

Finite two-loop amplitudes

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Celebrating Lance Dixon
Edinburgh · 24–26 June 2026

My first particle physics papers to study

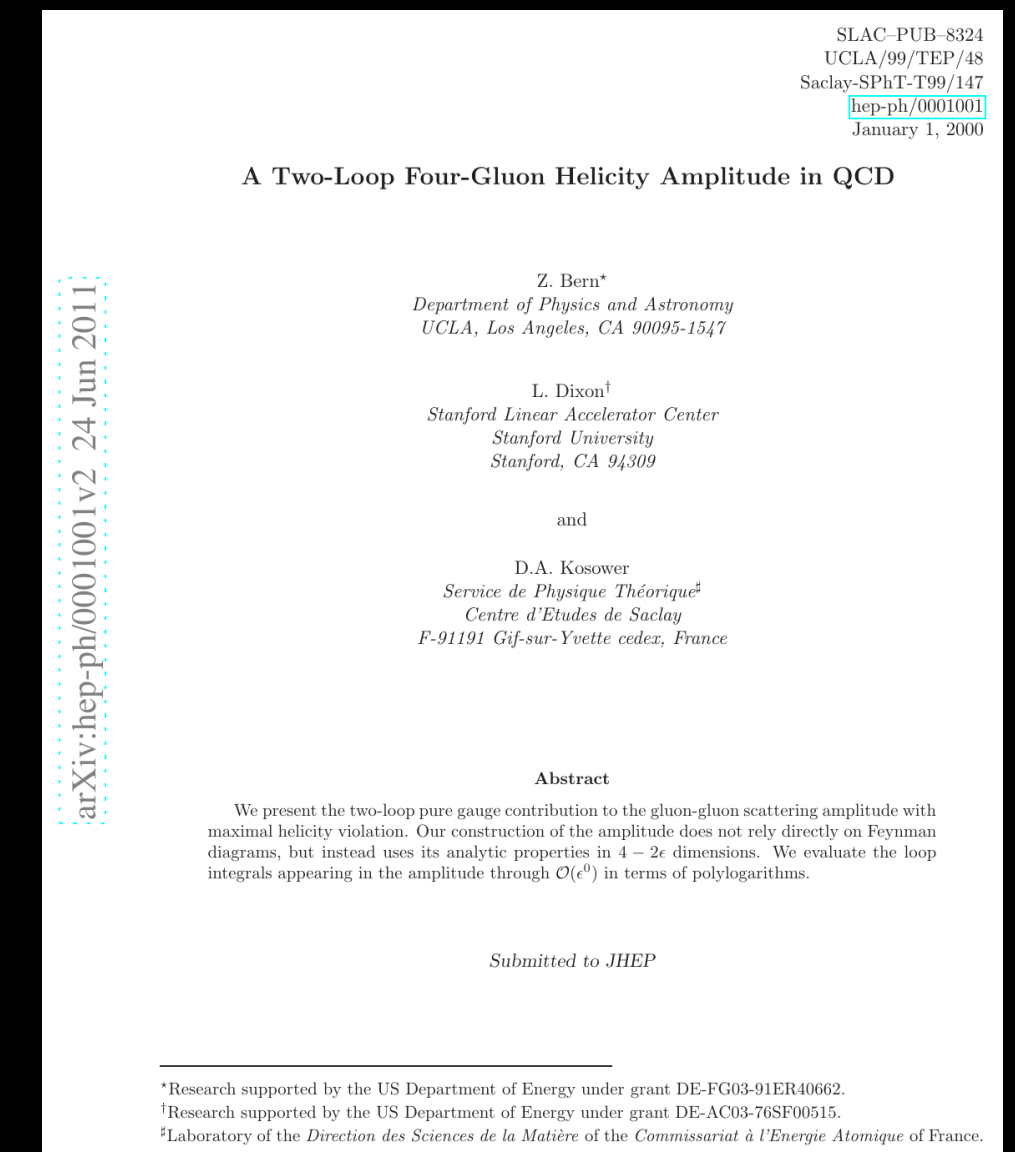
- Bern, Dixon, Kosower — one-loop integrals in dimensional regularization
- hep-ph/9212308 and hep-ph/9306240
- The beginning of my PhD.
- A lesson on alphabets: $\alpha \neq a$
- A nice feeling about the field of research I had just entered!

$$a_i = \frac{\alpha_i u_i}{\sum_{j=1}^n \alpha_j u_j}, \quad \text{no sum on } i,$$

$$a_n = \frac{\alpha_n \left(1 - \sum_{j=1}^{n-1} u_j\right)}{\sum_{j=1}^n \alpha_j u_j}.$$

The “millennium” paper

- Bern, Dixon & Kosower — a two-loop four-gluon helicity amplitude, from unitarity
- hep-ph/0001001 — the first arXiv paper of the new millennium (1 Jan 2000)
- A huge motivation: a new era in perturbative QCD and QFT
- Also a fright — I was working with Nigel Glover, Carlo Oleari and Maria Elena Tejeda-Yeomans towards two-loop QCD amplitudes, racing for my thesis...
- It worked out: the field blossomed, and Lance brought me to SLAC for my first postdoc.

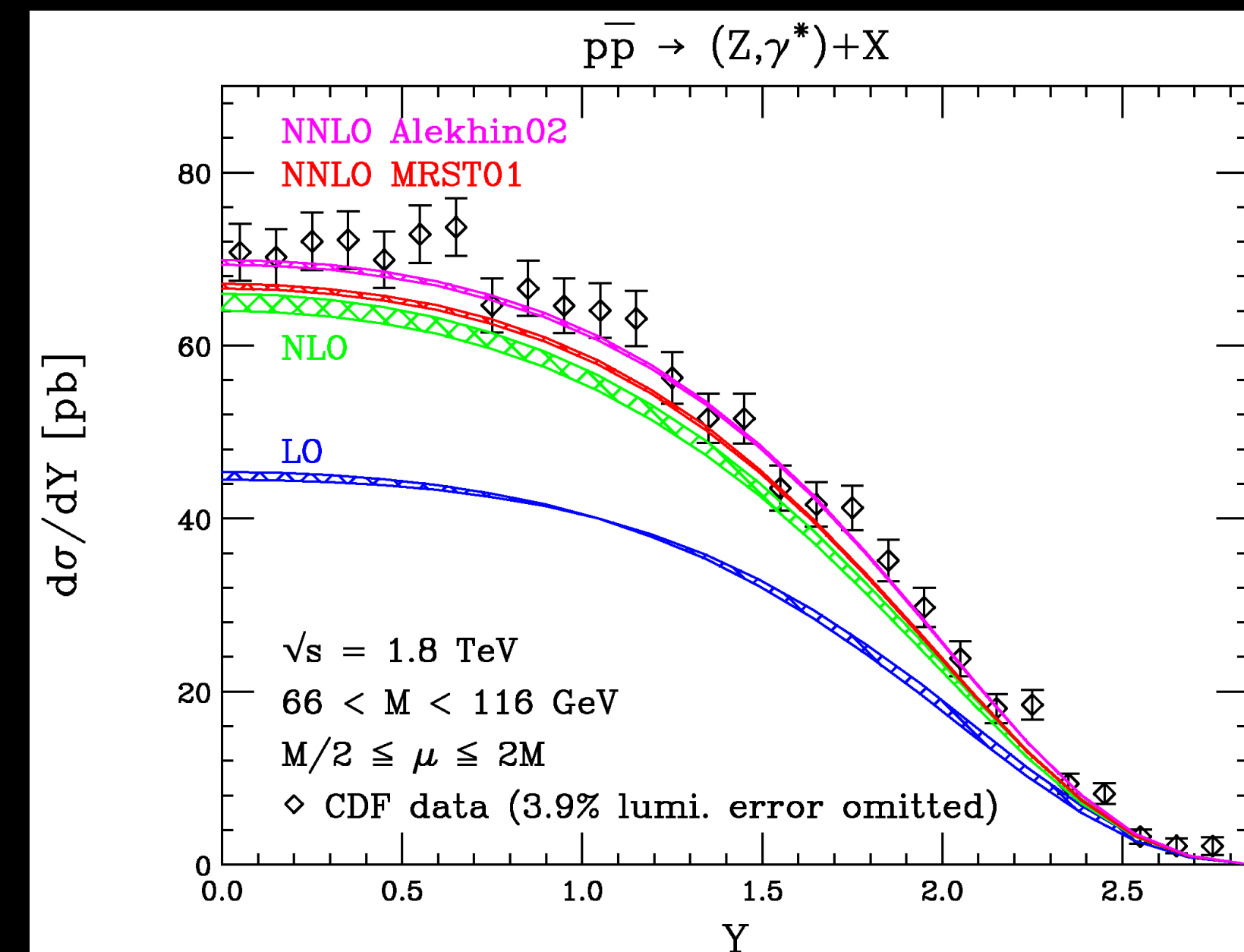
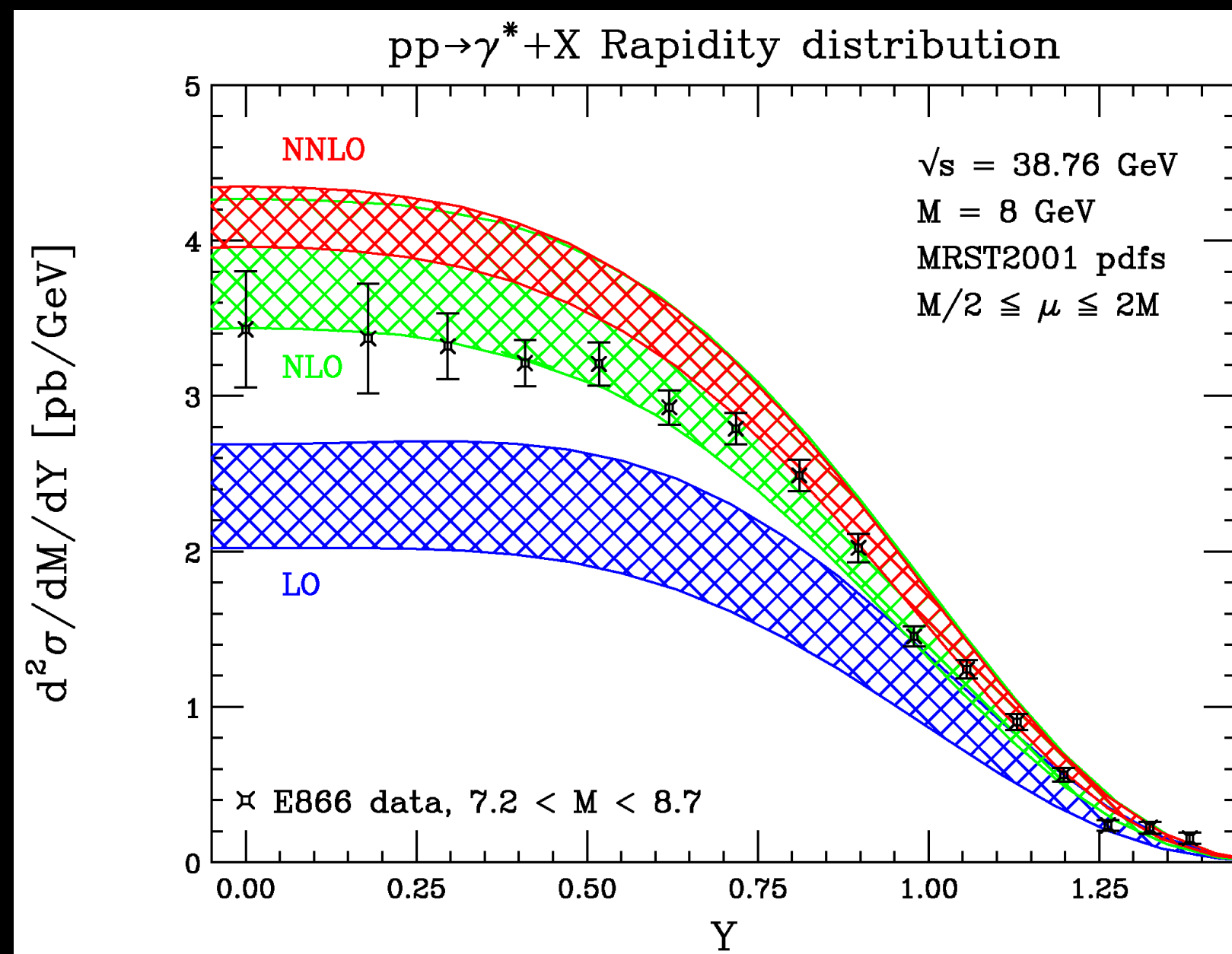


What Lance taught me at SLAC

- I arrived with a “next-goal” mindset: next integral, next amplitude, next cross-section
- SLAC, an unparalleled environment for developing the career of young physicists.
- Gained two good attitudes — passed on by Lance to generations of his students and postdocs:
 1. Look for deeper insight — the structure of finite two-loop remainders, and patterns that hold at all orders → Amplitudes is a profound research field, touching the foundations of QFT.
 2. Connect to experiment → precision phenomenology

Connecting to experiment: Drell-Yan at NNLO

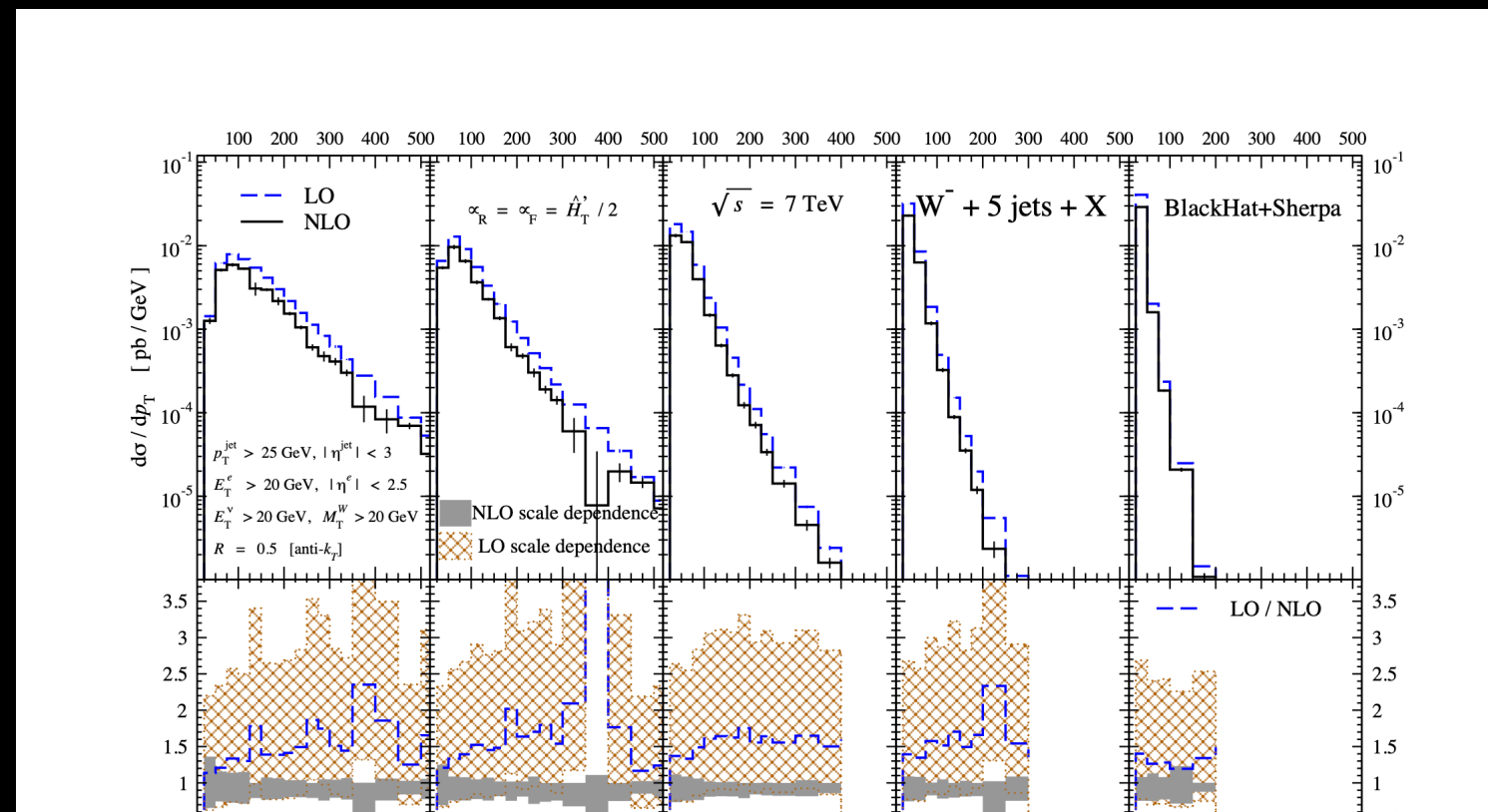
With Lance, Kirill and Frank



NNLO Drell-Yan rapidity distributions (hep-ph/0306192, hep-ph/0312266). Left: Fermilab E866/NuSea. Right: CDF Run I Z. Lance traced down the Tevatron points, and actually converted the experimentally measured distributions of E866 to the rapidity distribution that we could compare.

The NLO revolution

- Lance — a protagonist of automation of on-shell methods for one-loop amplitudes
- Extending NLO QCD to many external legs — e.g. W & Z + up to five jets (here, W + 5 jets)



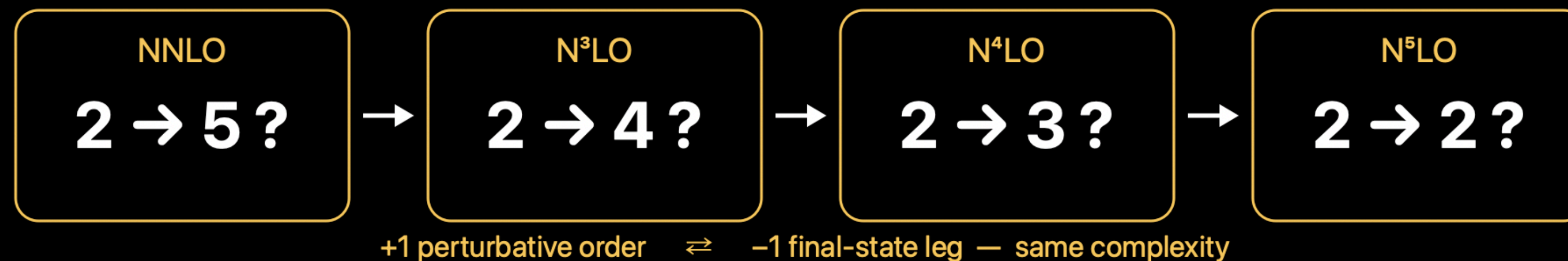
Review — On-Shell Methods in Perturbative QCD

Bern, Dixon, Kosower · *Annals Phys.* 322 (2007) 1587 · [0704.2798]

BlackHat · *Phys. Rev. D* 78 (2008) 036003 · [0803.4180]

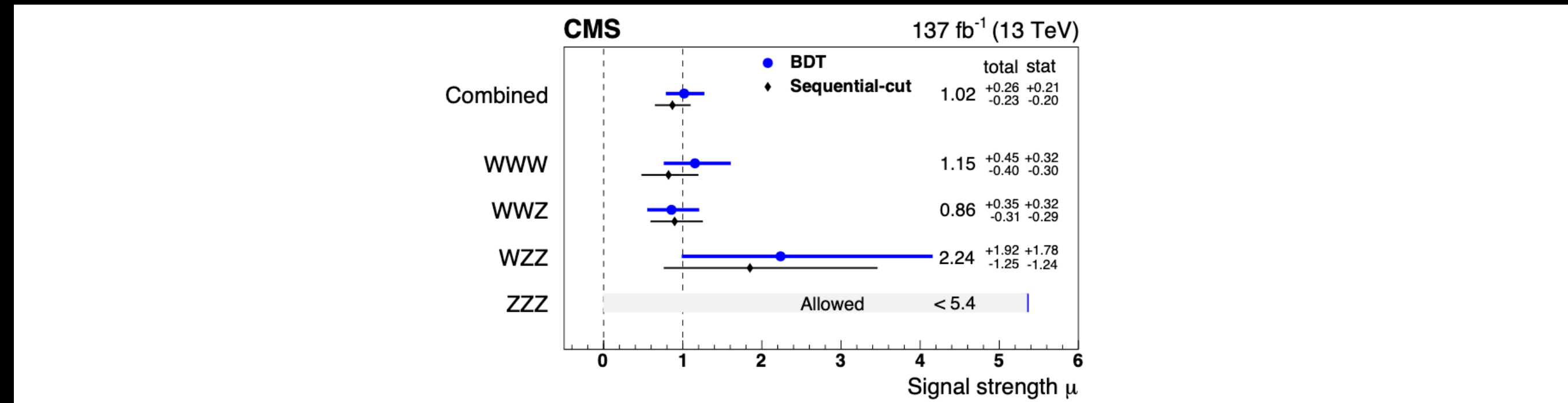
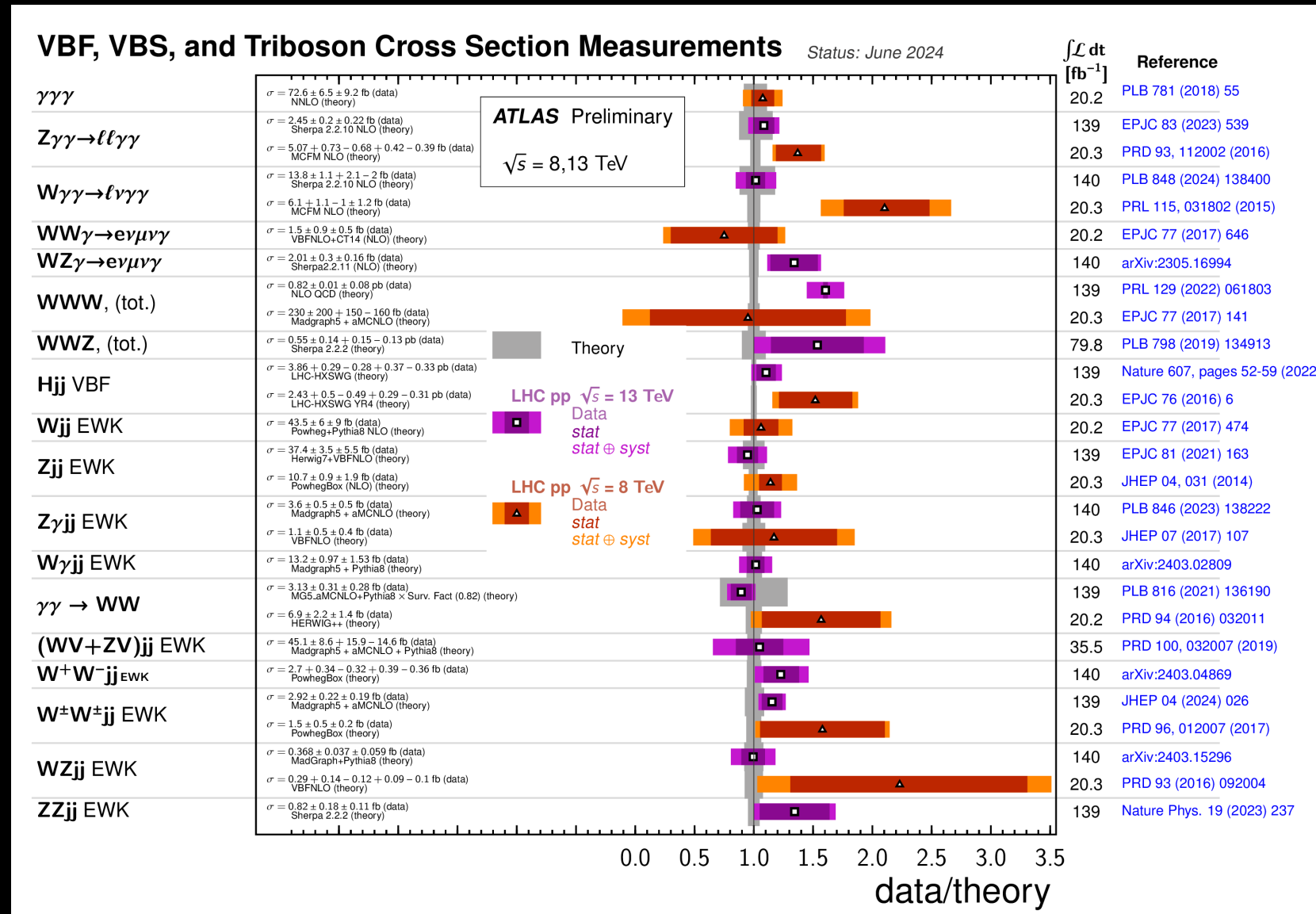
W + 5 jets · *Phys. Rev. D* 88 (2013) 014025 · [1304.1253]

This can be our inspiration.

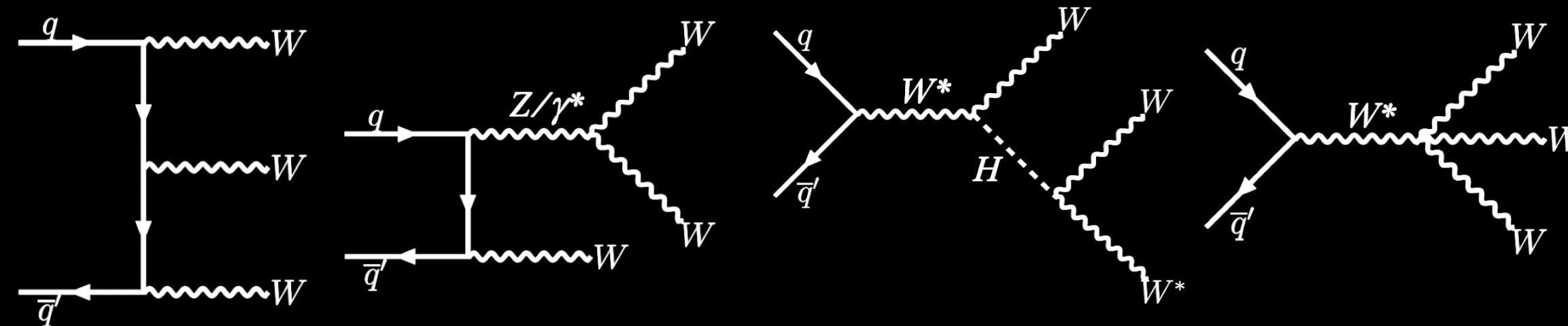


We need a framework that efficiently handles the complexity of perturbation theory.

“Rare” 2 → 3 LHC processes



HL-LHC forecast — triboson (VVV) is rare, statistically limited today: WWW to ~12 % (stat., ATLAS 139 fb⁻¹); the full VVV combination to ~25 % (CMS). With ≈ 6 ab⁻¹ (ATLAS \oplus CMS) the statistical uncertainty ($\propto 1/\sqrt{L}$) shrinks $\approx \sqrt{(6000/139)} \approx 7\times$ — toward the percent level. Precision then demands NNLO.



$$\sigma [pp \rightarrow VVV] \quad V = W, Z$$

The 2 \rightarrow 3 NNLO frontier

Two-loop leading-color QCD corrections for Higgs plus two-jet production in the heavy-top limit

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Vasily Sotnikov⁵

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²Paul Scherrer Institut, CH-5232 Villigen PSI, Switzerland

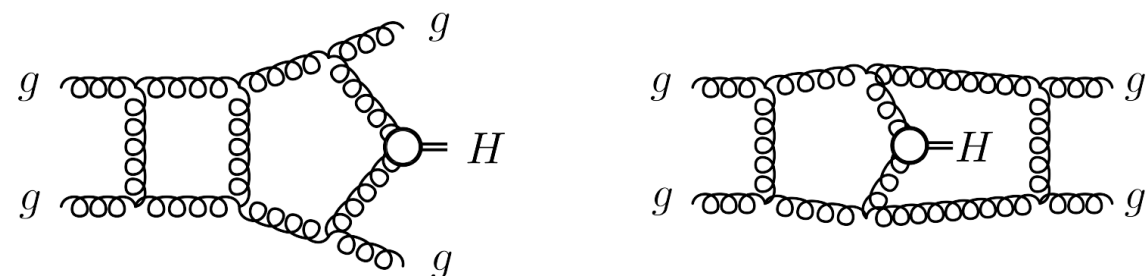
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vasily.sotnikov@physik.uzh.ch

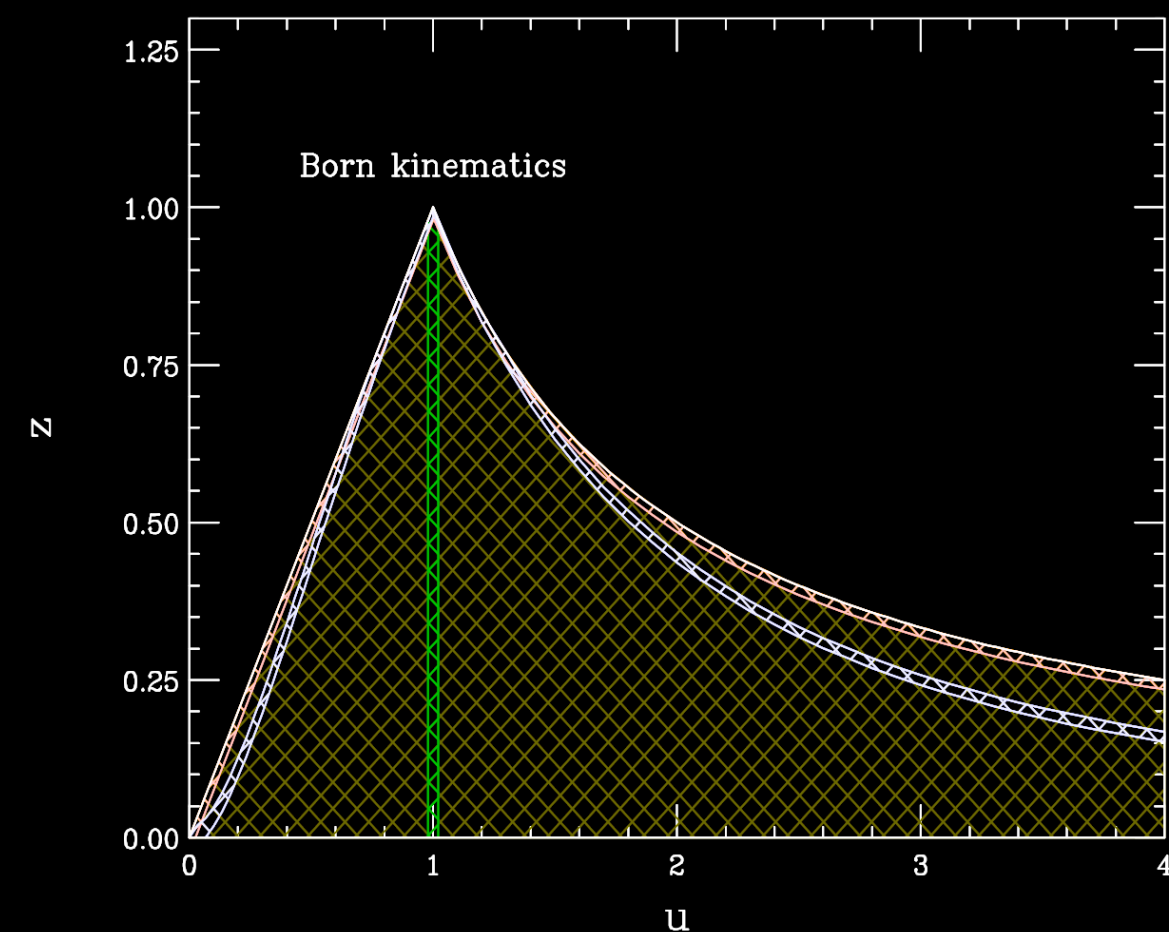
states contribute.



Channel	$H^{(1)[0]}$	$H^{(1)[1]}$	$H^{(2)[0]}$	$H^{(2)[1]}$	$H^{(2)[2]}$
$gg \rightarrow ggH$	21.60693008	-0.2042948925	758.1516119	-59.89542405	0.1057461955
$gg \rightarrow u\bar{u}H$	23.79850997	-0.8520056545	871.0125340	-73.34374616	-0.02463963835
$gu \rightarrow guH$	35.75432424	-1.667934932	1346.890607	-146.8303568	2.080699995
$\bar{u}u \rightarrow ggH$	-0.7439021475	-0.4143437544	323.4456429	-16.17511090	-1.000952210
$\bar{u}u \rightarrow d\bar{d}H$	13.52201772	-1.387398374	500.2142706	-45.24344286	-0.5366784223
$du \rightarrow duH$	65.11323369	-3.575719056	3180.810875	-369.2573793	8.083011923
$\bar{d}u \rightarrow \bar{d}uH$	38.89392891	-3.575719056	1532.625650	-254.1536590	8.083011923
$\bar{u}u \rightarrow u\bar{u}H$	31.68055000	-1.355979469	1049.884750	-103.8069713	0.7504319454
$uu \rightarrow uuH$	64.67929567	-3.497230387	3141.865903	-362.0898486	7.842910126

Table 3: Reference evaluations of hard functions for the different $pp \rightarrow Hjj$ partonic channels.

Side remark: spurious singularities in analytic computations.



- E.g., inside the region for the Drell-Yan NNLO rapidity distribution, individual terms in the hard functions blow — spurious power-law singularities (several powers of $1/(1-z)$)
- Extra singular strips beyond the edges: $u \rightarrow 1$, and the curves $z = [2u/(1+u)]^2$ and $z = [2/(1+u)]^2$
- The full polylogarithmic combination stays finite — the singularities cancel
- Lance built analytic “patching” functions for each strip to control the roundoff (Fig. 2, hep-ph/0312266)
- A much more tedious job for more complicated kinematics. Requires excellent understanding of all (physical and unphysical) singularities.

Two loop gauge theory amplitudes with direct integration

- Two-loop amplitudes with direct integration over loop momenta?
- Number of integrals is SIX.
- ... for all two-loop amplitudes and kinematic configurations.
- Understand fully the singular structure of QCD amplitudes at two loops.

$$A_2 \left(\{p_{ext_i}\}, \{M_i\} \right) = \int d^d k \int d^d l \mathcal{A}_2 \left(k, l, \{p_{ext_i}\}, \{M_i\} \right)$$

Monte-Carlo Integration?

$d \longrightarrow 4?$

Singularities

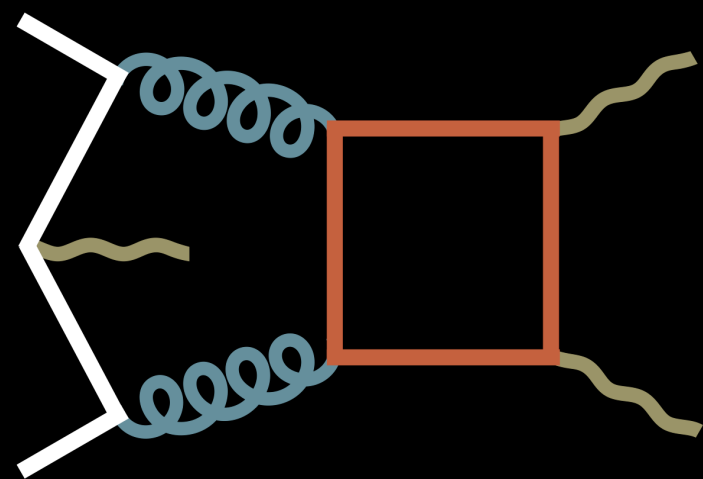
Direct numerical integration of loop momenta

A direct integration in momentum-space already reaches these two-loop amplitudes (and perhaps more general two-loop QCD). Dario Kermanschah & Matilde Vicini, with a single numerical-integration pipeline, a sweep of results for the most challenging fermion loop contributions — many for the first time:

Kermanschah & Vicini — diphoton / triphoton: JHEP 03 (2026) 004 [2510.18801];

$Z\gamma$ heavy-quark loops: [2603.03169]; axial triangles in 4D: [2603.03171];

NF at NNLO: JHEP 09 (2025) 213 [2407.18051].



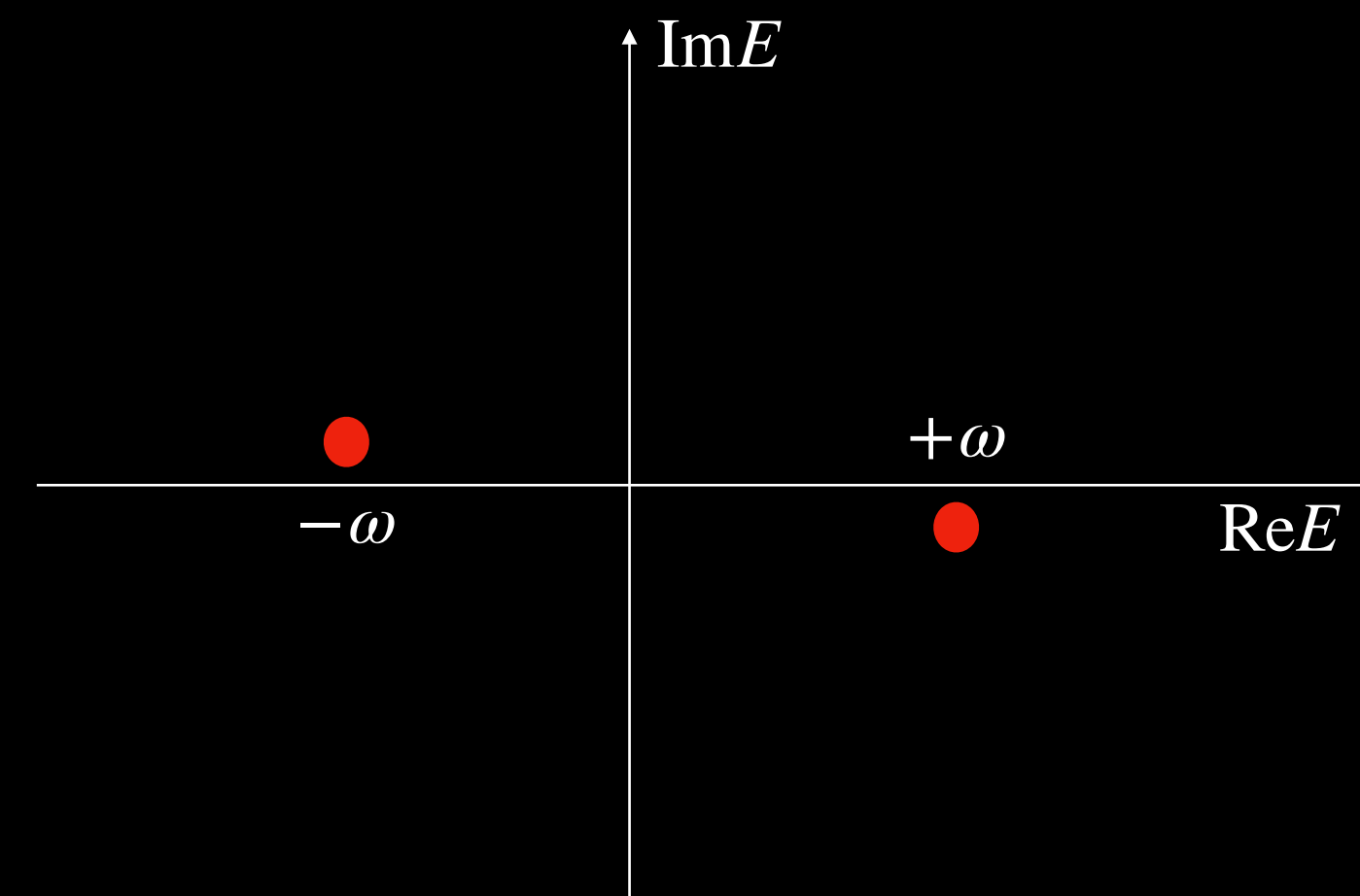
- No known analytic results, or master integrals.
- Seamless combined integration of loop and phase-space integrations.
- Process universality

Process	PSP	Exp.	Reference	Result	Δ [σ]	Δ [%]
$q\bar{q} \rightarrow \gamma\gamma$ $m = 0$	1	10^{+2}	-7.2127	-7.2574 ± 0.0458	0.976	0.631
	2	10^{+2}	-6.7813	-6.7649 ± 0.0421	0.391	0.622
	3	10^{+2}	-8.4533	-8.5154 ± 0.0558	1.113	0.655
$q\bar{q} \rightarrow \gamma^*\gamma^*$ $m = 0$	1	10^{+2}	-4.6281	-4.5791 ± 0.0347	1.411	0.758
	2	10^{+2}	-4.5407	-4.5137 ± 0.0352	0.768	0.780
	3	10^{+2}	-4.3357	-4.2968 ± 0.0339	1.149	0.788
$q\bar{q} \rightarrow \gamma^*\gamma_2^*$ $m = 0$	1	10^{+2}	-5.2774	-5.2742 ± 0.0087	0.363	0.164
	2	10^{+2}	-5.1052	-5.0972 ± 0.0087	0.913	0.170
	3	10^{+2}	-5.3523	-5.3433 ± 0.0110	0.821	0.206
$q\bar{q} \rightarrow \gamma\gamma\gamma$ $m = 0$	1	10^{-1}	-3.3918	-3.4066 ± 0.0159	0.929	0.467
	2	10^{+0}	-1.4313	-1.4335 ± 0.0101	0.217	0.707
	3	10^{-1}	-3.3987	-3.3930 ± 0.0167	0.341	0.493
$q\bar{q} \rightarrow \gamma^*\gamma^*\gamma^*$ $m = 0$	1	10^{-1}		-2.1869 ± 0.0209		0.956
	2	10^{-1}		-4.8302 ± 0.0481		0.996
	3	10^{-1}		-2.1826 ± 0.0208		0.951
$q\bar{q} \rightarrow \gamma\gamma$ $m > 0$	1	10^{+0}	4.0175	4.0709 ± 0.0358	1.491	0.879
	2	10^{+1}	-4.9939	-4.9853 ± 0.0098	0.883	0.196
	3	10^{+2}	-2.8525	-2.8432 ± 0.0075	1.236	0.264
$q\bar{q} \rightarrow \gamma^*\gamma^*$ $m > 0$	1	10^{+0}		4.0862 ± 0.0392		0.959
	2	10^{+1}		-5.3944 ± 0.0185		0.343
	3	10^{+2}		-2.9477 ± 0.0118		0.401
$q\bar{q} \rightarrow \gamma\gamma\gamma$ $m > 0$	1	10^{-2}		-5.5712 ± 0.0538		0.966
	2	10^{-1}		-3.6549 ± 0.0312		0.854
	3	10^{-2}		-3.5927 ± 0.0334		0.931

Singularities of Feynman diagrams and scattering amplitudes

$$\int_{-\infty}^{\infty} dE \dots \frac{\dots}{E^2 - \omega^2 + i\delta} = \int_{-\infty}^{\infty} dE \dots \frac{\dots}{\omega} \left(\frac{1}{E - \omega + i\delta} - \frac{1}{E + \omega - i\delta} \right)$$

- The poles can lie inside the domain of integration.

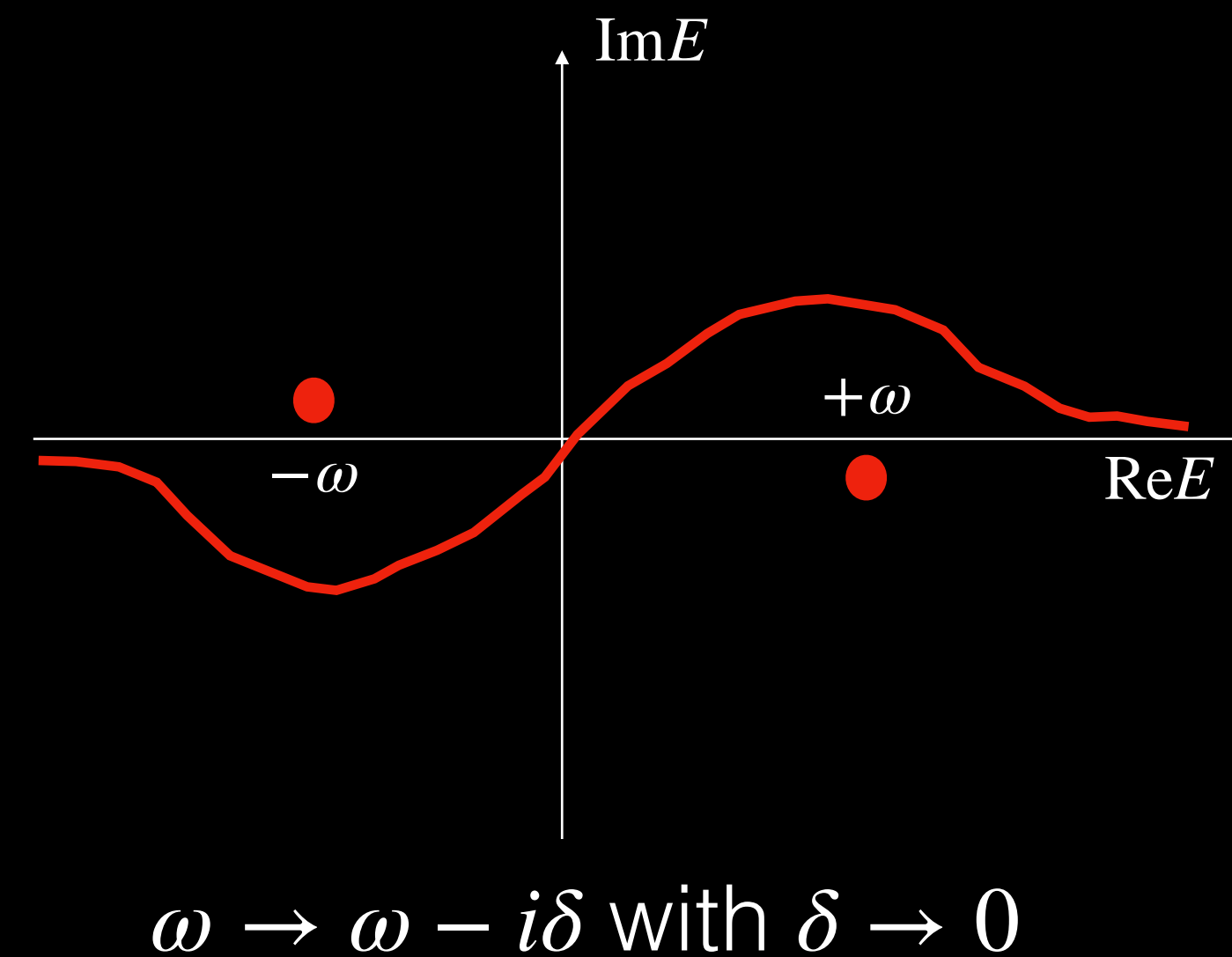


$\omega \rightarrow \omega - i\delta$ with $\delta \rightarrow 0$

Integrable Singularities

$$\int_{-\infty}^{\infty} dE \dots \frac{\dots}{E^2 - \omega^2 + i\delta} = \int_{-\infty}^{\infty} dE \dots \frac{\dots}{\omega} \left(\frac{1}{E - \omega + i\delta} - \frac{1}{E + \omega - i\delta} \right)$$

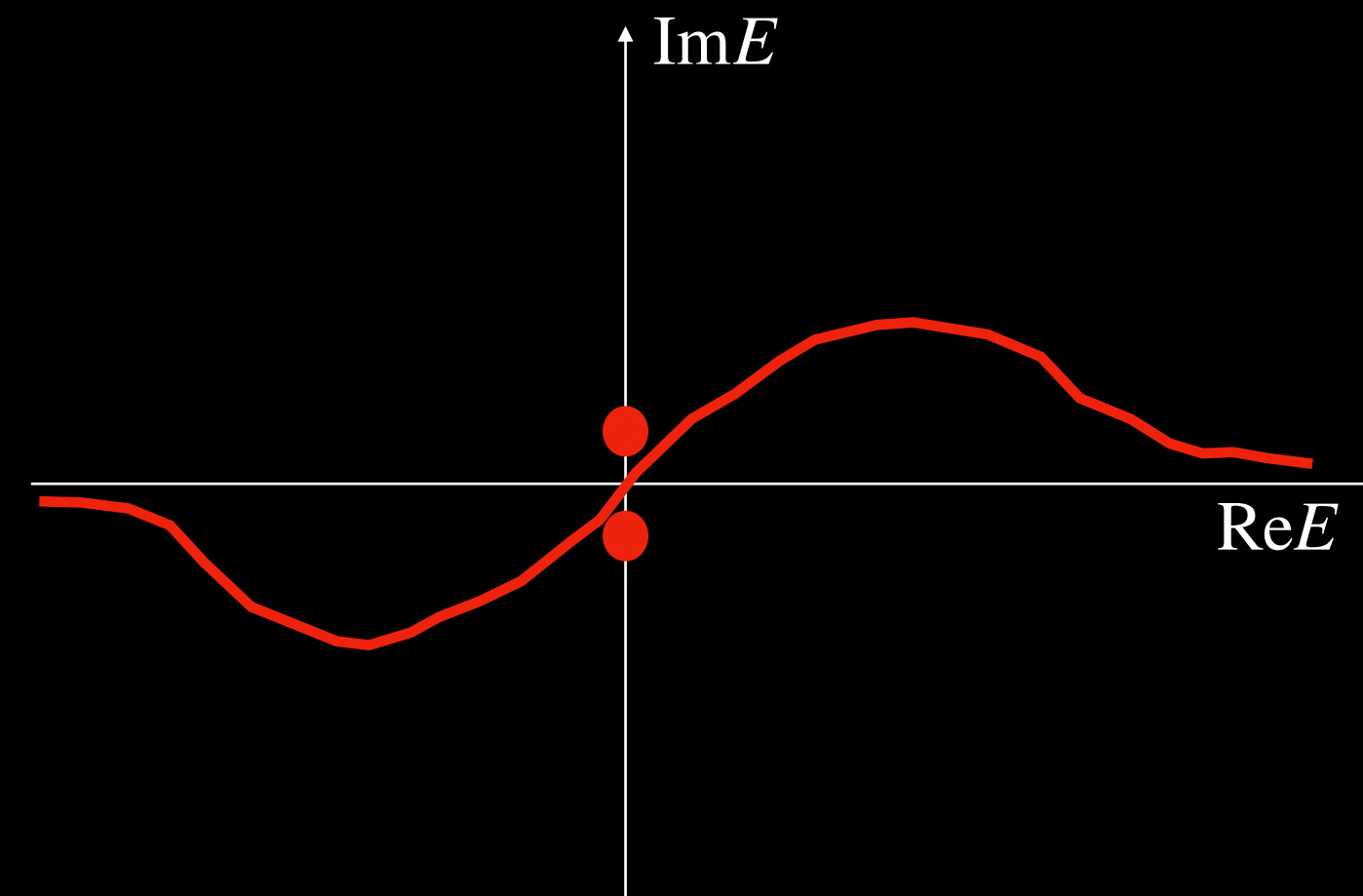
- The poles can lie inside the domain of integration.
- If we can deform the path of integration away from the poles, then they lead to no singularities



Soft massless particles

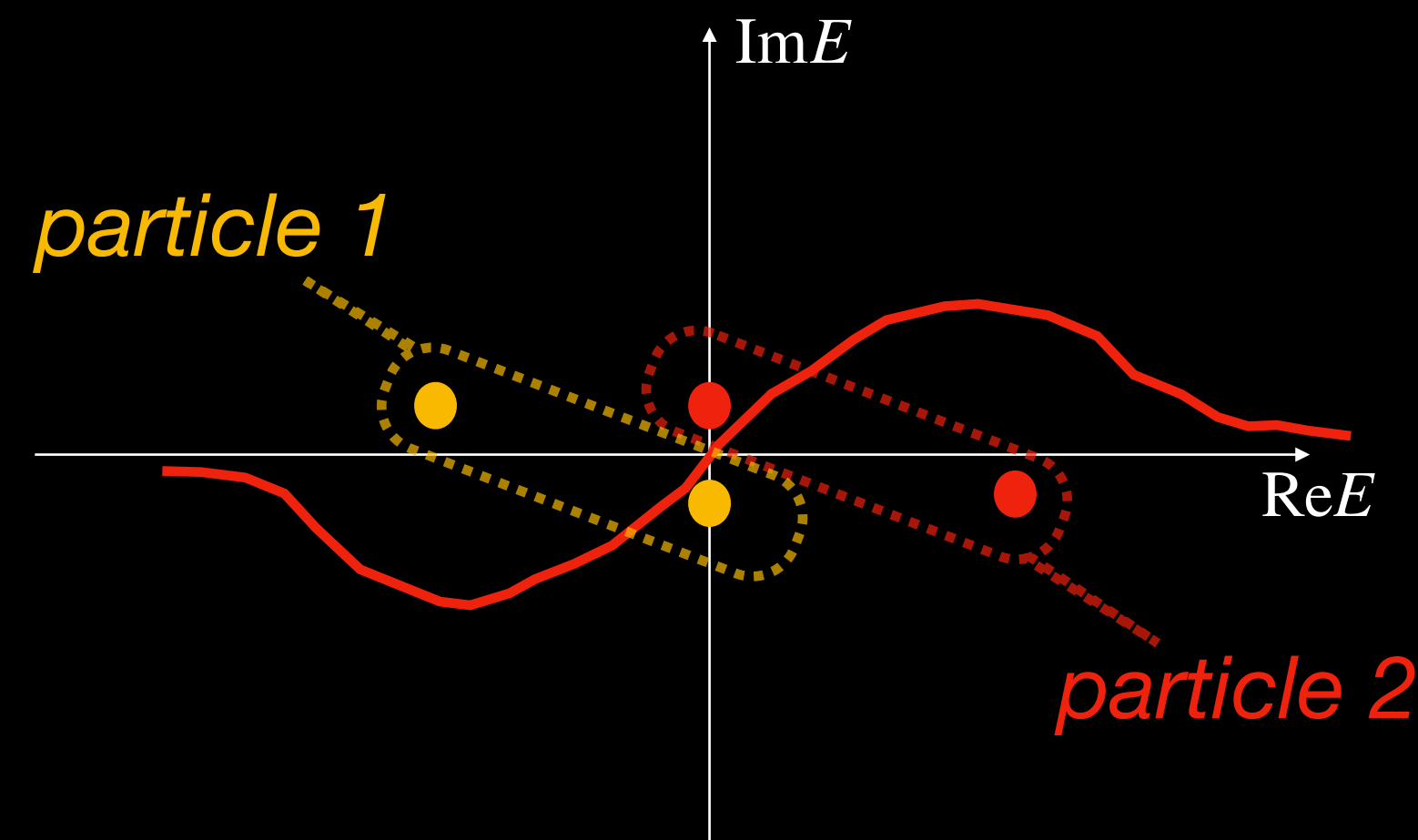
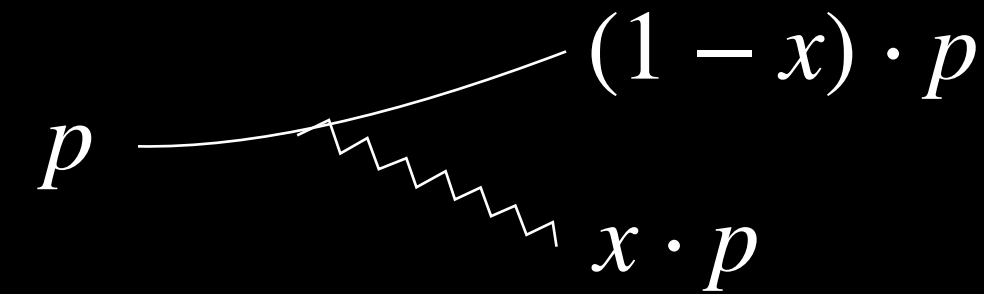
$$\int_{-\infty}^{\infty} dE \dots \frac{\dots}{(E + i\delta)(E - i\delta)}$$

- Poles due to soft massless particles.
- These singularities pinch the integration path from both sides.
- Condition for a TRUE INFINITY



Collinear massless particles

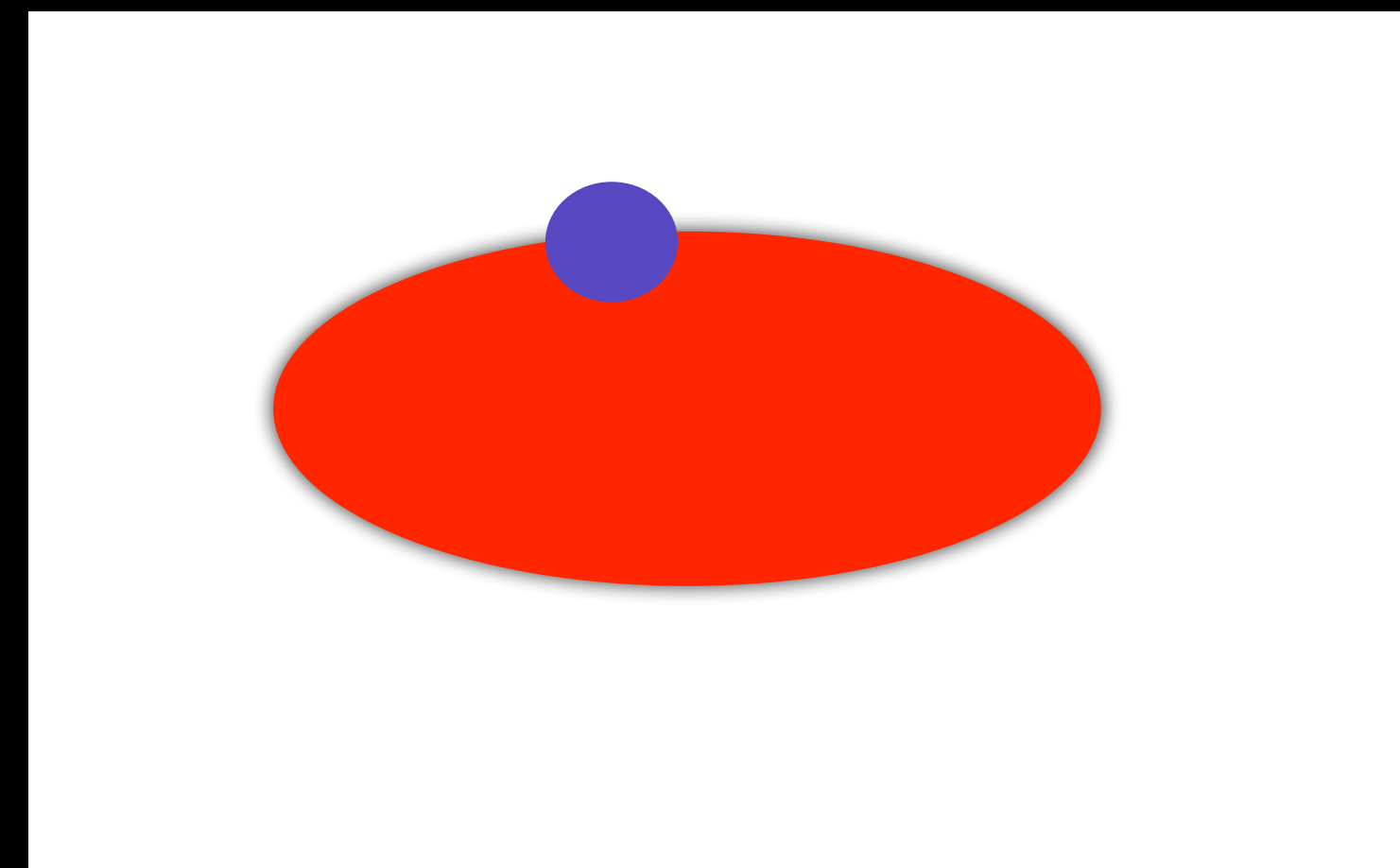
- A second source of infinities due to massless collinear particles.
- A singularity of one particle in the lower half-plane lines up with the singularity of a collinear particle in the higher half-plane.
- The singularities pinch the integration path from both sides.
- We cannot deform the path, a condition for a TRUE INFINITY!



Nested subtractions

- Singular regions are interconnected. How can we create systematically an approximation of the loop integrals in all singular regions?
- Order the singular regions by their “volume”

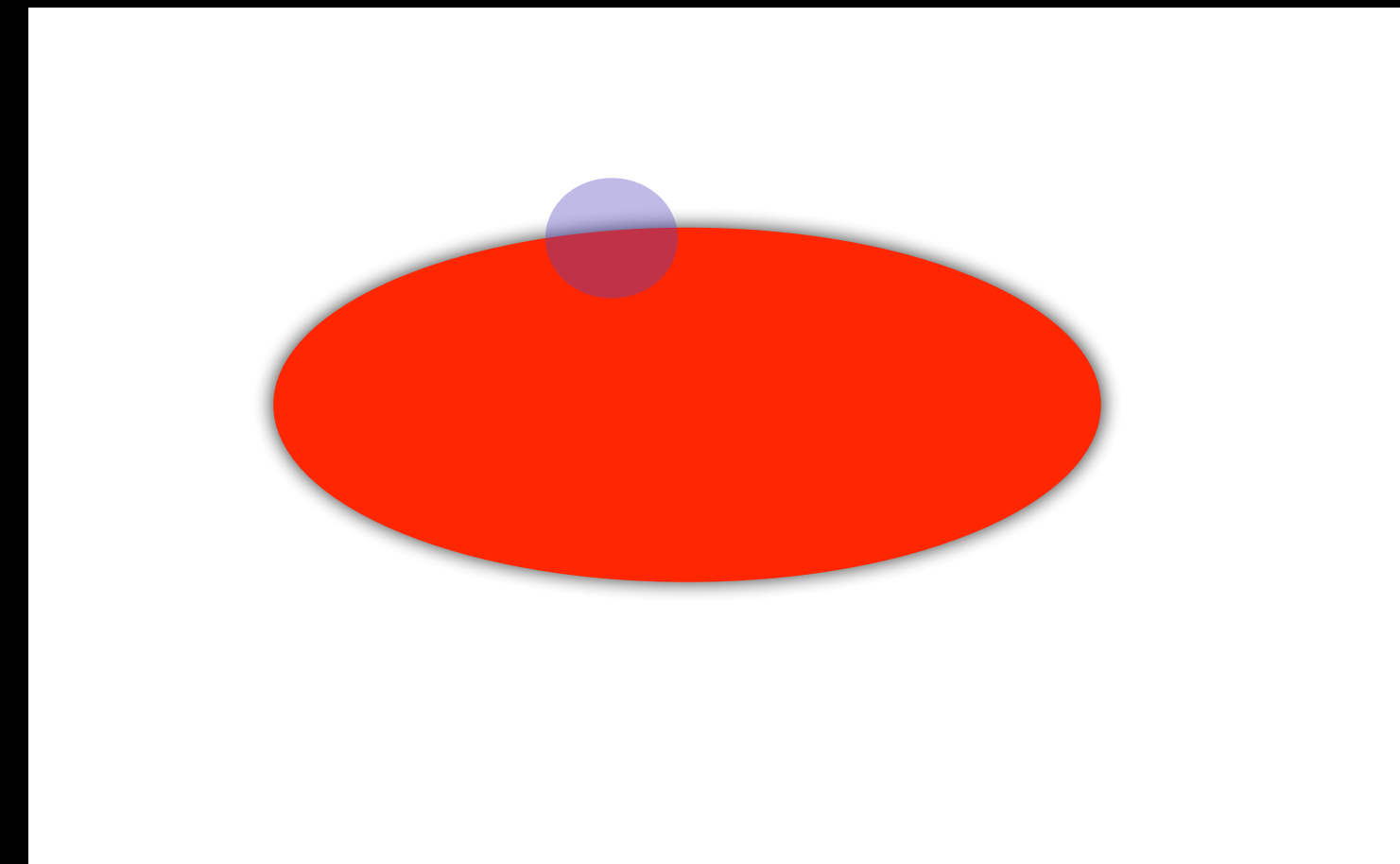
Ma; Erdogan, Sterman; Collins;
Collins, Soper, Sterman



Nested subtractions

- Singular regions are interconnected. How can we create systematically an approximation of the loop integrals in all singular regions?
- Order the singular regions by their “volume”
- Subtract an approximation of the integrand in the smallest volume
-

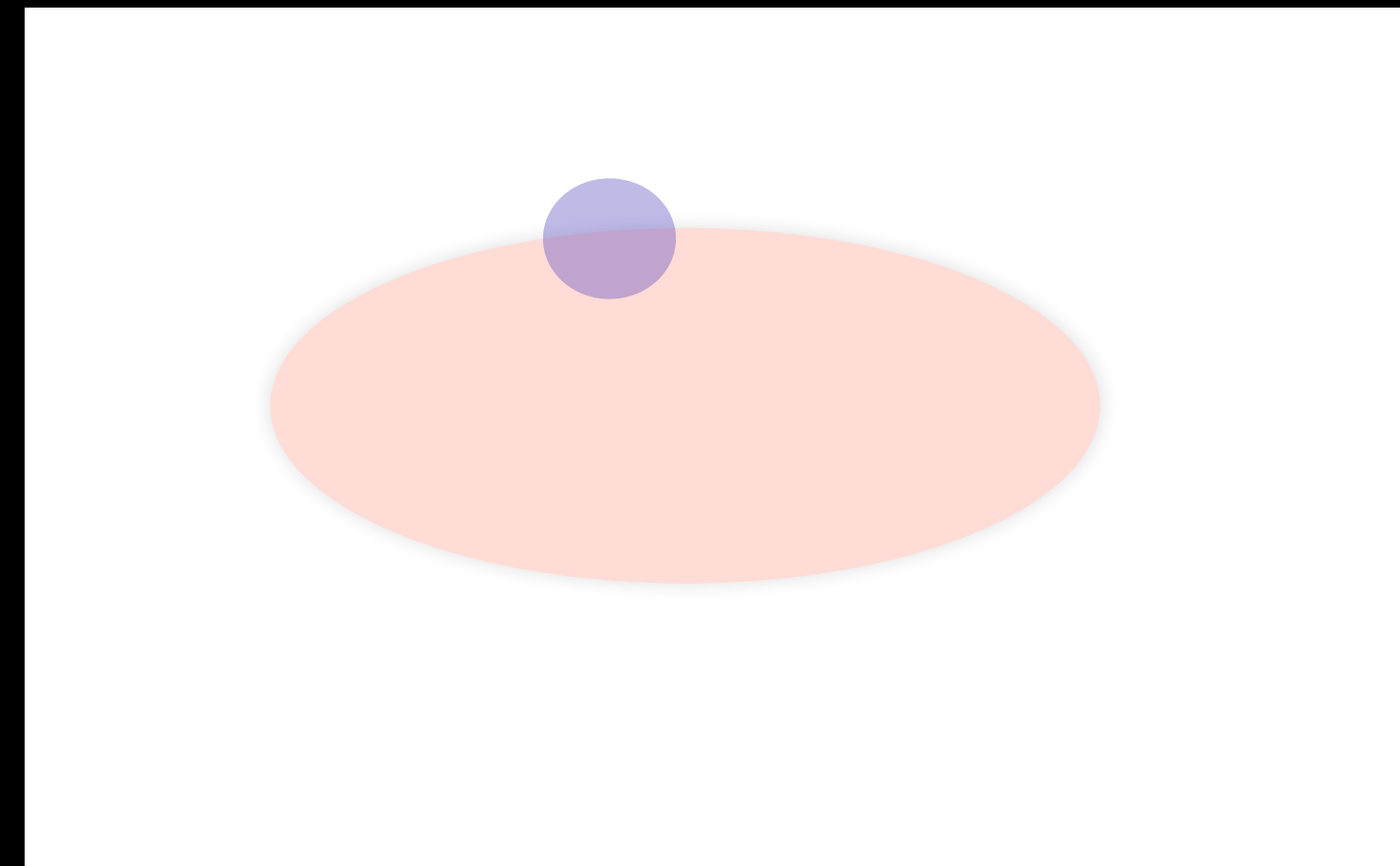
Ma; Erdogan, Sterman; Collins;
Collins, Soper, Sterman



Nested subtractions

- Singular regions are interconnected. How can we create systematically an approximation of the loop integrals in all singular regions?
- Order the singular regions by their “volume”
- Subtract an approximation of the integrand in the smallest volume
- Then, proceed to the next volume and repeat until there are no more singularities to remove.

Ma; Erdogan, Sterman; Collins;
Collins, Soper, Sterman

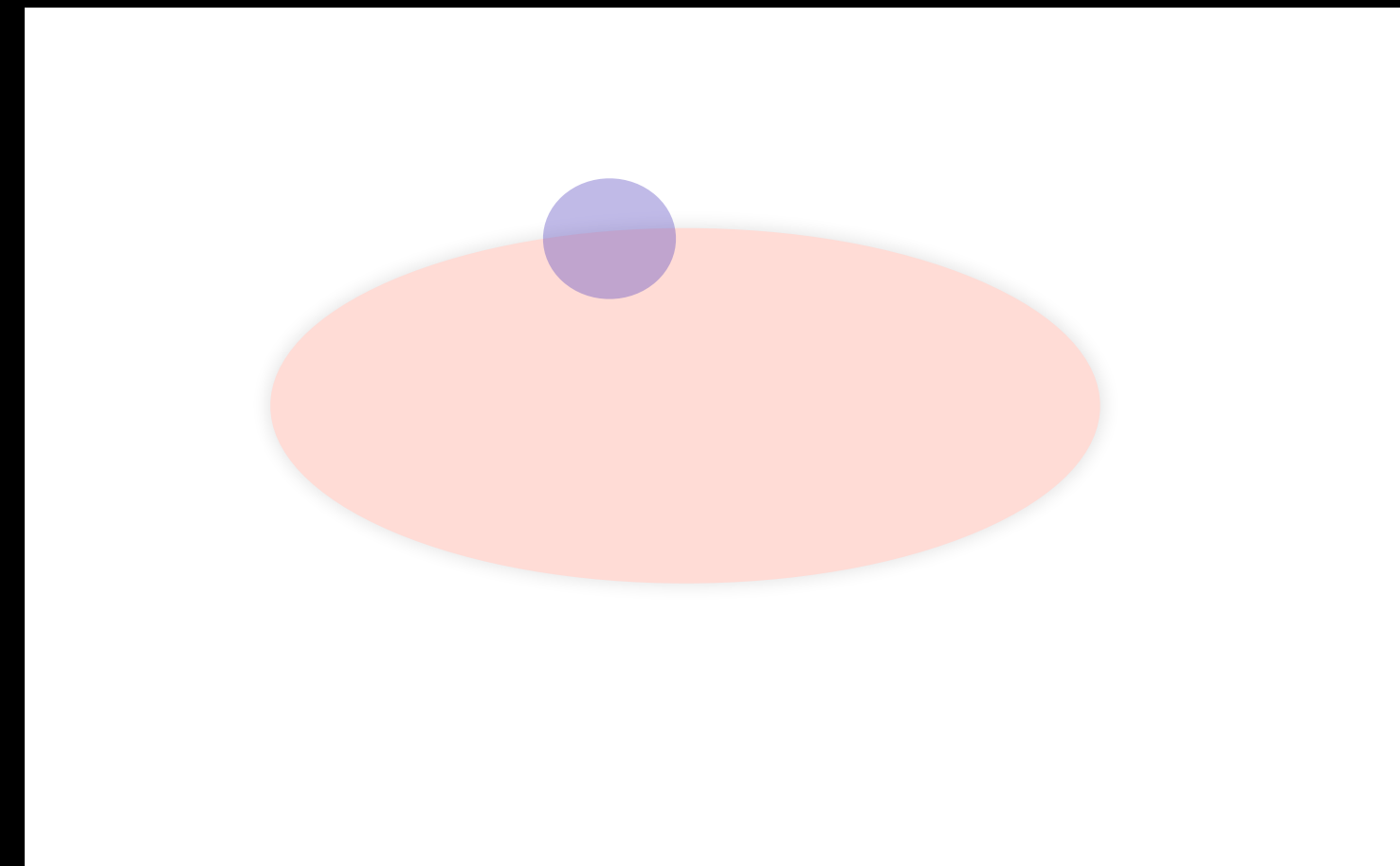


Nested subtractions

Ma; Erdogan, Sterman; Collins;
Collins, Soper, Sterman

$$R^{(n)} \gamma^{(n)} = \gamma^{(n)} + \sum_{N \in \mathcal{N}[\gamma^{(n)}]} \prod_{\rho \in N} (-t_\rho) \gamma^{(n)},$$

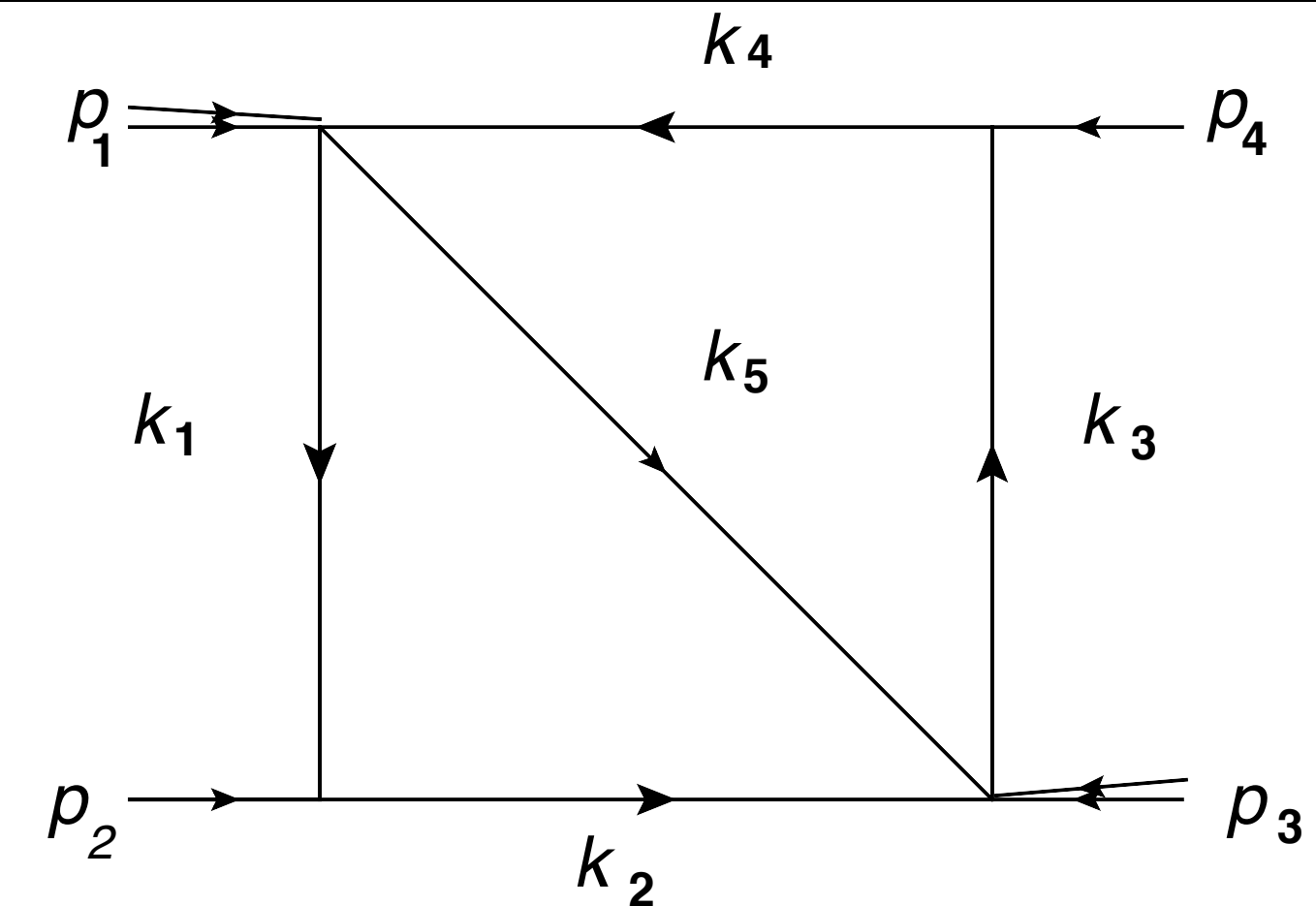
- The procedure of nested subtractions has a solution for the finite remainder at any loop order as a Forest formula (similarly to BPHZ of UV renormalization)



Constructing finite two-loop integrals

- The method of nested subtractions guarantees that we can always remove the infrared singularities
- ... of ANY integral at ANY order in perturbation theory.
- Subtractions can be made to take a simple form.
- Method demonstrated with examples at two-loops

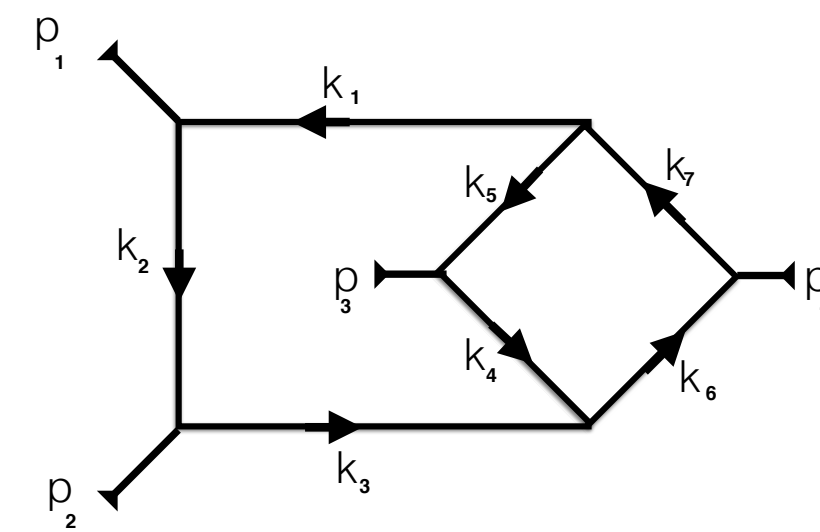
CA, G. Sterman *JHEP* 07 (2019) 056



$$\begin{aligned}
 D_{\text{box}}|_{\text{fin}} = & \int \frac{d^d k_1}{i\pi^{\frac{d}{2}}} \frac{d^d k_4}{i\pi^{\frac{d}{2}}} \left\{ \frac{1}{A_1 A_2 A_3 A_4 A_5} \right. \\
 & - \left[\frac{1}{A_1 A_2} - \frac{1}{(A_1 - \mu^2)(A_2 - \mu^2)} \right] \left[\frac{1}{A_3 A_4 A_5} \right]_{k_1 = -x_2 p_2} \\
 & - \left[\frac{1}{A_3 A_4} - \frac{1}{(A_4 - \mu^2)(A_3 - \mu^2)} \right] \left[\frac{1}{A_1 A_2 A_5} \right]_{k_4 = x_4 p_4} \\
 & \left. + \left[\frac{1}{A_1 A_2} - \frac{1}{(A_1 - \mu^2)(A_2 - \mu^2)} \right] \left[\frac{1}{A_5} \right]_{\substack{k_4 = x_4 p_4, \\ k_1 = -x_2 p_2}} \left[\frac{1}{A_3 A_4} - \frac{1}{(A_3 - \mu^2)(A_4 - \mu^2)} \right] \right\}
 \end{aligned}$$

Constructing finite two-loop integrals

- There is a lot of value in constructing finite integrands for Feynman integrals.
- And simplifications or elegance.
- Subtractions can be made to take a simple form.
- For example, the full two-loop crossed box has a mixed transcendentally. But the integration over our finite remainder gives uniform weight four...



$$s^3 X_{\text{box}}^{\text{fin}} = \frac{f_{X_{\text{box}}}(y)}{y} + \frac{f_{X_{\text{box}}}(1-y)}{1-y},$$

$$f_{X_{\text{box}}}(y) = [G_R(y) + i\pi G_I(y)] \log\left(\frac{\mu^2}{s}\right) + E_R(y) + i\pi E_I(y)$$

$$\begin{aligned} E_R(y) = & -8\pi^2 \text{Li}_2(y) + 8 \text{Li}_2(y) \log(1-y)^2 - 28 \log(y) \text{Li}_2(y) \log(1-y) - 18 \text{Li}_2(y) \log(y)^2 \\ & + 44 \text{Li}_3(y) \log(1-y) + 96 \text{Li}_3(y) \log(y) - 188 \text{Li}_4(y) + \frac{17}{36} \pi^4 + \frac{1}{12} \log(1-y)^4 \\ & + 7 \log(y) \log(1-y) \pi^2 - \frac{25}{6} \pi^2 \log(1-y)^2 - \frac{3}{2} \log(y)^2 \pi^2 + \log(y) \log(1-y)^3 \\ & + 44 S_{12}(y) \log(1-y) - 52 S_{12}(y) \log(y) + 84 S_{13}(y) + 88 S_{22}(y) - 44 \zeta_3 \log(1-y) \\ & - 4 \log(y) \zeta_3 - \frac{1}{4} \log(y)^4 + \log(y)^3 \log(1-y) - \frac{9}{2} \log(y)^2 \log(1-y)^2, \end{aligned}$$

QCD Amplitudes are SIMPLER than Feynman integrals

- In singular IR regions of loop momenta, virtual gauge bosons acquire (unphysical) longitudinal polarizations.
- The propagation of unphysical degrees of freedom is “prohibited” by Ward identities.
- Cancellations of IR singularities among Feynman diagrams.

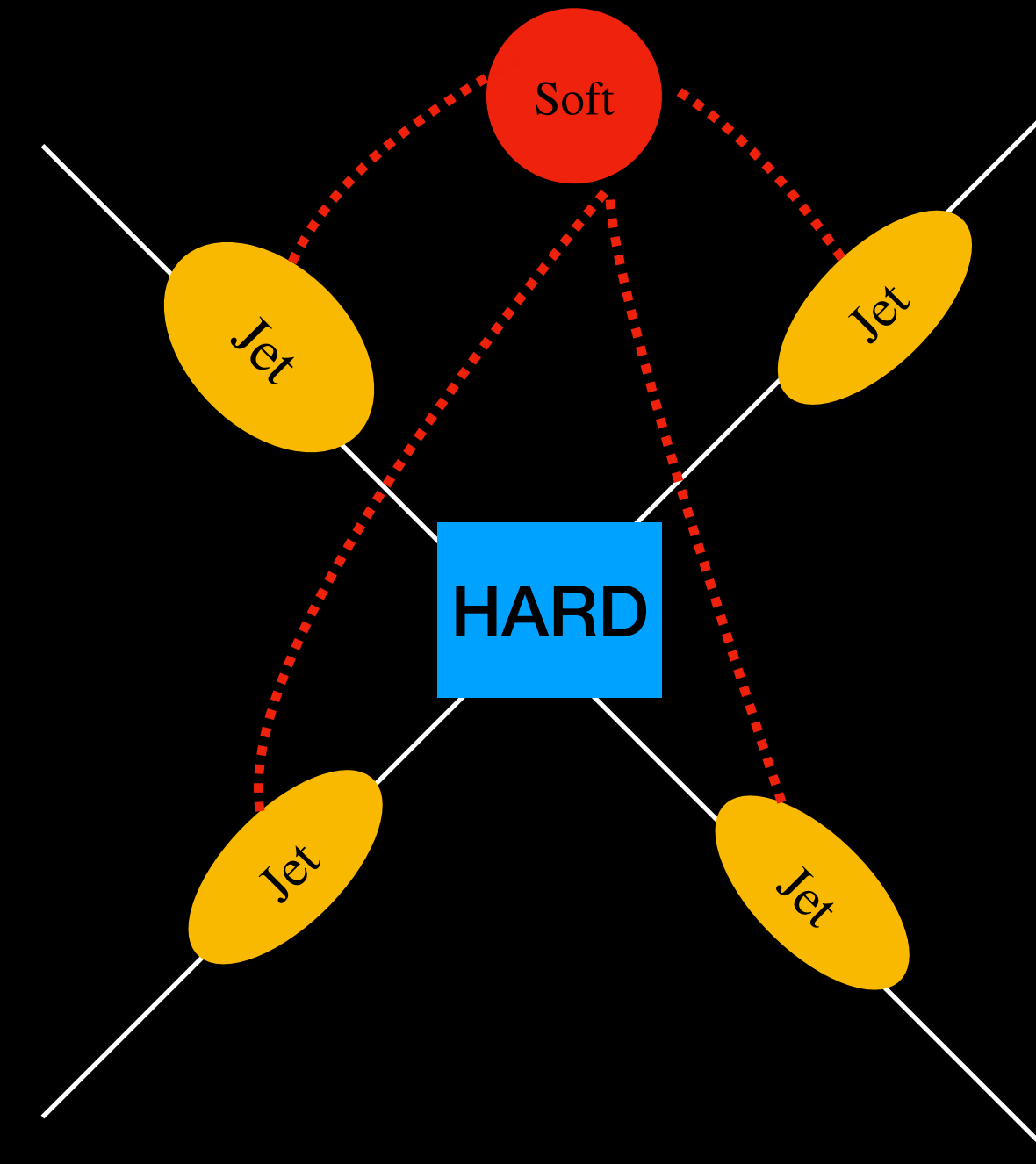
Infrared amplitude factorization

- UV Renormalized scattering amplitudes for well-separated final-states take a simple factorized form

$$Amplitude = \text{hard} \cdot \text{soft} \cdot \prod_i \text{Jet}_i.$$

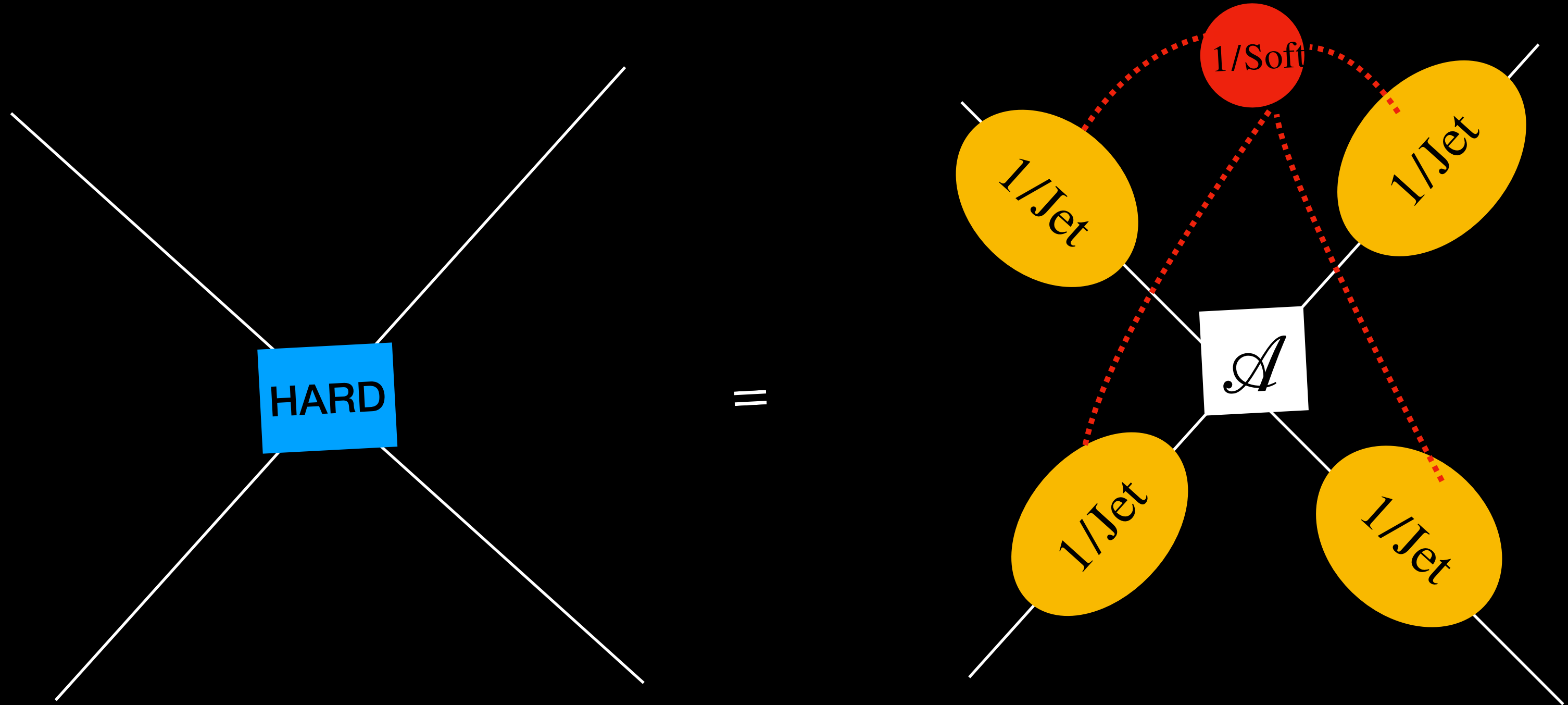
- “soft” and “jet” functions contain all divergences.

- These are universal functions. For any new process we should need to compute only the “hard” function.
- So far, we do not have a way to compute the “hard” function directly.
- But, what if we did?



*Ma;
Erdogan, Sterman; Feige,
Schwartz; Collins*

How could we imagine using factorisation?



An inverted factorization theorem

How could we imagine using factorization?

$$A = \int [dk] \mathcal{A}(k) = \int \mathcal{S} \prod_i \mathcal{I}_i \cdot \int [dk] \mathcal{A}(k) \cdot \mathcal{S}^{-1}(k) \cdot \prod_i \mathcal{I}_i^{-1}(k)$$

soft/collinear
Hard

Divergent
Finite

Analytic Integration in $D = 4 - 2\epsilon$,
known to at least three-loops

Numerical integration in
exactly $D = 4$.

Universal

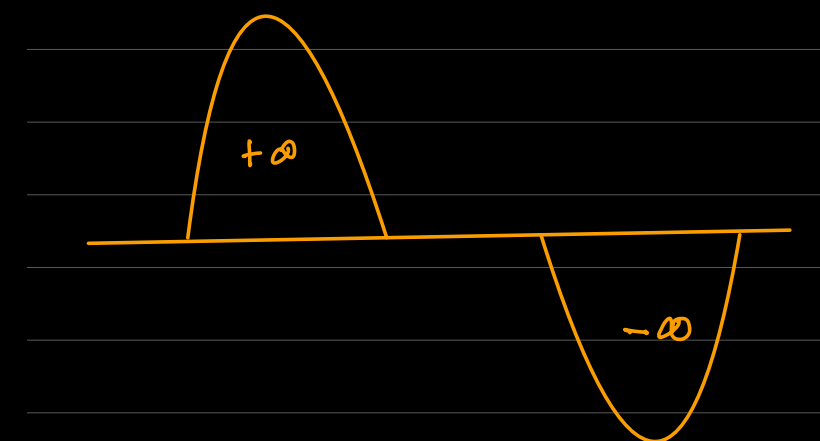
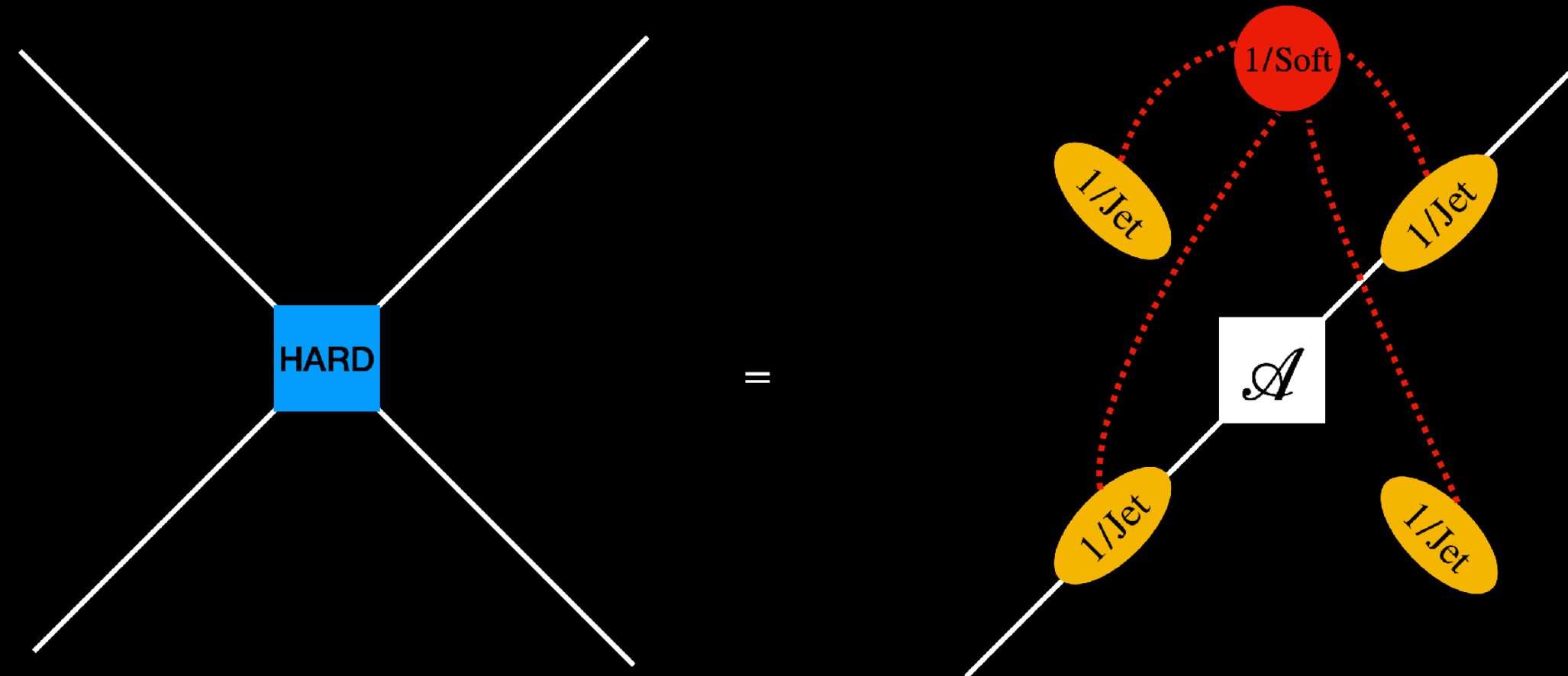
Process dependent

This procedure is universal...could be applied to any process, irrespectively of the complexity of its final state.

From factorisation we could identify, remove and integrate separately the singular parts of amplitudes order by order in perturbation theory:

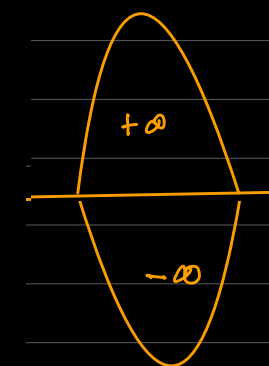
$$\mathcal{H}^{(0)} = \mathcal{A}^{(0)} \quad \mathcal{H}^{(1)} = \mathcal{A}^{(1)} - \mathcal{I}^{(1)}\mathcal{H}^{(0)} - \mathcal{S}^{(1)}\mathcal{H}^{(0)} \quad \mathcal{H}^{(2)} = \mathcal{A}^{(2)} - \mathcal{I}^{(1)}\mathcal{H}^{(1)} - \mathcal{S}^{(1)}\mathcal{H}^{(1)} - \mathcal{I}^{(2)}\mathcal{H}^{(0)} - \mathcal{S}^{(2)}\mathcal{H}^{(0)} + \mathcal{I}^{(1)}\mathcal{S}^{(1)}\mathcal{H}^{(0)} \quad \dots$$

Factorisation and integrands



Non-local cancellations

$$\int_0^{10} dx \left[\frac{1}{x-3+i0^+} - \frac{1}{x-7+i0^+} \right]$$



Local cancellations
Numerically integrable

$$\int_0^{10} dx \left[\frac{e^{-\frac{1}{(x-3)^2}}}{x-3+i0^+} - \frac{\cos(x-3)}{x-3+i0^+} \right]$$

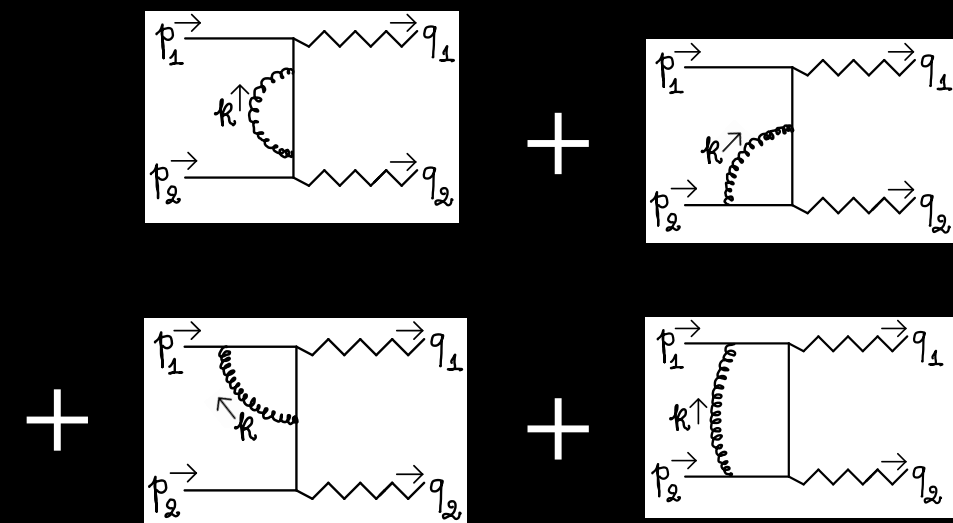
- In the integral expression of the process dependent “HARD” function, we need singularities to be cancelled locally, AT THE INTEGRAND.
- A naive construction leads to non-local cancellations.
- Integrands with non-local cancellations cannot be integrated numerically.
- To enable Monte-Carlo integration methods, can we ensure that ALL soft, collinear and ultraviolet singularities cancel point by point in the integrand?
- A challenge!

Locally finite integrands for electroweak production in quark annihilation

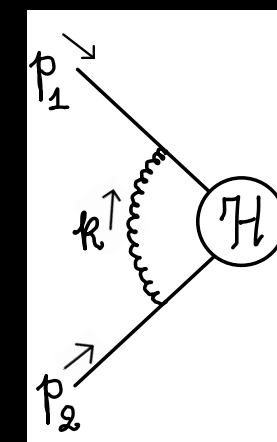
$$q + \bar{q} \rightarrow V_1 + V_2 + \dots V_n, \quad V_i = W, Z, \gamma^*$$

$$\mathcal{H}_{q\bar{q} \rightarrow ew}^{(1),R}(k) = \mathcal{M}_{q\bar{q} \rightarrow ew}^{(1),R}(k) - \mathcal{F}_{q\bar{q}}^{(1),R}(k) \left[\mathbf{P}_1 \widetilde{\mathcal{M}}_{q\bar{q} \rightarrow ew}^{(0)} \mathbf{P}_1 \right]$$

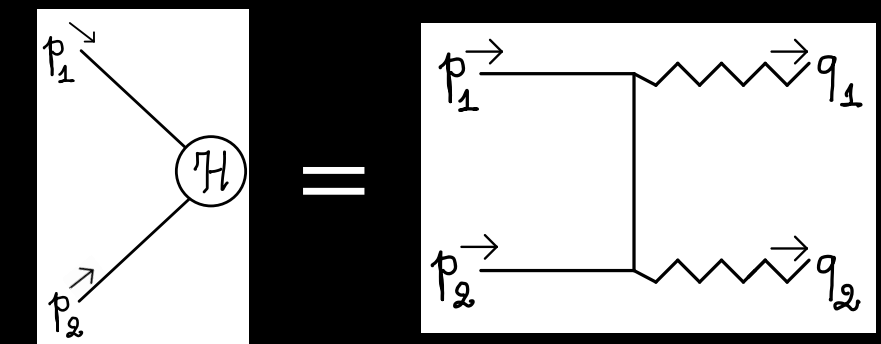
- Finite one-loop amplitude integrand
- Free of all IR and UV singularities locally
- Integrable in D=4 exactly



- One-loop amplitude integrand
- As derived with Feynman rules
- Momentum flow assignment



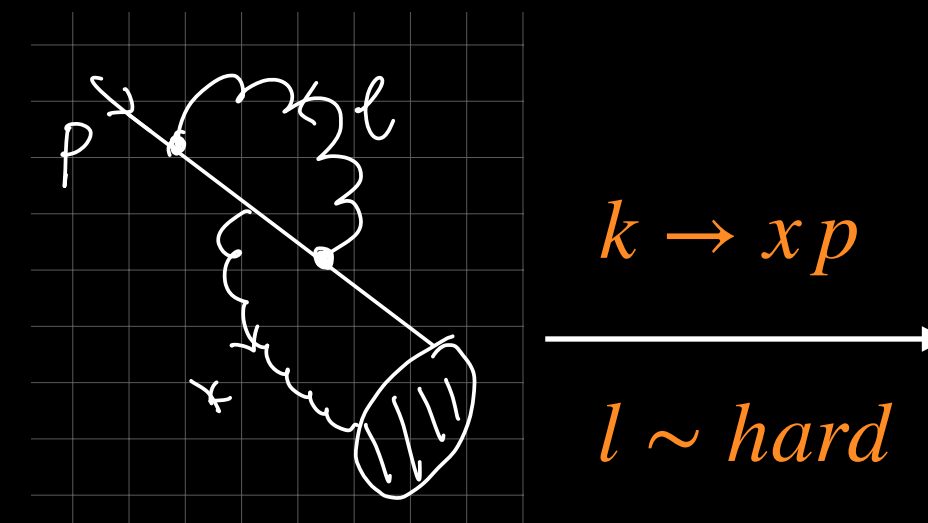
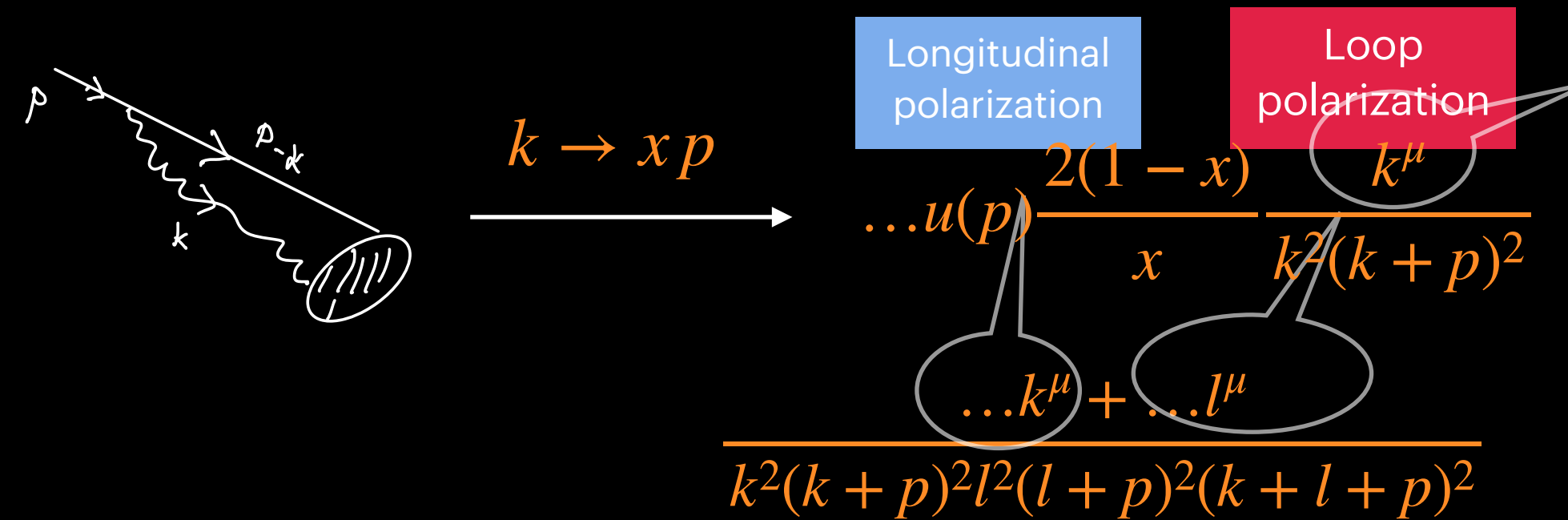
- One-loop amplitude integrand for simplest $2 \rightarrow 1$ process



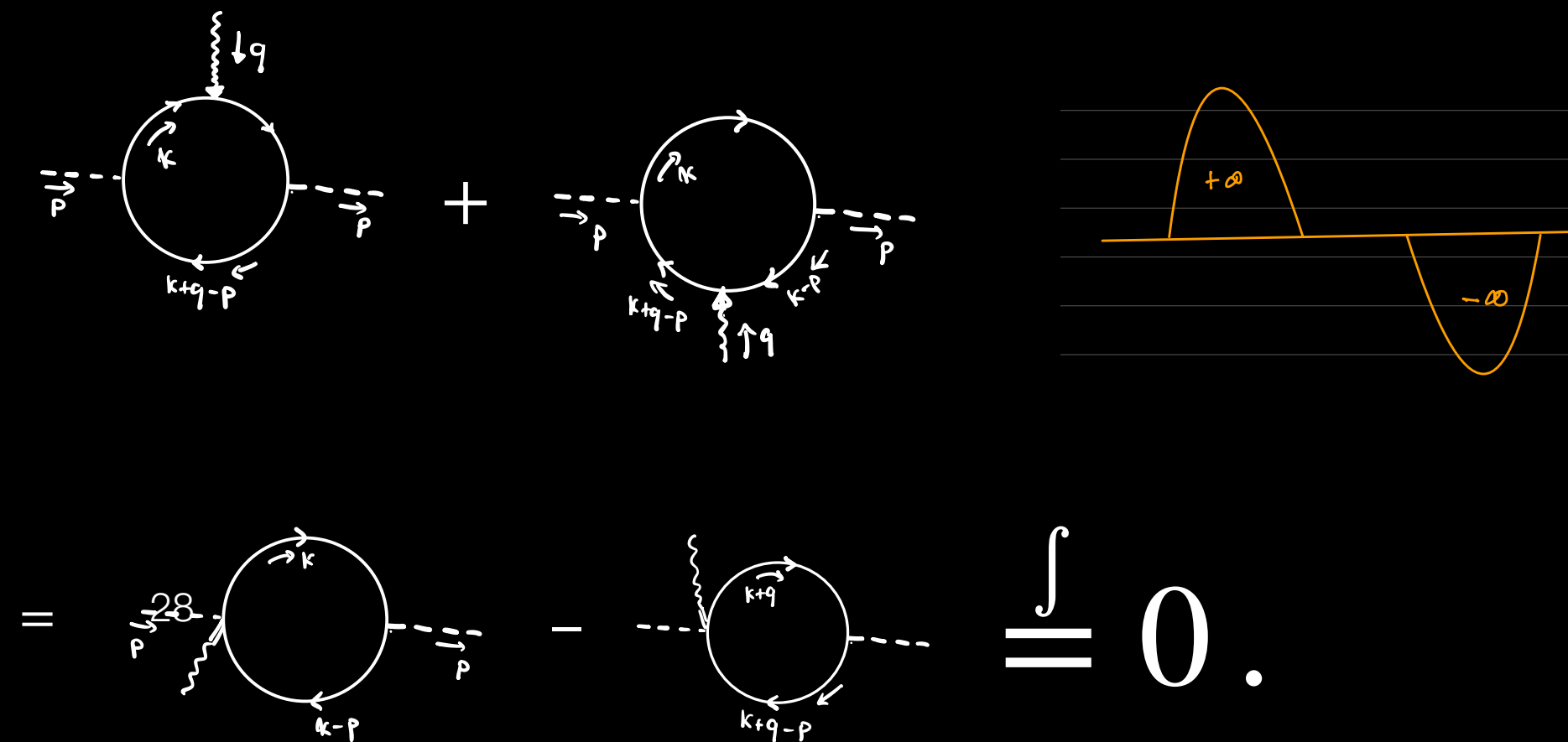
- The external current is the full tree-level amplitude
- I.e. the finite integrand of the previous perturbative order

Non-local cancellations?

- Collinear gluons off one-loop vertices acquire random polarisations.



- Ward identities generate non-local zeros.



Locally finite integrands for electroweak production in quark annihilation

$$q + \bar{q} \rightarrow V_1 + V_2 + \dots V_n, \quad V_i = W, Z, \gamma^*$$

- In singular IR regions of loop momenta, virtual gauge bosons acquire (unphysical) longitudinal polarizations.

$$\mathbf{M}_{q\bar{q} \rightarrow ew}^{(2),R}(k, l) = \mathcal{M}_{q\bar{q} \rightarrow ew}^{(2),R}(k, l) - \Delta \mathcal{M}_{q\bar{q} \rightarrow ew}^{(2),R}(k, l)$$

- The two-loop amplitude integrand as derived from Feynman diagrams

- “Shift” Counterterms

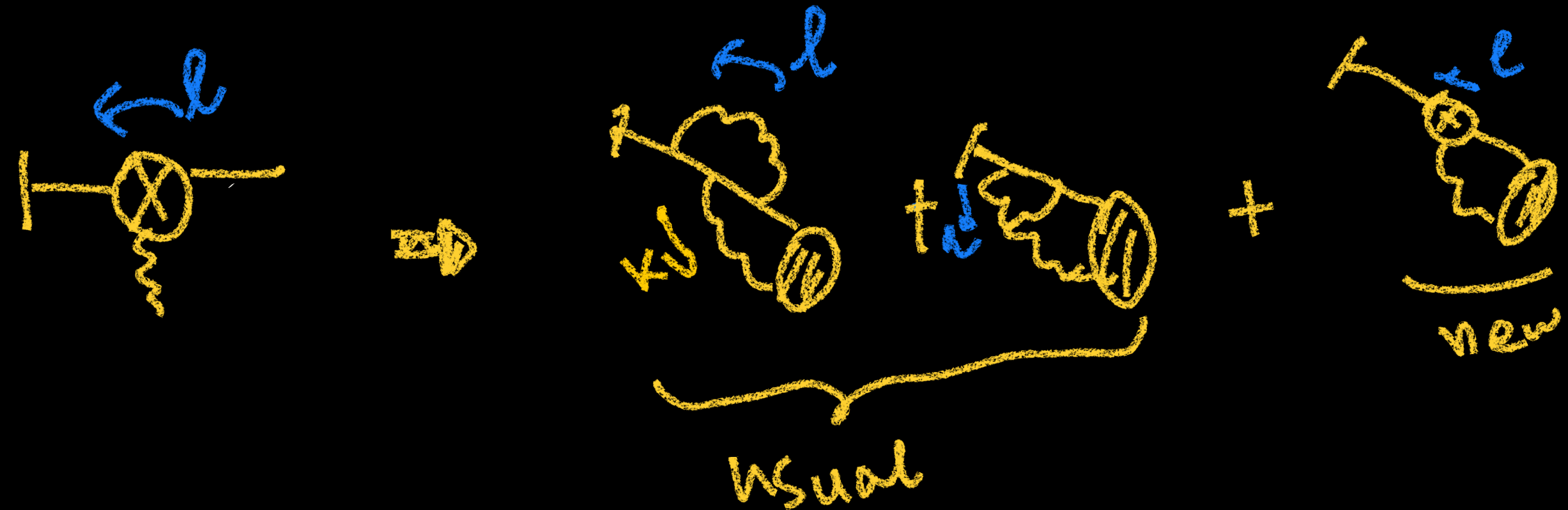
- Same scattering amplitude. “Shift” counterterms integrate to zero.

$$\int d^D k \int d^D l \Delta \mathcal{M}_{q\bar{q} \rightarrow ew}^{(2),R}(k, l) = 0$$

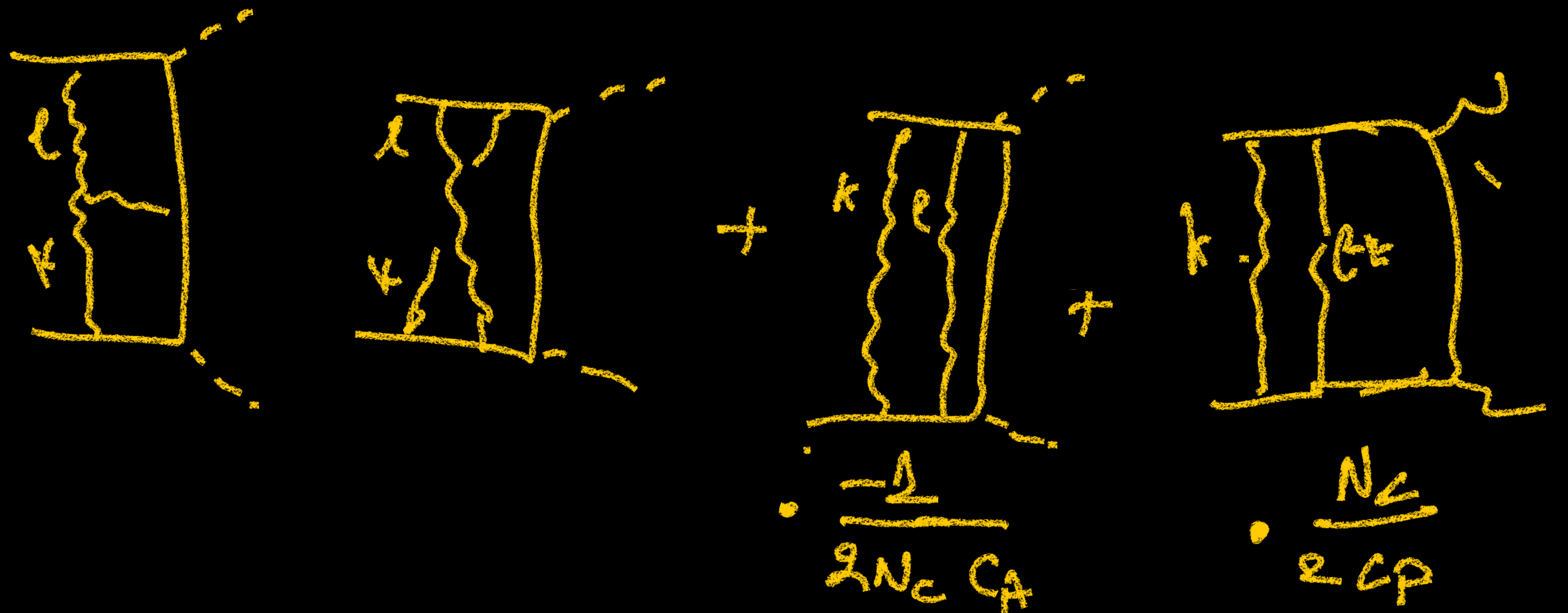
- All Ward identity cancellations are made local. All collinear gluons have longitudinal polarizations

What cures non-local cancellations?

- Novel Feynman rule
- Eliminates loop polarizations
- Vanishes upon integration



- Attribute different momentum routing to the various colour components of Feynman diagrams



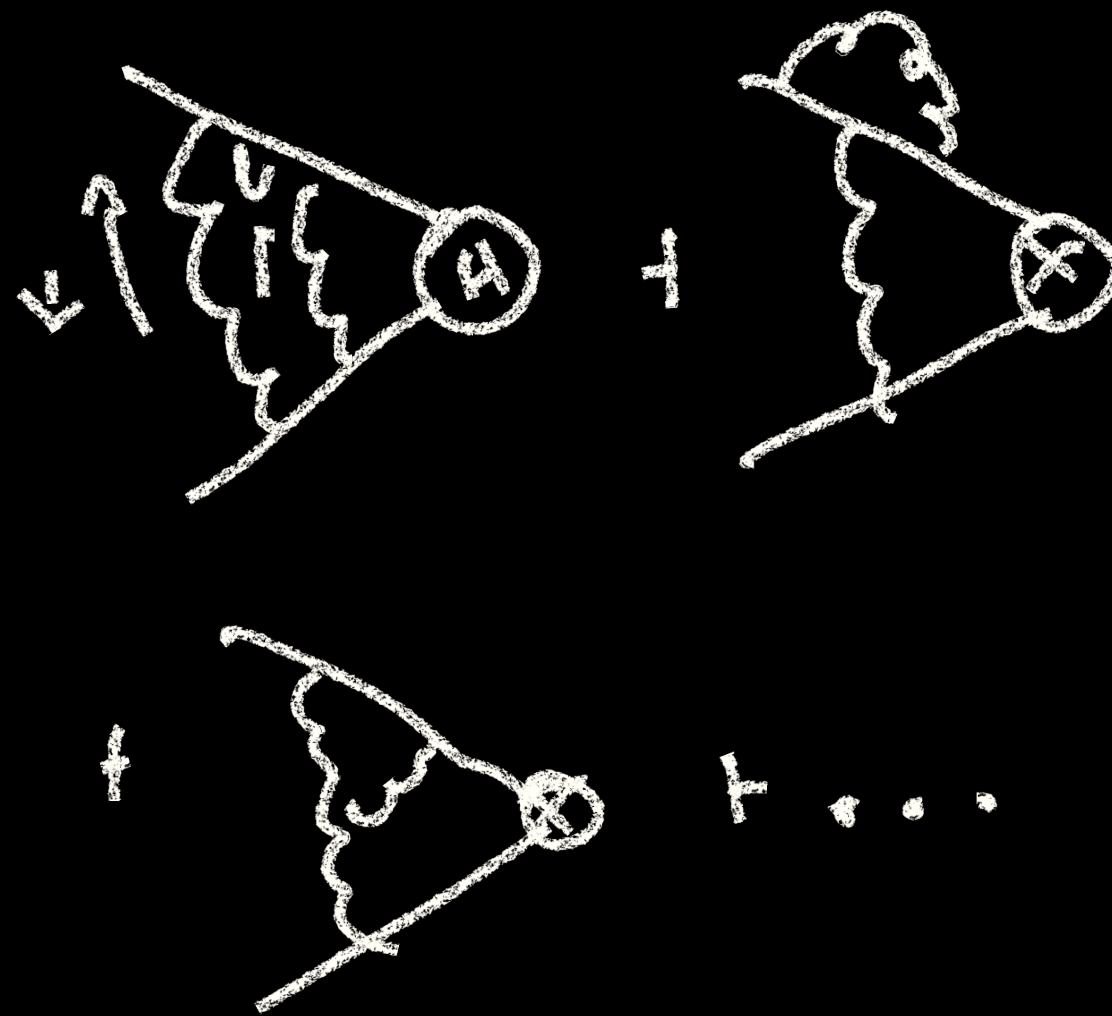
Locally finite integrands for electroweak production in quark annihilation

$$q + \bar{q} \rightarrow V_1 + V_2 + \dots V_n, \quad V_i = W, Z, \gamma^*$$

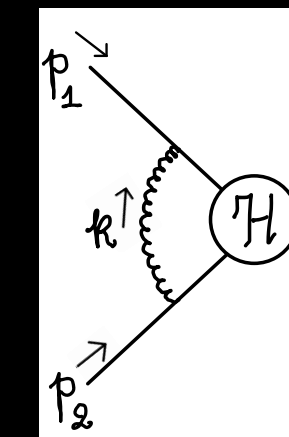
$$\mathcal{H}_{q\bar{q} \rightarrow ew}^{(2),R}(k, l) = \mathbf{M}_{q\bar{q} \rightarrow ew}^{(2),R}(k, l) - \mathcal{F}_{q\bar{q}}^{(2),R}(k, l) \left[\mathbf{P}_1 \widetilde{\mathcal{M}}_{q\bar{q} \rightarrow ew}^{(0)} \mathbf{P}_1 \right] - \mathcal{F}_{q\bar{q}}^{(1),R}(k) \left[\mathbf{P}_1 \widetilde{\mathcal{H}}_{q\bar{q} \rightarrow ew}^{(1),R}(l) \mathbf{P}_1 \right]$$

- Finite two-loop amplitude integrand
- Free of all IR and UV singularities locally
- Integrable in D=4 exactly

- Two-loop amplitude integrand
- As derived with Feynman rules
- AND “Shift” counterterms
- Momentum flow assignment



- Two-loop amplitude integrand for simplest $2 \rightarrow 1$ process



- One-loop amplitude integrand for simplest $2 \rightarrow 1$ process

- The external current is the finite one-loop amplitude integrand
- I.e. the finite integrand of the previous perturbative order

Realising the method

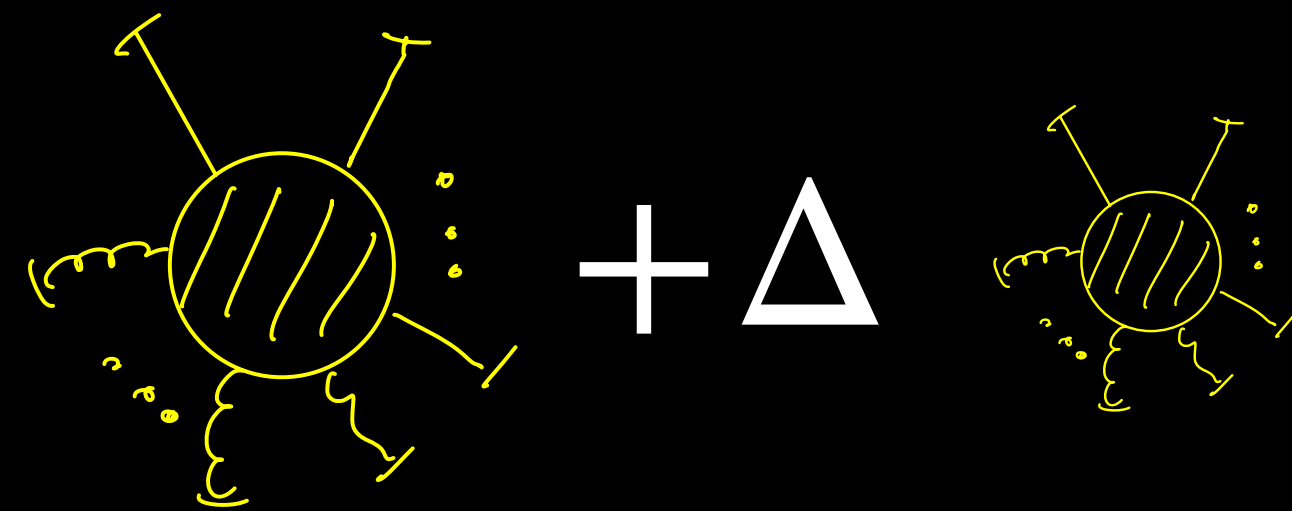
Constructions of finite two-loop integrands

- Removing infrared divergences from two-loop integrals
CA, Sterman (2018)
- Locally finite two-loop amplitudes for off-shell multi-photon production in electron-positron annihilation
CA, Haindl, Sterman, Yang, Zeng (2020)
- Locally finite two-loop QCD amplitudes from IR universality for electroweak production
CA, Sterman (2022)
- Locally finite two-loop amplitudes for electroweak production through gluon fusion
CA, Karlen, Sterman, Venkata (2024)
- General finite two-loop amplitude integrand for photoproduction in quark annihilation
CA, Karlen, Sahoo, Sterman, Vicini (2025)
- Local finiteness for real-virtual corrections to electroweak production in partonic collisions
CA, Karlen, Sterman, Ma (2026)

Local soft-collinear factorization for two-loop amplitudes

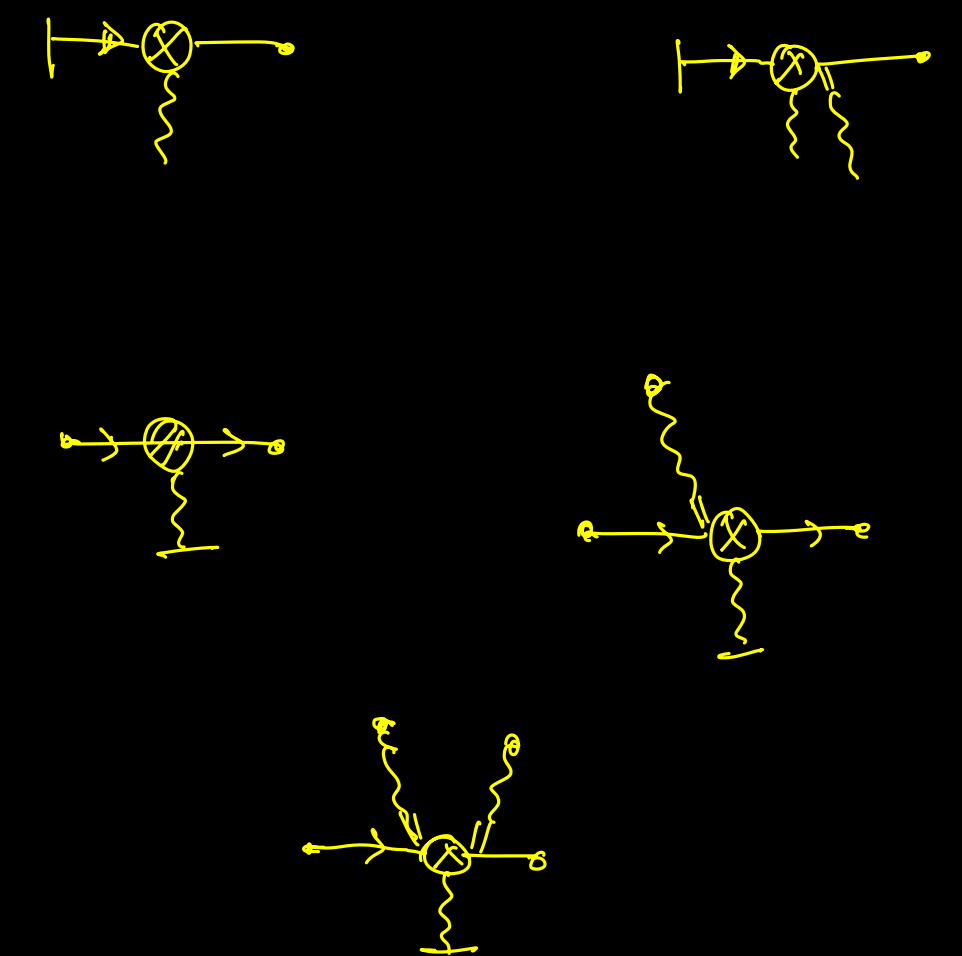
For generic processes in massless QED.

- In collaboration with Christian Biello, Andrea Favorito, Giulio Gambuti, Roshni Sahoo and George Sterman.



= S/C Finite

Δ – rules :



$$\Sigma \left(\text{diagram} + \Delta \text{diagram} \right)$$

The dawn of a new era!

- a quarter of a century into the millennium, Lance was very quick to apply Large Language Models in order to unveil patterns in amplitudes (2405.06107)
- Our capabilities to solve problems will be exploding.
- Testing our ideas will be more immediate.
- Already, genetic AI is trivializing implementations into code, of algorithms and tasks that occupied a big part of our time in tackling perturbative QCD problem.

Testing and implementing at a new speed

THEN · A.I.R. in Maple (2004)

- Set up the reduction of a topology
- Drive with Frank to the Stanford campus for a coffee or a burrito
- Come back a couple of hours later — check, feed the next input

A.I.R. (Anastasiou & Lazopoulos, hep-ph/0404258) — the Maple/
FORM silica

ine behind the Drell-Yan rapidity distribution

To be fair, I've benchmarked FAIR only against Reduze — not Kira, nor Bernhard Mistlberger's reducer (no RAM blow-up at N₃LO Higgs). FAIR is in the same ballpark; some are likely faster.

Our ability to test ideas is expanding fast — technical complexity is much less of an obstacle. Now that it is possible, maybe AI should combine the algorithmic ideas across all these Laporta reducers — freeing us for the big conceptual problems.

FAIR · Fast Automatic Integral Reduction — people.phys.ethz.ch/~babis/fair

LAST FEW DAYS with AI transcription · FAIR

- Fast Automatic Integral Reduction — standalone C++/FLINT, no CAS.
- The same topology ($gg \rightarrow H + 2$ partons, reverse unitarity)
- Reduced before I reached the Nespresso machine

Outputs to Maple / Mathematica / FORM / sympy



Lance,

In this new era, your leadership in the field is even more important.

Tackling tough problems.

Inspiring to think deeply, and to seek connections.

Solving perturbative QCD for all practical purposes is not such a far out goal.

Fully solving a four-dimensional quantum gauge theory may not be so far out either!

Schroedinger Lecture 2019

Monday, 27 May 2019, 14:15 h, ETH Zurich, Hönggerberg,
lecture hall HPH G3

Speaker: Prof. Lance Dixon, Stanford University, SLAC National
Accelerator Laboratory, USA

Particle Scattering and Number Theory

From the softest of interactions of a magnetic field with an electron, to the most violent collisions at the Large Hadron Collider, precision quantum field theory produces numbers and functions with interesting number-theoretic properties. In many examples a co-action principle holds, an invariance under a "cosmic" Galois group. I will provide several arenas in which this principle can be seen at work, including perhaps the richest set of theoretical data, scattering amplitudes in planar $N=4$ super-Yang-Mills theory.

Short courses on "The Hexagon Function Bootstrap for Planar $N=4$ Super-Yang-Mills Theory"

Monday, 3 June 2019, 10:30 h

Wednesday, 5 June 2019, 10:30 h

Friday, 7 June 2019, 10:30 h

ETH Zurich, Hönggerberg, lecture room HIT E 41.1

More Information: www.paulicenter.ch

Apero
after Monday
Lecture
(27 May 2019)



*Thank you
For the inspiration, collaboration and
support throughout my career
On behalf of my community, thank you for
the tireless help you always offered to
ETH and Zurich without hesitation.*

Happy birthday
From Zoltan and too

