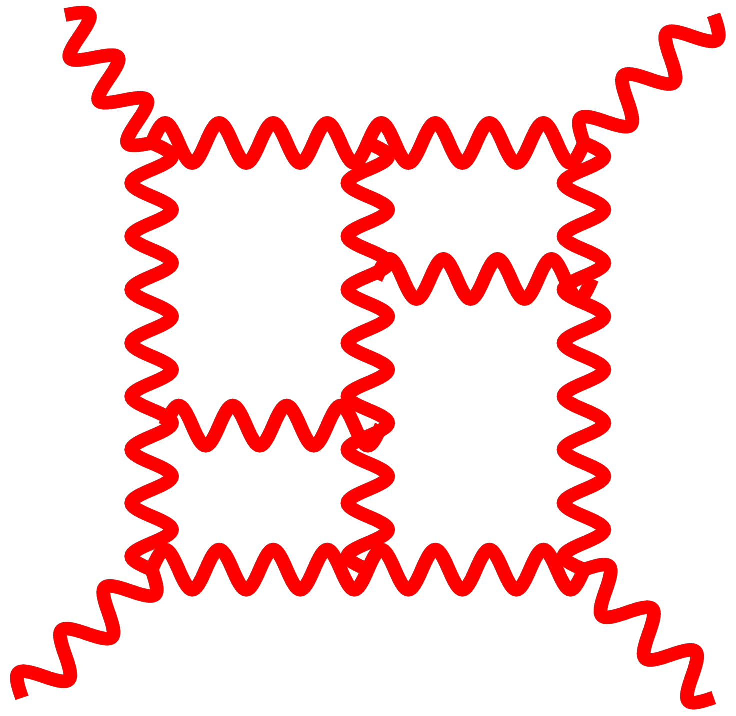
Gravity Adventures with Lance

Zvi Bern

LanceFest

June 25, 2026



UCLA Mani L. Bhaumik Institute for Theoretical Physics

Happy Birthday to Lance

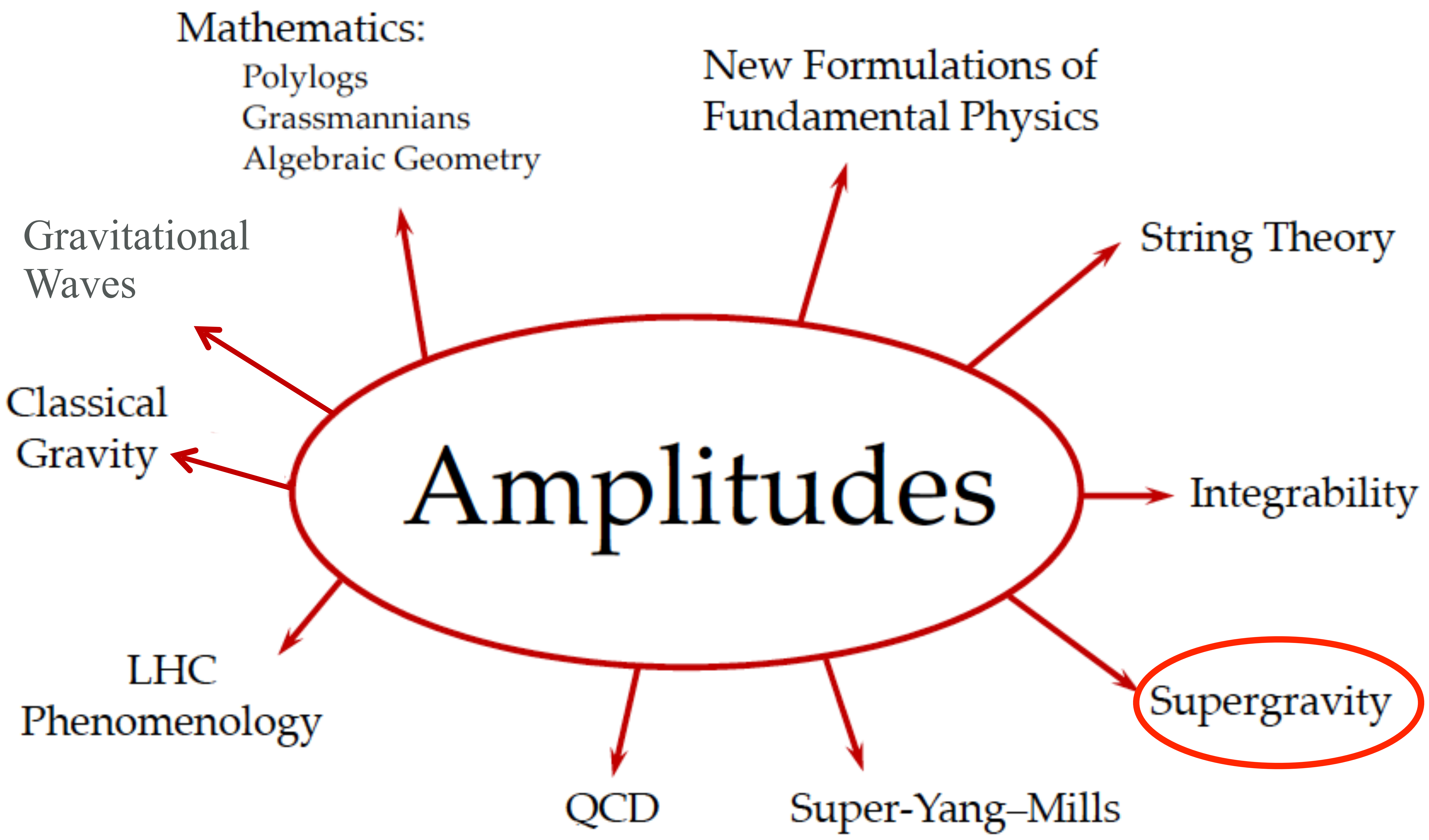
Organizers asked me to tell you about Lance's supergravity adventures.

Outline

1. Early days. Realizing the power of the KLT relations.
2. 1st round of supergravity papers. Two-loops four-point. One-loop all *n*.
3. 2nd round: UV studies of supergravity.
4. UV subtleties: "Divergences do not tell you if theory diverges"
5. Remaining UV Puzzles
   - Role of self-dual anomaly in UV divergences.
   - Efficient control of evanescent effects. Important issue at higher loops.
   - Origin of "enhanced UV cancellations"
   - Do $N \geq 5$ supergravities diverge? If so then at what loop order?
6. Conclusions

Today Many Directions

Looking at the broad range of talks, our field has blossomed in many directions.



We have seen many examples Lance's contributions to these directions in this workshop.

Young Believers ~1993

Zvi Bern

Lance Dixon

Dave Dunbar

David Kosower

$$A(1^-, 2^-, 3^+, \ldots, n^+) = i \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \cdots \langle n1 \rangle}$$

Parke, Taylor;
Mangano, Parke, Xu

In those days few appreciated significance of this formula.

In 1993, the field of "Amplitudes" was exactly four people.

Complications with Perturbative Gravity

Consider the Einstein gravity Lagrangian

curvature

Flat-space metric

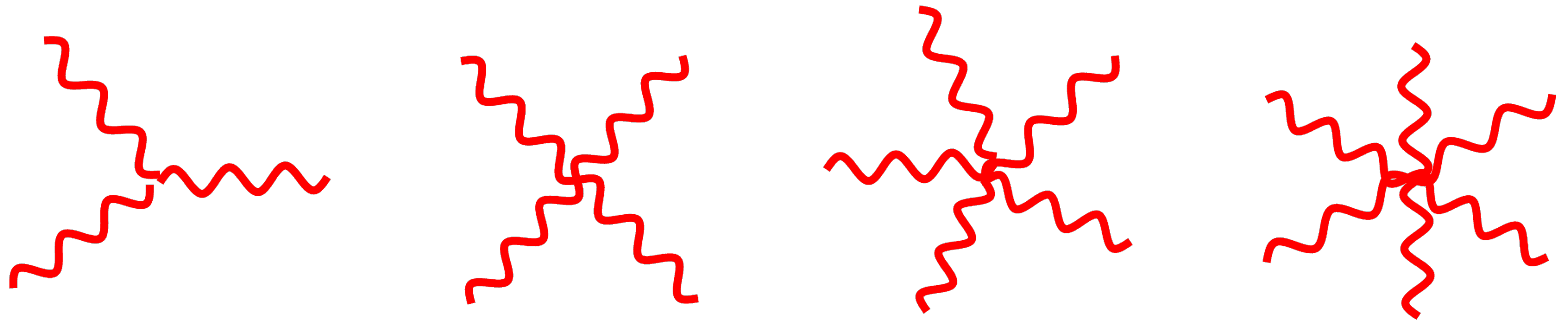
$$L_{\text{gravity}} = \frac{2}{\kappa^2}\sqrt{-g}\,R \qquad g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$$

graviton field

metric

$$\kappa^2 = 32\pi\, G_{\text{Newton}}$$

Infinite number of complicated interactions

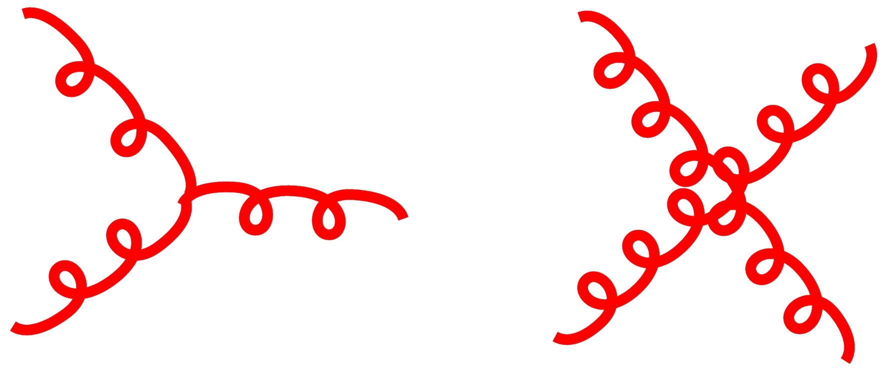


$+ \cdots$

Very nonlinear

Compare to gauge-theory Lagrangian on which QCD is based

$$L_{\text{YM}} = \frac{1}{g^2} F^2$$



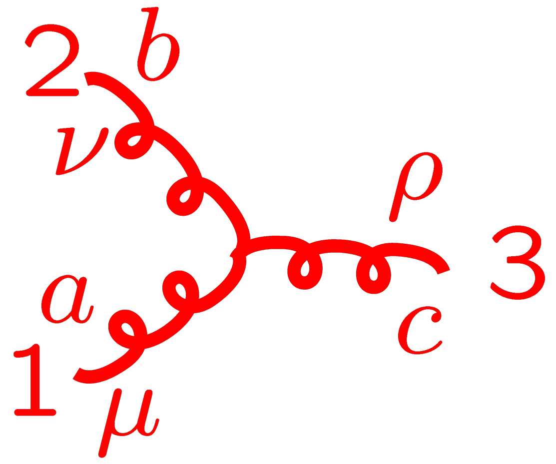
Only three and four point interactions

Gravity seems so much more complicated than gauge theory.

Gauge and gravity theories seem rather different.

Three-Point Interactions

Standard perturbative approach:



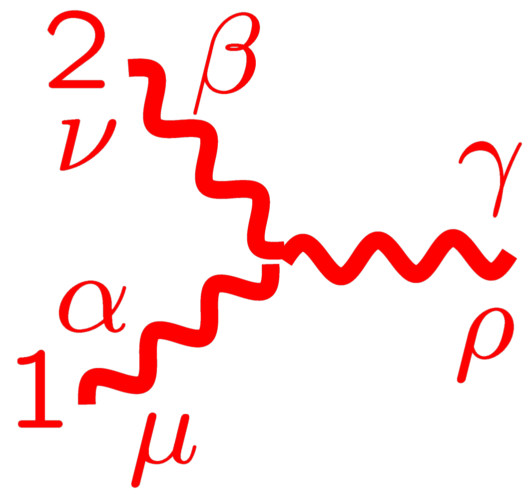
Three-gluon vertex from strong interactions:

$$V_{3\,\mu\nu\sigma}^{abc} = -g f^{abc}(\eta_{\mu\nu}(k_1-k_2)_\rho + \eta_{\nu\rho}(k_1-k_2)_\mu + \eta_{\rho\mu}(k_1-k_2)_\nu)$$

Three-graviton vertex:

$$k_i^2 = E_i^2 - \vec{k}_i^2 \neq 0$$

$$\begin{aligned} G_{3\mu\alpha,\nu\beta,\sigma\gamma}(k_1,k_2,k_3) = \\ \mathrm{sym}[-\tfrac{1}{2}P_3(k_1\cdot k_2\eta_{\mu\alpha}\eta_{\nu\beta}\eta_{\sigma\gamma}) - \tfrac{1}{2}P_6(k_{1\nu}k_{1\beta}\eta_{\mu\alpha}\eta_{\sigma\gamma}) + \tfrac{1}{2}P_3(k_1\cdot k_2\eta_{\mu\nu}\eta_{\alpha\beta}\eta_{\sigma\gamma}) \\ + P_6(k_1\cdot k_2\eta_{\mu\alpha}\eta_{\nu\sigma}\eta_{\beta\gamma}) + 2P_3(k_{1\nu}k_{1\gamma}\eta_{\mu\alpha}\eta_{\beta\sigma}) - P_3(k_{1\beta}k_{2\mu}\eta_{\alpha\nu}\eta_{\sigma\gamma}) \\ + P_3(k_{1\sigma}k_{2\gamma}\eta_{\mu\nu}\eta_{\alpha\beta}) + P_6(k_{1\sigma}k_{1\gamma}\eta_{\mu\nu}\eta_{\alpha\beta}) + 2P_6(k_{1\nu}k_{2\gamma}\eta_{\beta\mu}\eta_{\alpha\sigma}) \\ + 2P_3(k_{1\nu}k_{2\mu}\eta_{\beta\sigma}\eta_{\gamma\alpha}) - 2P_3(k_1\cdot k_2\eta_{\alpha\nu}\eta_{\beta\sigma}\eta_{\gamma\mu})] \end{aligned}$$

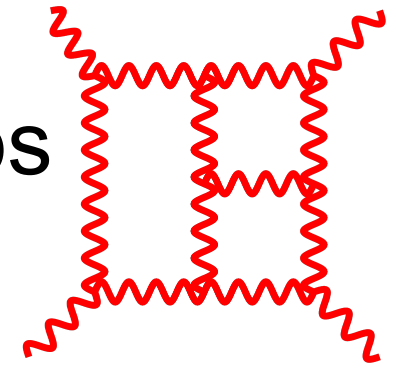


About 100 terms in three vertex

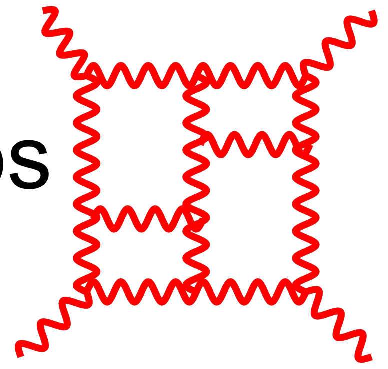
Naïve conclusion: Gravity is a nasty mess.

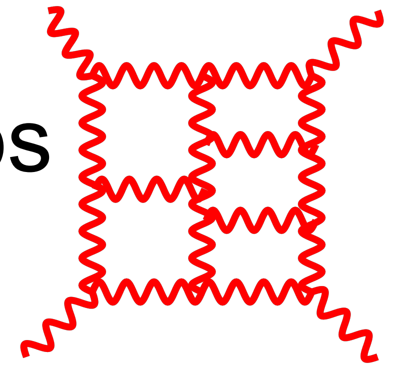
Feynman Diagrams for Gravity

We will be talking about high order processes

3 loops  $\sim 10^{20}$ TERMS

No surprise it has never been calculated via Feynman diagrams.

4 loops  $\sim 10^{26}$ TERMS

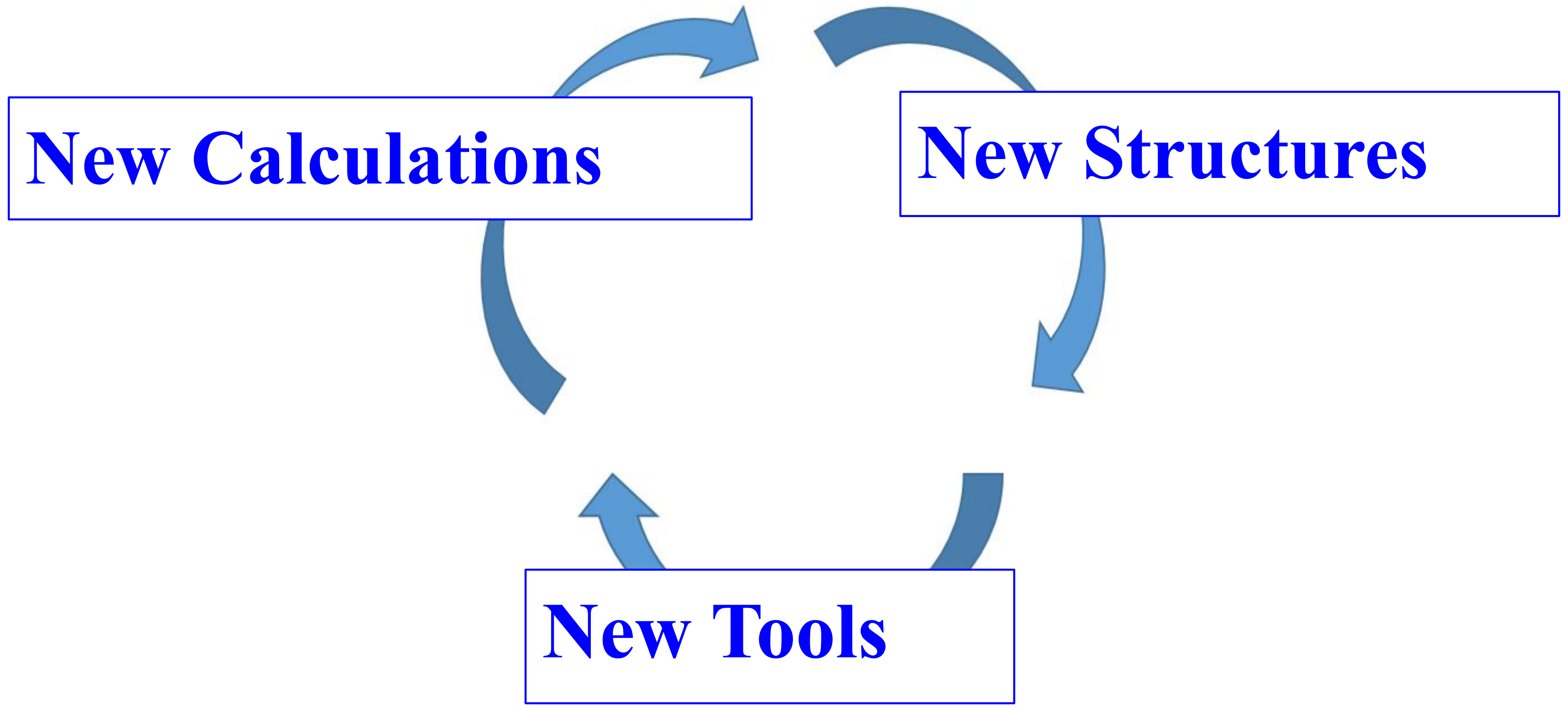
5 loops  $\sim 10^{31}$ TERMS

Spirals out of control

- Calculations to settle this seemed utterly hopeless!
- Seemed destined for dustbin of undecidable questions.

This is not a problem that can be done using ordinary methods.
Simplified gauges can help.
Supergravity should be simpler but it’s not, because extended superspace not easy.

The search for new structures.



A virtuous cycle.

- Key priority for new calculations is to uncover new and useful structures.
- Simultaneously push the state of the art for physical quantities of interest.

Examples of structure in this talk:

1. All-*n* gravity bootstraps.
2. Double copy. Converting gauge theory to gravity.

The Story of Supergravity Started in QCD

See David Kosower's talk

This was our first QCD amplitude together with Lance.

VOLUME 70, NUMBER 18 PHYSICAL REVIEW LETTERS 3 MAY 1993

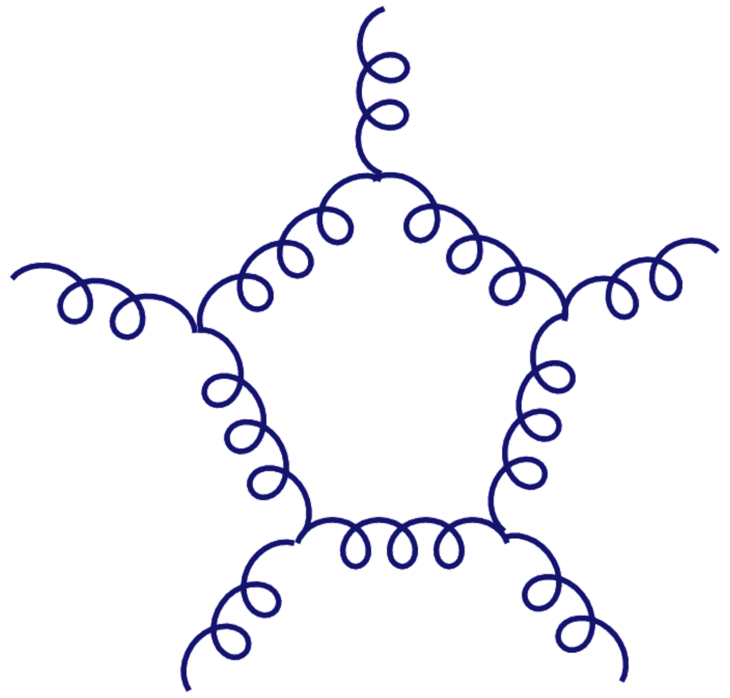
One-Loop Corrections to Five-Gluon Amplitudes

Zvi Bern[a]
Department of Physics, University of California, Los Angeles, Los Angeles, California 90024

Lance Dixon[b]
Stanford Linear Accelerator Center, Stanford, California 94309

David A. Kosower[c]
Theory Division, CERN, CH-1211 Geneva 23, Switzerland
(Received 1 March 1993)

We present the one-loop helicity amplitudes with five external gluons. The computation employs string-based methods, new techniques for performing tensor integrals, and improvements in the spinor helicity method.

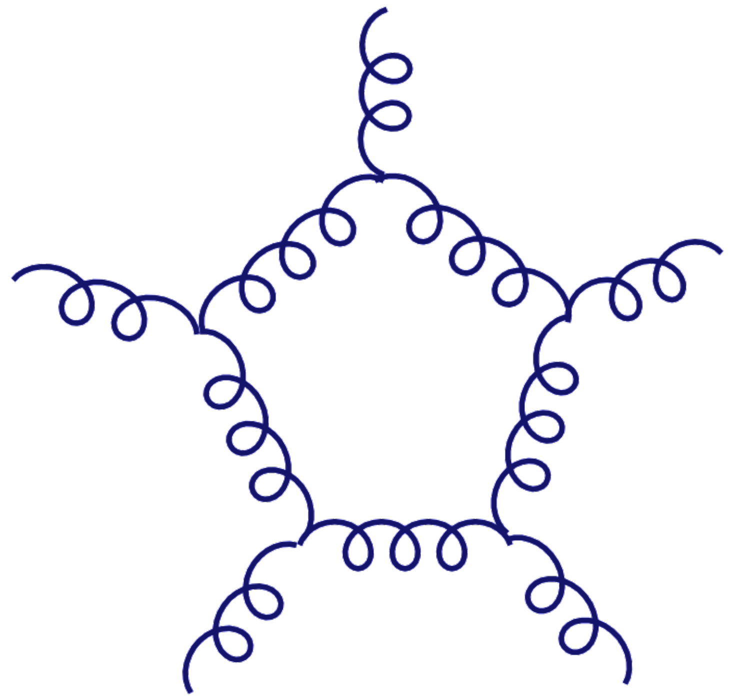


Our path to supergravity started with this paper

QCD Pure Gluon Theory Simplicity at One Loop

ZB, Dixon, Kosower 1993

Using string-based methods we obtained the one-loop QCD five-gluon.
Noticed convenient supersymmetric decomposition.



$$A_{5;1}^{[0]} = c_\Gamma \left(V^s A_5^{\rm tree} + iF^s \right) ,$$
$$A_{5;1}^{[1/2]} = -c_\Gamma \left((V^f + V^s) A_5^{\rm tree} + i(F^f + F^s) \right) ,$$
$$A_{5;1}^{[1]} = c_\Gamma \left((V^g + 4V^f + V^s) A_5^{\rm tree} + i(4F^f + F^s) \right)$$

Susy decompositions

***N* = 4 sYM**

$$V^g = -\frac{1}{\epsilon^2} \sum_{j=1}^{5} \left(\frac{\mu^2}{-s_{j,j+1}} \right)^{\epsilon} + \sum_{j=1}^{5} \ln \left(\frac{-s_{j,j+1}}{-s_{j+1,j+2}} \right) \ln \left(\frac{-s_{j+2,j-2}}{-s_{j-2,j-1}} \right) + \frac{5}{6}\pi^2$$

***N* = 4 sYM is simple!**
Seeds of supergravity right here.

***N* = 1 sYM**

$$V^f = -\frac{5}{2\epsilon} - \frac{1}{2} \left[\ln \left(\frac{\mu^2}{-s_{23}} \right) + \ln \left(\frac{\mu^2}{-s_{51}} \right) \right] - 2, \qquad V^s = -\frac{1}{3} V^f + \frac{2}{9}$$

$$F^f = -\frac{1}{2} \frac{\langle 1\,2\rangle^2 \left(\langle 2\,3\rangle [3\,4] \langle 4\,1\rangle + \langle 2\,4\rangle [4\,5] \langle 5\,1\rangle \right)}{\langle 2\,3\rangle \langle 3\,4\rangle \langle 4\,5\rangle \langle 5\,1\rangle} \frac{\mathrm{L}_0\left(\frac{-s_{23}}{-s_{51}}\right)}{s_{51}}$$

$$F^s = -\frac{1}{3} \frac{[3\,4] \langle 4\,1\rangle \langle 2\,4\rangle [4\,5] \left(\langle 2\,3\rangle [3\,4] \langle 4\,1\rangle + \langle 2\,4\rangle [4\,5] \langle 5\,1\rangle \right)}{\langle 3\,4\rangle \langle 4\,5\rangle} \frac{\mathrm{L}_2\left(\frac{-s_{23}}{-s_{51}}\right)}{s_{51}^3} - \frac{1}{3} F^f$$
$$- \frac{1}{3} \frac{\langle 3\,5\rangle [3\,5]^3}{[1\,2] [2\,3] \langle 3\,4\rangle \langle 4\,5\rangle [5\,1]} + \frac{1}{3} \frac{\langle 1\,2\rangle [3\,5]^2}{[2\,3] \langle 3\,4\rangle \langle 4\,5\rangle [5\,1]} + \frac{1}{6} \frac{\langle 1\,2\rangle [3\,4] \langle 4\,1\rangle \langle 2\,4\rangle [4\,5]}{s_{23} \langle 3\,4\rangle \langle 4\,5\rangle s_{51}}$$

Amazingly it fit into PRL

- **Loop-level helicity amplitudes are surprisingly simple.**
- **Maximally supersymmetric amplitudes simplest part**

Simplicity at one-loop in gauge theory.

ELSEVIER

Nuclear Physics B425 (1994) 217–260

NUCLEAR PHYSICS B

One-loop n-point gauge theory amplitudes, unitarity and collinear limits

Zvi Bern [a,1], Lance Dixon [b,2], David C. Dunbar [a,3], David A. Kosower [c,4]

[a] *Department of Physics, UCLA, Los Angeles, CA 90024, USA*
[b] *Stanford Linear Accelerator Center, Stanford University, Stanford, CA 94309, USA*
[c] *Service de Physique Théorique de Saclay, Centre d'Etudes de Saclay, F-91191 Gif-sur-Yvette cedex, France*

Received 11 March 1994; accepted for publication 6 April 1994

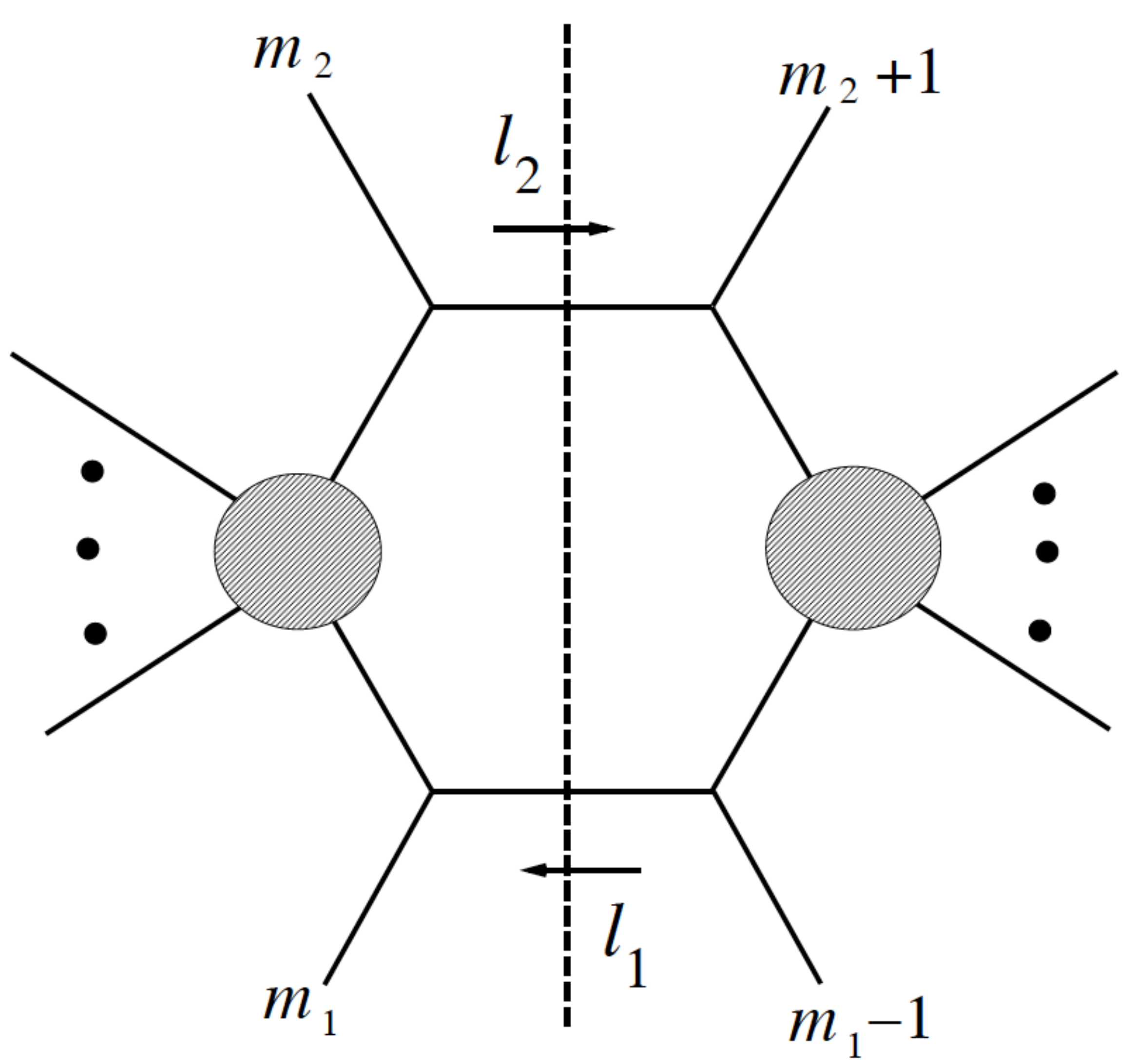
Five-gluon amplitude simplicity suggested that all-leg bootstrap is possible.
Parke-Taylor simplicity can be imported to create loops.

All-n at One Loop in Gauge Theory

Bern, Dixon, Dunbar, Kosower (1994)

Applied collinear limits and unitarity. Birth of unitarity method.

Key observation: simple dependence of propagators on loop momenta.



$$A^{\rm tree}(l_1^+,1^-,2^-,3^+,\ldots,m^+,l_3^+) = \frac{\langle 12\rangle^4}{\langle l_1 1\rangle\langle 12\rangle\cdots\langle m l_1\rangle\langle l_1 l_2\rangle}$$

No more than a hexagon integral needed for all *n* MHV.

N **= 4 susy**

$$V_n^g = \sum_{i=1}^n -\frac{1}{\epsilon^2}\left(\frac{\mu^2}{-t_i^{[2]}}\right)^\epsilon - \sum_{r=2}^{\lfloor n/2\rfloor -1}\sum_{i=1}^n \ln\left(\frac{-t_i^{[r]}}{-t_i^{[r+1]}}\right)\ln\left(\frac{-t_{i+1}^{[r]}}{-t_i^{[r+1]}}\right) + D_n + L_n + \frac{n\pi^2}{6}$$

D_n and L_n in terms of polylogs

Arbitrary number of external legs at loop level possible!

Kawai, Lewellen and Tye and Double Copy

Given our success in gauge theory Lance and I went to the SLAC library to get the KLT paper.

“Kids in a candy store”

A RELATION BETWEEN TREE AMPLITUDES
OF CLOSED AND OPEN STRINGS*

H. KAWAI, D.C. LEWELLEN and S.-H.H. TYE

Newman Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14853, USA

Received 11 October 1985

We derive a formula which expresses any closed string tree amplitude in terms of a sum o
the products of appropriate open string tree amplitudes. This formula is applicable to the heteroti

KLT Relation Between Gravity and Gauge Theory

KLT (1985)

Kawai-Lewellen-Tye string relations in low-energy limit:

gravity ↙ ↙ gauge-theory color ordered

$$M_4^{\rm tree}(1,2,3,4) = -is_{12}A_4^{\rm tree}(1,2,3,4)\,A_4^{\rm tree}(1,2,4,3)\,,$$

$$M_5^{\rm tree}(1,2,3,4,5) = is_{12}s_{34}A_5^{\rm tree}(1,2,3,4,5)\,A_5^{\rm tree}(2,1,4,3,5) + is_{13}s_{24}A_5^{\rm tree}(1,3,2,4,5)\,A_5^{\rm tree}(3,1,4,2,5)$$

Generalizes to explicit all-leg form. ZB, Dixon, Perelstein, Rozowsky

1. **Gravity is derivable from gauge theory. Standard Lagrangian methods offers no hint why this is possible.**

2. **It was obvious we would be able to translate gauge-theory advances to gravity.**

Gravity From Gauge Theory

The simplest constructions:

$$\mathcal{N}=8\ \text{sugra:}\quad (\mathcal{N}=4\ \text{sYM})\times(\mathcal{N}=4\ \text{sYM})$$
$$\mathcal{N}=5\ \text{sugra:}\quad (\mathcal{N}=4\ \text{sYM})\times(\mathcal{N}=1\ \text{sYM})$$
$$\mathcal{N}=4\ \text{sugra:}\quad (\mathcal{N}=4\ \text{sYM})\times(\mathcal{N}=0\ \text{sYM})$$

Spectrum controlled by simple tensor product of gauge theories.

Many other theories in double-copy story, including open and closed string theory, Einstein YM, NLSM, Dirac Born Infeld, Galileon and Z theory.

Cachazo, He, Yuan; Chen Du, Broedel, Schlotterer and Stieberger; Carrasco, Mafra, Schlotterer; Anastasiou, Bornsten, Duff; Duff, Hughs, Nagy; Johansson and Ochirov; Carrasco, Chiodaroli, Günaydin and Roiban; ZB, Davies, Dennen, Huang and Nohle; Nohle; Chiodaroli, Günaydin, Johansson, Roiban. A. Anastasiou, L. Borsten, M.J. Duff, M.J. Hughes, Marrani, Nagy, Zoccali.

Onwards to Multi-Loop Supergravity

Our First Paper on Gravity, $N = 8$ Supergravity at 2 Loops

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Nuclear Physics B 530 (1998) 401–456

NUCLEAR PHYSICS B

On the relationship between Yang–Mills theory and gravity and its implication for ultraviolet divergences

Z. Bern[a,1], L. Dixon[b,2], D.C. Dunbar[c,3], M. Perelstein[b,2], J.S. Rozowsky[a,1]

[a] Department of Physics, University of California at Los Angeles, Los Angeles, CA 90095-1547, USA
[b] Stanford Linear Accelerator Center, Stanford University, Stanford, CA 94309, USA
[c] Department of Physics, University of Wales Swansea, Swansea, SA2 8PP, UK

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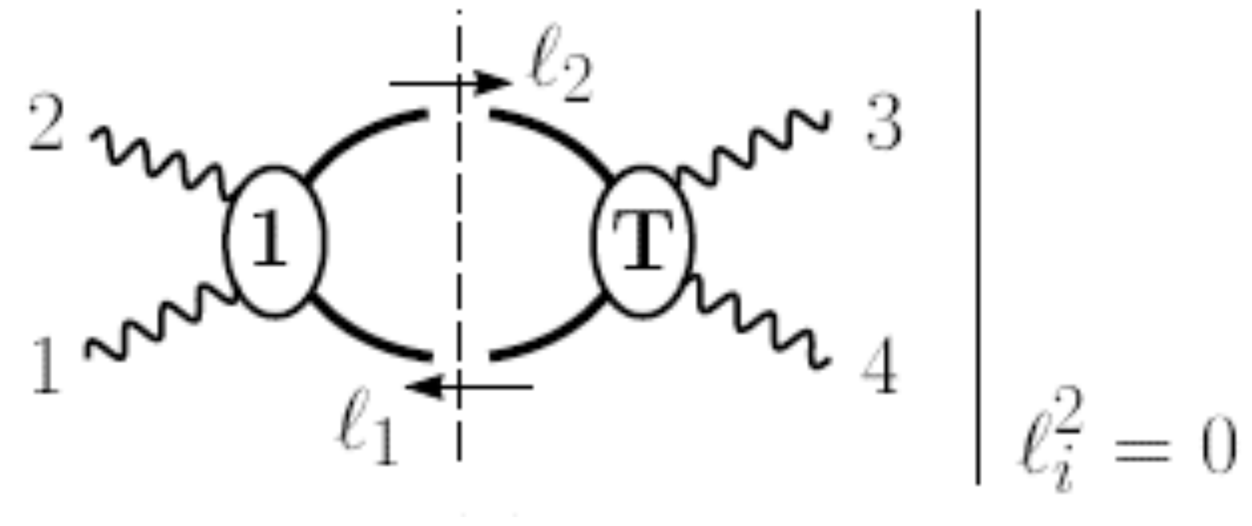
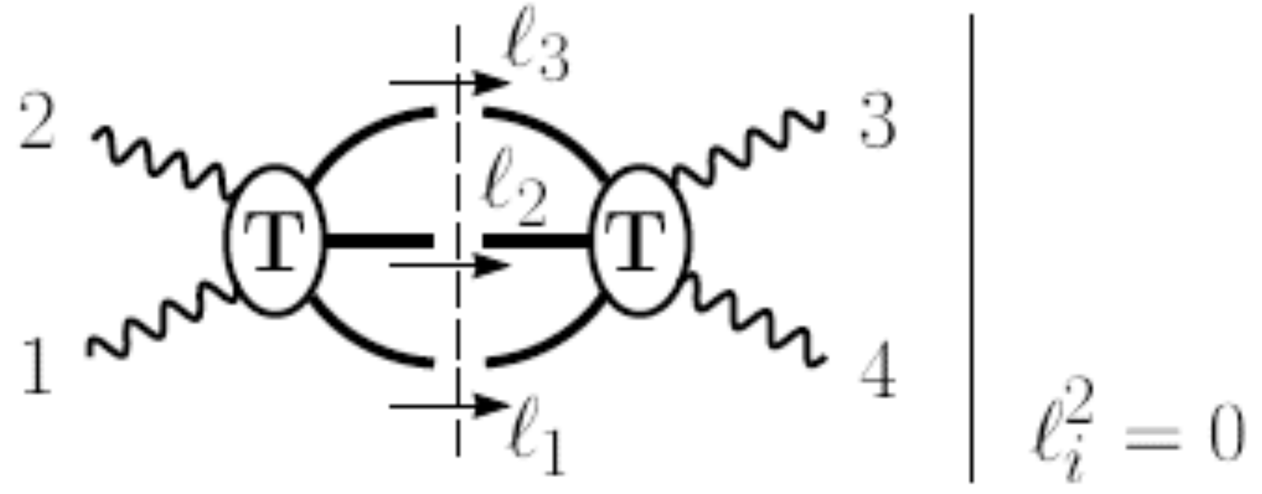
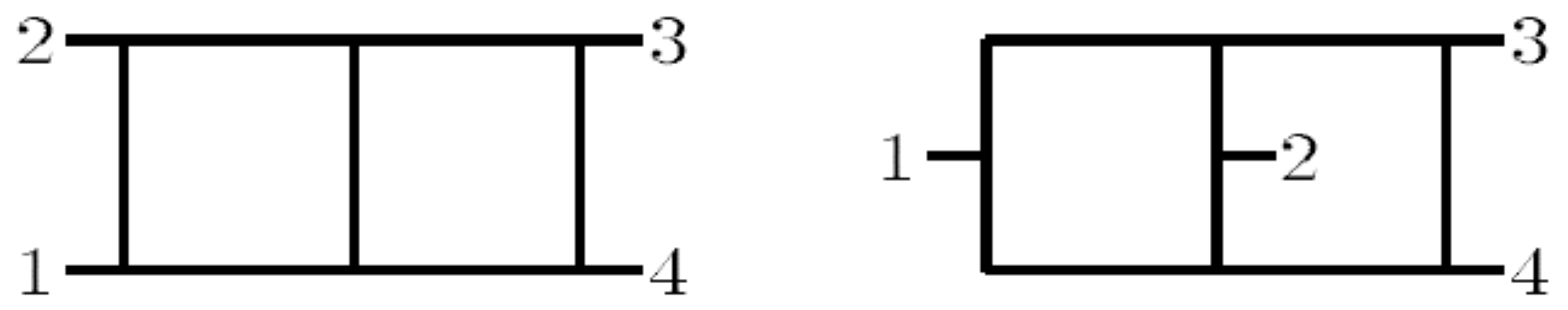




Note our early interest in UV divergences.

We could see patterns for higher loops, conjecturing 5-loop divergence.
More importantly, the paper made it clear we could calculate to high orders.

Two-Loop Four-Point Amplitudes



Two loop amplitudes in $N = 4$ sYM susy are simple!

Scalar double boxes

ZB, Yan, Rozowsky (1997);

$$\mathcal{A}_4^{\text{2-loop}}(1,2,3,4) = -g^6 s_{12}s_{23}\, A_4^{\text{tree}}(1,2,3,4)\Big(C^{\rm P}_{1234}\, s_{12}\, \mathcal{I}_4^{\text{2-loop,P}}(s_{12},s_{23}) + C^{\rm P}_{3421}\, s_{12}\, \mathcal{I}_4^{\text{2-loop,P}}(s_{12},s_{24}) + C^{\rm NP}_{1234}\, s_{12}\, \mathcal{I}_4^{\text{2-loop,NP}}(s_{12},s_{23}) + C^{\rm NP}_{3421}\, s_{12}\, \mathcal{I}_4^{\text{2-loop,NP}}(s_{12},s_{24}) + \text{cyclic}\Big)\,,$$

Integrals obtained later. Smirnov(1999); Tausk (1999)

Simplicity remains for integrated expressions!

$N = 8$ supergravity amplitudes just as simple!

ZB, Dixon, Dunbar, Rozowsky, Perelstein (1998)

$$\mathcal{M}_4^{\text{2-loop}}(1,2,3,4) = -i\Big(\frac{\kappa}{2}\Big)^6 [s_{12}s_{23}\, A_4^{\text{tree}}(1,2,3,4)]^2 \Big(s_{12}^2\, \mathcal{I}_4^{\text{2-loop,P}}(s_{12},s_{23}) + s_{12}^2\, \mathcal{I}_4^{\text{2-loop,P}}(s_{12},s_{24}) + s_{12}^2\, \mathcal{I}_4^{\text{2-loop,NP}}(s_{12},s_{23}) + s_{12}^2\, \mathcal{I}_4^{\text{2-loop,NP}}(s_{12},s_{24}) + \text{cyclic}\Big)$$

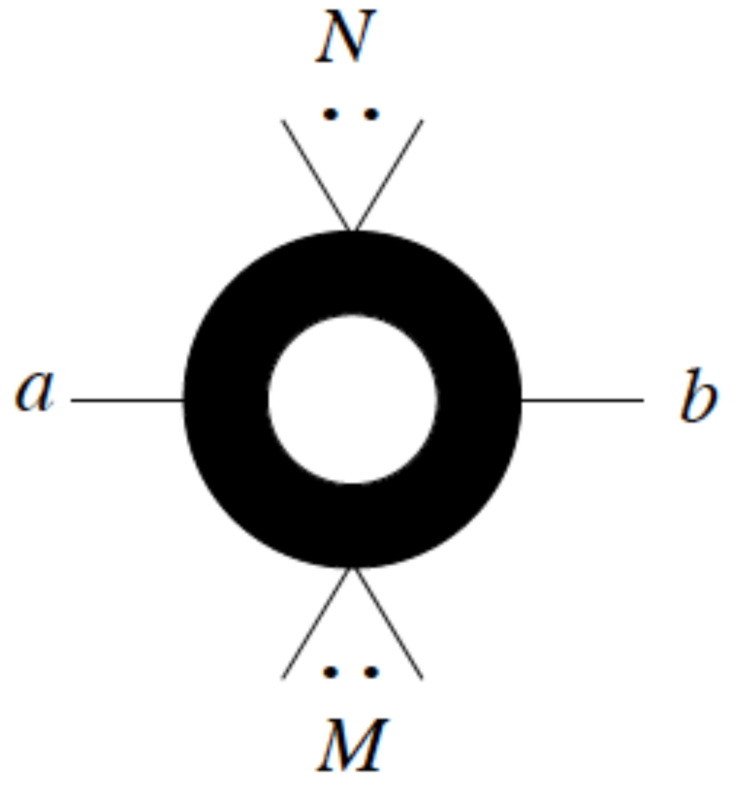
$$\mathcal{M}_4^{\text{2-loop},\ D=7-2\epsilon}\big|_{\text{pole}} = \frac{1}{2\epsilon\ (4\pi)^7}\frac{\pi}{3}(s^2+t^2+u^2) \times \Big(\frac{\kappa}{2}\Big)^6 \times stu M_4^{\text{tree}}$$

Gauge-theory simplicity imported to gravity.

MHV One-loop Gravity Amplitudes

ZB, Dixon, Rozowsky, Perelstein (1998)

We thought we could build all *n* gravity given gauge theory success.



But it was very hard to figure out.
Analytic properties seemed more complicated than gauge theory.

A loss of faith:

1) **No one cared about scattering amplitudes.**
2) **Even worse: No experimental relevance! (No crystal ball to look into the future)**

“Why am we working so hard on something no one cares about?”

An epiphany: “If you can work out a one loop all-*n* gravity amplitude, just do it.”
Impossible using ordinary methods. Almost magical.

It took about 1 year + 5 minute to figure out.

JUST DO IT

2nd Paper: MHV One-Loop Gravity Amplitudes

ELSEVIER

Nuclear Physics B 546 (1999) 423–479

NUCLEAR PHYSICS B

Multi-leg one-loop gravity amplitudes from gauge theory

Z. Bern [a,1], L. Dixon [b,2], M. Perelstein [b,2], J.S. Rozowsky [c,3]

[a] *Department of Physics, University of California at Los Angeles, Los Angeles, CA 90095-1547, USA*
[b] *Stanford Linear Accelerator Center, Stanford University, Stanford, CA 94309, USA*
[c] *Institute for Fundamental Theory, Department of Physics, University of Florida, Gainesville, FL 32611, USA*

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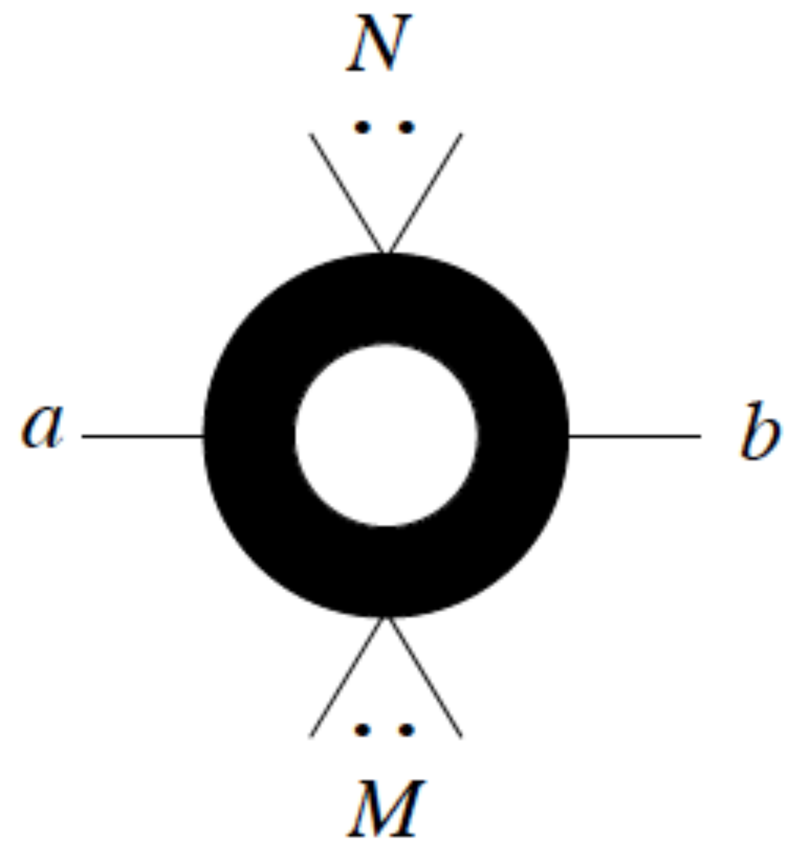
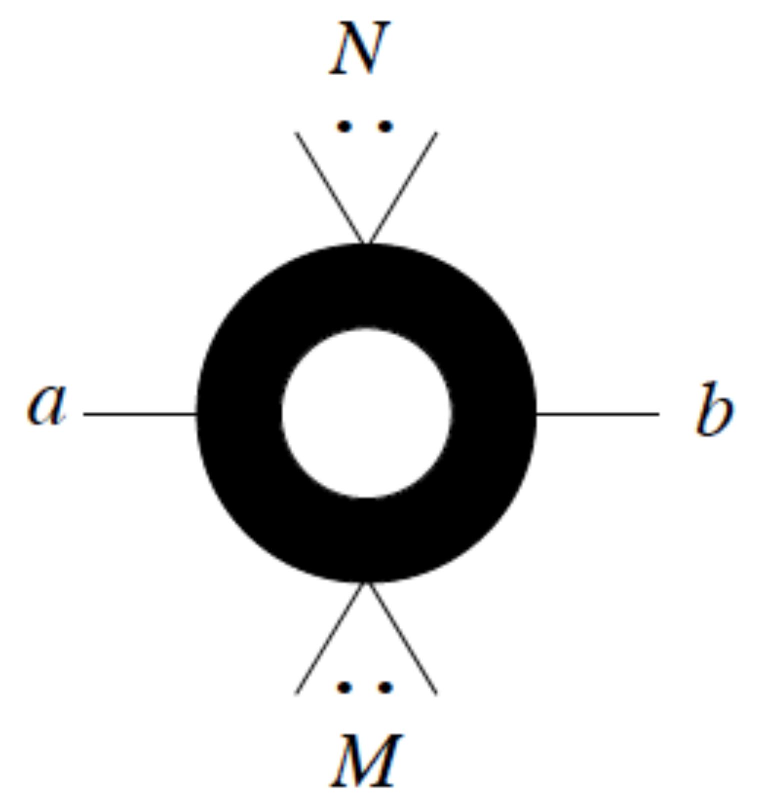


MHV One-loop Gravity Amplitudes

ZB, Dixon, Rozowsky, Perelstein (1998)



Pure gravity one-loop identical helicity:

$$M_n(1^+, 2^+, \ldots, n^+) = -\frac{i\,(-1)^n}{(4\pi)^2 \cdot 960} \sum_{\substack{1\le a<b\le n\\ M,N}} h(a,M,b)h(b,N,a)\,\mathrm{tr}^3[a\,M\,b\,N]$$

$$h(a,\{1,2,\ldots,n\},b) \equiv \frac{[1\,2]}{\langle 1\,2\rangle} \frac{\langle a^-|\not K_{1,2}|3^-\rangle\langle a^-|\not K_{1,3}|4^-\rangle\cdots\langle a^-|\not K_{1,n-1}|n^-\rangle}{\langle 2\,3\rangle\,\langle 3\,4\rangle\cdots\langle n-1,\,n\rangle\;\langle a\,1\rangle\,\langle a\,2\rangle\,\langle a\,3\rangle\cdots\langle a\,n\rangle\;\langle 1\,b\rangle\,\langle n\,b\rangle} + \mathcal{P}(2,3,\ldots,n),$$

"half-soft function"

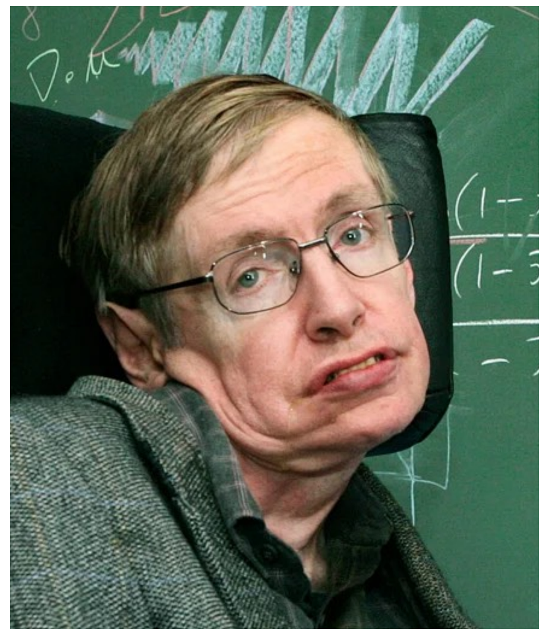
N = 8 supergravity MHV amplitude

$$M_n^{N=8}(1^-, 2^-, 3^+, \ldots, n^+) = \frac{(-1)^n}{8}\,\langle 1\,2\rangle^8 \sum_{\substack{1\le a<b\le n\\ M,N}} h(a,M,b)h(b,N,a)\,\mathrm{tr}^2[a\,M\,b\,N]\,\mathcal{I}_4^{aMbN}$$

All-*n* one-loop MHV gravity amplitudes are relatively simple (even if it was hard to derive back then).

Status of Supergravity Divergences 1985-2006

One year everyone believed that supergravity was finite. The next year the fashion changed and everyone said that supergravity was bound to have divergences even though none had actually been found. — ***Stephen Hawking, 1994***



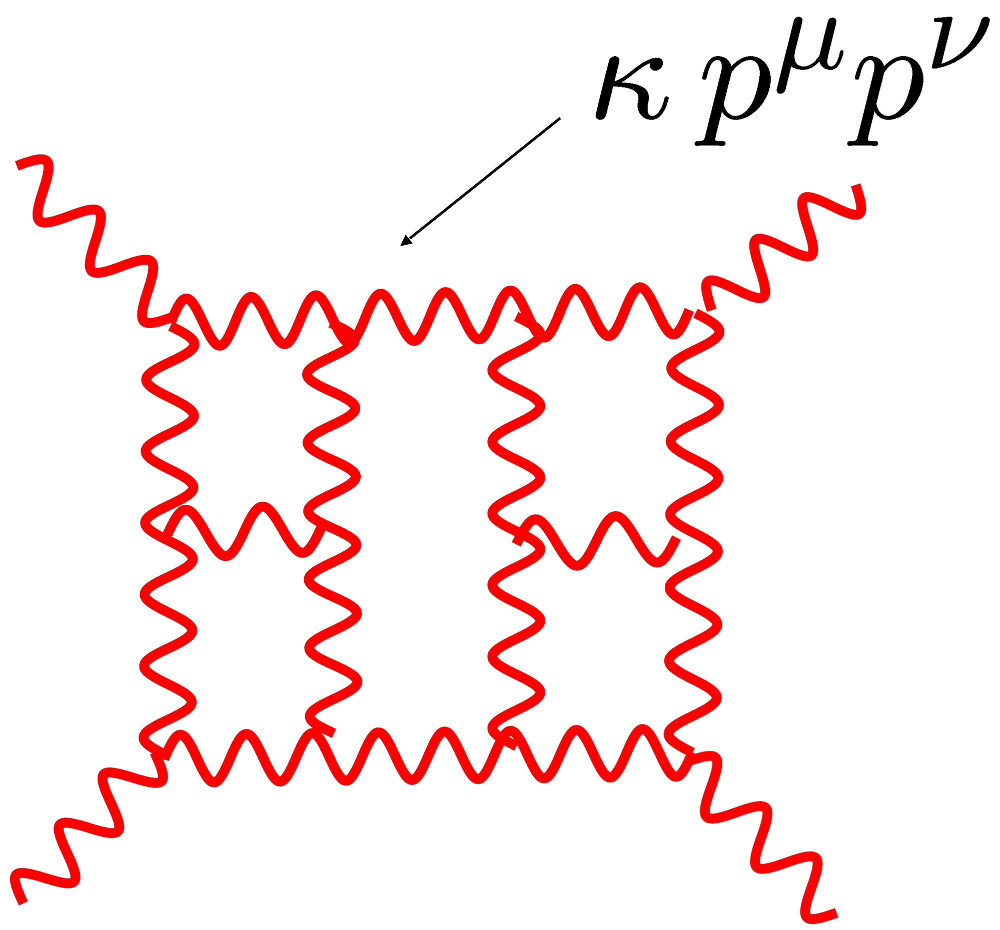
Consensus opinion until end of 2006 was that all supergravity theories expect to diverge at 3 loops.
But experts were actually cautious.

If a full superspace exists UV divergences cannot occur prior to 7 loops.
However, full off-shell superspace does not exist for $\mathcal{N} = 8$ supergravity.

Despite enormous progress, 30 years later, we are doing only a bit better than when Hawking complained

UV Behavior of Gravity

$$\kappa = \sqrt{32\pi G_N}$$ ⟵ Dimensionful coupling



Gravity: $$\int \prod_{i=1}^{L} \frac{d^D p_i}{(2\pi)^D} \frac{\cdots \kappa p_j^\mu p_j^\nu \cdots}{\text{propagators}}$$

Gauge theory: $$\int \prod_{i=1}^{L} \frac{d^D p_i}{(2\pi)^D} \frac{\cdots g p_j^\nu \cdots}{\text{propagators}}$$

- **Extra powers of loop momenta in numerator means integrals are badly behaved in the UV and must diverge at some loop order.**
- **Much more sophisticated power counting in supersymmetric theories but this is basic idea.**

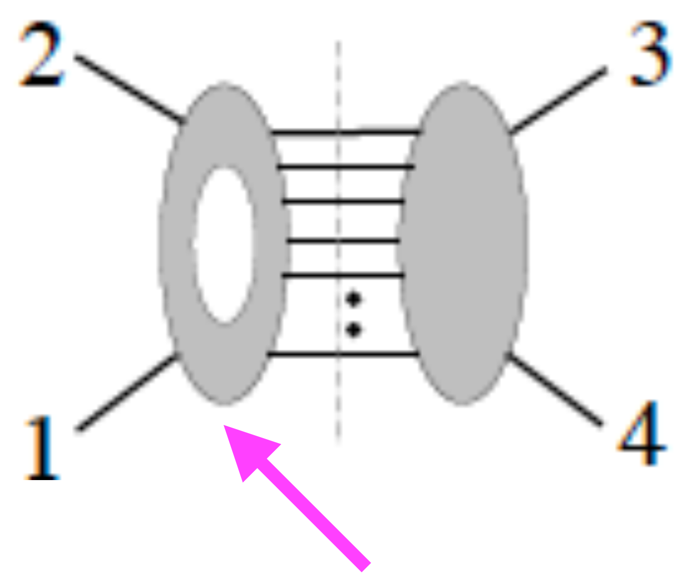
- **With more supersymmetry expect better UV properties.**
- **Need to worry about "hidden cancellations".**
- **$N = 8$ supergravity should have best behavior.** Cremmer, Julia, Scherk

The 2nd Era of Studying UV in Sugra

Is $\mathcal{N} = 8$ Supergravity Ultraviolet Finite?

Z. Bern[a], L. J. Dixon[b], R. Roiban[c]
[a]*Department of Physics and Astronomy, UCLA, Los Angeles, CA 90095-1547, USA*
[b]*Stanford Linear Accelerator Center, Stanford University, Stanford, CA 94309, USA*
[c]*Department of Physics, Pennsylvania State University, University Park, PA 16802, USA*

Conventional wisdom holds that no four-dimensional gravity field theory can be ultraviolet finite. This understanding is based mainly on power counting. Recent studies confirm that one-loop $\mathcal{N} = 8$ supergravity amplitudes satisfy the so-called "no-triangle hypothesis", which states that triangle and bubble integrals cancel from these amplitudes. A consequence of this hypothesis is that for any number of external legs, at one loop $\mathcal{N} = 8$ supergravity and $\mathcal{N} = 4$ super-Yang-Mills have identical superficial degrees of ultraviolet behavior in D dimensions. We describe how the unitarity method allows us to promote these one-loop cancellations to higher loops, suggesting that previous power counts were too conservative. We discuss higher-loop evidence suggesting that $\mathcal{N} = 8$ supergravity has the same degree of divergence as $\mathcal{N} = 4$ super-Yang-Mills theory and is ultraviolet finite in four dimensions. We comment on calculations needed to reinforce this proposal, which are feasible using the unitarity method.



ZB, Carrasco, Forde, Ita, Johansson

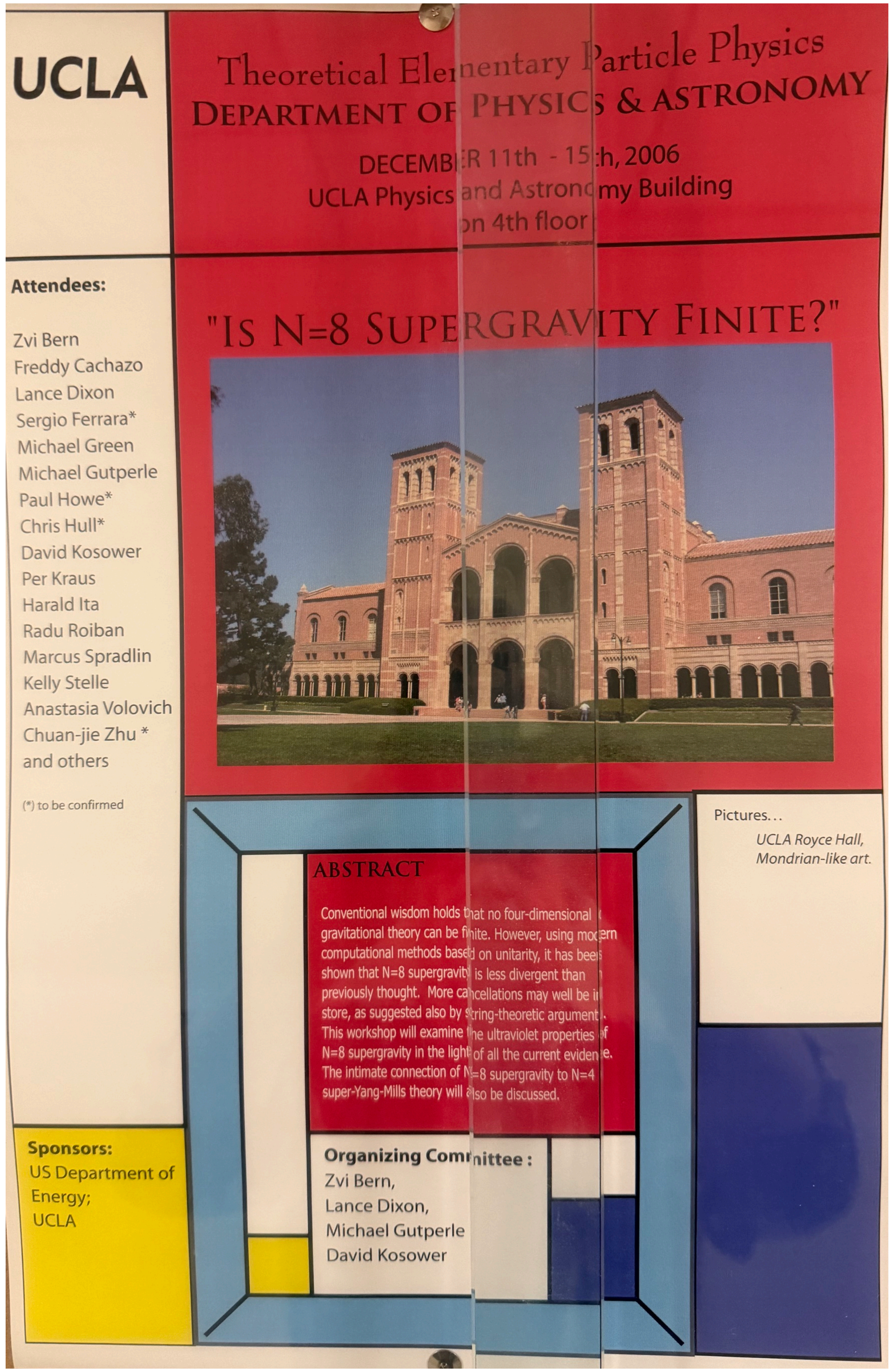
No real calculations!
Content: this cut seems to have a better than expected UV behavior!

Abstract says: "We comment on calculations needed …" not "We calculate …"

What kind of Lance paper is this?

How did it happen that Lance would want to write this paper?

How the Study of Supergravity Divergences Came Back to Life.



In 2006 the study of supergravity divergences was a dead field.

We were thinking about supergravity and divergences for much of 2005, but no action.

- In late 2005: DOE site visit at UCLA by P.K. Williams.
- Sergio Ferrara won the APS Heinemann Mathematical Physics Prize that very day!
- In my DOE proposal talk, I said "We think $N = 8$ supergravity might be UV finite"
- Sergio got really excited!
- PK got excited because Sergio was excited: "What can I do to help"
- My Answer: "$10K for a conference".
- PK called it "gambling money" but he gave all of it.

We had a conference and topic, but no new results from anyone!

Provocative workshop title to get people to attend.

It worked!

Supergravity UV Comes Back!

We got very lucky, thanks to Michael Green. Conference invitations stimulated two papers prior to meeting:

Preprint typeset in JHEP style - HYPER VERSION

hep-th/0610299
DAMTP-2006-102
SPHT06/127
UB-ECM-PF-06-30

arXiv:hep-th/0610299v4

Non-renormalisation Conditions in Type II String Theory and Maximal Supergravity

Michael B. Green
Department of Applied Mathematics and Theoretical Physics
Wilberforce Road, Cambridge CB3 0WA, UK
M.B.Green@damtp.cam.ac.uk

Jorge G. Russo
Institució Catalana de Recerca i Estudis Avançats (ICREA),
University de Barcelona, Facultat de Fisica
Av. Diagonal, 647, Barcelona 08028 SPAIN
jrusso@ub.edu

Pierre Vanhove
Service de Physique Théorique,
CEA/DSM/PhT, CEA/Saclay, Orme des Merisiers, CEA/Saclay
91191 Gif-sur-Yvette Cedex, France
pierre.vanhove@cea.fr

We got scooped by Michael Green and friends

$N = 8$ supergravity might be finite to all loops!

arXiv:hep-th/0611273v3

Ultraviolet properties of Maximal Supergravity

Michael B. Green
Department of Applied Mathematics and Theoretical Physics
Wilberforce Road, Cambridge CB3 0WA, UK

Jorge G. Russo
Institució Catalana de Recerca i Estudis Avançats (ICREA),
University of Barcelona, Av.Diagonal 647, Barcelona 08028 SPAIN

Pierre Vanhove
Service de Physique Théorique, CEA/Saclay, F-91191 Gif-sur-Yvette, France
(Dated: October 31, 2018)

$N = 8$ supergravity might be finite through 8 loops.

Now that we had competitors and a conference, we needed a paper.
Hence we wrote our un-Lance-like paper.

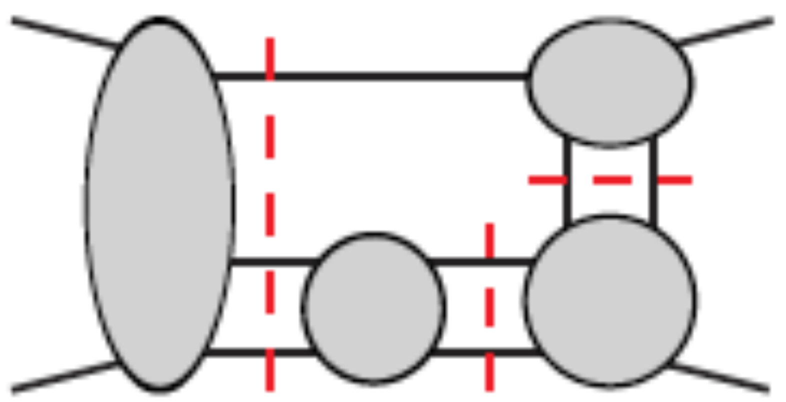
Supergravity UV came back to life after a 20-year nap.

A Parade of $N = 8$ Supergravity Calculations

Stimulated by the conference, we started calculating.

Maximal cut method + KLT.

Starting at 4 loops integrals complicated, needed IBP.



Various Lance-worthy papers on supergravity:

1) "Three-Loop Superfiniteness of $N = 8$ Supergravity" (2007) ZB, Carrasco, Dixon, Johansson, Kosower

2) "Manifest Ultraviolet Behavior for the Three-Loop Four-Point Amplitude of $N = 8$ Supergravity" (2008) ZB, Carrasco, Dixon, Johansson, Roiban

3) "The Ultraviolet Behavior of $N = 8$ Supergravity at Four Loops" (2009) ZB, Carrasco, Dixon, Johansson, Roiban

4) "Simplifying Multiloop Integrands and Ultraviolet Divergences of Gauge Theory and Gravity Amplitudes" (2012) ZB, Carrasco, Dixon, Johansson, Roiban

5) "Ultraviolet Properties of $N = 8$ Supergravity at Five Loops" (2018) ZB, Carrasco, Chen, Edison, Johansson, Parra-Martinez, Roiban, Zeng

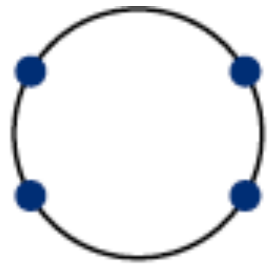
Summary of UV Properties

Green, Schwarz, Brink; ZB, Dixon, Dunbar, Perelstein, Rozowsky;Carrasco, Dixon, Johansson, Roiban; ZB, Carrasco, Chen, Edison, Johansson, Roiban, Parra-Martinez, Zeng

Summary of UV results or $N = 8$ sugra through five loops.

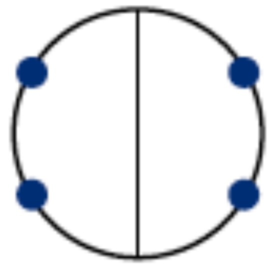
Vacuum diagrams represent UV divs.
Dots represent extra propagators.

$$\mathcal{M}_4^{(1)}\Big|_{\text{leading}} = -3\,\mathcal{K}_{\text{G}} \left(\frac{\kappa}{2}\right)^4$$

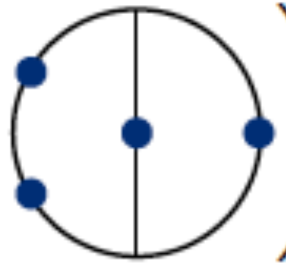


, $D_c = 8$

$$\mathcal{M}_4^{(2)}\Big|_{\text{leading}} = -8\,\mathcal{K}_{\text{G}} \left(\frac{\kappa}{2}\right)^6 (s^2+t^2+u^2)\left(\frac{1}{4}\right.$$



$$+\frac{1}{4}$$



$$\left.\right),\qquad D_c = 7$$

$$\mathcal{M}_4^{(3)}\Big|_{\text{leading}} = -60\,\mathcal{K}_{\text{G}} \left(\frac{\kappa}{2}\right)^8 stu\left(\frac{1}{6}\right.$$

$$+\frac{1}{2}$$

$$\left.\right),\qquad D_c = 6$$

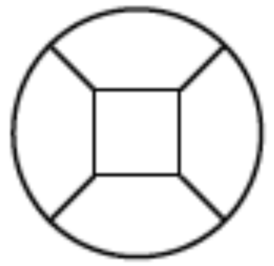
$$\mathcal{M}_4^{(4)}\Big|_{\text{leading}} = -\frac{23}{2}\,\mathcal{K}_{\text{G}} \left(\frac{\kappa}{2}\right)^{10} (s^2+t^2+u^2)^2\left(\frac{1}{4}\right.$$

$$+\frac{1}{2}$$

$$+\frac{1}{4}$$

$$\left.\right),\qquad D_c = \frac{11}{2}$$

$$\mathcal{M}_4^{(5)}\Big|_{\text{leading}} = -\frac{16\times 629}{25}\,\mathcal{K}_{\text{G}} \left(\frac{\kappa}{2}\right)^{12} (s^2+t^2+u^2)^2\left(\frac{1}{48}\right.$$



$$+\frac{1}{16}$$

$$\left.\right),\qquad D_c = \frac{24}{5}$$

We now have a lot of theoretical "data".
Vacuum integrals evaluated either analytically or numerically.

$N = 8$ supergravity: Where is First $D = 4$ UV Divergence?

3 loops $N = 8$	Green, Schwarz, Brink (1982); Howe and Stelle (1989); Marcus and Sagnotti (1985)	✗
5 loops $N = 8$	Bern, Dixon, Dunbar, Perelstein, Rozowsky (1998); Howe and Stelle (2003,2009)	✗
6 loops $N = 8$	Howe and Stelle (2003)	✗
7 loops $N = 8$	Grisaru and Siegel (1982); Bossard, Howe, Stelle (2009);Vanhove; Björnsson, Green (2010); Kiermaier, Elvang, Freedman(2010); Ramond, Kallosh (2010); Biesert et al (2010); Bossard, Howe, Stelle, Vanhove (2011)	?
8 loops N=8	Green, Russo, Vanhove (2006) Renata (1980s)	?
9 loops $N = 8$	Berkovits, Green, Russo, Vanhove (2009)	✗ ← **Retracted.**
3 loops $N = 4$	Bossard, Howe, Stelle, Vanhove (2011)	✗
4 loops $N = 5$	Bossard, Howe, Stelle, Vanhove (2011) Freedman, Kallosh	✗
5 loops $N = 5$	Kallosh, Yusuke (2023); Kallosh (2023)	?
4 loops $N = 4$	Vanhove and Tourkine (2012)	✓

"shut up and calculate"

ZB, Kosower, Carrasco, Dixon, Johansson, Roiban; ZB, Davies, Dennen, A. Smirnov, V. Smirnov; series of calculations.

This is what we are most interested in. (5 loops $N = 5$)

Special structure. Anomaly-like behavior of divergence. (4 loops $N = 4$)

- **Track record of predictions from bounding via symmetries not great.**
- **Conventional wisdom holds that it will diverge sooner or later.**

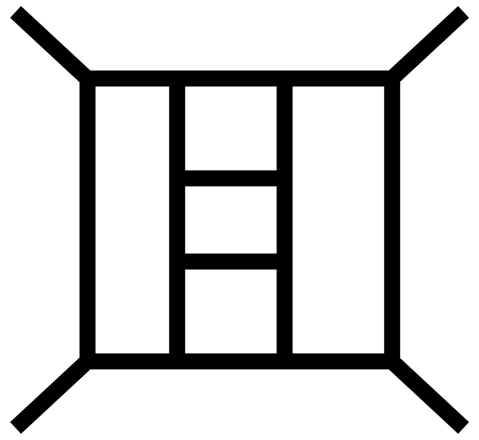
Stimulated work on Counterterms and Power Counting

Bossard, Howe, Stelle; Elvang, Freedman, Kiermaier; Green, Russo, Vanhove ; Green and Björnsson ;
Bossard , Hillmann and Nicolai; Ramond and Kallosh; Broedel and Dixon; Elvang and Kiermaier;
Beisert, Elvang, Freedman, Kiermaier, Morales, Stieberger; Bossard, Howe, Stelle, Vanhove, etc

- **Duality constraints on counterterms in $N = 8$ supergravity.** Bossard, Howe, Stelle; Beisert, Elvang, Freedman, Kiermaier, Morales, Stieberger
- **Expose all susy via Berkovits' pure-spinor formalism** Bjornsson and Green

Poor UV behavior, unless new types of cancellations between diagrams exist that are "not consequences of supersymmetry in any conventional sense". Bjornsson and Green

$N = 8$ sugra should diverge at 7 loops in $D = 4$.



Consensus agreement from all power-counting methods, including power counting within unitarity approach.

Symmetry Constraints on Divergences

Calculations stimulated studies to expose consequences of symmetries

Bossard, Howe, Stelle;
Beisert, Elvang, Freedman, Kiermaier, Morales, Stieberger;
Bjornsson, Green

$\mathcal{N} = 8$ supergravity

L										
3	~~R^4~~ $E_{7(7)}$									
4	~~D^2R^4~~	~~R^5~~								
5	~~D^4R^4~~ $E_{7(7)}$	~~D^2R^5~~	~~R^6~~							
6	~~D^6R^4~~ $E_{7(7)}$	~~D^4R^5~~	~~D^2R^6~~	~~R^7~~						
7	D^8R^4	~~D^6R^5~~	$E_{7(7)}$ ~~D^4R^6~~	~~D^2R^7~~	$E_{7(7)}$ ~~R^8~~	~~$\varphi^2D^2R^7$~~	$E_{7(7)}$ ~~φ^2R^8~~	~~$\varphi^4D^2R^7$~~	$E_{7(7)}$ ~~φ^4R^8~~	~~$\varphi^6D^2R^7$~~
8	$D^{10}R^4$	D^8R^5	D^6R^6	D^4R^7	D^2R^8	R^9	$\varphi^2D^2R^8$	φ^2R^9	$\varphi^4D^2R^8$	φ^4R^9
9	$D^{12}R^4$	$D^{10}R^5$	D^8R^6	D^6R^7	D^4R^8	D^2R^9	R^{10}	$\varphi^2D^2R^9$	φ^2R^{10}	$\varphi^4D^2R^9$
10	$D^{14}R^4$	$D^{12}R^5$	$D^{10}R^6$	D^8R^7	D^6R^8	D^4R^9	D^2R^{10}	R^{11}	$\varphi^2D^2R^{10}$	φ^2R^{11}

$\longrightarrow$ No MHV
$\longrightarrow$ No NMHV
$\longrightarrow$ No N^2MHV
$\longrightarrow$ No N^3MHV
$\longrightarrow$ No N^4MHV

Elvang, Freedman, Kiermaier

Divergences ruled out until 7 loops in N = 8 supergravity using know symmetry.
Do we know of any vanishing, not explained by known symmetries?

$\mathcal{N} = 5$ Supergravity Four Loop Cancellations

ZB, Davies and Dennen

We calculated four-loop divergence in $\mathcal{N} = 5$ supergravity.
Industrial strength software needed: FIRE5 and special purpose C++

$\mathcal{N} = 5$ sugra: ($\mathcal{N} = 4$ sYM) x ($\mathcal{N} = 1$ sYM)

Crucial help from (Smirnov)^2

$\mathcal{N} = 4$ sYM

$\mathcal{N} = 1$ sYM

Diagrams necessarily UV divergent.

$\mathcal{N} = 5$ supergravity has no divergence at four loops in $D = 4$.

Nontrivial example of an "enhanced cancellation".

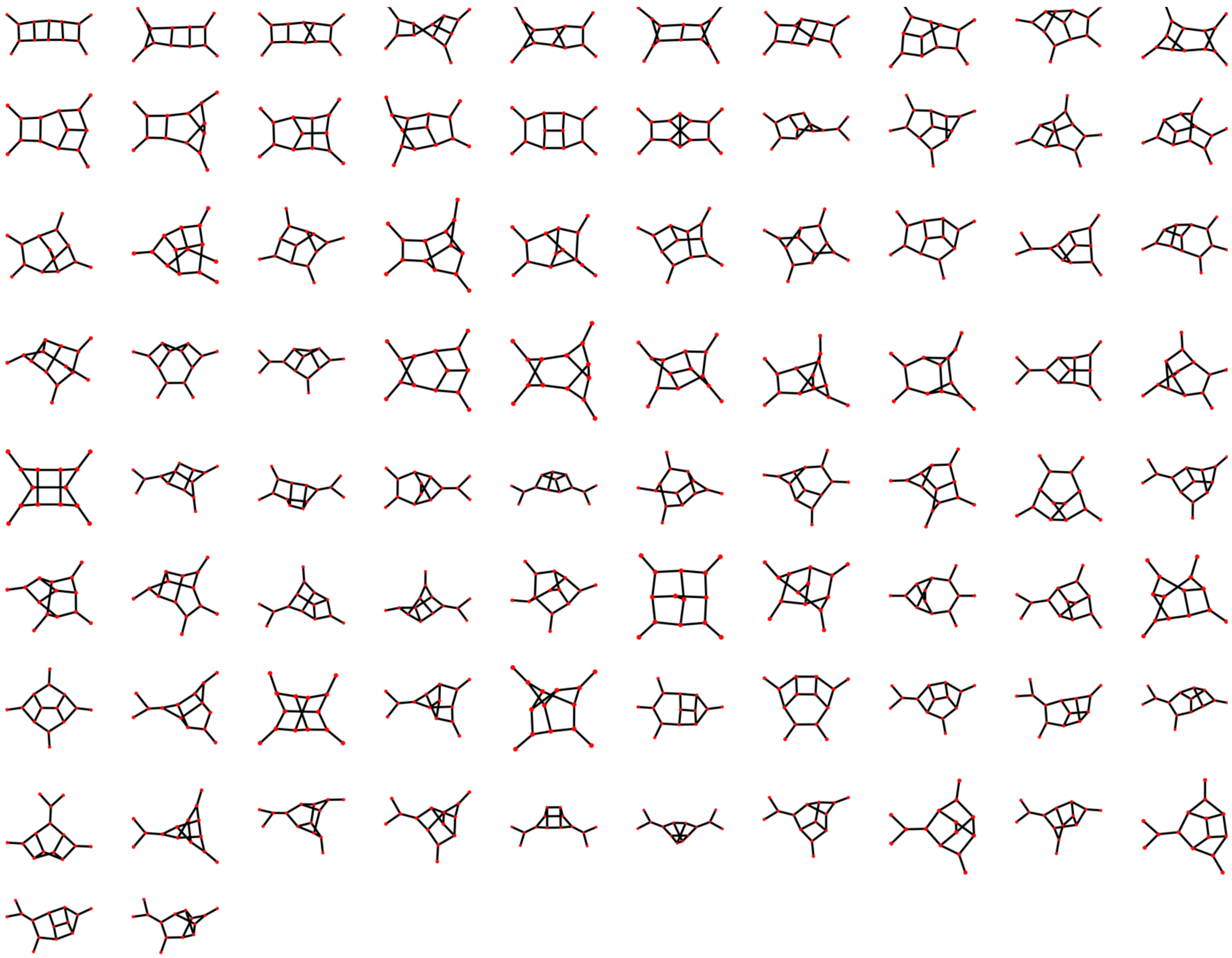
No Lorentz covariant formulation exists where diagrams expose the cancellations.

No anomaly and no standard-symmetry explanation known!

Freedman, Kallosh and Yamada

82 Nonvanishing Diagram Topologies via Double Copy

ZB, Carrasco, Dixon, Johansson, Roiban ($\mathcal{N} = 4$ sYM)
ZB, Davies and Dennen



$\mathcal{N} = 5$ Supergravity at Four Loops

Special purpose C++ and FIRE5

ZB, Davies and Dennen

graphs	(divergence) $\times\, u/(-i/(4\pi)^8\langle 12\rangle^2[34]^2 st A^{\rm tree}(\frac{\kappa}{2})^{10})$
1–30	$\frac{1}{\epsilon^4}\Big[\frac{7358585}{7962624}s^2+\frac{2561447}{2654208}st-\frac{872683}{1990656}t^2\Big]+\frac{1}{\epsilon^3}\Big[\frac{75972559}{35389440}s^2+\frac{240984061}{26542080}st+\frac{1302037}{1310720}t^2\Big]+\frac{1}{\epsilon^2}\Big[\zeta_3\big(-\frac{369234283}{11059200}s^2-\frac{257792411}{4915200}st-\frac{101847769}{14745600}t^2\big)+\zeta_2\big(\frac{7358585}{3981312}s^2+\frac{2561447}{1327104}st-\frac{872683}{995328}t^2\big)-\mathrm{S2}\big(\frac{1223621}{49152}s^2+\frac{46816475}{442368}st+\frac{2639903}{221184}t^2\big)+\frac{206093335871}{11466178560}s^2+\frac{320983191023}{3822059520}st+\frac{53309416589}{2866544640}t^2\Big]+\frac{1}{\epsilon}\Big[\zeta_5\big(-\frac{84777347}{368640}s^2+\frac{382194721}{1474560}st+\frac{417476581}{1474560}t^2\big)-\zeta_4\big(\frac{3062401}{2457600}s^2+\frac{3881051}{3276800}st-\frac{112081813}{29491200}t^2\big)+\zeta_3\big(\frac{28162691399797}{53747712000}s^2+\frac{19354492750651}{35831808000}st-\frac{22092683352811}{107495424000}t^2\big)-\zeta_2\big(\frac{70861961}{17694720}s^2+\frac{227180689}{13271040}st+\frac{105727243}{53084160}t^2\big)+\mathrm{T1ep}\big(-\frac{1223621}{663552}s^2-\frac{46816475}{5971968}st-\frac{2639903}{2985984}t^2\big)-\mathrm{S2}\big(\frac{11916028151}{5898240}s^2+\frac{72637733971}{13271040}st+\frac{17223563447}{53084160}t^2\big)+\mathrm{D6}\big(-\frac{9001177}{552960}s^2-\frac{264491}{10240}st-\frac{2610157}{552960}t^2\big)+\frac{110945914744727}{1146617856000}s^2+\frac{16989492195991}{127401984000}st-\frac{21362122998269}{573308928000}t^2\Big]$
31–60	$\frac{1}{\epsilon^4}\Big[-\frac{5502451}{2654208}s^2-\frac{3675877}{884736}st+\frac{11269}{497664}t^2\Big]+\frac{1}{\epsilon^3}\Big[\frac{38102993}{26542080}s^2-\frac{291607201}{106168320}st-\frac{565798829}{318504960}t^2\Big]+\frac{1}{\epsilon^2}\Big[\zeta_3\big(\frac{108955183}{2211840}s^2+\frac{653019571}{8847360}st+\frac{9453043}{1769472}t^2\big)+\zeta_2\big(-\frac{5502451}{1327104}s^2-\frac{3675877}{442368}st+\frac{11269}{248832}t^2\big)+\mathrm{S2}\big(\frac{16797481}{1327104}s^2+\frac{1172969}{16384}st+\frac{978427}{82944}t^2\big)-\frac{304243754383}{19110297600}s^2-\frac{2032063711381}{19110297600}st-\frac{257798086613}{7166361600}t^2\Big]+\frac{1}{\epsilon}\Big[\zeta_5\big(\frac{33327659}{122880}s^2+\frac{13276219}{24576}st+\frac{22251887}{184320}t^2\big)+\zeta_4\big(\frac{12299887}{1474560}s^2+\frac{258056147}{5898240}st+\frac{46913759}{5898240}t^2\big)+\zeta_3\big(-\frac{26846001990157}{42998169600}s^2-\frac{337106527201}{265420800}st-\frac{5298324906787}{42998169600}t^2\big)+\zeta_2\big(\frac{282283789}{39813120}s^2+\frac{975199319}{53084160}st+\frac{60394451}{159252480}t^2\big)+\mathrm{T1ep}\big(\frac{16797481}{17915904}s^2+\frac{1172969}{221184}st+\frac{978427}{1119744}t^2\big)+\mathrm{S2}\big(\frac{10516980893}{4976640}s^2+\frac{389045625329}{53084160}st+\frac{216032337589}{159252480}t^2\big)+\mathrm{D6}\big(\frac{503413}{23040}s^2+\frac{12342607}{552960}st+\frac{3661}{184320}t^2\big)-\frac{166777358259461}{1146617856000}s^2-\frac{565137511429117}{1146617856000}st-\frac{21629055712141}{191102976000}t^2\Big]$
61–82	$\frac{1}{\epsilon^4}\Big[\frac{285899}{248832}s^2+\frac{1058273}{331776}st+\frac{275869}{663552}t^2\Big]+\frac{1}{\epsilon^3}\Big[-\frac{380329649}{106168320}s^2-\frac{74703227}{11796480}st+\frac{124701919}{159252480}t^2\Big]+\frac{1}{\epsilon^2}\Big[\zeta_3\big(-\frac{1371419}{86400}s^2-\frac{236241539}{11059200}st+\frac{4326077}{2764800}t^2\big)+\zeta_2\big(\frac{285899}{124416}s^2+\frac{1058273}{165888}st+\frac{275869}{331776}t^2\big)+\mathrm{S2}\big(\frac{8120143}{663552}s^2+\frac{1893289}{55296}st+\frac{92293}{663552}t^2\big)-\frac{58867708103}{28665446400}s^2+\frac{71191292711}{3185049600}st+\frac{83016363427}{4777574400}t^2\Big]+\frac{1}{\epsilon}\Big[\zeta_5\big(-\frac{1520563}{36864}s^2-\frac{1178767861}{1474560}st-\frac{595491677}{1474560}t^2\big)-\zeta_4\big(\frac{6539029}{921600}s^2+\frac{313837819}{7372800}st+\frac{21665663}{1843200}t^2\big)+\zeta_3\big(\frac{20790944575597}{214990848000}s^2+\frac{6505876281371}{8957952000}st+\frac{70676991239557}{214990848000}t^2\big)+\zeta_2\big(-\frac{491377507}{159252480}s^2-\frac{66476563}{53084160}st+\frac{128393639}{79626240}t^2\big)+\mathrm{T1ep}\big(\frac{8120143}{8957952}s^2+\frac{1893289}{746496}st+\frac{92293}{8957952}t^2\big)+\mathrm{S2}\big(-\frac{14810628499}{159252480}s^2-\frac{19698937889}{10616832}st-\frac{10272602953}{9953280}t^2\big)+\mathrm{D6}\big(-\frac{616147}{110592}s^2+\frac{1939907}{552960}st+\frac{1299587}{276480}t^2\big)+\frac{9307894793789}{191102976000}s^2+\frac{206124003456599}{573308928000}st+\frac{21562322533673}{143327232000}t^2\Big]$

graphs	(divergence) $\times\, u/(-i/(4\pi)^8\langle 12\rangle^2[34]^2 st A^{\rm tree}(\frac{\kappa}{2})^{10})$
1–30	$\frac{1}{\epsilon^4}\Big[\frac{1052159}{995328}s^2+\frac{509789}{331776}st-\frac{121001}{497664}t^2\Big]+\frac{1}{\epsilon^3}\Big[\frac{9042569}{1474560}s^2+\frac{34360945}{1327104}st+\frac{73518401}{13271040}t^2\Big]+\frac{1}{\epsilon^2}\Big[\zeta_3\big(-\frac{11443919}{2764800}s^2+\frac{32520079}{552960}st+\frac{5836531}{230400}t^2\big)+\zeta_2\big(\frac{1052159}{497664}s^2+\frac{509789}{165888}st-\frac{121001}{248832}t^2\big)-\mathrm{S2}\big(\frac{637991}{6144}s^2+\frac{10978729}{27648}st+\frac{5080825}{55296}t^2\big)+\big(\frac{270806866183}{7166361600}s^2+\frac{89848068067}{597196800}st+\frac{218093645149}{7166361600}t^2\big)\Big]+\frac{1}{\epsilon}\Big[\zeta_5\big(\frac{100843}{360}s^2+\frac{17118043}{30720}st-\frac{30266471}{92160}t^2\big)+\zeta_4\big(\frac{11435323}{614400}s^2+\frac{232002227}{1843200}st+\frac{22211783}{460800}t^2\big)+\zeta_3\big(\frac{223300432349}{3359232000}s^2-\frac{178732984847}{716636160}st+\frac{951659436383}{53747712000}t^2\big)-\zeta_2\big(\frac{5492357}{245760}s^2+\frac{53468887}{663552}st+\frac{129714599}{6635520}t^2\big)+\mathrm{T1ep}\big(-\frac{637991}{82944}s^2-\frac{10978729}{373248}st-\frac{5080825}{746496}t^2\big)+\mathrm{S2}\big(-\frac{5700088747}{3686400}s^2-\frac{69470348491}{16588800}st-\frac{713512871}{6635520}t^2\big)+\mathrm{D6}\big(-\frac{357421}{43200}s^2-\frac{2891743}{230400}st-\frac{470219}{138240}t^2\big)-\frac{3571506237341}{28665446400}s^2-\frac{1611591325291}{5971968000}st+\frac{2301084608777}{143327232000}t^2\Big]$
31–60	$\frac{1}{\epsilon^4}\Big[-\frac{150715}{82944}s^2-\frac{668333}{221184}st-\frac{7213}{1990656}t^2\Big]+\frac{1}{\epsilon^3}\Big[-\frac{68021833}{13271040}s^2-\frac{36852103}{1327104}st-\frac{298377299}{39813120}t^2\Big]+\frac{1}{\epsilon^2}\Big[\zeta_3\big(-\frac{36448033}{2764800}s^2-\frac{455889533}{2764800}st-\frac{82059281}{1382400}t^2\big)+\zeta_2\big(-\frac{150715}{41472}s^2-\frac{668333}{110592}st-\frac{7213}{995328}t^2\big)+\mathrm{S2}\big(\frac{13910839}{165888}s^2+\frac{1340033}{4096}st+\frac{26303855}{331776}t^2\big)-\frac{68286245653}{2388787200}s^2-\frac{20649690431}{119439360}st-\frac{351701043553}{7166361600}t^2\Big]+\frac{1}{\epsilon}\Big[\zeta_5\big(-\frac{2362679}{9216}s^2-\frac{178668311}{92160}st-\frac{1268313}{10240}t^2\big)+\zeta_4\big(-\frac{124344121}{1843200}s^2-\frac{491722333}{1843200}st-\frac{68141309}{921600}t^2\big)-\zeta_3\big(\frac{630084012997}{53747712000}s^2-\frac{1250670277213}{663552000}st-\frac{6913218302303}{13436928000}t^2\big)+\zeta_2\big(\frac{352368061}{19906560}s^2+\frac{35509679}{663552}st+\frac{227699801}{19906560}t^2\big)+\mathrm{T1ep}\big(\frac{13910839}{2239488}s^2+\frac{1340033}{55296}st+\frac{26303855}{4478976}t^2\big)+\mathrm{S2}\big(\frac{188312318729}{99532800}s^2+\frac{110749829741}{16588800}st+\frac{5056299197}{3981312}t^2\big)+\mathrm{D6}\big(\frac{1220779}{76800}s^2+\frac{44791}{6912}st-\frac{1159831}{230400}t^2\big)+\frac{2755666297013}{28665446400}s^2+\frac{5622513975899}{35831808000}st-\frac{196197363193}{1769472000}t^2\Big]$
61–82	$\frac{1}{\epsilon^4}\Big[\frac{756421}{995328}s^2+\frac{985421}{663552}st+\frac{163739}{663552}t^2\Big]+\frac{1}{\epsilon^3}\Big[-\frac{1670161}{1658880}s^2+\frac{415193}{221184}st+\frac{4863881}{2488320}t^2\Big]+\frac{1}{\epsilon^2}\Big[\zeta_3\big(\frac{110861}{6400}s^2+\frac{16293841}{153600}st+\frac{9408019}{276480}t^2\big)+\zeta_2\big(\frac{756421}{497664}s^2+\frac{985421}{331776}st+\frac{163739}{331776}t^2\big)+\mathrm{S2}\big(\frac{1657459}{82944}s^2+\frac{7734025}{110592}st+\frac{4181095}{331776}t^2\big)-\frac{8243516153}{895795200}s^2+\frac{558349337}{24883200}st+\frac{11133949867}{597196800}t^2\Big]+\frac{1}{\epsilon}\Big[\zeta_5\big(-\frac{1094509}{46080}s^2+\frac{63657091}{46080}st+\frac{5210161}{11520}t^2\big)+\zeta_4\big(\frac{11254769}{230400}s^2+\frac{129860053}{921600}st+\frac{23717743}{921600}t^2\big)-\zeta_3\big(\frac{2745647960587}{53747712000}s^2+\frac{3654260151947}{2239488000}st+\frac{5720906529119}{10749542400}t^2\big)+\zeta_2\big(\frac{11564107}{2488320}s^2+\frac{2244901}{82944}st+\frac{40360999}{4976640}t^2\big)+\mathrm{T1ep}\big(\frac{1657459}{1119744}s^2+\frac{7734025}{1492992}st+\frac{4181095}{4478976}t^2\big)+\mathrm{S2}\big(-\frac{420043}{1215}s^2-\frac{825589625}{331776}st-\frac{5785239343}{4976640}t^2\big)+\mathrm{D6}\big(-\frac{210731}{27648}s^2+\frac{4196129}{691200}st+\frac{1457647}{172800}t^2\big)+\frac{33976742047}{1194393600}s^2+\frac{4046536311847}{35831808000}st+\frac{212357840779}{2239488000}t^2\Big]$

Adds up to zero: no divergence. Enhanced cancellations!

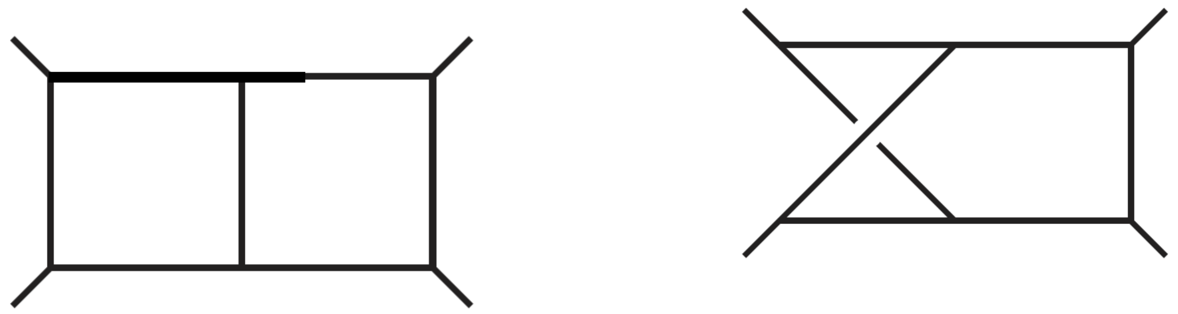
No standard (super)symmetry explanation exists.

Freedman, Kallosh and Yamada

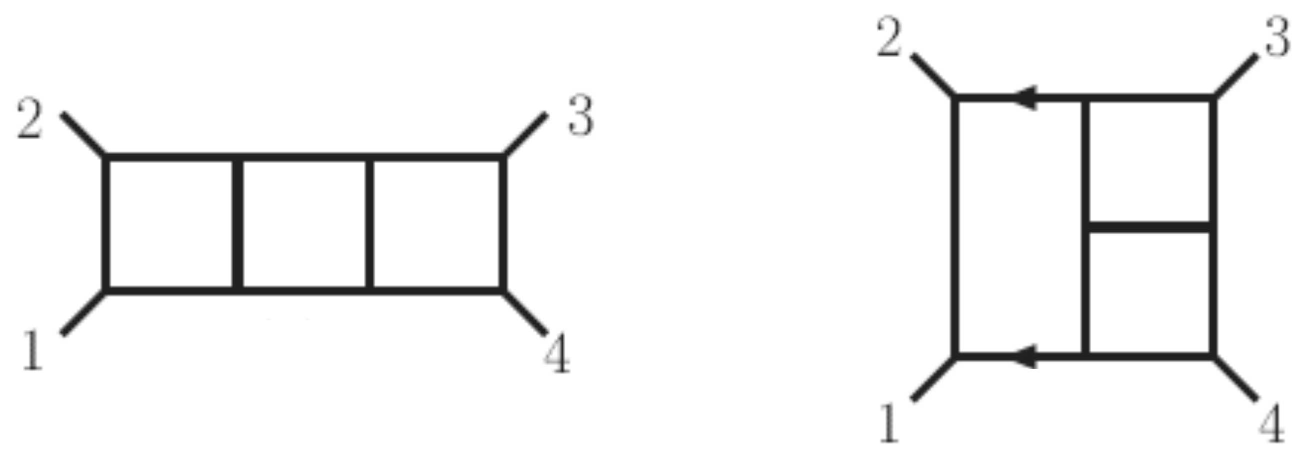
UV Divergence of $N = 4$ Supergravity

At 2, 3 loops 4-point amplitudes finite

Boucher-Veronneau, Dixon; ZB, Davies, Dennen



Finite, as exected



Finite, in disagreement with consensus

First divergence is at four loops :

ZB, Davies, Dennen, Smirnov, Smirnov

$$\mathcal{M}^{\text{4-loop}}\Big|_{\text{div.}}^{D=4-2\epsilon} = \frac{1}{(4\pi)^8}\frac{1}{\epsilon}\left(\frac{\kappa}{2}\right)^{10}\frac{1}{144}(1-264\zeta_3)\,\mathcal{T}$$

Non trivial kinematic tensor

The divergence is present in sectors that would vanish if not for an anomaly in the SU(1,1) duality symmetry.

Carrasco, Kallosh, Tseytlin and Roiban

- **An anomaly appears behind the 4 loop divergence.**
- **Hard to study because of high loop order. Want a simpler example.**

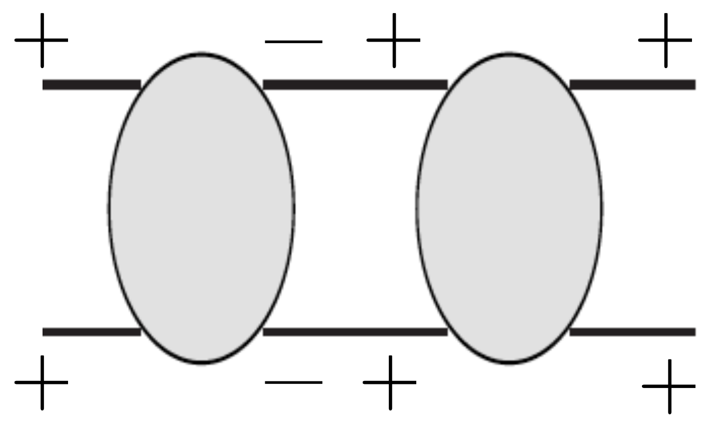
One-Loop Anomaly of Self Dual Theories

Self dual anomaly plays an important role in UV story.

Same anomaly mentioned in Morales' talk

Gauge theory: $$A^{1\text{-loop}} = \frac{i}{96\pi^2}(N_b - N_f)\frac{[12][34]}{\langle 12\rangle\langle 34\rangle}$$

Gravity: $$M^{1\text{-loop}} = \frac{i}{(4\pi)^2}\frac{(N_b - N_f)}{240}\left(\frac{[12][34]}{\langle 12\rangle\langle 34\rangle}\right)^2(s^2+t^2+u^2)$$



- **The self-dual theory is integrable and has an infinite number of conserved currents.**
- **The one-loop amplitude in self dual theory non-vanishing because of an anomaly.**

Bardeen (1995); D. Cangemi (1996)

- **Note: just like chiral anomaly integrand vanishes in strictly $D = 4$.**

This anomaly is tied to the U(1) anomaly of $N = 4$ supergravity, via double copy. Four loops make it hard to study

Two loop divergence in pure Einstein gravity is tied to the same anomaly and easier to study.

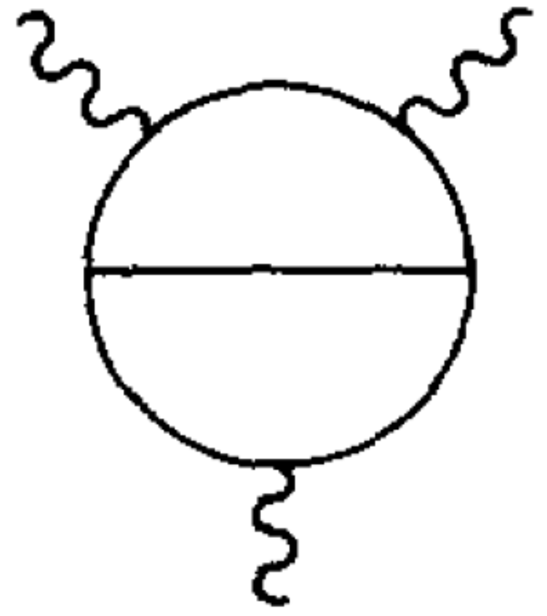
Weird Pure Gravity Adventures with Lance: Two-Loop Pure Gravity

Goroff and Sagnotti (1986); Van de Ven (1992); ZB, Cheung, Chi, Davies, Dixon, Nohle

Using standard MS-bar prescription Goroff and Sagnotti showed showed Einstein gravity diverges at 2 loops.

$D = 4 - 2\epsilon$

$$\mathcal{L}^{R^3} = \frac{209}{2880} \frac{1}{(4\pi)^4} \frac{1}{2\epsilon} \sqrt{-g}\, R^{\alpha\beta}{}_{\gamma\delta} R^{\gamma\delta}{}_{\rho\sigma} R^{\rho\sigma}{}_{\alpha\beta}$$



Definitely correct. 3 groups using completely different techniques.

In one of the strangest adventures I've had with Lance:

The above result does *not* tell you if pure gravity should be considered UV divergent!

Essential problem is the coefficient of the divergence is scheme dependent.
"Evanescent Chaos"

Einstein Gravity Divergences

PRL 115, 211301 (2015) PHYSICAL REVIEW LETTERS week ending 20 NOVEMBER 2015

Evanescent Effects can Alter Ultraviolet Divergences in Quantum Gravity without Physical Consequences

Zvi Bern,[1,2,3] Clifford Cheung,[2] Huan-Hang Chi,[4] Scott Davies,[1] Lance Dixon,[2,4] and Josh Nohle[1]
[1]*Department of Physics and Astronomy, University of California at Los Angeles, Los Angeles, California 90095, USA*
[2]*Walter Burke Institute for Theoretical Physics, California Institute of Technology, Pasadena, California 91125, USA*
[3]*Department of Physics, CERN Theory Division, CH-1211 Geneva 23, Switzerland*
[4]*SLAC National Accelerator Laboratory, Stanford University, Stanford, California 94309, USA*
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Evanescent operators such as the Gauss-Bonnet term have vanishing perturbative matrix elements in exactly $D = 4$ dimensions. Similarly, evanescent fields do not propagate in $D = 4$; a three-form field is in this class, since it is dual to a cosmological-constant contribution. In this Letter, we show that evanescent operators and fields modify the leading ultraviolet divergence in pure gravity. To analyze the divergence, we compute the two-loop identical-helicity four-graviton amplitude and determine the coefficient of the associated (nonevanescent) R^3 counterterm studied long ago by Goroff and Sagnotti. We compare two pairs of theories that are dual in $D = 4$: gravity coupled to nothing or to three-form matter, and gravity coupled to zero-form or to two-form matter. Duff and van Nieuwenhuizen showed that, curiously, the one-loop trace anomaly—the coefficient of the Gauss-Bonnet operator—changes under p-form duality transformations. We concur and also find that the leading R^3 divergence changes under duality transformations. Nevertheless, in both cases, the physical renormalized two-loop identical-helicity four-graviton amplitude can be chosen to respect duality. In particular, its renormalization-scale dependence is unaltered.

Note: Evanescent effect is proportional to ϵ/ϵ where $\epsilon = (4 - D)/2$

Pure Einstein Gravity Divergences

Standard finiteness argument for 1 loop finiteness or pure gravity:

't Hooft and Veltman (1974)

$\cancel{R^2}$ $\cancel{R_{\mu\nu}^2}$ $\cancel{R_{\mu\nu\rho\sigma}^2}$

Counterterms vanish by equation of motion and can be eliminated by field redefinition.

In $D = 4$ flat space Gauss-Bonnet theorem eliminates Riemann square term.

Pure gravity divergence is total derivative:

$$\mathcal{L}^{\rm GB} = \frac{1}{(4\pi)^2}\frac{53}{90\epsilon}(R^2 - 4R_{\mu\nu} + R_{\mu\nu\rho\sigma}^2)$$

$$\int d^4x\, \sqrt{-g}(R^2 - 4R_{\mu\nu} + R_{\mu\nu\rho\sigma}^2) = 0$$

Capper and Duff (1974)
Tsao (1977); Critchley (1978)
Gibbons, Hawking, Perry (1978)
Goroff and Sagnotti (1986)
Bornsen and van de Ven (2009)

No available counterterm in $D = 4$

't Hooft and Veltman (1974)

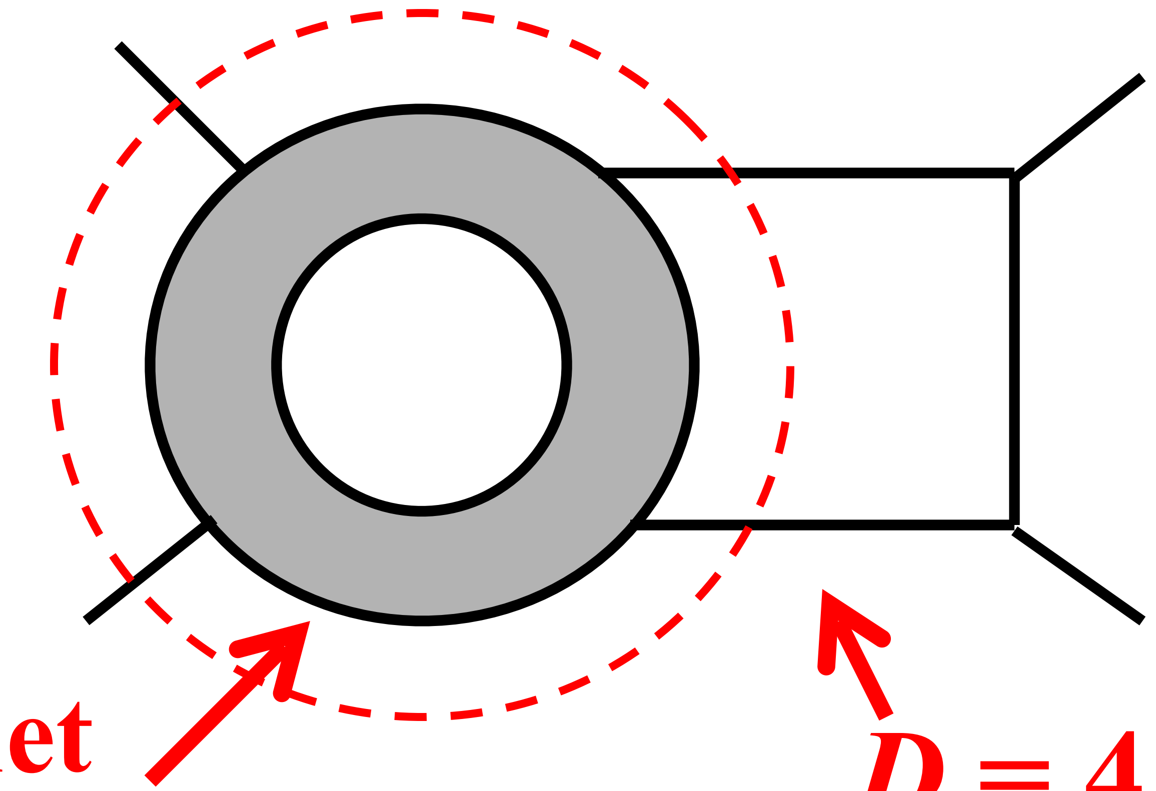
Dimensional regularization makes it subtle.

Capper and Kimber (1980)

Subdivergences?

ZB, Cheung, Chi, Davies, Dixon, Nohle

The integrand has subdivergences



Representative diagram.

Gauss-Bonnet subdivergence

$D = 4$, no subdivergences

$D \neq 4$, subdivergences!

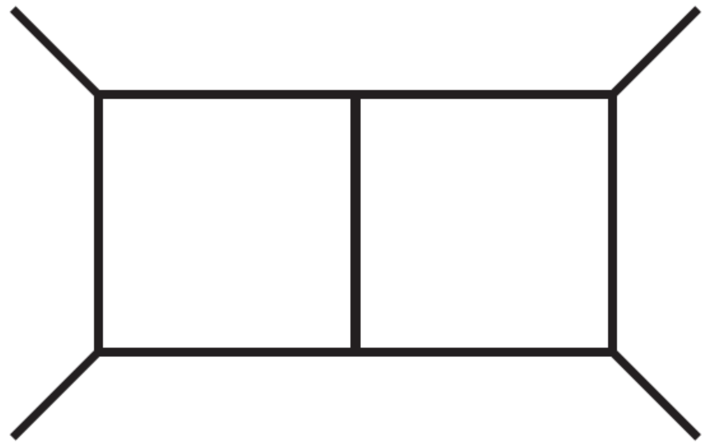
A strange phenomenon: no one loop divergences, yet there are one-loop subdivergences! Part of evanescent chaos in gravity.

- **To match the G&S result, need to subtract subdivergences.**
- **We used counterterm method. G&S subtracted integral by integral.**

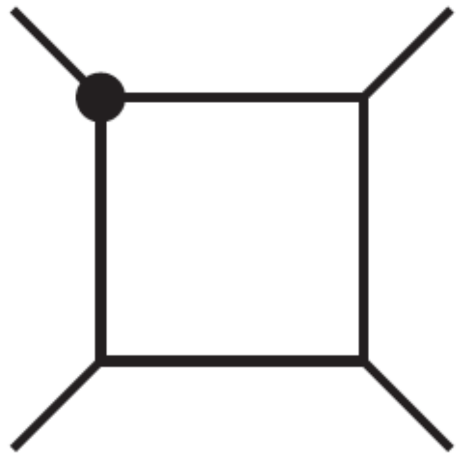
Two Loop Identical Helicity Amplitude

ZB, Cheung, Chi, Davies, Dixon and Nohle; Abreu, Febres Cordero, Ita, Jaquier, Page, Ruf, Sotnikov; Bern, Chen, Dixon, Herrman, Morales, Roiban, Ruf (to appear)

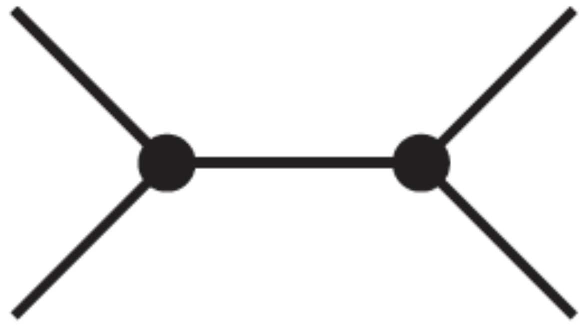
bare:



single GB counterterm:



double GB counterterm:



Also, subtract known IR singularity

Weinberg; Naculich, Nastase, Schnitzer; Akhoury, Saotome, Sterman

Full two-loop identical helicity amplitude in MS-bar scheme:

Goroff and Sagnotti divergence

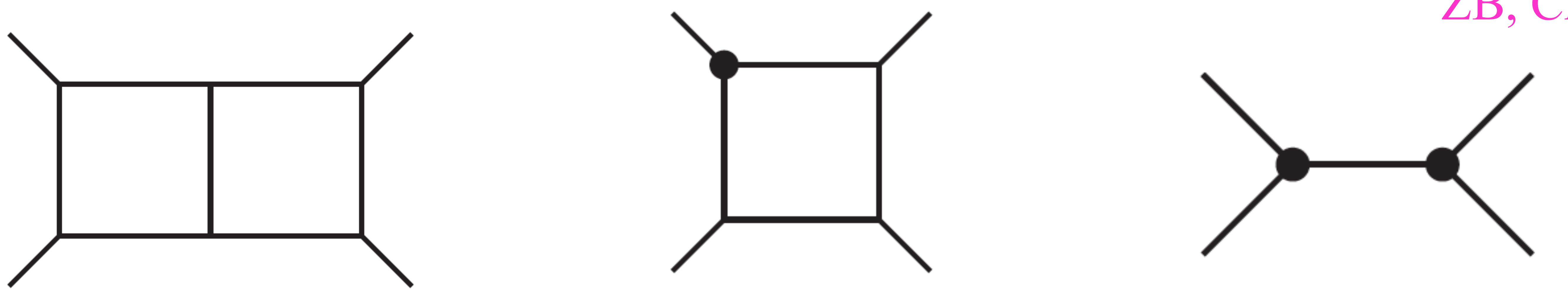
$$M^{2\text{-loop}}_{++++} = i\,\frac{e^{-2\gamma_E\epsilon}}{(4\pi)^{4-2\epsilon}}\left(\frac{\kappa}{2}\right)^6\left(\frac{[12][34]}{\langle 12\rangle\langle 34\rangle}\right)^2\left[stu\left(\frac{209}{24\epsilon}+\frac{117617}{21600}\right)-\frac{1}{60}(f_{stu}+f_{tus}+f_{ust})\right]+\text{IR div.}$$

$$f_{stu} = s(3t^2+3u^2-2s^2)\log(s/\mu^2)$$

This is in MS-bar scheme

Two Loop Identical Helicity Divergence

ZB, Cheung, Chi, Davies, Dixon and Nohle



2 loop bare — **single GB counterterm** — **double GB counterterm:**

$$\mathcal{K} = \frac{i}{(4\pi)^4} \left(\frac{\kappa}{2}\right)^6 \left(\frac{[12][34]}{\langle 12\rangle\langle 34\rangle}\right)^2 stu$$

$$\mathcal{M}_4^{\text{2-loop}}\Big|_{\text{div.}} = -\frac{1}{\epsilon}\frac{3431}{5400}\mathcal{K}$$

$$\mathcal{M}_4^{\text{1-loop GB}}\Big|_{\text{div.}} = \frac{1}{\epsilon}\frac{689}{675}\mathcal{K}$$

$$\mathcal{M}_4^{\text{tree GB}^2}\Big|_{\text{div.}} = \frac{1}{\epsilon}\frac{5618}{675}\mathcal{K}$$

$$\mathcal{M}_4^{\text{total}}\Big|_{\text{div.}} = \frac{1}{\epsilon}\frac{209}{24}\mathcal{K}$$

Goroff and Sagnotti translated to four points

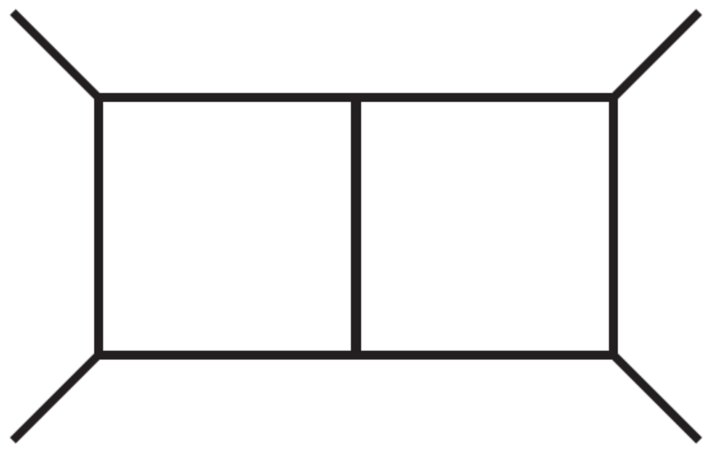
- **Goroff and Sagnotti divergence reproduced.**
- **GB counterterm contributes at two loops even in flat space.**

Closer Look at UV Divergence

ZB, Cheung, Chi, Davies, Dixon and Nohle

For pure gravity:

$$M^{2\text{-loop}}_{++++}\Big|_{\text{div.}} = i\,\frac{e^{-2\gamma_E\epsilon}}{(4\pi)^{4-2\epsilon}}\left(\frac{\kappa}{2}\right)^6\left(\frac{[12][34]}{\langle 12\rangle\langle 34\rangle}\right)^2 stu\left(\frac{209}{24\epsilon}-\frac{1}{4}\log(\mu^2)\right)$$



- **Note simplicity of $\log(\mu^2)$ coefficient compared to UV poles**
- **Value of the UV divergence is scheme dependent and can be set to *any* value.**
- **Adding evanescent fields and operators $_\mu$changes divergence but not $\log(\mu^2)$**

- **Note: Because $\log(\mu^2)$ coefficient is non-zero we should consider pure gravity divergent.**
- **Goroff and Sagnotti conclusion correct despite the evanescent chaos.**

For general minimally coupled matter:

$$M^{2\text{-loop}}_{++++}\Big|_{\text{div.}} = i\,\frac{e^{-2\gamma_E\epsilon}}{(4\pi)^{4-2\epsilon}}\left(\frac{\kappa}{2}\right)^6\left(\frac{[12][34]}{\langle 12\rangle\langle 34\rangle}\right)^2 stu\left(\frac{C}{\epsilon}-\frac{N_b-N_f}{8}\log(\mu^2)\right)$$

N_b is number of bosonic states.
N_f is number of fermionic states.
C depends on theory and scheme.

$\log(\mu^2)$ term scheme independent. No UV regulator needed! Use unitarity! $\log(\mu^2/s_i)$

Dunbar, Jehu, Perkins; ZB, Chi, Dixon, Edison; Caron-Huot and Wilhelm

Evanescent Fields and UV

Pure Gravity:

$$\mathcal{M}^{(2)}(1^+,2^+,3^+,4^+) = \mathcal{N}\left\{\frac{1}{\epsilon}\frac{209}{24}stu + \frac{117617}{21600}stu + \frac{1}{10}stu\log(-s) - \frac{1}{60}s^3\log(-s) + \frac{1}{120}(s^2+t^2+u^2)s\log(-s)^2 + \text{perms}\right\}$$

Gravity + 3 Form (evanescent):

$$\mathcal{M}^{(2)}(1^+,2^+,3^+,4^+) = \mathcal{N}\left\{\frac{1}{\epsilon}\frac{29}{24}stu + \frac{411617}{21600}stu + \frac{1}{10}stu\log(-s) - \frac{1}{60}s^3\log(-s) + \frac{1}{120}(s^2+t^2+u^2)s\log(-s)^2 + \text{perms}\right\}$$

Divergences are different but logarithms are identical!
No physical effect! The 3 form is a Cheshire Cat field.

Using a scheme with or without the 3 form changes the divergence but not the physics.
UV divergence doesn't tell you if theory is UV divergent!

Recent New Results

Celestial Holography, Soft Theorems ↔ asymptotic symmetries; infinite dimensional algebra

Strominger; Paterski, Shao, Strominger; He, Lysov, Mitra, Strominger

A subalgebra — the CCA — captures collinear limits that are universa between SD & full theory

$$\mathcal{O}_{h_1,\bar{h}_1}(z_1,\bar{z}_1)\mathcal{O}_{h_2,\bar{h}_2}(z_2,\bar{z}_2) \sim \frac{1}{z_{12}}\sum_p \sum_{m=0}^{\infty} C_p^{(m)}\left(\bar{h}_1,\bar{h}_2\right) \bar{z}_{12}^{p+m}\,\bar{\partial}^m \mathcal{O}_{h_1+h_2-1,\bar{h}_1+\bar{h}_2+p}(z_2,\bar{z}_2)$$

Anomaly free theories: Choose special matter fermion representation to cancel the anomaly

Costello; Costello, Paquette; Dixon and Morales

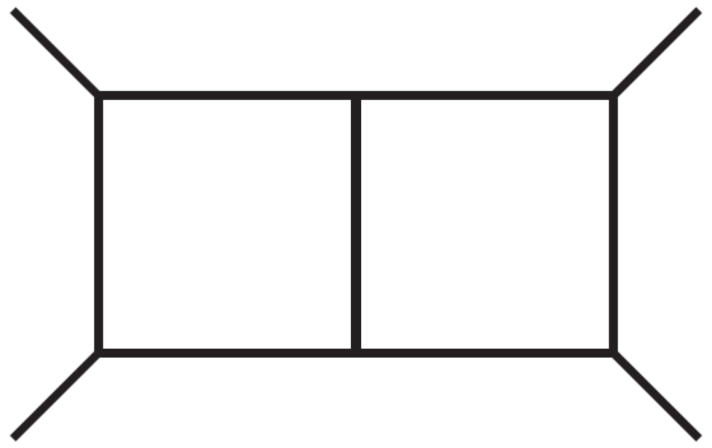
An anomaly free theory $R_0 \equiv 8F \oplus 8\bar{F} \oplus \wedge^2 F \oplus \wedge^2 \bar{F}$

In terms of Dirac quarks
8 fund rep + 2-index anti sym tensor rep

See Morales' talk

$$\mathcal{A}^{\text{2-loop}}_{4,\text{sdYM}} = \frac{g^6}{(4\pi)^4}\rho\left[\left(12N - 4\frac{s^2+4st+t^2}{st} - \frac{24}{N}\right)\left(\text{tr}(1234)+\text{tr}(1432)\right) + \left(24+\frac{24}{N}\right)\text{tr}(12)\text{tr}(34)\right] + \mathcal{C}(234),$$

$$\rho = i\frac{[12][34]}{\langle 12\rangle\langle 34\rangle}$$



- **Papers bootstrapped to all *n*.**
- **Need to remove IR singularities.**

An obvious question is whether same holds in gravity. Choose $N_f = N_b$

Bern, Chen, Dixon, Herrman, Morales, Roiban, Ruf (in progress)

Open Questions and Conclusions

1. **What is the role of the anomaly in UV divergences?**
2. **What is the mechanism behind enhanced cancellations, and how far do they go?**
3. **If we cancel the self-dual anomaly, can we make finite theories?**
4. **What is the fate of $\mathcal{N} = 8$ supergravity UV?**
   - **Recent progress for $\mathcal{N} = 4$ sYM at 6 loops (needed to go on in $\mathcal{N} = 8$)** Carrasco, Edison, Johansson
   - **Unfortunately, need 7, 8 loops in $\mathcal{N} = 8$ gravity!**
   - **$\mathcal{N} = 5$ supergravity at 5 loops is a better bet. 4 loops already has enhanced cancellations.**
5. **What is the best way to formulate high-loop UV calculations to be scheme independent?**
   - **Target $\log(s_i/\mu^2)$. Use unitarity.** Dunbar, Jehu, Perkins; ZB, Chi, Dixon, Edison; Caron-Huet, Wilhelm
6. **Anomaly free theories and the identical helicity sector: How far can we go?**

Bern, Chen, Dixon, Herrman, Morales, Roiban, Ruf (in progress)

Despite all the fun we had with Lance to understand UV of supergravity more remains to be done.

Happy Birthday to Lance