Charge correlators in deep inelastic scattering

Frank Petriello

Northwestern University

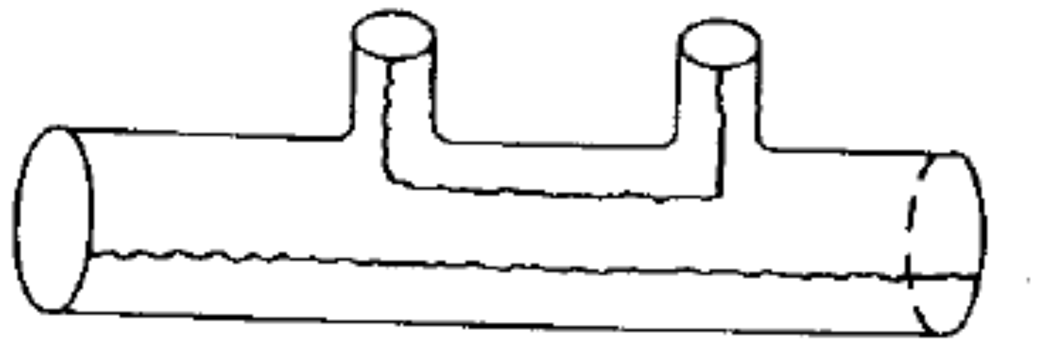
LanceFest

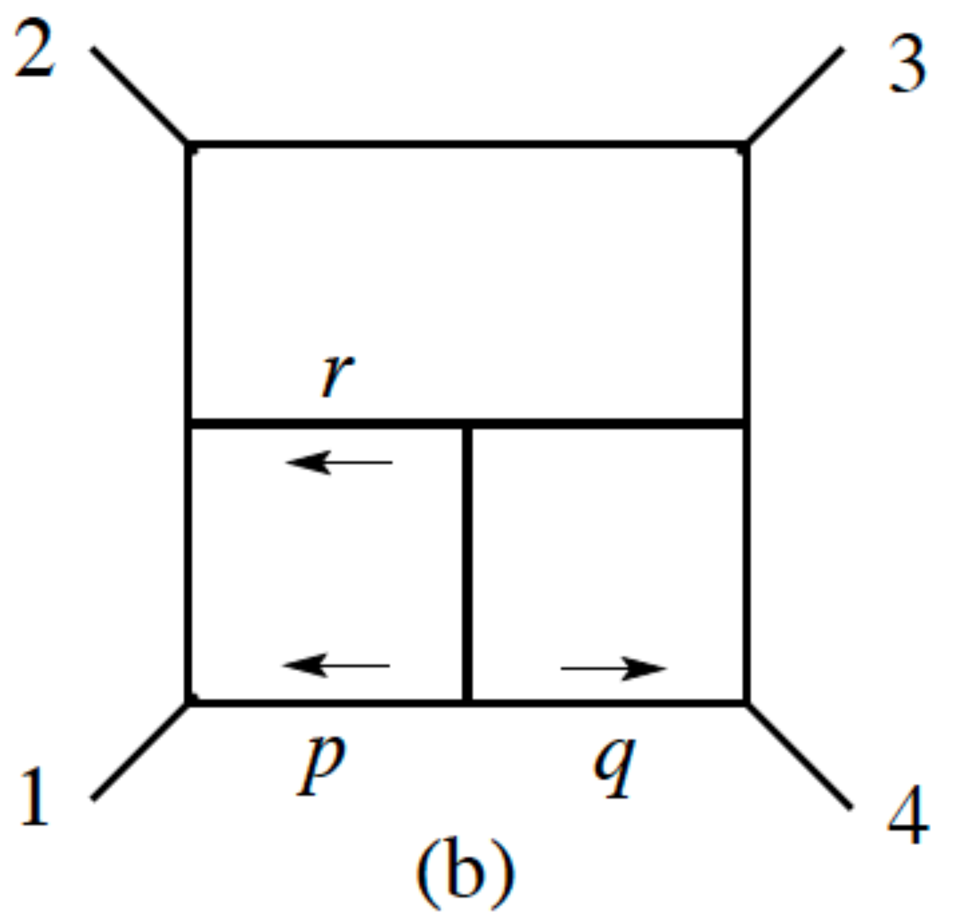
June 25, 2026

Outline and Instructions

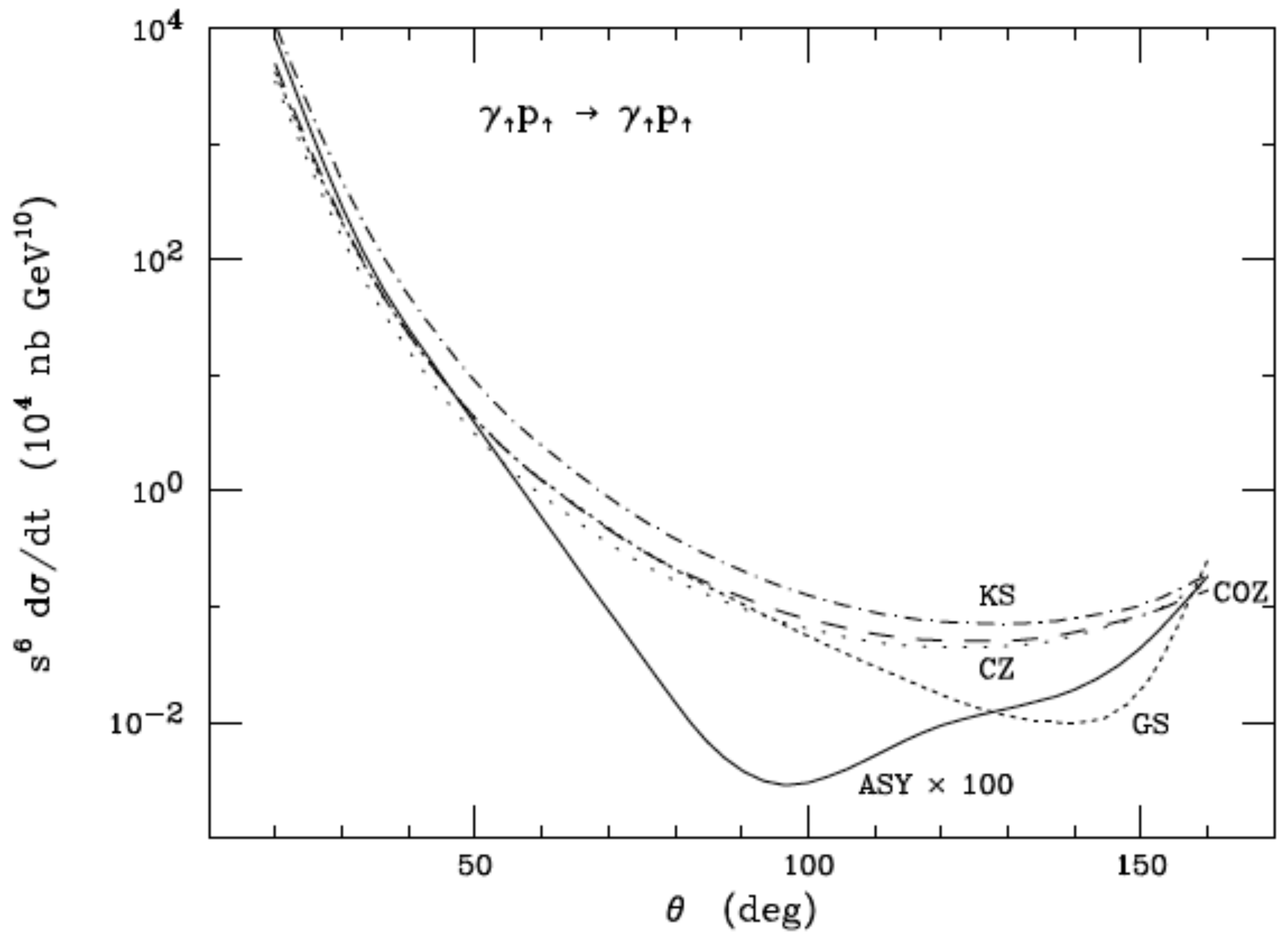
- “I would suggest that your talk can cover a mix of current research, and where this work (or related past work) comes into contact with Lance”
- Sounds difficult, but Lance’s breadth and influence makes this an easy task

Dixon, Harvey, Vafa, Witten NPB 261 (1985)



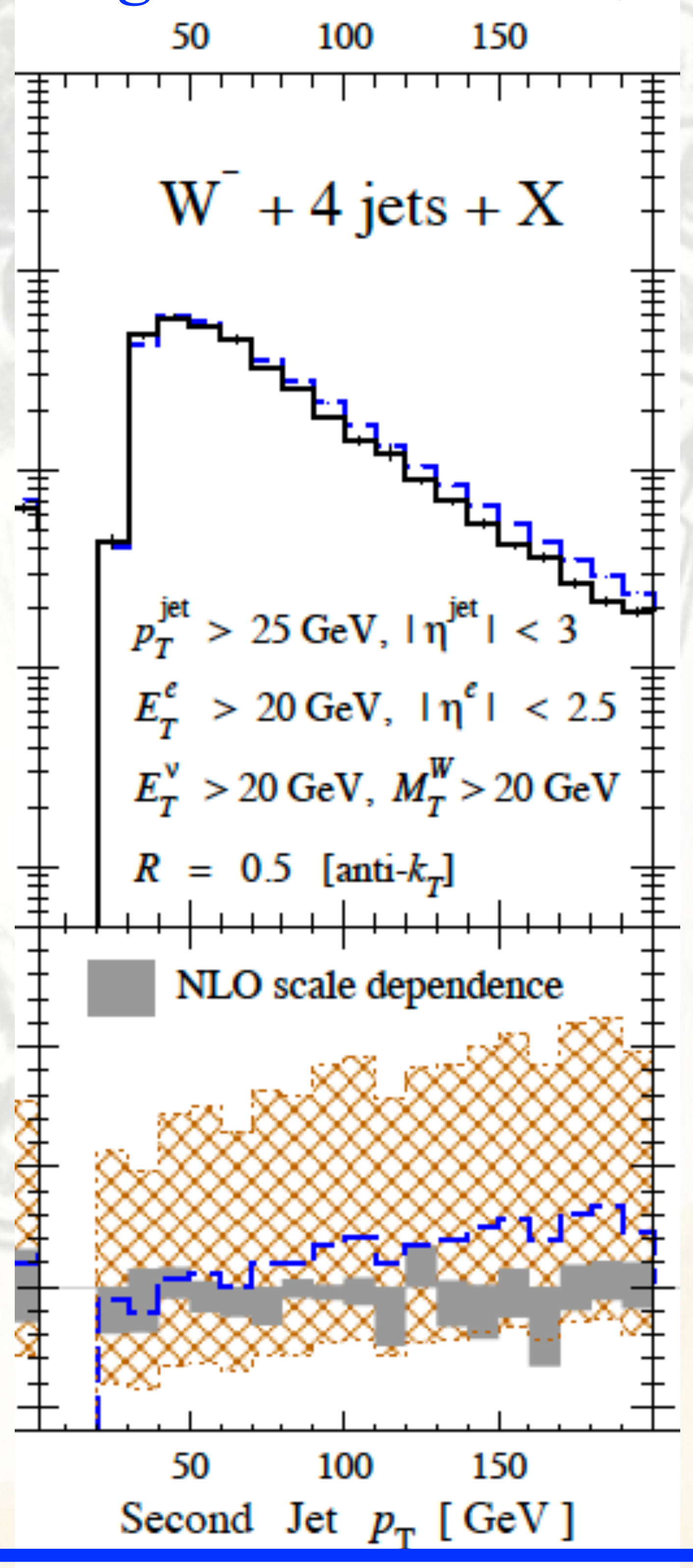


Brooks, Dixon PRD 62 (2000)



$$\frac{q^{-n^2/2}\,\Sigma q^{(n+n)^2/2}}{\Pi(1-q^n)} = \prod_{n>0} (1 + q^{n+n+1/2})(1 + q^{n-n+1/2})$$

C. Berger et al PRL 106 (2011)



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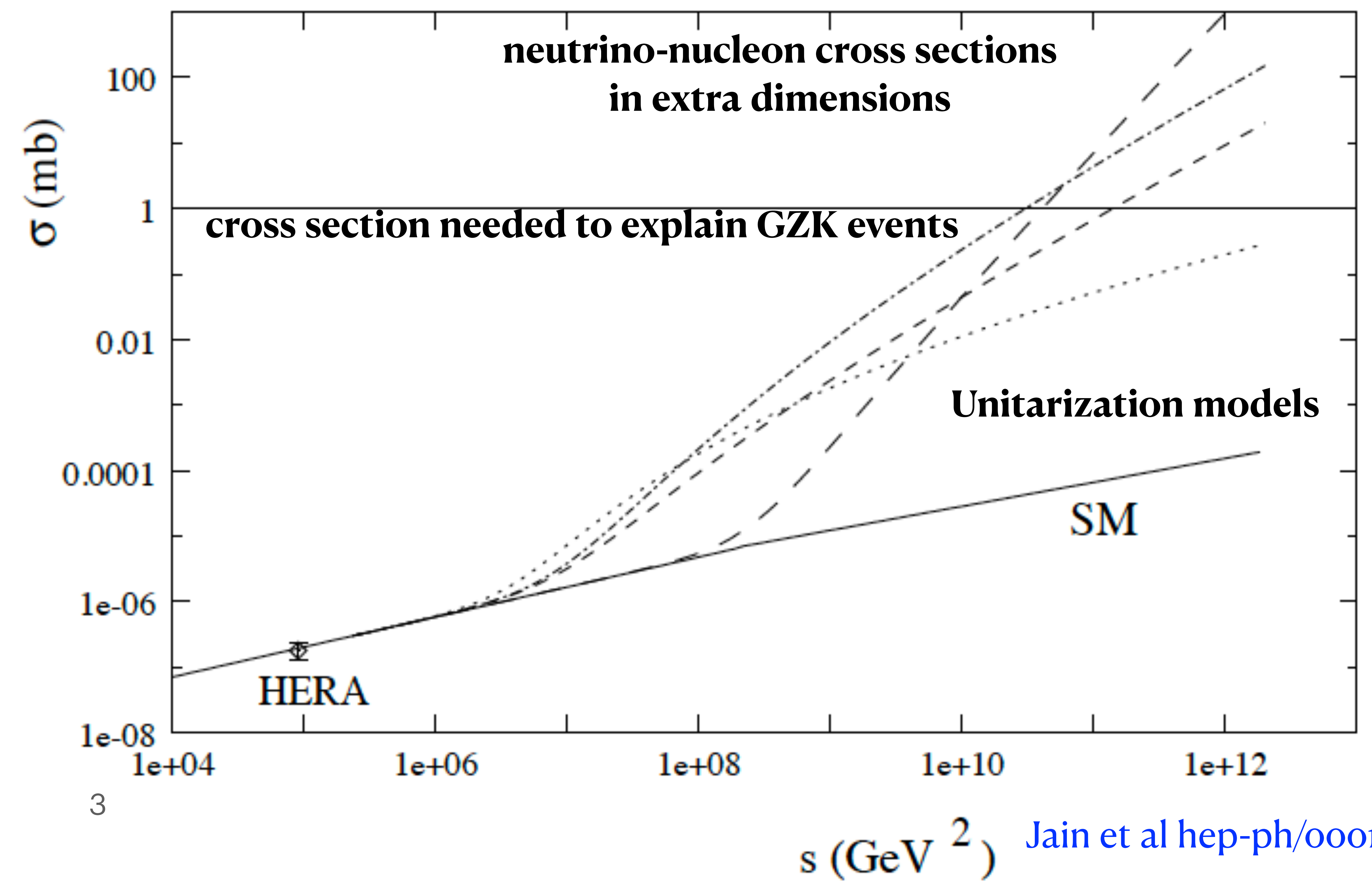
My first suggested project from Lance as a grad student in 2000 was in astro-particle physics:

The idea was to follow up on this work, check feasibility, think about low-x physics… never completed.



Jain et al hep-ph/0001031

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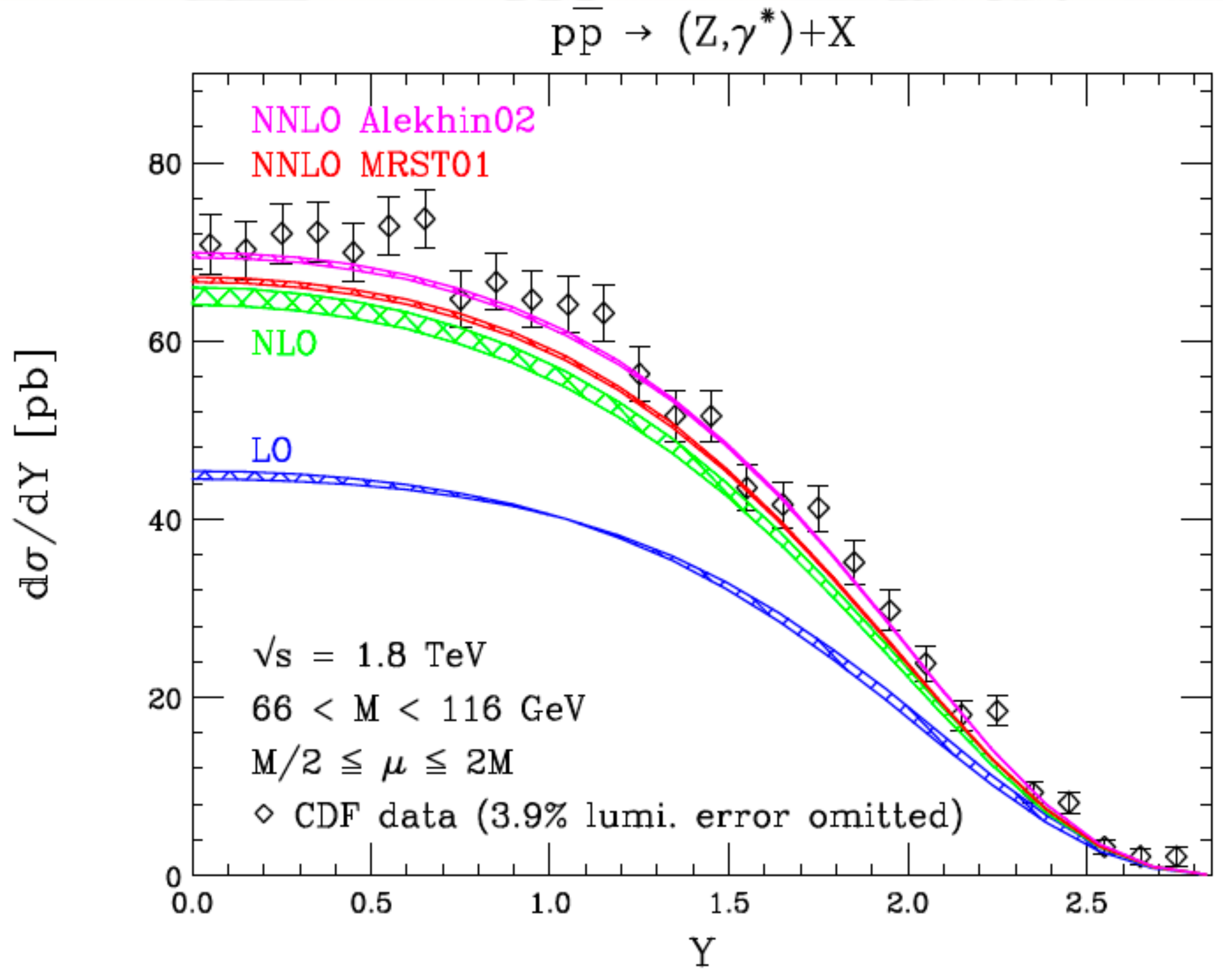
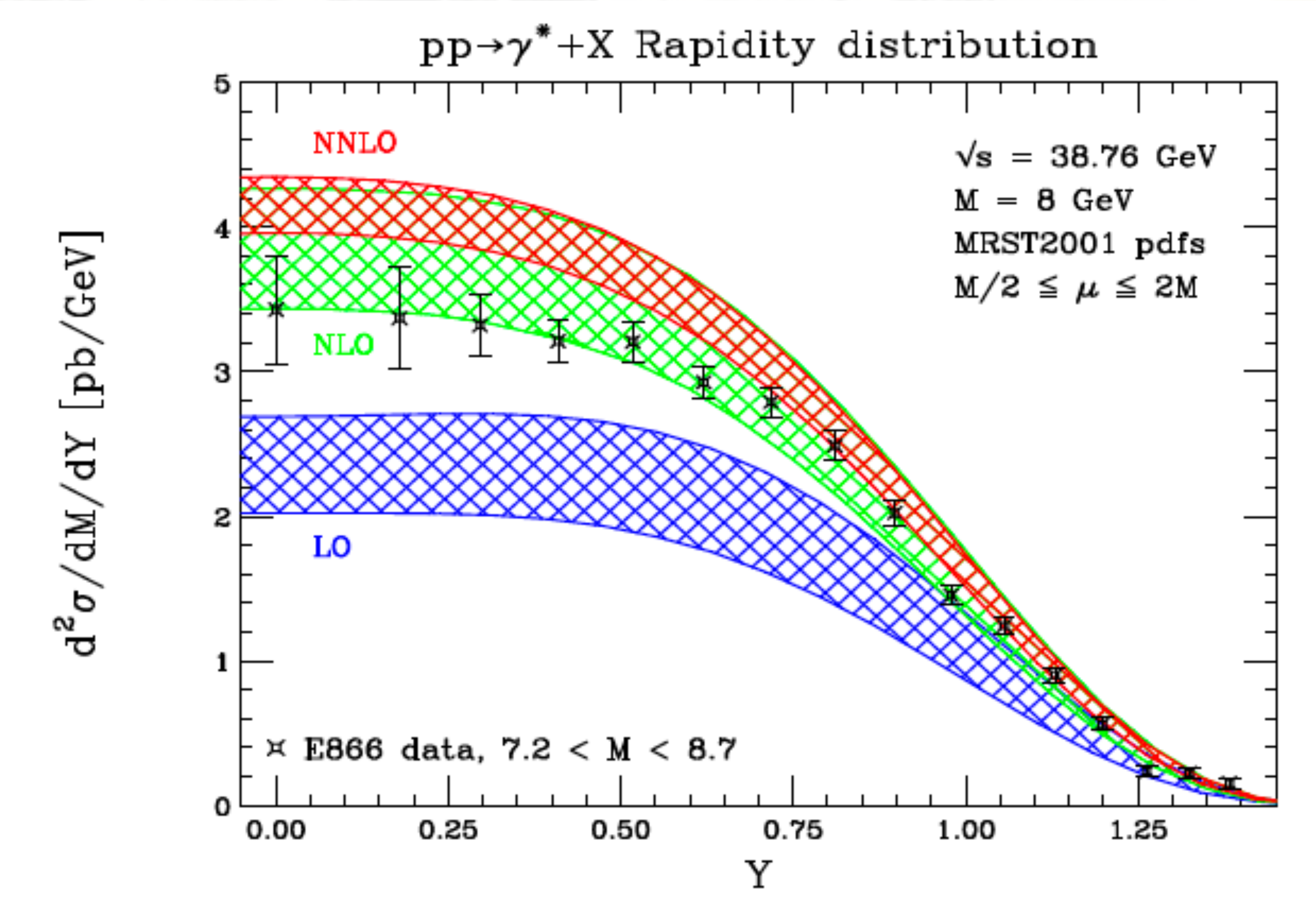
Eventually we published together on Drell-Yan at NNLO:

"Putting theory and experiment on the same plot"

Anastasiou, Dixon, Melnikov FP PRL 91 (2003)



Anastasiou, Dixon, Melnikov FP PRD 69 (2004)



Outline and Instructions

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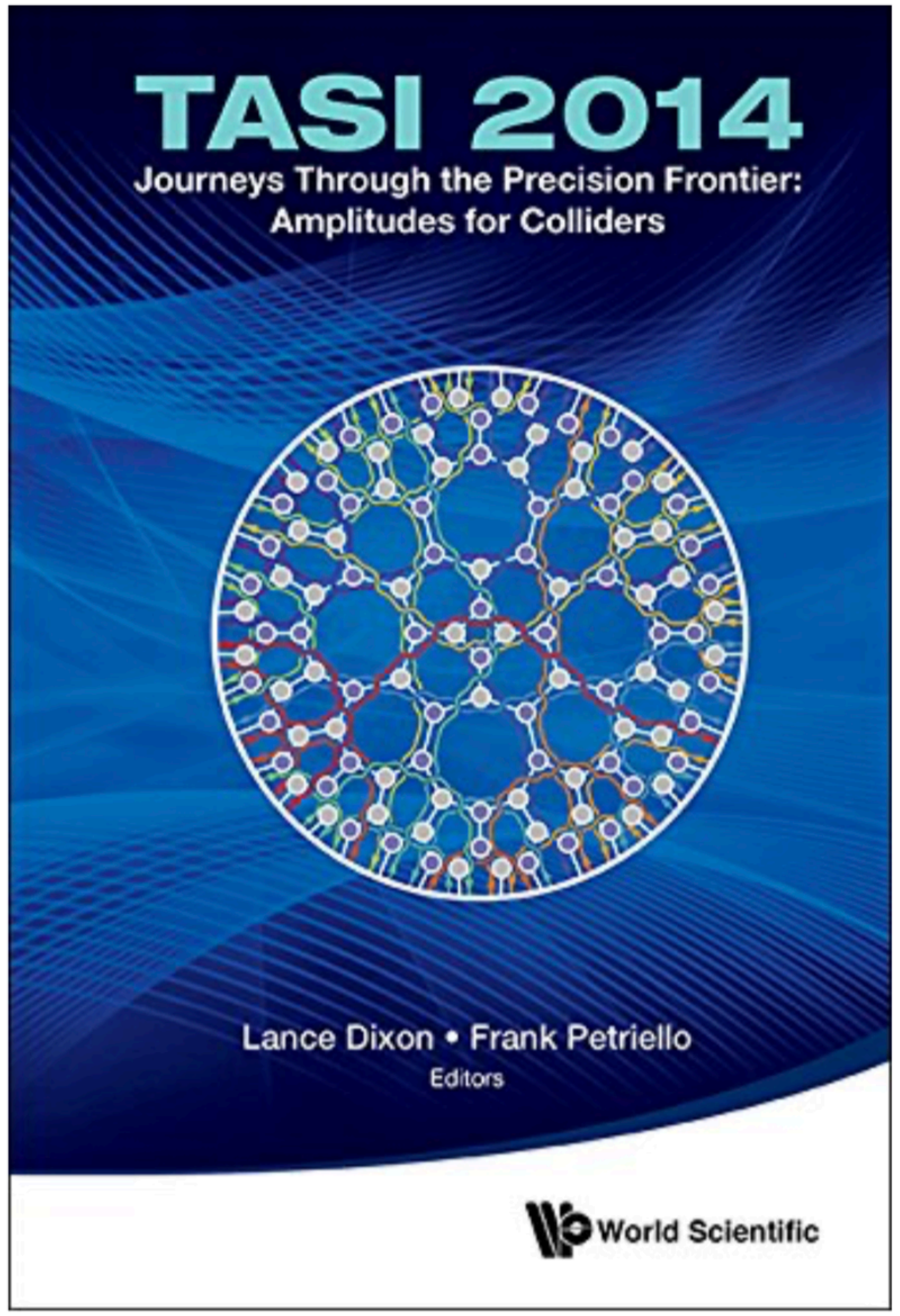
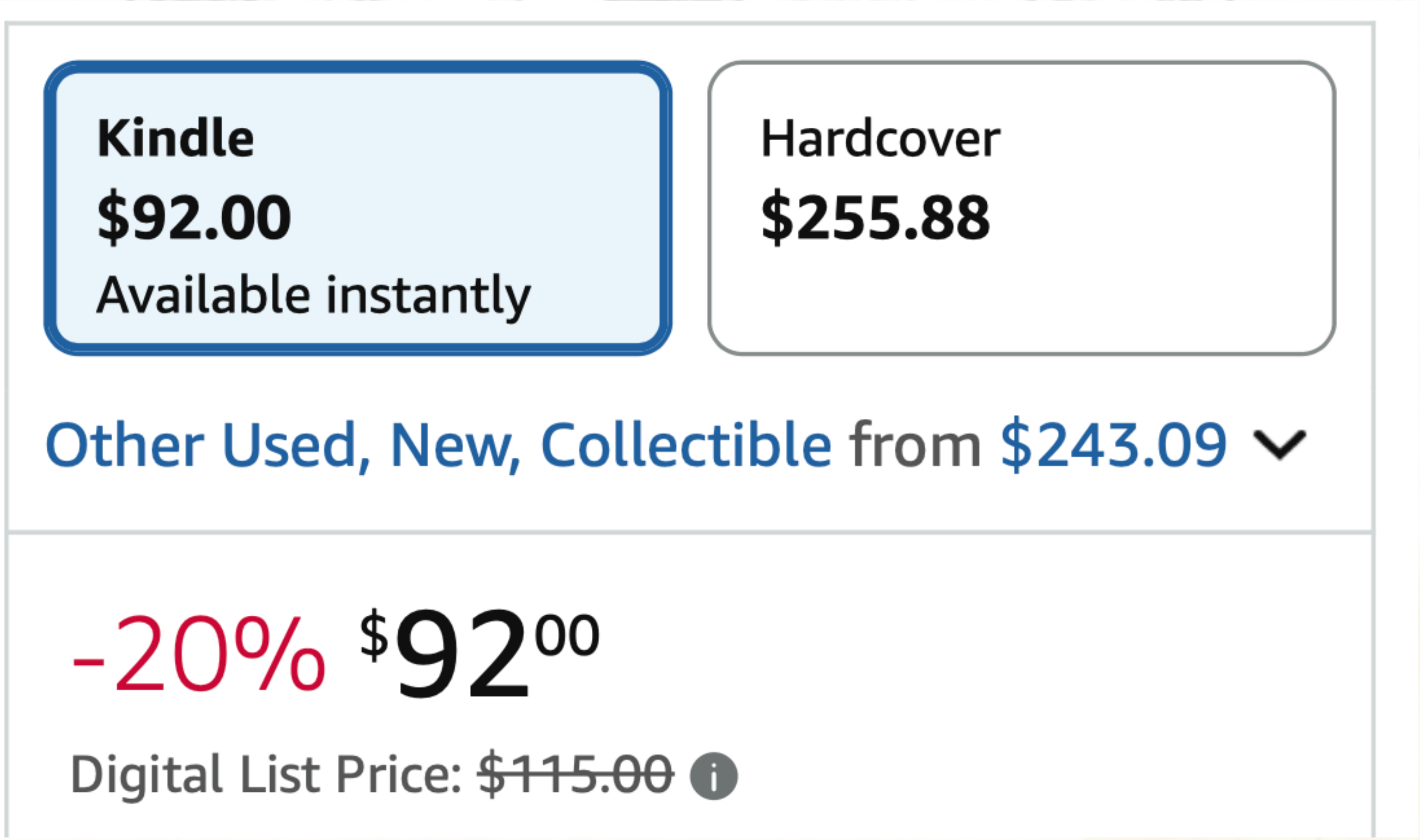
And organized TASI 2014 together:





Lance's stats

Inspired by yesterday's talk

Strings on Orbifolds #1
Lance J. Dixon (Princeton U.), Jeffrey A. Harvey (Princeton U.), C. Vafa (Princeton U.), Edward Witten (Princeton U.) (Jul, 1985)
Published in: *Nucl.Phys.B* 261 (1985) 678-686
DOI cite claim reference search 1,900 citations

Strings on Orbifolds. 2. #2
Lance J. Dixon (Princeton U.), Jeffrey A. Harvey (Princeton U.), C. Vafa (Harvard U.), Edward Witten (Princeton U.) (Feb, 1986)
Published in: *Nucl.Phys.B* 274 (1986) 285-314
pdf DOI cite claim reference search 1,782 citations

One loop n point gauge theory amplitudes, unitarity and collinear limits #3
Zvi Bern (UCLA), Lance J. Dixon (SLAC), David C. Dunbar (UCLA), David A. Kosower (Saclay) (Mar 7, 1994)
Published in: *Nucl.Phys.B* 425 (1994) 217-260 • e-Print: hep-ph/9403226 [hep-ph]
pdf links DOI cite claim reference search 1,689 citations

High precision QCD at hadron colliders: Electroweak gauge boson rapidity distributions at NNLO #4
Charalampos Anastasiou (SLAC), Lance J. Dixon (SLAC), Kirill Melnikov (Hawaii U.), Frank Petriello (SLAC and Johns Hopkins U.) (Dec, 2003)
Published in: *Phys.Rev.D* 69 (2004) 094008 • e-Print: hep-ph/0312266 [hep-ph]
pdf links DOI cite claim reference search 1,445 citations

Fusing gauge theory tree amplitudes into loop amplitudes #5
Zvi Bern (UCLA), Lance J. Dixon (SLAC), David C. Dunbar (UCLA), David A. Kosower (Saclay) (Sep, 1994)
Published in: *Nucl.Phys.B* 435 (1995) 59-101 • e-Print: hep-ph/9409265 [hep-ph]
pdf links DOI cite claim reference search 1,267 citations

The D0 Detector #6
D0 Collaboration • S. Abachi et al. (Jul, 1993)
Published in: *Nucl.Instrum.Meth.A* 338 (1994) 185-253
pdf links DOI cite claim reference search 1,184 citations

Foundational string theory

Modern understanding of one-loop multi-leg amplitudes

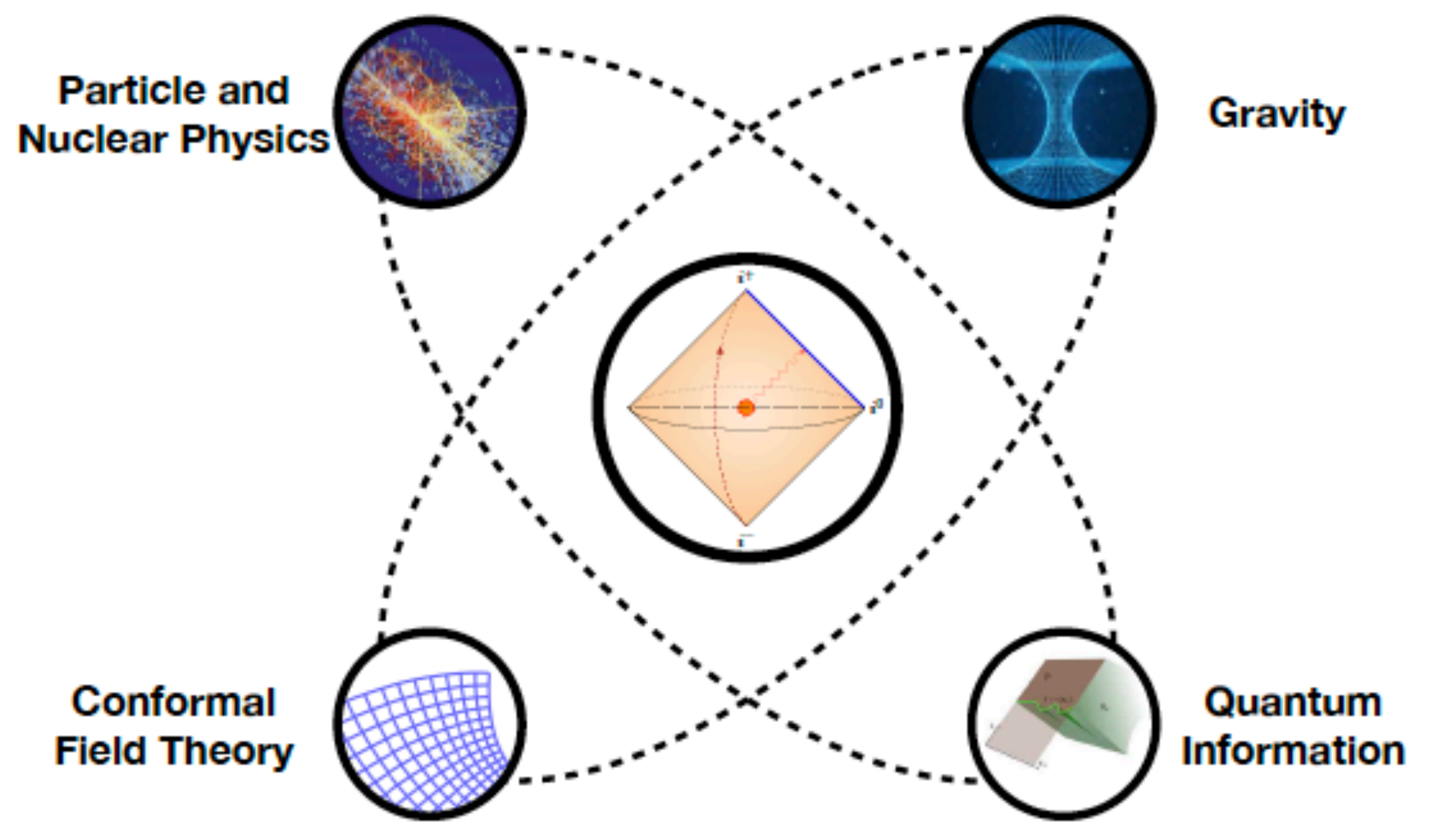
The beginning of differential QCD at NNLO

Moonlighting as an experimentalist?

Energy correlators

- "I would suggest that your talk can cover a mix of current research, and where this work (or related past work) comes into contact with Lance"
- Lance's influence is so pervasive I can take literally the last paper I've written, and Lance has influenced this area.

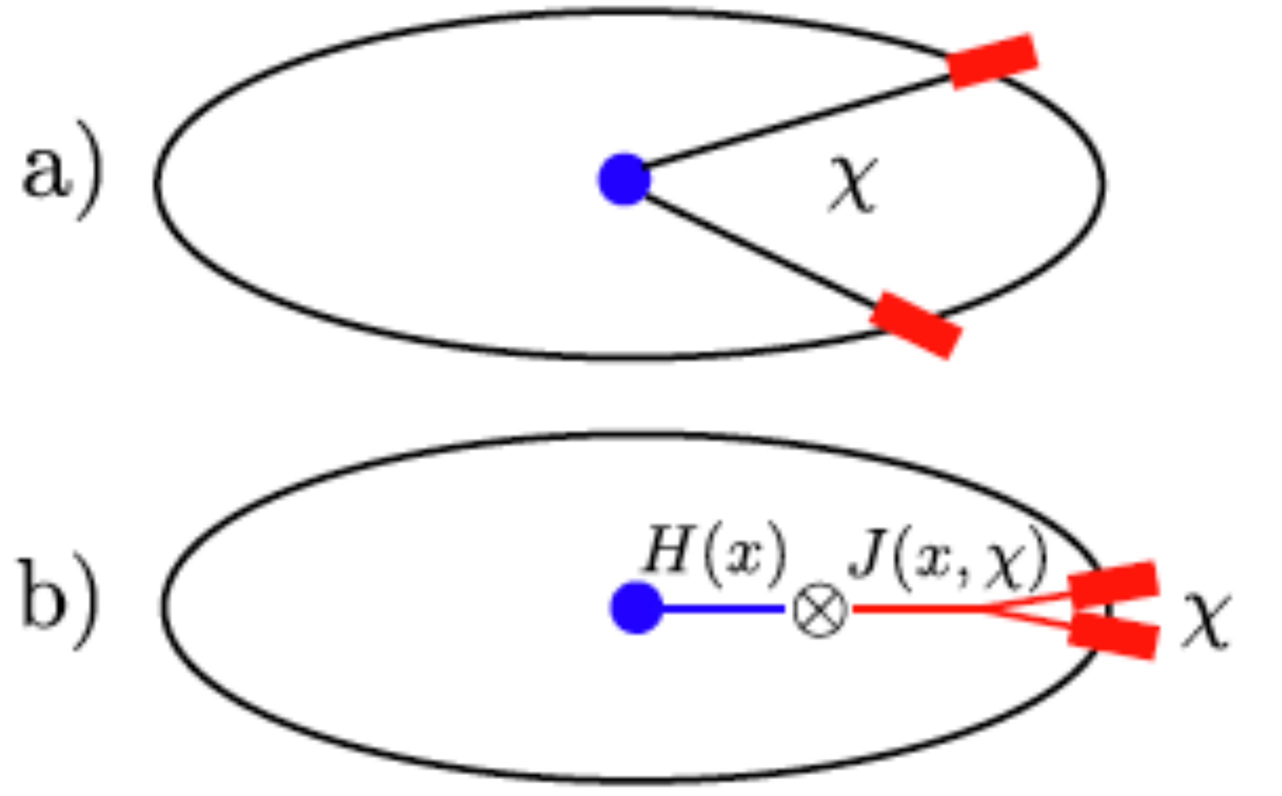
Moult, Zhu 2506.09119



Dixon, Moult, Zhu PRD 100 (2019)

5

$$\frac{1}{\sigma_0}\int_0^1 dz\,\frac{d\sigma_H^{\mathrm{bulk}}}{dz}\bigg|_{a_s^2} = n_f^2\left(-\frac{6}{5}\zeta_2+\frac{4371}{500}\right)+C_F n_f\left(-\frac{104}{15}\zeta_3+\frac{23}{10}\zeta_2-\frac{42509}{12000}\right) + C_A n_f\left(\frac{64}{15}\zeta_3+\frac{3334}{225}\zeta_2-\frac{191416183}{1620000}\right)+C_A^2\left(-62\zeta_4+\frac{44}{3}\zeta_3-\frac{8213}{450}\zeta_2+\frac{122348527}{405000}\right). \qquad (24)$$



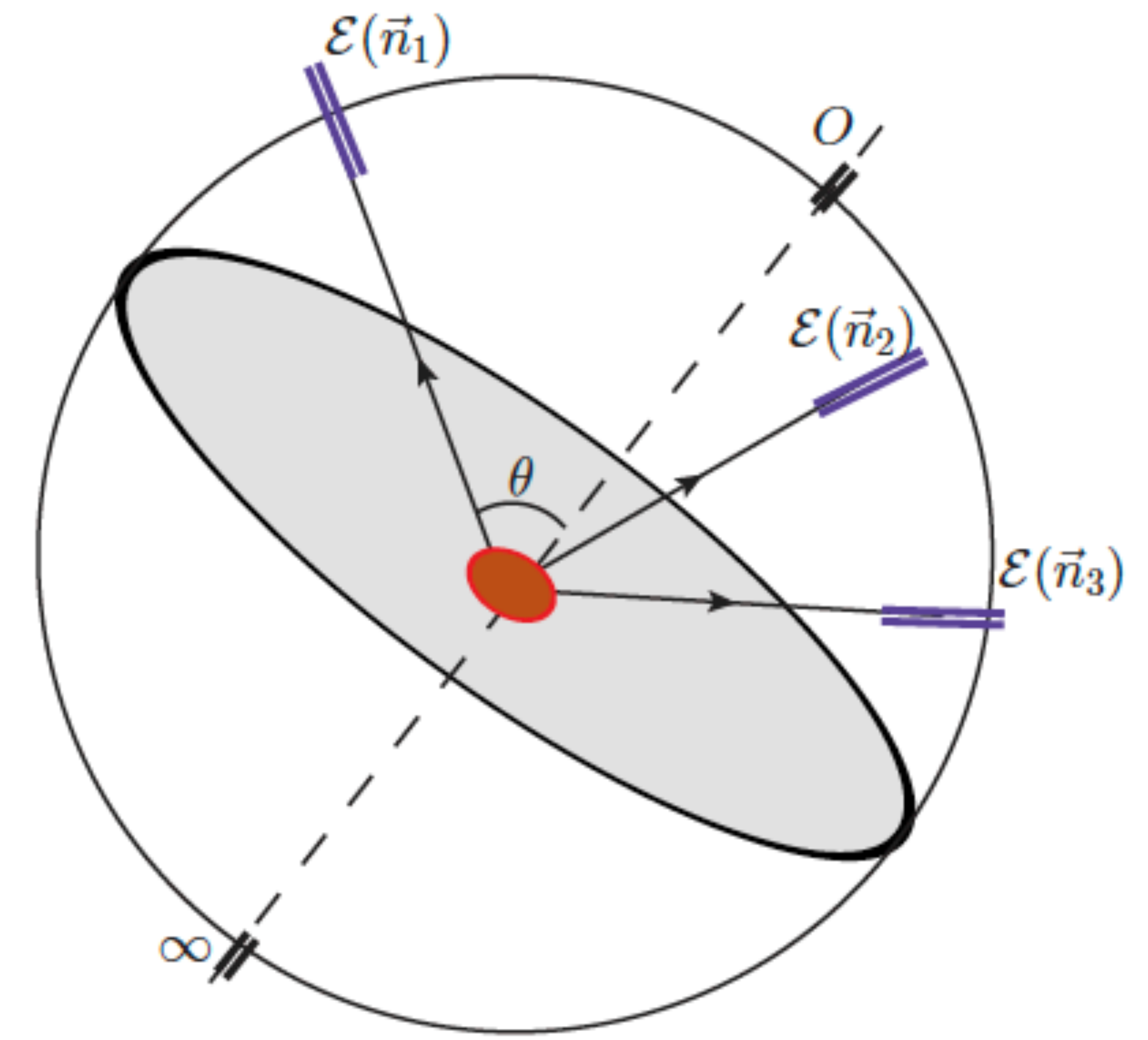
Energy-energy correlators in the collinear limit

Energy correlators

- Energy correlators provide a beautiful connection between formal theory, experiment and the field of perturbative calculations in focus here.

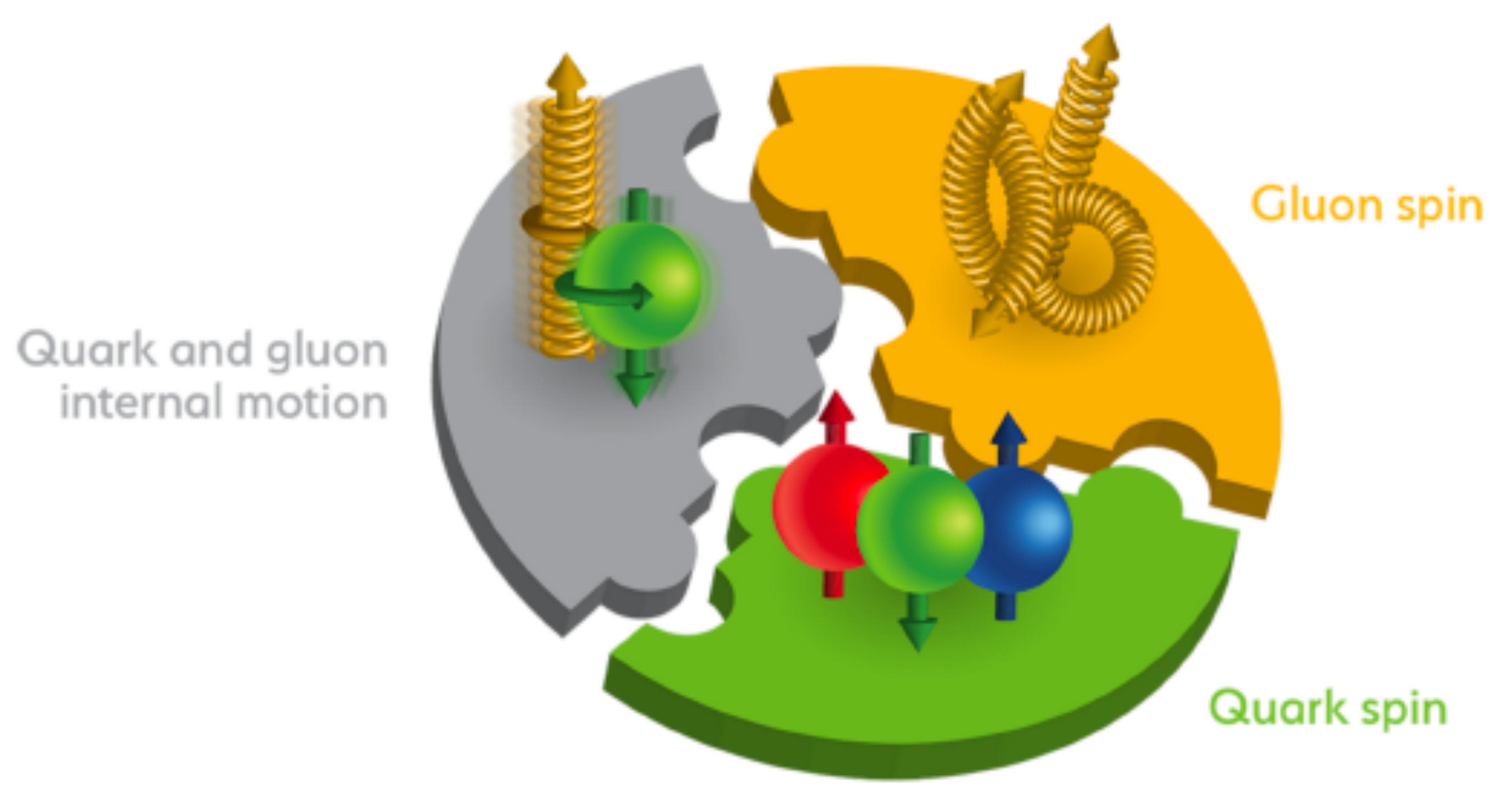
$$\mathcal{E}(\hat{n}) = \lim_{r\to\infty} \int_0^\infty \mathrm{d}t\, r^2 n_i T_{0i}(t, r\hat{n})$$

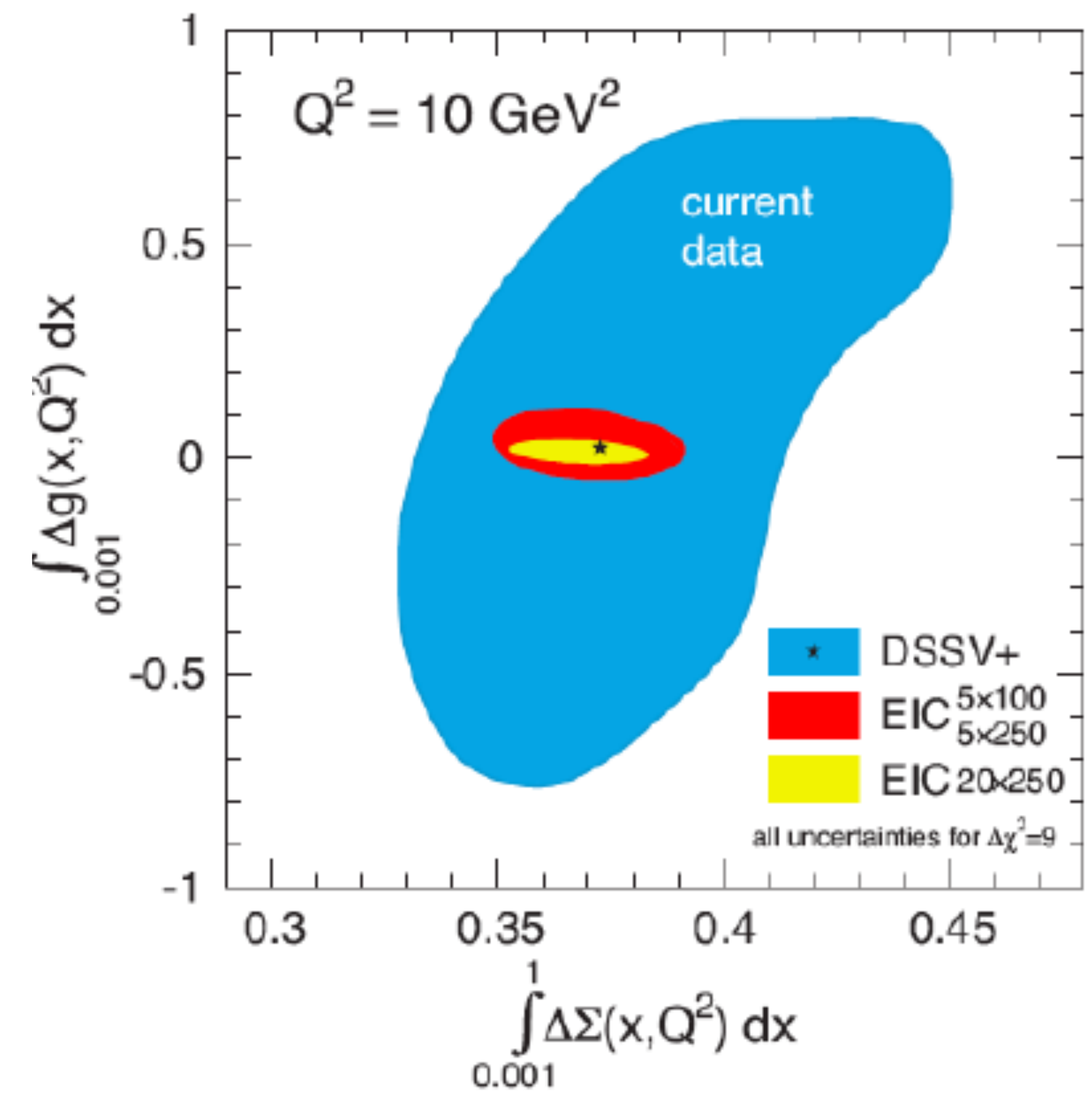
$$\int \mathrm{d}^4x\, e^{\mathrm{i}Q\cdot x} \langle 0|J(x)\mathcal{E}(\hat{n}_1)\mathcal{E}(\hat{n}_2)\cdots\mathcal{E}(\hat{n}_k)J(0)|0\rangle \equiv \langle \mathcal{E}(\hat{n}_1)\mathcal{E}(\hat{n}_2)\cdots\mathcal{E}(\hat{n}_k)\rangle\,,$$



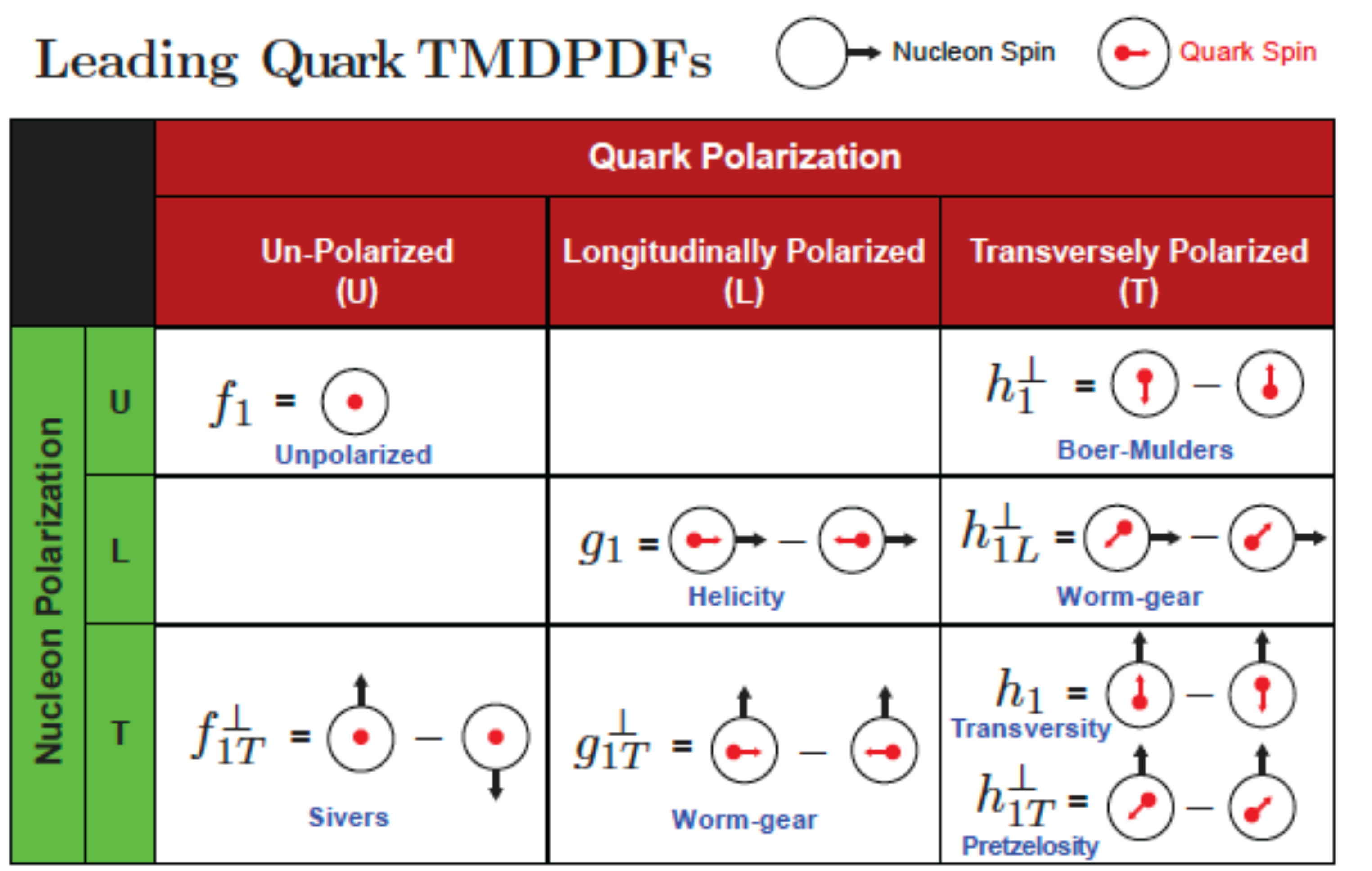
Spin structure of the proton

- Major focus of upcoming Electron-Ion Collider (EIC) program is the spin structure of the proton.





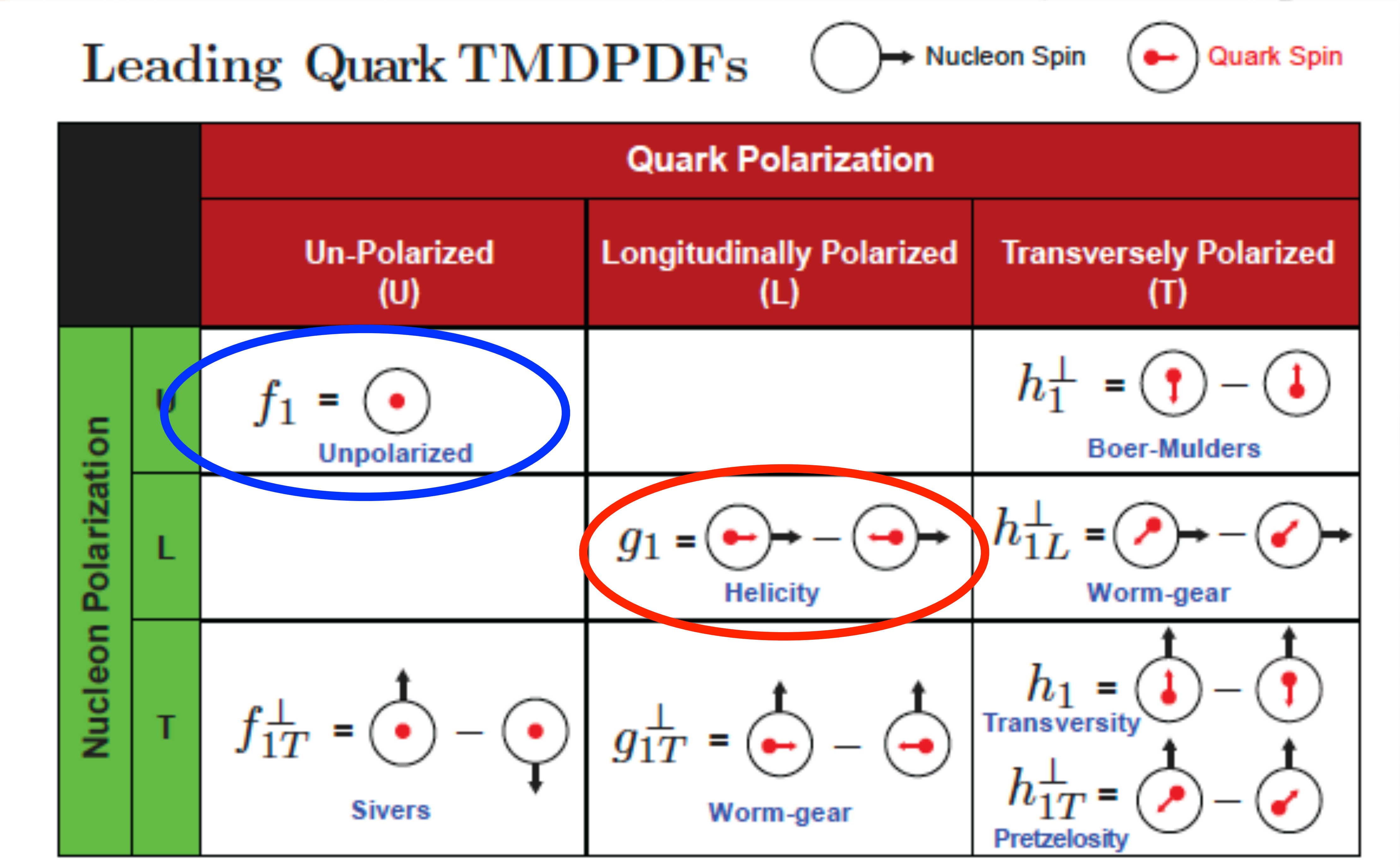
Boussarie et al TMD Handbook



Many possible measurements that reveal aspects of proton structure

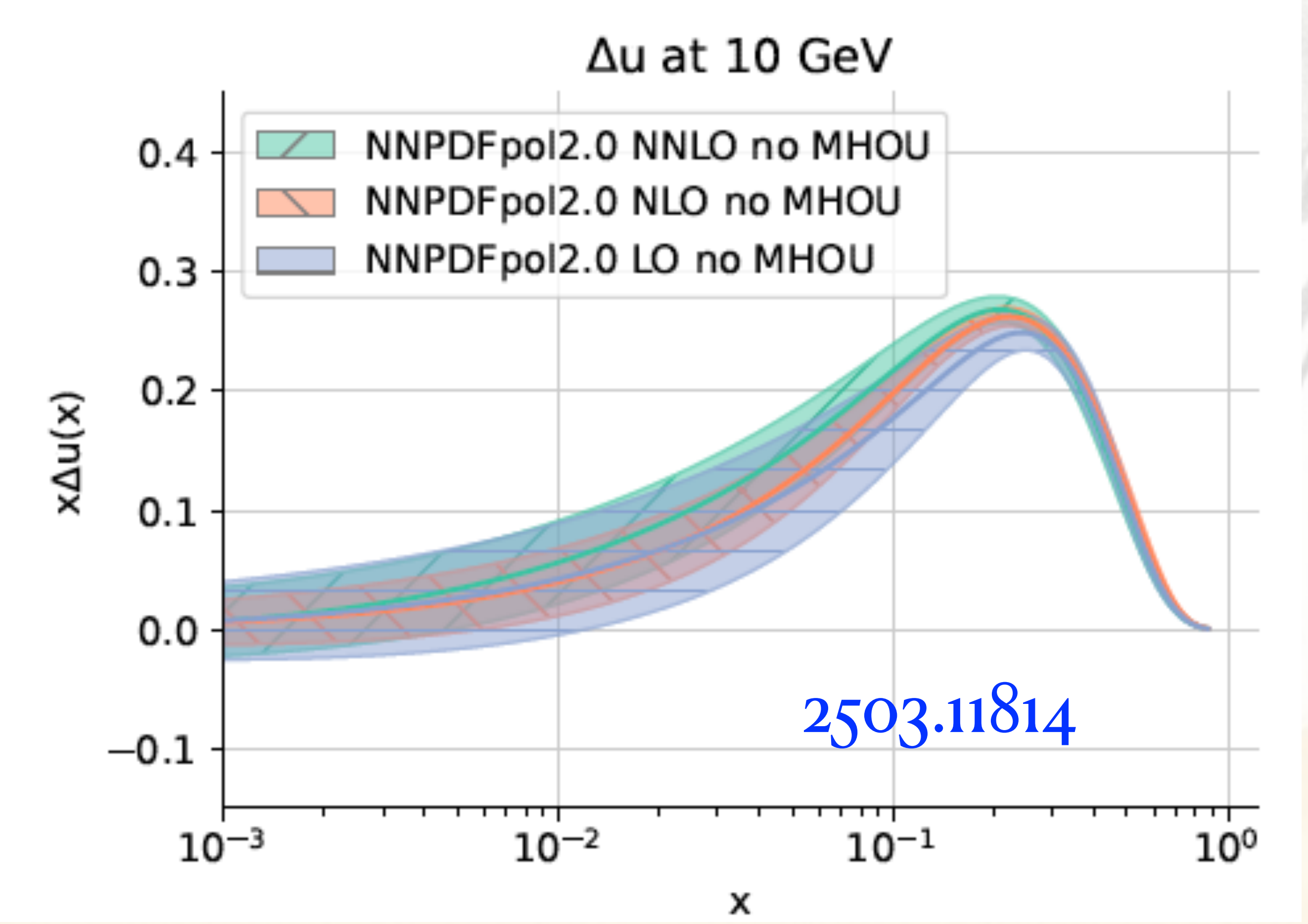
Collinear structure of the proton

- Transversity distribution is the most poorly-determined leading-twist distribution function. Impacts potential BSM measurements of SMEFT dipole coefficients at the EIC.



Unpolarized collinear PDFs well-known: NNPDF, MMHT, CT, etc.

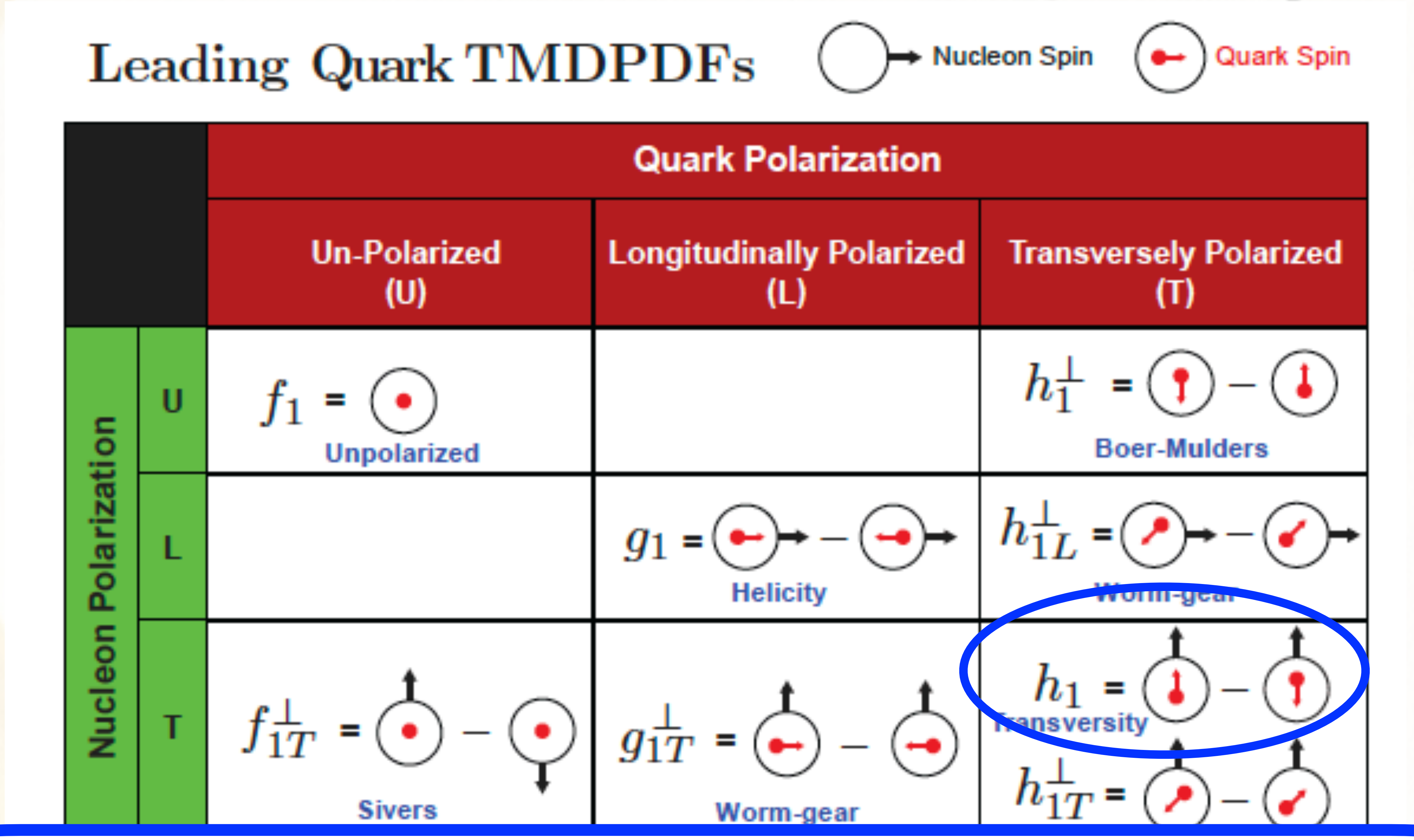
Longitudinally-polarized PDFs also fairly well developed



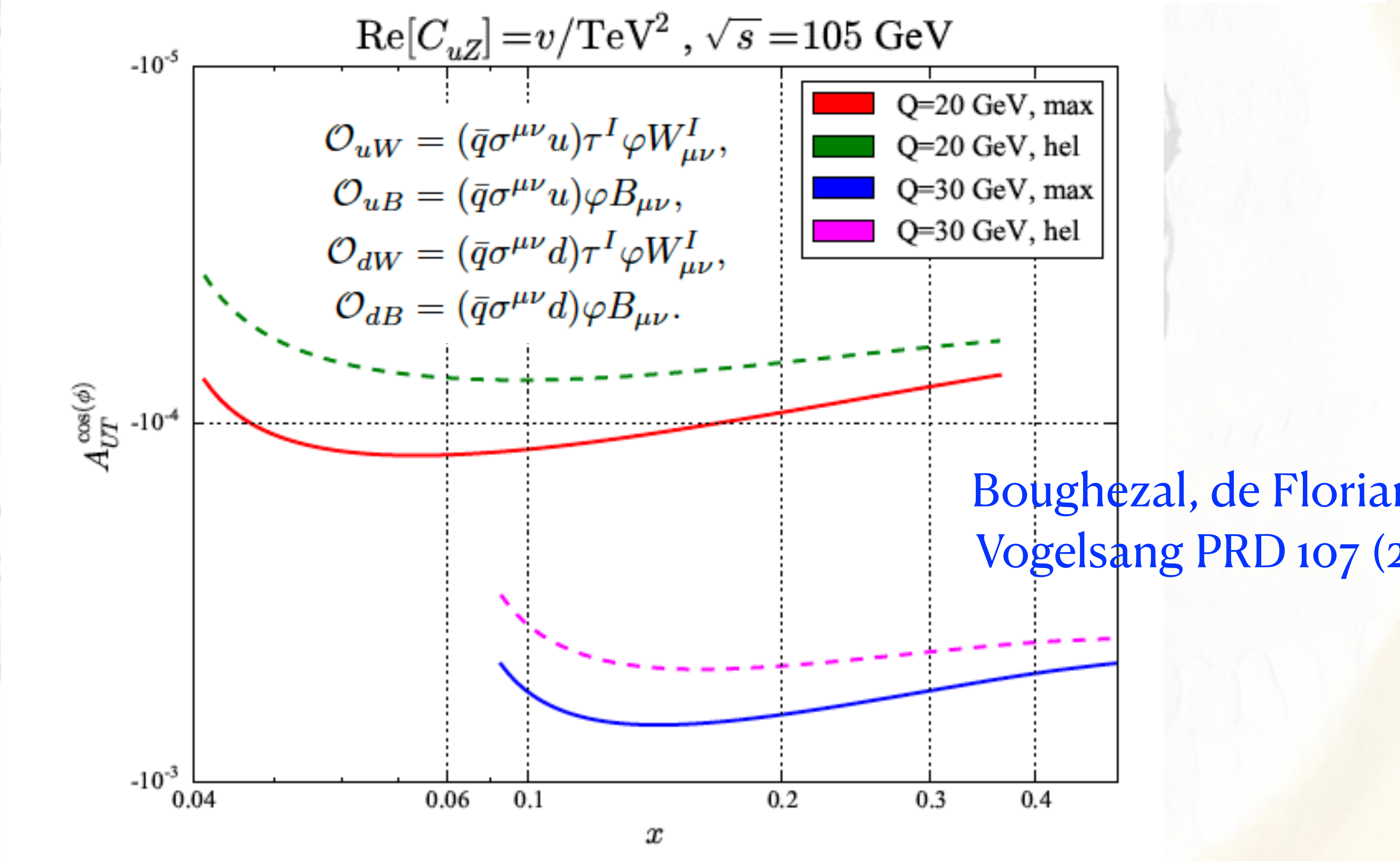
Transverse structure of the proton

- Transversity distribution is the most poorly-determined leading-twist distribution function. Impacts potential BSM measurements of SMEFT dipole coefficients at the EIC.

$$|h_1^q(x,Q^2)| \leq \frac{1}{2}\left[f_{q/p}(x,Q^2) + g_{1L}^q(x,Q^2)\right]$$



Need transversity distributions for future dipole-moment searches at a future EIC

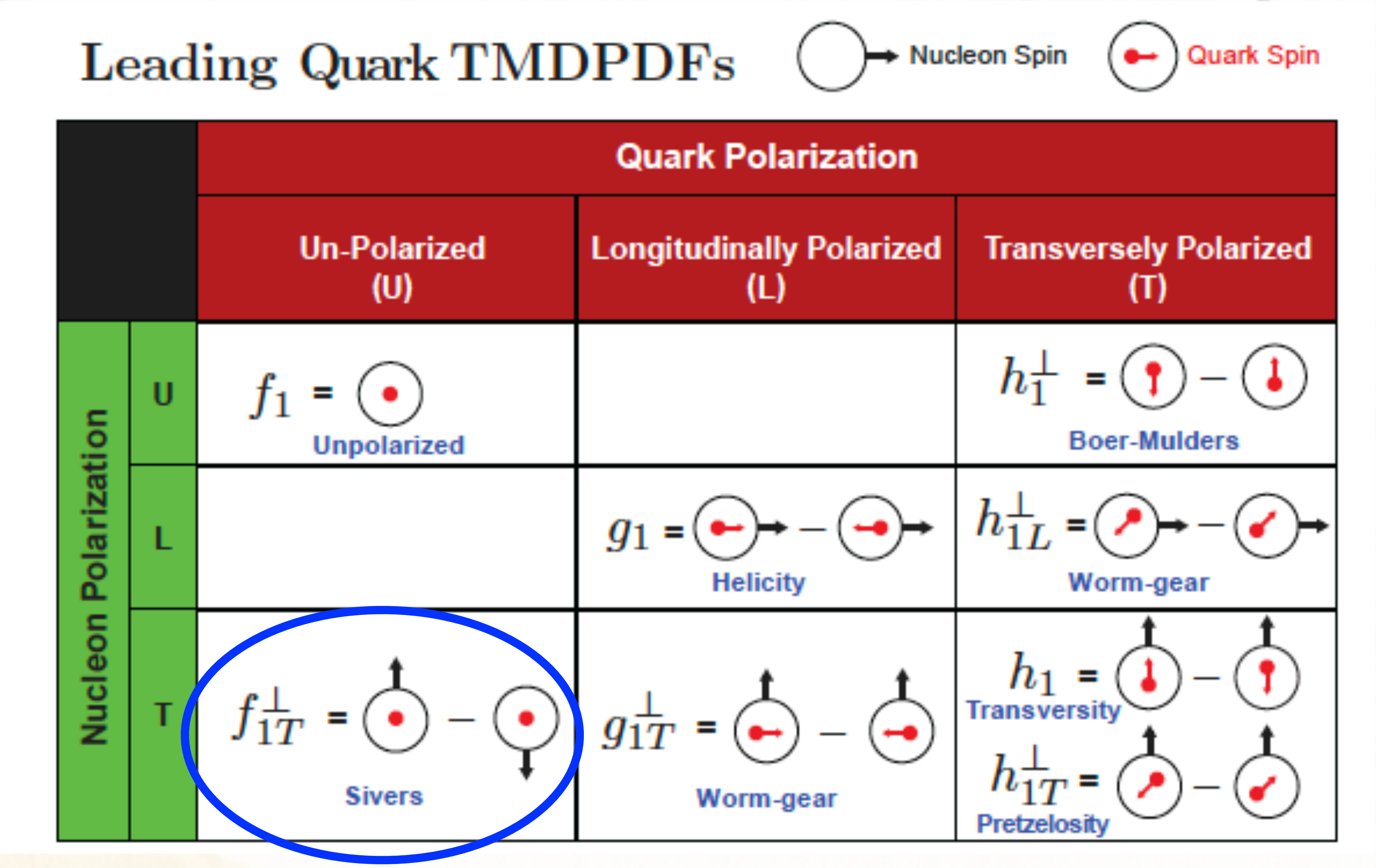


Boughezal, de Florian, FP Vogelsang PRD 107 (2023)

Order-of-magnitude difference in two different ways of treating the Soffer bound

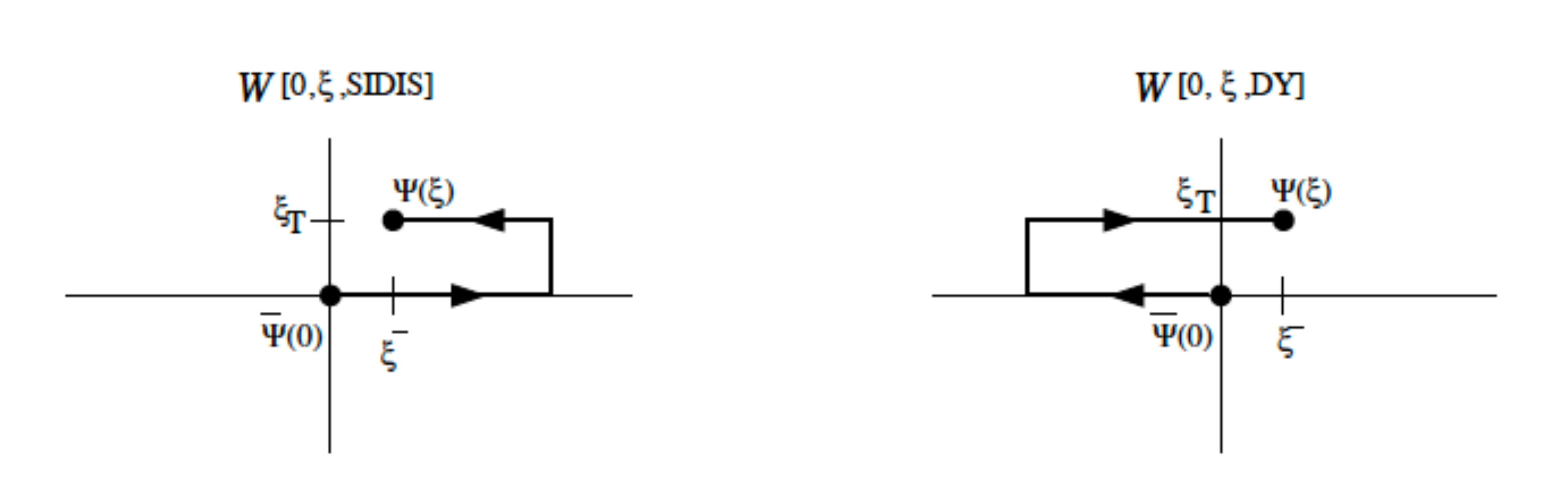
Transverse structure of the proton

- Sivers asymmetry arises from interaction of quarks with nucleon remnants and is sensitive to how gauge links change under initial-to-final state crossing.



$$f_{1T}^{\perp}(x,k_T)_{[\text{SIDIS}]} = -f_{1T}^{\perp}(x,k_T)_{[\text{DY}]}$$

$$\Phi^q(x,p_T) \equiv \int \frac{d\xi^- d^2\xi_T}{2(2\pi)^3} e^{ip\cdot\xi}\langle P,S_T|\bar{\psi}_q(0)\gamma_\mu n_-^\mu \mathcal{W}[0,\xi;\text{process}]\psi_q(\xi)|P,S_T\rangle\Big|_{\xi^+=0}$$
$$= f_1^q(x,p_T^2) + f_{1T}^{\perp q}(x,p_T^2)\frac{\varepsilon_{\mu\nu\rho\sigma}n_-^\mu n_+^\nu p_T^\rho S_T^\sigma}{M_N},$$



Efremov et al hep-ph/0412353

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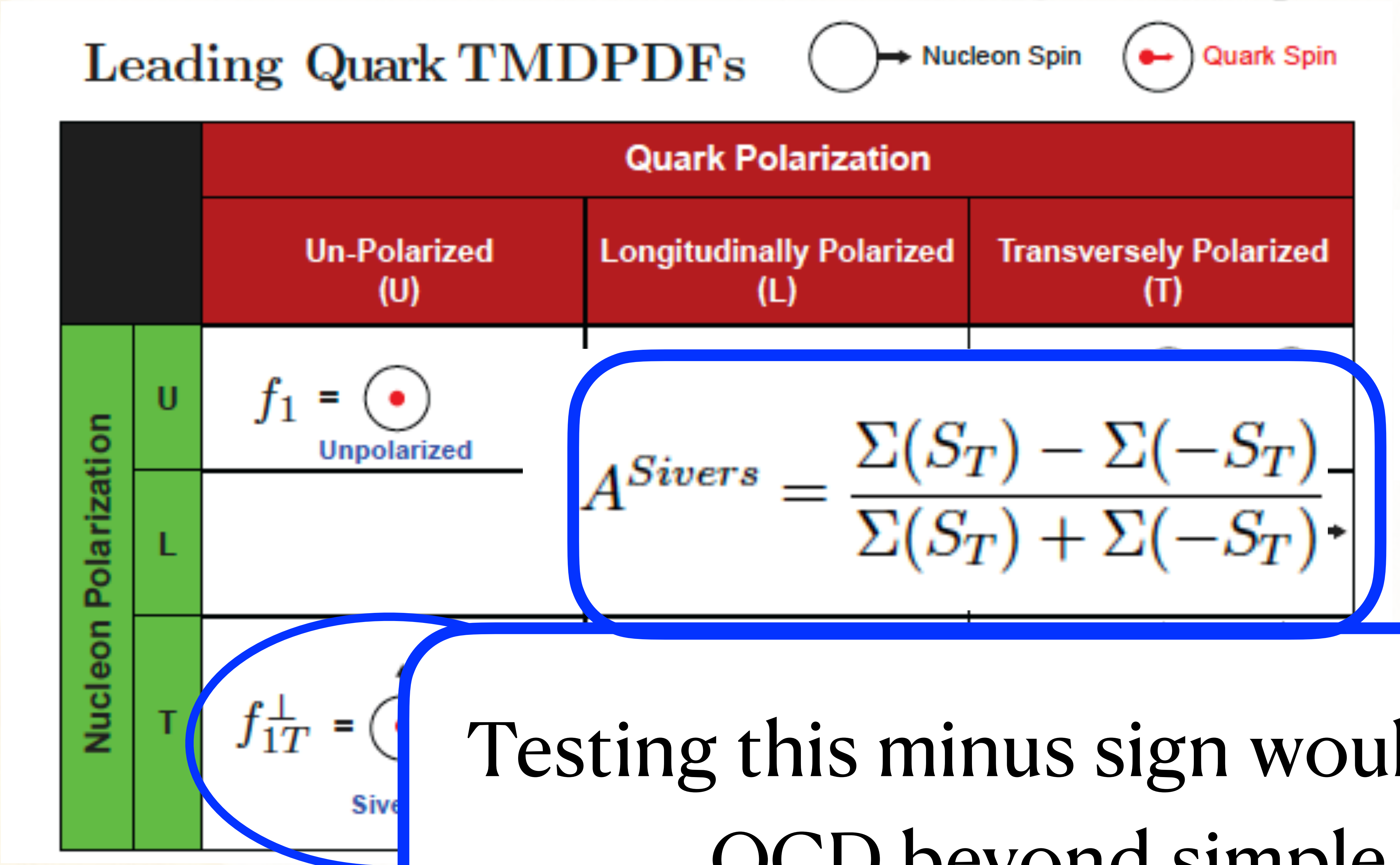
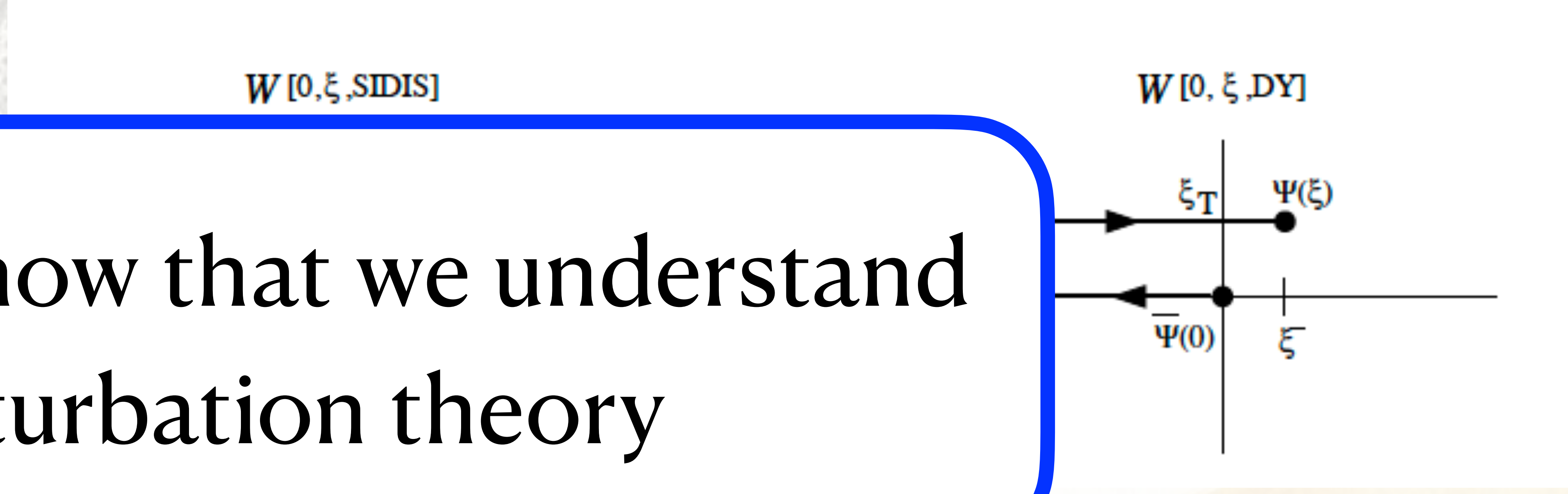


$$A^{Sivers} = \frac{\Sigma(S_T) - \Sigma(-S_T)}{\Sigma(S_T) + \Sigma(-S_T)}$$

$$f_{1T}^{\perp}(x, k_T)_{[\mathrm{SIDIS}]} = -f_{1T}^{\perp}(x, k_T)_{[\mathrm{DY}]}$$

$$\Phi^q(x, \mathrm{p}_T) \equiv \int \frac{\mathrm{d}\xi^- \mathrm{d}^2\xi_T}{2(2\pi)^3} e^{ip\cdot\xi} \langle P, S_T | \bar{\psi}_q(0) \gamma_\mu n_-^\mu \mathcal{W}[0, \xi; \mathrm{process}] \psi_q(\xi) | P, S_T \rangle \Big|_{\xi^+=0}$$
$$= f_1^q(x, \mathrm{p}_T^2) + f_{1T}^{\perp q}(x, \mathrm{p}_T^2) \frac{\epsilon_{\mu\nu\rho\sigma} n_-^\mu n_+^\nu p_T^\rho S_T^\sigma}{M_N},$$



Testing this minus sign would show that we understand QCD beyond simple perturbation theory

Efremov et al hep-ph/0412353

Transverse structure of the proton

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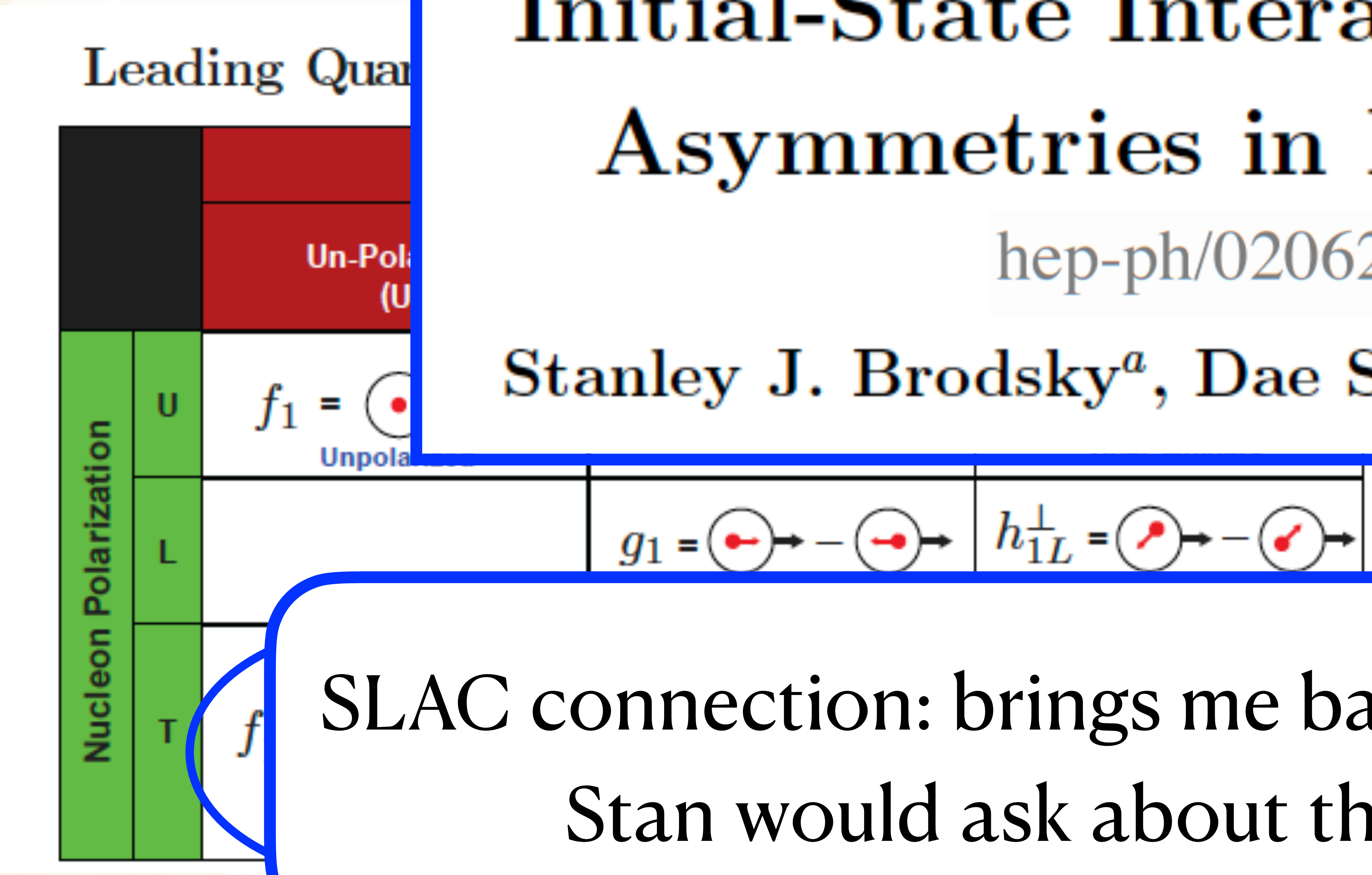


Initial-State Interactions and Single-Spin Asymmetries in Drell-Yan Processes *

hep-ph/0206259v1 25 Jun 2002

Stanley J. Brodsky[a], Dae Sung Hwang[a,b], and Ivan Schmidt[c]

SLAC connection: brings me back to my student days at SLAC when Stan would ask about the BLM scale during a seminar

Efremov et al hep-ph/0412353

Transverse structure of the proton

- Both the Sivers asymmetry and the possibility of probing dipole-moment new physics at the EIC are through single-spin asymmetries, and arise from the behavior of the SM under discrete symmetries.

Recall the transformations of quantum operators under parity and time-reversal:

$$P\, c a_{\vec{p}}^{s} P^{-1} = c a_{-\vec{p}}^{s}$$

$$T\, c a_{\vec{p}}^{s} T^{-1} = c^{*} a_{-\vec{p}}^{-s}$$

c is a c-number; time reversal is an anti-unitary operator

It is useful to also consider a linear transformation related to time-reversal invariance, often called "naive" time-reversal (Sivers 1996):

$$A_t\, c a_{\vec{p}}^{s} A_t^{-1} = c a_{-\vec{p}}^{-s}$$

Note that the combined transformation PA_t leaves a particle momentum unchanged but flips its spin; single-spin asymmetries are necessarily odd under this transformation. Need a phase from a loop to get this effect.

Transverse structure of the proton

- Something I learned about many years ago from Lance, and got advice from when we were working on the transverse SSA probes of dipole moments.

From: Dixon, Lance
Sent: Monday, September 16, 2002 2:50 PM
To: Wagner, Stephen Robert <stevew@slac.stanford.edu>
Cc: Petriello, Frank John <frankjp@slac.stanford.edu>; Melnikov, Kirill <melnikov@slac.stanford.edu>
Subject: KKMC 4.13
mailto:melnikov@slac.stanford.edu

Hi Steve,

We have been thinking a little bit more about your problem of the QED radiative corrections to the e+e- -> mu+mu- forward-backward. But I wouldn't say we've made a lot of progress yet, except perhaps for one result of a literature search, which you may or may not be aware of already:

For your problem, it seems that KORALB has been superceded by KK4.13, which includes not only the O(alpha) QED forward-backward asymmetry, but also log-enhanced higher order terms. See hep-ph/9912214 for the Comp. Phys. Commun. manuscript, and hep-ph/0101246 for a short description of it.
In particular, table 1 of hep-ph/0101246 contains a comparison of KORALB vs. KK4.13 for ISR-FSR interference, indicating that resummation of alpha * L (L = ln(s/m_e^2) I think) is being performed by KK4.13.
Section 4.3 also has a short discussion of ISR-FSR interference, though in the context of LEP2.

$$D(\theta) = \frac{d\sigma(\theta)}{d\Omega} - \frac{d\sigma(\pi-\theta)}{d\Omega}$$

LD
Dixon, Lance J.
May 27, 2022 at 6:25 PM
RE: QED question
To: Francis John Petriello

Hi Frank,

Yes, both North and South Chicagoland visits were delightful! Thanks a lot for taking me to lunch at the faculty place.

The answer to your physics question (and thanks for asking) is: Fritz Berends et al, more specifically, from 1973(!),

https://urldefense.com/v3/__https://inspirehep.net/literature/83686__;!!Dq0X2DkFhyF93HkjWTBQKhk!SF15xznQ8XOznJXpzeWQqYC1w9nfnXJXt8IoPA7Cz64V3l9qgKN0MU-SIHJb6okeb1CkOXPH-V0dNCDOJbUnG6eYH9kWuak$

I thought I remembered that Berends had a later paper where he did more analytically, but I can't find it now. Anyway, I would take a skim through papers by Berends (and also not by Berends) that refer to this paper, in order to see if there is a more up-to-date treatment somewhere.

α^3-CONTRIBUTION TO THE ANGULAR ASYMMETRY IN $e^+e^- \rightarrow \mu^+\mu^-$

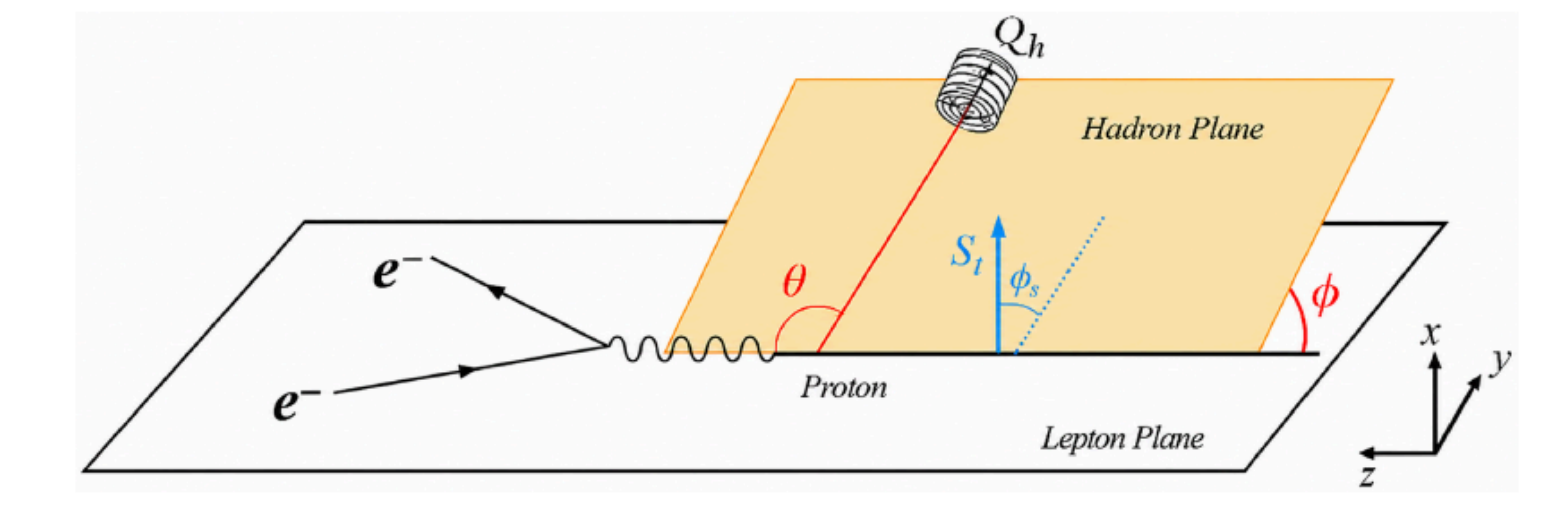
F.A. BERENDS and K.J.F. GAEMERS
Instituut-Lorentz, University of Leiden, Leiden, Netherlands

R. GASTMANS*
Instituut voor Theoretische Fysica, University of Leuven, Leuven, Belgium

Received 25 June 1973

Two-point charge correlator (TPCC)

- Can extend the idea of energy correlators to consider charge correlators as well. This idea will help us access the Sivers asymmetry in DIS. First introduced in electron-positron collisions.



Monni, Vita, Xu, Zhu 2508.00977

$$\mathrm{QQC}(\chi) = \sum_{a,b} \int \mathrm{d}\sigma_{e^+e^- \to a,b+X} \frac{Q_a Q_b}{\sigma_{\mathrm{tot}}} \delta(\cos\chi - \vec{n}_a \cdot \vec{n}_b)$$

Contrast this with the energy-energy correlator:

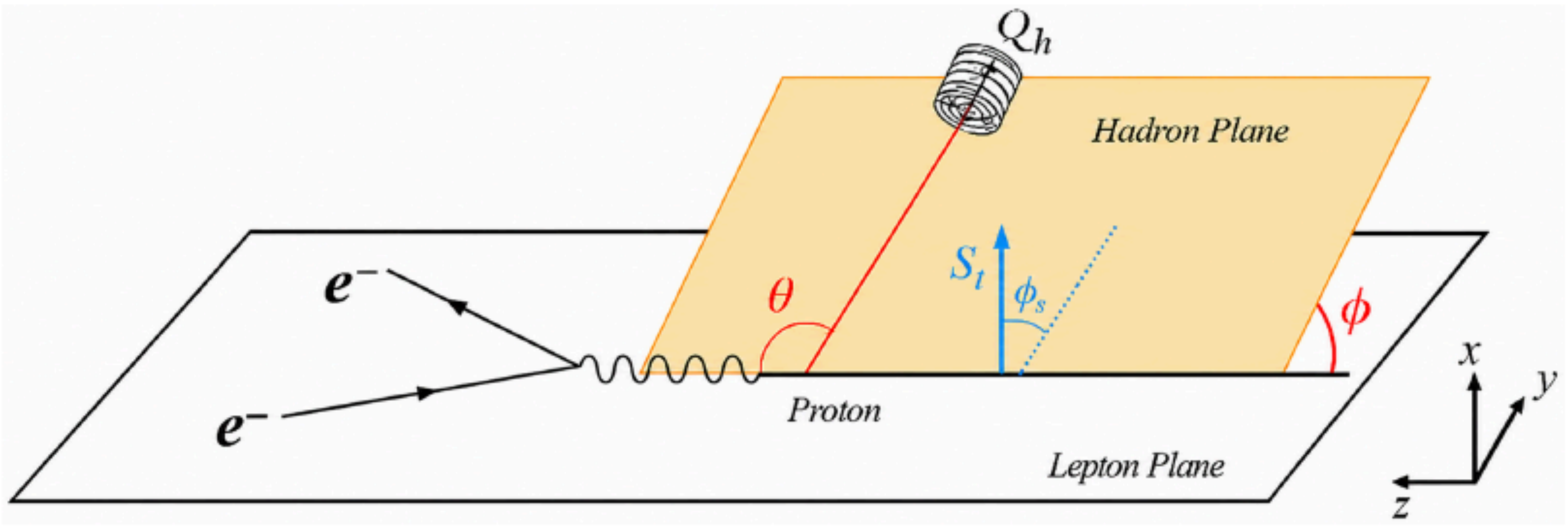
$$\mathrm{EEC}(z) = \frac{1}{\sigma} \sum_{i,j} \int d\sigma \, \frac{E_i E_j}{Q^2} \delta\left(z - \frac{1-\cos\chi_{ij}}{2}\right)$$

Moult, Zhu 2506.09119

EEC is naturally IR-safe in the soft limit; not the case for the TPCC

One-point charge correlator (OPCC)

- We can extend the idea to one-point charge correlators for the purpose of measurements in future DIS experiments such as the EIC.



$$\Sigma(\vec{n}, x_B, Q^2) = \sum_h \int d\sigma_{ep^\uparrow \to eh+X} \, Q_h \, \delta(\vec{n} - \vec{n}_h)$$

Different systematics than other measurements

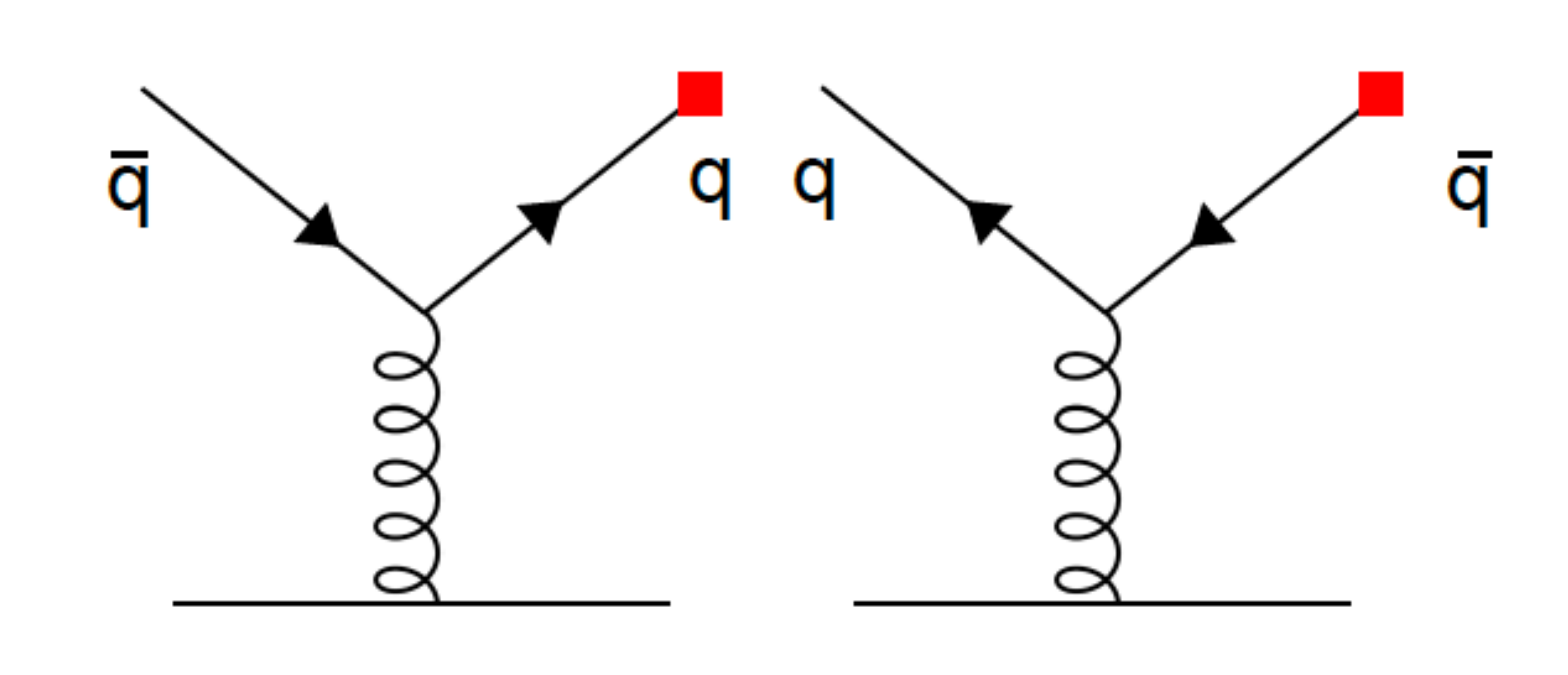
- No neutral hadron reconstruction needed
- No energy measurement

For transverse-spin observables can write as:

$$\Sigma(\theta, \phi) = \sum_i \int d\sigma_{ep \to e+i+X} \, Q_i \, \Theta(\theta - \theta_i) \delta(\phi - \phi_i)$$

One-point charge correlator (OPCC)

- We can extend the idea to one-point charge correlators for the purpose of measurements in future DIS experiments such as the EIC.



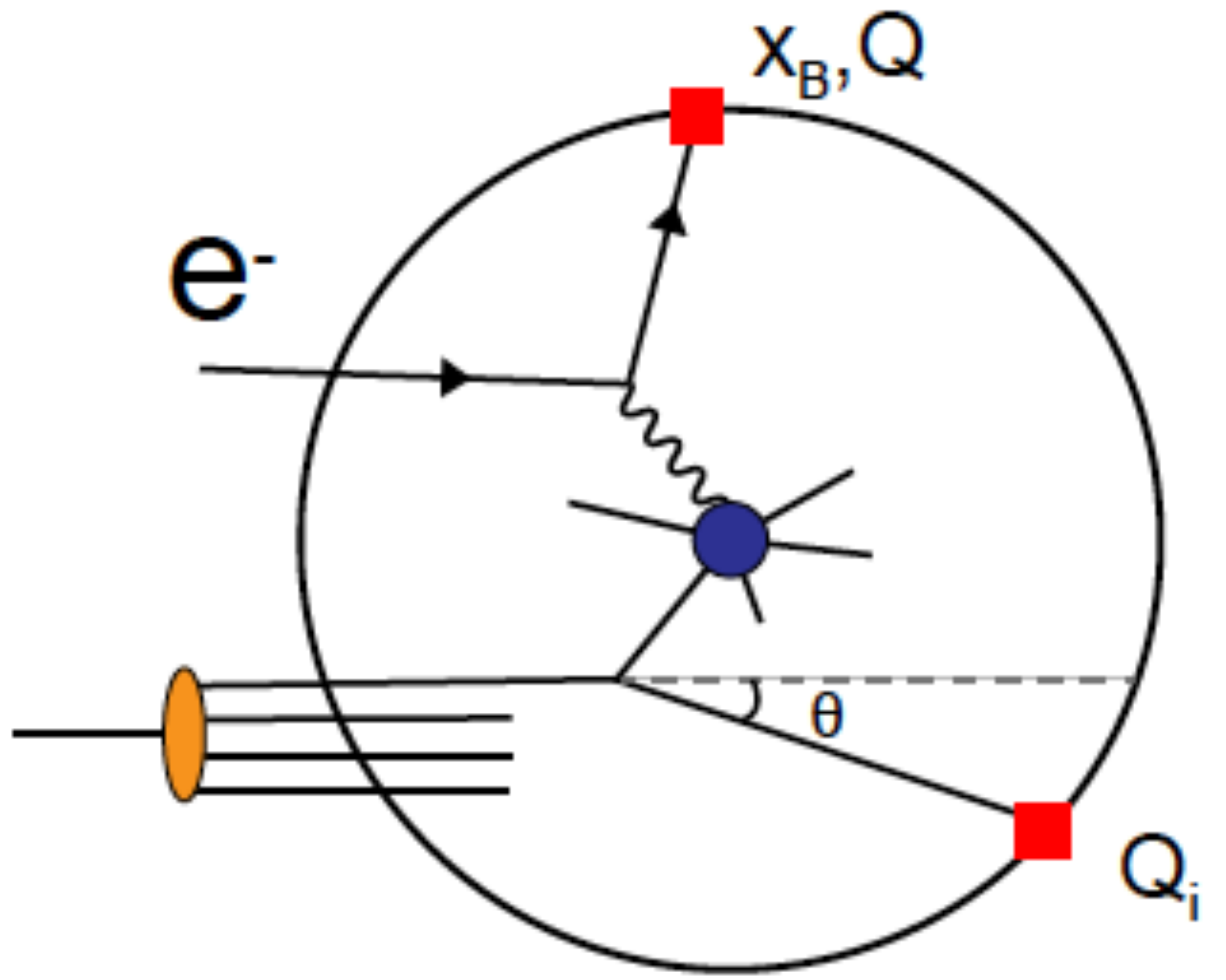
Monni, Vita, Xu, Zhu 2508.00977

Non-local cancellation; charge conservation forces IR safety after summing over quark+antiquark

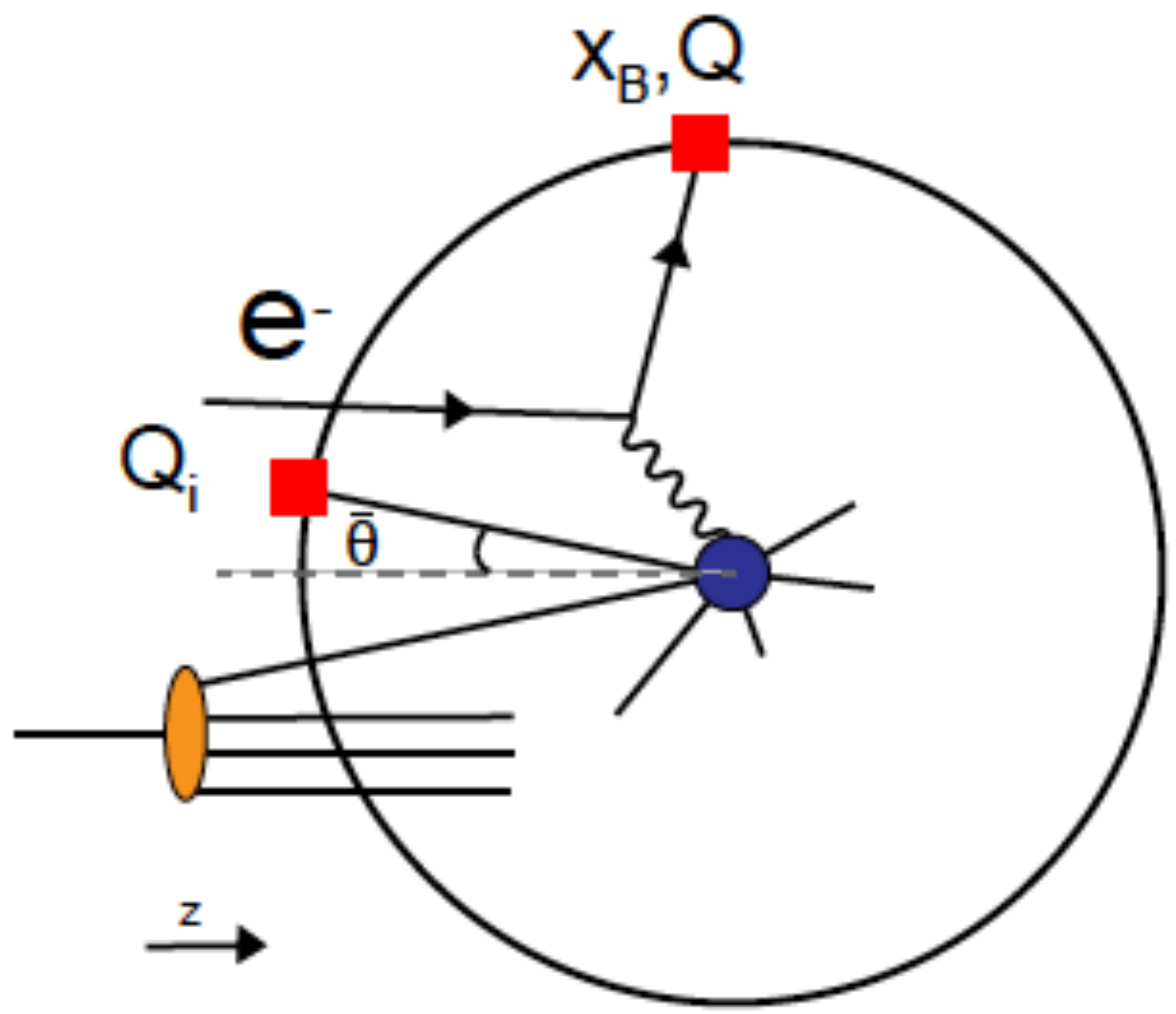
Matrx elements symmetric under q to qbar; charge-operator insertion anti-symmetric; soft poles cancel in the sum

One-point charge correlator (OPCC)

- Two physical limits of the Breit-frame polar angle are of interest.



Target fragmentation: $\theta \ll 1$



TMD region: $\bar{\theta} = \pi - \theta \ll 1$

Description in the TMD region

- One can factorize the OPCC in the TMD region using the SCET framework.

$$A^{Sivers} = \frac{\Sigma(S_T) - \Sigma(-S_T)}{\Sigma(S_T) + \Sigma(-S_T)}$$

Hallmark of Sivers asymmetry

$$d\sigma = F_{UU} + |S_\perp| \sin(\phi - \phi_S) F_{UT}^{\sin(\phi-\phi_S)}$$

$$F_{UT} = \sigma_0 H(Q^2,\mu) \frac{\bar{\theta} Q^2}{4} \int_0^\infty \frac{b^2\, db}{4\pi} J_1\left(b\frac{Q\bar{\theta}}{2}\right) \times \sum_f e_f^2 J_{f,\mathcal{Q}}(b,\mu,\nu) f_{1T}^{\perp,f}(x_B,b,\mu,\nu) S(b,\mu,\nu)$$

$$J_{f,\mathcal{Q}} = \sum_i \int dz\, Q_i \mathcal{I}_{i\leftarrow f}(z,b)$$

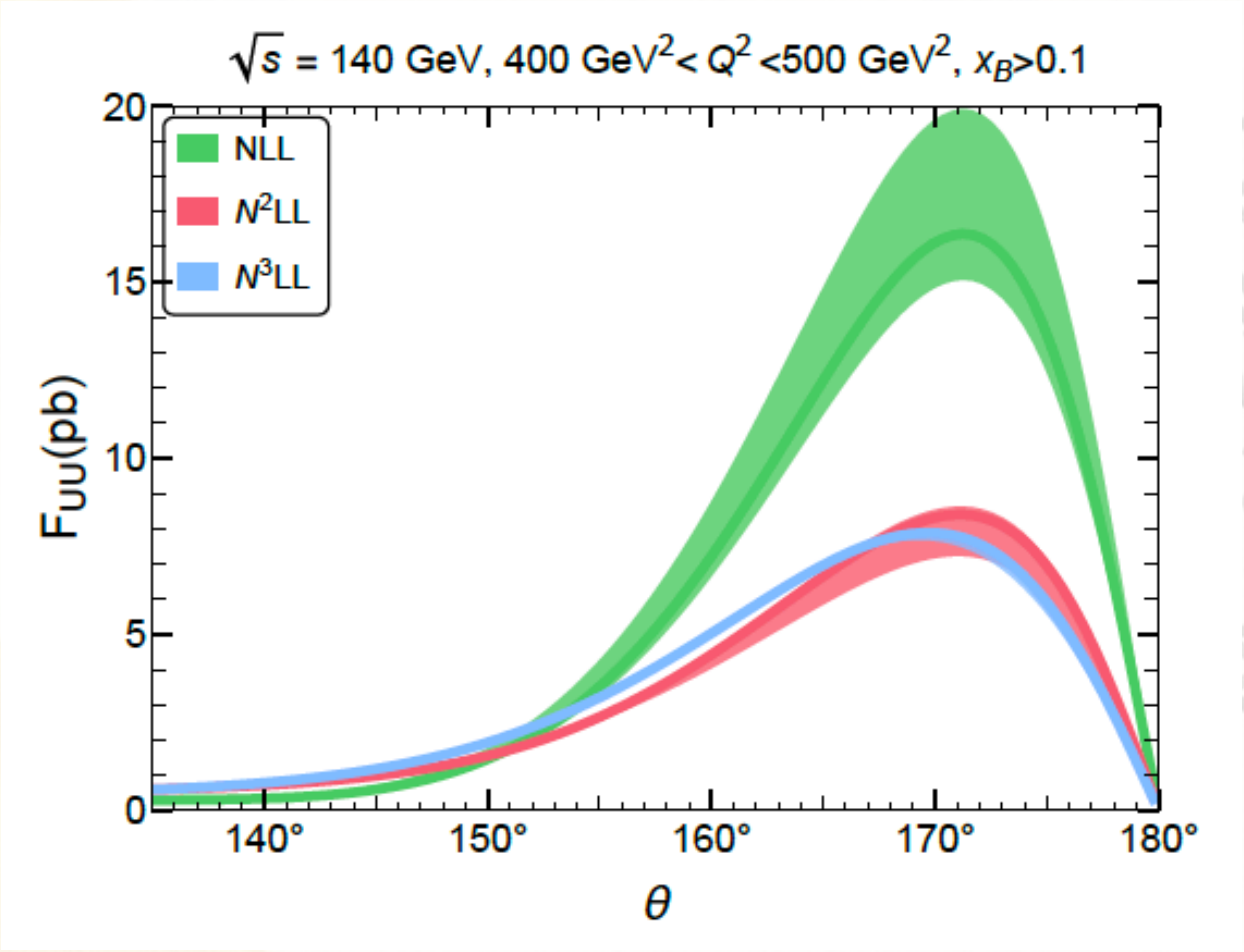
Matches to the twist-3 Qiu-Sterman function (quark-gluon correlation in proton)

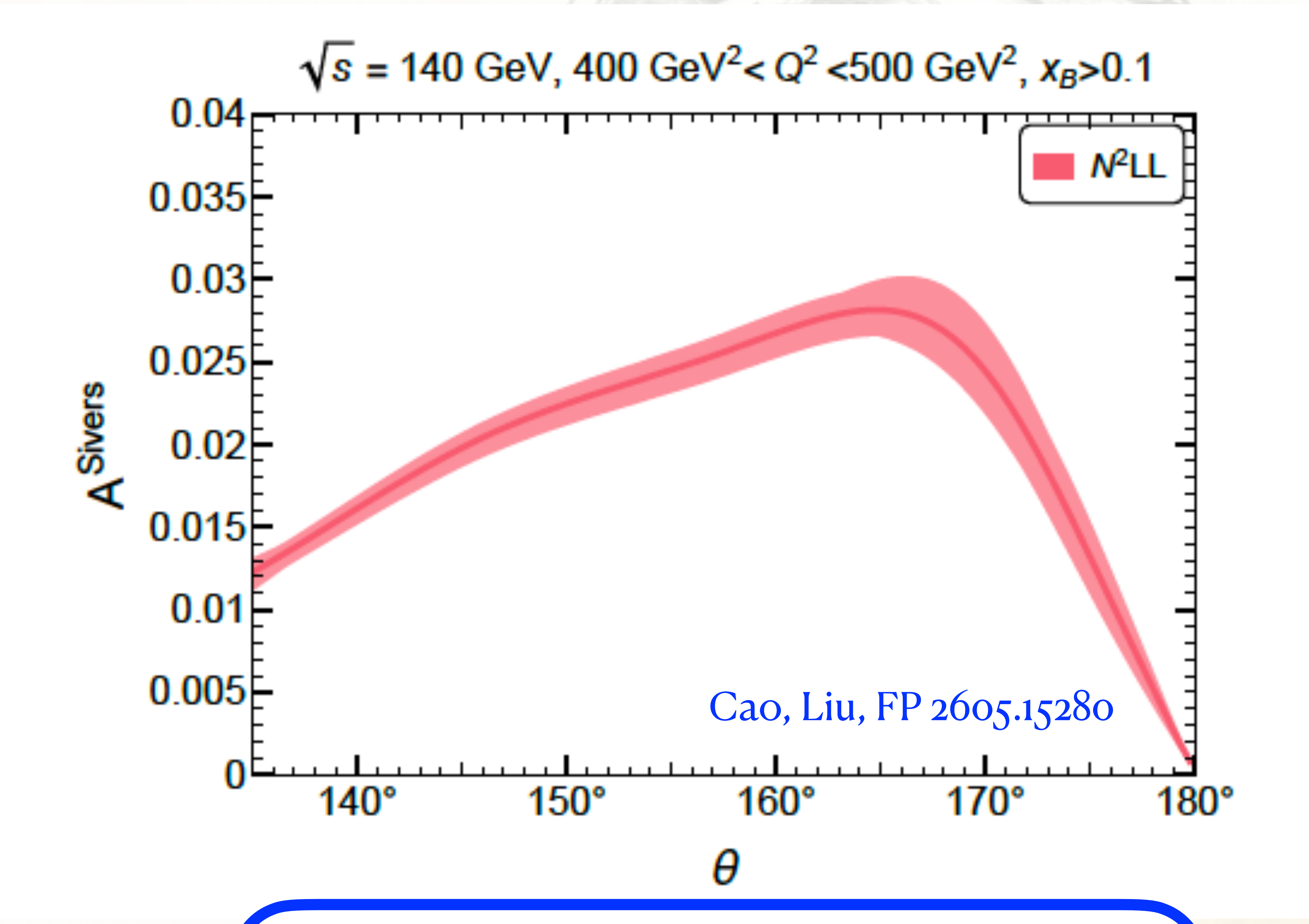
TMD soft function

Perturbative matching coefficient

Numerics in the TMD region

- Stable perturbation theory for the OPCC, and a sizable Sivers asymmetry at a future EIC.





$$b^* = b/\sqrt{1+b^2/b_{\max}^2}$$

$$\exp\left(-\frac{g_2}{2}\ln\frac{Q}{Q_0}\ln\frac{b}{b_*} - g_1 b^2\right)$$

Sizable and predictable effect at future electron-ion colliders

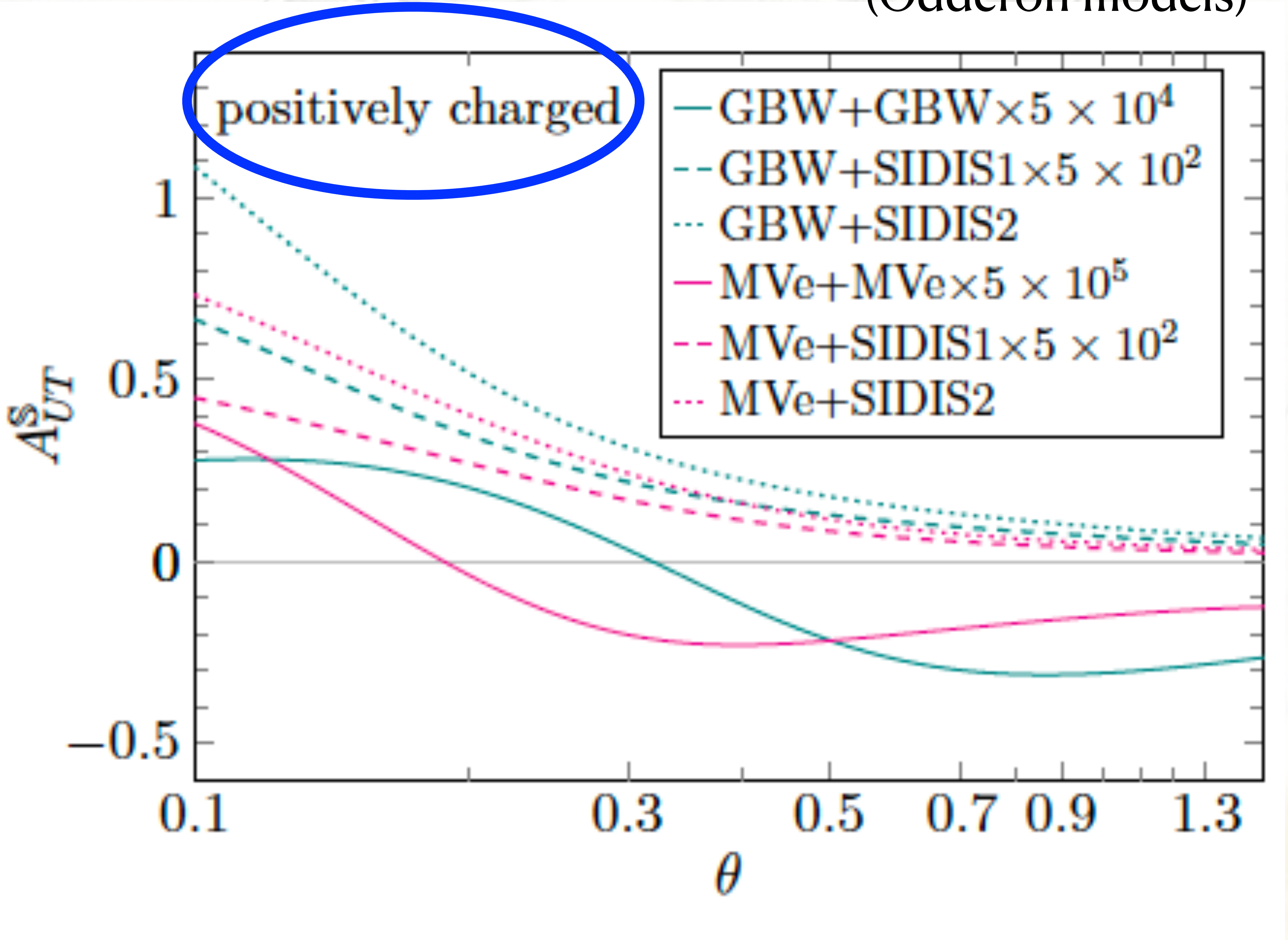
Description in the TFR

- In the target fragmentation region factorizes into the partonic DIS cross section and a nucleon charge correlator. Similar to nucleon energy correlator also proposed to study various aspects of nucleon structure.

$$\Sigma(\theta) = \sum_{i=q,g} \int \frac{dz}{z} \hat{\sigma}_i \left(\frac{x_B}{z}, Q^2 \right) f_{i,\mathrm{QC}}(z, \theta)$$

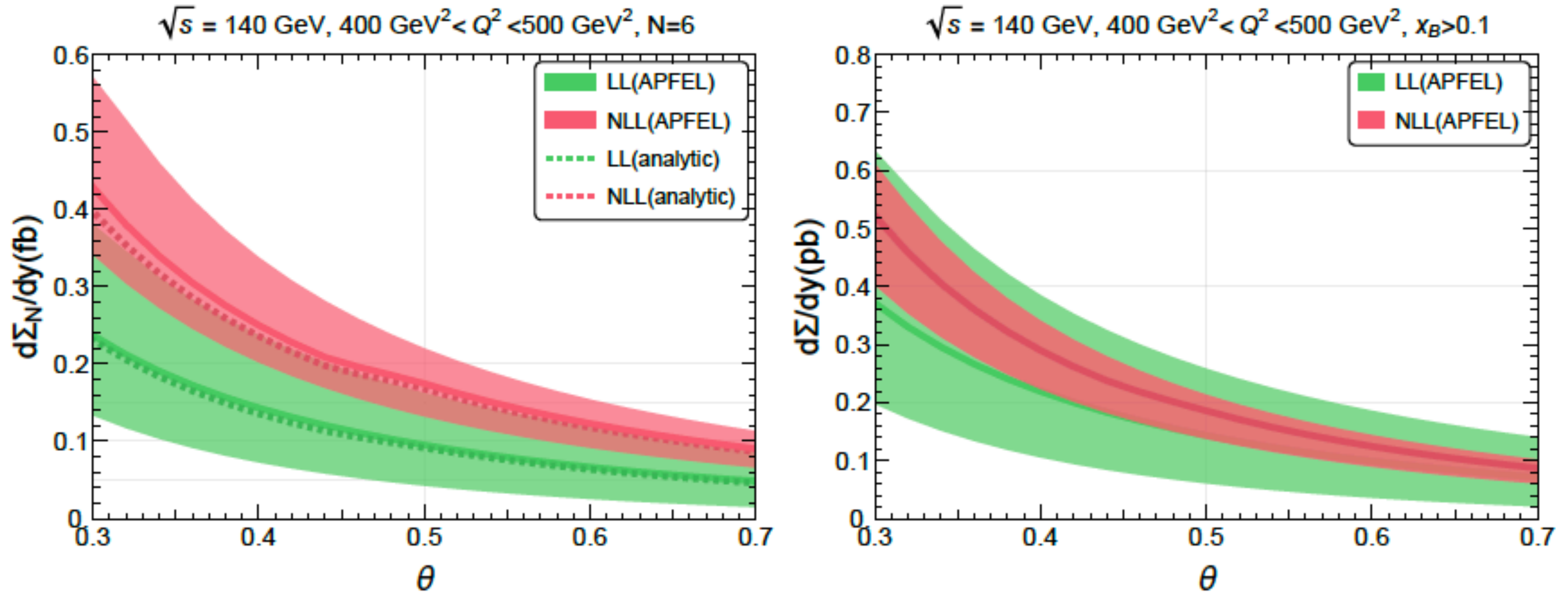
The NEC often has restriction to a charged subset, which is natural here. Expect lower experimental uncertainty here without energy reconstruction.

(Odderon models)



Mantysaari, Tawabutr, Tong PRD 112 (2025)

Numerics in the TFR



Cao, FP to appear

$$f_{i,\mathrm{QC}}(z,\theta) = Q_P f_i(z) - \int_z^1 \frac{d\xi}{\xi} I_{ij}\left(\frac{z}{\xi},\theta\right) f_j(\xi)$$

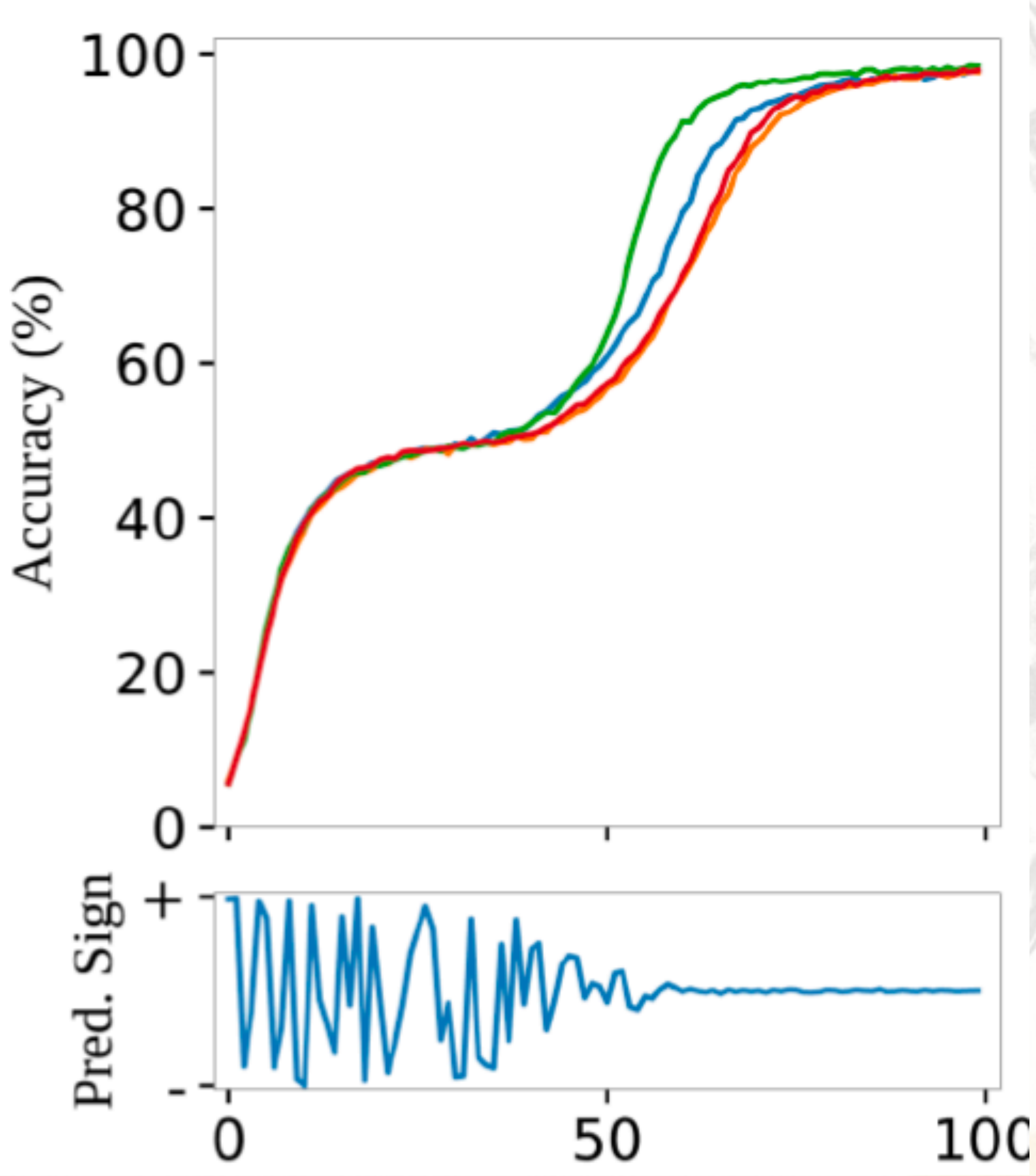
Matches to the standard collinear PDF;
excellent perturbative behavior

Conclusions

- Also from the organizers: "As far as I'm concerned, jokes are also encouraged!"

- Train model to "learn" symbol of $F_3^{(L)}$ using a subset of the 5 million terms at $L = 6$ loops

Lance has recently been working on teaching AI and LLMs to predict coefficients in N=4 SYM amplitudes



Cai et al Mach. Learn. Sci. Tech. 5 (2024)

Conclusions

- Also from the organizers: "As far as I'm concerned, jokes are also encouraged!"

SPACE, PHYSICS, AND MATH

Twenty Questions, Ten Million Synapses

Forget chess -- this AI has mastered a classic parlor game.

KAREN SCHROCK • JULY 28, 2006

Q: Is it an animal, vegetable, mineral or other?

A: Other.

Old British TV show had experts examine an object and guess. Early neural net training used the same concept through the 20Q project

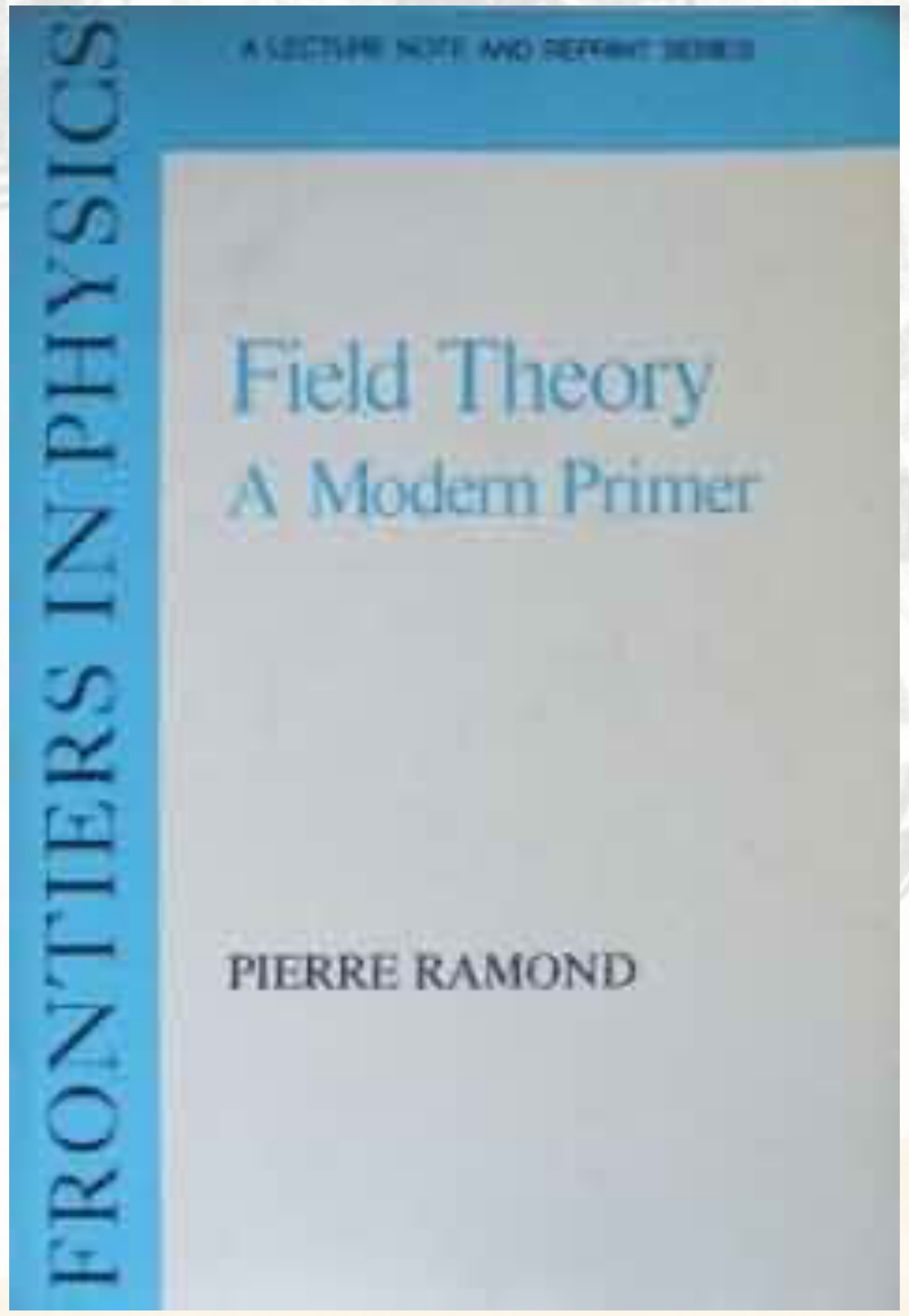
[Q2] Is 20Q an Object? **MAYBE**

20Q is an object, a website, a company, and a phenomenon. It first gained popularity as an online game (20Q.net) where users log onto the website and play against an artificial intelligence (A.I.) foe. Players think of an animal, vegetable, mineral, or other object and 20Q guesses what the player is thinking in twenty questions or less. And, the more people play, the more the game "learns." 20Q.net gets 1,000,000 impressions per month, has asked more than one and a half billion questions and played its 79,000,000th game in December 2010!

Conclusions

- Also from the organizers: "As far as I'm concerned, jokes are also encouraged!"

This is always what I felt Lance was doing when I went to ask him a question as a student!




Happy Birthday Lance!
B(z) = C_F^2 B_lc(z) + C_F(C_A - 2C_F) B_nlc(z) + C_F N_f T_f B_Nf(z)
"Lance likes getting a number"