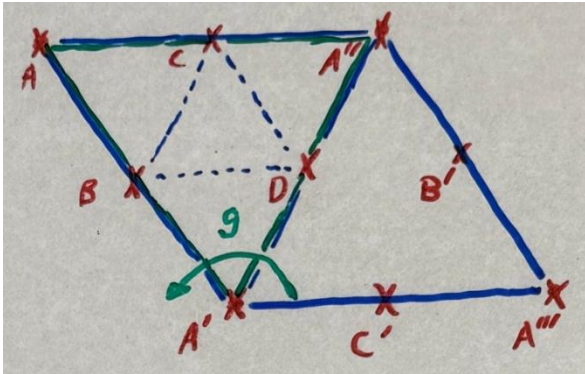
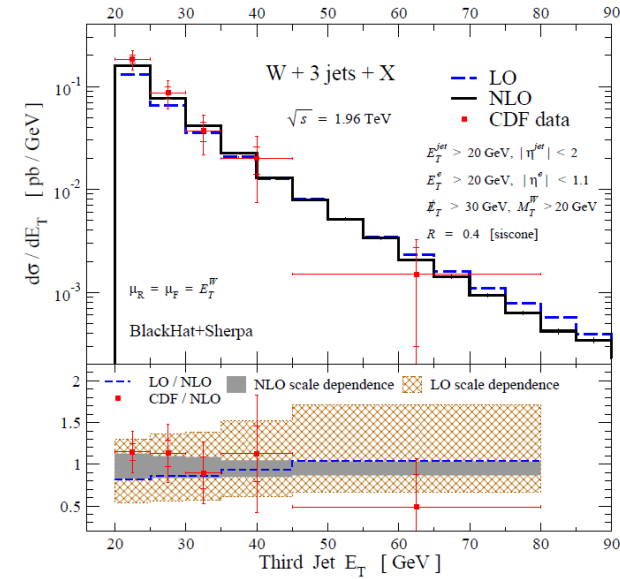


# From Orbifolds to Amplitudes



“In Scotland,  
talking about yourself  
is considered the  
eighth deadly sin”  
– Janice Galloway

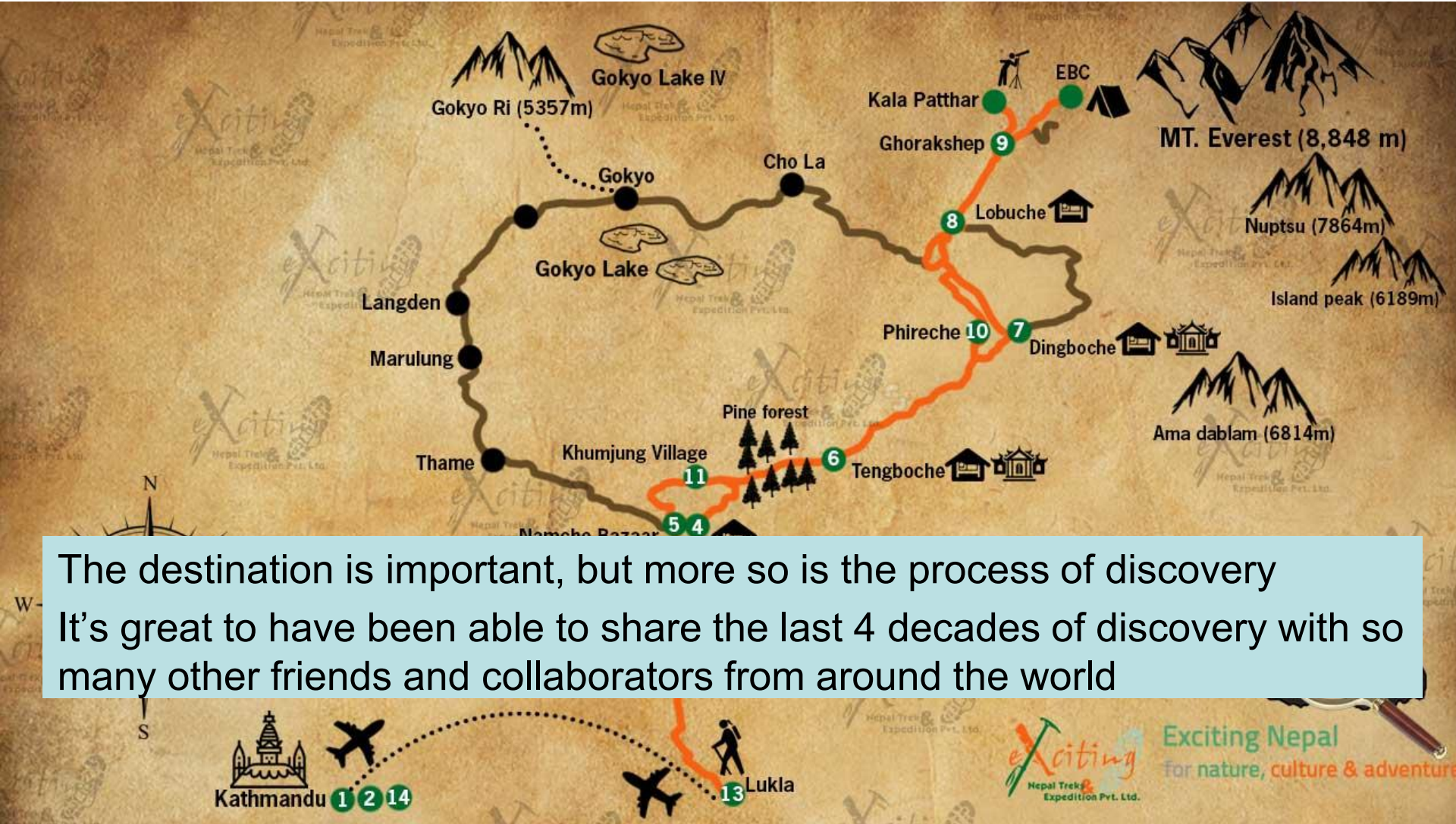
Lance Dixon (SLAC)



University of Edinburgh Physics Colloquium  
26 June 2026



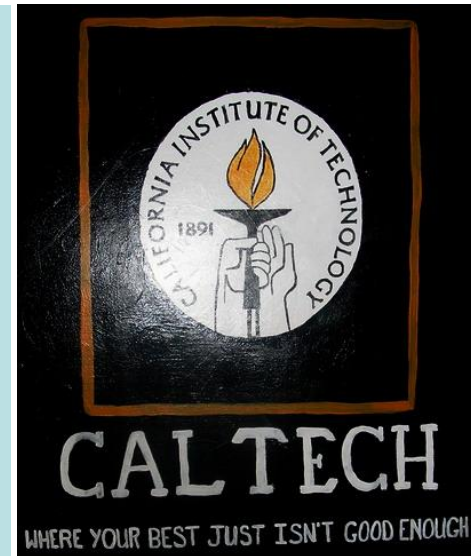
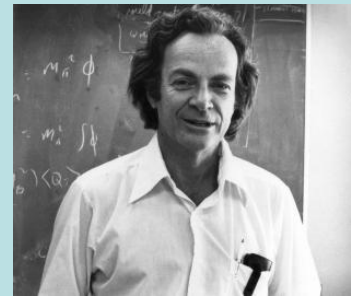
# Physics is a Journey



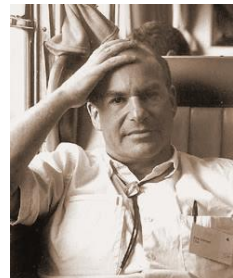
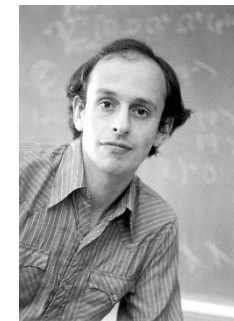
The destination is important, but more so is the process of discovery  
It's great to have been able to share the last 4 decades of discovery with so many other friends and collaborators from around the world

# Physics + Applied Math Major at CalTech

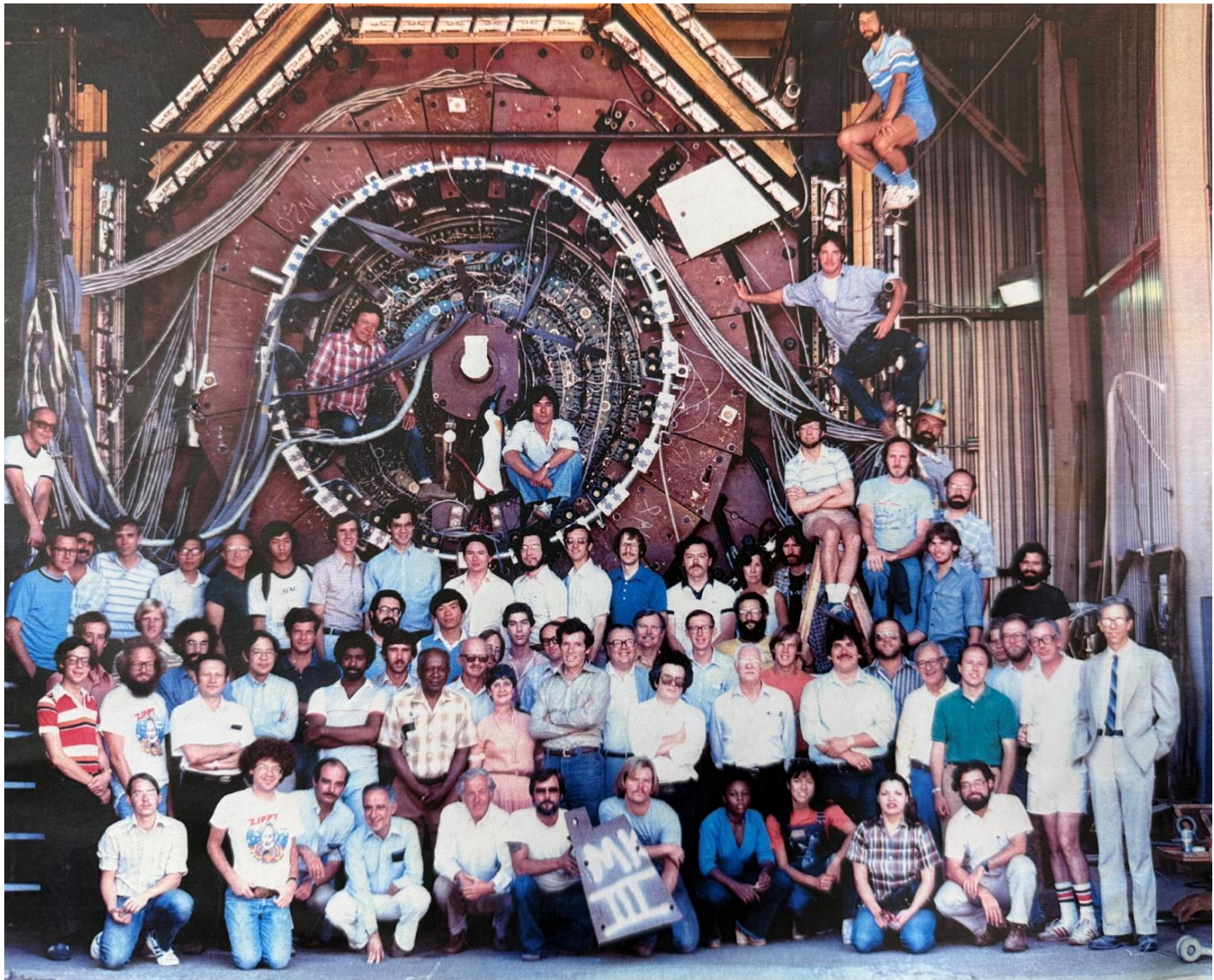
- 1978 - 1982
- Took many courses, worked hard (in between sports)
- Encountered Richard Feynman five times, 2.1 courses



- Physics 1 and QFT from David Politzer
- Colloquium on string theory by John Schwarz



# Summer of 1981: Mark III experiment at SLAC

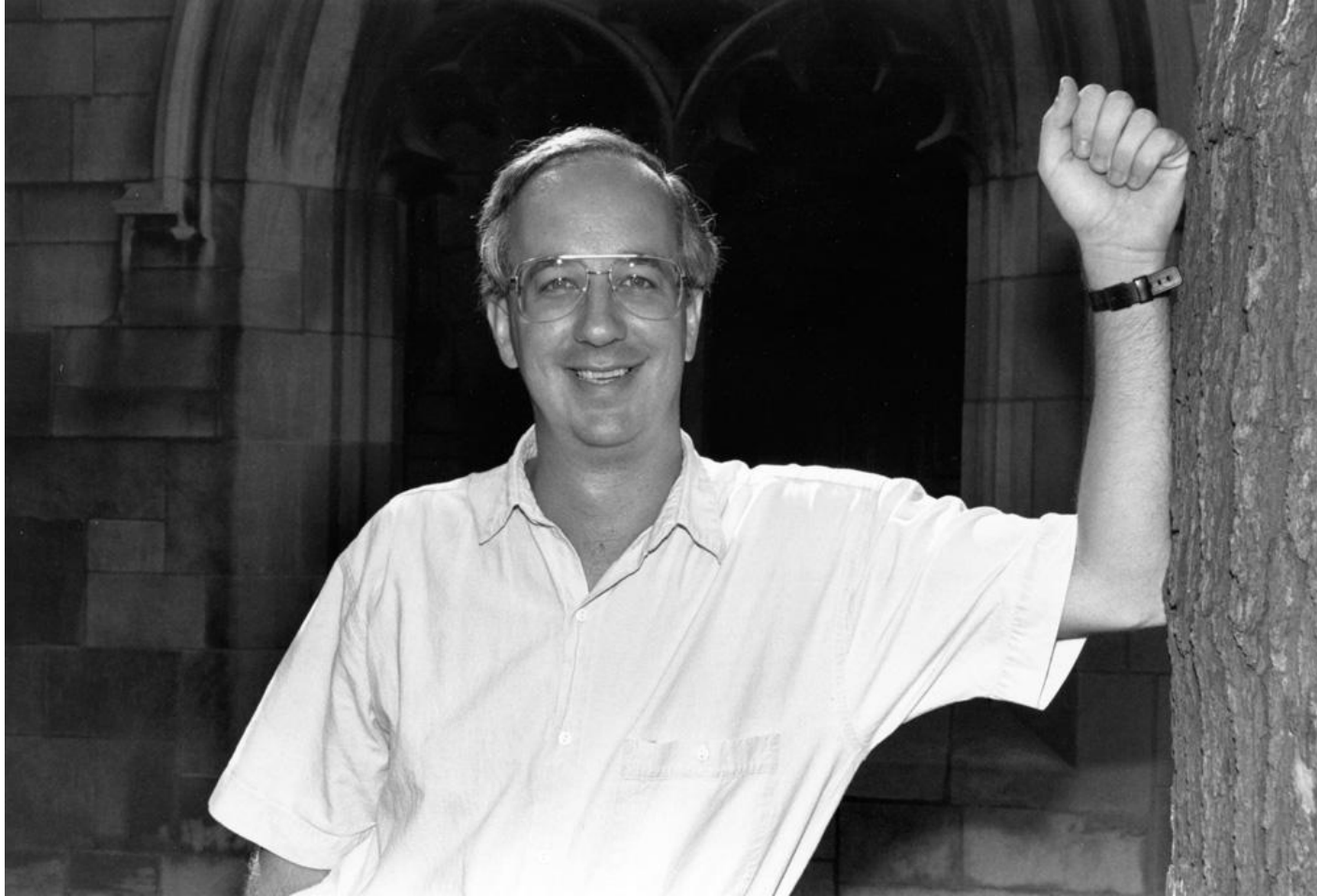


# Physics PhD at Princeton

- 1982 - 1986
- Took a few courses, including electronics, machine shop, generals experiment in biophysics
- Worked hard (in between sports)
- QFT from David Gross
- Became Jeff Harvey's PhD student in 1984 (after brief discussions with Ed Witten and Ian Affleck)



# Indebted to Jeff



# First project: Magnetic monopoles

- Exercise in understanding collective coordinate quantization of soliton solutions

**iNSPIRE** HEP literature ▼ 🔍 Help Submit

## Spherically Symmetric Monopoles and Dyons With Spin

[Lance J. Dixon \(Princeton U.\)](#)  
Jun, 1984

15 pages  
Published in: *Nucl.Phys.B* 248 (1984) 90-104  
Published: 1984  
DOI: [10.1016/0550-3213\(84\)90588-1](https://doi.org/10.1016/0550-3213(84)90588-1)  
Report number: PRINT-84-0548 (PRINCETON)  
View in: [AMS MathSciNet](#)

[📄 cite](#) [📄 claim](#) [🔍 reference search](#) [🔄 6 citations](#)

### Citations per year

Year	Citations
1985	2
1986	1
1987	0
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2014	0
2015	0
2016	0
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2021	0
2022	0
2023	1

Abstract: (Elsevier)  
The collective coordinate method of semiclassical quantization is used to show that the dyonic excitations of certain magnetic monopoles found in unified gauge theories may have nonzero spin, even when the classical monopole fields are spherically symmetric. The magnetic monopoles in the SU(5) model which have three and six times the minimum Dirac magnetic charge provide examples of this phenomenon. The usual relation between spin and statistics is verified for the dyons.

# Superstring Theory

- Working on second soliton project in Sept. 1984 when Jeff told me the news out of Aspen:
- **$D = 10$  gauge & gravitational anomalies cancel for gauge groups  $SO(32)$  and  $E_8 \times E_8$**   
Green, Schwarz, Phys. Lett. B 149 (1984) 117
- Jeff: “We’ll all be doing string theory now”
- Read: “Superstring Theory”, J. Schwarz, Phys. Rept. 89 (1982) 223
- Within a couple of months, Jeff, David, Emil Martinec and Ryan Rohm had constructed the **heterotic string**  
Gross, Harvey Martinec, Rohm, Phys. Rev. Lett. 54 (1985) 502

# From 10 to 4 dimensions

- Kaluza-Klein compactifications:  $\mathbb{R}^{1,3} \times \mathcal{M}^6$
- $\mathcal{N} = 1$  supersymmetry, vacuum stability
  - $\mathcal{M}$  should be Ricci-flat manifold with  $SU(3)$  holonomy
  - Calabi-Yau manifold
- Embed  $SU(3)$  spin connection into gauge connection,  
$$SU(3) \subset E_8$$
  - 4d gauge group  $E_6 \times E_8$  (*hidden*)
  - $E_6 \supset SO(10) \supset SU(5)$  → grand unified theory with gravity!  
Candelas, Horowitz, Strominger, Witten, March 1985, NPB258, 46
- But Calabi-Yau's required a lot of algebraic geometry, so Jeff and I were looking for simpler 6d manifolds  $\mathcal{M}$

# Orbifolds

- James Lepowsky (Rutgers) was on sabbatical at IAS writing the longer version of the FLM (Moonshine) Module



RESEARCH ARTICLE | MATHEMATICS |

[f](#) [X](#) [🦋](#) [in](#) [✉](#)

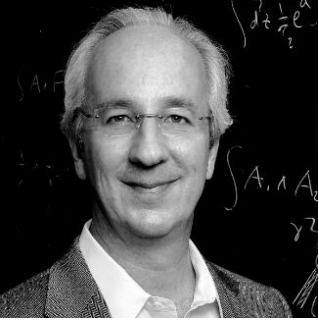
## A natural representation of the Fischer-Griess Monster with the modular function $J$ as character

I. B. Frenkel, J. Lepowsky, and A. Meurman [Authors Info & Affiliations](#)

May 15, 1984 | 81 (10) 3256-3260 | <https://doi.org/10.1073/pnas.81.10.3256>

which one could call an asymmetric orbifold,  $\mathbb{T}^{24}/\mathbb{Z}_2$ , where the lattice associated with  $\mathbb{T}^{24}$  was the Leech lattice

- Jeff and I were working out the properties of “twisted strings” with this case as a guide. Jeff talked to Ed Witten at tea. He found out that Ed and Cumrun Vafa were working on very similar things, so we joined forces.



## STRINGS ON ORBIFOLDS\*

L DIXON<sup>1</sup>, J A HARVEY, C VAFA and E WITTEN

*Joseph Henry Laboratories, Princeton University, Princeton, NJ 08540, USA*

Received 26 June 1985

String propagation on the quotient of a flat torus by a discrete group is considered. We obtain an exactly soluble and more or less realistic method of string compactification.

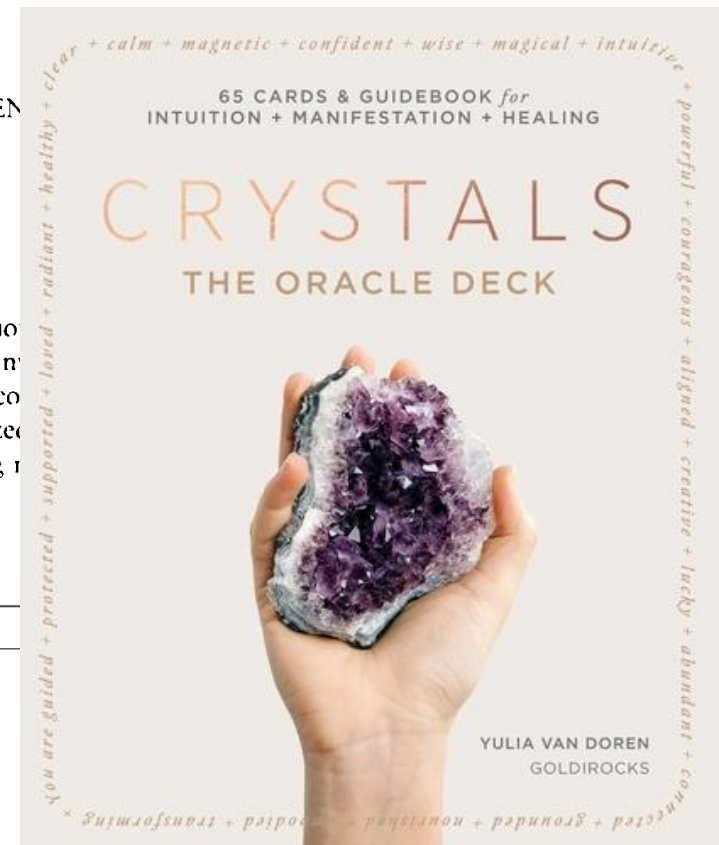
## STRINGS ON ORBIFOLDS (II)

L. DIXON,<sup>1†</sup> J. HARVEY,<sup>2†</sup> C. VAFA<sup>3</sup> and E. WITTEN

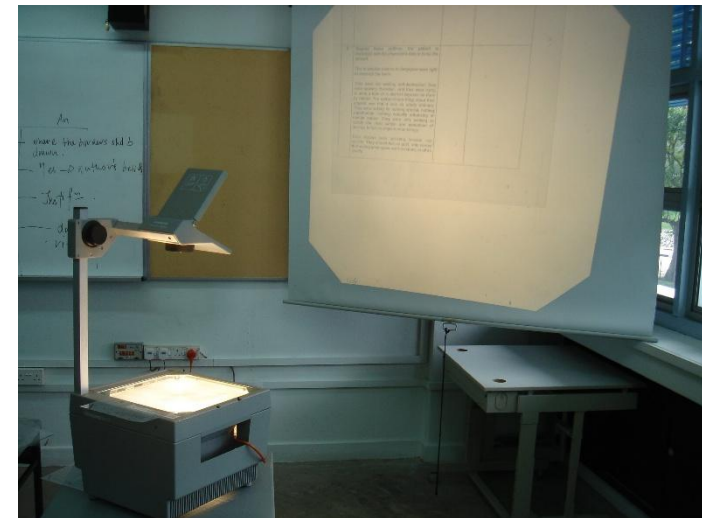
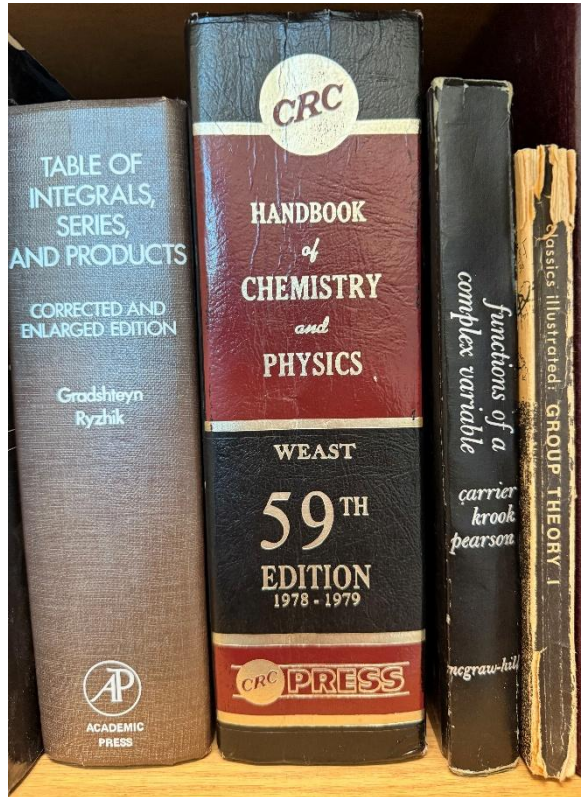
Received 12 March 1986

A general framework for analyzing string propagation on the quotient of a flat torus by a discrete group is presented. The conditions necessary for modular invariance of the tachyons are discussed. Models with four or fewer generations which can be embedded in the spin connection in the gauge group are analyzed. Generalizations which may lead to more phenomenologically promising models are discussed.

I learned 6 dimensional crystallography as part of (II) in particular!



# Theoretical Physics Tools in 1980s



# Strings on Orbifolds

or:

Symmetry Breaking in  
String Theories via Twisting  
of Boundary Conditions

[Harvey, Vafa, Witten, LD]

Why use orbifolds?

- Models are:
  - quasi-realistic — small numbers of generations of 4-d chiral fermions,...
  - tractable — can do string pert. theory for arbitrary radius  $R$  of the orbifold — ~~in principle~~
  - (Dine-Seiberg suggest  $R \sim \sqrt{\alpha'}$   
↔  $\sigma$ -model strongly coupled)
- Check non-ren. of superpotential in pert. thy, ...

# From my job talk

Delivered at Harvard in Fall 1985

Included **proof by exhaustion** that there were **no** (2,2) orbifolds with exactly **3 generations** of  $27$ s of  $E_6$

LD, Harvey, Vafa, Witten,  
NPB261 (1985) 678,  
NPB274 (1986) 285

## What is an Orbifold?

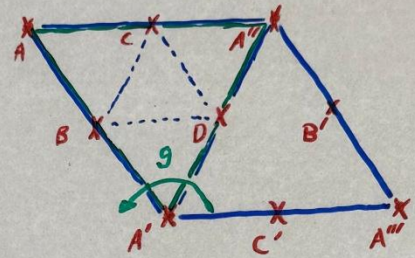
- Given manifold  $M$ , finite subgroup  $P$  of the group of isometries of  $M$ .
- Orbifold  $M/P$  is space of orbits of points in  $M$  under  $P$ : identify  $x \equiv Px$  for  $x \in M, P \in P$ .

- Here, take  $M$  to be a torus (or flat space) (can also let  $\langle A_i^i \rangle, \langle B_{ij} \rangle \neq 0$ )  
Torus defines a lattice  $\Lambda$  of identified points in  $\mathbb{R}^n$ .

Isometries are:  $g = (\theta, v)$

$SO(n)$  rotations in  $\text{Aut}(\Lambda)$   $\uparrow$  translations on torus

Example:  $M = T^2$ , 2-d hexagonal lattice



$$g = (e^{2\pi i \cdot \frac{1}{2} J_{12}}, \vec{0})$$

$P = \mathbb{Z}_2 \rightarrow$  tetrahedron

- Orbifold fails to be a manifold at fixed points of  $g$  ("x")
- Orbifold has discrete holonomy group ( $\mathbb{Z}_2$  above)
- Can repair singularities to form (sometimes!) a Calabi-Yau manifold ( $SU(n)$  holonomy)

Above Example: Remove 4 disks from  $T^2$  (Euler character  $\chi=1$ ), divide by  $P = \mathbb{Z}_2$ , patch 4 disks back in

$$\Rightarrow \chi = \frac{0-4}{2} + 4 = 2 \quad (\text{sphere})$$

- Orbifolds with " $SU(3)$ " holonomy  
 $\Leftrightarrow$  singular limits of Calabi-Yau spaces (with or without spin connection embedded in gauge connection — i.e.,  $(2,2)$  or  $(0,2)$  orbifold models) Perturb away from orbifold
- Orbifolds without " $SU(3)$ " holonomy  $(0,0)$  (Break SUSY!) can be classically stable vacua.  
 ("zero"-dimensional "orbifolds" of this kind  $\Leftrightarrow$  10-dimensional vacuum of the heterotic string without spacetime SUSY.)  
 e.g.  $O(16) \times O(16)$  string

Jeff and I used orbifold methods to find a nonsupersymmetric, tachyon-free heterotic string vacuum in 10 dimensions  
 LD, Harvey, NPB274 (1986) 93;  
 Alvarez-Gaume, Ginsparg, Moore, Vafa, PLB171 (1986) 155;  
 Seiberg, Witten, NPB276 (1986) 272

My initiation into TeX



# Conformal field theory of orbifolds



LD, Friedan, Martinec, Shenker, NPB282 (1987) 13

Introduced to 2d CFT methods by Dan Friedan, Emil Martinec, Steve Shenker

Twist fields  $\sigma$  defined **implicitly** by putting cuts into string coordinate  $X^\mu$

Emphasis on **"bootstrapping"** answer from analyticity and differential equations

4  $\mathbb{Z}_3$  Twists on Closed VM Orbifold

-To calculate  $\langle \sigma_+(e_1) \sigma_+(e_2) \sigma_-(e_3) \sigma_-(e_4) \rangle$ ,  
 first calculate:  

$$g(z, z') \equiv \frac{\langle \frac{1}{2} \partial_z X(z) \partial_{z'} \bar{X}(z') \prod \sigma_i(e_i) \rangle}{\langle \prod \sigma_i(e_i) \rangle}$$
 (Green's function in presence of twist operators)

Asymptotic conditions on  $g(z, z')$ :

(1)  $-\frac{1}{2} \partial_z X(z) \partial_{z'} \bar{X}(z') \sim \frac{1}{(z-z')^2} + T(z) + \dots$   
 $\Rightarrow g(z, z') \sim \frac{1}{(z-z')^2} + \text{finite as } z \rightarrow z'$

(2) Expand  $\partial_z X(z)$  around  $e_1$ , say.  
 $\partial_z X(z) = \sum a_n (z-e_1)^{-n-1}$   
 But  $X((z-e_1)e^{2\pi i}) = e^{-2\pi i/3} X(z-e_1)$   
 $\Rightarrow n \in \text{integers} - \frac{1}{3} \Rightarrow \partial_z X(z) \sigma_-(e_1) | 0 \rangle = (z-e_1)^{-1/3} a_{-1/3} \sigma_-(e_1) | 0 \rangle$

Denote the 3 sheets by:

$$0 = \Delta_\alpha X_{\mu\alpha} = \oint_{C_\alpha} [dz \partial_z X(z) + d\bar{z} \partial_{\bar{z}} X(\bar{z})]$$

$$\Rightarrow 0 = \oint_{C_\alpha} [dz g(z, z') + d\bar{z} \bar{g}(\bar{z}, \bar{z}')] \quad (*)$$

Here  

$$\bar{g}(\bar{z}, \bar{z}') \equiv \frac{\langle -\frac{1}{2} \partial_{\bar{z}} X(\bar{z}) \partial_{\bar{z}'} \bar{X}(\bar{z}') \prod \sigma_i(e_i) \rangle}{\langle \prod \sigma_i(e_i) \rangle}$$

$$= B \frac{[(\bar{z}-e_2)(\bar{z}-e_3)(\bar{z}'-e_2)(\bar{z}'-e_3)]^{-1/3}}{[(\bar{z}-e_1)(\bar{z}-e_3)(\bar{z}'-e_1)(\bar{z}'-e_3)]^{-2/3}}$$
 (no  $z \rightarrow z'$  singularity)

- Integrals in (\*) have form of hypergeometric functions,  ${}_2F_1(\frac{1}{3}, \frac{2}{3}; 1; X)$

$X = \frac{(e_1-e_2)(e_3-e_4)}{(e_1-e_3)(e_2-e_4)}$  is "anharmonic ratio" (SL(2, C) invariant)

# Also importance of factorization in multiple channels

Use: ~~Milne~~

$$\int d\omega \wedge d\bar{\omega} = \text{Im} \left[ \sum_{i=1}^g A_i B_i^* \right]$$

$$A_i = \oint_{c_i} \phi d\omega, \quad B_i = \oint_{c_i} \phi d\bar{\omega}$$

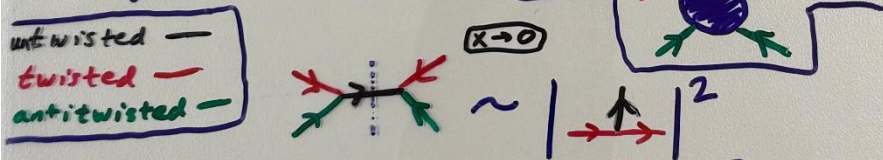
generators of a canonical homology basis for the genus  $g$  Riemann surface covering the cut  $\mathbb{Z}$ -plane

$$d\omega = dz (z-e_1)^{-1/3} (z-e_2)^{-1/3} (z-e_3)^{-2/3} (z-e_4)^{-2/3}$$

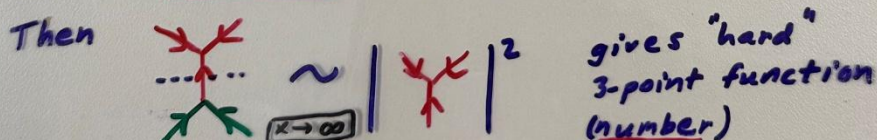
$\omega$  is a holomorphic 1-form on this surface

So  $\hat{S}_{g,2}$  will also involve  ${}_2F_1(\frac{1}{3}, \frac{2}{3}; 1; x)$

-Now, factorize on poles of 4-point function to get 3 point functions:



use to normalize 4 point function



To do:

- Add fermions in NSR formulation  
 $[\Sigma = X + \theta \Psi]$   
 should be able to extend the above to superspace. (Holomorphic forms in superspace, ...)  $\Psi$  zero modes?

- Nonabelian twists: (Martinec)

Need to find matrix

$$g_{IJ}(z, z') = \frac{\langle \partial_z X^I \partial_{z'} X^J \prod_i \sigma_i(e_i) \rangle}{\langle \prod_i \sigma_i(e_i) \rangle}$$

With correct asymptotics.

- Excited states:  $\alpha_{-113} |0\rangle_t$ , etc.  
 (need for <sup>some</sup> massless states for heterotic string on orbifold)

Modifies  $z \rightarrow e_i$  behavior of  $g(z, z')$   
 $\sim (z - e_i)^{-4/3}$

# → SLAC as postdoc



- **Generalize** from orbifolds to arbitrary 2d SCFTs with right central charge, etc., to serve as string compactifications
- Try to prove **general results**

# Type II Superstring No-Go Theorem

LD, Kaplunovsky, Vafa,  
NPB294 (1987) 43

Could get remarkably close to Standard Model using asymmetric orbifolds, but not quite there.

Results for Type II (Closed) Superstring:  
Can have: 11

- (a)  $G_{4D} \supseteq SU(3) \times SU(2) \times U(1)$   
- but  $\dim(G_{4D}) \leq 18$
- (b) "Massless" fermions in non-self-conjugate representations of  $G_{4D}$ ,  
i.e. parity violation
- (c) "Massless"  $\mathbb{Z}_2$ 's of  $SU(3)$
- (d) "Massless"  $\mathbb{Z}_2$ 's of  $SU(2)$
- ~~(e) All of the above~~

Note:  
Omitting  $U(1)$  from  $G_{4D} \Rightarrow$   
can have  $\mathbb{Z}_2$ 's of  $SU(3)$  and  $\mathbb{Z}_2, \mathbb{Z}_3$ 's of  $SU(2) \cong SO(3)$   
 $\Rightarrow$  Experimental prediction from type II strings: Georgi-Glashow model?  
**NO NEUTRAL CURRENTS**

$\hat{c}^{int} \geq 4 + \frac{5}{3} + 1 = 6\frac{2}{3}$   
 $> 6$   
"Loopholes?" (a) Strong coupling vacuum? (b)  $\langle (R, R)_{\text{sector}} \rangle?$

Fortunately we left loopholes for when D-branes came along later

# More general CFT statements



with FMS & Tom Banks

## Phenomenology and Conformal Field Theory Or Can String Theory Predict the Weak Mixing Angle?

Tom Banks (UC, Santa Cruz), Lance J. Dixon (SLAC), Daniel Friedan (Chicago U., EFI and Chicago U.), Emil J. Martinec (Chicago U., EFI and Chicago U.)  
Jul, 1987

14 pages  
Published in: *Nucl.Phys.B* 299 (1988) 613-626  
Published: 1988  
DOI: [10.1016/0550-3213\(88\)90551-2](https://doi.org/10.1016/0550-3213(88)90551-2)  
Report numbers: SLAC-PUB-4377

## Constraints on String Vacua with Space-Time Supersymmetry

Tom Banks (UC, Santa Cruz), Lance J. Dixon (Princeton U.)  
Feb, 1988

16 pages  
Published in: *Nucl.Phys.B* 307 (1988) 93-108  
Published: 1988  
DOI: [10.1016/0550-3213\(88\)90523-8](https://doi.org/10.1016/0550-3213(88)90523-8)  
Report numbers: PUPT-1086, SCIPP-8805  
View in: [AMS MathSciNet](#), [KEK scanned document](#)

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[reference search](#) [561 citations](#)

Abstract: (Elsevier)

We examine the consequences of extended spacetime supersymmetry for classical superstring vacua with four dimensions uncompactified.  $N = 2$  spacetime supersymmetry implies that the "internal"  $N = 1$  superconformal algebra with central charge  $c = 6$  splits into a piece with  $c = 4$  which has  $N = 4$  superconformal invariance, and a piece with  $c = 2$  which is constructed from two free dimension 1 2 superfields.  $N = 4$  spacetime supersymmetry requires that the entire  $c = 6$  algebra be represented by six free superfields. Using the world-sheet properties of  $N = 1$  spacetime supersymmetric classical vacua, we show that spacetime supersymmetry cannot be continuously broken within a family of classical vacua. Finally, we argue that the effective field theories for classical vacua of superstring theories (whether spacetime supersymmetric or not) have no continuous global symmetries - all continuous symmetries are gauged.

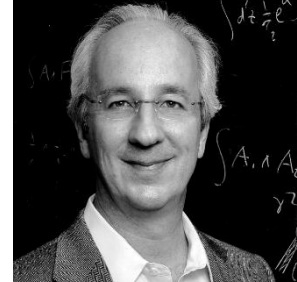
Citations per year



Included argument that global symmetries are forbidden in (some) theories of quantum gravity



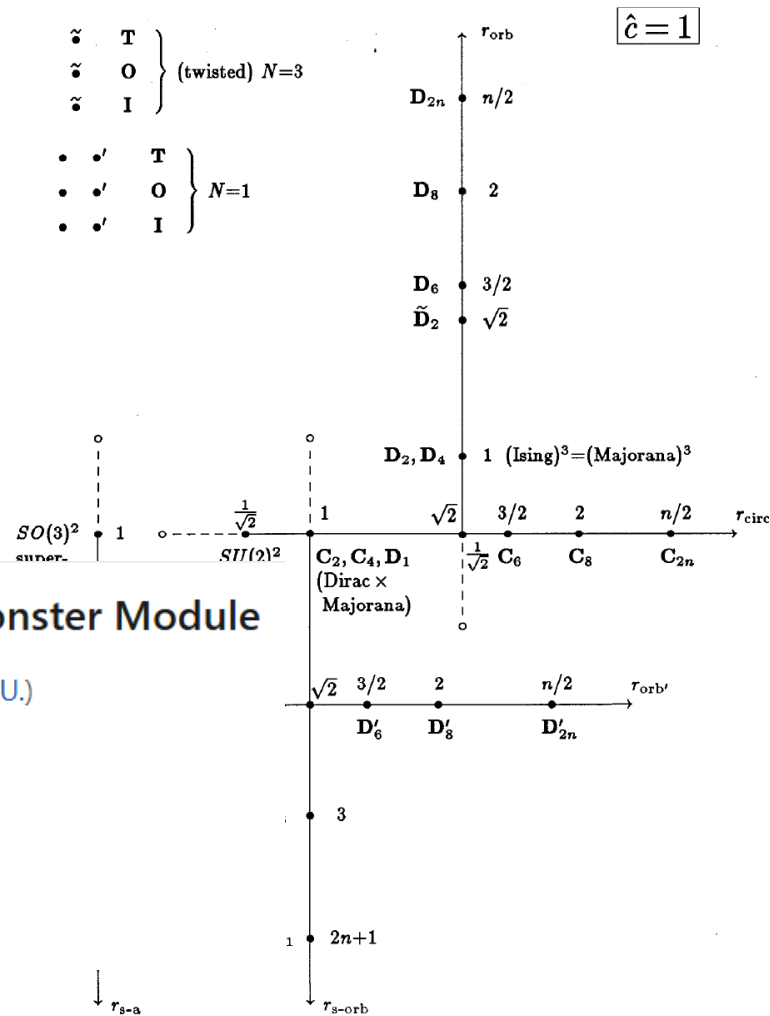
# Return to Princeton (but not for long)



- Worked with Jeff and Paul Ginsparg on  $\hat{c} = 1$  SCFT archipelago

LD, Ginsparg, Harvey NPB306 (1988) 470

- and showed that the FLM monster module had superconformal symmetry



## Beauty and the Beast: Superconformal Symmetry in a Monster Module

Lance J. Dixon (Princeton U.), Paul H. Ginsparg (Harvard U.), Jeffrey A. Harvey (Princeton U.)

Apr, 1988

30 pages

Published in: *Commun.Math.Phys.* 119 (1988) 221-241

DOI: [10.1007/BF01217740](https://doi.org/10.1007/BF01217740)

Report numbers: HUTP-88-A013, PUPT-1088

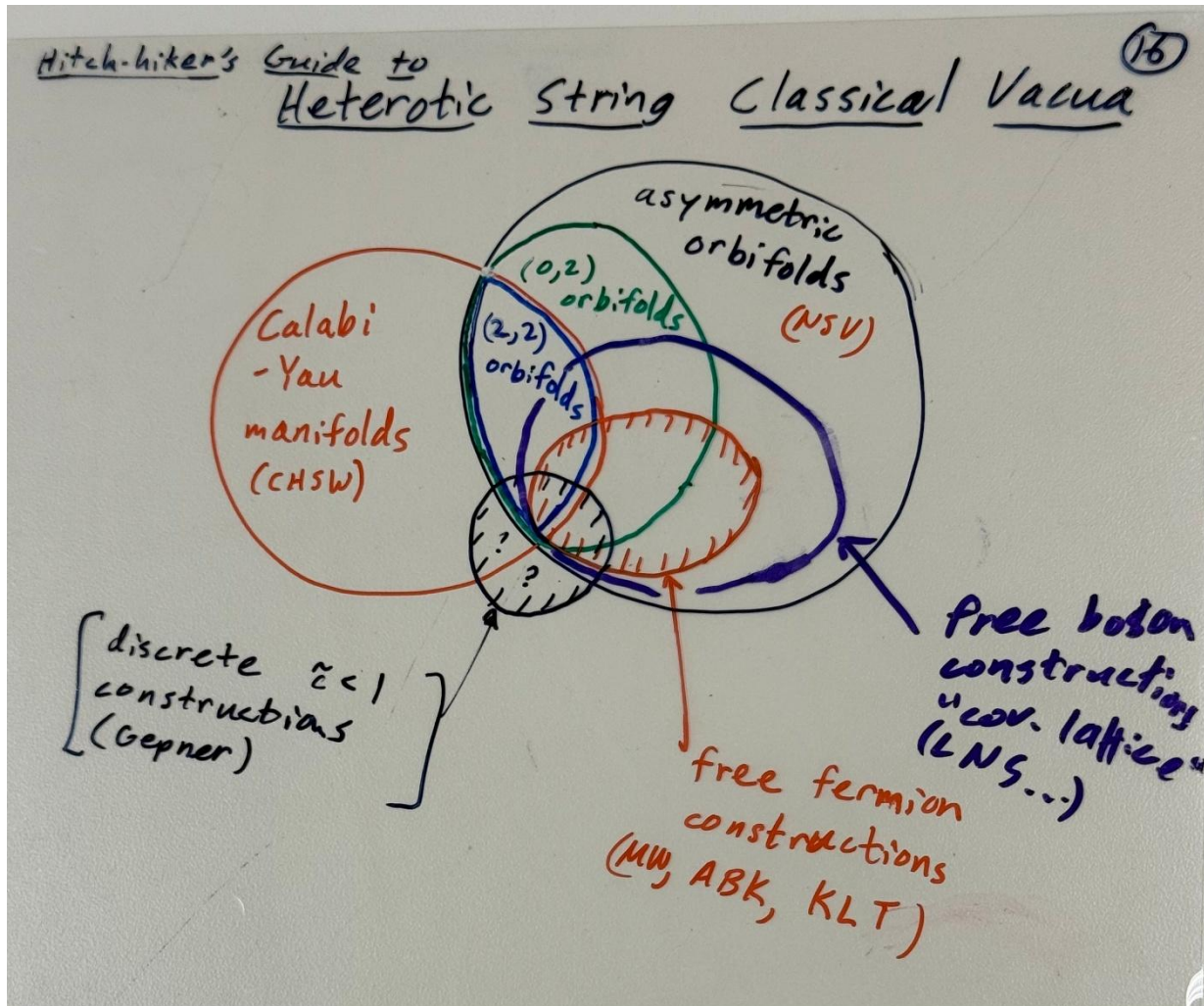


# Return to SLAC



- Collaboration with Vadim Kaplunovsky and Jan Louis
- (special) geometry in EFTs for superstring moduli
- Moduli-dependence of corrections to relations between gauge couplings in stringy GUTs
- Also discussions about fixing the dilaton using multiple gauge groups condensing
- Around this time I started moving toward phenomenology, as the number of string vacua  
 $\rightarrow > 10^3 = 1,000$

# String Landscape circa 1989



# Started talking to experimentalists more

Mark III Evening Seminar

Lance Dixon

will speak on

## Super Strings

April 5, Wednesday 1989

Walter Toki's place  
950 Mears Court  
Stanford, CA  
493-5139

7:30 refreshments  
8:00 seminar

## A Brief History of String Theory <sup>①</sup>

- or -

Why are those theorists spending  
so much time at the Planck scale?  
in 10 dimensions?  
in 2 dimensions??

### Some reasons:

(1) Unify the standard model (plus gravity),  
predict  $\sin^2\theta_w$ , fermion masses & mixings, etc.  
In principle, no dimensionless numbers, only  $M_{Pl}$

(2) Finite theory of quantum gravity  
→ understand structure of spacetime  
at the Planck scale

(3) Lots of pretty mathematics

# Met Zvi and David!



## The Computation of Loop Amplitudes in Gauge Theories

Zvi Bern (Pittsburgh U.), David A. Kosower (Fermilab)

Jun, 1991

106 pages

Published in: *Nucl.Phys.B* 379 (1992) 451-561

Published: 1992

DOI: [10.1016/0550-3213\(92\)90134-W](https://doi.org/10.1016/0550-3213(92)90134-W)

Report numbers: FERMILAB-PUB-91-111-T, PITT-91-05



- Now it can be told: I was the referee (of their color paper)
- Perfect opportunity to leverage my string theory background & try to do something relevant for experiment!
- 63, 44 published papers together (+ a bunch of proceedings)

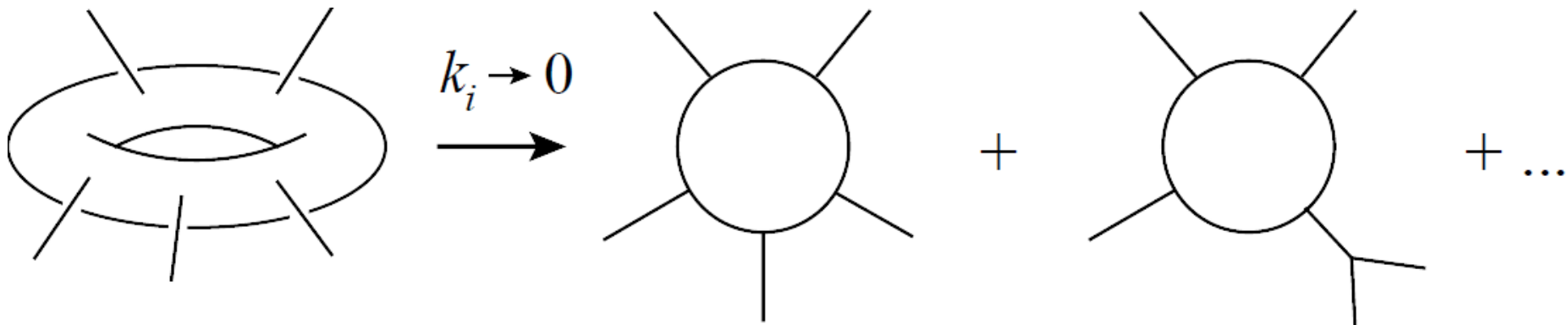
# SLAC theory circa 1991



# String-based rules for QCD

Bern, Kosower NPB 379 (1992) 451

- Zvi and David used string theory, because it has **all Feynman diagrams in one package**:



- Low-energy limit (large string tension limit)  
→ Bern-Kosower rules.
- Computed  $gg \rightarrow gg$  at 1-loop in terms of **helicity amplitudes**
- I joined Zvi and David and we pushed to one more leg.

# For 2 $\rightarrow$ 3 needed pentagon integrals

Bern, LD, Kosower, 9212308, 9306240

- Took a detour down the integral road
- Reductions, IBPs, and differential equations, all in parameter space ...
- Represent dimensionally regulated **multi-point integrals** in terms of **boxes** + ...

The diagram shows a pentagon integral on the left, which is equal to a sum over  $i=1$  to  $5$  of a box integral with two external lines labeled  $i-1$  and  $i$ , plus a term  $\epsilon$  multiplied by a pentagon integral with the dimensionality  $D=6-2\epsilon$  indicated inside.

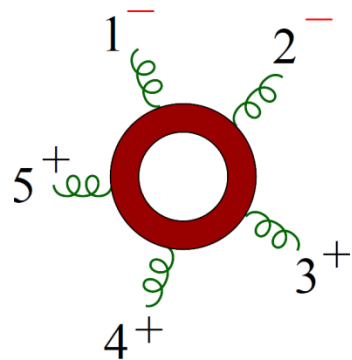
$$\text{Pentagon} = \sum_{i=1}^5 \text{Box}(i-1, i) + \epsilon \text{Pentagon}(D=6-2\epsilon)$$

# A new result: $gg \rightarrow ggg$

Bern, LD, Kosower, 9302280

- First 2  $\rightarrow$  3 loop amplitude. “Just complicated enough.”
- Could start to see clearly how it was organized.

$$L_k(r) \propto \ln(r)$$



$$V^f = -\frac{5}{2\epsilon} - \frac{1}{2} \left[ \ln\left(\frac{\mu^2}{-s_{23}}\right) + \ln\left(\frac{\mu^2}{-s_{51}}\right) \right] - 2, \quad V^s = -\frac{1}{3}V^f + \frac{2}{9}$$

$$F^f = -\frac{1}{2} \frac{\langle 12 \rangle^2 (\langle 23 \rangle [34] \langle 41 \rangle + \langle 24 \rangle [45] \langle 51 \rangle)}{\langle 23 \rangle \langle 34 \rangle \langle 45 \rangle \langle 51 \rangle} \frac{L_0\left(\frac{-s_{23}}{-s_{51}}\right)}{s_{51}}$$

$$F^s = -\frac{1}{3} \frac{[34] \langle 41 \rangle \langle 24 \rangle [45] (\langle 23 \rangle [34] \langle 41 \rangle + \langle 24 \rangle [45] \langle 51 \rangle)}{\langle 34 \rangle \langle 45 \rangle} \frac{L_2\left(\frac{-s_{23}}{-s_{51}}\right)}{s_{51}} - \frac{1}{3}F^f$$

Only log arguments:

$s_{23}$  and  $s_{51}$

– not  $s_{12}$ ,  $s_{34}$  or  $s_{45}$

Why??

$$-\frac{1}{3} \frac{\langle 35 \rangle [35]^3}{[12] [23] \langle 34 \rangle \langle 45 \rangle [51]} + \frac{1}{3} \frac{\langle 12 \rangle [35]^2}{[23] \langle 34 \rangle \langle 45 \rangle [51]} + \frac{1}{6} \frac{\langle 12 \rangle [34] \langle 41 \rangle \langle 24 \rangle [45]}{s_{23} \langle 34 \rangle \langle 45 \rangle s_{51}}$$

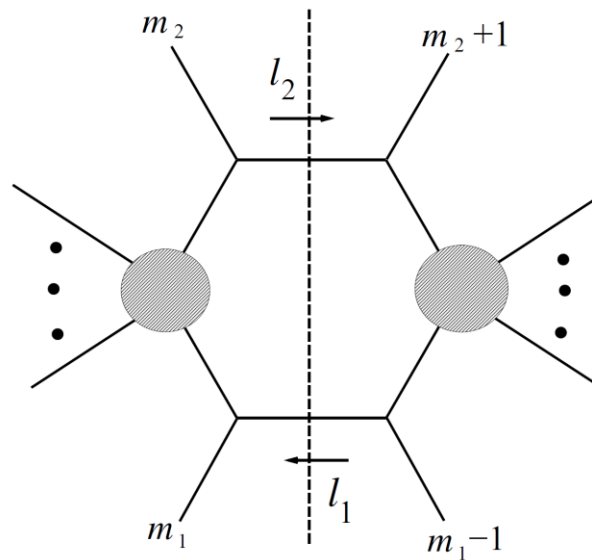
denominators related to collinear limits

# Unitarity method

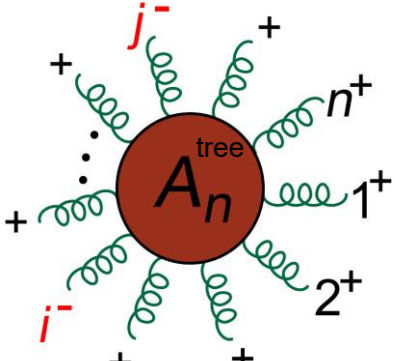


Bern, LD, Dunbar, Kosower, 9403226, 9409265

- Testing ground for a new method: N=4 super-Yang-Mills theory
- Could get an infinite number of (MHV) loop amplitudes simply by multiplying together 2 (Parke-Taylor) tree amplitudes!!



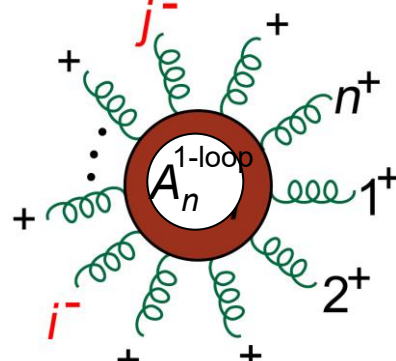
# From trees to loops

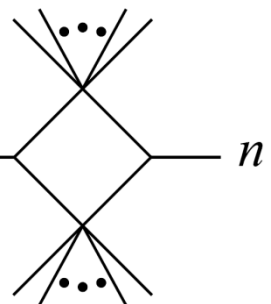


$$A_n^{\text{tree}} = \frac{\langle ij \rangle^4}{\langle 12 \rangle \langle 23 \rangle \cdots \langle n1 \rangle}$$

Parke-Taylor formula (1986)

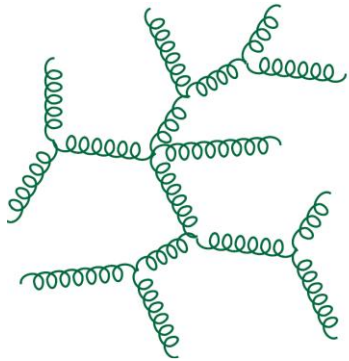
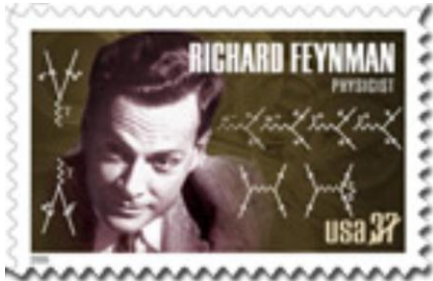
## N=4 SYM



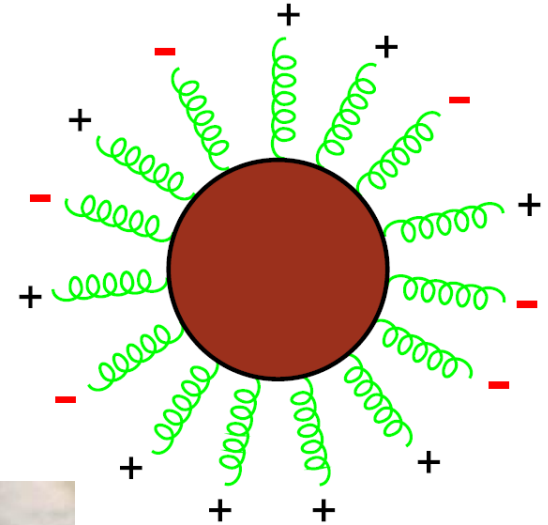
$$A_n^{\text{1-loop}} = \frac{\langle ij \rangle^4}{\langle 12 \rangle \langle 23 \rangle \cdots \langle n1 \rangle} \sum_{m,n} m$$


BDDK (1994)

# Granularity vs. Fluidity

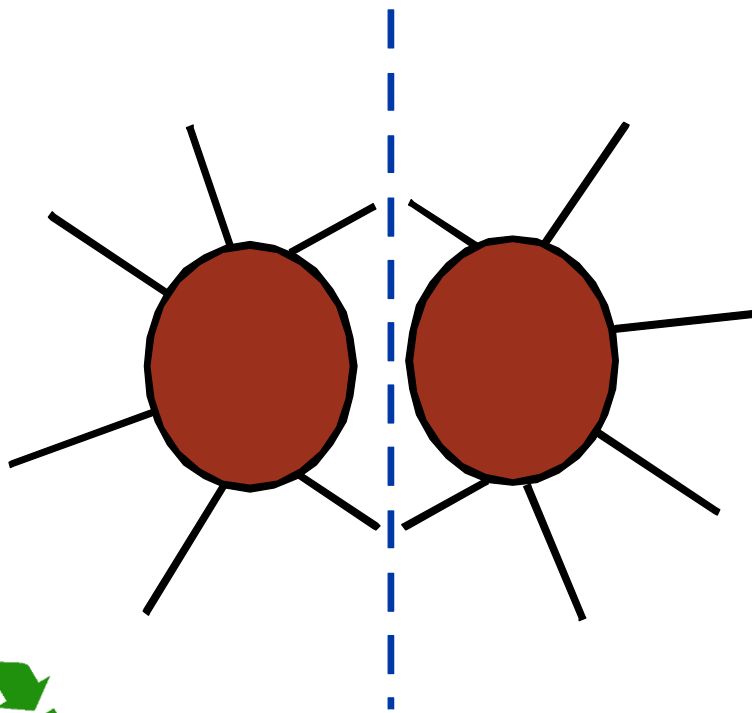


+ ...

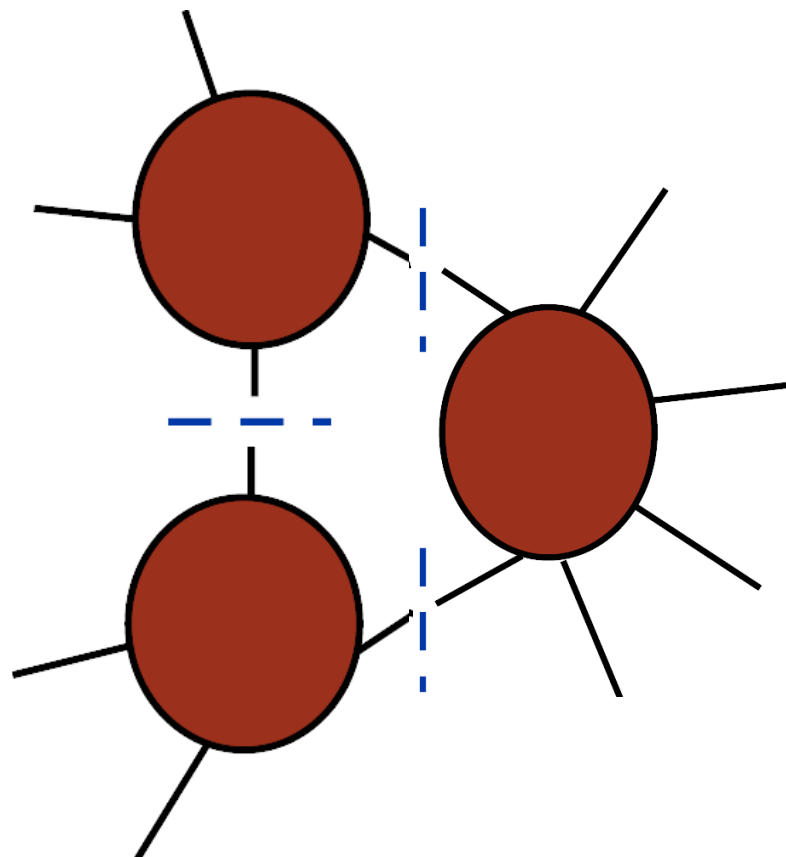


# Branch cut information → Generalized Unitarity (One-loop Fluidity)

**Ordinary unitarity:**  
put 2 particles on shell,  
with real momenta



**Generalized unitarity:**  
put 3 or 4 particles on shell,  
complex momenta



**Trees recycled into loops!**

# The BlackHat Collaboration (circa 2010)



## **BlackHat:**

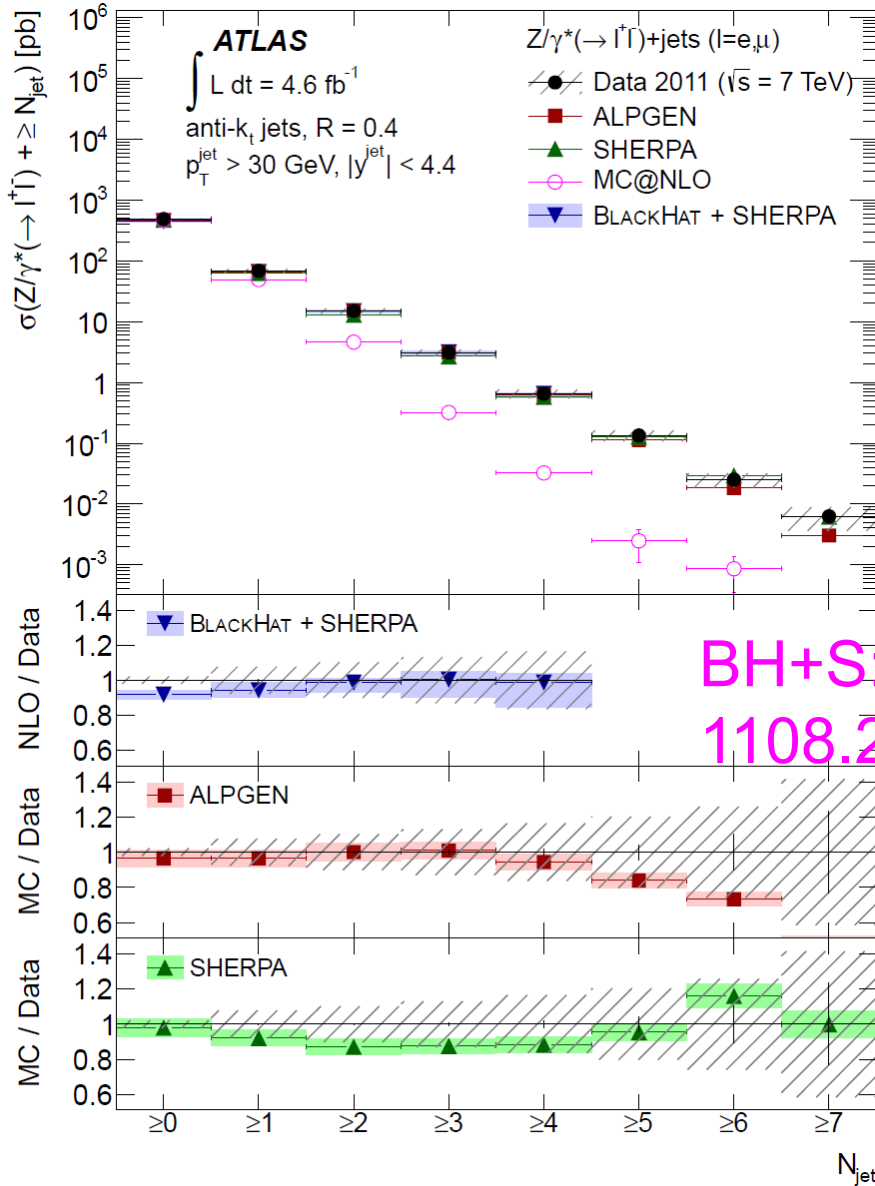
Berger, Bern, Diana, LD,  
Febres Cordero,  
Forde, Gleisberg, Höche, Ita,  
Kosower, Lo Presti, Maître, Ozeren

## **Other key contributors (t < 2014):**

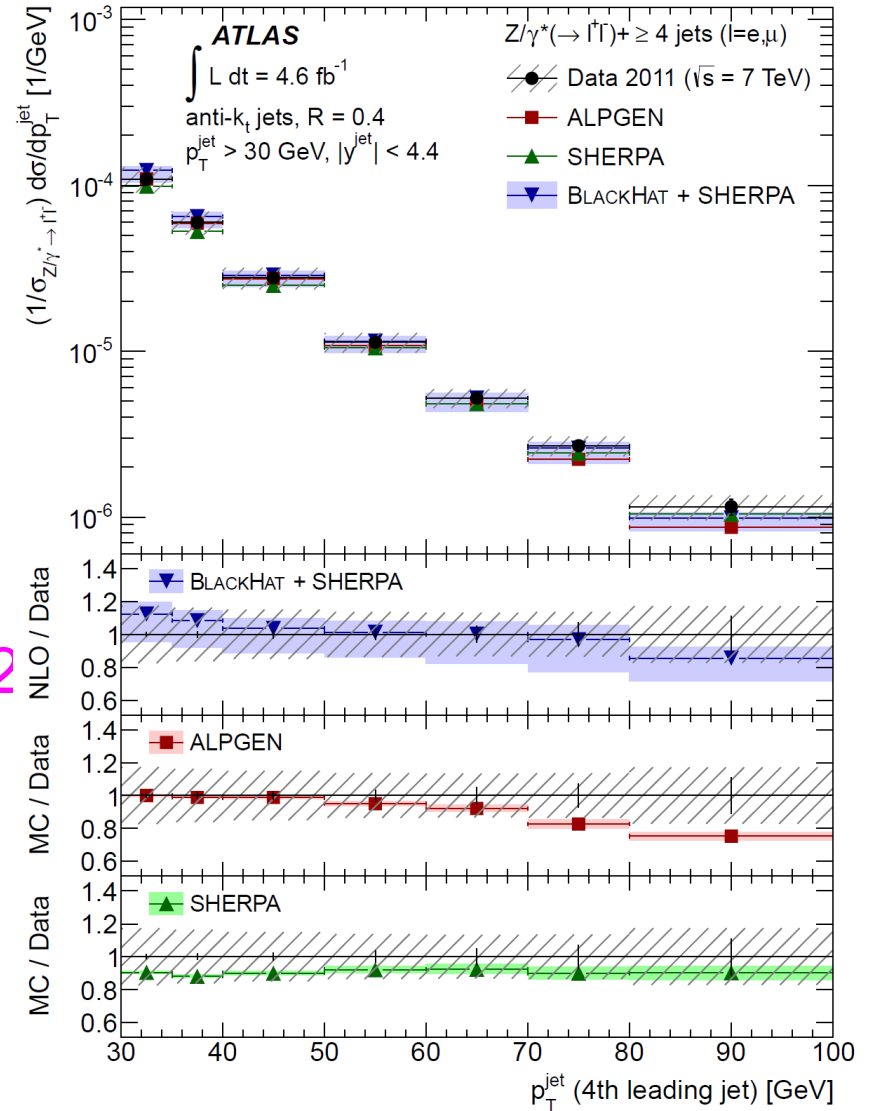
Anastasiou, Badger, Bevilacqua, Biedermann, Britto, Cachazo, Cascioli, Czakon, Dunbar, Ellis, Feng, Frederix, Frixione, Garzelli, Giele, Hirschi, Kardos, Kunszt, Maltoni, Mastrolia, Melnikov, Ossola, Papadopoulos, Pittau, Pozzorini, Reiter, Schulze, Tramontano, Uwer, van Hameren, Weinzierl, Winter, Witten, Worek, Zanderighi, ...

# NLO $pp \rightarrow Z + 1,2,3,4$ jets vs. ATLAS 2011 data

ATLAS 1304.7098



BH+S:  
1108.222



# Back to 1990s



- Amplitudes in EFTs, namely 5-point matrix elements of  $\text{tr}G^3$  – because 4-point was orthogonal to massless 4-point QCD amplitudes  
[LD, Shadmi, 9312363](#)
- Inspired by discussions with SLD QCD experimentalists @ Friday coffee & donuts, computed naïve T-odd QCD observables benefitting from  $Z^0$  polarization (phases!)  
[Brandenburg, LD, Shadmi, 9505355](#)



# Forward to 2003

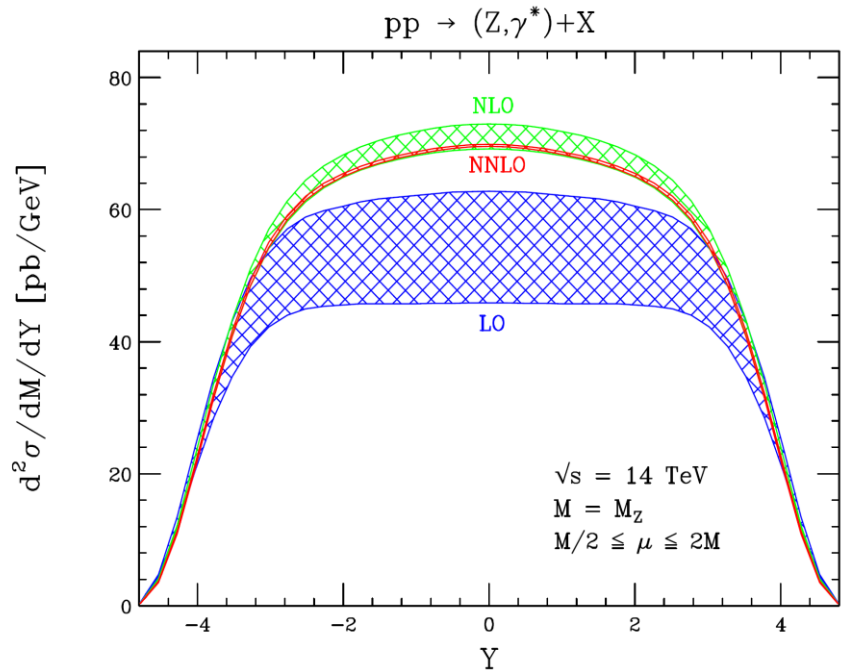
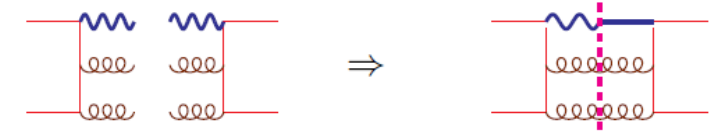


Anastasiou, LD, Melnikov, Petriello, 0306192, 0312266

- Use unitarity in **reverse**: represent cross sections as forward scattering amplitudes  
 → apply multi-loop IBP methods

Anastasiou, Melnikov, 0207004

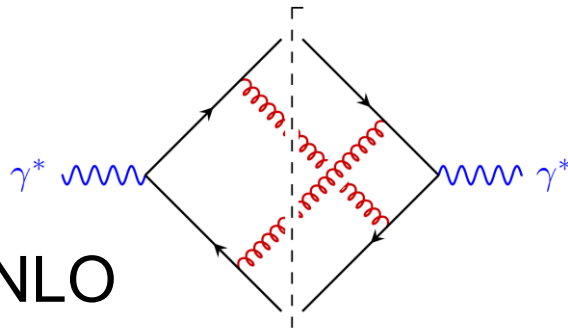
- **New**: Could apply to **differential** cross sections if observable was simple enough, like rapidity  $Y$  of an electroweak gauge boson  
 → **Vrap** program for NNLO W/Z



# Same method for $e^+e^-$ EEC



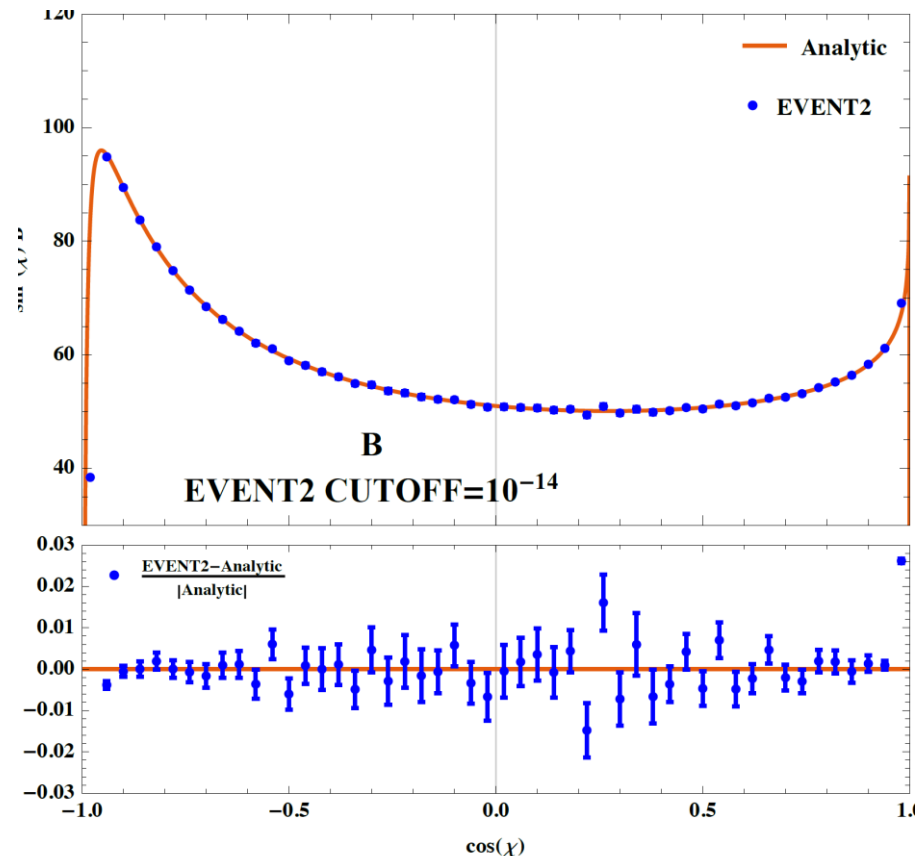
LD, Luo, Shtabovenko, Yang, H.-X. Zhu, 1801.03219



- Analytic NLO
  - Understanding small angle limit of formula
- collinear log resummation with HuaXing Zhu and Ian Moutl
- 1905.01310



L. Dixon From Orbifolds to Amplitudes



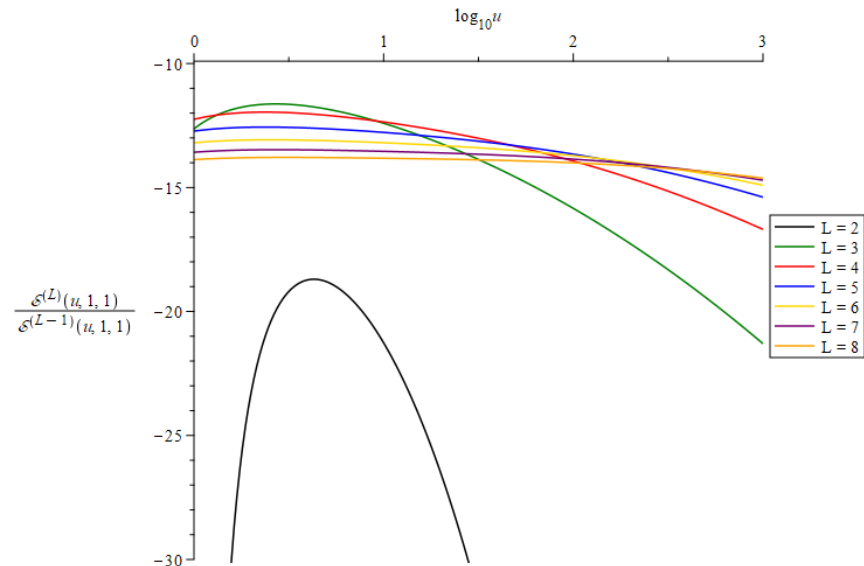


# Back to N=4: Amplitude Bootstrap

LD, Drummond, Henn, 1108.4461, 1111.1704 + many, many more

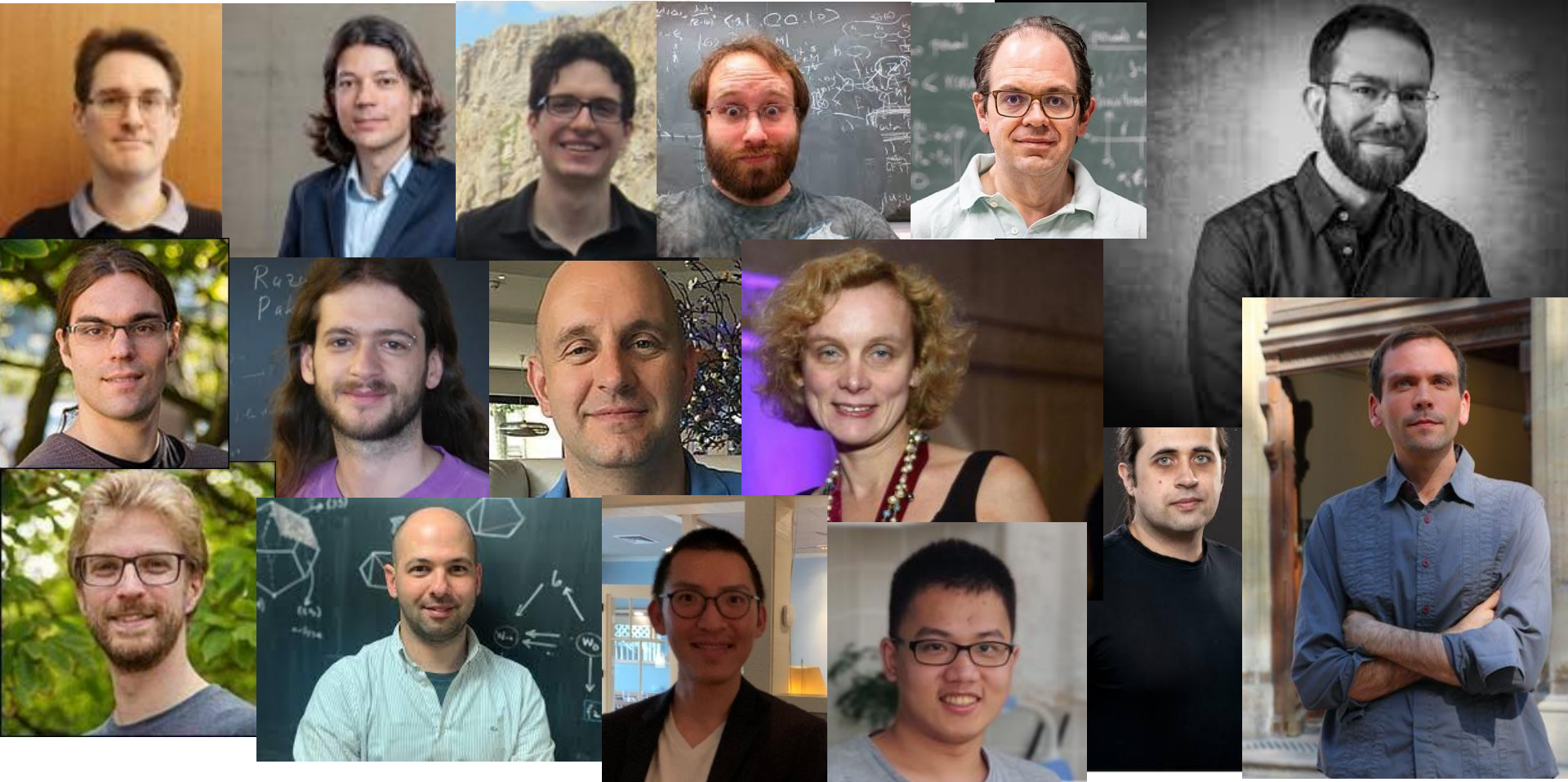
- Tired of doing integrals?
- Then just write the answer down (with enough constraints)
- Works very well with the symbol of multiple polylogarithms

Goncharov, Spradlin, Vergu, Volovich, 1006.5703



Up to 8 loops!  
LD, Liu, 2308.08199

# The fellowship of the amplitude bootstrap



L. Dixon

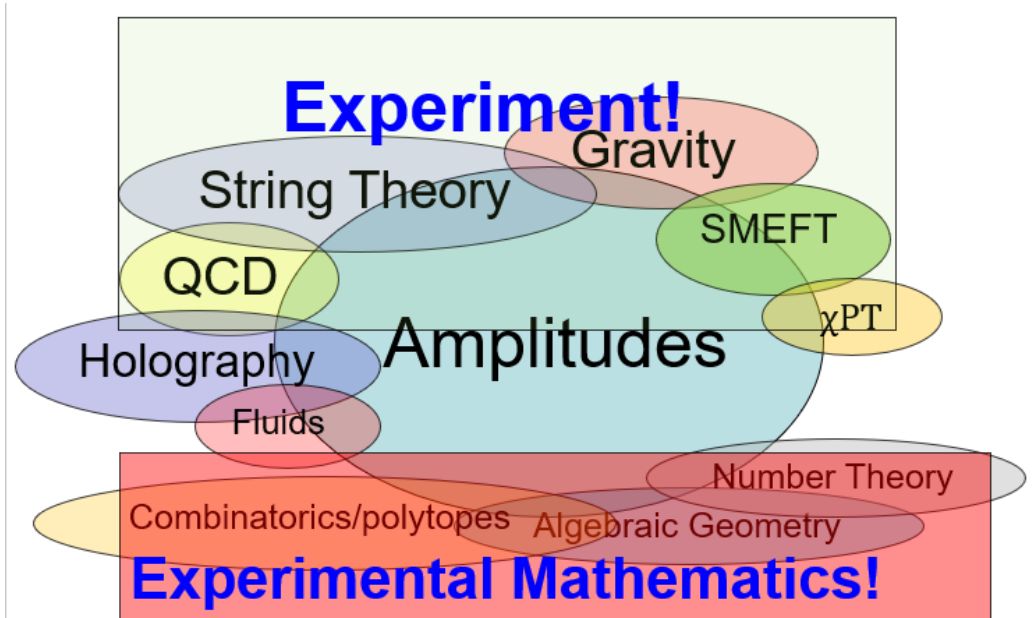
From Orbifolds to Amplitudes

Edinburgh Colloquium 26/6/26

# A Few Lessons Learned

- March to your own drummer

- Be **T** shaped!!



- Timing is everything
- Don't put lists of unfinished projects on your blackboard (desk drawer is OK)

# Closing Thoughts

Science is a noble undertaking: humans want to understand their place in the cosmos.

Science leads to technology that has vastly improved our way of life (but can be a double-edged sword).

The economic payoff of fundamental physics comes increasingly far down the road, if at all.

We depend on enlightened governments and increasingly, philanthropy, to support our research.

I was very fortunate to go into science at a time when governmental support was strong.

I hope that we can return to a collective realization of the importance of science, for both the economy and the soul.

# Very Special Thanks

All of you

All speakers this week

Claude [Duhr]

Claude [Anthropic]

Mayling

Anastasia, Yael, Matt, Donal

Andrew

Nature never deceives us;  
it is always we who deceive ourselves.

—JEAN-JACQUES ROUSSEAU



