WIMP Meets ALP: Coherent Freeze-out of Dark Matter

Maxim Perelstein (Cornell)

@LanceFest Workshop, Edinburgh

June 26, 2026

based on arXiv: [2511.16731], Phys. Rev. Lett. 136, 201004 (2026)

in collaboration with Steven Ferrante and Bingrong Yu

My Days as Lance’s grad student, Seven World Cups ago…

France wins the 1998 World Cup (SLAC watch party chez Lance)

Soccer on SLAC Lawn, August 1998

Standing:

- Eva Silverstein
- Damien Pierce
- James Wells
- Mihir Worah
- Nima Arkani-Hamed
- Maxim Perelstein
- Travis Brooks
- Apostolos Pilaftsis

Front row:

- Jose Pelaez
- Gino Isidori
- Lance Dixon
- Yuval Grossman
- Barak Kol

hep-th/9802162

SLAC–PUB–7751
UCLA/98/TEP/03
SWAT-98-183
February, 1998

On the Relationship between Yang-Mills Theory and Gravity and its Implication for Ultraviolet Divergences

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Abstract

String theory implies that field theories containing gravity are in a certain sense 'products' of gauge theories. We make this product structure explicit up to two loops for the relatively simple case of $N = 8$ supergravity four-point amplitudes, demonstrating that they are 'squares' of $N = 4$ super-Yang-Mills amplitudes. This is accomplished by obtaining an explicit expression for the D-dimensional two-loop contribution to the four-particle S-matrix for $N = 8$ supergravity, which we compare to the corresponding $N = 4$ Yang-Mills result. From these expressions we also obtain the two-loop ultraviolet divergences in dimensions $D = 7$ through $D = 11$. The analysis relies on the unitarity cuts of the two theories, many of which can be recycled from a one-loop computation. The two-particle cuts, which may be iterated to all loop orders, suggest that squaring relations between the two theories exist at any loop order. The loop-momentum power-counting implied by our two-particle cut analysis indicates that in four dimensions the first four-point divergence in $N = 8$ supergravity should appear at five loops, contrary to the earlier expectation, based on superspace arguments, of a three-loop counterterm.

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[1] Research supported in part by the US Department of Energy under grant DE-FG03-91ER40662

[2] Research supported by the US Department of Energy under grant DE-AC03-76SF00515.

[3] Research supported in part by the Leverhulme Foundation.

hep-th/9811140

SLAC–PUB–7992
UCLA/98/TEP/28
UFIFT–HEP–98–33
November, 1998

Multi-Leg One-Loop Gravity Amplitudes from Gauge Theory

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Abstract

By exploiting relations between gravity and gauge theories, we present two infinite sequences of one-loop n-graviton scattering amplitudes: the 'maximally helicity-violating' amplitudes in $N = 8$ supergravity, and the 'all-plus' helicity amplitudes in gravity with any minimally coupled massless matter content. The all-plus amplitudes correspond to self-dual field configurations and vanish in supersymmetric theories. We make use of the tree-level Kawai-Lewellen-Tye (KLT) relations between open and closed string theory amplitudes, which in the low-energy limit imply relations between gravity and gauge theory tree amplitudes. For $n \leq 6$, we determine the all-plus amplitudes explicitly from their unitarity cuts. The KLT relations, applied to the cuts, allow us to extend to gravity a previously found 'dimension-shifting' relation between (the cuts of) the all-plus amplitudes in gauge theory and the maximally helicity-violating amplitudes in $N = 4$ super-Yang-Mills theory. The gravitational version of the relation lets us determine the $n \leq 6$ $N = 8$ supergravity amplitudes from the all-plus gravity amplitudes. We infer the two series of amplitudes for all n from their soft and collinear properties, which can also be derived from gauge theory using the KLT relations.

PACS: 04.50.+h; 04.65+e;04.60.-m
Keywords: Unitarity; Cutting; Supersymmetry; Supergravity; One-loop

Submitted to Nuclear Physics B

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[3] Research supported by the US Department of Energy under grants DE-FG03-91ER40662 and DE-FG02-97ER41029.

November 26, 2024

SLAC-PUB-7801
SU-ITP-98/28
CPTH–S608.0498
IC/98/39
hep-ph/9804398

arXiv:hep-ph/9804398v1 24 Apr 1998

New Dimensions at a Millimeter to a Fermi and Superstrings at a TeV

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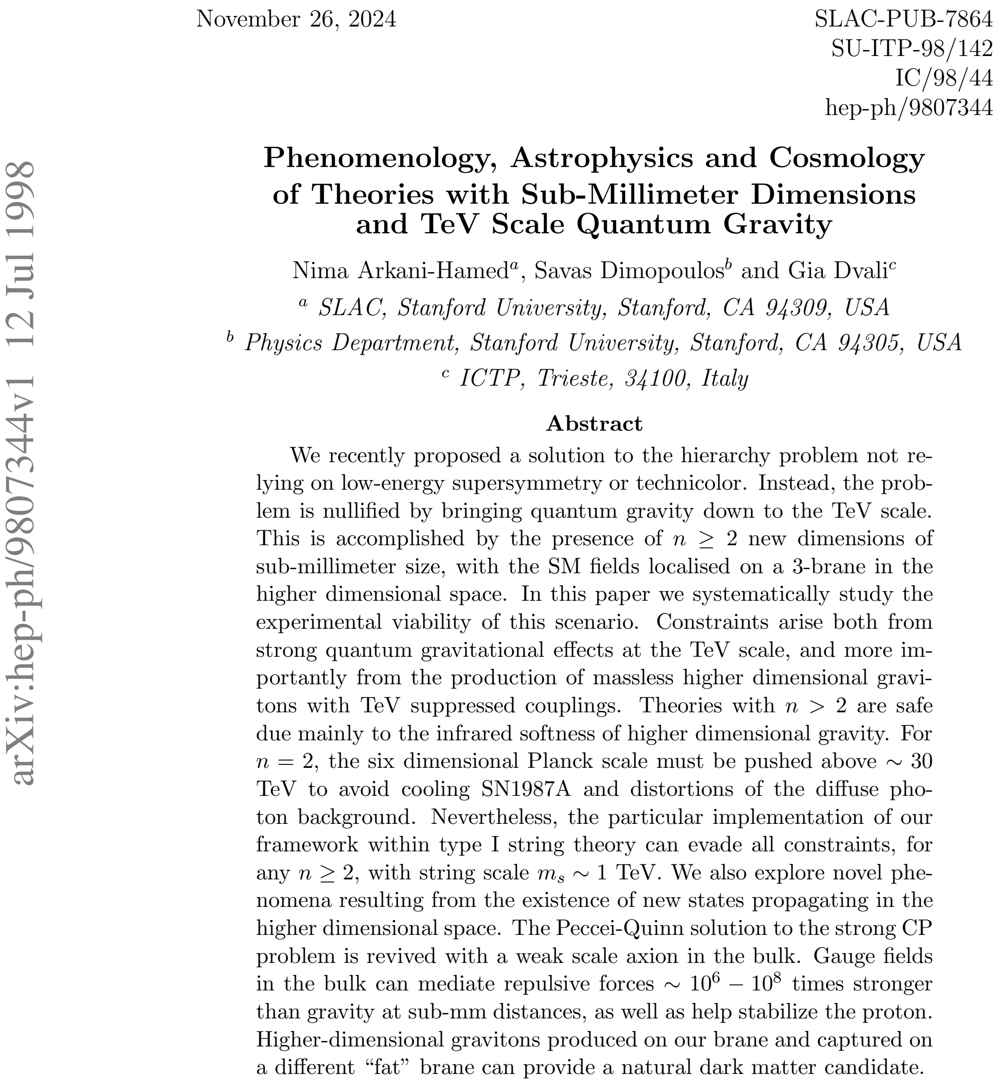
Abstract

Recently, a new framework for solving the hierarchy problem has been proposed which does not rely on low energy supersymmetry or technicolor. The gravitational and gauge interactions unite at the electroweak scale, and the observed weakness of gravity at long distances is due the existence of large new spatial dimensions. In this letter, we show that this framework can be embedded in string theory. These models have a perturbative description in the context of type I string theory. The gravitational sector consists of closed strings propagating in the higher-dimensional bulk, while ordinary matter consists of open strings living on D3-branes. This scenario raises the exciting possibility that the LHC and NLC will experimentally study ordinary aspects of string physics such as the production of narrow Regge-excitations of all standard model particles, as well more exotic phenomena involving strong gravity such as the production of black holes. The new dimensions can be probed by events with large missing energy carried off by gravitons escaping into the bulk. We finally discuss some important issues of model building, such as proton stability, gauge coupling unification and supersymmetry breaking.



November 26, 2024

SLAC-PUB-7864
SU-ITP-98/142
IC/98/44
hep-ph/9807344

arXiv:hep-ph/9807344v1 12 Jul 1998

Phenomenology, Astrophysics and Cosmology of Theories with Sub-Millimeter Dimensions and TeV Scale Quantum Gravity

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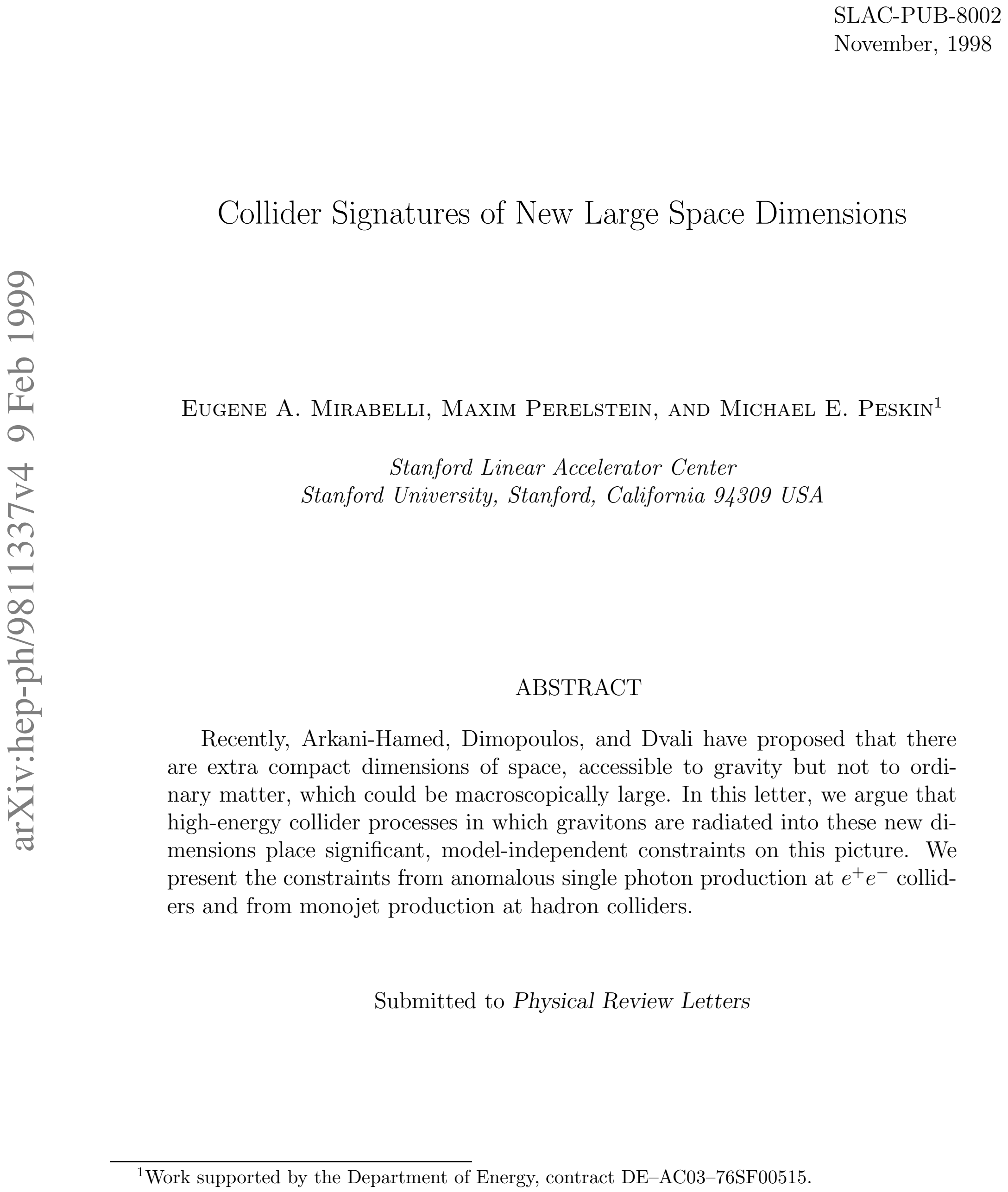
Abstract

We recently proposed a solution to the hierarchy problem not relying on low-energy supersymmetry or technicolor. Instead, the problem is nullified by bringing quantum gravity down to the TeV scale. This is accomplished by the presence of $n \geq 2$ new dimensions of sub-millimeter size, with the SM fields localised on a 3-brane in the higher dimensional space. In this paper we systematically study the experimental viability of this scenario. Constraints arise both from strong quantum gravitational effects at the TeV scale, and more importantly from the production of massless higher dimensional gravitons with TeV suppressed couplings. Theories with $n > 2$ are safe due mainly to the infrared softness of higher dimensional gravity. For $n = 2$, the six dimensional Planck scale must be pushed above ~ 30 TeV to avoid cooling SN1987A and distortions of the diffuse photon background. Nevertheless, the particular implementation of our framework within type I string theory can evade all constraints, for any $n \geq 2$, with string scale $m_s \sim 1$ TeV. We also explore novel phenomena resulting from the existence of new states propagating in the higher dimensional space. The Peccei-Quinn solution to the strong CP problem is revived with a weak scale axion in the bulk. Gauge fields in the bulk can mediate repulsive forces $\sim 10^6 - 10^8$ times stronger than gravity at sub-mm distances, as well as help stabilize the proton. Higher-dimensional gravitons produced on our brane and captured on a different "fat" brane can provide a natural dark matter candidate.

Lance's remark following seminar by Nima: LEP constraints on mono-photon + missing energy may kill this idea



SLAC-PUB-8002
November, 1998

Collider Signatures of New Large Space Dimensions

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Stanford University, Stanford, California 94309 USA

ABSTRACT

Recently, Arkani-Hamed, Dimopoulos, and Dvali have proposed that there are extra compact dimensions of space, accessible to gravity but not to ordinary matter, which could be macroscopically large. In this letter, we argue that high-energy collider processes in which gravitons are radiated into these new dimensions place significant, model-independent constraints on this picture. We present the constraints from anomalous single photon production at e^+e^- colliders and from monojet production at hadron colliders.

Submitted to *Physical Review Letters*

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[1] Work supported by the Department of Energy, contract DE–AC03–76SF00515.

New research direction for me
(for the next 28 years):
BSM phenomenology

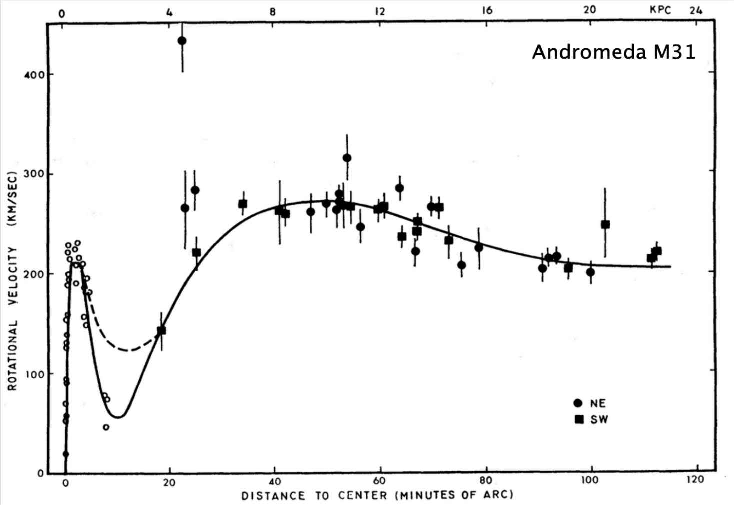
Thank you Lance for your amazingly generous, unwavering support and advice
(even as our research paths diverged a bit)!!!

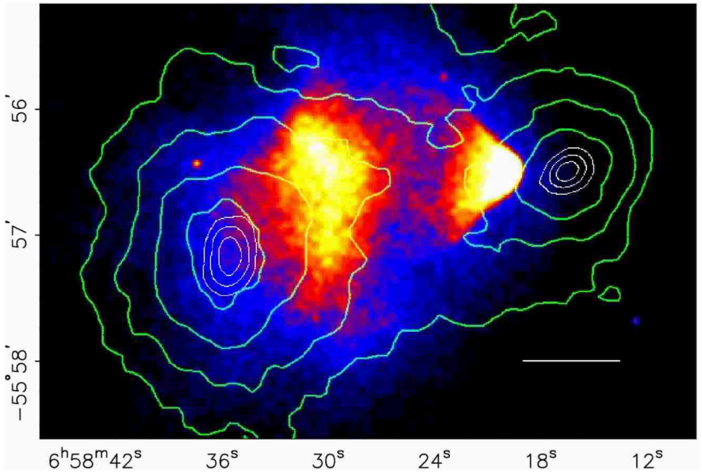
Outline of the talk

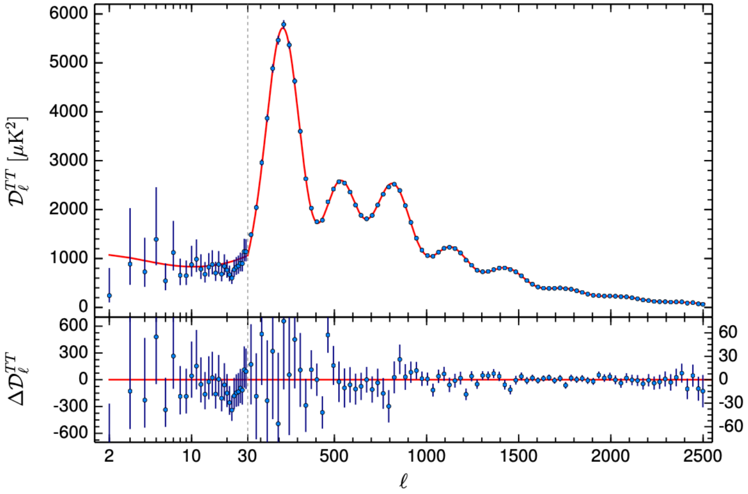
- **Motivation**
- Framework and dynamics
- Phenomenology
- Conclusions

Evidence for dark matter

- A lot of evidence for DM from astrophysics and cosmology
- Precise measurement of present DM abundance on cosmological scale







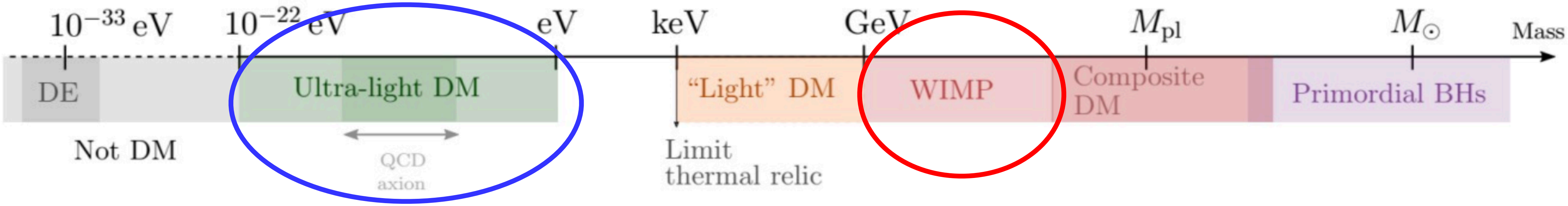
Galaxy rotation curve

Bullet cluster

CMB anisotropies

Particle nature of dark matter

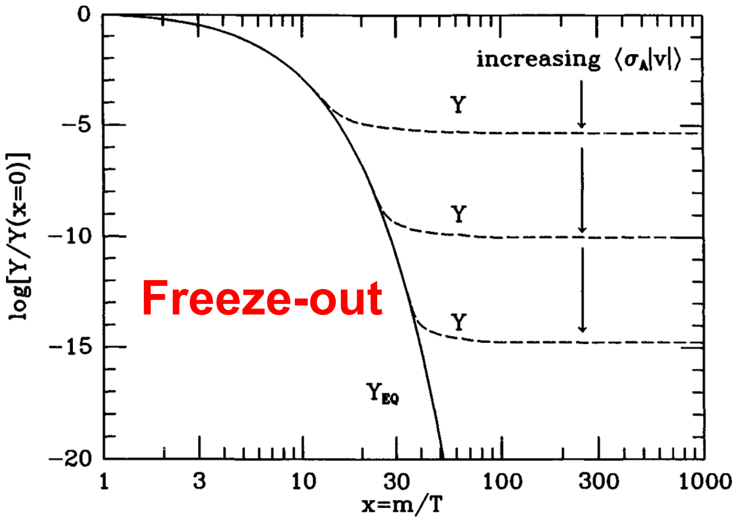
- No DM candidate in the Standard Model. We know very little about the microscopic nature of dark matter



- For many reasons, **Weakly-Interacting Massive Particle (WIMP)** and **Axion-Like Particle (ALP)** are two leading candidates for particle dark matter -- they exhibit sharply **different** cosmological histories

Cosmic history of WIMP

- Thermalized at high temperatures, decoupled later

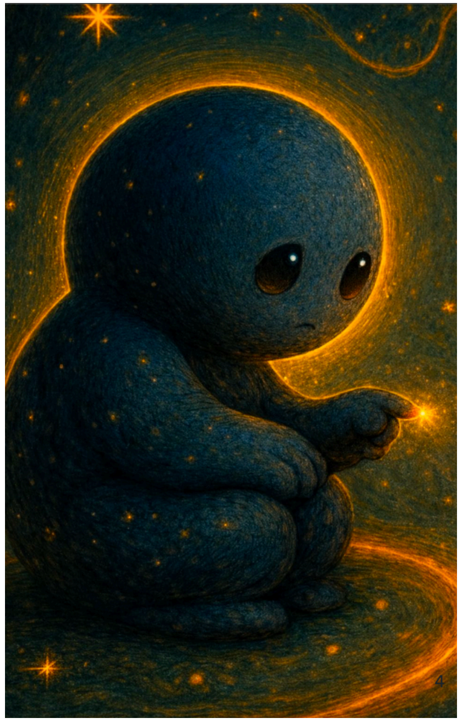


$$\frac{\mathrm{d}n_\chi}{\mathrm{d}t} + 3Hn_\chi = -\langle\sigma v\rangle\left(n_\chi^2 - n_{\chi,\mathrm{eq}}^2\right)$$

- Relic abundance insensitive to the initial conditions
- Determined mainly by the annihilation cross section

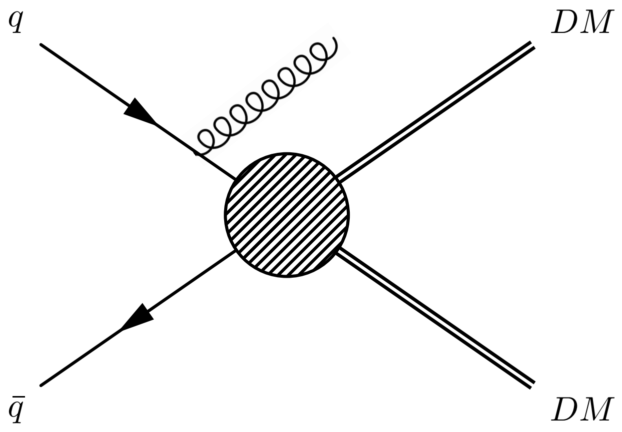
Pure weak-scale physics, “WIMP miracle”

Strong experimental constraints (LHC - MET+X, Direct Detection)

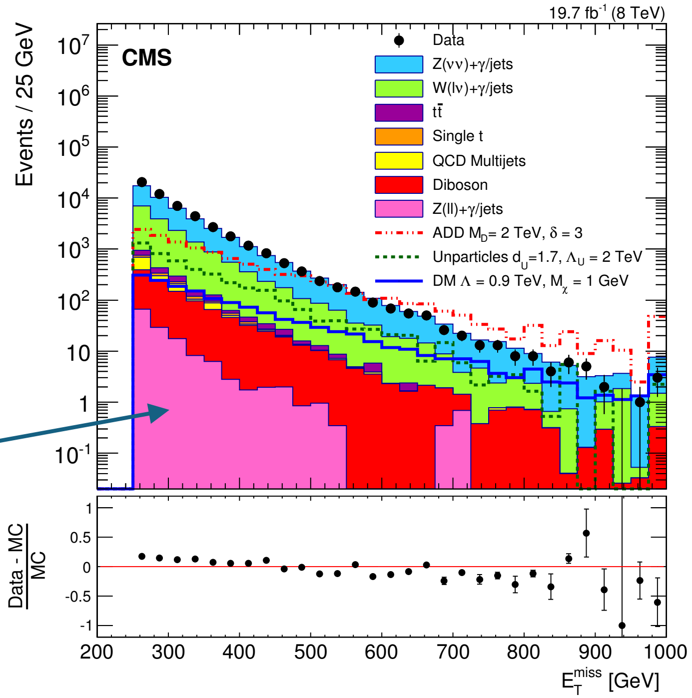


[Image is AI-generated]

LHC Searches for WIMPs



Related to cross section in BE using soft/collinear factorization [Birkedal, Matchev, MP, ’04]

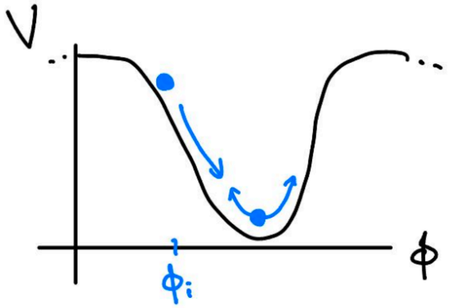


Thanks Lance for all the backgrounds!

Figure 3: Missing transverse energy $E_{\mathrm{T}}^{\mathrm{miss}}$ after all selections for data and SM backgrounds. The processes contributing to the SM background are from simulation, normalised to the estimation from data using the $E_{\mathrm{T}}^{\mathrm{miss}}$ threshold of 500 GeV. The error bars in the lower panel represent the statistical uncertainty. Overflow events are included in the last bin.

Cosmic history of ALP

- Feeble coupling, never thermalized, described by a **homogeneous**, **classical** field



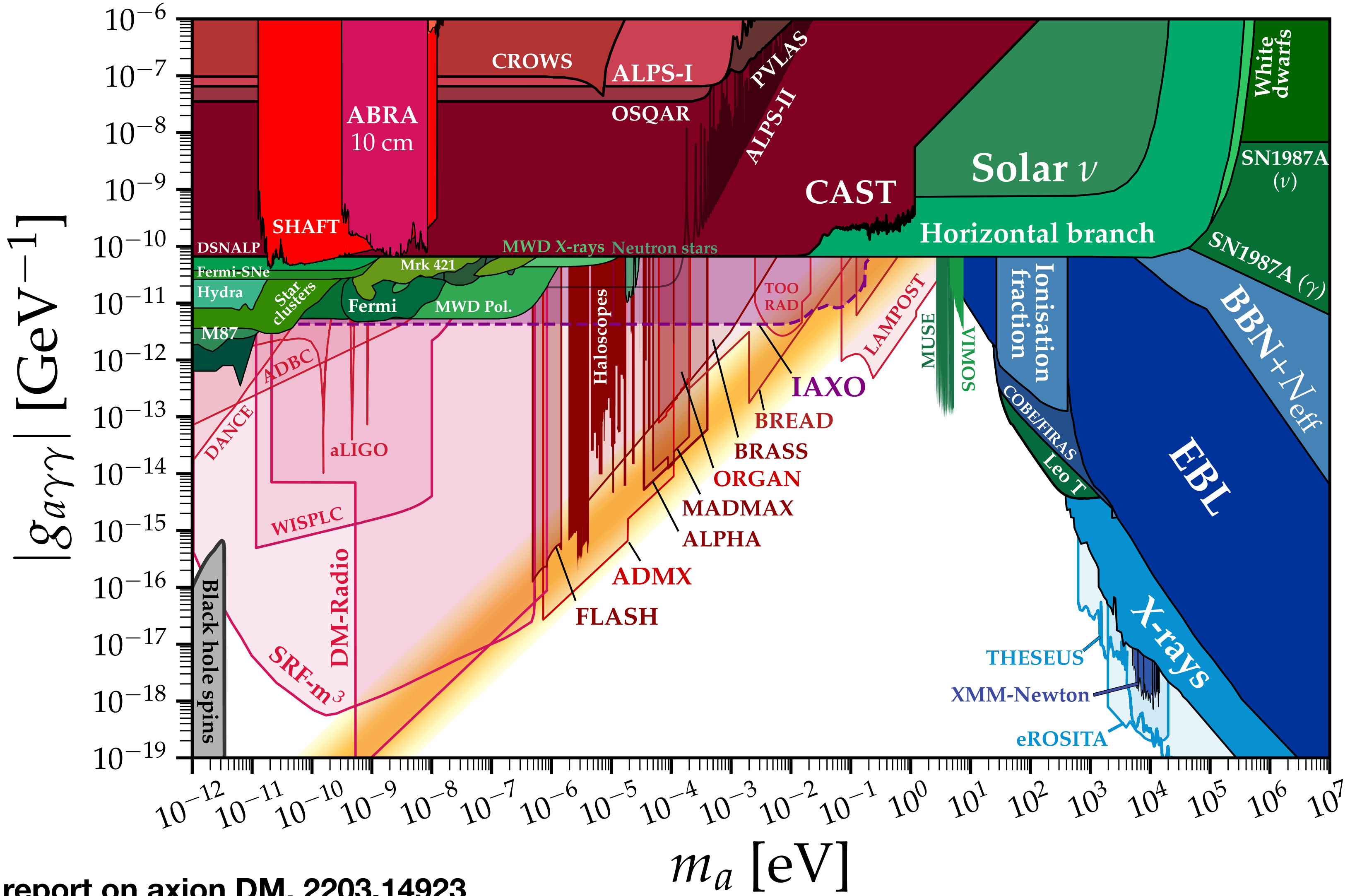
- Cosmological evolution is determined by solving the EOM of a scalar field in the FRW background

- Initial field value **misaligned**, relic abundance depends on both initial field value and mass: $\frac{\Omega_\phi h^2}{0.12} \sim \left(\frac{\phi_i}{10^{14}\ \mathrm{GeV}}\right)^2 \left(\frac{m_\phi}{10^{-10}\ \mathrm{eV}}\right)^{1/2}$

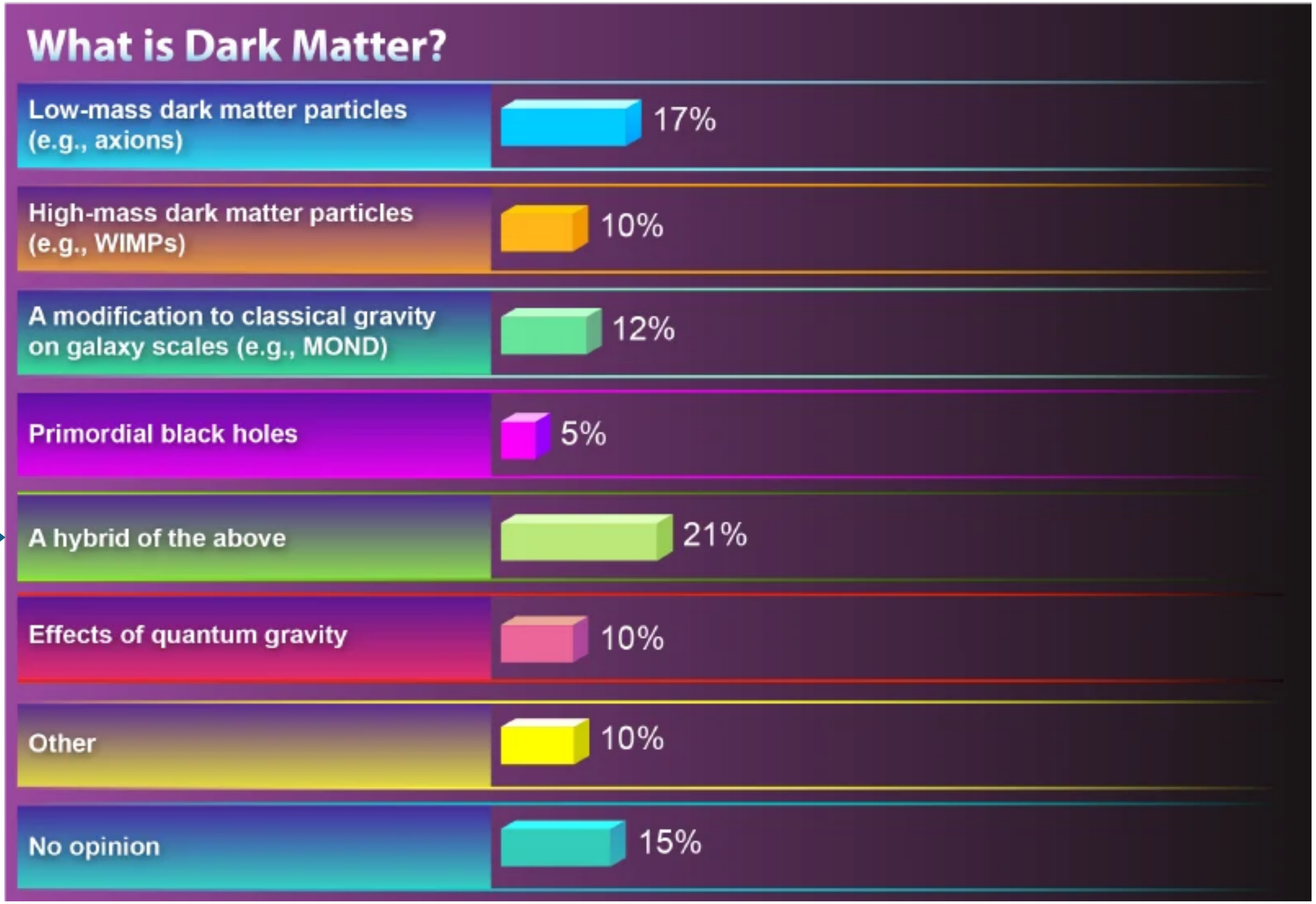
[Image is AI-generated]

Sensitive to initial conditions set at T >> weak scale

Experimental Searches for ALPs



Source: Snowmass report on axion DM, 2203.14923

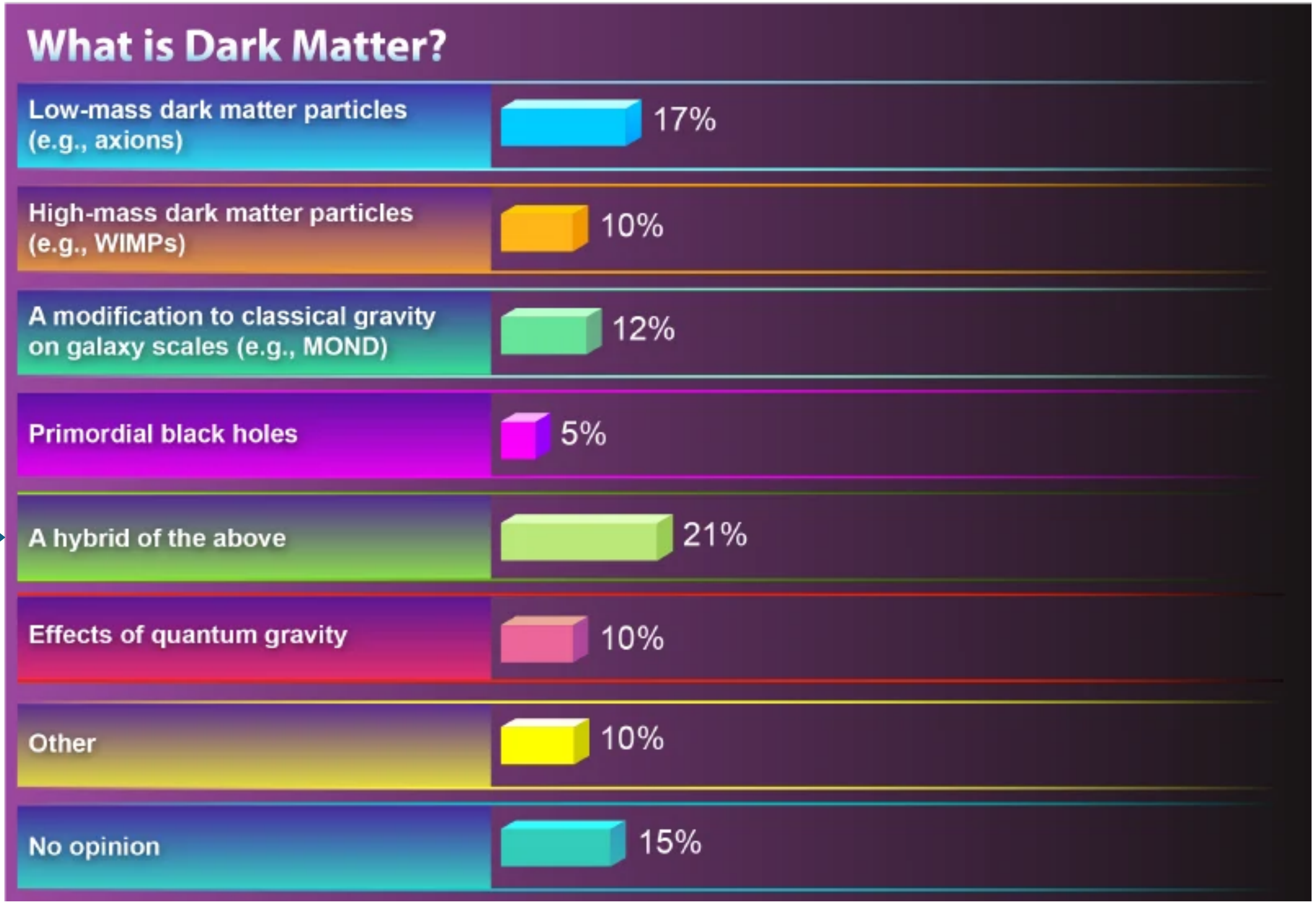


https://physics.aps.org/articles/v19/34

~10^4 papers

~10^5 papers

O(1) papers



https://physics.aps.org/articles/v19/34

What about WIMP + ALP?

$$\mathcal{L} =$$


Would a tiny coupling lead to a completely different cosmology?

[Image is AI-generated]

WIMP Meets ALP: Coherent Freeze-Out

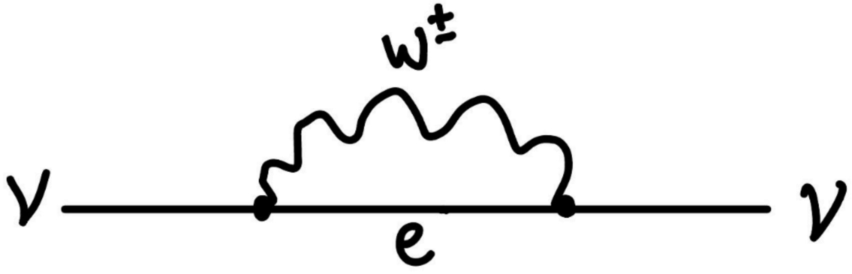
- Naively, you might think they would evolve independently, since ALP is **not** thermalized and momentum exchange is negligible
- However, physics is much richer thanks to the **coherent forward scattering** between these two sectors

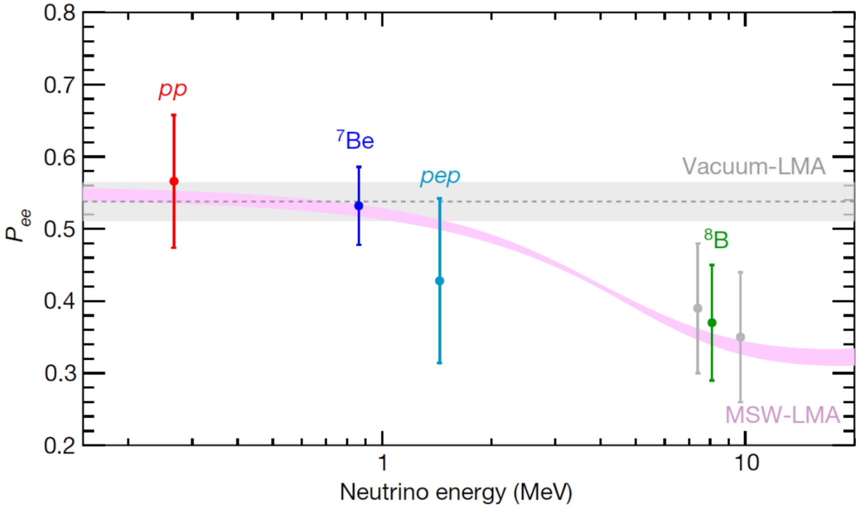
[Image is AI-generated]

Coherent forward scattering

- No momentum exchange, but modify **dispersion relation** of scattering particles

- Example in the Standard Model: **MSW effect** of neutrino oscillation



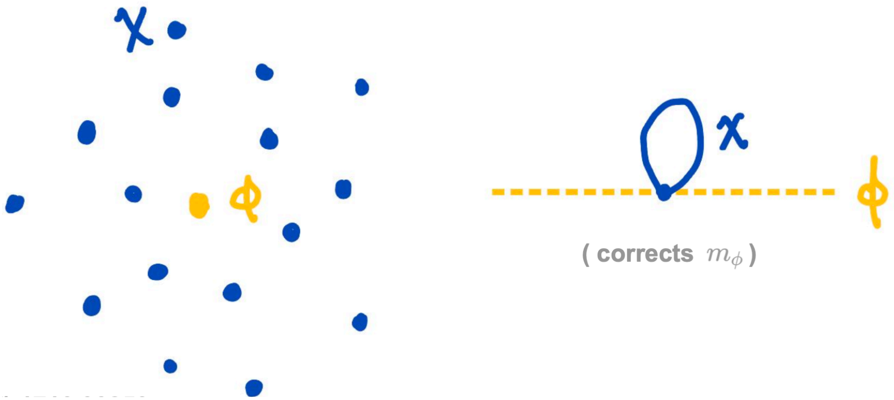


[hot topic in Summer 1998,
one of many many explained to me by Lance
after a SLAC seminar]

change neutrino oscillation probability by O(1),
crucial for solving the solar neutrino missing puzzle

Coherent forward scattering for WIMP + ALP

- Due to coherent forward scattering, both of their dispersion relations are modified by the medium of the other one, schematically,



ALP interacts with WIMP thermal bath

$$m^2_{\phi,\text{eff}} = m^2_\phi + \delta m^2_\phi$$

WIMP interacts with ALP classical background

$$m_{\chi,\text{eff}} = m_\chi + \delta m_\chi$$

Coherent forward scattering for WIMP + ALP

- Due to coherent forward scattering, both of their dispersion relations are modified by the medium of the other one
- WIMP Boltzmann equation and ALP equations of motion become a coupled system and need to be solved together
  - The effect is much more significant than one might naively expect. In the following, I will show it quantitatively via an explicit example

Outline of the talk

- Motivation
- **Framework and dynamics**
- Phenomenology
- Conclusions

The setup

- We consider an effective **quadratic** coupling between a fermion χ (the WIMP) and a real (pseudo-)scalar ϕ (the ALP):

$$\mathcal{L} = \frac{1}{\Lambda}\bar{\chi}\chi\frac{\phi^2}{2}$$

- This is the **leading non-derivative** coupling for a pNGB

- Motivated by the QCD axion coupling to nucleons (at scales below $\Lambda_{\rm QCD}$)

including the sign!

The QCD axion story

- Low-energy QCD (non-linear sigma model) contains a coupling

$$\mathcal{L} \sim \frac{m_q}{f_\pi^2}\,\pi^2\,\bar{N}N$$

- Pion-axion mixing

$$\pi \to \pi + \frac{f_\pi}{f_a}\,a \qquad \Rightarrow \qquad \mathcal{L} \sim \frac{1}{\Lambda}\,a^2\,\bar{N}N$$

- This is the **leading non-derivative** coupling for a pNGB

The setup

- We consider an effective **quadratic** coupling between a fermion χ (the WIMP) and a real (pseudo-)scalar ϕ (the ALP):

$$\mathcal{L} = \frac{1}{\Lambda}\overline{\chi}\chi\frac{\phi^2}{2}$$

- Possible UV origin: confinement of some dark SU(N), with χ being the dark nucleon and ϕ being the pNGB, analogous to the nucleon and the QCD axion

- In this talk, we take the EFT point of view and do not specify the UV origin of this coupling

Separation of scales

$$\mathcal{L} = \frac{1}{\Lambda}\bar{\chi}\chi\frac{\phi^2}{2}$$

- We assume the hierarchy among three relevant scales

$$\Lambda \gg m_\chi \gg m_\phi \qquad\qquad m_\chi \sim \text{electroweak scale}$$

- The coupling is assumed to be small enough to prevent the ALP from ever being thermalized via its scattering with the WIMP

$$\Gamma \sim \frac{T^3}{\Lambda^2} \ll H \sim \frac{T^2}{M_{\rm Pl}} \quad \Rightarrow \quad \Lambda \gg \sqrt{T_{\rm RH} M_{\rm Pl}}$$

Effective potential at finite T

$$x \equiv m_\chi / T$$

$$\varphi \equiv \phi / \sqrt{m_\chi \Lambda}$$

$$\gamma \equiv |1 - \varphi^2/2|$$

- At finite T, the WIMP thermal bath modifies the ALP potential

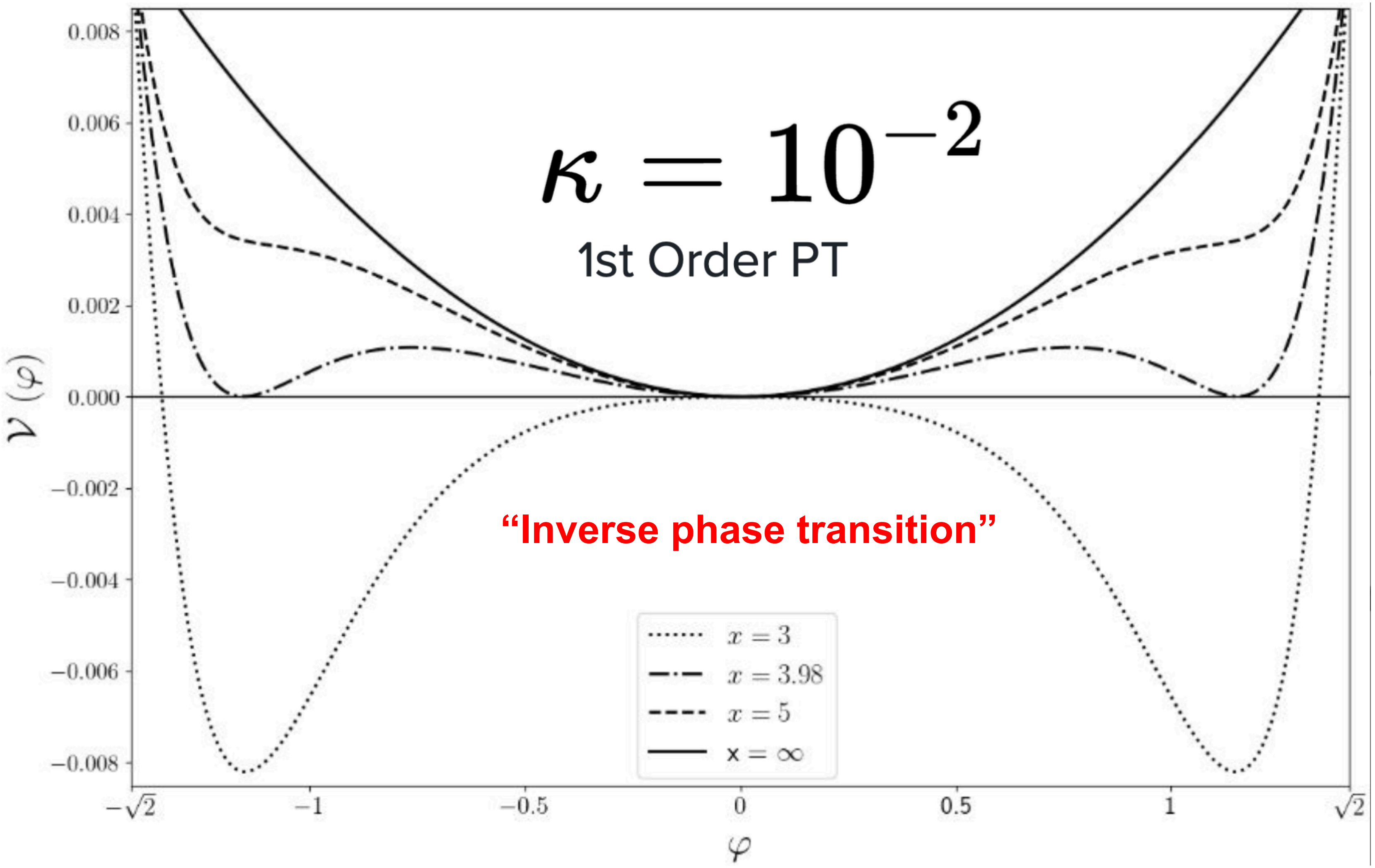
$$\mathcal{V}(\varphi, x) = \boxed{-\frac{\varphi^2}{2}\frac{\gamma^2 K_1(\gamma x)}{x}} + \boxed{\kappa\frac{\varphi^2}{2}}$$

thermal mass term **bare mass term**

$$\kappa \sim \frac{m_\phi^2 \Lambda}{m_\chi^3}$$

$$m_{\phi,\mathrm{eff}}^2 = m_\phi^2 - \frac{\langle \bar{\chi}\chi \rangle_T}{\Lambda}$$

- **High-T:** symmetry is broken, ALP obtains a VEV, shifting the WIMP mass
  Low-T: symmetry is restored, VEV returns to zero, WIMP freezes out



$$\kappa = 10^{-2}$$

1st Order PT

"Inverse phase transition"

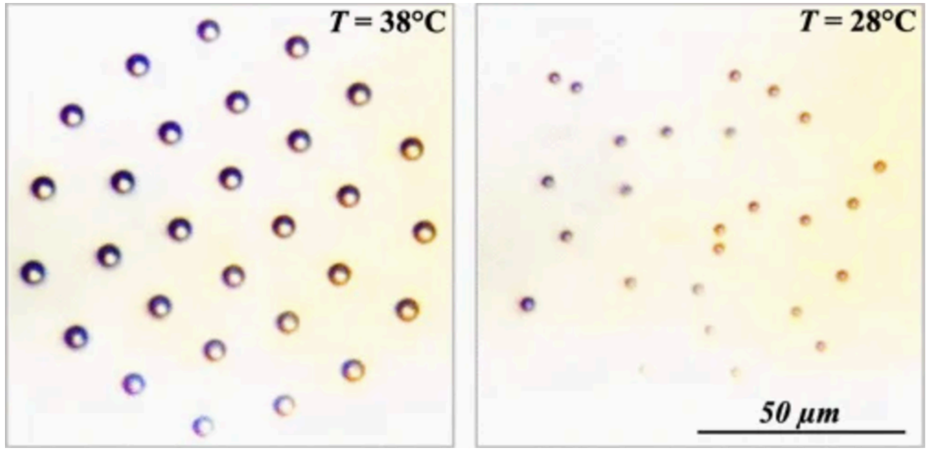
$$x = m_\chi / T$$

$$\kappa \sim \frac{m_\phi^2 \Lambda}{m_\chi^3}$$

- **High-T:** symmetry is broken, ALP obtains a VEV, shifting the WIMP mass
  Low-T: symmetry is restored, VEV returns to zero, WIMP freezes out

Why inverse phase transition

- Thermodynamics tend to minimize the free energy
- Normal phase transition: the symmetric (disordered) phase has a larger entropy than the broken (ordered) phase
- Inverse phase transition (our case): the broken phase has a larger entropy due to the suppression of the WIMP mass by the ALP VEV



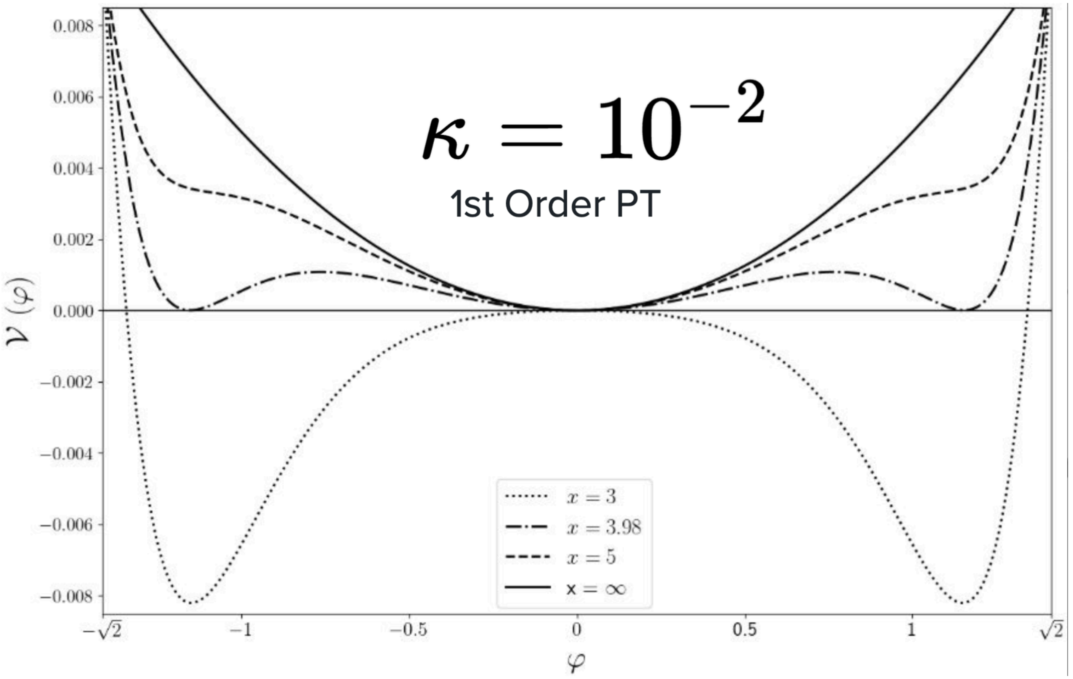
inverse melting

$$m_{\chi,\mathrm{eff}} = \left| m_\chi - \frac{\phi^2}{2\Lambda} \right|$$

$$\mathcal{V}(\varphi, x) = -\frac{\varphi^2}{2}\frac{\gamma^2 K_1(\gamma x)}{x} + \kappa\frac{\varphi^2}{2}$$

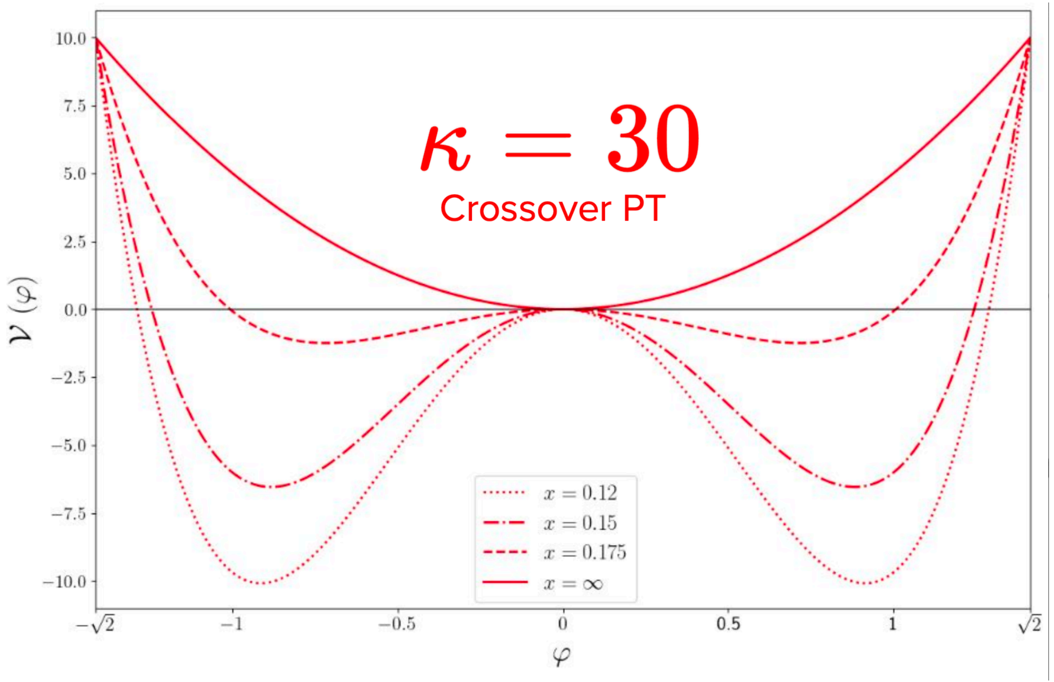
Orders of phase transition

- Using Ginzburg-Landau theory, we find $\kappa_c \sim 0.27$ splits FOPT and crossover



$\kappa = 10^{-2}$

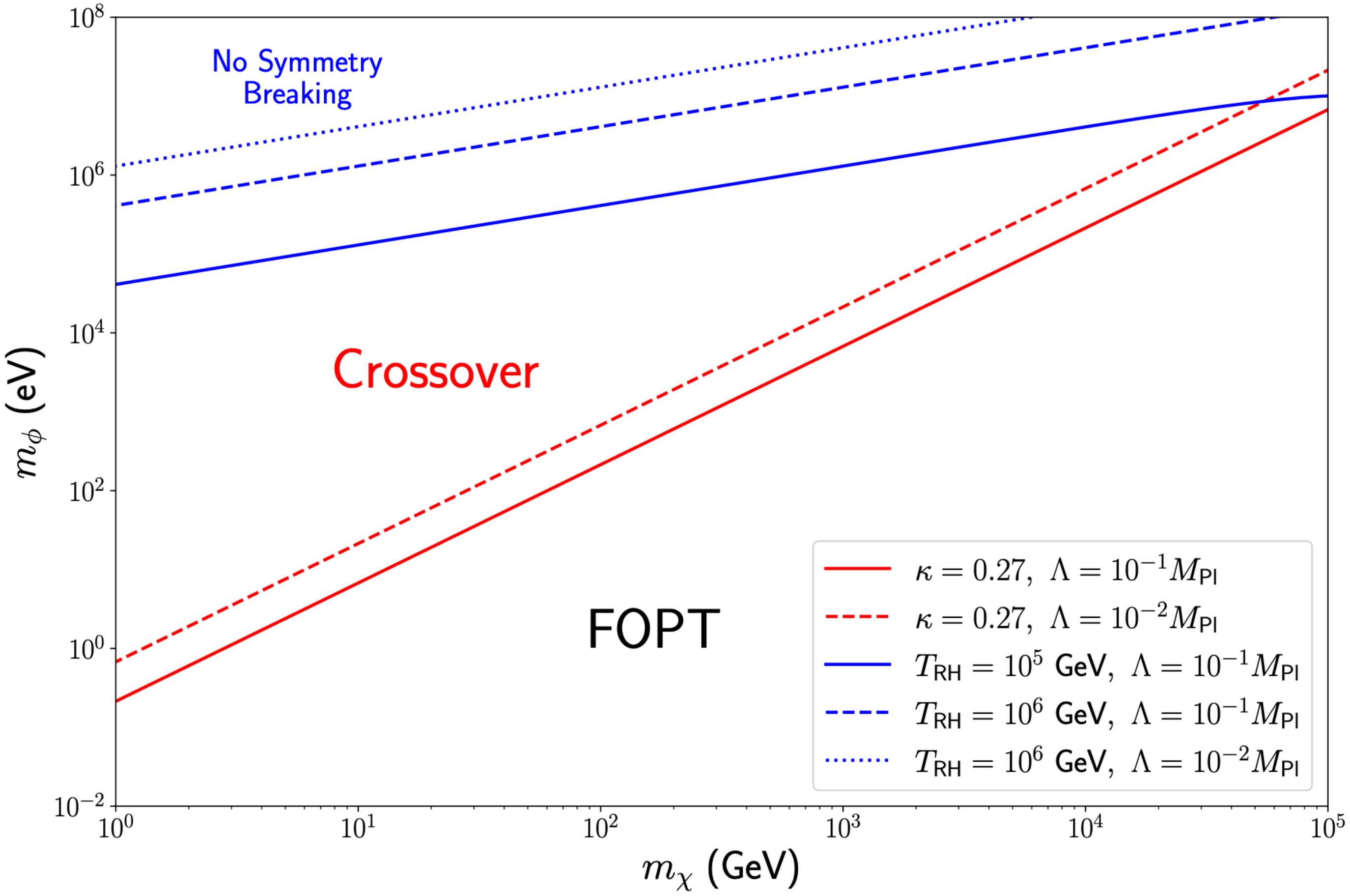
1st Order PT



$\kappa = 30$

Crossover PT

Classification of the parameter space



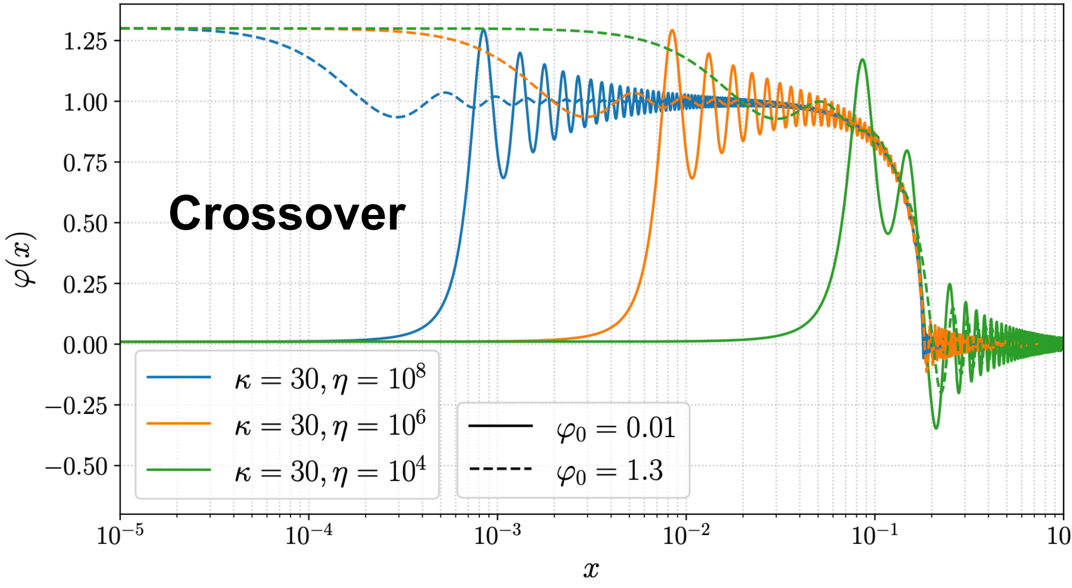
$$\kappa \sim \frac{m_\phi^2 \Lambda}{m_\chi^3}$$

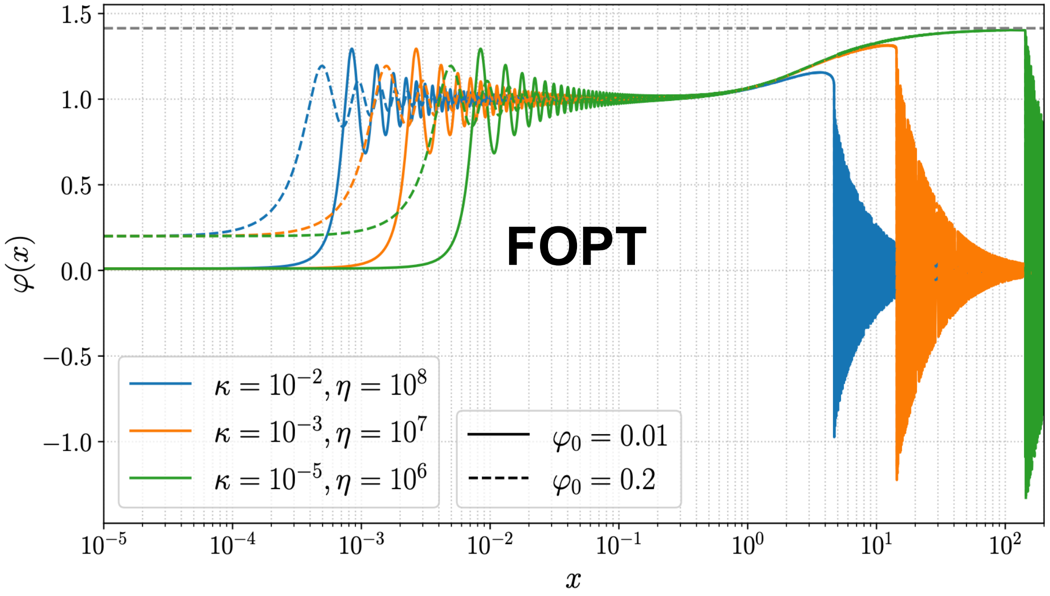
$$\kappa \gtrsim 0.27:\ \text{Crossover}$$

$$\kappa \lesssim 0.27:\ \text{FOPT}$$

Dynamics and phenomenology are very different in these two regimes

$$\varphi'' + \frac{2}{x}\varphi' - \eta x\left(1 - \frac{\varphi^2}{2}\right)\left[\left(1-\varphi^2\right)K_1(\gamma x) + \frac{\varphi^2}{2}\gamma x K_0(\gamma x)\right]\varphi + \eta\kappa x^2\varphi = 0$$





- Crossover regime: ALP evolution is nearly adiabatic, decouples earlier, does not affect freeze-out
- FOPT regime: ALP evolution is **non-adiabatic**, decouples later, affects WIMP freeze-out

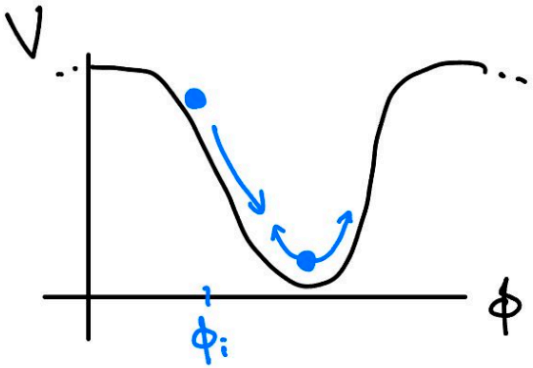
Outline of the talk

- Motivation
- Framework and dynamics
- **Phenomenology**
- Conclusions

Crossover regime

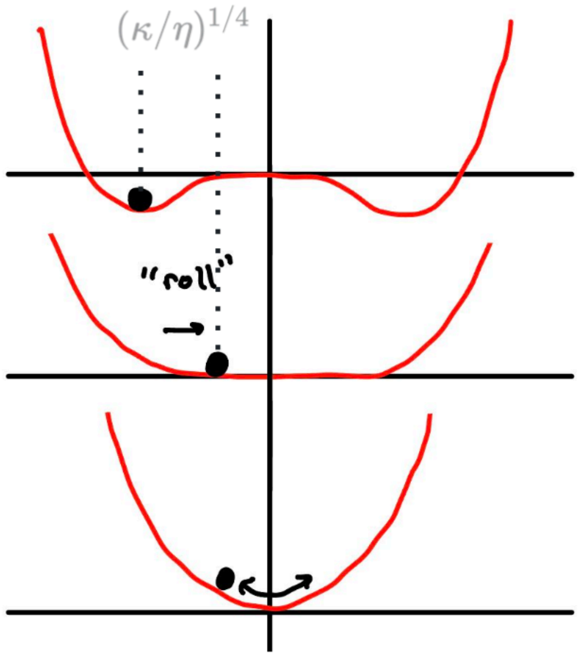
Relic abundance of ALP

- Usual misalignment: φ_i ~ UV dependent

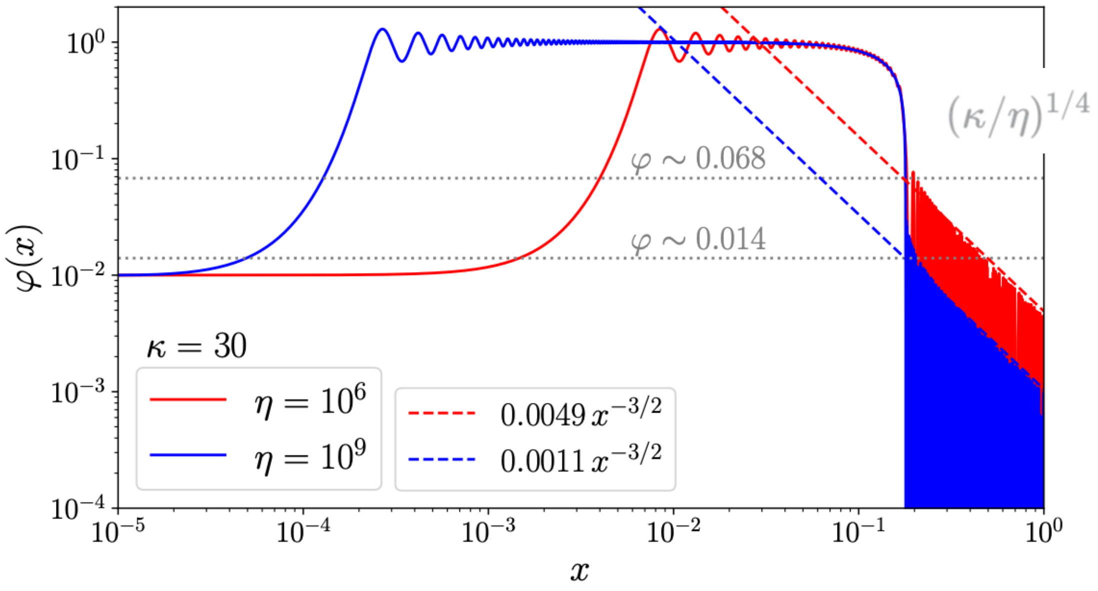


- Our case: $\varphi_i \sim \varphi_* (\kappa/\eta)^{1/4}$ fixed by thermal dynamics

$$\rho_\phi(x_0) \approx m_\phi^2 \phi^2(x_c) \left(\frac{\kappa}{\eta}\right)^{\frac{1}{2}} \left(\frac{x_c}{x_0}\right)^3 \frac{g_{*S}(x_0)}{g_{*S}(x_c)}$$



Relic abundance of ALP



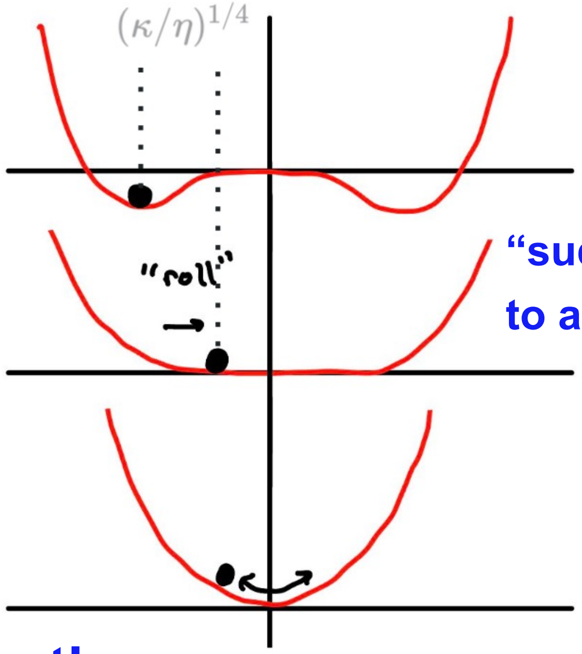
- Usual misalignment: φ_i ~ UV dependent

$$\frac{\Omega_\phi h^2}{0.12} \sim \left(\frac{\phi_i}{10^{14}\ \mathrm{GeV}}\right)^2 \left(\frac{m_\phi}{10^{-10}\ \mathrm{eV}}\right)^{1/2}$$

- Our case: $\varphi_i \sim \varphi_* (\kappa/\eta)^{1/4}$ **fixed by thermal dynamics**

$$\rho_\phi(x_0) \approx m_\phi^2 \phi^2(x_c) \left(\frac{\kappa}{\eta}\right)^{\frac{1}{2}} \left(\frac{x_c}{x_0}\right)^3 \frac{g_{*S}(x_0)}{g_{*S}(x_c)}$$

$$\boxed{\Omega_\phi \approx 0.3 \left(\frac{m_\chi}{10\ \mathrm{GeV}}\right)^{3/2} \left(\frac{\Lambda}{0.1 M_{\mathrm{Pl}}}\right)^{1/2}}$$



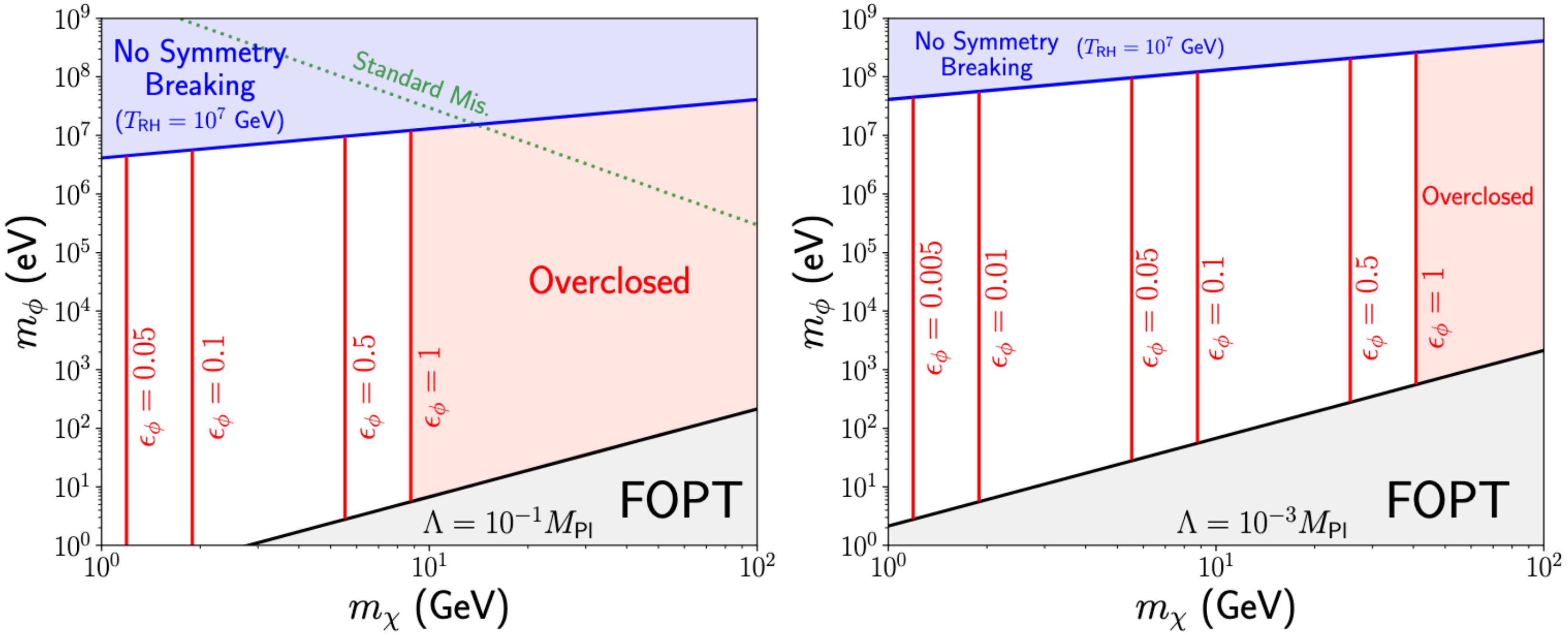
"sudden drop" due to adiabatic invariant

Surprisingly, the ALP mass also drops out!

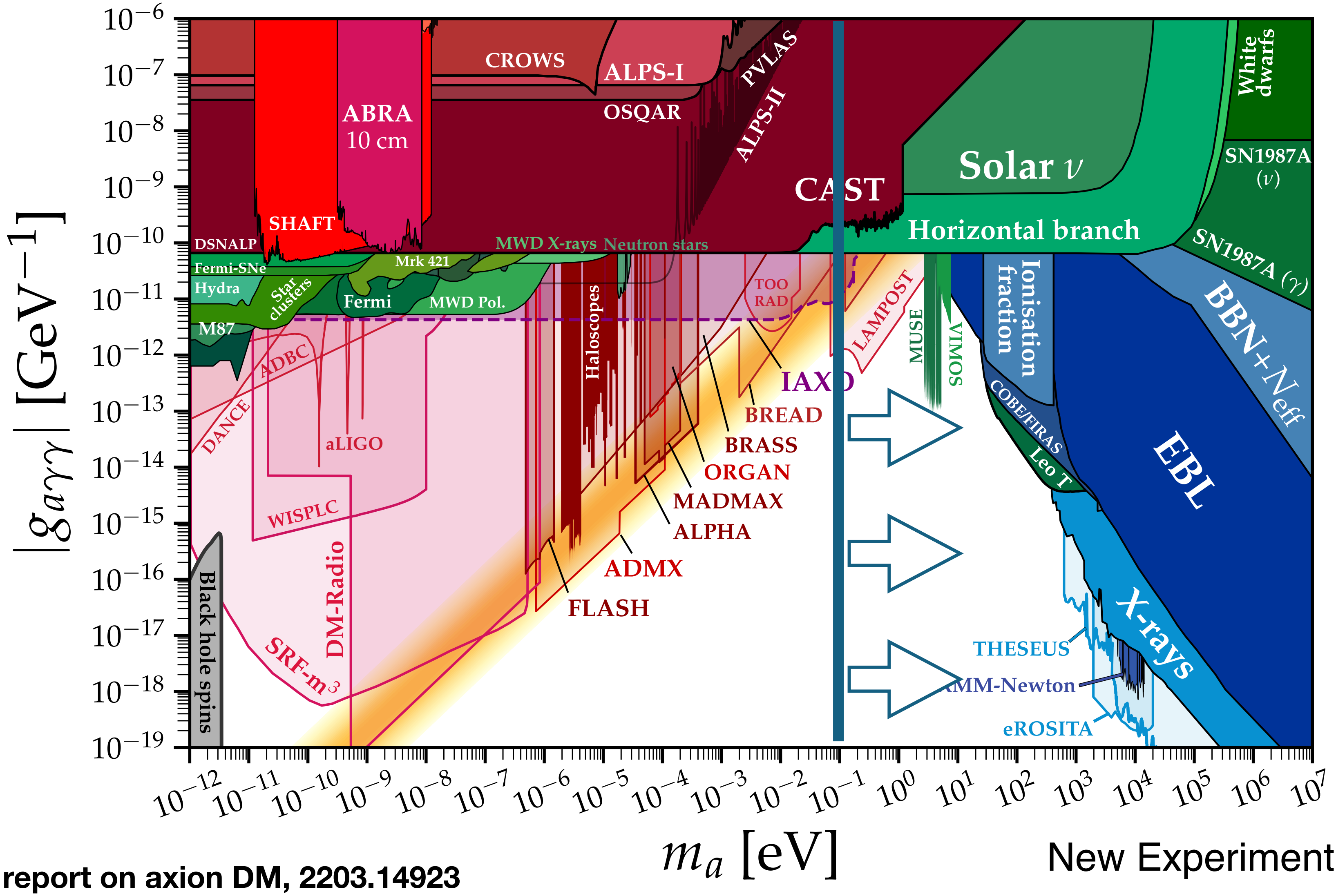
The "ALP miracle"

$$\Omega_\phi \approx 0.3\left(\frac{m_\chi}{10\ \mathrm{GeV}}\right)^{3/2}\left(\frac{\Lambda}{0.1 M_{\mathrm{Pl}}}\right)^{1/2}$$

- For $m_\chi \sim$ weak scale, $\Lambda \sim$ Planck scale, the ALP obtains correct relic abundance
- This is largely insensitive to both the initial ALP field value and the ALP mass



Experimental Searches for ALPs



Source: Snowmass report on axion DM, 2203.14923

New Experimental Ideas Needed!

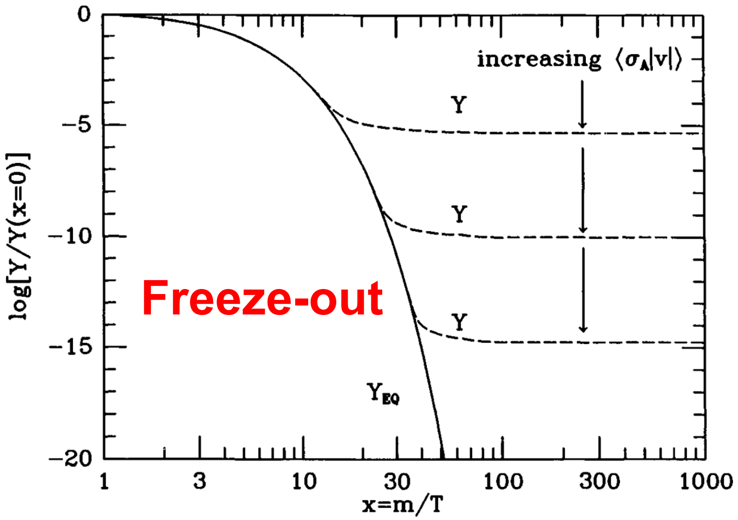
First-order phase transition regime

Normal Cosmic history of WIMP

- Thermalized at high temperatures, decoupled later

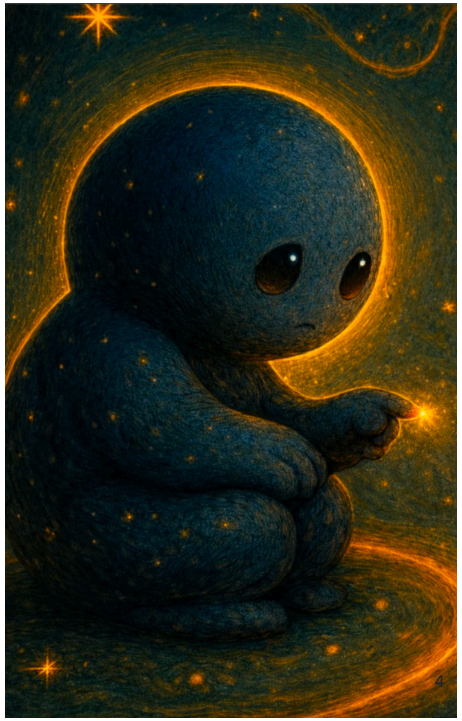
$$\frac{\mathrm{d}n_\chi}{\mathrm{d}t} + 3Hn_\chi = -\langle\sigma v\rangle\left(n_\chi^2 - n_{\chi,\mathrm{eq}}^2\right)$$

- Relic abundance insensitive to the initial conditions



Pure weak-scale physics, “WIMP miracle”

Strong experimental constraints (LHC - MET+X, Direct Detection)



[Image is AI-generated]

ALP Effect: Evolution of the vacuum

- Given the potential, for $\kappa \ll 1$ and $x \gg 1$, the symmetry-breaking vacuum evolves as

$$\varphi_* = \sqrt{2\left(1 - \frac{c_1}{x}\right)}$$

$$c_1 \approx 1.33 \text{ is the root of } c_1 K_0(c_1) = K_1(c_1)$$

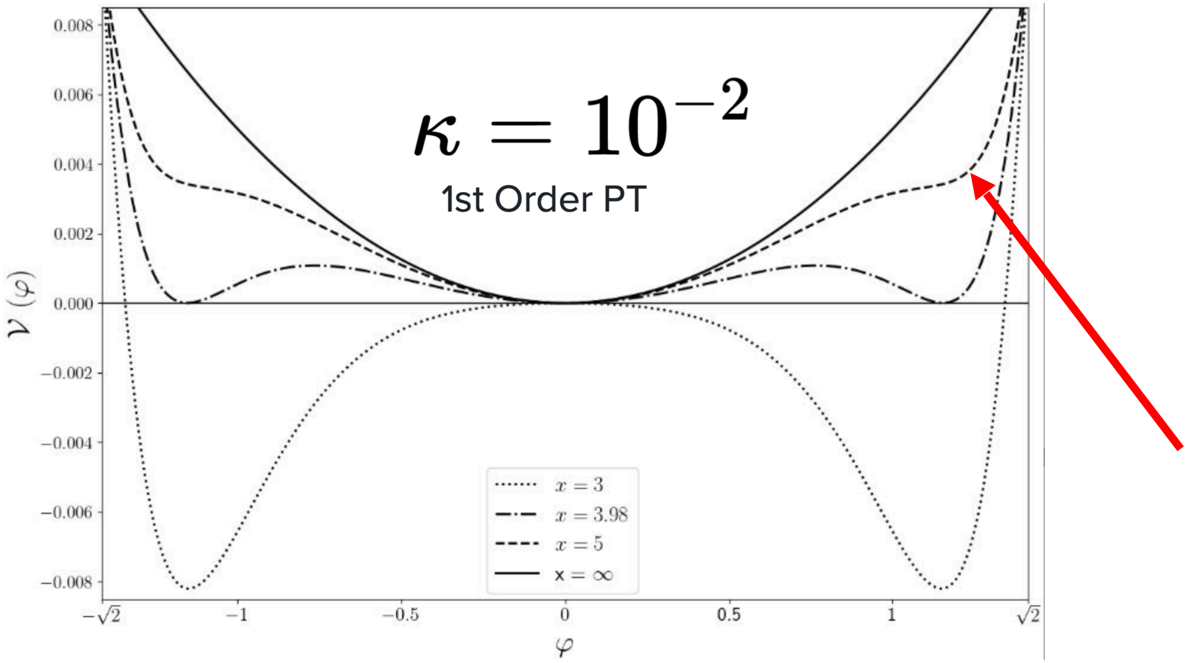
- The evolving vacuum reduces the effective WIMP mass as temperature decreases

$$\boxed{m_{\chi,\text{eff}} = \left(1 - \varphi_*^2/2\right) m_\chi \sim T}$$

- This makes the WIMP stay in equilibrium even at $x \equiv \frac{m_\chi}{T} \gg O(25)$

No Boltzmann suppression at large x!

Coherent freeze-out

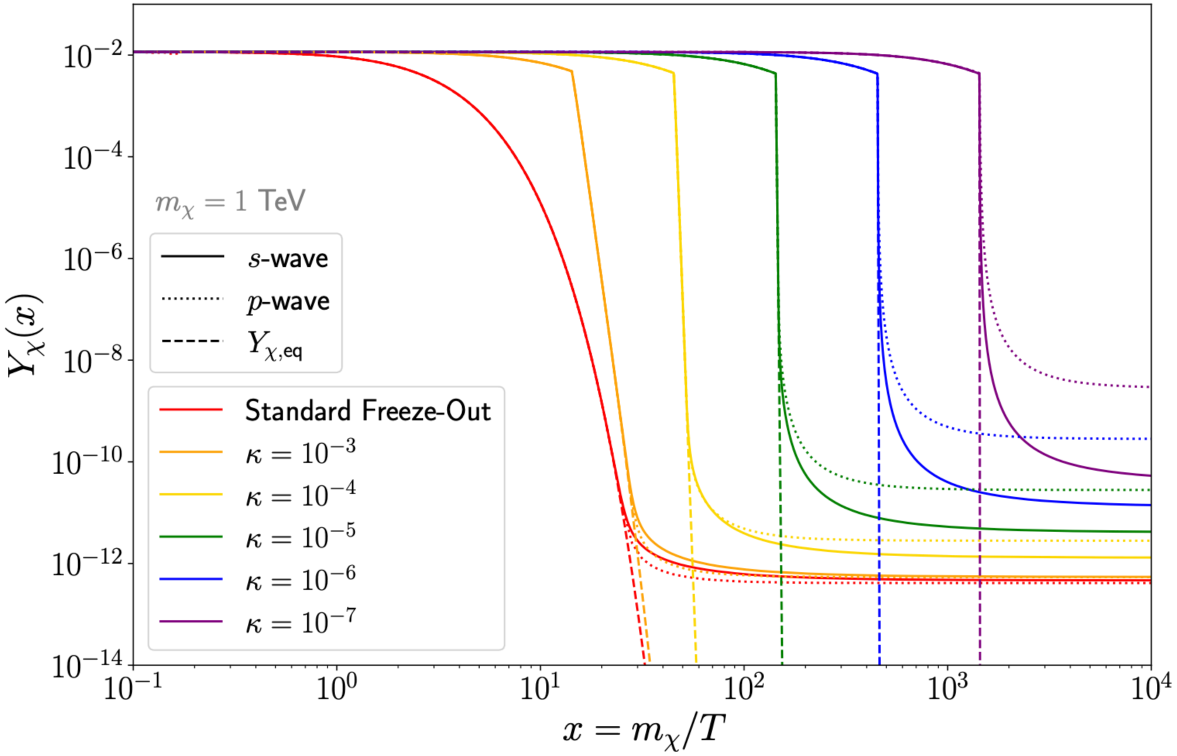


WIMP mass drops rapidly

- Normal freeze-out: triggered by universe expansion, typically occurs at $x_{\rm fo} \sim 25$
- Coherent freeze-out: triggered by phase transition, can occur at $x_{\rm cfo} \gg 25$

Relic abundance of coherent freeze-out

- Because freeze-out is delayed, the relic abundance is enhanced thanks to less redshift between freeze-out and present day



Relic abundance of coherent freeze-out

- To reproduce the measured relic abundance, need to increase the WIMP annihilation cross section

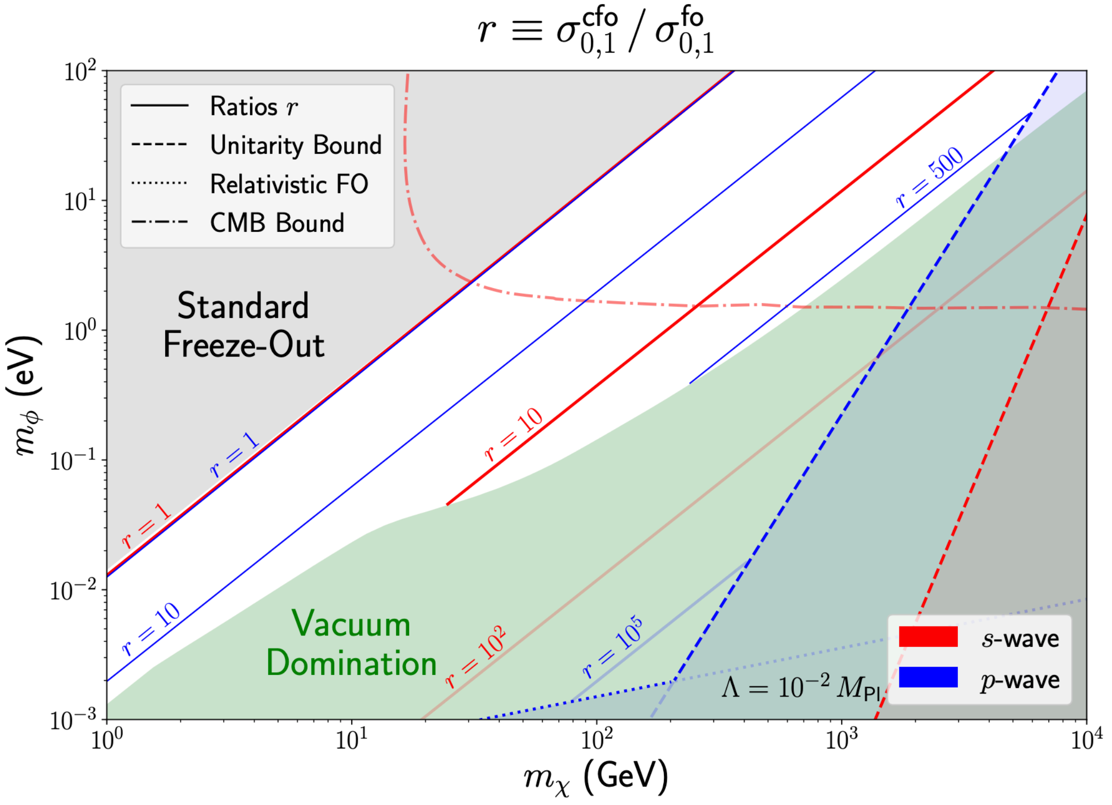
- Quantatively, we find $\frac{Y^{\rm cfo}_{\chi,\infty}}{Y^{\rm fo}_{\chi,\infty}} = \frac{14}{9}\frac{x_{\rm cfo}}{x_{\rm fo}}\frac{\sigma_0^{\rm fo} + 3\sigma_1^{\rm fo}/x_{\rm fo}}{\sigma_0^{\rm cfo} + 21\sigma_1^{\rm cfo}/(10x_{\rm cfo})}$

- To get the same yield, we have

s-wave: $\sigma_0^{\rm cfo}/\sigma_0^{\rm fo} \sim x_{\rm cfo}/x_{\rm fo}$

p-wave: $\sigma_1^{\rm cfo}/\sigma_1^{\rm fo} \sim (x_{\rm cfo}/x_{\rm fo})^2$

Enhancement of annihilation cross section



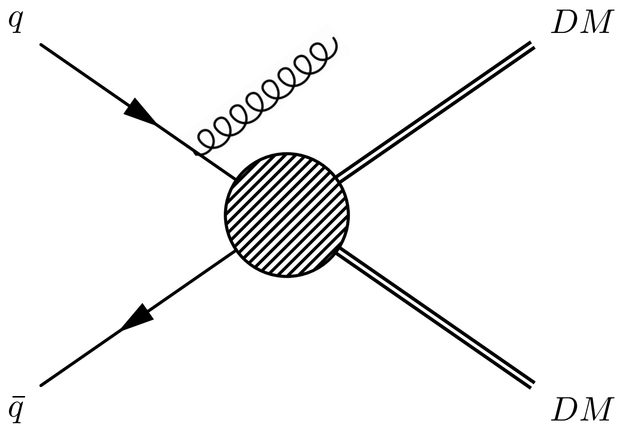
s-wave: enhanced up to 30

p-wave: enhanced up to 10^3

- Typically in models, higher cross section means lower mass
- Lower-mass WIMPs are less constrained at the LHC due to higher backgrounds

[work in progress]

LHC Searches for WIMPs



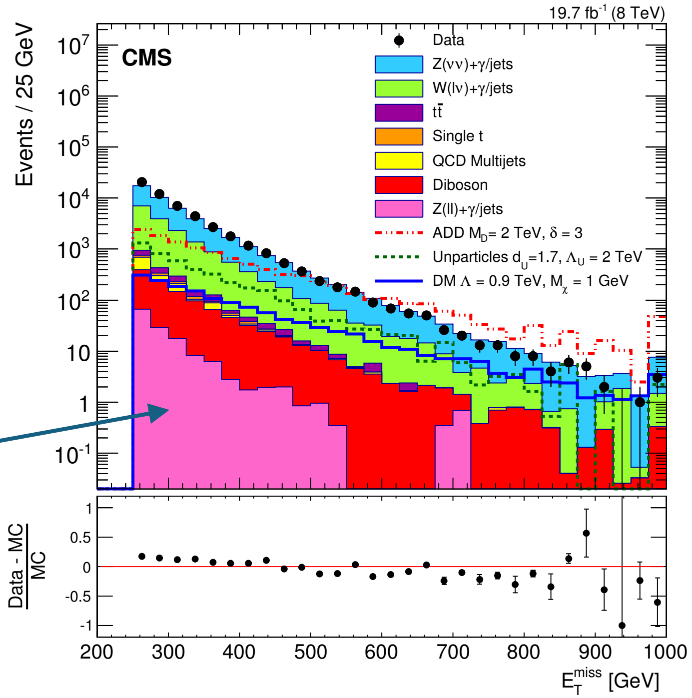


Figure 3: Missing transverse energy E_T^{miss} after all selections for data and SM backgrounds. The processes contributing to the SM background are from simulation, normalised to the estimation from data using the E_T^{miss} threshold of 500 GeV. The error bars in the lower panel represent the statistical uncertainty. Overflow events are included in the last bin.

Thanks Lance for
all the backgrounds!

Conclusions

- Even a tiny (Planck-suppressed) coupling between WIMP and ALP can substantially modify their cosmological histories through coherent forward scattering

- This dramatically changes the predicted properties of WIMP and ALP dark matter particles, such as their masses and annihilation cross sections

- This in turn leads to important consequences for experimental searches for particle dark matter, both axion-like and WIMP-like

Happy Birthday, Lance!

and many thanks for

- Teaching me a ton of physics
- Teaching me how to be a physicist
- Unwavering support
- Calculating the backgrounds
- …

Backup slides

Modification of dispersion relation

- Both masses are shifted due to the coherent forward scattering inside the other medium

$$\langle\bar{\chi}\chi\rangle_T = g_\chi \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}\frac{m_{\chi,\mathrm{eff}}}{E_\mathbf{k}} f_\chi(\mathbf{k})$$

$$m^2_{\phi,\mathrm{eff}} = m^2_\phi - \frac{\langle\bar{\chi}\chi\rangle_T}{\Lambda}$$

$$m_{\chi,\mathrm{eff}} = \left| m_\chi - \frac{\phi^2}{2\Lambda} \right|$$

- The dynamics of the two sectors are therefore **coupled**

The full one-loop thermal potential

$$V_{\text{full}}(\phi, T) = -\frac{g_\chi}{2\pi^2} T^4 \int_0^\infty \mathrm{d}y\, y^2 \log\left[1 + e^{-\sqrt{y^2 + m_{\chi,\text{eff}}^2/T^2}}\right] + \frac{1}{2} m_\phi^2 \phi^2 \equiv \frac{g_\chi}{2\pi^2} m_\chi^4 \mathcal{V}_{\text{full}}(\varphi, x)$$

$$\mathcal{V}_{\text{full}}(\varphi, x) = -\frac{1}{x^4} \int_0^\infty \mathrm{d}y\, y^2 \log\left[1 + e^{-\sqrt{y^2 + \gamma^2 x^2}}\right] + \kappa \frac{\varphi^2}{2}$$

$$\kappa \equiv \frac{2\pi^2}{g_\chi} \frac{m_\phi^2 \Lambda}{m_\chi^3}$$

EOM with the full potential

$$\kappa \equiv \frac{2\pi^2}{g_\chi} \frac{m_\phi^2 \Lambda}{m_\chi^3}$$

$$\eta \equiv \frac{g_\chi}{2\pi^2} \frac{1}{1.66^2 g_*} \frac{M_{\rm Pl}^2}{m_\chi \Lambda}$$

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V(\phi, T)}{\partial \phi} = 0$$

$$\varphi'' + \frac{2}{x}\varphi' - \eta\, x\, \varphi\, \mathrm{sgn}\left(1 - \frac{\varphi^2}{2}\right) \sum_{n=1}^{\infty} \left[\frac{(-1)^{n-1}}{n} \gamma^2 K_1(n\gamma x)\right] + \eta\, \kappa\, x^2 \varphi = 0$$

Dark matter in the FOPT regime

- Freeze-out of WIMP is delayed in a *dynamical* way, allowing for orders of magnitude enhancement of its annihilation cross section while still yielding the correct relic abundance

- In this regime, however, ALP only behaves as a spectator field. In order not to overclose the Universe, it must decay to radiation after WIMP freeze-out. Therefore, dark matter consists solely of the WIMP in the FOPT regime

More calculations in the crossover regime

- Symmetry is restored at $x_c \equiv 1/\sqrt{\kappa}$

$$\varphi'' + \frac{2}{x}\varphi' - \eta\left(1 - \kappa x^2\right)\varphi + \eta\,\varphi^3 = 0$$

- The adiabatic approximation holds when $\eta\left(1 - \kappa x^2\right) \gg 1/x^2$

- We define $x_1 = x_c + \delta x$ the time when adiabaticity is restored, i.e., when effective mass = Hubble friction

$$\left|\eta\left(1 - \kappa x_1^2\right)\right| \equiv 1/x_1^2$$

- For $\eta \gg \kappa$ (which always holds), the loss of adiabaticity is extremely brief $\delta x / x_c \approx \kappa/(2\eta)$

- For $x > x_1$, we have adiabatic invariant

$$m_{\phi,\mathrm{eff}}(x)\phi^2(x)\,a^3(x) = m_{\phi,\mathrm{eff}}(x_1)\phi^2(x_1)\,a^3(x_1)$$

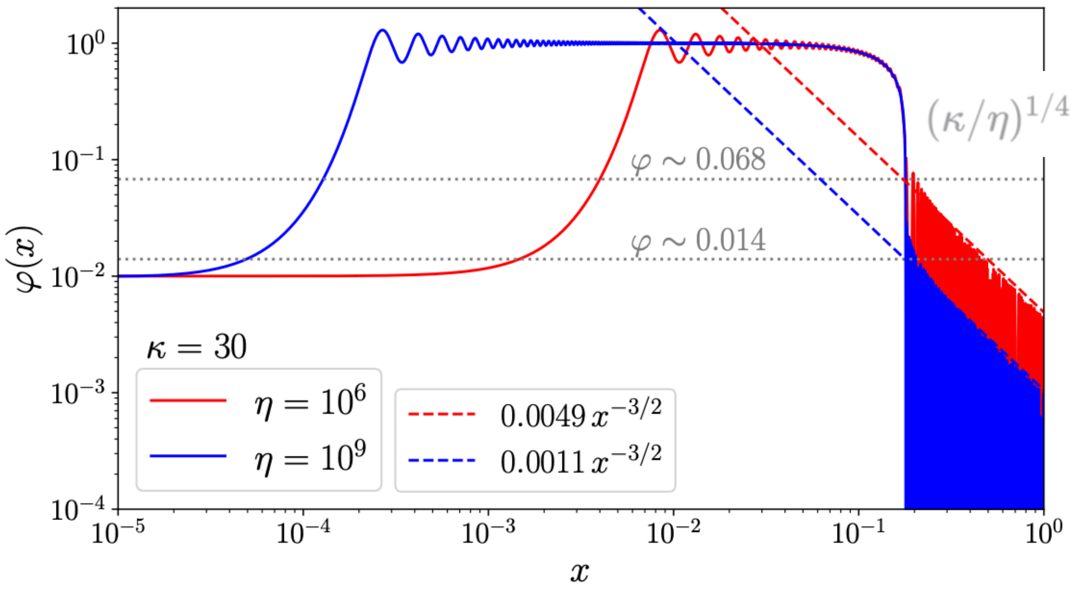
More calculations in the crossover regime

- Near $x_c \approx x_1$, the field undergoes a sudden drop due to the adiabatic condition

$$\frac{\phi^2(x_0)\, a^3(x_0)}{\phi^2(x_1)\, a^3(x_1)} = \frac{H(x_1)}{m_\phi} \approx \frac{H(x_c)}{m_\phi} = \left(\frac{\kappa}{\eta}\right)^{\frac{1}{2}} \ll 1$$

- So, the present-day energy density is given by

$$\rho_\phi(x_0) \approx m_\phi^2 \phi^2(x_c) \left(\frac{\kappa}{\eta}\right)^{\frac{1}{2}} \left(\frac{x_c}{x_0}\right)^3 \frac{g_{*S}(x_0)}{g_{*S}(x_c)}$$



More calculations in the crossover regime

- So, the present-day energy density is given by

$$\rho_\phi(x_0) \approx m_\phi^2 \phi^2(x_c) \left(\frac{\kappa}{\eta}\right)^{\frac{1}{2}} \left(\frac{x_c}{x_0}\right)^3 \frac{g_{*S}(x_0)}{g_{*S}(x_c)}$$

$$\begin{aligned}
\rho_\phi(x_0) &= m_\phi^2 \times 2m_\chi\Lambda \times \frac{H(x_c)}{m_\phi} \times \frac{x_c^3}{x_0^3} \times \frac{g_{*S}(x_0)}{g_{*S}(x_c)} \\
&= m_\phi^2 \times 2m_\chi\Lambda \times \frac{1.66\sqrt{g_*(x_c)}\, m_\chi^2}{M_{\rm Pl}\, m_\phi} \times \frac{x_c}{x_0^3} \times \frac{g_{*S}(x_0)}{g_{*S}(x_c)} \\
&= \cancel{m_\phi^2} \times 2m_\chi\Lambda \times \frac{1.66\sqrt{g_*(x_c)}\, m_\chi^2}{M_{\rm Pl}\, \cancel{m_\phi}} \times \frac{\pi}{2\sqrt{3}} \left(\frac{g_\chi}{2\pi^2}\right)^{1/2} \frac{m_\chi^{3/2}}{\Lambda^{1/2} \cancel{m_\phi} x_0^3} \times \frac{g_{*S}(x_0)}{g_{*S}(x_c)} \\
&= 1.66\sqrt{\frac{g_\chi g_*(x_c)}{6}}\, \frac{g_{*S}(x_0)}{g_{*S}(x_c)}\, \frac{m_\chi^{3/2}\Lambda^{1/2}T_0^3}{M_{\rm Pl}} \,.
\end{aligned}$$

ALP mass precisely drops out in the relic abundance!