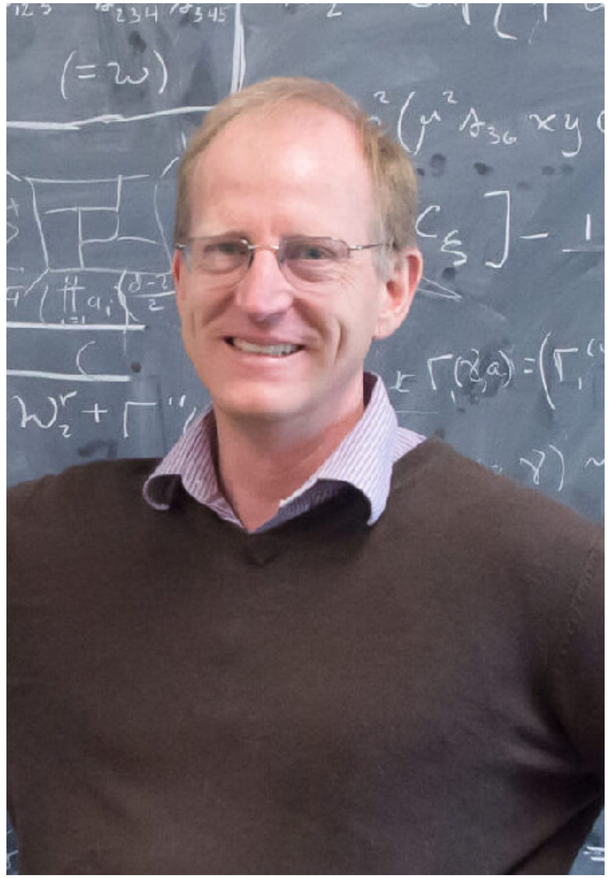
Anomalies and Inspirations

Learning from the universe’s most perfect microscopic structures

John Joseph M. Carrasco, Lancefest 2026




HAPPY
Birthday


L'OSTERIA


"This is really the most incredible thing that has happened in my lifetime"
Peter Higgs 4 July 2012
ATLAS
2011-12
√s = 7-8 TeV
Local p0

SLAC–PUB–7106
hep-ph/9601359
January, 1996

CALCULATING SCATTERING AMPLITUDES EFFICIENTLY*

LANCE DIXON

Stanford Linear Accelerator Center
Stanford University, Stanford, CA 94309

ABSTRACT

We review techniques for more efficient computation of perturbative scattering amplitudes in gauge theory, in particular tree and one-loop multi-parton amplitudes in QCD. We emphasize the advantages of (1) using color and helicity information to decompose amplitudes into smaller gauge-invariant pieces, and (2) exploiting the analytic properties of these pieces, namely their cuts and poles. Other useful tools include recursion relations, special gauges and supersymmetric rearrangements.

Invited lectures presented at the Theoretical Advanced Study Institute in Elementary Particle Physics (TASI 95): QCD and Beyond Boulder, CO, June 4-30, 1995

*Research supported by the US Department of Energy under grant DE-AC03-76SF00515.

Winter 2006

Zvi's Amplitudes Class

UCLA/07/TEP/3 Saclay-SPhT–T07/010 SLAC–PUB–12341 Zürich ZU-TH 2/07

Three-Loop Superfiniteness of $\mathcal{N} = 8$ Supergravity

Z. Bern[a], J. J. Carrasco[a], L. J. Dixon[b], H. Johansson[a], D. A. Kosower[c,d] and R. Roiban[e]

[a] *Department of Physics and Astronomy, UCLA, Los Angeles, CA 90095-1547, USA*
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We construct the three-loop four-point amplitude of $\mathcal{N} = 8$ supergravity using the unitarity method. The amplitude is ultraviolet finite in four dimensions. Novel cancellations, not predicted by traditional superspace power-counting arguments, render its degree of divergence in D dimensions to be no worse than that of $\mathcal{N} = 4$ super-Yang-Mills theory — a finite theory in four dimensions. Similar cancellations can be identified at all loop orders in certain unitarity cuts, suggesting that $\mathcal{N} = 8$ supergravity may be a perturbatively finite theory of quantum gravity.

PACS numbers: 11.15.Bt, 11.25.Db, 11.25.Tq, 11.55.Bq, 12.38.Bx

14 Feb 2007

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They were really good notes!

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UCLA/07/TEP/16 SLAC–PUB–12609

Unexpected Cancellations in Gravity Theories

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(Dated: July, 2007)

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UCLA/08/TEP/24 SLAC–PUB–13361

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UCLA/08/TEP/40

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UCLA/09/TEP/41

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UCLA/09/TEP/09/47 SLAC–PUB–13608

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UCLA/10/TEP/102 Saclay/IPhT–T10/044

Perturbative Quantum Gravity from Gauge Theory

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UCLA/07/TEP/04 ZU-TH 12/07 Saclay/SPhT–T07/050

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UCLA/07/TEP/16 SLAC–PUB–12609

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UCLA/09/TEP/09/47 SLAC–PUB–13608

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PRINCIPLES OF
MATHEMATICAL ANALYSIS

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2017

SU-ITP-11/51 SLAC–PUB–14784

Color-kinematics duality and double-copy construction for amplitudes from higher-dimension operators

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May 8 2013

SU-ITP-11/51 SLAC–PUB–14784

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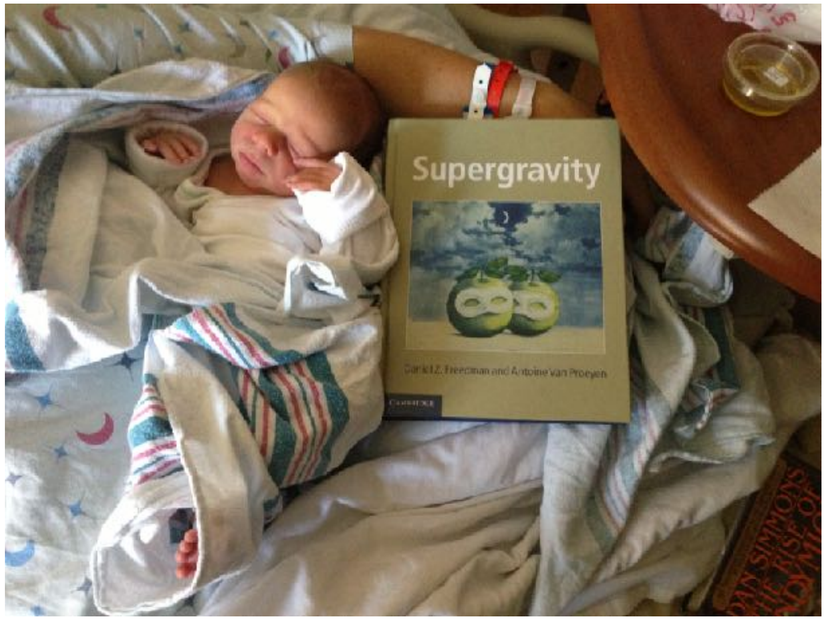
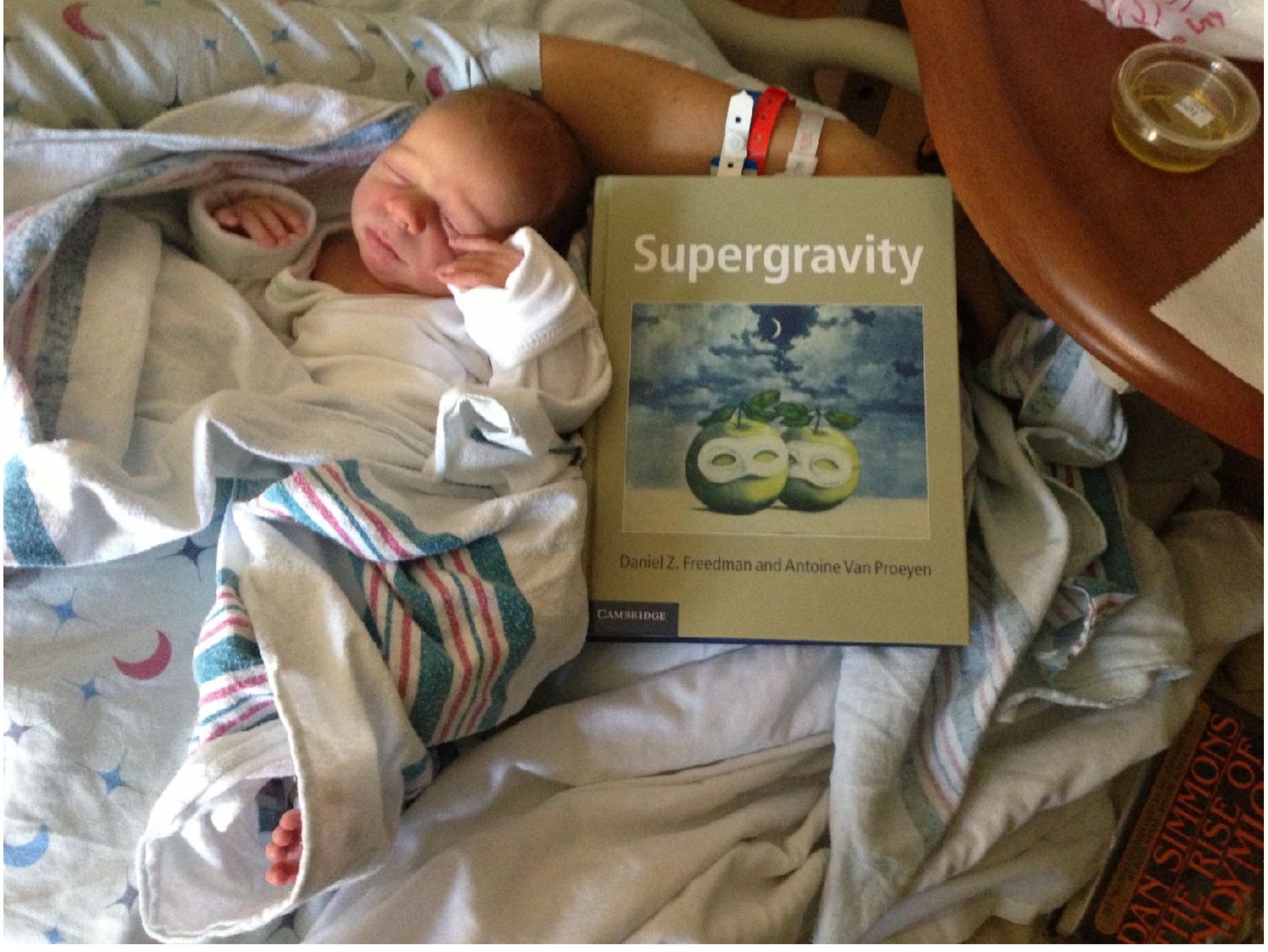
Abstract




Supergravity
Daniel Z. Freedman and Antoine Van Proeyen
CAMBRIDGE

Color-dual EFT!

SU-ITP-11/51 SLAC–PUB–14784

Color-kinematics duality and double-copy construction for amplitudes from higher-dimension operators

Johannes Broedel[1] and Lance J. Dixon[2]

[1]*Stanford Institute for Theoretical Physics and Department of Physics, Stanford University, Stanford, CA 94305, USA*

[2]*SLAC National Accelerator Laboratory, Stanford University, Stanford, CA 94309, USA*

Abstract

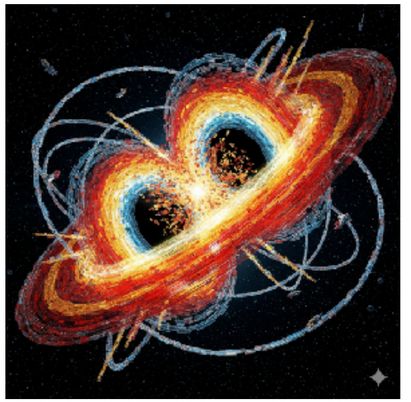
First deformation: F^3

Single-insertion color-dual to f^{abc}

Higher order CP-stripped string doesn't satisfy "BCJ" relations, e.g. $O(\alpha^2) \sim s\,t\,A(s,t)$

This was the first real hint that there's generic dimension color-dual beyond antisymmetric adjoint. Color-dressed: $O(\alpha^2) \sim d^{abcd} \times s\,t\,A(s,t)$

Are there any gravitational predictions we can't get from gauge theory?



The Entire Four-Graviton EFT from the Duality Between Color and Kinematics

John Joseph M. Carrasco [1,2] Sai Sasank Chava [1] Alex Edison [1,2] Eliseu Kloster [3,1,2] and Suna Zekioğlu [1]

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Abstract

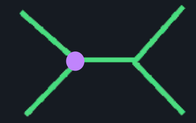
The Complete Four-Point Color Algebra

ff Sector

Antisymmetric-Adjoint (traditional BCJ)

- Antisymmetry: $n(ij;kl) = -n(ji;kl)$
- Jacobi: $n_s - n_t - n_u = 0$

df Sector



Mixed-Symmetry

- Sym left, antisym right vertex
- Mixed Jacobi: $n_s + n_t + n_u = 0$
- Numerators are gauge invariant!

dd Sector

Symmetric

- Symmetric vertices, no Jacobi
- All 3 graph structures independent
- Numerators gauge invariant graph-by-graph

d_4 Sector

Permutation Invariant (contact)

- Fully S_4 invariant quartic vertex
- Redundant: spanned by dd basis
- Useful for composing N-copies

A Finite Basis of Color-Dual Building Blocks

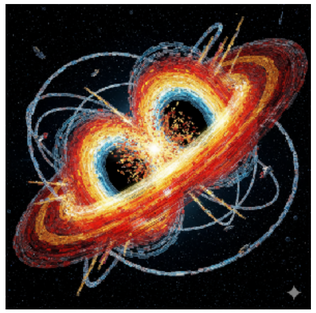
Mass Dim	ff	df	dd	d_4
2	1	0	0	0
4	3	0	4	2
6	3	0	7	3
8	1	1	7	2
10	0	1	3	0
12	0	1	0	0
Total	**8**	**3**	**21**	**7**

Despite infinite tower of higher-derivative operators, the number of genuinely distinct building blocks is finite.

All higher-dimension numerators are generated by multiplying these blocks by permutation-invariant scalars $P(s,t,u)$.

Equivalence class:

$$n_1 \sim n_2 \iff \mathscr{A}_1 = P(s,t,u) \times \mathscr{A}_2$$



Independent Classification: All 4-Graviton Amplitudes

Universal numerator: $\mathcal{N} \equiv \mathcal{A} \times (stu)$

$\mathcal{N}$ must be: fully permutation invariant, linear in graviton polarizations, diffeomorphism invariant

Mass dim	$\lvert \mathcal{N} \rvert$	$\lvert \mathcal{G} \rvert$
6	1	1
8	6	6
10	10	9
12	17	10
14	19	3
≥16	grows	0

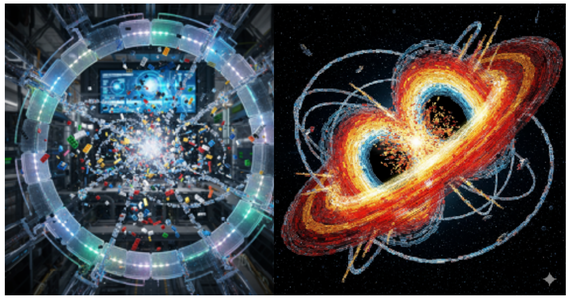
29 fundamental gravitational polynomials $\mathcal{G}$ ($\mathcal{N}$ mod scalar perm)— no new structures beyond mass dimension 14!

Einstein-Hilbert: mass dim 8
($\mathcal{N}^8 = stu \times \mathcal{A}^{\mathrm{GR}}$)

R^3 Lovelock: mass dim 12
(contact, non-vanishing only D > 6)

A finite number of gravitational fundamentals generates the entire infinite tower

The Universal Double Copy Spans All 4-Graviton Amplitudes



4-Gluon Double Copy

$(gggg) \otimes (gggg)$

$ff + dd$ sectors spans all $\mathcal{G}$ with mass dim ≥ 8

Scalar-Gluon Triple Copy

$(ggss) \otimes (sggg) \otimes (gsgg)$

Only needed for $\mathcal{G}^{(6)}$: Lovelock R^3 in $D \geq 6$

Every 4-point gravitational interaction factorizes into gauge-theory building blocks governed by color-kinematics duality

In D ≤ 6: double copy alone is sufficient — no triple copy needed

I should say — this line has leveraged tons of additional ideas starting from:

Polylogarithms, Multiple Zeta Values and Superstring Amplitudes

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[b] *Department of Applied Mathematics and Theoretical Physics, Cambridge CB3 0WA, United Kingdom,*

[c] *Max-Planck-Institut für Gravitationsphysik, Albert-Einstein-Institut, 14476 Potsdam, Germany,*

[d] *Max-Planck-Institut für Physik*

Abelian Z-theory: NLSM amplitudes and α'-corrections from the open string

John Joseph M. Carrasco[a,c], Carlos R. Mafra[b,c] and Oliver Schlotterer[d]

[a] *Institut de Physique Théorique, CEA–Saclay, F–91191 Gif-sur-Yvette cedex, France*

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[d] *Max–Planck–Institut für Gravitationsphysik, Albert–Einstein–Institut, Am Mühlenberg 1, 14476 Potsdam, Germany*

In this paper we derive the tree-level S-matrix of the effective theory of Goldstone bosons known as the non-linear sigma model (NLSM) from string theory. This novel connection relies on a recent realization of tree-level open-superstring S-matrix predictions as a *double copy* of super-Yang–Mills theory with Z-theory — the collection of putative scalar effective field theories encoding all the α'-dependence of the open superstring. Here we identify the color-ordered amplitudes of the NLSM as the low-energy limit of abelian Z-theory. This realization also provides natural higher-derivative corrections to the NLSM amplitudes arising from higher powers of α' in the abelian Z-theory amplitudes, and through double copy also to Born–Infeld and Volkov–Akulov theories. The Kleiss–Kuijf and Bern–Carrasco–Johansson relations obeyed by Z-theory amplitudes thereby apply to all α'-corrections of the NLSM. As such we naturally obtain a cubic-graph parameterization for the abelian Z-theory predictions whose kinematic numerators obey the duality between color and kinematics to all orders in α'.

August 2016

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Non-abelian Z-theory: Berends–Giele recursion for the α'-expansion of disk integrals

Carlos R. Mafra[a,b] and Oliver Schlotterer[c]

[a] *STAG Research Centre and Mathematical Sciences, University of Southampton, UK*

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[c] *Max–Planck–Institut für Gravitationsphysik,*

Prepared for submission to JHEP

Semi-abelian Z-theory: NLSM+ϕ^3 from the open string

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[a] *Institut de Physique Théorique, CEA–Saclay, F–91191 Gif-sur-Yvette cedex, France*

[b] *STAG Research Centre and Mathematical Sciences, University of Southampton, UK*

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Abstract: We continue our investigation of Z-theory, the second double-copy component of open-string tree-level interactions besides super-Yang–Mills (sYM). We show that the amplitudes of the extended non-linear sigma model (NLSM) recently considered by Cachazo, Cha, and Mizera are reproduced by the leading α'-order of Z-theory amplitudes in the semi-abelian limit. The extension refers to a coupling of NLSM pions to bi-adjoint scalars, and the semi-abelian limit refers to a *partial* symmetrization over one of the color orderings that characterize the Z-theory amplitudes. Alternatively, the partial symmetrization corresponds to a mixed interaction among abelian and non-abelian states in the underlying open-superstring amplitude. We simplify these permutation sums via monodromy relations which greatly in-

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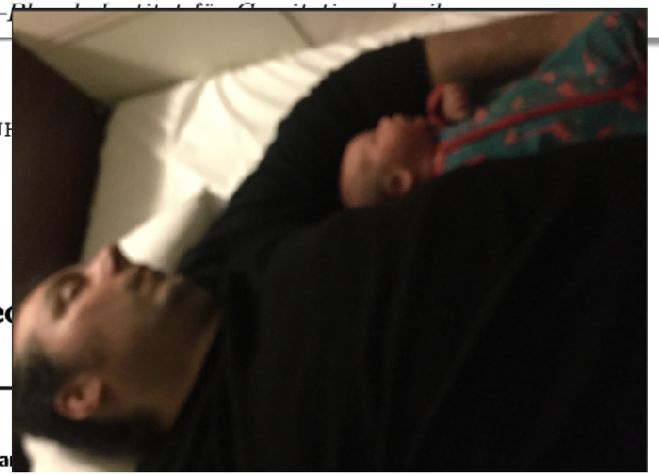
[c] *Max–Pl*



Prepared for submission to JH

29 Oct 2016

Semi-abelian Z-the

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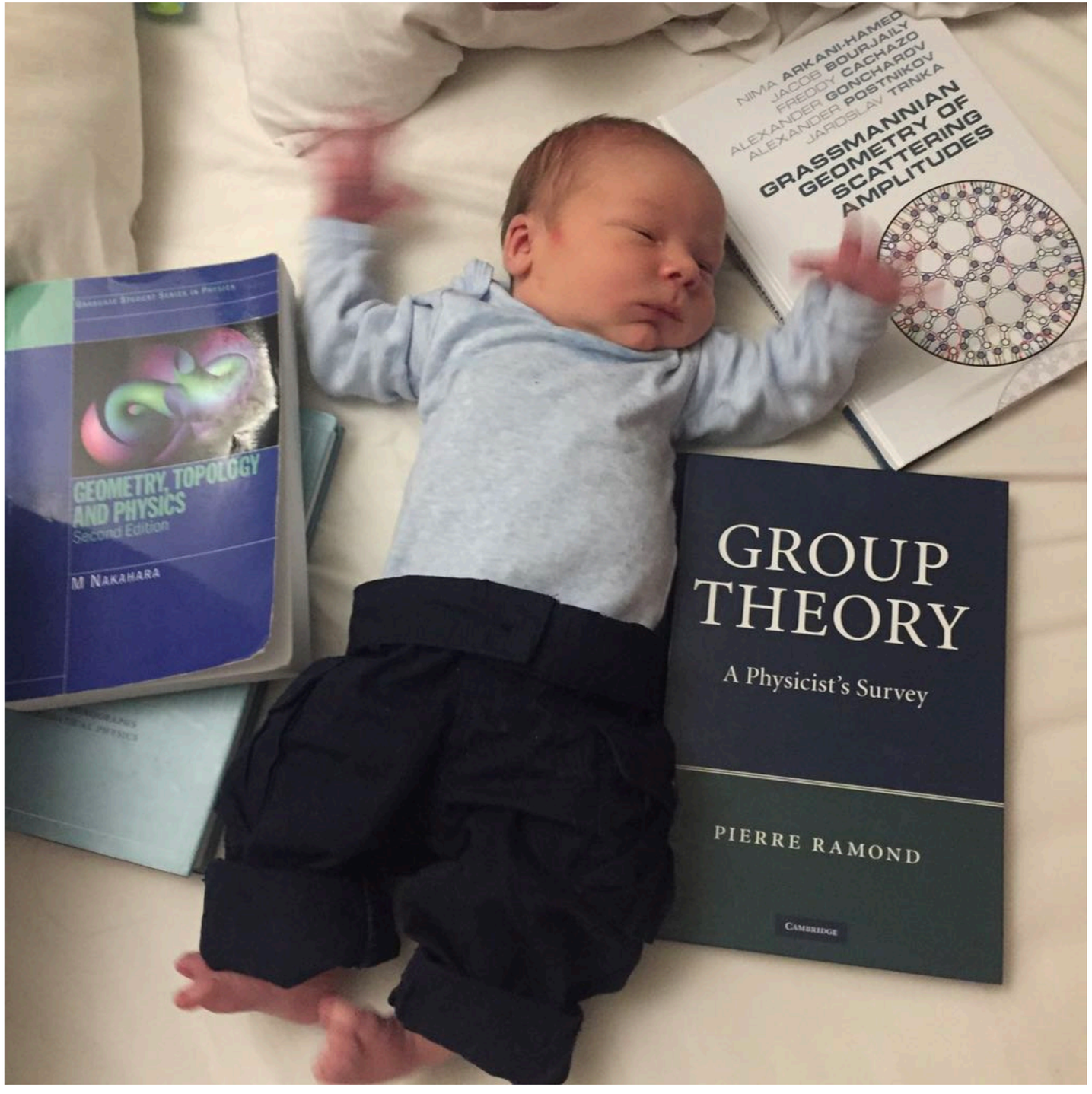
NIMA ARKANI-HAMED
JACOB BOURJAILY
FREDDY CACHAZO
ALEXANDER GONCHAROV
ALEXANDER POSTNIKOV
JAROSLAV TRNKA
GRASSMANNIAN GEOMETRY OF SCATTERING AMPLITUDES
GEOMETRY, TOPOLOGY AND PHYSICS
Second Edition
M NAKAHARA
GROUP THEORY
A Physicist's Survey
PIERRE RAMOND
CAMBRIDGE

The Color-Dual Fates of F^3, R^3, and $\mathcal{N} = 4$ Supergravity

John Joseph M. Carrasco,[1,2] Matthew Lewandowski,[1] and Nicolas H. Pavao[1]

[1] *Department of Physics and Astronomy, Northwestern University, Evanston, Illinois 60208, USA*
[2] *Institut de Physique Théorique, Universite Paris Saclay, CEA, CNRS, F-91191 Gif-sur-Yvette, France*

We find that the duality between color and kinematics can be used to inform the high energy behavior of effective field theories. Namely, we demonstrate that the massless gauge theory of Yang-Mills deformed by a higher-derivative F^3 operator cannot be tree-level color-dual while consistently factorizing without a tower of additional four-point counterterms with rigidly fixed Wilson coefficients that reaches to the ultraviolet (UV). We find through explicit calculation a suggestive resummation, namely that their amplitudes are consistent with the α' expansion of those generated by the $(DF)^2$ + YM theory, a known color-dual theory where the F^2 term has been given a mass squared proportional to $1/\alpha'$. As a result, considering consistent double-copy construction as a physical principle implies that an F^3-based color-dual resolution of the UV divergence in $\mathcal{N} = 4$ supergravity comes at the cost of field-theoretic locality. Similarly, when double-copying F^3 with itself, double-copy consistency lifts R^3 gravity to a family of gravity theories with an all-order tower of higher-derivative corrections, which includes the closed bosonic string as a standard adjoint-type double-copy.

Oct 2023

Zvi told us that the 4-loop divergence of $\mathcal{N} = 4$ SG is related to an anomaly.

SU-ITP-13/04
Imperial-TP-AT-2013-02

On the $U(1)$ duality anomaly and the S-matrix of $\mathcal{N} = 4$ supergravity

J.J.M. Carrasco[a], R. Kallosh[b], R. Roiban[c] and A.A. Tseytlin[d,1]

[a,b] *Stanford Institute for Theoretical Physics and Department of Physics, Stanford University, Stanford, CA 94305-4060, USA*

[c] *Department of Physics, The Pennsylvania State University, University Park, PA 16802 , USA*

[d] *Blackett Laboratory, Imperial College, London SW7 2AZ, U.K.*

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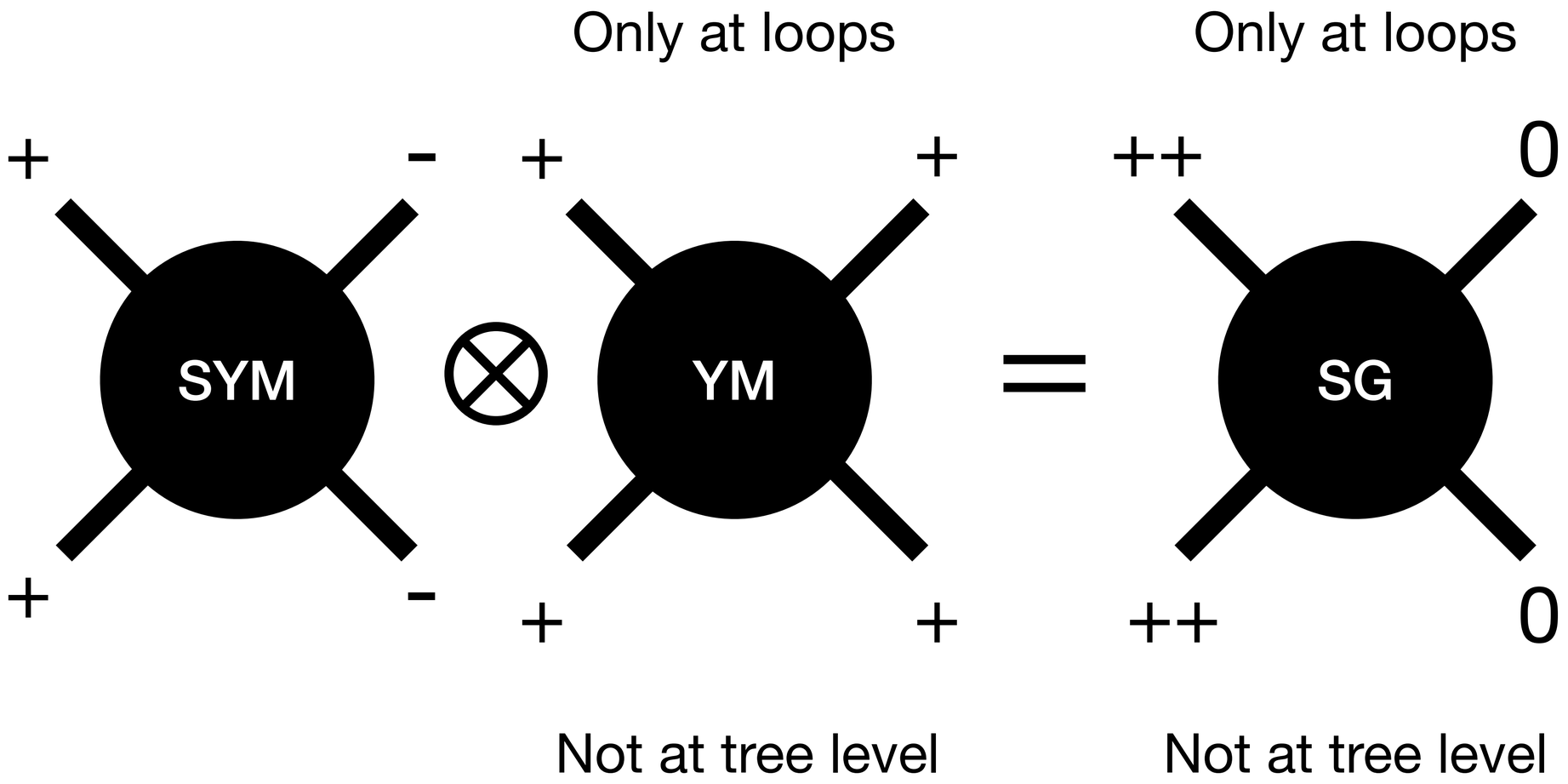

Target Anomalous Behavior with F^3 double copies

Curvature-Squared Multiplets, Evanescent Effects and the U(1) Anomaly in $\mathcal{N} = 4$ Supergravity

Zvi Bern[ab], Alex Edison[a], David Kosower[cd] and Julio Parra-Martinez[ab]

[a] *Mani L. Bhaumik Institute for Theoretical Physics, Department of Physics and Astronomy, University of California at Los Angeles, Los Angeles, CA 90095, USA*

[b] *Kavli Institute for Theoretical Physics, University of California, Santa Barbara, CA 93106, USA*

[c] *School of Natural Sciences, Institute for Advanced Study, 1 Einstein Drive, Princeton, New Jersey 08540*

UCLA/17/TEP/107

Cancelling the $U(1)$ Anomaly in the S-matrix of $\mathcal{N} = 4$ Supergravity

Zvi Bern[a], Julio Parra-Martinez[a] and Radu Roiban[b]

[a] *Mani L. Bhaumik Institute for Theoretical Physics, UCLA Department of Physics and Astronomy, Los Angeles, CA 90095, USA*

[b] *Institute for Gravitation and the Cosmos, Pennsylvania State University, University Park, PA 16802, USA*

$\mathcal{N} = 4$ supergravity is understood to contain a $U(1)$ anomaly which manifests itself via the nonvanishing of loop-level scattering amplitudes that violate a tree-level charge conservation rule. In this letter we provide detailed evidence that at one loop such anomalous amplitudes can be set to zero by the addition of a finite local counterterm. We suggest that the cancellation happens through a four-dimensional version of the Green-Schwarz mechanism. We show that the same counterterm also cancels evanescent contributions which play an important role in the analysis of ultraviolet divergences in dimensionally regularized gravity. These cancellations call for a reanalysis of the four-loop ultraviolet divergences previously found in this theory without the addition of such counterterms.

PACS numbers: 04.65+e, 11.15.Bt, 11.25.Db, 12.60.Jv

Citing exactly amplitudes from:

SU-ITP-11/51 SLAC–PUB–14784

Color-kinematics duality and double-copy construction for amplitudes from higher-dimension operators

Johannes Broedel[1] and Lance J. Dixon[2]

[1]*Stanford Institute for Theoretical Physics and Department of Physics, Stanford University, Stanford, CA 94305, USA*

[2]*SLAC National Accelerator Laboratory, Stanford University, Stanford, CA 94309, USA*

Abstract

SLAC–PUB–6416
December 1993
(T)

Testing Gluon Self-Interactions in Three Jet Events at Hadron Colliders*

Lance Dixon and Yael Shadmi

Stanford Linear Accelerator Center
Stanford, CA 94309

Abstract

The effective operator $\mathrm{tr}(G^3)$ is the only dimension-6 gluonic operator that cannot be related to four-quark operators. A peculiar property of this operator is that it does

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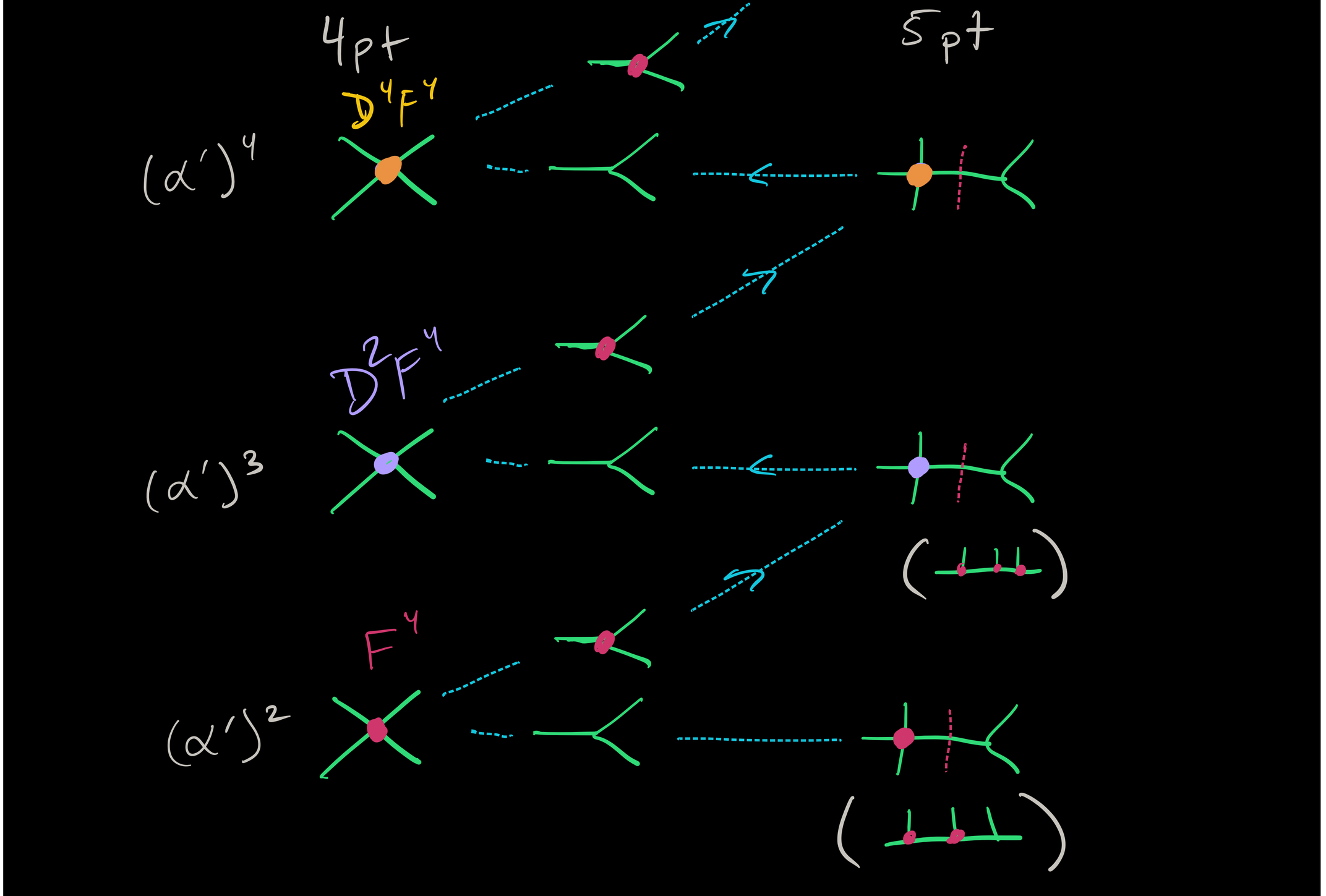
4pt
5pt
D^4F^4
$(\alpha')^4$
D^2F^4
$(\alpha')^3$
F^4
$(\alpha')^2$

Double copy consistency lifts $YM + F^3 \to (DF)^2 + YM$

Conformal Gravity from Gauge Theory

Henrik Johansson,[a,b,c] **Josh Nohle**[b]

[a] *Department of Physics and Astronomy, Uppsala University, Box 516, SE-75120 Uppsala, Sweden*
[b] *Nordita, KTH Royal Institute of Technology and Stockholm University, Roslagstullsbacken 23, SE-10691 Stockholm, Sweden*
[c] *Kavli Institute for Theoretical Physics, University of California, Santa Barbara, CA 93106, USA*

E-mail: henrik.johansson@physics.uu.se, josh.nohle@gmail.com

Abstract: We use the duality between color and kinematics to obtain scattering amplitudes in non-minimal conformal $\mathcal{N} = 0, 1, 2, 4$ (super)gravity theories. Generic tree amplitudes can be constructed from a double copy between (super-)Yang-Mills theory and a new gauge theory built entirely out of dimension-six operators. The latter theory is marginal in six dimensions and contains modes with a wrong-sign propagator, echoing the behavior of conformal gravity. The dimension-six Lagrangian is uniquely determined by demanding that its scattering amplitudes obey the color-kinematics duality. The conformal gravity amplitudes obtained from the double copy are compared with the Berkovits-Witten twistor string and shown to agree up to at least eight points in the MHV sector. Our construction can be generalized in a number of ways. Adding scalars to the dimension-six theory gives Maxwell-Weyl gravity, and further adding ϕ^3 self-interactions among these scalars gives Yang-Mills-Weyl gravity. The latter is identified with Witten's twistor string for maximal $\mathcal{N} = 4$ supersymmetry. Deforming the dimension-six theory by adding a Yang-Mills term, m^2F^2, gives a gauge theory that interpolates between marginal $D = 6$ and $D = 4$ theories. The corresponding double copy gives an interpolation between conformal gravity and Einstein gravity.

Double copy consistency lifts $YM + F^3 \to (DF)^2 + YM$

But this theory famously has a tachyon pole so unitarity is violated. Double copy with N=4 SG doesn't cure this — still have tachyon pole: "twisted heterotic string." Is all hope lost?

UUITP–17/21
NORDITA 2021-030

Scattering Massive String Resonances through Field-Theory Methods

Max Guillen[a], Henrik Johansson[a,b], Renann Lipinski Jusinskas[c] and Oliver Schlotterer[a]
[a] *Department of Physics and Astronomy, Uppsala University, Box 516, 75120 Uppsala, Sweden*
[b] *Nordita, Stockholm University and KTH Royal Institute of Technology, Hannes Alfvéns väg 12, 10691 Stockholm, Sweden and*
[c] *Institute of Physics of the Czech Academy of Sciences & CEICO Na Slovance 2, 18221 Prague, Czech Republic*

We present a new method, exact in α', to explicitly compute string tree-level amplitudes involving one massive state and any number of massless ones. This construction relies on the so-called twisted heterotic string, which admits only gauge multiplets, a gravitational multiplet, and a single massive supermultiplet in its spectrum. In this simplified model, we determine the moduli-space integrand of all amplitudes with one massive state using Berends–Giele currents of the gauge multiplet. These integrands are then straightforwardly mapped to gravitational amplitudes in the twisted heterotic string and to the corresponding massive amplitudes of the conventional type-I and type-II superstrings.

Double copy consistency lifts $YM + F^3 \to (DF)^2 + YM$

But this theory famously has a tachyon pole so unitarity is violated. Double copy with N=4 SG doesn't cure this — still have tachyon pole: "twisted heterotic string." Is all hope lost?

Not at all!! One natural solution is to simply demand DC-consistent level matching (get rid of any mass that isn't shared on both sides of the double copy).

Do we know any ways? Sure: SV-promotion ($Z \otimes^{\alpha'} Z$).

What do you get?

Double copy consistency lifts $YM + F^3 \to (DF)^2 + YM$

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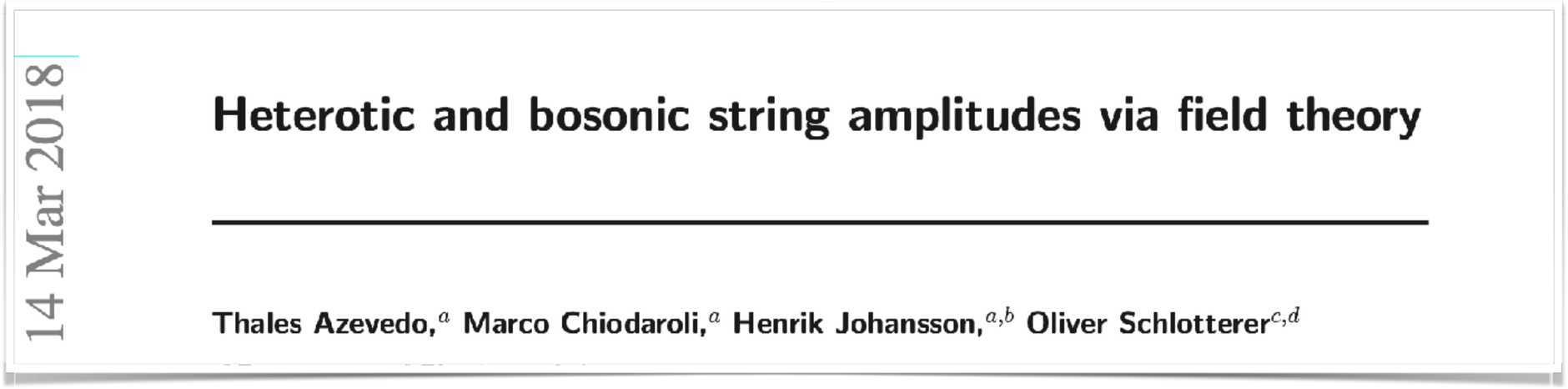
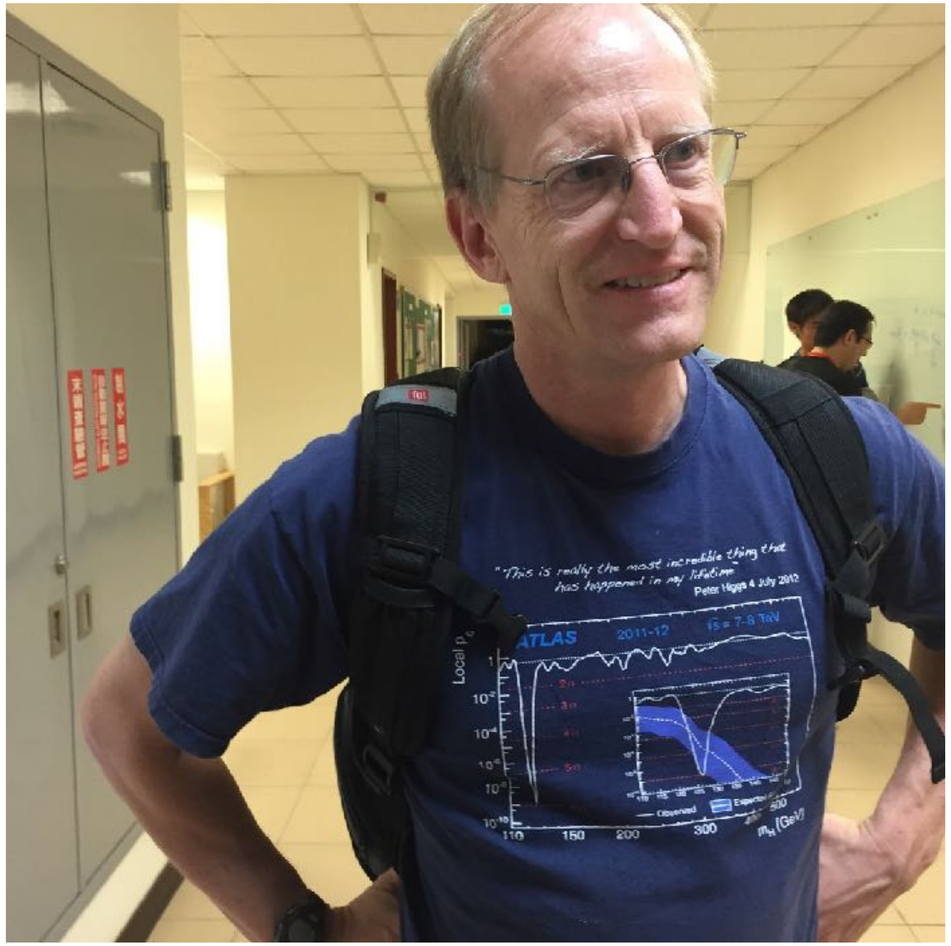
What do you get? Gravitational amplitudes in the Heterotic String :-)





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Abstract

The Double-Copy Root of Hawking Thermality

John Joseph M. Carrasco[1,2] and Yaxi Chen[1,2]

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(Dated: November 4, 2025)

The Hawking radiation spectrum from a collapsing null shell can be derived via the double copy of a simpler gauge theory calculation. Analyzing the non-abelian Yang-Mills root of this process, we demonstrate that the radiation spectrum is thermal in the color charge eigenvalue λ, not energy. Considering the $SU(N_c)$ gauge theory in the large N_c limit, we find the differential spectrum $\mathrm{d}N/\mathrm{d}\lambda$ is a product of the gravitationally familiar Planck like factor and the color phase space density, modeled here as the Wigner semicircle from random matrix theory. This reveals that apparent energy thermality in gravity is the direct dual of charge thermality in its underlying non-abelian gauge theory.

INTRODUCTION

Classical solutions in General Relativity (GR), such as the Schwarzschild metric, can be viewed as an infinite sum of tree-level graviton diagrams sourced by their own self-interaction [1]. This perturbative picture suggests an immense complexity that is miraculously resummed in the exact solution. The classical double copy [2–4] reveals this is no miracle, but a direct consequence of gravity's structure as the relativistic *quantum double copy* [5–8] of a non-abelian Yang-Mills (YM) theory.

This principle is clearly manifest in metrics of the Kerr-Schild form, $g_{\mu\nu} = \eta_{\mu\nu} + \phi_g k_\mu k_\nu$. For a null, geodesic vector k_μ, the Einstein tensor linearizes, $G_{\mu\nu}[g] = G^{(1)}_{\mu\nu}[h]$, meaning the full non-linear field equations reduce to a free wave equation. The infinite tower of graviton interactions collapses, and the solution behaves as if generated by a single, simple propagator. This simplicity is a direct consequence of the double copy, which guarantees that every operator in the Einstein-Hilbert action is completely specified by YM operator data [9, 10].

This correspondence is realized in the YM root. There, a specific gauge choice allows a potential with a fixed color orientation, $A^a_\mu = c^a \phi k_\mu$, to also behave as a simple propagator; it becomes an exact classical solution because its non-linear self-interactions vanish identically via the color algebra ($f^{c_1 c_2 a} c^{c_1} c^{c_2} c^{c_3} = 0$ if c is aligned along a fixed color direction). The Kerr-Schild metric is therefore not merely analogous to a double copy; it is the literal double copy of the Yang-Mills propagator in the presence of an abelianized source. The geometric properties that linearize Einstein's equations are the precise dual of the gauge choice that reduces Yang-Mills to its fundamental propagator.

While the classical solution appears abelian, the underlying root theory of general relativity must be non-abelian Yang-Mills. The duality relies on the kinematic numerators of YM amplitudes; indeed, their cubic and quartic gauge self-interactions are sufficient to generate all operators in GR through the double copy. Attempting to double-copy QED yields linearized gravity only. It has a consistent metric interpretation but none of the self-interaction that is the hallmark of Einstein-Hilbert. The celebrated generation of Newton from Coulomb is possible only because their non-relativistic potentials are described solely by mediator exchange in two-to-two scattering. Einstein requires Yang-Mills.

In a remarkable paper [11] of this past year, Aoude, O'Connell, and Sergola recovered the apparently thermal spectrum of Hawking radiation by emphasizing the on-shell nature of the original [12] calculation, setting the stage for a modern S-matrix perspective. The physical foundation of their approach lies in the principles of quantum field theory on a dynamic background. For a collapsing shell, the initial vacuum state ($|0_{\rm in}\rangle$) is not an eigenstate of the final Hamiltonian and evolves under the S-matrix into a superposition of states containing real, outgoing particles — a formal description of particle creation from vacuum fluctuations.

Ref. [11] reminds us that we can compute the spectrum of this created radiation by considering the evolution of a single probe state. This probe is a computational tool: we calculate its scattering amplitude to characterize its dynamical interaction with the background responsible for particle creation. The calculation begins by computing a three-point tree-level amplitude for a probe scattering against the Vaidya background. By exponentiating this result via the Lippmann-Schwinger equation, the authors capture the exact result within the eikonal limit[13], which re-sums the leading soft contributions to all orders.

The crucial result of this calculation is a logarithmic eikonal phase, $\chi(v_0) \propto \log(-v_0)$. The argument, v_0, represents the probe trajectory's initial time offset $v_0 < 0$ relative to the moment of collapse. This phase is not specific to the probe but is a universal imprint left on *any* quantum mode by the extreme time-delay near the forming horizon. It is this mathematical structure that encodes the mixing of positive and negative frequency

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Nonperturbative double copy: worldline instantons, color thermality, and backreaction

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Abstract: We present a first-principles, non-perturbative worldline instanton analysis of vacuum decay in the non-abelian Yang-Mills root of a Schwarzschild background. We recover the gauge theory color-thermal spectrum as a topological winding mode. The double copy maps vacuum response from the gauge theory directly to the gravity theory. Furthermore, the decay exponent acquires a universal quadratic correction from color charge conservation, showing that the double copy correctly captures the non-linear backreaction as required for unitarity.

The Question

Hawking radiation is a quantum effect of fields in a curved background.

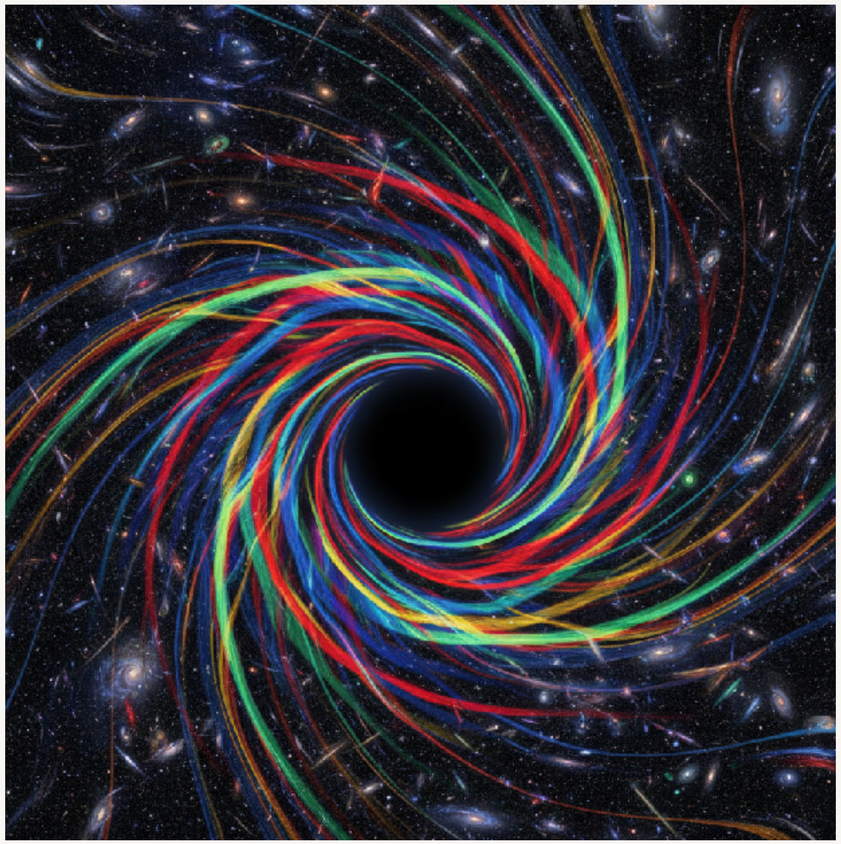
A collapsing shell forms a black hole. Quantum vacuum fluctuations near the forming horizon produce real, thermal radiation with temperature:

$$T_H = 1/(8\pi G_N M)$$

The spectrum is thermal in energy — the Planck distribution, with the temperature set by the mass of the black hole.

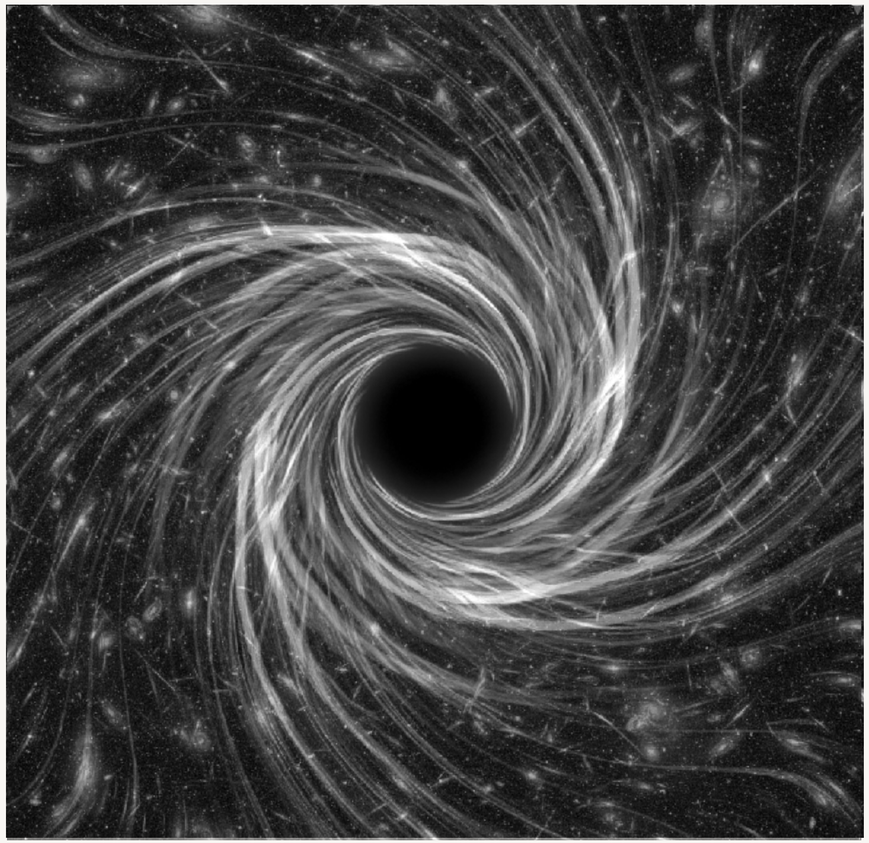
What is the Yang-Mills avatar of this thermality?

Particle Production from (semi)-Classical Backgrounds



Coherent Classical Color Source (Yang-Mills)

Charge ~ **Q**



Schwarzschild Black Hole (Gravity)

Charge ~ **M**

Hawking Radiation from Scattering

Vaidya metric: a collapsing null shell of mass M activating at $v = t + r = 0$:

$$g_{\mu\nu} = \eta_{\mu\nu} + \Theta(v)\frac{2G_N M}{r}k_\mu k_\nu$$

Consider an eikonal probe scalar skimming near the forming horizon.

The probe doesn't change momentum — but accumulates a *time-dependent* phase:

- Tree-level 3-point amplitude for probe in background
- Loop corrections exponentiate → eikonal S-matrix
- The result: a logarithmic eikonal phase:

$\chi(v_0) \propto -E\log(-v_0/\mu)$. (log from integrating 4D $1/r$ background along probe trajectory)

Logarithmic eikonal phase → Fourier transform → thermality

Physical Picture

The log phase is the universal imprint of the extreme time-delay near the forming horizon.

Equivalent to Bogoliubov mixing of positive/negative frequency modes → particle creation.

Einstein Requires Yang-Mills

Why can't we just double-copy QED?

QED (Abelian)

Double-copying QED gives linearized gravity only. Consistent metric, but no self-interaction.

Newton from Coulomb works because it's just propagator exchange.

Yang-Mills (Non-abelian)

The cubic and quartic self-interactions of YM generate all operators in GR through the double copy.

Full E-H, including the nonlinear self-interaction. **Even when the solution abelianizes, the theory remains non-abelian.**

The duality relies on the kinematic numerators of non-abelian gauge theory.
The root of Einstein gravity is Yang-Mills — not QED.

This distinction is not just structural — it will be essential for thermodynamics.

Ilderton, Landed, Rajeev [2510.25852] **Aoude, O'Connell, Sergola, White [2510.25866]**

The Yang-Mills Root: Root-Vaidya

Collapsing shell of color charge Q in root-Vaidya gauge:

$$A^a_\mu(x) = \Theta(v)\left(\frac{gQ}{r}\right)\hat{c}^a k_\mu$$

- Source abelianizes → fix color orientation → effectively abelian dynamics.
- k_μ is null ($k^2 = 0$) → only the 3-point contributes!
- Probe: adjoint scalar in eigenstate of $c^a T^a$ with **eigenvalue λ**
  Path ordering trivializes → color algebra reduces to scalar λ
- Same computation as Gravity but $E \to \lambda$ in eikonal phase

Crucial Simplification

All non-abelian structure reduces to a single eigenvalue λ

Log phase produces a thermal spectrum *in color!*

$\chi(v_0) \propto -\lambda \log(-v_0/\mu)$. (log from integrating the 4D $1/r$ background along probe trajectory)

Logarithmic eikonal phase → Fourier transform → thermality (from λ-spectrum)

The Gauge Eikonal Phase: Energy Drops Out

The eikonal phase is the integral of the potential along the probe's worldline.

The worldline gives $r(\sigma) \propto E|\sigma| \implies \int d\sigma \frac{c_3 n_3}{r(\sigma)} \to \int \frac{dr}{E} \frac{c_3 n_3}{r} \to \int \frac{dr}{\cancel{E}} \frac{c_3 \cancel{E}}{r}.$ $(n_3 \propto E)$

$$\chi(v_0) = -\beta \log(-v_0/\mu) \qquad \beta = c_3 = gQ\lambda$$

No Dependence on probe energy E

Yang-Mills: $\beta \propto \lambda$

The phase depends on color eigenvalue, not energy. **Thermality lives in λ!**

Gravity: $\beta \propto E$

The double copy replaces $c_3 \to n_3$, promoting $c_3 n_3 \to n_3^2 \propto E^2$

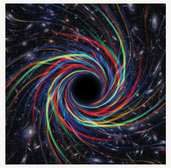
Energy thermality is emergent, not universal!

Final radiation amplitude from FT time dependent phase $e^{iS(v_0)}$ wrt radiated energy:

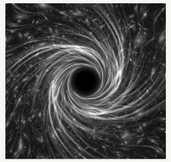
$$\mathcal{A}(E) = \int_{-\infty}^{0} dv_0\, e^{iEv_0} e^{i\beta \log(-v_0/\mu)}$$

$$|\mathcal{A}(E)|^2 \propto E^{-2} |\Gamma(1 + i\beta)|^2$$

$$\beta_{YM} \sim gQ\lambda$$



$$\beta_{GR} \sim G_N M E$$



$$\boxed{\frac{dN}{dE} \propto \frac{1}{E^2} \frac{\pi\beta}{\sinh(\pi\beta)}}$$

From Energy Spectrum → Color Spectrum

$$\frac{dN}{dE} \propto \frac{1}{E^2} \frac{\pi g Q \lambda}{\sinh(\pi g Q \lambda)}$$

Initially Bremsstrahlung-like in Energy

But we can make an inclusive observable in color if we understand the distribution of eigenvalues for the gauge group. As an example consider phase space density for large N_c from Random Matrix Theory, the Wigner Semicircle of eigenvalue distribution radius R:

$$\rho(\lambda) \propto \sqrt{R^2 - \lambda^2} \implies \frac{dN}{d\lambda} \propto \frac{\pi g Q \lambda}{\sinh(\pi g Q \lambda)} \times \sqrt{R^2 - \lambda^2}$$

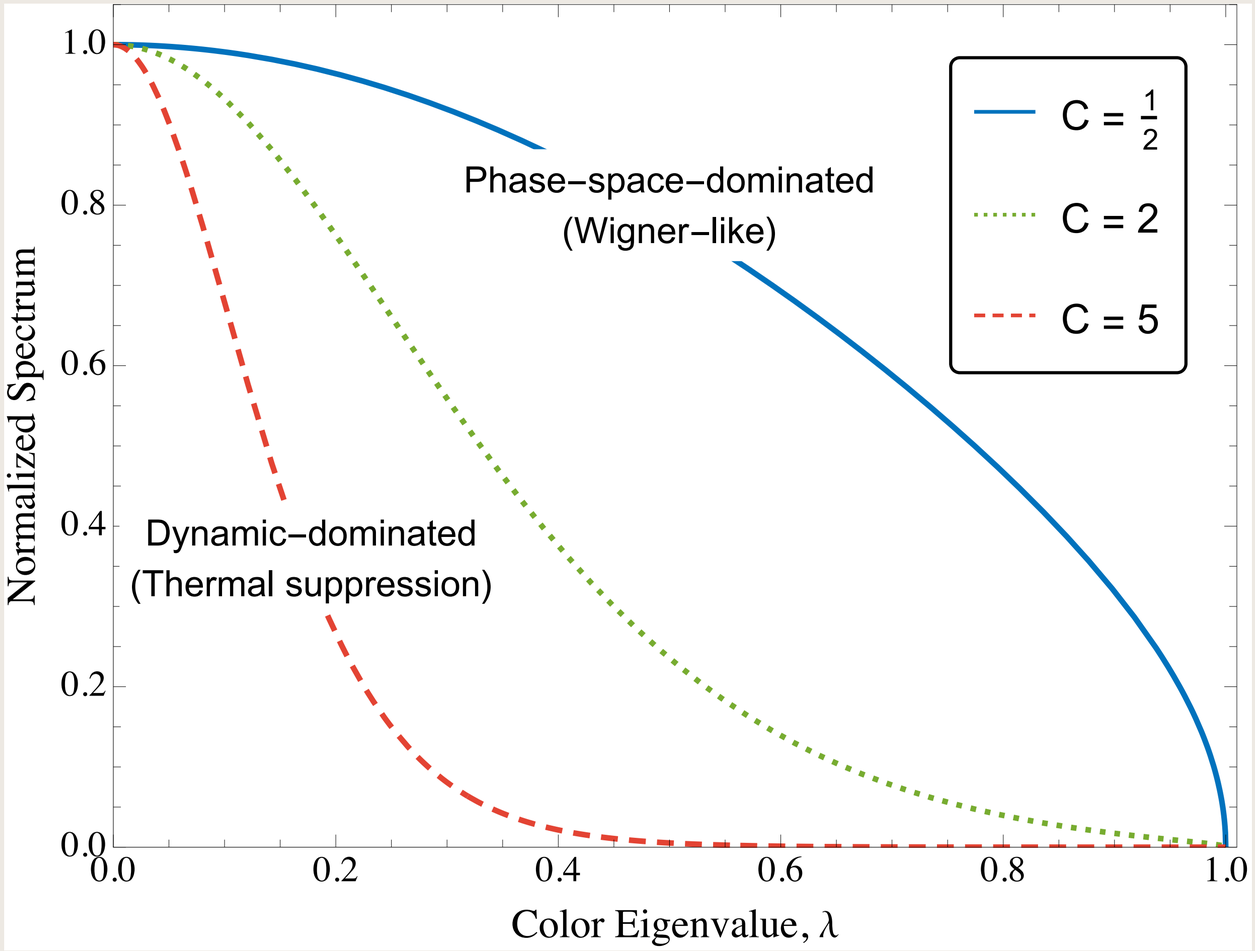
Yang-Mills radiation looks classical in energy but thermal in color!

Phase Space vs Dynamics

$$\frac{dN}{d\lambda} \propto \frac{C\lambda}{\sinh(\pi C\lambda)} \times \sqrt{R^2 - \lambda^2}$$

Dynamical Factor (thermal)

Phase Space Factor (Wigner semicircle)



Large N_c : Wigner semicircle from RMT gives the density of color eigenvalue states.

The observable spectrum is controlled by the ratio C / R :

- Weak coupling (C ≪ R):
  Phase space dominates → Wigner semicircle
- Strong coupling (C ≫ R):
  Dynamics dominates → Planck-like spectrum

Can We See This Non-Perturbatively?

So far, we have derived color thermality from real-time eikonal resummation.
But that raises a question: is this truely non-perturbative?
Is color thermality a true semiclassical feature, or just an artifact of the eikonal approximation?

We will now rederive the same physics from a completely different starting point!

One-Loop Effective Action

Vacuum-to-vacuum amplitude

$$Z = \langle 0_{\text{out}} | 0_{\text{in}} \rangle = \exp(i\,\Gamma[A])$$

Worldline Path Integral

Euclidean closed worldlines
Saddle points = instantons
Mediating pair creation

Vacuum Decay Rate

$$\text{Im}(\Gamma) \sim e^{-X}$$

$$X = \oint p_r\, dr$$

Contour integral = instanton action

Established equivalence to real-time Bogoliubov description: Kim (2000), Dunne et al. (2006, 2010)

We will reproduce the same thermal structure—but now as a semiclassical saddle.

Backreaction: Parikh-Wilczek from Color Conservation

The source has finite charge. As a particle radiates, conservation of color charge depletes it:

$$Q \to Q' \equiv Q - \lambda$$

Integrating the differential cost over the build-up of emitted charge gives a quadratic correction:

Yang-Mills

$$X = 4\pi g\,|\lambda|\,(Q - |\lambda|/2)$$

JJMC, Chen, Pavao, Seifi [2601.17884]

Gravity

$$X = 8\pi G_N \omega (M - \omega/2)$$

Parikh, Wilczek [hep-th/9907001]

Rep-theoretic: Take $Q \sim \sqrt{C_2(R)}$ and charge conserved. With, e.g. $C_2(R) = \frac{a}{2}Q^2$,

$$\Delta C_2 = C_2(R) - C_2(R') \propto \frac{a}{2}\left(Q^2 - (Q-\lambda)^2\right) = a\,(Q\lambda - \lambda^2/2)$$

- Linear term → Boltzmann suppression (thermal)
- Quadratic term → deviation from thermality (correlations for unitarity)
- Sign: enhancement of decay rate (just like Hawking)

The gravitational dual: $\Delta C_2 \propto \Delta(Q^2) \leftrightarrow \Delta S_{\text{BH}} \propto \Delta(M^2)$

The Double Copy Dictionary

Yang-Mills		Gravity
Color charge Q	⟺	Mass M
Color eigenvalue λ	⟺	Energy ω
Quadratic Casimir $C_2 \sim Q^2$	⟺	Horizon area $A \sim M^2$
Color temperature $T_c = 1/(4\pi g Q)$	⟺	Hawking temperature $T_h = 1/(8\pi G_n M)$
$X = 4\pi g \lvert\lambda\rvert (Q - \lvert\lambda\rvert/2)$	⟺	$X = 8\pi G_N \omega (M - \omega/2)$
ΔC_2 (rep transition)	⟺	ΔS_{BH} (Bekenstein-Hawking)

In exponent X: Linear term → Temperature (thermal) · Quadratic term → Backreaction

Executive Summary

1. Gravity is the double copy of Yang-Mills — a **weak-weak** duality with a sharp state/operator map.

2. The YM root of Hawking radiation reveals a **novel gauge phenomenon**: radiation from coherent color sources is thermal in the color eigenvalue λ.

3. Confirmed by **two complementary methods**: eikonal S-matrix and worldline instantons.

4. Backreaction maps precisely to **Parikh-Wilczek**: horizon thermodynamics = double copy of color thermodynamics.

5. Only the **non-abelian** root is consistent with gauge thermality —
Einstein requires Yang-Mills, even for thermodynamics.

[2511.01832] PRL 136 (2026) • [2601.17884] JHEP 05 (2026) 177

Liz, Kathleen, Shain, Emrys, and Patrick all send their love and warm wishes, they wish they could be here!

Wonderful to be surrounded by friends

I can’t wait to go on many more adventures….

Happy Birthday Lance and Thank You!