Energy Correlators From MeV to TeV

Ian Moult
Yale

TASI 2013

A brief introduction to modern amplitude methods

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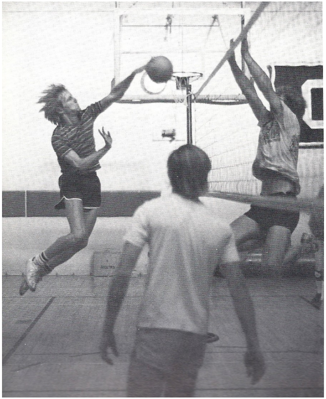
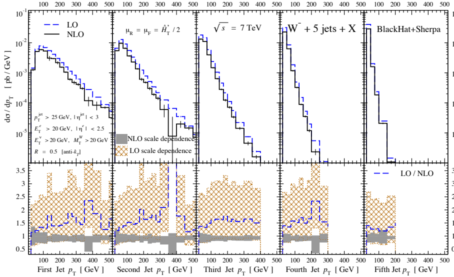
ABSTRACT: I provide a basic introduction to modern helicity amplitude methods, including color organization, the spinor helicity formalism, and factorization properties. I also describe the BCFW (on-shell) recursion relation at tree level, and explain how similar ideas — unitarity and on-shell methods — work at the loop level. These notes are based on lectures delivered at the 2012 CERN Summer School and at TASI 2013.

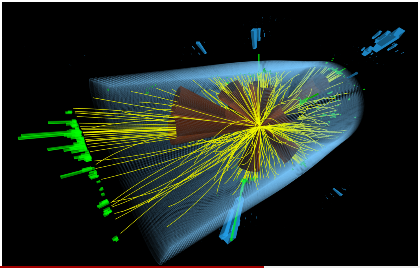
Approximate Reenactment



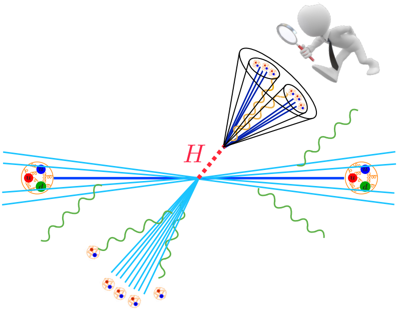
From Jets to Jet Substructure

Jet Kinematic Distributions



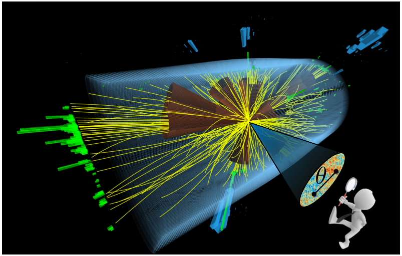


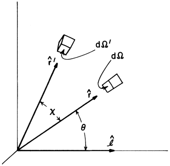
Jet Substructure

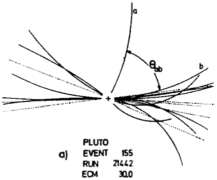


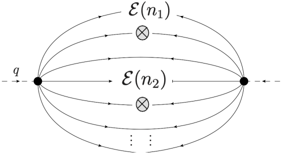
Energy Correlators

- Energy Correlators are a particularly simple class of observables originally proposed in e^+e^- colliders (Ellis et al. 1977).

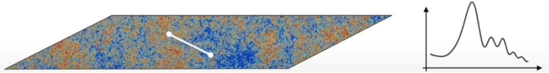








- Analogous to correlators in the CMB.

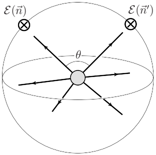


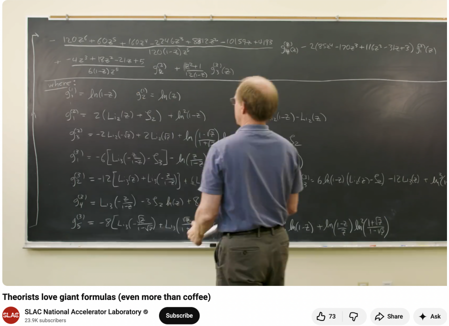
- Never measured at hadron colliders. Not on the radar of my generation (or at least of me).

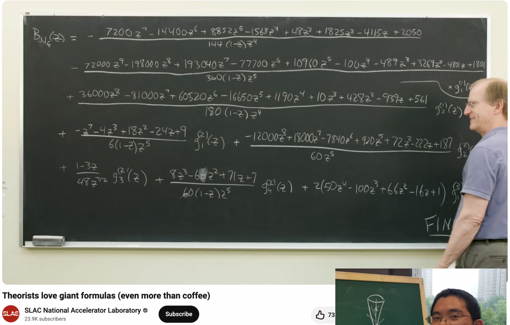
Cross Section Level Observables Can Be Beautiful

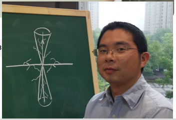
- Energy correlators provide an example of an analytically understandable collider observable.

$$z = (1 - \cos(\theta))/2$$



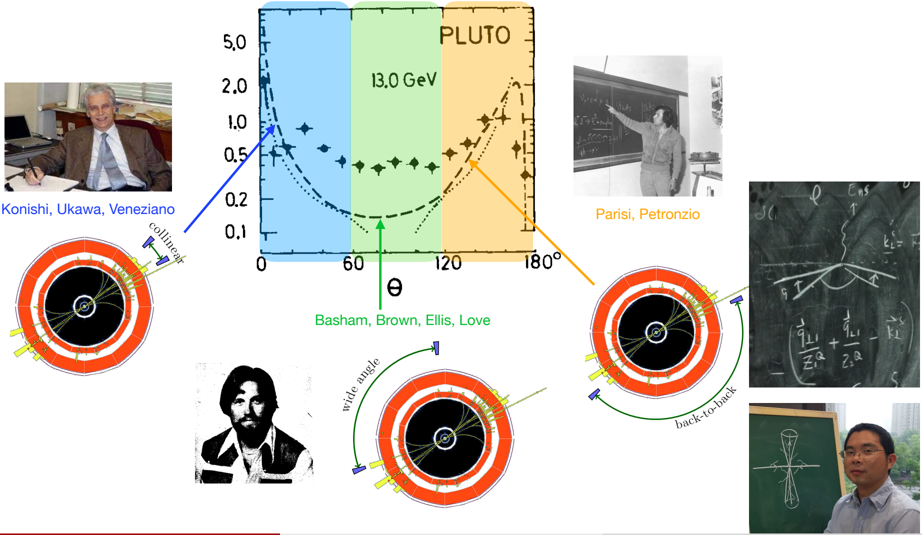






History

- Long history of calculations of energy correlators in kinematic limits:



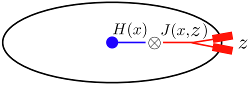
Collinear Limit of the EEC

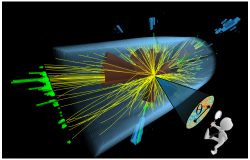
- In a conformal theory, the energy correlator exhibits a powerlaw scaling z^{γ}, in the collinear limit (Hofman, Maldacena).
- Using renormalization group techniques, we provided a description of the collinear limit of the EEC in non-conformal theories, such as real world QCD:

The Collinear Limit of the Energy-Energy Correlator

Lance J. Dixon,[1] Ian Moult,[2,3] and Hua Xing Zhu[4]

$$\Sigma(z, \ln\frac{Q^2}{\mu^2}, \mu) = \int_0^1 dx\, x^2 \vec{J}(\ln\frac{zx^2Q^2}{\mu^2}, \mu) \cdot \vec{H}(x, \frac{Q^2}{\mu^2}, \mu)\,.$$





$$\frac{d\vec{J}(\ln\frac{zQ^2}{\mu^2}, \mu)}{d\ln\mu^2} = \int_0^1 dy\, y^2 \vec{J}(\ln\frac{zy^2Q^2}{\mu^2}, \mu) \cdot \widehat{P}_T(y, \mu)$$

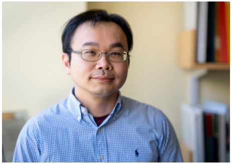
$$\frac{d\vec{H}(x, \frac{Q^2}{\mu^2}, \mu)}{d\ln\mu^2} = -\int_x^1 \frac{dy}{y} \widehat{P}_T(y, \mu) \cdot \vec{H}\left(\frac{x}{y}, \frac{Q^2}{\mu^2}, \mu\right)$$

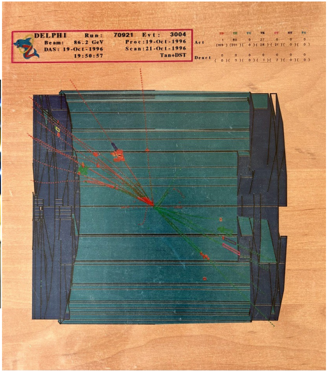
- Scaling in the collinear limit is a clean experimental observable, potentially measurable inside jets at the LHC.

Few months at SLAC, then cut short by COVID

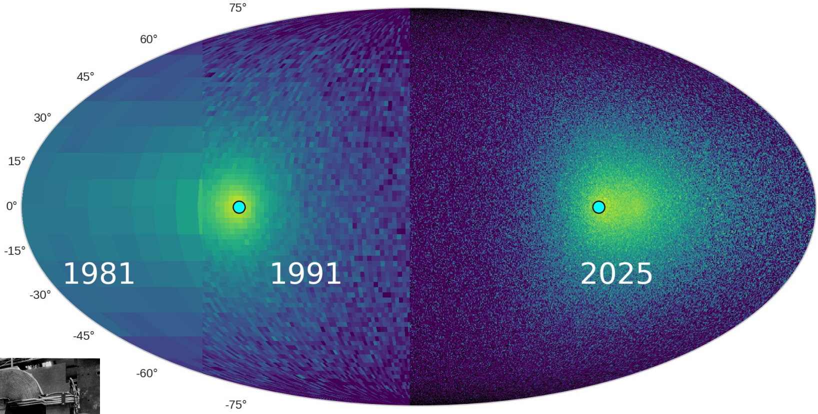
Energy Correlators in Real World

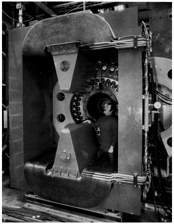
- Energy Correlators had not received experimental attention since the 90s.
- Worked with YenJie Lee to measure them in archival LEP data.

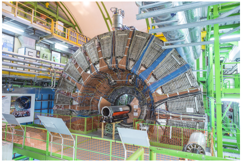


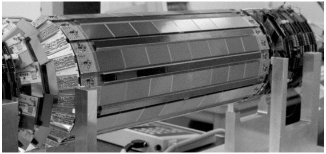


QCD Correlations in Focus



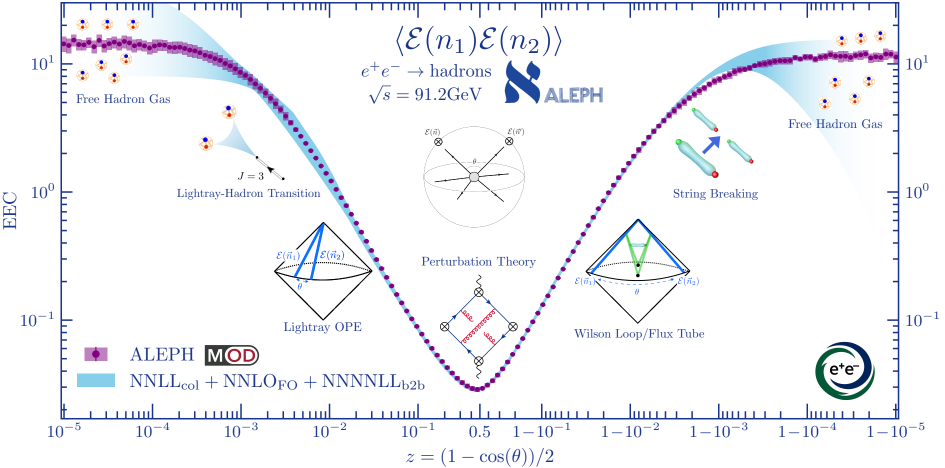




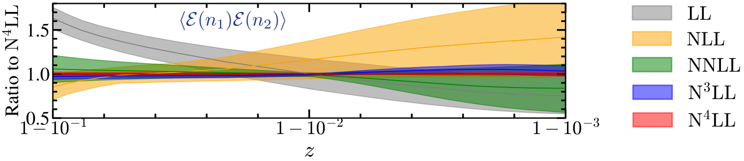


Energy Correlators from Partons to Hadrons

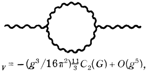
- High resolution image of correlations in QCD energy flux: Beautiful playground for testing understanding of QCD.

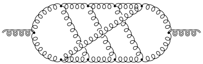


Impact of Advances in Perturbative QCD

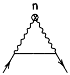


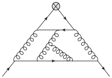
- β functions



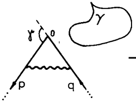


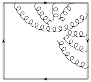
- Twist-2 operators



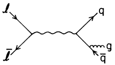


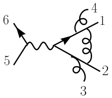
- Cusped Wilson loops





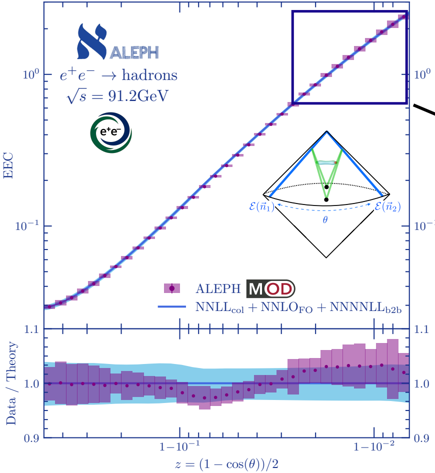
- Scattering Amplitudes

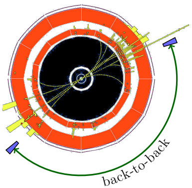


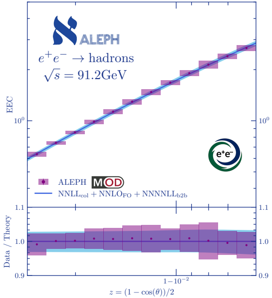


Back-to-Back Limit

- Back-to-Back limit: theory error matches experiment.

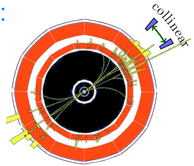




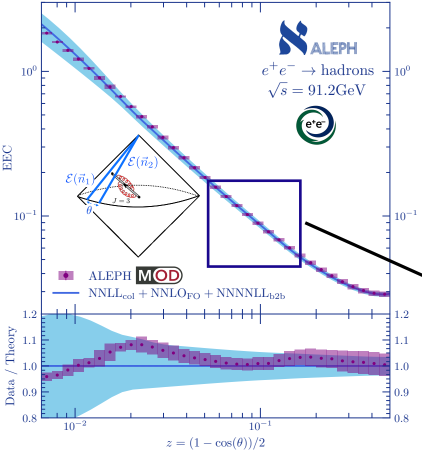


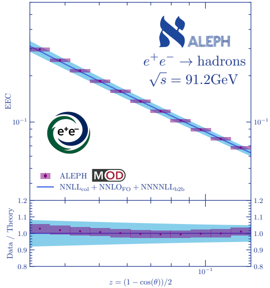
Collinear Limit

- Testing the formula we derived with Lance:



$$\mathcal{E}(\hat{n}_1)\mathcal{E}(\hat{n}_2) \sim \frac{1}{\theta^{2-\gamma(3)}}\mathcal{D}^{n=3}_{\tau=2}(\hat{n}_1) + \cdots$$

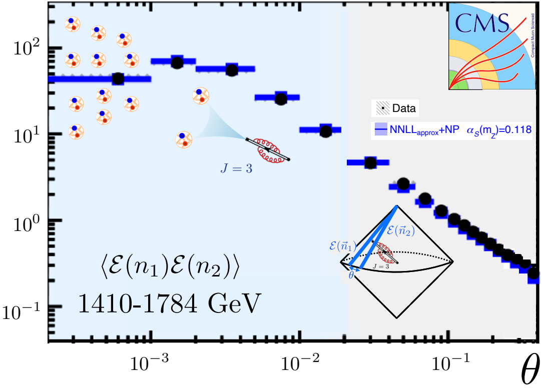


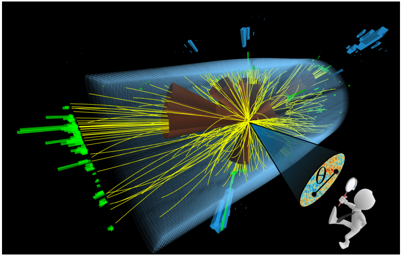


- Experimental errors beating theory. Still more work to do!

Scaling in Jet Substructure

- Measurements by CMS up to 1784 GeV compared to calculations

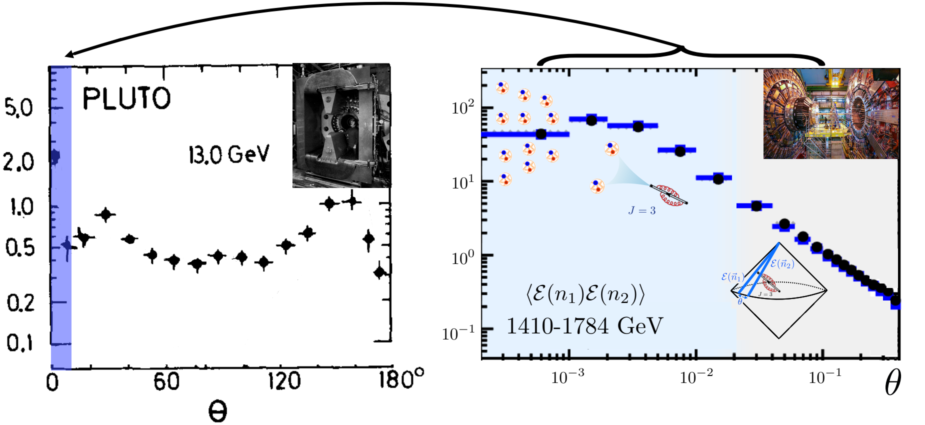




- Clean scaling in complex hadronic environment!

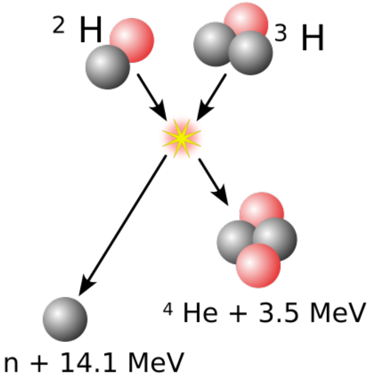
Scaling in Jet Substructure

- A comparison with the first measurement of the Energy Correlators



- Lots of fun exploring theory + phenomenological applications.
- Thanks to Lance for getting us started in this direction!

Non-Relativistic Conformal Collider Physics: Explosions of Mini-Neutron Bombs

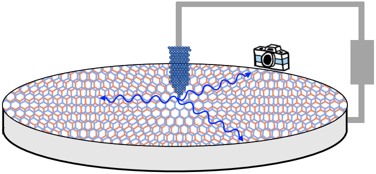


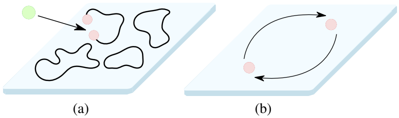
The LANCE enhanced radiation warhead

w/ C.H. Chang, S.D. Chowdhury, D.T. Son

Colliders in Other Systems

- General “collider” paradigm is not unique to relativistic systems.
- Many experimentally realizable systems are non-relativistic/ exhibit non-isotropic scaling in space/time.





Wilczek et al.
Sachdev et al.
....

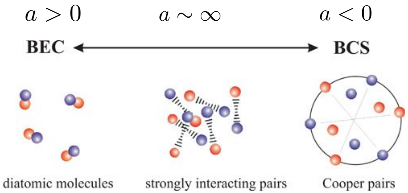
- Can formalizing such measurements in terms of detector operators enable the identification of universal signatures in strongly coupled systems?

Non-Relativistic CFTs

- Non-relativistic CFTs provide a universal description for non-relativistic systems interacting via a contact interaction, tuned to infinite scattering length.
- Recall that for an s-wave scattering, the cross section can be written:

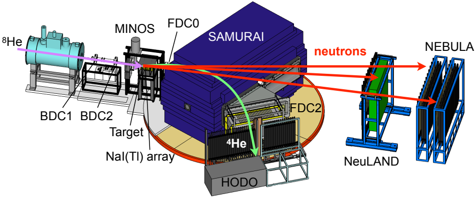
$$\sigma = \frac{4\pi}{k^2 + k^2 \cot^2 \delta_0}\,, \qquad\qquad k \cot \delta_0(k) = -\frac{1}{a} + \frac{1}{2} r_e k^2 + \cdots$$

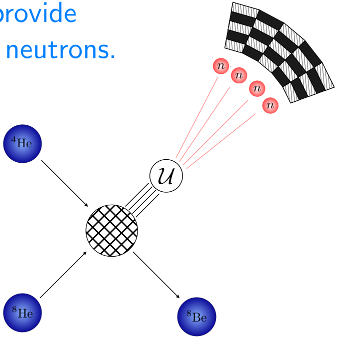
- In the limit $a \to \infty$ and $r_e \to 0$, $\sigma \sim 4\pi/k^2$ is scale invariant.
- Few and many body physics can be realized experimentally, most famously in the BEC-BCS crossover: "fermions at unitarity"



Mini Neutron Bombs

- Neutrons are famously fine tuned, $a = -18.9$ fm and $r_e = 2.75$ fm, allowing access to few body physics of NRCFTs.
- Experiments studying neutron rich matter provide data for "neutron bombs" (jets) of up to 6 neutrons.

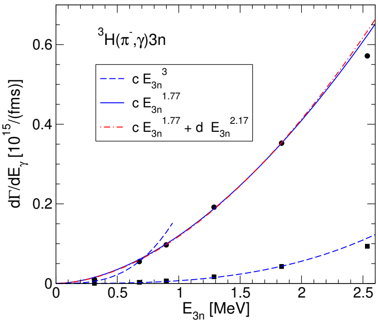


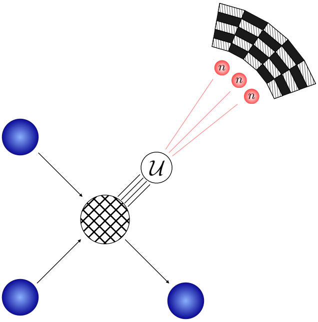


$$^3\mathrm{H} + {}^3\mathrm{H} \to {}^4\mathrm{He} + 2n\,,$$
$$^7\mathrm{Li} + {}^7\mathrm{Li} \to {}^{11}\mathrm{C} + 3n\,,$$
$$^4\mathrm{He} + {}^8\mathrm{He} \to {}^8\mathrm{Be} + 4n\,.$$

Production Rate

- Hammer and Son studied the "jet rate".
- Exhibits a scaling with the exactly computable non-integer scaling dimension of an NRCFT three-body operator.

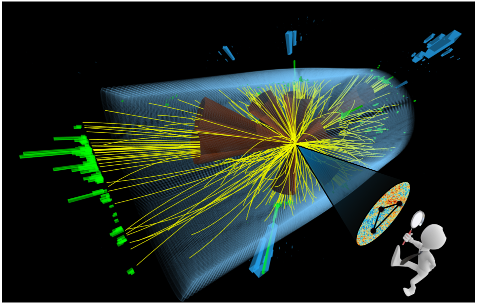


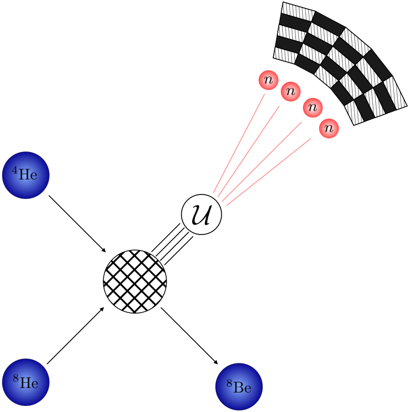


$$\frac{d\sigma}{dE} \sim (E_0 - E)^{1.77272\cdots}.$$

Slow Neutron Jet Substructure

- Neutrons exhibit interesting correlations in velocity and angle.
- Can we compute "jet substructure" observables in NRCFTs?

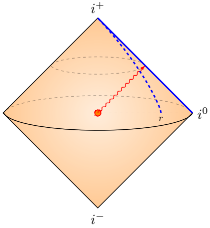




Detector Operators in NRCFTs

- Would like to develop a language to compute asymptotic observables from local correlators in NRCFTs.

Relativistic



$$\mathcal{E}(\vec{n}) = \lim_{r\to\infty} r^2 \int_0^\infty dt\; n^i T_{0i}(t, r\vec{n})$$

$$\mathcal{E}(n)|0\rangle = 0\,,$$
$$[\mathcal{E}(n_1), \mathcal{E}(n_2)] = 0\,,$$
$$\langle\Psi|\mathcal{E}(n)|\Psi\rangle \geq 0,$$

Non-Relativistic

Non-Relativistic CFT Primer

- NRCFTs are invariant under a non-isotropic ($z = 2$) scaling
$$(t, x^i) \to (\lambda^2 t, \lambda x^i)$$
- Scale invariance is often enhanced to the full Schrodinger group
  - Space+time translations: P_0, P_i
  - Galilean Boosts: K_i
  - Spatial Rotations: M_{ij}
  - Special Conformal transformations: C_0
  - Dilatations: D
- Algebra has a central element, the particle number, N (mass)
- Local operators are characterized by their particle number, $N_{\mathcal{O}}$, scaling dimension, $\Delta_{\mathcal{O}}$, and spin, l:

$$[N, \mathcal{O}(0)] = N_{\mathcal{O}}\mathcal{O}(0), \quad [D, \mathcal{O}(0)] = i\Delta_{\mathcal{O}}\mathcal{O}(0), \quad [M_{ij}, \mathcal{O}(0)] = S^l_{ij} \cdot \mathcal{O}(0),$$

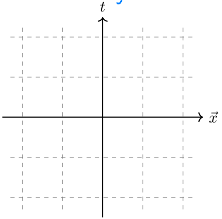
- Examples: Free particles, fermions at unitarity in 3+1 d, anyons in 2+1d, null reductions

Detector Operators in NRCFTs

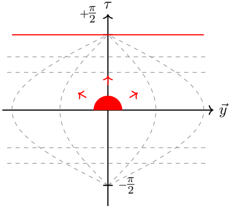
- In a non-relativistic theory, asymptotic fluxes can be measured as a function of *both angle and velocity*
- We can define an energy detector operator in terms of the number operator

$$\mathcal{E}_v(\hat{n}) = \frac{v}{2}\lim_{r\to\infty} r^3 n\left(\frac{r}{v}, r\hat{n}\right)$$

- Easily understood in "harmonic trap" coordinates:



Flat space



Harmonic trap

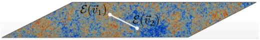
$$t = \tan(\tau), \qquad \tau = \arctan(t),$$

$$\leftrightarrow$$

$$\vec{x} = \vec{y}\sec(\tau), \qquad \vec{y} = \frac{\vec{x}}{\sqrt{1+t^2}},$$

- Behave as local operators in a d-1 scale invariant, Euclidean theory in "velocity space":

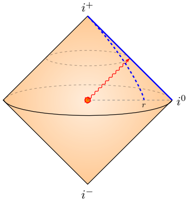
$$\mathcal{E}_v(\hat{n}) = \frac{v^4}{2}\tilde{n}\left(\tau = \frac{\pi}{2}, v\hat{n}\right)$$

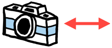


Detector Operators

- Interesting formal analogy between massless local operators, $:(\phi^\dagger)^n\phi^n:$ in NRCFTs and lightray operators in relativistic theories.
- Intuitively understood via null-reduction.

Relativistic

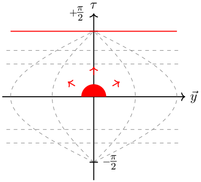


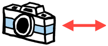


$$\mathcal{E}(\vec{n}) = \lim_{r\to\infty} r^2 \int_0^\infty dt\ n^i T_{0i}(t, r\vec{n})$$

$$\mathcal{E}(n)|0\rangle = 0\,,$$
$$[\mathcal{E}(n_1), \mathcal{E}(n_2)] = 0\,,$$
$$\langle\Psi|\mathcal{E}(n)|\Psi\rangle \geq 0,$$

Non-Relativistic



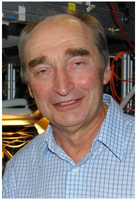


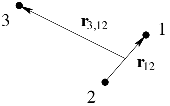
$$\mathcal{E}_v(\hat{n}) = \frac{v}{2} \lim_{r\to\infty} r^3 n\left(\frac{r}{v}, r\hat{n}\right)$$

$$\mathcal{E}_v(n)|0\rangle = 0\,,$$
$$[\mathcal{E}_{v_1}(n_1), \mathcal{E}_{v_2}(n_2)] = 0\,,$$
$$\langle\Psi|\mathcal{E}_v(n)|\Psi\rangle \geq 0,$$

Exact Three-Body Spectra

- Local operators with particle number $N_{\mathcal{O}}$ can be defined by their matrix elements, expressed in terms of N-body wavefunctions with Bethe-Peierls boundary conditions.
- Remarkably, the three body sector was solved by Efimov. Wavefunctions and operator dimensions of all three-body operators known analytically.





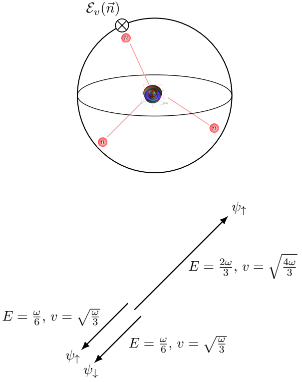
$l=0$	$l=1$
2.16622	1.77272
5.12735	4.35825
7.11448	5.71643
8.83225	8.05319

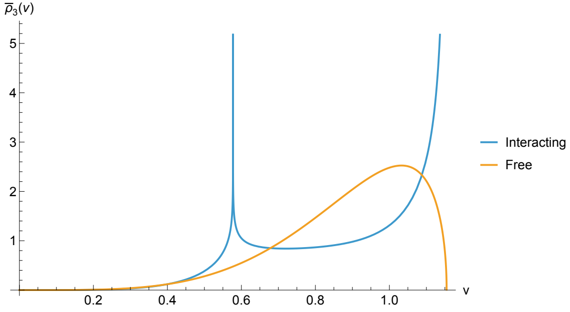
$$l=0: \qquad s\cos\left(\frac{\pi}{2}s\right)+\frac{4}{\sqrt{3}}\sin\left(\frac{\pi}{6}s\right)=0,$$

$$l=1: \qquad (s^2-1)\sin\left(\frac{\pi}{2}s\right)+\frac{4}{\sqrt{3}}s\cos\left(\frac{\pi}{6}s\right)-4\sin\left(\frac{\pi}{6}s\right)=0.$$

Energy Correlators in Three-Body States

- Using Efimov's solution we can compute the one-point functions $\langle \mathcal{E}_v(\vec{n}) \rangle$ in different three-body states.



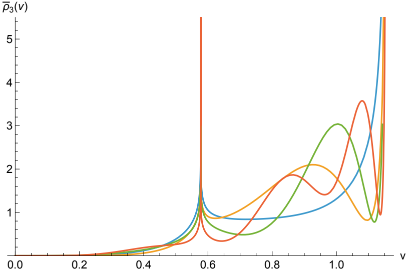


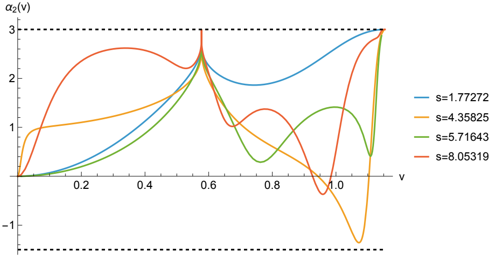
- Distribution strongly modified by interactions, producing enhanced two neutron "jet state".

Excited Three-Body States and Conformal Collider Bounds

- We can also compute the result in excited and higher spin states:

$$\langle \mathcal{E}_v(\theta) \rangle = \frac{\bar{\rho}_3(v)}{4\pi} \left(1 + \alpha_2(v)(\cos^2\theta - \tfrac{1}{3})\right)$$



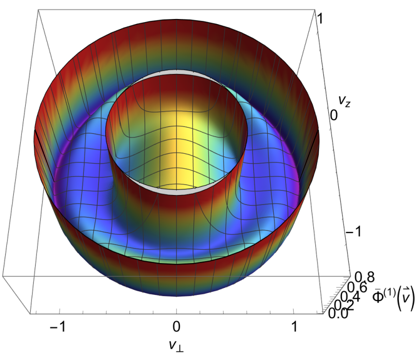


- Positivity of the $\mathcal{E}_v(\theta)$ operator implies a non-relativistic Hofman-Maldacena bound:

$$-\frac{3}{2} \leq \alpha_2(v) \leq 3$$

Celestial Wavefunctions

- Correlators can be computed from a "celestial" multi-body wavefunction, $\Psi(v_1, v_2, \cdots, v_n)$, in velocity space at timelike infinity.

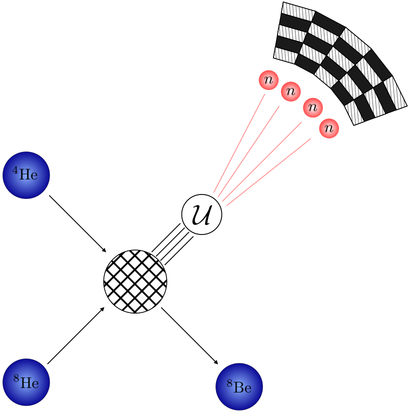


- Provides a simple, solvable example, structurally similar to cosmological correlators or celestial holography.

Future Directions

- Higher point correlation functions, e.g. $\langle \mathcal{E}(\vec{v}_1)\mathcal{E}(\vec{v}_2)\rangle$ can easily be computed in three body states.
- Detector OPE predicts a universal scaling as two neutrons are brought together in velocity space:

$$\mathcal{E}(\vec{v}_1)\mathcal{E}(\vec{v}_2) = \sum \frac{C_i}{|\vec{v}_1 - \vec{v}_2|^{\Delta_i}} \mathcal{D}(v_1)$$



- Most importantly, hope to be able to compare with data soon!

Thanks for the support, collaboration, and inspiration!