observe that the three-gluon form factor of the chiral part of the stress-tensor multiplet in planar $\mathcal{N} = 4$ super-Yang-Mills theory is dual to the six-gluon MHV amplitude on its parity-preserving surface. Up to a simple variable substitution, the map between these two quantities is given by the antipode operation defined on polylogarithms (as part of their Hopf algebra structure), whic... ▽ More

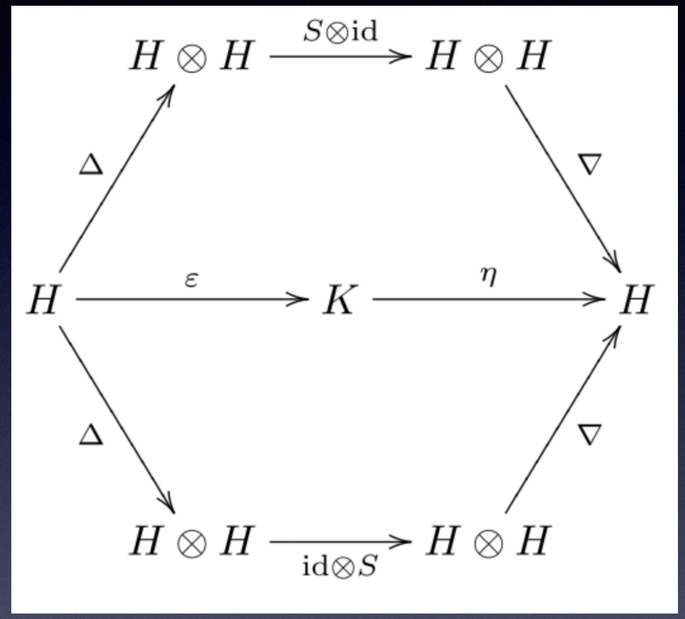
   Submitted 22 March, 2022; **v1** submitted 12 December, 2021; **originally announced** December 2021.

   Comments: 5+2 pages, 2 figures, 2 tables. v2: added further checks at kinematic points, some clarifications and references; version to appear in PRL

   Report number: SLAC-PUB-17637

   Journal ref: Phys.Rev.Lett. 128 (2022) 11, 111602

Hopf algebra



- Δ is the coproduct.
- S is the antipode.

- We have de Rham versions of the hypergeometric functions that form a Hopf algebra. [Brown, Dupont]
- Antipode of a 2F1 doesn't have to be a single 2F1, just something generated in the algebra.
- In our example, it recombines very nicely.
- Why? Anything to do with Z-sums? [cf. Moch, Uwer, Weinzierl]

- This example has extra symmetry.

$$ {}_2F_1(ig, -ig, 1, x) = ig \int_0^1 dt\, t^{-1+ig}(1-t)^{-ig}(1-xt)^{ig} $$

- The exponents of t and $(1-t)$ sum to an integer.
- Therefore, integrals over $(0,1)$ and $(1/x, \infty)$ have the same symbol.
- Now it makes sense!

The antipode exchanges canonical forms and their dual cycles.

$$\mathcal{S}\left[\begin{pmatrix} \int_{\gamma_{0,1}} \Phi\psi_{0,\infty} & \int_{\gamma_{0,1}} \Phi\psi_{1/x,\infty} \\ \int_{\gamma_{1/x,1}} \Phi\psi_{0,\infty} & \int_{\gamma_{1/x,1}} \Phi\psi_{1/x,\infty} \end{pmatrix}\right] = \mathcal{S}\left[\begin{pmatrix} \int_{\gamma_{1/x,\infty}} \Phi^{-1}\psi_{0,\infty} & \int_{\gamma_{1/x,\infty}} \Phi^{-1}\psi_{1/x,\infty} \\ \int_{\gamma_{0,\infty}} \Phi^{-1}\psi_{0,\infty} & \int_{\gamma_{0,\infty}} \Phi^{-1}\psi_{1/x,\infty} \end{pmatrix}\right]$$

$$\Phi = t^{ig}(1-t)^{-ig}(1-xt)^{ig}.$$

$$\gamma_{v,w} = (v,w), \quad \psi_{v,w} = d\log\frac{t-v}{t-w}, \quad \psi_{v,\infty} = d\log(t-v).$$

Hypergeometric Antipode

$$S\left[\int_{\gamma_i} \Phi\psi_j\right] = \int_{\gamma_j} \Phi^{-1}\psi_i \,.$$

- ψ_i is the canonical form associated to γ_i.
- For a general ${}_2F_1$ expanded around integer parameters, we would decompose the integrand into a basis of canonical forms.

$$S\left[{}_2F_1(ig,-ig,1,x)^{\mathfrak{d}r}\right] = {}_2F_1(-ig,ig,1,1-x)^{\mathfrak{d}r}$$

Let’s keep talking!

Thank you for every kind of collegiality.

Happy birthday, Lance!