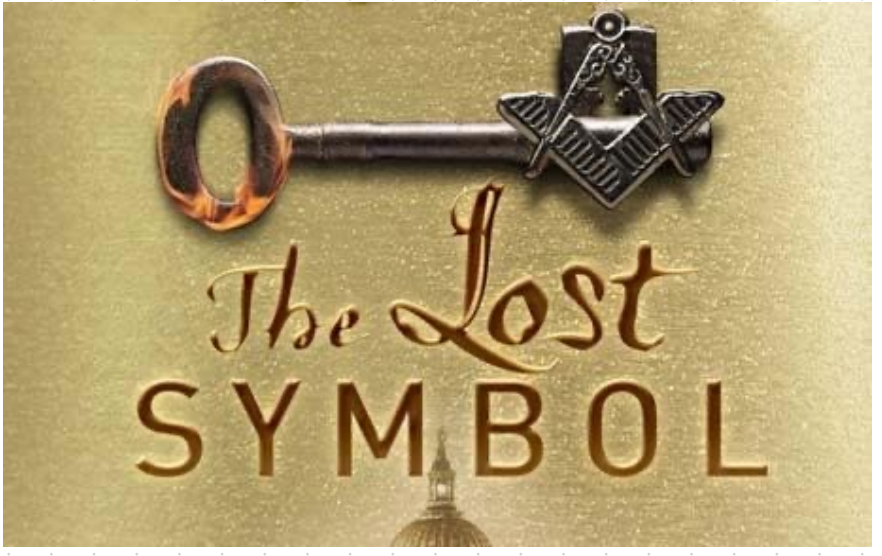

Lancefest Workshop, Edinburgh 2026

James Drummond

Planar Amplitudes in Maximally Supersymmetric Yang-Mills Theory

C. Anastasiou and L. Dixon
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94309, USA

Z. Bern
Department of Physics and Astronomy, UCLA, Los Angeles, California 90095-1547, USA

D. A. Kosower
Service de Physique, Centre d'Etudes de Saclay, F-91191 Gif-sur-Yvette cedex, France
(Received 7 September 2003; published 18 December 2003)

The collinear factorization properties of two-loop scattering amplitudes in dimensionally regulated $N = 4$ super-Yang-Mills theory suggest that, in the planar ('t Hooft) limit, higher-loop contributions can be expressed entirely in terms of one-loop amplitudes. We demonstrate this relation explicitly for the two-loop four-point amplitude and, based on the collinear limits, conjecture an analogous relation for n-point amplitudes. The simplicity of the relation is consistent with intuition based on the anti–de Sitter/conformal field theory correspondence that the form of the large-N_c L-loop amplitudes should be simple enough to allow a resummation to all orders.

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Iteration of planar amplitudes in maximally supersymmetric Yang-Mills theory at three loops and beyond

Zvi Bern
Department of Physics and Astronomy, UCLA, Los Angeles, California 90095-1547, USA

Lance J. Dixon
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94309, USA

Vladimir A. Smirnov
Nuclear Physics Institute of Moscow State University, Moscow 119992, Russia
(Received 25 June 2005; published 4 October 2005)

We compute the leading-color (planar) three-loop four-point amplitude of $N = 4$ supersymmetric Yang-Mills theory in $4 - 2\epsilon$ dimensions, as a Laurent expansion about $\epsilon = 0$ including the finite terms. The amplitude was constructed previously via the unitarity method, in terms of two Feynman loop integrals, one of which has been evaluated already. Here we use the Mellin-Barnes integration technique to evaluate the Laurent expansion of the second integral. Strikingly, the amplitude is expressible, through the finite terms, in terms of the corresponding one- and two-loop amplitudes, which provides strong evidence for a previous conjecture that higher-loop planar $N = 4$ amplitudes have an iterative structure. The infrared singularities of the amplitude agree with the predictions of Sterman and Tejeda-Yeomans based on resummation. Based on the four-point result and the exponentiation of infrared singularities, we give an exponentiated Ansatz for the maximally helicity-violating n-point amplitudes to all loop orders. The $1/\epsilon^2$ pole in the four-point amplitude determines the soft, or cusp, anomalous dimension at three loops in $N = 4$ supersymmetric Yang-Mills theory. The result confirms a prediction by Kotikov, Lipatov, Onishchenko and Velizhanin, which utilizes the leading-twist anomalous dimensions in QCD computed by Moch, Vermaseren and Vogt. Following similar logic, we are able to predict a term in the three-loop quark and gluon form factors in QCD.

DOI: 10.1103/PhysRevD.72.085001 PACS numbers: 11.15.Bt, 11.25.Db, 11.25.Tq, 11.55.Bq

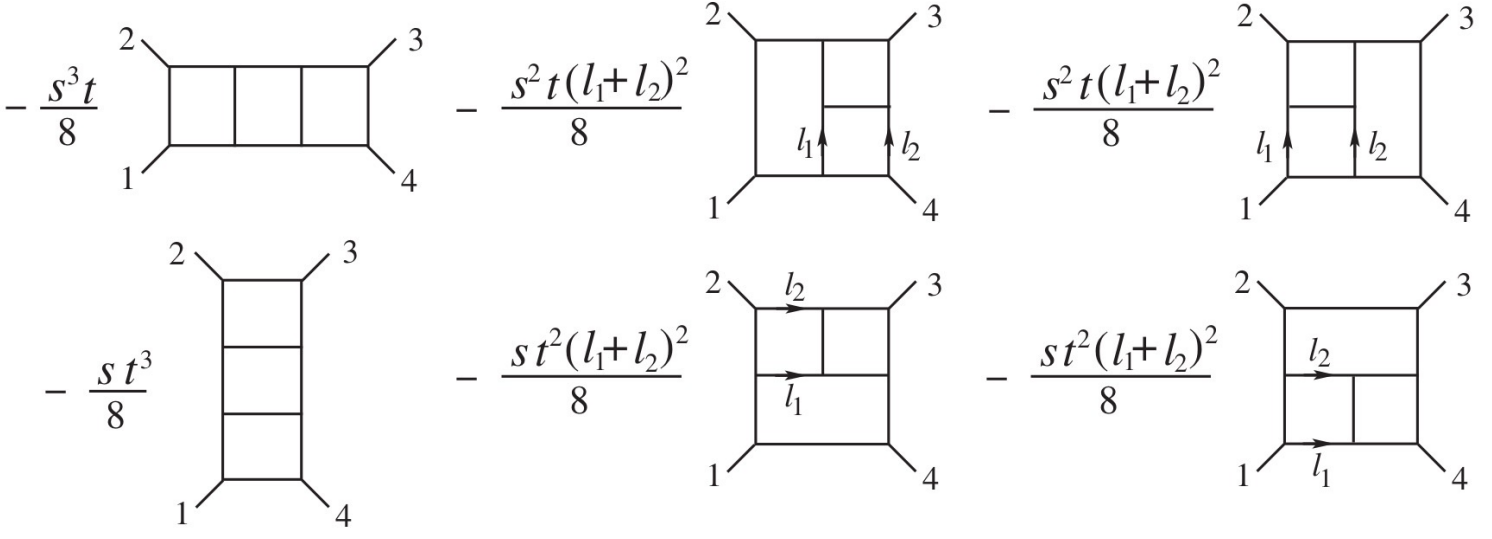


FIG. 3. Mondrian diagrams for the three-loop four-point MSYM planar amplitude given in Eq. (2.5). The second and third diagrams have identical values, as do the fifth and sixth. The factors of $(l_1 + l_2)^2$ denote numerator factors appearing in the integrals, where l_1 and l_2 are the momenta carried by the lines marked by arrows.

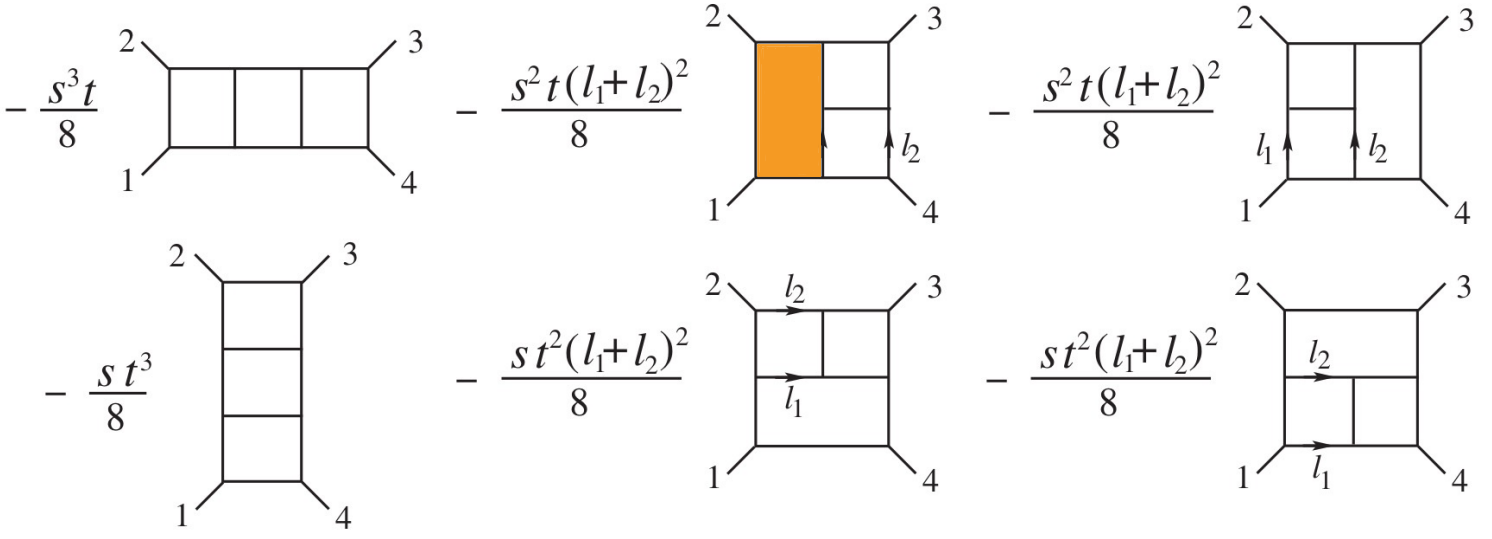


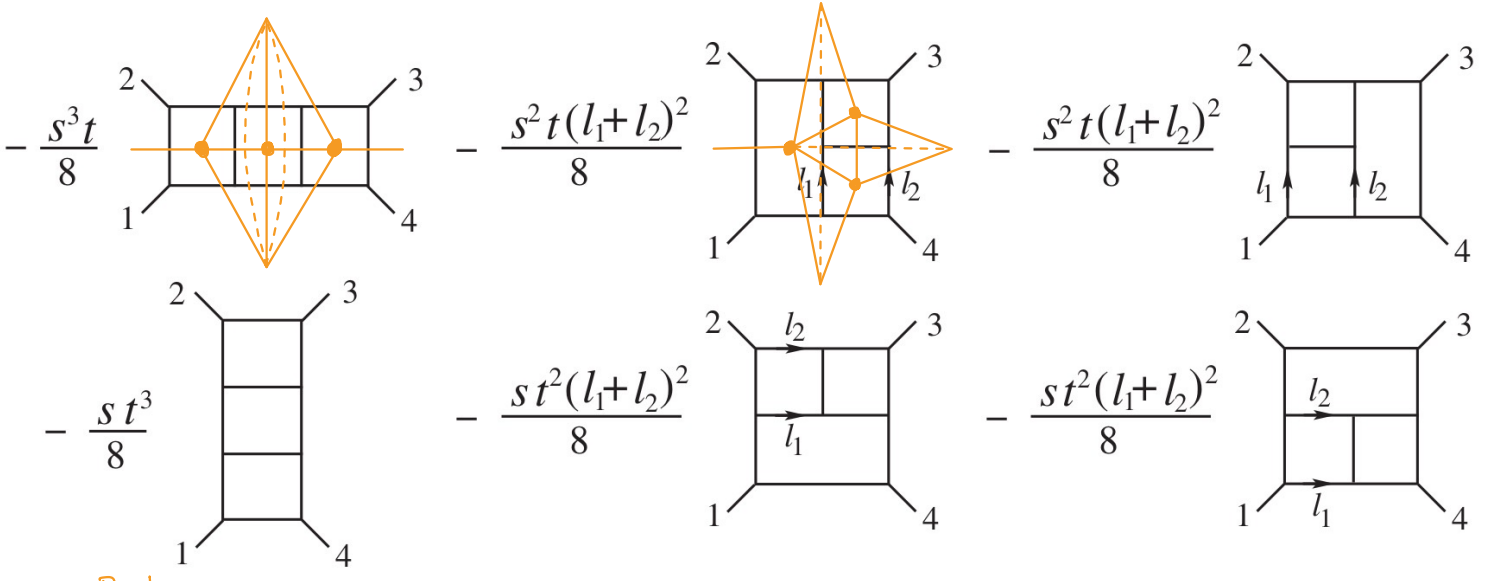
FIG. 3. Mondrian diagrams for the three-loop four-point MSYM planar amplitude given in Eq. (2.5). The second and third diagrams have identical values, as do the fifth and sixth. The factors of $(l_1 + l_2)^2$ denote numerator factors appearing in the integrals, where l_1 and l_2 are the momenta carried by the lines marked by arrows.

"The realisation of equivalent relationships is of the highest importance for life"

– Piet Mondrian

Dual conformal integrals:

(JMD, Henn, Smirnov, Sokatchev '06)



Dual

FIG. 3. Mondrian diagrams for the three-loop four-point MSYM planar amplitude given in Eq. (2.5). The second and third diagrams have identical values, as do the fifth and sixth. The factors of $(l_1 + l_2)^2$ denote numerator factors appearing in the integrals, where l_1 and l_2 are the momenta carried by the lines marked by arrows.

$(p_i = x_{i+1} - x_i)$ If $p_i^2 \neq 0$ then $\Phi^{(3)} = \Psi^{(3)}$ But $p_i^2 = 0 \rightarrow$ divergences ... ?

Gluon scattering amplitudes at strong coupling

Luis F. Alday[ab] **and Juan Maldacena**[b]

[a] *Institute for Theoretical Physics and Spinoza Institute, Utrecht University, 3508 TD Utrecht, The Netherlands*
[b] *School of Natural Sciences, Institute for Advanced Study Princeton, NJ 08540, U.S.A.*
E-mail: alday@phys.uu.nl, malda@ias.edu

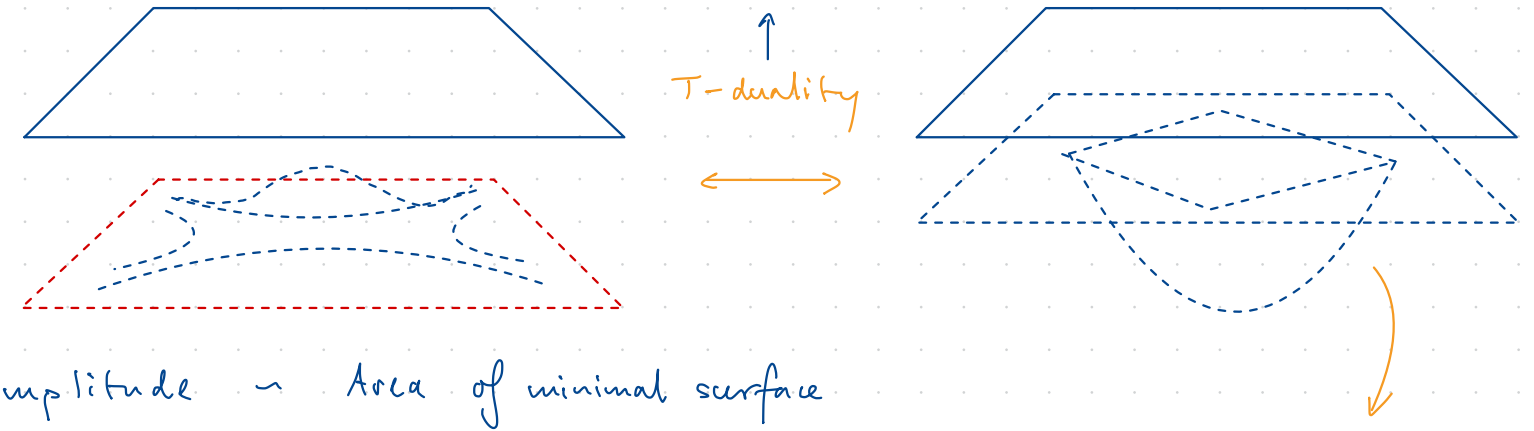
Abstract: We describe how to compute planar gluon scattering amplitudes at strong coupling in $\mathcal{N} = 4$ super Yang Mills by using the gauge/string duality. The computation boils down to finding a certain classical string configuration whose boundary conditions are determined by the gluon momenta. The results are infrared divergent. We introduce the gravity version of dimensional regularization to define finite quantities. The leading and subleading IR divergencies are characterized by two functions of the coupling that we compute at strong coupling. We compute also the full finite form for the four point amplitude and we find agreement with a recent ansatz by Bern, Dixon and Smirnov.

Keywords: Strong Coupling Expansion, Supersymmetric gauge theory, AdS-CFT Correspondence.

JHEP06(2007)064

Alday, Maldacena '07 : (c.f. Piet Mondrian)

$p_i = x_{i+1} - x_i$



Amplitude ~ Area of minimal surface

Light-like Wilson loop $x_1, \ldots, x_4$

$$A_4 \sim W_4 \sim \left[\prod_i D_i\right] \exp\left\{\Gamma_{cusp} F_4\right\}$$

strong coupling: matches ABDK/BDS exponentiation

Perhaps surprising: $A_4 \sim W_4$ at one loop (JMD, Kordhamsky, Sokatchev '07)

$A_n^{MHV} \sim W_n$ " " " (Brandhuber, Heslop, Travaglini '07)

(MHV) Amplitude / WL duality $\Rightarrow$ Amplitude must have symmetries of WL

$\downarrow$

Dual conformal symmetry

$\Rightarrow$ (Dual) conformal Ward identity: (JMD, Henn, Korchemsky, Sokatchev '07)

$$K_\mu F_n = \sum_i \left(x_{i\mu} x_i^\nu \partial_{i\nu} - \tfrac{1}{2} x_i^2 \partial_{i\mu} \right) F_n$$

$$= \Gamma_{\text{cusp}} \sum_i \left(2x_{i\mu} - x_{i+1\mu} - x_{i-1\mu} \right) \log x^2_{i-1,i+1}$$

$\Longrightarrow$ ABDK/BDS exponentiation for $n = 4, 5$

$$F_n = \Gamma_{\text{cusp}} f_n + R_n$$

f_n: can compute from one-loop answer

R_n: 'remainder'-function of cross-ratios, e.g.

$$u = \frac{x_{13}^2 x_{46}^2}{x_{14}^2 x_{36}^2} \qquad v = \frac{x_{24}^2 x_{15}^2}{x_{25}^2 x_{14}^2} \qquad w = \frac{x_{35}^2 x_{26}^2}{x_{36}^2 x_{25}^2}$$

for $n = 6$.

So does amplitude / WL duality hold more generally?

What about exponentiation?

- Alday Maldacena WL computation for large n
- 2-loop hexagon WL computation (numerical) ⟶ $R_6^{(2)} \neq 0$ for WL
  (JMD, Henn, Korchemsky, Sokatchev) '07
- Bartels, Lipatov, Sabio Vera looked at multi-Regge kinematics '08

contradictions with exponentiation

⇓

- Bern, Dixon, Kosower, Roiban, Spradlin, Vergu, Volovich '08
- JMD, Henn, Korchemsky, Sokatchev '08

Amp / WL duality holds for $n=6$ at two loops.

Lots of fun comparing numerics!

kinematic point	(u_1, u_2, u_3)	$R_A - R_A^0$	$R_W - R_W^0$
$K^{(1)}$	$(1/4, 1/4, 1/4)$	-0.018 ± 0.023	$< 10^{-5}$
$K^{(2)}$	$(0.547253,\ 0.203822,\ 0.881270)$	-2.753 ± 0.015	-2.7553
$K^{(3)}$	$(28/17, 16/5, 112/85)$	-4.7445 ± 0.0075	-4.7446
$K^{(4)}$	$(1/9, 1/9, 1/9)$	4.12 ± 0.10	4.0914
$K^{(5)}$	$(4/81, 4/81, 4/81)$	10.00 ± 0.50	9.7255

These two columns agree!

2008 : I met Lance, David & Zvi at ENS

during a visit to Paris by Emery, Johannes & myself.

Among other things we discussed what happens for non-MHV amplitudes ...

Eventually → Dual superconformal symmetry (JMD, Henn, Korchemsky, Sokatchev)

Fermionic T-duality (Berkovits, Maldacena)

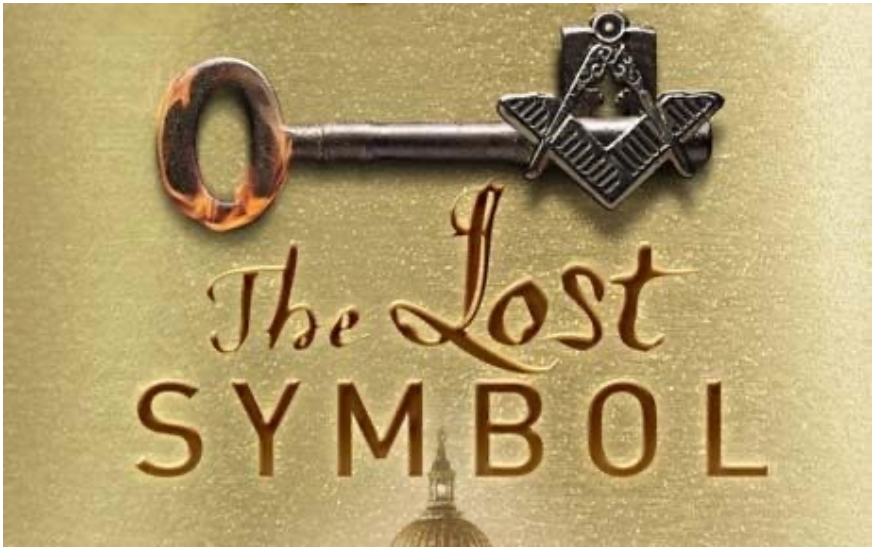
Super Wilson loops (Mason, Skinner) (Caron-Huot)

So, what is $R_6^{(2)}$?

- Del Duca, Duhr, Smirnov obtained many-page expression from WL diagrams ⟶ 'Goncharov polylogarithms' '09

- Goncharov, Spradlin, Vergu, Volovich '10 used <u>symbol</u> to simplify result into a couple of lines involving logs, Li_2, Li_3, Li_4. !

$$R_6^{(2)}(u_1,u_2,u_3) = \sum_{i=1}^{3}\left[L_4(x_i^+,x_i^-) - \tfrac{1}{2}Li_4(1-1/u_i)\right] - \frac{1}{8}\left(\sum_{i=1}^{3} Li_2(1-\tfrac{1}{u_i})\right)^2 + \frac{J^4}{24} + \frac{\pi^2 J^2}{12} + \frac{\pi^4}{72}$$



"Harrison Ford in Harris tweed"

- Dan Brown

Typical professor of symbology ⟶

Symbol

(Chen, Gonchanov, Brown, ...)

Consider class of functions defined via

$$d f^{(k)} = \sum_{a \in A} f_a^{(k-1)} \, d\log a$$

$$S[f^{(k)}] = \sum_{a \in A} S[f_a^{(k-1)}] \otimes a$$

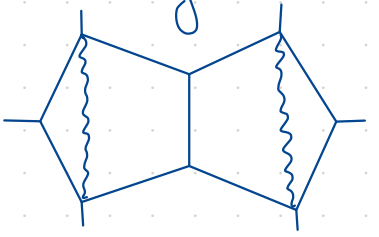
$$= \sum_{\vec{a}} C_{\vec{a}} \, [a_1 \otimes \cdots \otimes a_k] \qquad a_i \in A$$

monodromies act here ↑ ↑ derivatives act here

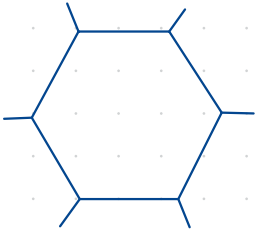
2011 @ CERN

└ Lance visited 2010/2011

I was chatting to Johannes Henn about integrals like:



Johannes was already in contact with Lance on 6d integral:



We naturally ended up discussing together ...

Idea: Bootstrap the symbol!

Note
$$df^{(k)} = \sum_a f_a^{(k-1)} \, d\log a$$

$$d^2 f^{(k)} = \sum_a df_a^{(k-1)} \wedge d\log a$$

$$= \sum_{a,b} f_{ab}^{(k-2)} \, d\log b \wedge d\log a$$

$$= \sum_{a,b} f_{ab}^{(k-2)} \, Y_{ab}^{m} \, W_m^{(2)}$$

$W_m^{(2)}$: basis of independent 2-forms

(integrability constraint) $\stackrel{!}{=} 0$

So build integrable symbols recursively

Assume weight $(k-1)$ basis known:

$$S_i^{(k-1)} = \sum_{j,a} M_{ij}^{(k-1)}{}_a \left[S_j^{(k-2)} \otimes a \right]$$

Build weight (k): $S^{(k)} = \sum_{j,a} C_{ja} \left[S_j^{(k-1)} \otimes a \right]$

$$= \sum_{j,a} C_{ja} \sum_{l,b} M_{jl}^{(k-1)}{}_b \left[S_l^{(k-2)} \otimes b \otimes a \right]$$

Impose integrability: $\sum_{ja} C_{ja} \sum_{l,b} M_{jl}^{(k-1)}{}_b \, Y^m_{ba} \, S_l^{(k-1)} = 0$

i.e. compute kernel of $\mathcal{M}_{(lm),(ja)} = \sum_b M_{jl}^{(k-1)}{}_b \, Y^m_{ba}$

compute kernel of $\mathcal{M}_{(\ell m),(j a)} = \sum_b M^{(k-1)}_{j \ell b} Y^m_{b a}$

Solution: $M^{(k)}_{i j a}$ s.t. $S^{(k)}_i = \sum M^{(k)}_{i j a} \left[S^{(k-1)}_j \otimes a \right]$

- simple linear algebra problem
- can impose constraints on initial entry (unitarity cuts)

  final entry (diff eqs, Supersymmetry)

Hypothesis: alphabet unchanged from two loops.

$\{u, v, w, 1-u, 1-v, 1-w, y_u, y_v, y_w\}$ 9 letters.

Bootstrapping the three-loop hexagon

Lance J. Dixon,[a,b] **James M. Drummond**[a,c] **and Johannes M. Henn**[d,e]

[a] *PH-TH Division, CERN, CH – 1211 Geneva 23, Switzerland*

[b] *SLAC National Accelerator Laboratory, 2575 Sand Hill Road, Menlo Park, CA 94025, U.S.A.*

[c] *LAPTH, Université de Savoie, CNRS, B.P. 110, F-74941 Annecy-le-Vieux Cedex, France*

[d] *Institut für Physik, Humboldt-Universität zu Berlin, Newtonstraße 15, D-12489 Berlin, Germany*

[e] *Kavli Institute for Theoretical Physics, University of California, Santa Barbara, CA 93106, U.S.A.*

E-mail: lance@slac.stanford.edu, drummond@lapp.in2p3.fr, jmhenn@ias.edu

Later fixed by Caron-Huot, He '12

Abstract: We consider the hexagonal Wilson loop dual to the six-point MHV amplitude in planar $\mathcal{N} = 4$ super Yang-Mills theory. We apply constraints from the operator product expansion in the near-collinear limit to the symbol of the remainder function at three loops. Using these constraints, and assuming a natural ansatz for the symbol's entries, we determine the symbol up to just two undetermined constants. In the multi-Regge limit, both constants drop out from the symbol, enabling us to make a non-trivial confirmation of the BFKL prediction for the leading-log approximation. This result provides a strong consistency check of both our ansatz for the symbol and the duality between Wilson loops and MHV amplitudes. Furthermore, we predict the form of the full three-loop remainder function in the multi-Regge limit, beyond the leading-log approximation, up to a few constants representing terms not detected by the symbol. Our results confirm an all-loop prediction for the real part of the remainder function in multi-Regge $3 \to 3$ scattering. In the multi-Regge limit, our result for the remainder function can be expressed entirely in terms of classical polylogarithms. For generic six-point kinematics other functions are required.

Also simple to perform analytic continuation

Analytic result for the two-loop six-point NMHV amplitude in $\mathcal{N} = 4$ super Yang-Mills theory

Lance J. Dixon,[a] **James M. Drummond**[b,c] **and Johannes M. Henn**[d,e]

[a] *SLAC National Accelerator Laboratory, Stanford University, Stanford, CA 94309, U.S.A.*

[b] *PH-TH Division, CERN, Geneva, Switzerland*

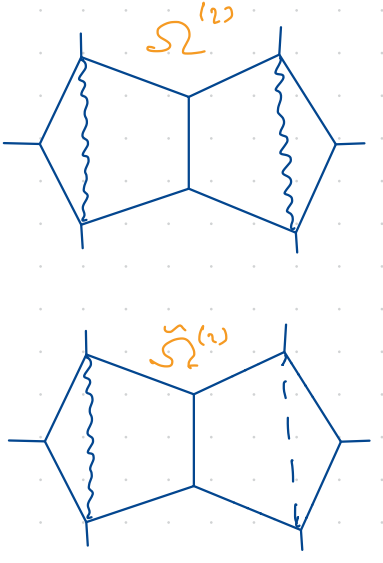
[c] *LAPTH, Université de Savoie, CNRS, B.P. 110, F-74941 Annecy-le-Vieux Cedex, France*

[d] *Humboldt-Universität zu Berlin, Newtonstraße 15, 12489 Berlin, Germany*

[e] *Institute for Advanced Study, Princeton, NJ 08540, U.S.A.*

E-mail: lance@slac.stanford.edu, drummond@lapp.in2p3.fr, jmhenn@ias.edu

Abstract: We provide a simple analytic formula for the two-loop six-point ratio function of planar $\mathcal{N} = 4$ super Yang-Mills theory. This result extends the analytic knowledge of multi-loop six-point amplitudes beyond those with maximal helicity violation. We make a natural ansatz for the symbols of the relevant functions appearing in the two-loop amplitude, and impose various consistency conditions, including symmetry, the absence of spurious poles, the correct collinear behaviour, and agreement with the operator product expansion for light-like (super) Wilson loops. This information reduces the ansatz to a small number of relatively simple functions. In order to fix these parameters uniquely, we utilize an explicit representation of the amplitude in terms of loop integrals that can be evaluated analytically in various kinematic limits. The final compact analytic result is expressed in terms of classical polylogarithms, whose arguments are rational functions of the dual conformal cross-ratios, plus precisely two functions that are not of this type. One of the functions, the loop integral $\Omega^{(2)}$, also plays a key role in a new representation of the remainder function $\mathcal{R}_6^{(2)}$ in the maximally helicity violating sector. Another interesting feature at two loops is the appearance of a new (parity odd) $\times$ (parity odd) sector of the amplitude, which is absent at one loop, and which is uniquely determined in a natural way in terms of the more familiar (parity even) $\times$ (parity even) part. The second non-polylogarithmic function, the loop integral $\tilde{\Omega}^{(2)}$, characterizes this sector. Both $\Omega^{(2)}$ and $\tilde{\Omega}^{(2)}$ can be expressed as one-dimensional integrals over classical polylogarithms with rational arguments.



Single-valued harmonic polylogarithms and the multi-Regge limit

'12

Lance J. Dixon,[a] Claude Duhr[b] and Jeffrey Pennington[a]

Hexagon functions and the three-loop remainder function

'13

Lance J. Dixon,[a] James M. Drummond,[b,c] Matt von Hippel[a,d] and Jeffrey Pennington[a]

The four-loop remainder function and multi-Regge behavior at NNLLA in planar $\mathcal{N} = 4$ super-Yang-Mills theory

'14

Lance J. Dixon,[a] James M. Drummond,[b,c,d] Claude Duhr[c] and Jeffrey Pennington[a]

Bootstrapping an NMHV amplitude through three loops

'14

Lance J. Dixon[a] and Matt von Hippel[b]

The four-loop six-gluon NMHV ratio function

'15

Lance J. Dixon,[a,b] Matt von Hippel[c] and Andrew J. McLeod[a]

Bootstrapping a Five-Loop Amplitude Using Steinmann Relations

Simon Caron-Huot,[1,2] Lance J. Dixon,[3,4] Andrew McLeod,[3] and Matt von Hippel[5]
[1]*Niels Bohr International Academy & Discovery Center, Niels Bohr Institute, University of Copenhagen, 2100 Copenhagen, Denmark*
[2]*Department of Physics, McGill University, 3600 Rue University, Montréal, Québec, Canada H3A 2T8*
[3]*SLAC National Accelerator Laboratory, Stanford University, Stanford, California 94309, USA*
[4]*Kavli Institute for Theoretical Physics, UC Santa Barbara, Santa Barbara, California 93106, USA*
[5]*Perimeter Institute for Theoretical Physics, Waterloo, Ontario N2L 2Y5, Canada*
(Received 9 September 2016; published 5 December 2016)

The analytic structure of scattering amplitudes is restricted by Steinmann relations, which enforce the vanishing of certain discontinuities of discontinuities. We show that these relations dramatically simplify the function space for the hexagon function bootstrap in planar maximally supersymmetric Yang-Mills theory. Armed with this simplification, along with the constraints of dual conformal symmetry and Regge exponentiation, we obtain the complete five-loop six-particle amplitude.

$a = \frac{u}{vw}$ $b = \frac{v}{uw}$ $c = \frac{w}{uv}$ then $[a \otimes b \otimes \dots]$ forbidden

Heptagons from the Steinmann cluster bootstrap

Lance J. Dixon,[a] **James Drummond,**[b] **Thomas Harrington,**[c] **Andrew J. McLeod,**[a] **Georgios Papathanasiou**[a,d] **and Marcus Spradlin**[c]

[a] *SLAC National Accelerator Laboratory, Stanford University, Stanford, CA 94309, U.S.A.*

[b] *School of Physics & Astronomy, University of Southampton, Highfield, Southampton, SO17 1BJ, United Kingdom*

[c] *Department of Physics, Brown University, Providence, RI 02912, U.S.A.*

[d] *DESY Theory Group, DESY Hamburg, Notkestraße 85, D-22607 Hamburg, Germany*

E-mail: lance@slac.stanford.edu, j.m.drummond@soton.ac.uk, thomas_harrington@brown.edu, ajmcleod@stanford.edu, georgios@slac.stanford.edu, marcus_spradlin@brown.edu

Abstract: We reformulate the heptagon cluster bootstrap to take advantage of the Steinmann relations, which require certain double discontinuities of any amplitude to vanish. These constraints vastly reduce the number of functions needed to bootstrap seven-point amplitudes in planar $\mathcal{N} = 4$ supersymmetric Yang-Mills theory, making higher-loop contributions to these amplitudes more computationally accessible. In particular, dual superconformal symmetry and well-defined collinear limits suffice to determine uniquely the symbols of the three-loop NMHV and four-loop MHV seven-point amplitudes. We also show that at three loops, relaxing the dual superconformal ($\bar{Q}$) relations and imposing dihedral symmetry (and for NMHV the absence of spurious poles) leaves only a single ambiguity in the heptagon amplitudes. These results point to a strong tension between the collinear properties of the amplitudes and the Steinmann relations.

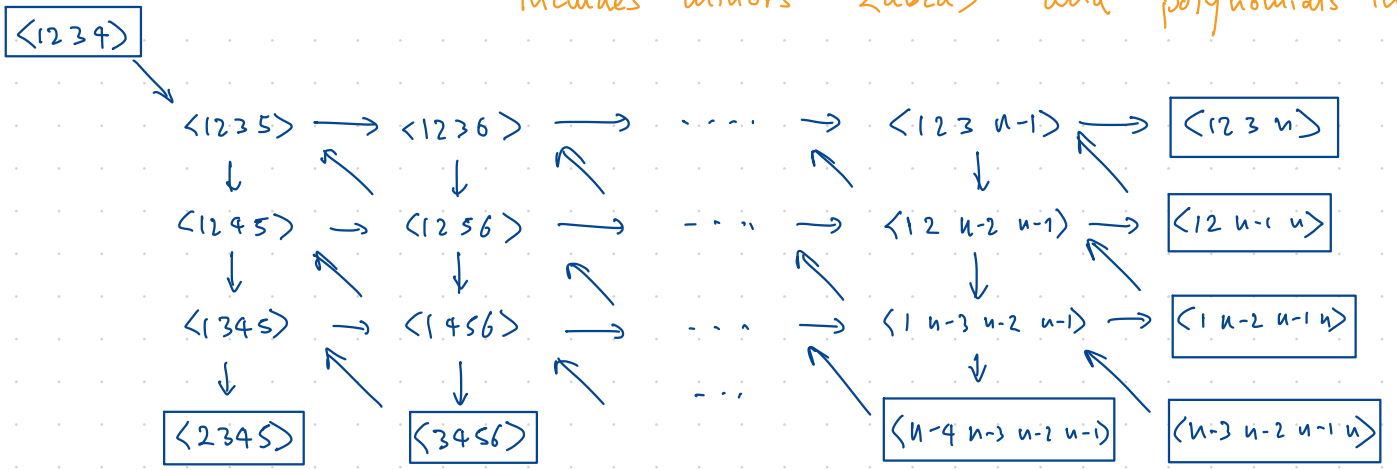
Alphabets & Cluster Algebras

Golden, Goncharov, Spradlin, Vergu, Volovich (2013)

Observed results for $L=2$ consistent with:

alphabet $\mathcal{A} \subseteq$ { cluster variables for $Gr(4,n)$ cluster algebra }

includes minors $\langle abcd \rangle$ and polynomials in minors.

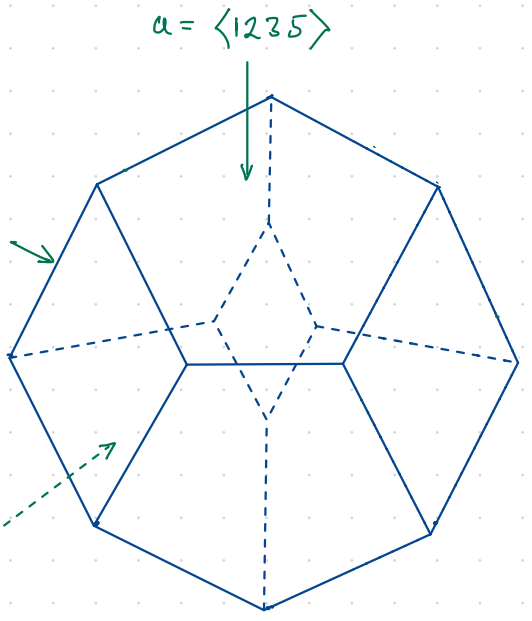


Example $Gr(4,6) \cong Gr(2,6)$

$\langle 12 \rangle \longleftrightarrow \langle 3456 \rangle$

etc.

$\mathcal{X}(a,a') = \dfrac{\langle 1234\rangle\langle 1256\rangle}{\langle 1236\rangle\langle 1245\rangle}$



$\langle 1235\rangle \otimes \langle 1246\rangle$ forbidden

C.A. (JMD, Foster, Gürdoğan '17)

$\cdots \otimes \langle 1235\rangle \otimes \dfrac{\langle 1234\rangle\langle 1256\rangle}{\langle 1236\rangle\langle 1245\rangle} \otimes \langle 1246\rangle \otimes \cdots$

Similar for heptagon.

The cosmic Galois group and extended Steinmann relations for planar $\mathcal{N} = 4$ SYM amplitudes

Simon Caron-Huot,[a] Lance J. Dixon,[b,c,d,e] Falko Dulat,[b] Matt von Hippel,[f,g] Andrew J. McLeod[b,c,g] and Georgios Papathanasiou[c,h]

Folding Amplitudes into Form Factors: An Antipodal Duality

Lance J. Dixon,[1] Ömer Gürdoğan,[2] Andrew J. McLeod,[3,4,5] and Matthias Wilhelm[5]
[1]*SLAC National Accelerator Laboratory, Stanford University, Stanford, California 94309, USA*
[2]*School of Physics and Astronomy, University of Southampton, Southampton SO17 1BJ, United Kingdom*
[3]*CERN, Theoretical Physics Department, 1211 Geneva 23, Switzerland*
[4]*Mani L. Bhaumik Institute for Theoretical Physics, Department of Physics and Astronomy, UCLA, Los Angeles, California 90095, USA*
[5]*Niels Bohr International Academy, Niels Bohr Institute, Blegdamsvej 17, 2100 Copenhagen Ø, Denmark*

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We observe that the three-gluon form factor of the chiral part of the stress-tensor multiplet in planar $\mathcal{N} = 4$ super-Yang-Mills theory is dual to the six-gluon MHV amplitude on its parity-preserving surface. Up to a simple variable substitution, the map between these two quantities is given by the antipode operation defined on polylogarithms (as part of their Hopf algebra structure), which acts at symbol level by reversing the order of letters in each term. We provide evidence for this duality through seven loops.

$a_1 \otimes \cdots \otimes a_k$

$\longrightarrow \; a_k \otimes \cdots \otimes a_1$

Two-Loop Five-Point Amplitude in $\mathcal{N} = 4$ Super-Yang-Mills Theory

Samuel Abreu,[1] Lance J. Dixon,[2] Enrico Herrmann,[2] Ben Page,[3] and Mao Zeng[4]
[1]*Center for Cosmology, Particle Physics and Phenomenology (CP3), Université Catholique de Louvain, 1348 Louvain-La-Neuve, Belgium*
[2]*SLAC National Accelerator Laboratory, Stanford University, Stanford, California 94039, USA*
[3]*Institut de Physique Théorique, CEA, CNRS, Université Paris-Saclay, F-91191 Gif-sur-Yvette cedex, France*
[4]*Institut für Theoretische Physik, Eidgenössische Technische Hochschule Zürich, Wolfgang-Pauli-Strasse 27, 8093 Zürich, Switzerland*

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We compute the symbol of the two-loop five-point scattering amplitude in $\mathcal{N} = 4$ supersymmetric Yang-Mills theory, including its full color dependence. This requires constructing the symbol of all two-loop five-point nonplanar massless master integrals, for which we give explicit results.

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The two-loop five-point amplitude in $\mathcal{N} = 8$ supergravity

Samuel Abreu,[a] **Lance J. Dixon,**[b,c] **Enrico Herrmann,**[b] **Ben Page**[d] **and Mao Zeng**[e]

A few days ago

Eight loop form factors, amplitudes and patterns in planar $\mathcal{N} = 4$ super-Yang-Mills theory

Lance J. Dixon[a,*] **and Zhenjie Li**[a]

[a] *SLAC National Accelerator Laboratory, Stanford, CA 94309, USA*

E-mail: lance@slac.stanford.edu, munuxilee@gmail.com

The simplest nontrivial amplitude in planar $\mathcal{N} = 4$ super-Yang-Mills theory is six-gluon scattering in the maximally-helicity-violating configuration. It has been computed to 8 loops with the help of antipodal duality, which relates it to the three-point form factor of a protected operator, the chiral stress tensor super-multiplet, represented also as $\mathrm{tr}\phi^2$. In this talk, we describe the computation to 8 loops of another three-point form factor, for the operator $\mathrm{tr}\phi^3$. This form factor lives in the same restricted space of polylogarithms as the $\mathrm{tr}\phi^2$ form factor. We also report on all-order patterns for sequences of coefficients in the symbols of these polylogarithmic results, for the leading discontinuity of the $\mathrm{tr}\phi^3$ form factor.

Thank you Lance!

and Happy Birthday!