Lance (and I) in Reggeland

Vittorio Del Duca

INFN LNF

LanceFest Edinburgh 26 June 2026

Lance

- intellectual breadth

Lance

- intellectual breadth
- smartness
- speed

Lance

- intellectual breadth
- smartness
- speed
- integrity

Lance has 5 papers and one review about Regge theory

Single-valued harmonic polylogarithms and the multi-Regge limit — see James's talk

Lance J. Dixon (SLAC), Claude Duhr (Zurich, ETH), Jeffrey Pennington (SLAC) (Jul, 2012)

Published in: *JHEP* 10 (2012) 074 • e-Print: 1207.0186 [hep-th]

The BFKL equation, Mueller-Navelet jets and single-valued harmonic polylogarithms

Vittorio Del Duca (Frascati), Lance J. Dixon (SLAC), Claude Duhr (Zurich, ETH and Durham U., IPPP), Jeffrey Pennington (SLAC) (Sep 25, 2013)

Published in: *JHEP* 02 (2014) 086 • e-Print: 1309.6647 [hep-ph]

The four-loop remainder function and multi-Regge behavior at NNLLA in planar N = 4 super-Yang-Mills theory — see James's talk

Lance J. Dixon (SLAC), James M. Drummond (Annecy, LAPTH and CERN and Southampton U.), Claude Duhr (Durham U., IPPP), Jeffrey Pennington (SLAC) (Feb 13, 2014)

Published in: *JHEP* 06 (2014) 116 • e-Print: 1402.3300 [hep-th]

Heptagon functions and seven-gluon amplitudes in multi-Regge kinematics

Lance J. Dixon (SLAC), Yu-Ting Liu (SLAC), Julian Miczajka (Humboldt U., Berlin and Munich, Max Planck Inst.) (Oct 21, 2021)

Published in: *JHEP* 12 (2021) 218 • e-Print: 2110.11388 [hep-th]

One-loop central-emission vertex for two gluons in $\mathcal{N} = 4$ super Yang-Mills theory

Emmet P. Byrne (U. Edinburgh, Higgs Ctr. Theor. Phys.), Vittorio Del Duca (Zurich, ETH and Zurich U.), Lance J. Dixon (SLAC), Einan Gardi (U. Edinburgh, Higgs Ctr. Theor. Phys.), Jennifer M. Smillie (U. Edinburgh, Higgs Ctr. Theor. Phys.) (Apr 26, 2022)

Published in: *JHEP* 08 (2022) 271 • e-Print: 2204.12459 [hep-ph]

The SAGEX review on scattering amplitudes Chapter 15: The multi-Regge limit

Vittorio Del Duca (Zurich, ETH), Lance J. Dixon (Zurich, ETH) (Mar 24, 2022)

Published in: *J.Phys.A* 55 (2022) 44, 443016 • e-Print: 2203.13026 [hep-th]

- Lance and I share 6 papers, and that review

- I will talk about some of those papers

- one (DDM) was inspired by Regge limit of QCD but deals with colour

New color decompositions for gauge amplitudes at tree and loop level

Vittorio Del Duca (INFN, Turin), Lance J. Dixon (SLAC), Fabio Maltoni (Turin U.) (Oct, 1999)

Published in: *Nucl.Phys.B* 571 (2000) 51-70 • e-Print: hep-ph/9910563 [hep-ph]

see also Dave's talk

- another was inspired by Regge limit of N=4 SYM
but deals with jet production in the Regge limit of QCD

The BFKL equation, Mueller-Navelet jets and single-valued harmonic polylogarithms #4

Vittorio Del Duca (Frascati), Lance J. Dixon (SLAC), Claude Duhr (Zurich, ETH and Durham U., IPPP), Jeffrey Pennington (SLAC) (Sep 25, 2013)

Published in: *JHEP* 02 (2014) 086 • e-Print: 1309.6647 [hep-ph]

- 1985: Parke-Taylor *n*-gluon amplitude

- 1990's: following Parke-Taylor, in the Western world Amplitudes (in the bulk) were a prerogative of Bern, Dixon, Dunbar, Kosower

see David's history lecture

- 1985: Parke-Taylor *n*-gluon amplitude

- 1990's: following Parke-Taylor, in the Western world Amplitudes (in the bulk) were a prerogative of Bern, Dixon, Dunbar, Kosower

see David's history lecture

- Beyond the Iron Curtain, Gribov, Lipatov & friends had been doing Amplitudes since the 60's, although in the Regge limit

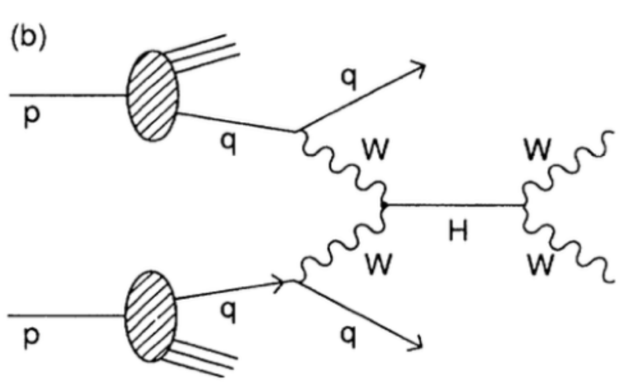
- 1990-93: I was a postdoc at SLAC, where Lance & Bjorken were
- 1991-93: "Bj" Bjorken was interested in the inter-jet radiation pattern in Higgs + 2-jet production from Vector Boson Fusion which naturally occurred with large rapidity intervals

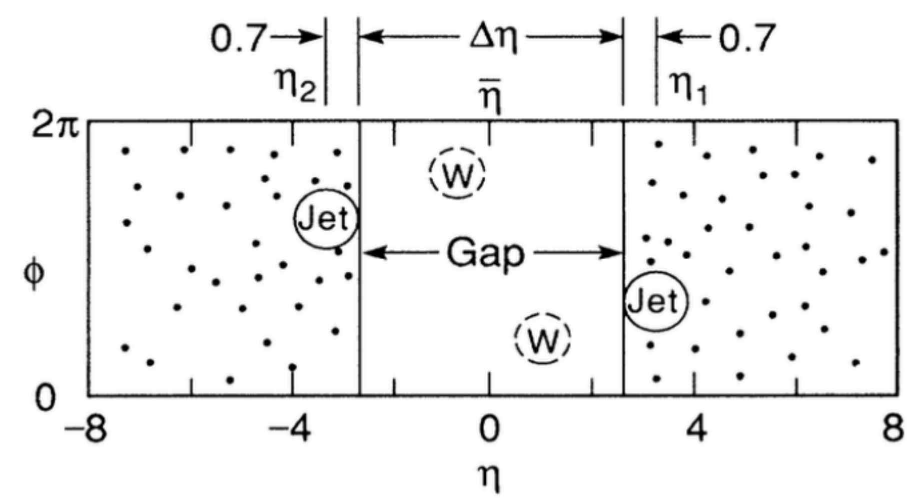
PHYSICAL REVIEW D VOLUME 47, NUMBER 1 1 JANUARY 1993

Rapidity gaps and jets as a new-physics signature in very-high-energy hadron-hadron collisions

J. D. Bjorken
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94309
(Received 30 March 1992)

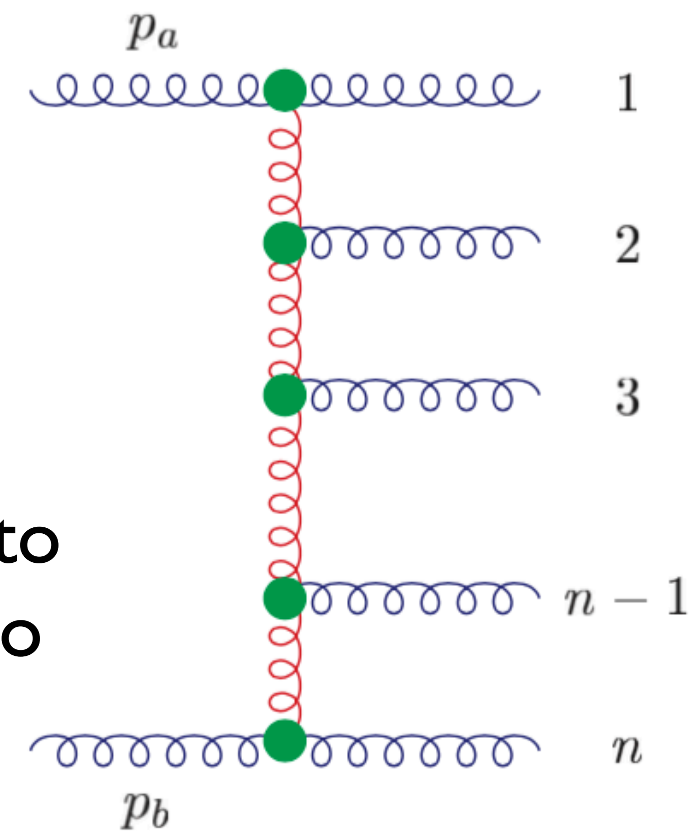
In hadron-hadron collisions, production of Higgs bosons and other color-singlet systems can occur via fusion of electroweak bosons, occasionally leaving a "rapidity gap" in the underlying-event structure.





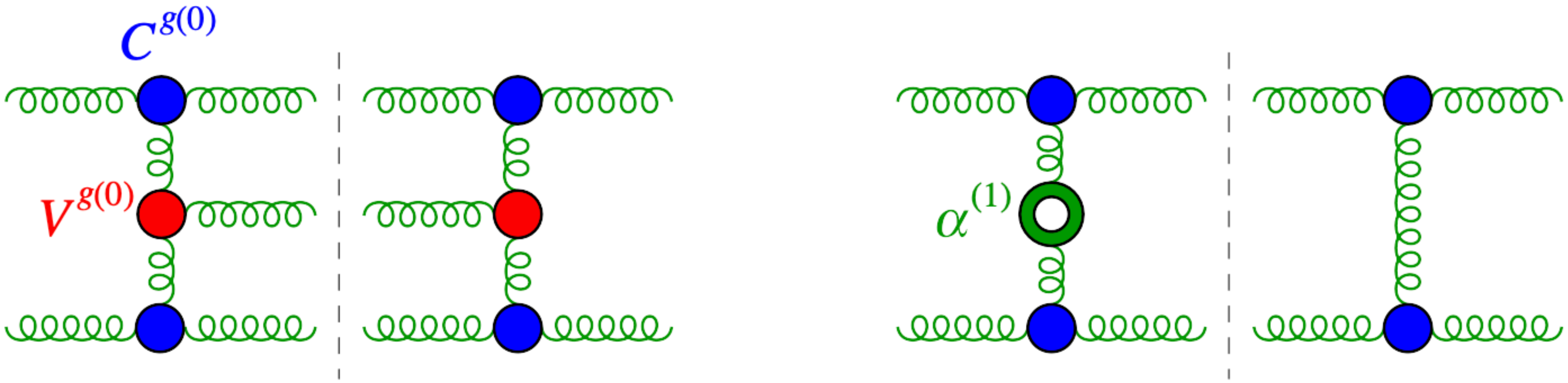
- Bj encouraged me to look at the inter-jet radiation pattern in jet production at large rapidity intervals → Mueller-Navelet jets → BFKL

Balitski Fadin Kuraev Lipatov

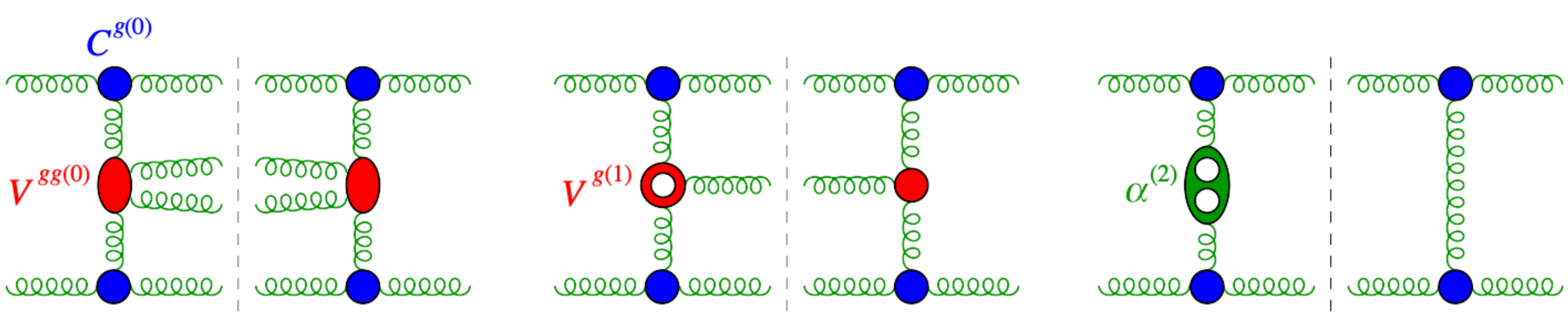


- BFKL is a resummation of multiple gluon radiation out of the gluon exchanged in the *t* channel
- the Leading Logarithmic (BFKL 1976-77) and Next-to-Leading Logarithmic (Fadin-Lipatov 1998) contributions in log(s/|t|) of the radiative corrections to the gluon propagator in the *t* channel are resummed to all orders in α_s
- the resummation yields an integral (BFKL) equation for the evolution of the gluon propagator in 2-dim transverse momentum space
- the BFKL equation is obtained in the limit of strong rapidity ordering of the emitted gluons, with no ordering in transverse momentum - *multi-Regge kinematics* (MRK)
- the solution is a Green's function of the momenta flowing in and out of the gluon ladder exchanged in the *t* channel

LO kernel



NLO kernel

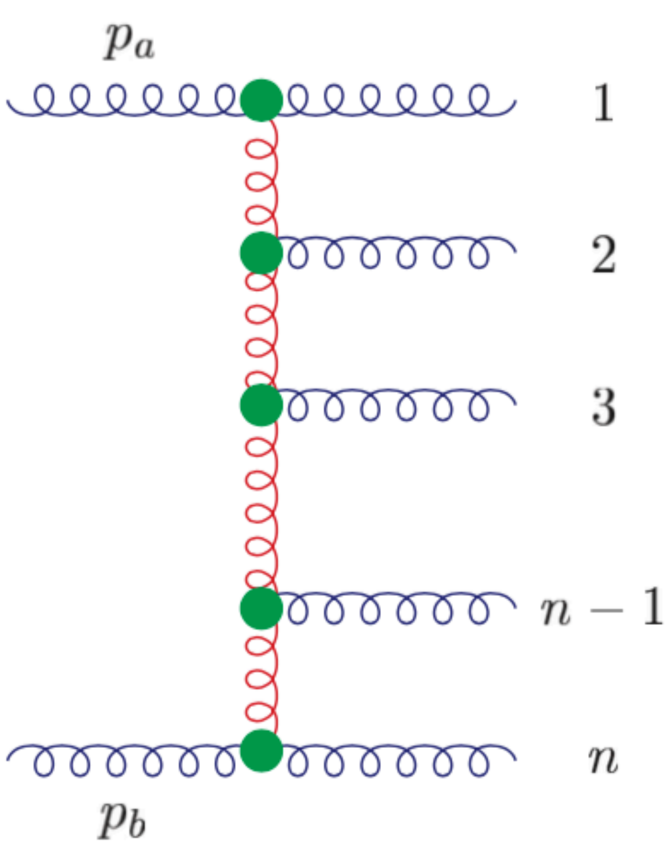


n gluons in multi-Regge limit of QCD

in the multi-Regge limit

$$p_1^+ \gg p_2^+ \gg \ldots \gg p_n^+ \,, \qquad |p_{1\perp}| \simeq |p_{2\perp}| \simeq \ldots \simeq |p_{n\perp}|$$

n-gluon amplitude has a factorised ladder form



$$\begin{aligned}
\mathcal{M}^{(0)}_{n\,g} &= s\left[g_s (F^{d_1})_{ac_1}\, C^{g(0)}(p_a^{\nu_a}, p_1^{\nu_1}))\right] \\
&\times \frac{1}{t_1}\left[g_s (F^{d_2})_{c_1 c_2}\, V^{g(0)}(q_1, p_2^{\nu_2}, q_2)\right] \\
&\cdot \\
&\cdot \\
&\cdot \\
&\times \frac{1}{t_{n-2}}\left[g_s (F^{d_{n-1}})_{c_{n-2} c_{n-1}}\, V^{g(0)}(q_{n-2}, p_{n-1}^{\nu_{n-1}}, q_{n-1})\right] \\
&\times \frac{1}{t_{n-1}}\left[g_s (F^{d_n})_{b c_{n-1}}\, C^{g(0)}(p_b^{\nu_b}, p_n^{\nu_n})\right]
\end{aligned}$$

Fadin Kuraev Lipatov 1976

$$(F^c)_{ab} = i\sqrt{2} f^{acb}$$

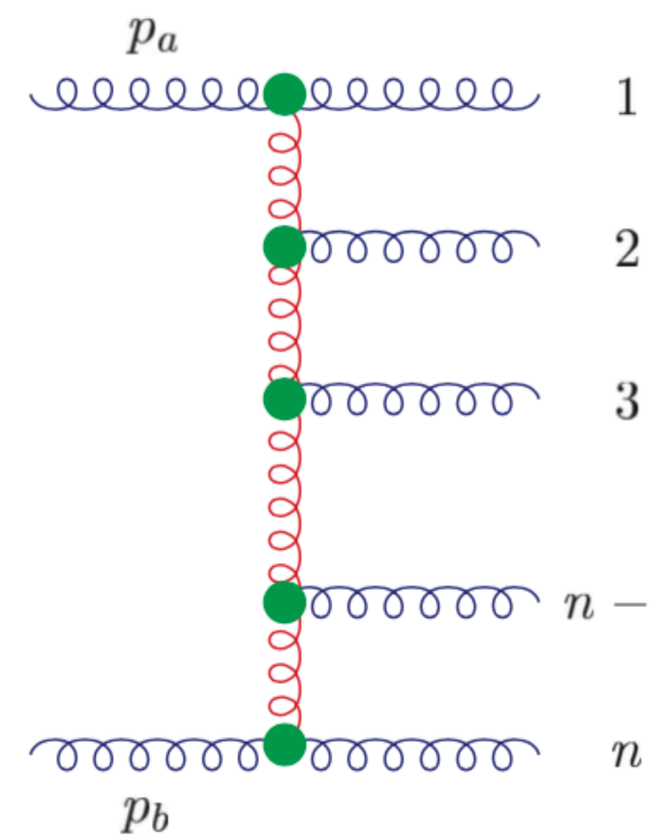
- in those years, I proved the equivalence between Parke-Taylor and Fadin-Kuraev-Lipatov *n*-gluon amplitudes in the multi-Regge limit

PHYSICAL REVIEW D VOLUME 52, NUMBER 3 1 AUGUST 1995

Equivalence of the Parke-Taylor and the Fadin-Kuraev-Lipatov amplitudes in the high-energy limit

Vittorio Del Duca
Deutsches Elektronen-Synchrotron DESY, D-22603 Hamburg, Germany
(Received 15 March 1995)

We give a unified description of tree-level multigluon amplitudes in the high-energy limit. We represent the Parke-Taylor amplitudes and the Fadin-Kuraev-Lipatov amplitudes in terms of color configurations that are ordered in rapidity on a two-sided plot. We show that for the helicity configurations they have in common the Parke-Taylor amplitudes and the Fadin-Kuraev-Lipatov amplitudes coincide.



using an identity between adjoint and fundamental colour factors

$$F^{a_1 a_2 x_1} F^{x_1 a_3 x_2} \cdots F^{x_{n-3} a_{n-1} a_n} = Tr\left(T^{a_1}\left[T^{a_2}, \left[T^{a_3}, \left[\ldots, \left[T^{a_{n-1}}, T^{a_n}\right], \ldots\right]\right]\right]\right)$$
$$= \left(F^{a_2} \cdots F^{a_{n-1}}\right)_{a_1 a_n}$$

Carl Schmidt and I used Zvi-Lance-David's one-loop 5-gluon amplitude

VOLUME 70, NUMBER 18 PHYSICAL REVIEW LETTERS 3 MAY 1993

One-Loop Corrections to Five-Gluon Amplitudes

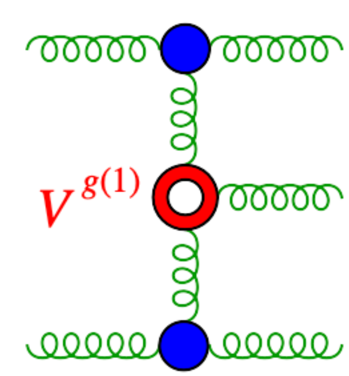
Zvi Bern[(a)]
Department of Physics, University of California, Los Angeles, Los Angeles, California 90024

Lance Dixon[(b)]
Stanford Linear Accelerator Center, Stanford, California 94309

David A. Kosower[(c)]
Theory Division, CERN, CH-1211 Geneva 23, Switzerland
(Received 1 March 1993)

see David's talk

to extract in the multi-Regge limit
the one-loop corrections to the gluon CEV,
which enter the NLO corrections to the BFKL kernel



PHYSICAL REVIEW D, VOLUME 59, 074004

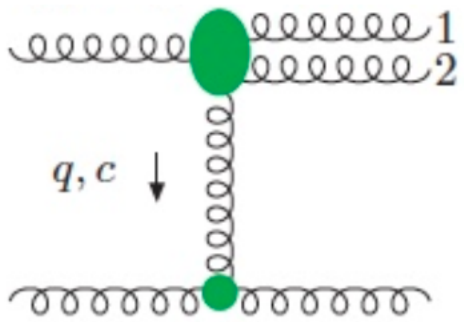
Virtual next-to-leading corrections to the Lipatov vertex

Vittorio Del Duca*
Particle Physics Theory Group, Department of Physics and Astronomy, University of Edinburgh, Edinburgh EH9 3JZ, Scotland, United Kingdom

Carl R. Schmidt
Department of Physics and Astronomy, Michigan State University, East Lansing, Michigan 48824
(Received 2 October 1998; published 16 February 1999)

next-to-multi-Regge limit

$p_1^+ \simeq p_2^+ \gg p_3^+ \qquad |p_{1\perp}| \simeq |p_{2\perp}| \simeq |p_{3\perp}|$



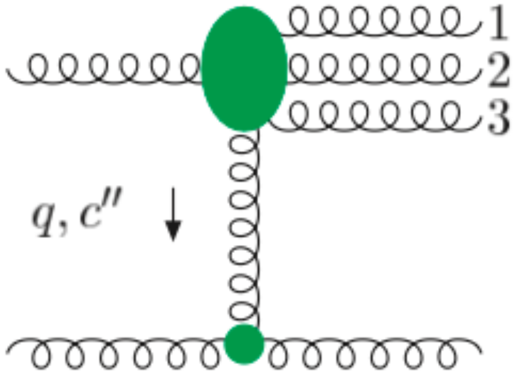
n-gluon amplitude keeps a factorised ladder form

$$\mathcal{M}_{5g}^{(0)} = s \left[g_s^2 \sum_{\sigma \in S_2} (F^{d_{\sigma_1}} F^{d_{\sigma_2}})_{ac} \, A^{gg(0)}(p_a^{\nu_a}; p_{\sigma_1}^{\nu_{\sigma_1}}, p_{\sigma_2}^{\nu_{\sigma_2}}) \right] \times \frac{1}{t} \left[g_s (F^{d_3})_{cb} C^{g(0)}(p_b^{\nu_b}; p_3^{\nu_3}) \right]$$

Fadin Lipatov 1989

next-to-next-to-multi-Regge limit

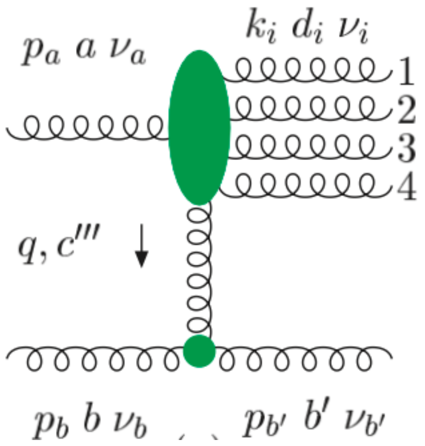
$$p_1^+ \simeq p_2^+ \simeq p_3^+ \gg p_4^+ \qquad |p_{1\perp}| \simeq |p_{2\perp}| \simeq |p_{3\perp}| \simeq |p_{4\perp}|$$



$$\mathcal{M}_{6g}^{(0)} = s \left[g_s^3 \sum_{\sigma \in S_3} (F^{d_{\sigma_1}} F^{d_{\sigma_2}} F^{d_{\sigma_3}})_{ac} \, A^{ggg(0)}(p_a^{\nu_a}; p_{\sigma_1}^{\nu_{\sigma_1}}, p_{\sigma_2}^{\nu_{\sigma_2}}, p_{\sigma_3}^{\nu_{\sigma_3}}) \right] \times \frac{1}{t} \left[g_s (F^{d_4})_{cb} C^{g(0)}(p_b^{\nu_b}; p_4^{\nu_4}) \right]$$

next-to-next-to-next-to-multi-Regge limit

$$p_1^+ \simeq p_2^+ \simeq p_3^+ \simeq p_4^+ \gg p_5^+ \qquad |p_{1\perp}| \simeq |p_{2\perp}| \simeq |p_{3\perp}| \simeq |p_{4\perp}| \simeq |p_{5\perp}|$$



$$\mathcal{M}_{7g}^{(0)} = s \left[g_s^4 \sum_{\sigma \in S_4} (F^{d_{\sigma_1}} F^{d_{\sigma_2}} F^{d_{\sigma_3}} F^{d_{\sigma_4}})_{ac} \, A^{gggg(0)}(p_a^{\nu_a}; p_{\sigma_1}^{\nu_{\sigma_1}}, p_{\sigma_2}^{\nu_{\sigma_2}}, p_{\sigma_3}^{\nu_{\sigma_3}}, p_{\sigma_4}^{\nu_{\sigma_4}}) \right] \times \frac{1}{t} \left[g_s (F^{d_5})_{cb} C^{g(0)}(p_b^{\nu_b}; p_5^{\nu_5}) \right]$$

Frizzo Maltoni VDD 1999

- note that for the maximal "next-to", i.e. (next-to)$^{n-4}$ the allowed colour ladders are (n-3)!

- we made the ansatz, checked through *n*=7 with Kleiss-Kuijf relations

$$A_n^{tree}(1,\{\alpha\},n,\{\beta^T\}) = (-1)^{n_\beta} \sum_{\sigma\in\{\alpha\}\sqcup\sqcup\{\beta\}} A_n^{tree}(1,\sigma(\{\alpha\},\{\beta\}),n)$$

that the *n*-gluon amplitude in the bulk display the colour decomposition

$$M_n^{tree}(1_g,\ldots,n_g) = g^{n-2} \sum_{\sigma\in S_{n-2}} (F^{a_{\sigma_2}}\cdots F^{a_{\sigma_{n-1}}})_{a_1 a_n} A_n^{tree}(1,\sigma(2),\ldots,\sigma(n-1),n)$$

with (*n*-2)! colour-ordered amplitudes

Frizzo Maltoni VDD hep-ph/9909464

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Nuclear Physics B 568 (2000) 211–262

NUCLEAR PHYSICS B

www.elsevier.nl/locate/npe

Factorization of tree QCD amplitudes in the high-energy limit
and in the collinear limit

Vittorio Del Duca [a], Alberto Frizzo [b], Fabio Maltoni [b]

[a] *Istituto Nazionale di Fisica Nucleare, Sezione di Torino, via P. Giuria, 1, 10125 Turin, Italy*
[b] *Dipartimento di Fisica Teorica, Università di Torino, via P. Giuria, 1, 10125 Turin, Italy*

Received 29 September 1999; accepted 11 October 1999

- could one extend that colour decomposition beyond tree level?
- at this point, we wrote to Lance …
- Lance entered the picture and moved at a lightning speed
- we provided a few proofs of the new tree colour decomposition

- for arbitrary *n*, we linked the new colour decomposition to Kleiss-Kuijf relations, so that proving the former was equivalent to inserting the latter into the standard colour decomposition

- we used a multi peripheral graphical notation of colour factors

together with Jacobi identities

see Dave's talk

and the identity between adjoint and fundamental colour factors

$$F^{a_1 a_2 x_1} F^{x_1 a_3 x_2} \cdots F^{x_{n-3} a_{n-1} a_n} = Tr\Big(T^{a_1}\Big[T^{a_2}, \big[T^{a_3}, \big[\ldots, \big[T^{a_{n-1}}, T^{a_n}\big], \ldots\big]\Big]\Big)$$

$$= (F^{a_2} \cdots F^{a_{n-1}})_{a_1 a_n}$$

Dixon Maltoni VDD hep-ph/9910563

- we (Lance) provided a new one-loop colour decomposition for *n* gluons

$$M_n^{1-\mathrm{loop}}(1_g,\ldots,n_g) = g^n \sum_{\sigma\in S_{n-1}/\mathcal{R}} \Big[\mathrm{tr}(F^{a_{\sigma_1}}\cdots F^{a_{\sigma_n}}) A_{n;1}^{[1]}(\sigma_1,\ldots,\sigma_n) + 2n_f \mathrm{tr}(T^{a_{\sigma_1}}\cdots T^{a_{\sigma_n}}) A_{n;1}^{[1/2]}(\sigma_1,\ldots,\sigma_n) \Big]$$

which did not feature double traces,
like the standard one-loop colour decomposition

$$M_n^{1-\mathrm{loop}}(1_g,\ldots,n_g) = g^n \Big[N_c \sum_{\sigma\in S_n/Z_n} \mathrm{tr}(T^{a_{\sigma_1}}\cdots T^{a_{\sigma_n}}) A_{n;1}^{[1]}(\sigma_1,\ldots,\sigma_n) + \sum_{c=2}^{[n/2]+1} \sum_{\sigma\in S_n/S_{n;c}} \mathrm{tr}(T^{a_{\sigma_1}}\cdots T^{a_{\sigma_{c-1}}})\, \mathrm{tr}(T^{a_{\sigma_c}}\cdots T^{a_{\sigma_n}}) A_{n;c}(\sigma_1,\ldots,\sigma_n) + n_f \sum_{\sigma\in S_n/Z_n} \mathrm{tr}(T^{a_{\sigma_1}}\cdots T^{a_{\sigma_n}}) A_{n;1}^{[1/2]}(\sigma_1,\ldots,\sigma_n) \Big]$$

- likewise, we (Lance) provided a new one-loop colour decomposition for *n* gluons with an external qqbar pair

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Nuclear Physics B 571 (2000) 51–70

NUCLEAR PHYSICS B

www.elsevier.nl/locate/npe

New color decompositions for gauge amplitudes at tree and loop level

Vittorio Del Duca [a,1], Lance Dixon [b,2], Fabio Maltoni [c]

[a] *Istituto Nazionale di Fisica Nucleare, Sezione di Torino, via P. Giuria, 1 10125 - Torino, Italy*
[b] *Stanford Linear Accelerator Center, Stanford University, Stanford, CA 94309, USA*
[c] *Dipartimento di Fisica Teorica, Università di Torino, via P. Giuria, 1 10125 - Torino, Italy*

Received 8 November 1999; accepted 20 December 1999

Abstract

Recently, a color decomposition using structure constants was introduced for purely gluonic tree amplitudes, in a compact form involving only the linearly independent subamplitudes. We give two proofs that this decomposition holds for an arbitrary number of gluons. We also present and prove similar decompositions at one loop, both for pure gluon amplitudes and for amplitudes with an external quark–antiquark pair.

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Nuclear Physics B 571 (2000) 51–70

NUCLEAR PHYSICS B

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435 citations

~ #20 on Lance's Hit Parade

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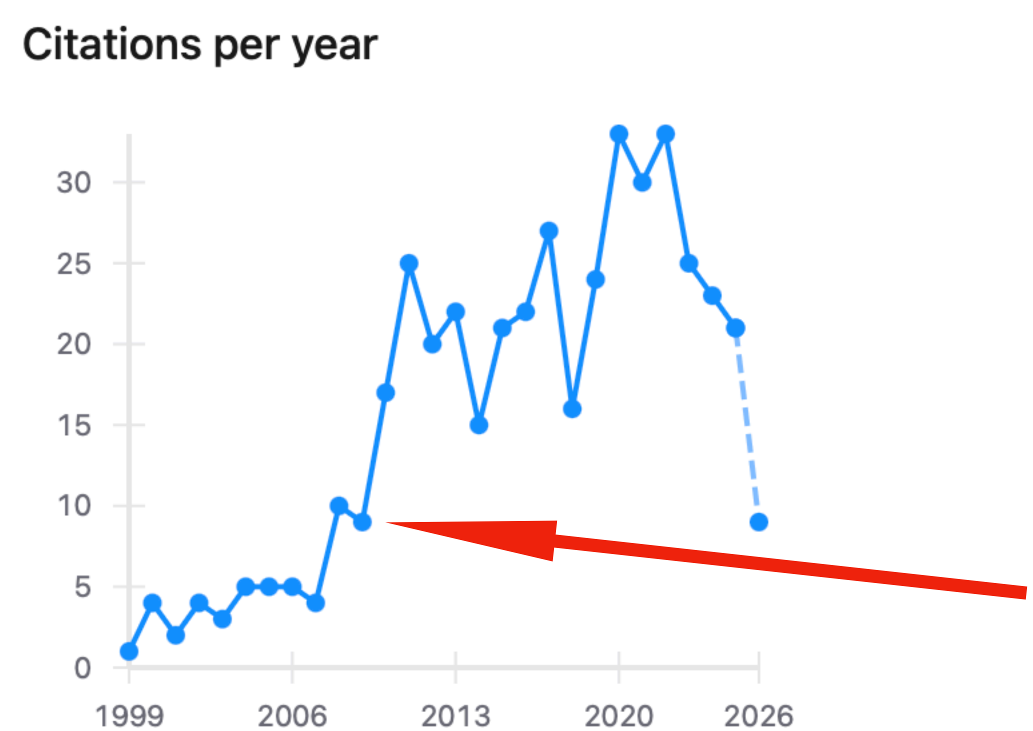

2008-2009

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Nuclear Physics B 571 (2000) 51–70

NUCLEAR PHYSICS B

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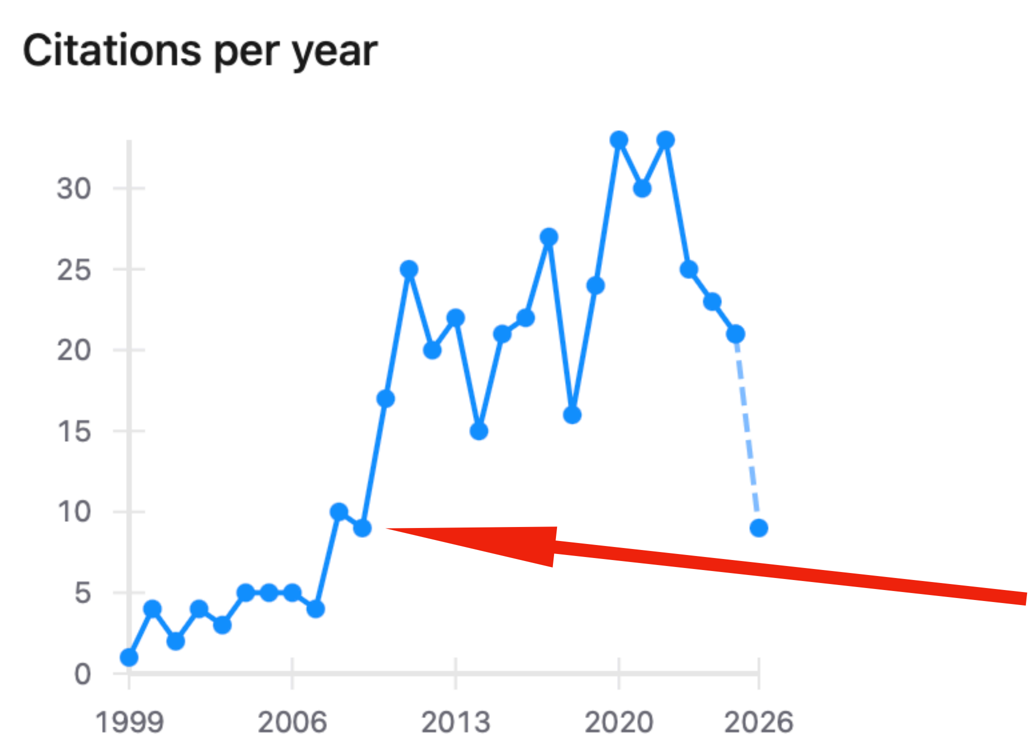
colour-kinematics duality

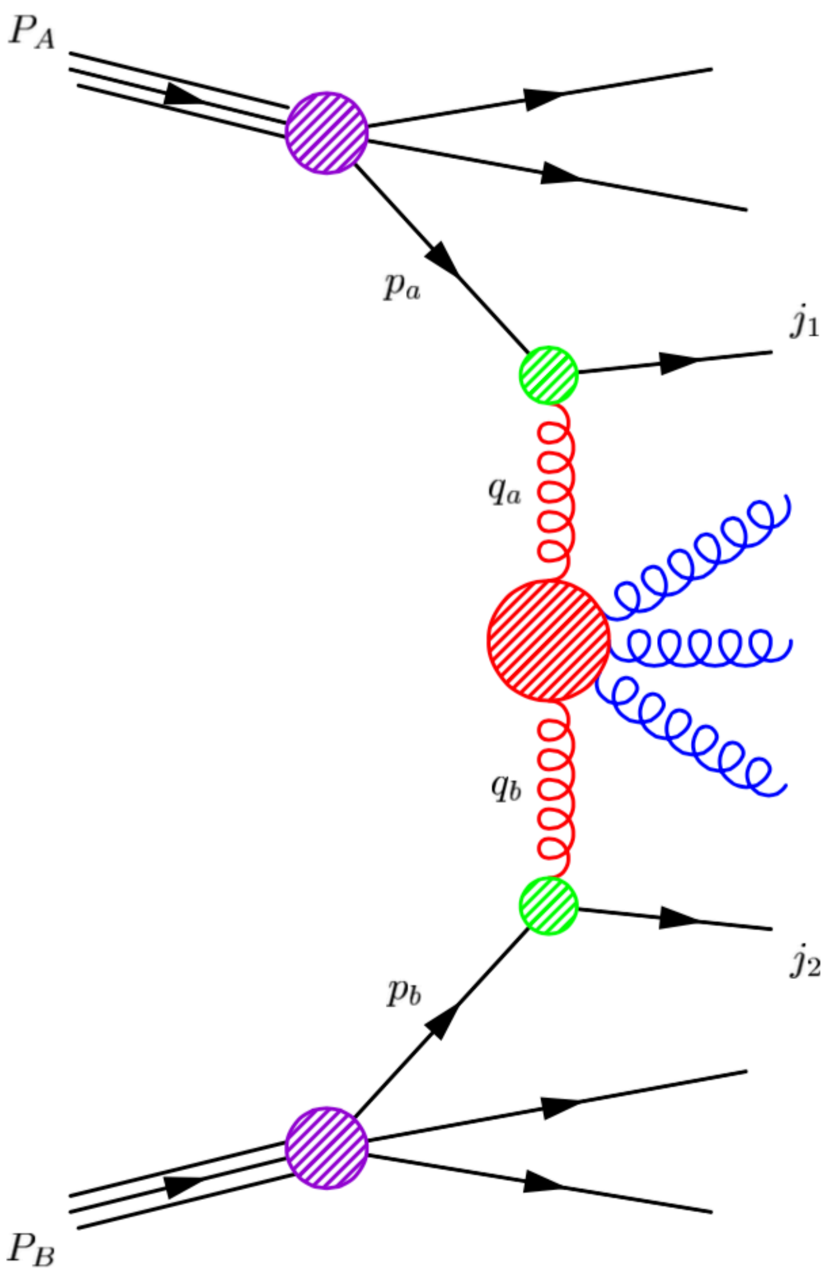
Bern Carrasco Johansson 2008



2008-2009

DDM at BDK Galileo Medal ceremony

Dijet production in the Regge limit of QCD



Dijet production cross section with two tagging jets in the forward and backward directions

$p_a = x_a P_A \quad p_b = x_b P_B$ incoming parton momenta

S: hadron centre-of-mass energy

$s = x_a x_b S$: parton centre-of-mass energy

E_{Tj}: jet transverse energies

$$\Delta y = |y_{j_1} - y_{j_2}| \simeq \log \frac{s}{E_{Tj_1} E_{Tj_2}}$$

is the rapidity interval between the tagging jets

gluon radiation is considered in MRK and resummed through the LL BFKL equation

model inter-jet radiation through BFKL ladder

the cross section for dijet production at large rapidity intervals $\Delta y = y_1 - y_2 = \ln\left(\frac{\hat{s}}{-t}\right) \gg 1$

with $\hat{s} = x_a x_b S\,, \quad t = -\sqrt{p_{1\perp}^2 p_{2\perp}^2}$

$$\frac{d\hat{\sigma}_{gg}}{dp_{1\perp}^2 dp_{2\perp}^2 d\phi_{jj}} = \frac{\pi}{2}\left[\frac{C_A\alpha_s}{p_{1\perp}^2}\right] f(\vec{q}_{1\perp}, \vec{q}_{2\perp}, \Delta y)\left[\frac{C_A\alpha_s}{p_{2\perp}^2}\right]$$

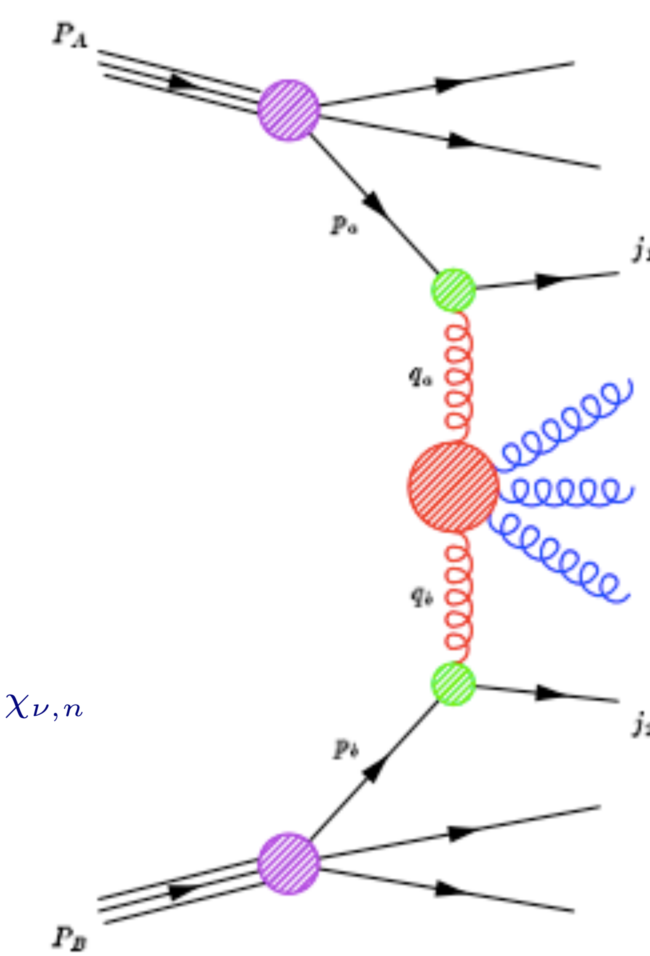
is described through the BFKL Green's function

$$f(\vec{q}_{1\perp}, \vec{q}_{2\perp}, \Delta y) = \frac{1}{(2\pi)^2\sqrt{q_{1\perp}^2\, q_{2\perp}^2}}\sum_{n=-\infty}^{+\infty} e^{in\phi}\int_{-\infty}^{+\infty} d\nu\left(\frac{q_{1\perp}^2}{q_{2\perp}^2}\right)^{i\nu} e^{\eta\,\chi_{\nu,n}}$$

with $\eta \equiv \frac{C_A\,\alpha_s}{\pi}\Delta y$ and ϕ the angle between q_1^2 and q_2^2

and the LL BFKL eigenvalue

$$\chi_{\nu,n} = -2\gamma_E - \psi\left(\frac{1}{2} + \frac{|n|}{2} + i\nu\right) - \psi\left(\frac{1}{2} + \frac{|n|}{2} - i\nu\right)$$



Mueller Navelet 1987

Mueller-Navelet jets

- azimuthal angle distribution ($\phi_{jj} = \phi - \pi$)

$$\frac{d\hat{\sigma}_{gg}}{d\phi_{jj}} = \frac{\pi (C_A \alpha_s)^2}{2E_\perp^2} \left[\delta(\phi_{jj} - \pi) + \sum_{k=1}^{\infty} \left(\sum_{n=-\infty}^{\infty} \frac{e^{in\phi}}{2\pi} f_{n,k} \right) \eta^k \right]$$

with $f_{n,k} = \frac{1}{2\pi} \frac{1}{k!} \int_{-\infty}^{\infty} d\nu \, \frac{\chi_{\nu,n}^k}{\nu^2 + \frac{1}{4}}$

Mueller-Navelet evaluated the inclusive dijet cross section up to 5 loops

- the dijet cross section is $\hat{\sigma}_{gg} = \frac{\pi (C_A \alpha_s)^2}{2E_\perp^2} \sum_{k=0}^{\infty} f_{0,k} \, \eta^k$ Mueller Navelet 1987

with

$$\begin{aligned}
f_{0,0} &= 1 \,, \\
f_{0,1} &= 0 \,, \\
f_{0,2} &= 2\zeta_2 \,, \\
f_{0,3} &= -3\zeta_3 \,, \\
f_{0,4} &= \frac{53}{6} \zeta_4 \,, \\
f_{0,5} &= -\frac{1}{12} (115\zeta_5 + 48\zeta_2\zeta_3)
\end{aligned}$$

Mueller-Navelet jets and SVHPLs

- The singlet LL BFKL ladder in QCD, and thus the dijet cross section in the high-energy limit, can also be expressed in terms of SVHPLs, i.e. in terms of single-valued iterated integrals on $\mathcal{M}_{0,4}$

  Dixon Duhr Pennington VDD 2013

- Mueller & Navelet evaluated analytically the inclusive dijet cross section up to 5 loops. We evaluated it analytically up to 13 loops

- Also, we could evaluate analytically the dijet cross section differential in the jet transverse energies or the azimuthal angle between the jets (up to 6 loops)

Multi-Regge limit of $\mathcal{N}$=4 SYM

- In planar $\mathcal{N}$=4 SYM, 4-pt and 5-pt amplitudes are expressed through the BDS ansatz

  Bern Dixon Smirnov 2005

- In the Euclidean region (where all Mandelstam invariants are negative), amplitudes in MRK factorise completely in terms of building blocks which are expressed in terms of Regge poles and can be determined to all orders through the 4-pt and 5-pt amplitudes.
  Thus the remainder functions R vanish at all points

  Duhr Glover VDD 2008

- After analytic continuation to some regions of the Minkowski space, the amplitudes develops cuts which are described by a dispersion relation for octet exchange, which is similar to the singlet BFKL equation in QCD

  Bartels Lipatov Sabio-Vera 2008

  see James's talk

- In particular, 6-pt amplitudes at weak coupling can be expressed in terms of single-valued harmonic polylogarithms

  Dixon Duhr Pennington 2012

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Single-valued harmonic polylogarithms and the multi-Regge limit

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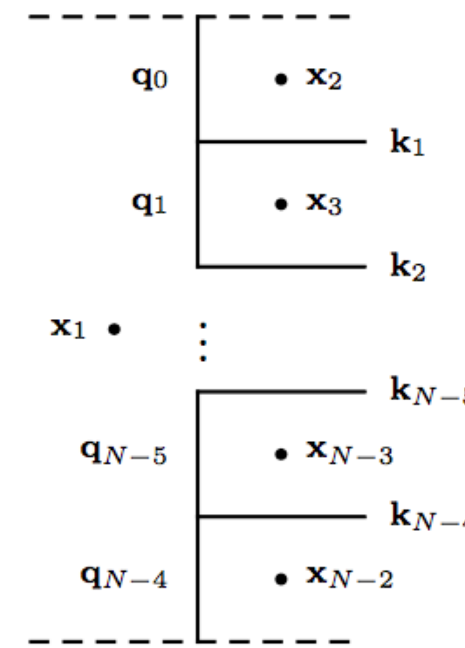
Abstract: We argue that the natural functions for describing the multi-Regge limit of six-gluon scattering in planar $\mathcal{N} = 4$ super Yang-Mills theory are the single-valued harmonic polylogarithmic functions introduced by Brown. These functions depend on a single

JHEP10(2012)

Moduli space of Riemann spheres



- in MRK, the light-cone momenta are strongly ordered, the non-trivial dynamics is in the *n*-2 transverse momenta

- dual conformal invariance in transverse momentum space implies dependence on *n*-5 cross ratios of the transverse momenta

$$z_i = \frac{(\mathbf{x}_1 - \mathbf{x}_{i+3})\,(\mathbf{x}_{i+2} - \mathbf{x}_{i+1})}{(\mathbf{x}_1 - \mathbf{x}_{i+1})\,(\mathbf{x}_{i+2} - \mathbf{x}_{i+3})} = -\frac{\mathbf{q}_{i+1}\,\mathbf{k}_i}{\mathbf{q}_{i-1}\,\mathbf{k}_{i+1}} \qquad i = 1, \ldots, n-5$$

- $\mathcal{M}_{0,p}$ = space of configurations of *p* points on the Riemann sphere

- Because we can fix 3 points at 0, 1, ∞, its dimension is dim($\mathcal{M}_{0,p}$)= *p*-3

- $\mathcal{M}_{0,n-2}$ is the space of MRK of planar *N*=4 SYM, with dim($\mathcal{M}_{0,n-2}$) = *n*-5

- Its coordinates can be chosen to be the z_i's, i.e. the cross ratios of the transverse momenta

Drummond Druc Duhr Dulat Marzucca Papathanasiou Verbeek VDD 2016

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Multi-Regge kinematics and the moduli space of Riemann spheres with marked points

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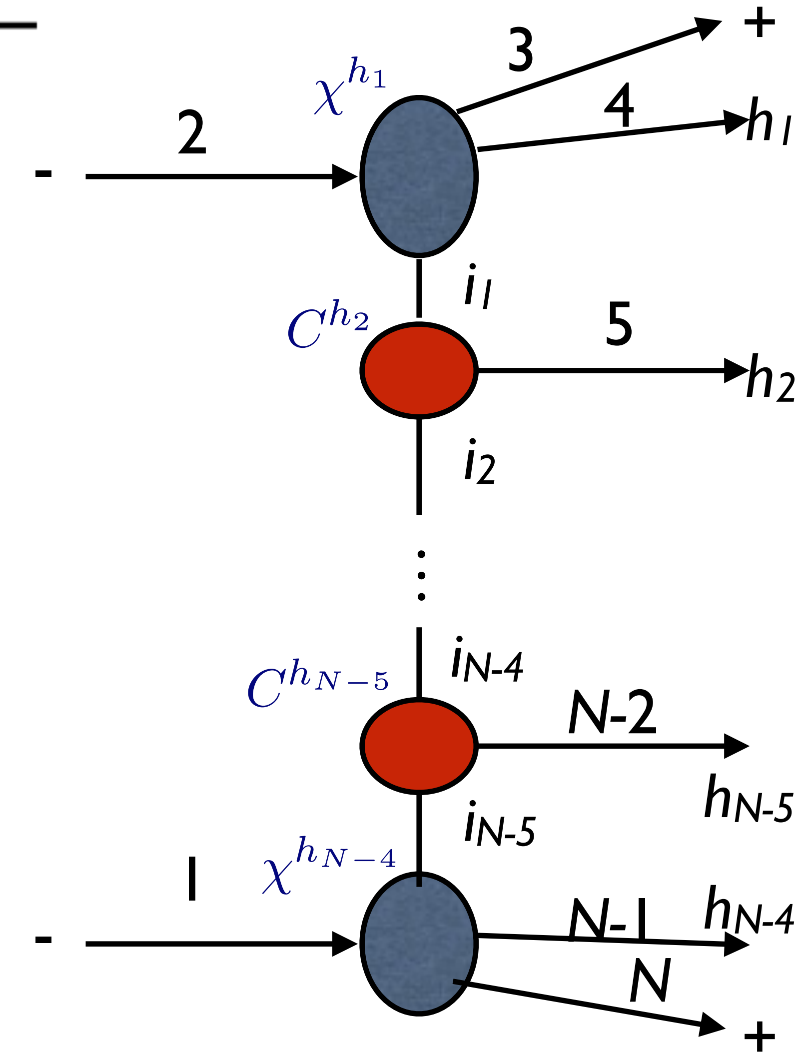

Abstract: We show that scattering amplitudes in planar $\mathcal{N} = 4$ Super Yang-Mills in multi-Regge kinematics can naturally be expressed in terms of single-valued iterated integrals on the moduli space of Riemann spheres with marked points. As a consequence, scat-

Iterated integrals on $\mathcal{M}_{0,n-2}$

- on $\mathcal{M}_{0,n-2}$, the singularities are associated to degenerate configurations when two points merge $x_i \to x_{i+1}$
  i.e. when momentum p_i becomes soft $p_i \to 0$

- iterated integrals on $\mathcal{M}_{0,p}$ can be written as multiple polylogarithms

  Brown 2006

➡ amplitudes in MRK can be written in terms of multiple polylogarithms

- analytic structure of amplitudes is constrained by unitarity and the optical theorem $\mathrm{Disc}(M) = iMM^\dagger$

- massless amplitudes may have branch points when Mandelstam invariants vanish $s_{ij} \to 0$ or become infinite $s_{ij} \to \infty$, but branch cuts are constrained by unitarity

- the coproduct of an amplitude is related to unitarity
- for massless amplitudes

  $$\Delta(M) = \ln(s_{ij}) \otimes \ldots$$

- in particular, for amplitudes in MRK

  $$\Delta(M) = \ln|\mathbf{x}_i - \mathbf{x}_j|^2 \otimes \ldots$$

- except for the soft limit $p_i \to 0$, in MRK the transverse momenta never vanish

  $|\mathbf{x}_i - \mathbf{x}_j|^2 \neq 0$ ⟹ single-valued functions

- therefore, n-point amplitudes of planar N=4 SYM in MRK can be written in terms of single-valued iterated integrals on $\mathscr{M}_{0,n-2}$

  Drummond Druc Duhr Dulat Marzucca Papathanasiou Verbeek VDD 2016

- for n=6, iterated integrals on $\mathscr{M}_{0,4}$ are harmonic polylogarithms
  thus, 6-point amplitudes in MRK of can be written in terms of single-valued harmonic polylogarithms (SVHPL)

  Dixon Duhr Pennington 2012

An all-order any-legs amplitude in MRK

PHYSICAL REVIEW LETTERS **124,** 161602 (2020)

All-Order Amplitudes at any Multiplicity in the Multi-Regge Limit

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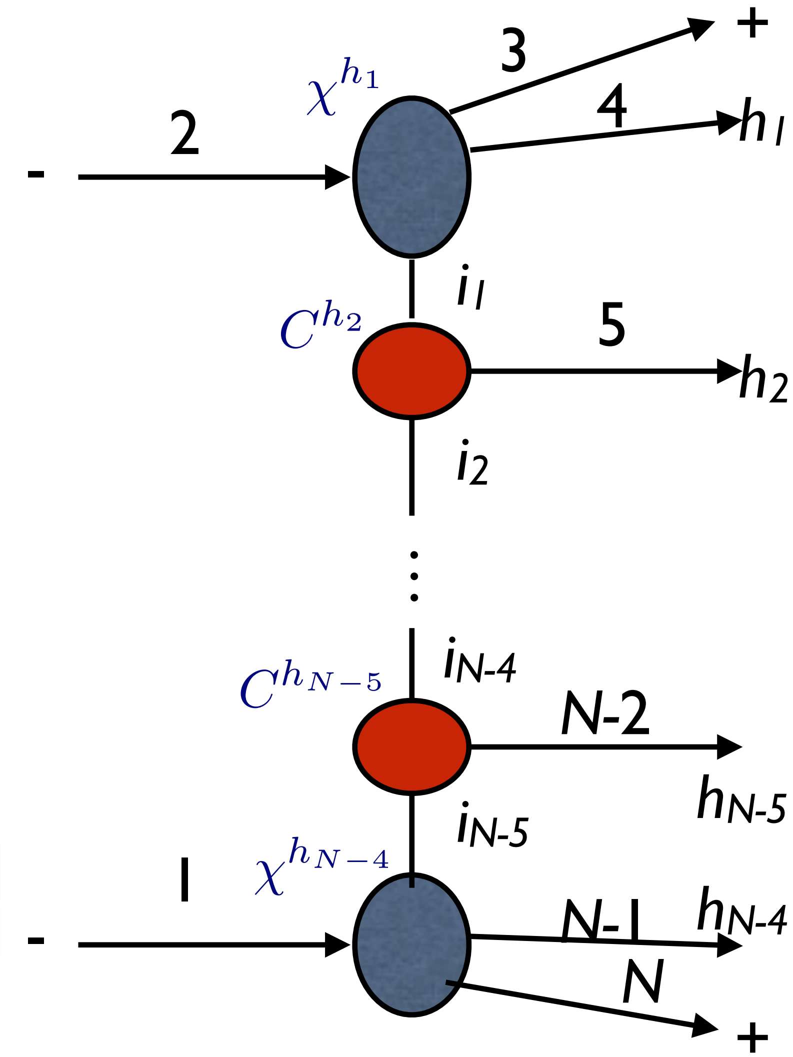
We propose an all-loop expression for scattering amplitudes in planar $\mathcal{N} = 4$ super Yang-Mills theory in multi-Regge kinematics valid for all multiplicities, all helicity configurations, and arbitrary logarithmic accuracy. Our expression is arrived at from comparing explicit perturbative results with general expectations from the integrable structure of a closely related collinear limit. A crucial ingredient of the analysis is an all-order extension for the central emission vertex that we recently computed at next-to-leading logarithmic accuracy. As an application, we use our all-order formula to prove that all amplitudes in this theory in multi-Regge kinematics are single-valued multiple polylogarithms of uniform transcendental weight.



Lance checked it agrees with the 7-pt amplitude through 4 loops in MRK

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Heptagon functions and seven-gluon amplitudes in multi-Regge kinematics

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Abstract: We compute all $2 \to 5$ gluon scattering amplitudes in planar $\mathcal{N} = 4$ super-Yang-Mills theory in the multi-Regge limit that is sensitive to the non-trivial ("long") Regge cut. We provide the amplitudes through four loops and to all logarithmic accuracy at leading power, in terms of single-valued multiple polylogarithms of two variables. To obtain these results, we leverage the function-level results for the amplitudes in the Steinmann cluster bootstrap. To high powers in the series expansion in the two variables, our results agree with the recently conjectured all-order *central emission vertex* used in the Fourier-Mellin representation of amplitudes in multi-Regge kinematics. Our results therefore provide a resummation of the Fourier-Mellin residues into single-valued polylogarithms, and constitute an important cross-check between the bootstrap approach and the all-orders multi-Regge proposal.

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Back to Mueller-Navelet jets and SVHPLs

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The BFKL equation, Mueller-Navelet jets and single-valued harmonic polylogarithms

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Abstract: We introduce a generating function for the coefficients of the leading logarithmic BFKL Green's function in transverse-momentum space, order by order in α_S, in terms of single-valued harmonic polylogarithms. As an application, we exhibit fully analytic azimuthal-angle and transverse-momentum distributions for Mueller-Navelet jet cross sections at each order in α_S. We also provide a generating function for the total cross section valid to any number of loops.

BFKL Green's function and single-valued functions

- use complex transverse momentum $\tilde{q}_k \equiv q_k^x + i q_k^y$

  and a complex variable $z \equiv \frac{\tilde{q}_1}{\tilde{q}_2}$

  the Green's function can be expanded into a power series in $\eta_\mu = \overline{\alpha}_\mu \, y$

$$f^{LL}(q_1, q_2, \eta_\mu) = \frac{1}{2}\delta^{(2)}(q_1 - q_2) + \frac{1}{2\pi\sqrt{q_1^2\, q_2^2}} \sum_{k=1}^{\infty} \frac{\eta_\mu^k}{k!}\, f_k^{LL}(z)$$

  where the coefficient functions f_k are given by the Fourier-Mellin transform

$$f_k^{LL}(z) = \mathcal{F}\left[\chi_{\nu n}^k\right] = \sum_{n=-\infty}^{+\infty} \left(\frac{z}{\bar{z}}\right)^{n/2} \int_{-\infty}^{+\infty} \frac{d\nu}{2\pi}\, |z|^{2i\nu}\, \chi_{\nu n}^k$$

  - the f_k have a unique, well-defined value for every ratio of the magnitudes of the two jet transverse momenta and angle between them.
    So, they are real-analytic functions of z

Azimuthal angle distribution

- this allows us to write the azimuthal angle distribution as

$$\frac{d\hat{\sigma}_{gg}}{d\phi_{jj}} = \frac{\pi (C_A \alpha_s)^2}{2E_\perp^2} \left[\delta(\phi_{jj} - \pi) + \sum_{k=1}^{\infty} \frac{a_k(\phi_{jj})}{\pi} \eta^k \right]$$

where the contribution of the k^{th} loop is

$$a_k(\phi_{jj}) = \int_0^\infty \frac{d|w|}{|w|} f_k(w, w^*) = \frac{\text{Im}\, A_k(\phi_{jj})}{\sin \phi_{jj}}$$

with

$$\begin{aligned}
A_1(\phi_{jj}) &= -\frac{1}{2} H_0 \,, \\
A_2(\phi_{jj}) &= H_{1,0} \,, \\
A_3(\phi_{jj}) &= \frac{2}{3} H_{0,0,0} - 2H_{1,1,0} + \frac{5}{3}\zeta_2 H_0 - i\pi\,\zeta_2 \,, \\
A_4(\phi_{jj}) &= -\frac{4}{3} H_{0,0,1,0} - H_{0,1,0,0} - \frac{4}{3} H_{1,0,0,0} + 4H_{1,1,1,0} - \zeta_2 \left(2H_{0,1} + \frac{10}{3} H_{1,0} \right) + \frac{4}{3}\zeta_3\, H_0 + i\pi \left(2\zeta_2 H_1 - 2\zeta_3 \right) , \\
A_5(\phi_{jj}) &= -\frac{46}{15} H_{0,0,0,0,0} + \frac{8}{3} H_{0,0,1,1,0} + 2H_{0,1,0,1,0} + 2H_{0,1,1,0,0} + \frac{8}{3} H_{1,0,0,1,0} + 2H_{1,0,1,0,0} \\
&\quad + \frac{8}{3} H_{1,1,0,0,0} - 8H_{1,1,1,1,0} - \zeta_2 \left(\frac{33}{5} H_{0,0,0} - 4H_{0,1,1} - 4H_{1,0,1} - \frac{20}{3} H_{1,1,0} \right) \\
&\quad - \zeta_3 \left(2H_{0,1} + \frac{8}{3} H_{1,0} \right) + \frac{217}{15}\zeta_4 H_0 + i\pi \left[\zeta_2 \left(\frac{10}{3} H_{0,0} - 4H_{1,1} \right) + 4\zeta_3 H_1 - \frac{10}{3}\zeta_4 \right]
\end{aligned}$$

where $H_{i,j,\ldots} \equiv H_{i,j,\ldots}(e^{-2i\phi_{jj}})$

Dixon Duhr Pennington VDD 2013

Transverse momentum distribution

$$\frac{d\hat{\sigma}_{gg}}{dp_{1\perp}^2\, dp_{2\perp}^2} = \frac{\pi (C_A \alpha_s)^2}{2 p_{1\perp}^2 p_{2\perp}^2} \left[\delta(p_{1\perp}^2 - p_{2\perp}^2) + \frac{1}{2\pi\, \sqrt{p_{1\perp}^2\, p_{2\perp}^2}}\, b(\rho;\eta) \right]$$

where $\rho = |w|$ $\qquad b(\rho;\eta) = \dfrac{2\,\pi\,\rho}{1-\rho^2} \displaystyle\sum_{k=1}^{\infty} B_k(\rho)\, \eta^k$

with

$$\begin{aligned}
B_1(\rho) &= 1\,,\\
B_2(\rho) &= -\frac{1}{2}\, H_0 - 2H_1\,,\\
B_3(\rho) &= \frac{1}{6} H_{0,0} + 2H_{0,1} + H_{1,0} + 4H_{1,1}\,,\\
B_4(\rho) &= -\frac{1}{24} H_{0,0,0} - \frac{4}{3} H_{0,0,1} - H_{0,1,0} - 4H_{0,1,1} - \frac{1}{3} H_{1,0,0} - 4H_{1,0,1} - 2H_{1,1,0} - 8H_{1,1,1} + \frac{1}{3}\,\zeta_3\,,\\
B_5(\rho) &= \frac{1}{120} H_{0,0,0,0} + \frac{2}{3} H_{0,0,0,1} + \frac{2}{3} H_{0,0,1,0} + \frac{8}{3} H_{0,0,1,1} + \frac{1}{3} H_{0,1,0,0} + 4H_{0,1,0,1}\\
&\quad + 2H_{0,1,1,0} + 8H_{0,1,1,1} + \frac{1}{12} H_{1,0,0,0} + \frac{8}{3} H_{1,0,0,1} + 2H_{1,0,1,0} + 8H_{1,0,1,1}\\
&\quad + \frac{2}{3} H_{1,1,0,0} + 8H_{1,1,0,1} + 4H_{1,1,1,0} + 16H_{1,1,1,1} + \zeta_3 \left(-\frac{1}{12} H_0 - \frac{2}{3} H_1 \right),
\end{aligned}$$

where $H_{i,j,\ldots} \equiv H_{i,j,\ldots}(\rho^2)$

Dixon Duhr Pennington VDD 2013

Mueller-Navelet dijet cross section reloaded

- the MN dijet cross section is $$\hat{\sigma}_{gg} = \frac{\pi (C_A \alpha_s)^2}{2E_\perp^2} \sum_{k=0}^{\infty} f_{0,k}\, \eta^k$$

the first 5 loops were computed by Mueller-Navelet.
We computed it through the 13 loops

Dixon Duhr Pennington VDD 2013

$$f_{0,6} = \frac{13}{4}\zeta_3^2 + \frac{3737}{120}\zeta_6\,,$$

$$f_{0,7} = -\frac{87}{5}\zeta_3\zeta_4 - \frac{116}{9}\zeta_2\zeta_5 - \frac{3983}{144}\zeta_7\,,$$

$$f_{0,8} = -\frac{37}{75}\zeta_{5,3} + \frac{64}{15}\zeta_2\zeta_3^2 + \frac{369}{20}\zeta_5\zeta_3 + \frac{50606057}{453600}\zeta_8\,,$$

$$f_{0,9} = -\frac{139}{60}\zeta_3^3 - \frac{15517}{252}\zeta_6\zeta_3 - \frac{3533}{63}\zeta_4\zeta_5 - \frac{557}{15}\zeta_2\zeta_7 - \frac{5215361}{60480}\zeta_9\,,$$

$$f_{0,10} = -\frac{2488}{4725}\zeta_{5,3}\zeta_2 - \frac{94721}{211680}\zeta_{7,3} + \frac{1948}{105}\zeta_4\zeta_3^2 + \frac{2608}{105}\zeta_2\zeta_5\zeta_3 + \frac{12099}{224}\zeta_7\zeta_3 + \frac{1335931}{47040}\zeta_5^2 + \frac{25669936301}{63504000}\zeta_{10}$$

$$\begin{aligned} f_{0,11} = {} & \frac{62}{315}\zeta_{5,3}\zeta_3 + \frac{83}{120}\zeta_{5,3,3} - \frac{2872}{945}\zeta_2\zeta_3^3 - \frac{13211}{672}\zeta_5\zeta_3^2 - \frac{661411}{3024}\zeta_8\zeta_3 \\ & - \frac{242776937}{725760}\zeta_{11} - \frac{605321}{3024}\zeta_5\zeta_6 - \frac{2583643}{16200}\zeta_4\zeta_7 - \frac{28702763}{340200}\zeta_2\zeta_9\,, \end{aligned}$$

$$\begin{aligned} f_{0,12} = {} & \frac{74711}{162000}\zeta_{5,3}\zeta_4 - \frac{13793}{7560}\zeta_{6,4,1,1} + \frac{3965011}{793800}\zeta_{7,3}\zeta_2 - \frac{33356851}{4082400}\zeta_{9,3} \\ & + \frac{252163}{181440}\zeta_3^4 + \frac{620477}{10080}\zeta_6\zeta_3^2 + \frac{8101339}{75600}\zeta_4\zeta_5\zeta_3 + \frac{342869}{3780}\zeta_2\zeta_7\zeta_3 \\ & + \frac{101571047}{680400}\zeta_9\zeta_3 + \frac{71425871}{1587600}\zeta_2\zeta_5^2 + \frac{904497401571619}{620606448000}\zeta_{12} + \frac{484414571}{2721600}\zeta_5\zeta_7\,, \end{aligned}$$

$$\begin{aligned} f_{0,13} = {} & \frac{4513}{1890}\zeta_{5,3}\zeta_5 + \frac{27248}{23625}\zeta_{5,3,3}\zeta_2 - \frac{97003}{235200}\zeta_{5,5,3} + \frac{13411}{75600}\zeta_{7,3}\zeta_3 \\ & + \frac{7997743}{12700800}\zeta_{7,3,3} - \frac{187318}{14175}\zeta_4\zeta_3^3 - \frac{125056}{4725}\zeta_2\zeta_5\zeta_3^2 - \frac{17411413}{302400}\zeta_7\zeta_3^2 \\ & - \frac{5724191}{100800}\zeta_5^2\zeta_3 - \frac{1874972477}{2376000}\zeta_{10}\zeta_3 - \frac{2418071698069}{2235340800}\zeta_{13} \\ & - \frac{2379684877}{6048000}\zeta_{11}\zeta_2 - \frac{297666465053}{523908000}\zeta_6\zeta_7 - \frac{1770762319}{2494800}\zeta_5\zeta_8 - \frac{229717224973}{628689600}\zeta_4\zeta_9 \end{aligned}$$

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The analytic structure and the transcendental weight of the BFKL ladder at NLL accuracy

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Abstract: We study some analytic properties of the BFKL ladder at next-to-leading logarithmic accuracy (NLLA). We use a procedure by Chirilli and Kovchegov to construct the NLO eigenfunctions, and we show that the BFKL ladder can be evaluated order by order in the coupling in terms of certain generalised single-valued multiple polylogarithms recently introduced by Schnetz. We develop techniques to evaluate the BFKL ladder at any loop order, and we present explicit results up to five loops. Using the freedom in defining the matter content of the NLO BFKL eigenvalue, we obtain conditions for the BFKL ladder in momentum space at NLLA to have maximal transcendental weight. We observe that, unlike in moment space, the result in momentum space in $\mathcal{N} = 4$ SYM is not identical to the maximal weight part of QCD, and moreover that there is no gauge theory with this property. We classify the theories for which the BFKL ladder at NLLA has maximal weight in terms of their field content, and we find that these theories are highly constrained: there are precisely four classes of theories with this property involving only fundamental and adjoint matter, all of which have a vanishing one-loop beta function and a matter content that fits into supersymmetric multiplets. Our findings indicate that theories which have maximal weight are highly constrained and point to the possibility that there is a connection between maximal transcendental weight and superconformal symmetry.

JHEP10(2017)001

BFKL eigenvalue at NLLA

- At NLLA in QCD and in *N*=4 SYM, the eigenvalue is

Fadin Lipatov 1998
Kotikov Lipatov 2000, 2002
Chirilli Kovchegov 2013
Duhr Marzucca Verbeek VDD 2017

$$\omega_{\nu n}^{(1)} = \frac{1}{4}\,\delta_{\nu n}^{(1)} + \frac{1}{4}\,\delta_{\nu n}^{(2)} + \frac{1}{4}\,\delta_{\nu n}^{(3)} + \gamma_K^{(2)}\chi_{\nu n} - \frac{1}{8}\beta_0\chi_{\nu n}^2 + \frac{3}{2}\zeta_3$$

with one-loop beta function and two-loop cusp anomalous dimension

$$\beta_0 = \frac{11}{3} - \frac{2N_f}{3N_c} \qquad \gamma_K^{(2)} = \frac{1}{4}\left(\frac{64}{9} - \frac{10N_f}{9N_c}\right) - \frac{\zeta_2}{2}$$

and with

$$\delta_{\nu n}^{(1)} = \partial_\nu^2\chi_{\nu n} \qquad \chi_{\nu n} = \omega_{\nu n}^{(0)}$$

$$\delta_{\nu n}^{(2)} = -2\Phi(n,\gamma) - 2\Phi(n,1-\gamma)$$

$$\delta_{\nu n}^{(3)} = -\frac{\Gamma(\frac{1}{2}+i\nu)\Gamma(\frac{1}{2}-i\nu)}{2i\nu}\left[\psi\left(\frac{1}{2}+i\nu\right) - \psi\left(\frac{1}{2}-i\nu\right)\right]$$
$$\times\left[\delta_{n0}\left(3 + \left(1+\frac{N_f}{N_c^3}\right)\frac{2+3\gamma(1-\gamma)}{(3-2\gamma)(1+2\gamma)}\right) - \delta_{|n|2}\left(\left(1+\frac{N_f}{N_c^3}\right)\frac{\gamma(1-\gamma)}{2(3-2\gamma)(1+2\gamma)}\right)\right]$$

$\Phi(n,\gamma)$ is a sum over linear combinations of ψ functions
and γ is a shorthand $\gamma = 1/2 + i\nu$

In blue we labeled the terms which occur only in QCD,
in red the ones which occur in QCD and in *N*=4 SYM

Fourier-Mellin transform

- At NLLA, the BFKL ladder is

$$f^{NLL}(q_1, q_2, \eta_{s_0}) = \frac{1}{2\pi\sqrt{q_1^2 q_2^2}} \sum_{k=1}^{\infty} \frac{\eta_{s_0}^k}{k!} f_{k+1}^{NLL}(z) \qquad \eta_{s_0} = \overline{\alpha}_S(s_0)\, y$$

with coefficients given by the Fourier-Mellin transform

$$f_k^{NLL}(z) = \mathcal{F}\left[\omega_{\nu n}^{(1)}\, \chi_{\nu n}^{k-2}\right] = \sum_{n=-\infty}^{+\infty} \left(\frac{z}{\bar{z}}\right)^{n/2} \int_{-\infty}^{+\infty} \frac{d\nu}{2\pi} |z|^{2i\nu}\, \omega_{\nu n}^{(1)}\, \chi_{\nu n}^{k-2} \qquad \chi_{\nu n} = \omega_{\nu n}^{(0)}$$

using the explicit form of the eigenvalue

$$\omega_{\nu n}^{(1)} = \frac{1}{4}\delta_{\nu n}^{(1)} + \frac{1}{4}\delta_{\nu n}^{(2)} + \frac{1}{4}\delta_{\nu n}^{(3)} + \gamma_K^{(2)}\chi_{\nu n} - \frac{1}{8}\beta_0 \chi_{\nu n}^2 + \frac{3}{2}\zeta_3$$

the coefficients can be written as

$$f_k^{NLL}(z) = \frac{1}{4} C_k^{(1)}(z) + \frac{1}{4} C_k^{(2)}(z) + \frac{1}{4} C_k^{(3)}(z) + \gamma_K^{(2)} f_{k-1}^{LL}(z) - \frac{1}{8}\beta_0 f_k^{LL}(z) + \frac{3}{2}\zeta_3 f_{k-2}^{LL}(z)$$

with $\quad C_k^{(i)}(z) = \mathcal{F}\left[\delta_{\nu n}^{(i)}\, \chi_{\nu n}^{k-2}\right]$

the weight of f^{NLL}_k is

weight(f^{NLL}_k)= $\quad k \qquad k \qquad 0 \le w \le k \qquad k-2 \le w \le k \qquad k-1 \qquad k$

SV functions

- $C_k^{(1)}(z)$ are SVHPLs of uniform weight k with singularities at $z=0$ and $z=1$

  $C_k^{(3)}(z)$ are MPLs of type $G(a_1,\ldots,a_n;|z|)$ with $a_k \in \{-i,0,i\}$

  they are SV functions of z because they have no branch cut on the positive real axis, and have weight $0 \leq w \leq k$

  For $C_k^{(2)}(z)$ one needs Schnetz' generalised SVMPLs with singularities at

  $$z = \frac{\alpha\,\bar{z}+\beta}{\gamma\,\bar{z}+\delta}\,, \qquad \alpha,\beta,\gamma,\delta \in \mathbb{C}$$

  Schnetz 2016

  then one can show that $C_k^{(2)}(z)$ are Schnetz' generalised SVMPLs

  $\mathcal{G}(a_1,\ldots,a_n;z)$ with singularities at $a_i \in \{-1,0,1,-1/\bar{z}\}$

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- In moment space, the maximal weight of the BFKL eigenvalue and of the anomalous dimensions of the leading twist operators which control the Bjorken scaling violations in QCD is the same as the corresponding quantities in N=4 SYM

  Kotikov Lipatov 2000, 2002
  Kotikov Lipatov Velizhanin 2003

  Interestingly, in transverse momentum space at NLLA, the maximal weight of the BFKL ladder in QCD is *not* the same as the one of the ladder in *N*=4 SYM

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BFKL ladder in a generic SU(N$_c$) gauge theory

- one can consider the BFKL eigenvalue at NLLA in a SU(N$_c$) gauge theory with scalar or fermionic matter in arbitrary representations

$$\omega^{(1)}_{\nu n} = \frac{1}{4}\delta^{(1)}_{\nu n} + \frac{1}{4}\delta^{(2)}_{\nu n} + \frac{1}{4}\delta^{(3)}_{\nu n}(\tilde N_f, \tilde N_s) + \frac{3}{2}\zeta_3 + \gamma^{(2)}(\tilde n_f, \tilde n_s)\,\chi_{\nu n} - \frac{1}{8}\beta_0(\tilde n_f, \tilde n_s)\,\chi^2_{\nu n}$$

Kotikov Lipatov 2000

with $\beta_0(\tilde n_f, \tilde n_s) = \frac{11}{3} - \frac{2\tilde n_f}{3N_c} - \frac{\tilde n_s}{6N_c}$ $\qquad \gamma^{(2)}(\tilde n_f, \tilde n_s) = \frac{1}{4}\left(\frac{64}{9} - \frac{10\,\tilde n_f}{9N_c} - \frac{4\,\tilde n_s}{9N_c}\right) - \frac{\zeta_2}{2}$

$$\tilde n_f = \sum_R n_f^R T_R \qquad \tilde n_s = \sum_R n_s^R T_R \qquad \mathrm{Tr}(T_R^a T_R^b) = T_R\,\delta^{ab} \qquad T_F = \frac{1}{2}$$

$\tilde n_s(\tilde n_f) =$ number of scalars (Weyl fermions) in the representation *R*

$$\delta^{(3)}_{\nu n}(\tilde N_f, \tilde N_s) = \delta^{(3,1)}_{\nu n}(\tilde N_f, \tilde N_s) + \delta^{(3,2)}_{\nu n}(\tilde N_f, \tilde N_s)$$

with $\tilde N_x = \frac{1}{2}\sum_R n_x^R\, T_R\,(2C_R - N_c)\,, \qquad x = f, s$

- Necessary and sufficient conditions for a SU(N$_c$) gauge theory to have a BFKL ladder of maximal weight are:

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— the one-loop beta function must vanish
— the two-loop cusp AD must be proportional to ζ_2
— $\delta^{(3,2)}_{\nu n}$ must vanish $\rightarrow$ $2\tilde N_f = N_c^2 + \tilde N_s$

⇒ There is no theory whose BFKL ladder has uniform maximal weight which agrees with the maximal weight terms of QCD

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Matter in the fundamental and in the adjoint

- We solve the conditions above for matter in the fundamental *F* and in the adjoint *A* representations. We obtain:

$$2\,n_f^F = n_s^F \qquad\qquad 2\,n_f^A = 2 + n_s^A$$

which describes the spectrum of a gauge theory with *N* supersymmetries and $n^F = n_f^F$ chiral multiplets in *F* and $n^A = n_f^A - N$ chiral multiplets in *A*

- There are four solutions to those conditions

$\mathcal{N}$	4	2	1	1
n_A	0	0	0	2
n_F	0	$4N_c$	$6N_c$	$2N_c$

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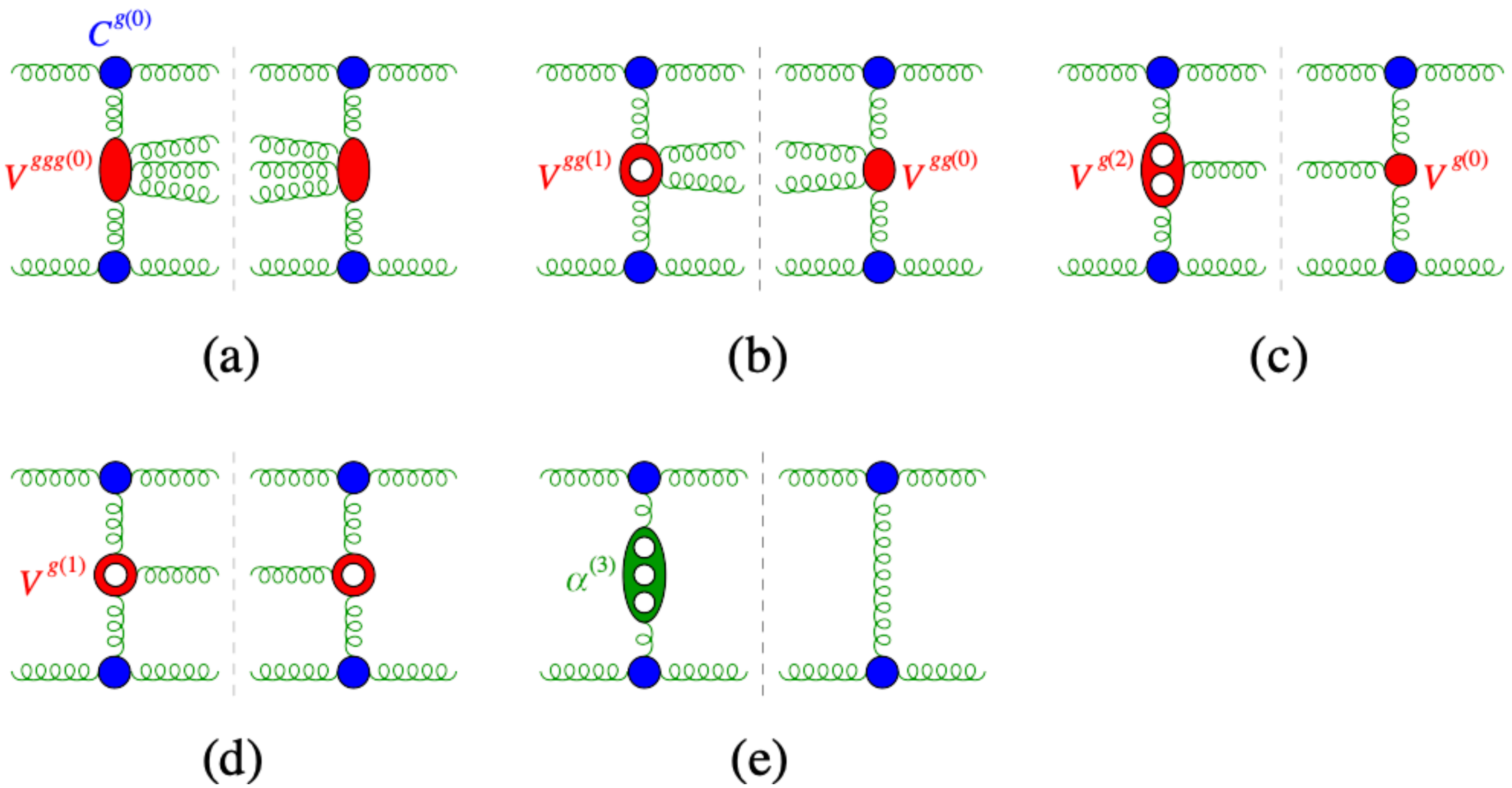
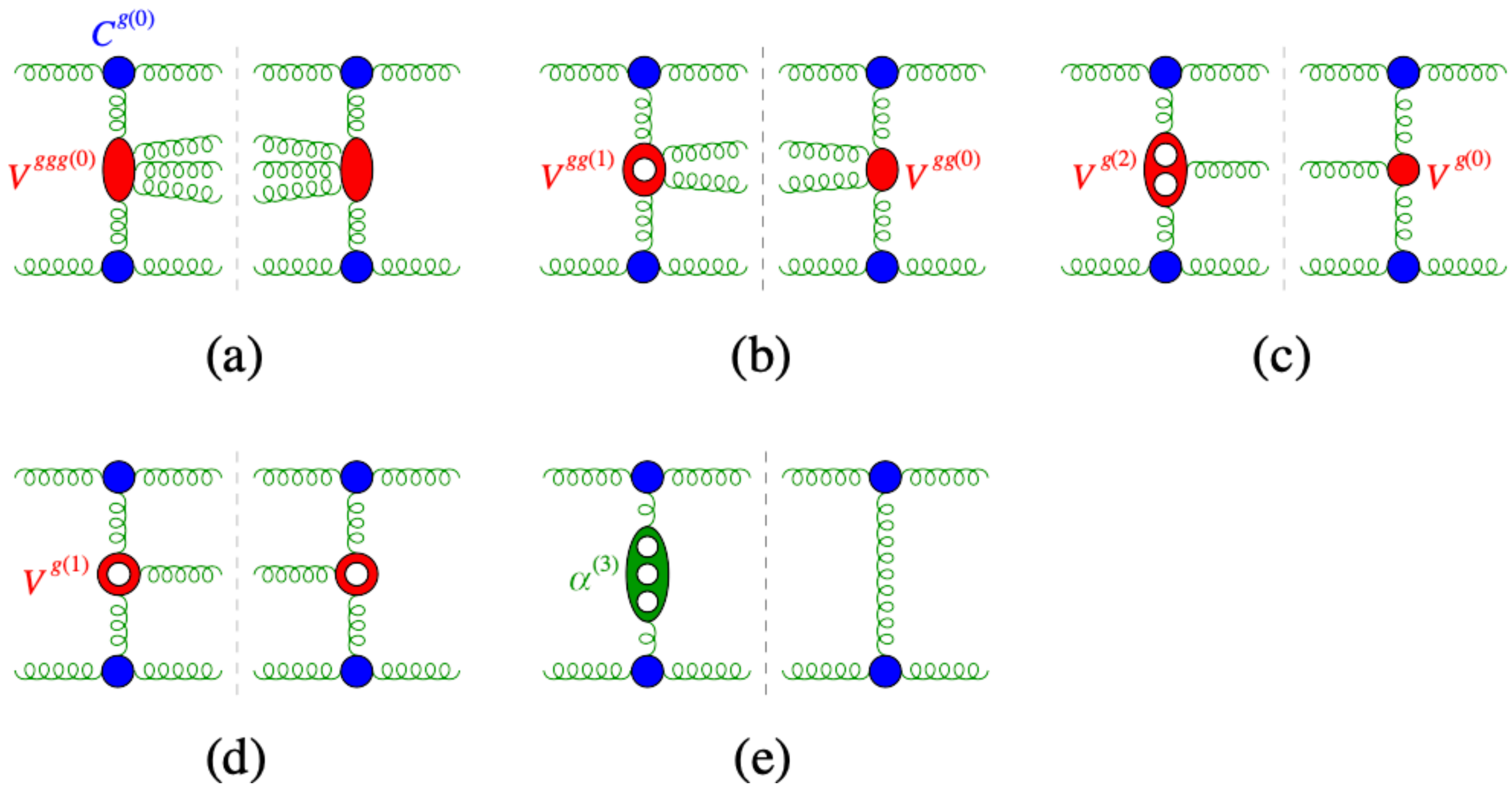
— the first is *N*=4 SYM
— the second is N=2 superconformal QCD with $N_f = 2N_c$ hypermultiplets
— the third is *N*=1 superconf. QCD

- because the one-loop beta function is fixed by matter loops in gluon self-energies, we are only sensitive to the matter content of a theory, and not to its details (like scalar potential or Yukawa couplings)

NNLO kernel



NNLO kernel



An Introduction to leading and next-to-leading BFKL

Gavin P. Salam (INFN, Milan) (Oct, 1999)

Published in: *Acta Phys.Polon.B* 30 (1999) 3679-3705 • Contribution to: 39th Cracow School of Theoretical Physics: Strong Interactions at Low and High Energies • e-Print: hep-ph/9910492 [he ph]

4. Beyond NLL

The solution is bound to lie with higher orders. Shortly after preliminary results on χ_1 had appeared, it was suggested that stable predictions might be obtained by inclusion of the NNLL and NNNLL terms [26]. But remembering that the LL calculation took about a year, and the NLL calculation ten years, a reasonable estimate for the time to calculate the NNLL terms might lie somewhere between an arithmetic (19 years) and a geometric (100 years) extrapolation. Even were these contributions to be calculated, there is actually no guarantee that the resulting series would converge for the values of $\bar{\alpha}_s$ of interest!

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One-loop central-emission vertex for two gluons in $\mathcal{N}=4$ super Yang-Mills theory

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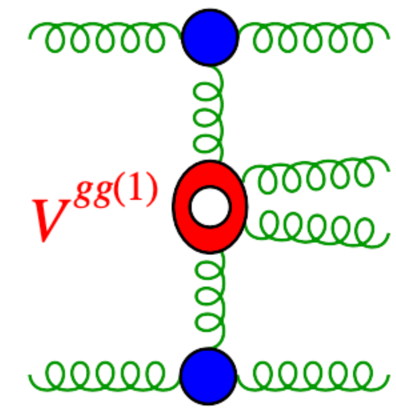
Abstract: A necessary ingredient for extending the BFKL equation to next-to-next-to-leading logarithmic (NNLL) accuracy is the one-loop central emission vertex (CEV) for two gluons which are not strongly ordered in rapidity. Here we consider the one-loop six-gluon amplitude in $\mathcal{N}=4$ super Yang-Mills (SYM) theory in a central next-to-multi-Regge kinematic (NMRK) limit, we show that its dispersive part factorises in terms of the two-gluon

JHEP08(2022)271



- we used Lance's one-loop 6-gluon amplitudes

- these days, I'm doing gravity amplitudes

- no collinear graviton divergences in the eikonal limit

Weinberg 1965

- no collinear graviton divergences for fixed-angle scattering

Akhoury Saotome Sterman 2011

- no collinear graviton divergences in forward scattering?

likely not, nonetheless
soft-collinear gravitons are important

ELSEVIER

Nuclear Physics B 546 (1999) 423–479

NUCLEAR PHYSICS B

Multi-leg one-loop gravity amplitudes from gauge theory

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Received 26 November 1998; accepted 8 January 1999

Abstract

By exploiting relations between gravity and gauge theories, we present two infinite sequences of one-loop n-graviton scattering amplitudes: the 'maximally helicity-violating' amplitudes in $N = 8$ supergravity, and the 'all-plus' helicity amplitudes in gravity with any minimally coupled massless matter content. The all-plus amplitudes correspond to self-dual field configurations and vanish in supersymmetric theories. We make use of the tree-level Kawai–Lewellen–Tye (KLT) relations between open and closed string theory amplitudes, which in the low-energy limit imply relations between gravity and gauge theory tree amplitudes. For $n \leqslant 6$, we determine the all-plus amplitudes explicitly from their unitarity cuts. The KLT relations, applied to the cuts, allow us to extend to gravity a previously found 'dimension-shifting' relation between (the cuts of) the all-plus amplitudes in gauge theory and the maximally helicity-violating amplitudes in $N = 4$ super-Yang–Mills theory. The gravitational version of the relation lets us determine the $n \leqslant 6$ $N = 8$ supergravity amplitudes from the all-plus gravity amplitudes. We infer the two series of amplitudes for all n from their soft and collinear properties, which can also be derived from gauge theory using the KLT relations.

Amarcord

Academy Award-winning movie by F. Fellini

A m'arcord = I remember (*romagnolo* dialect)

Amarcord

Academy Award-winning movie by F. Fellini

A m’arcord = I remember (*romagnolo* dialect)

- in the 1990’s

  no mobile phones

  no digital cameras

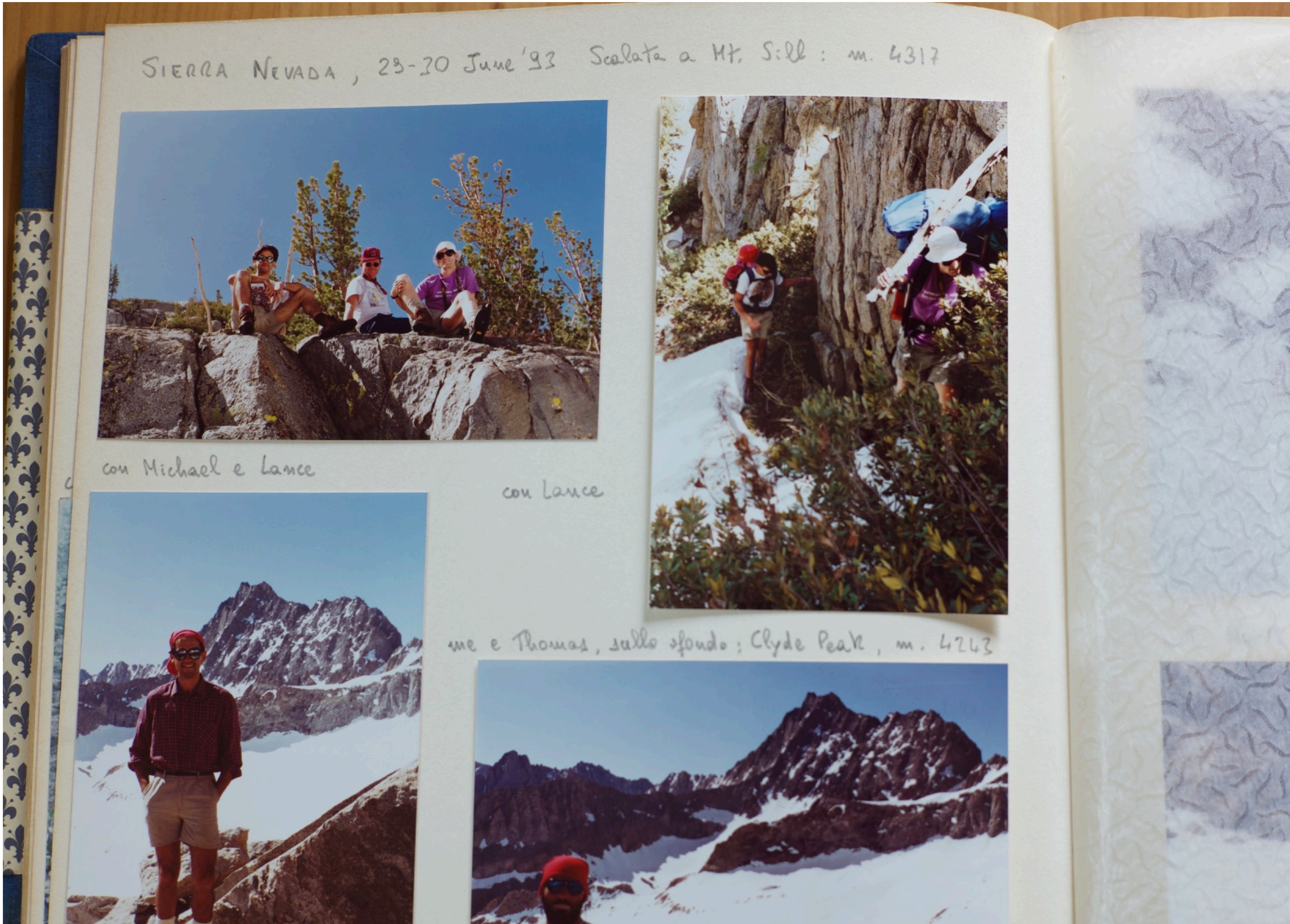
Sierra Nevada The Minarets 4 July 1992

Tomas

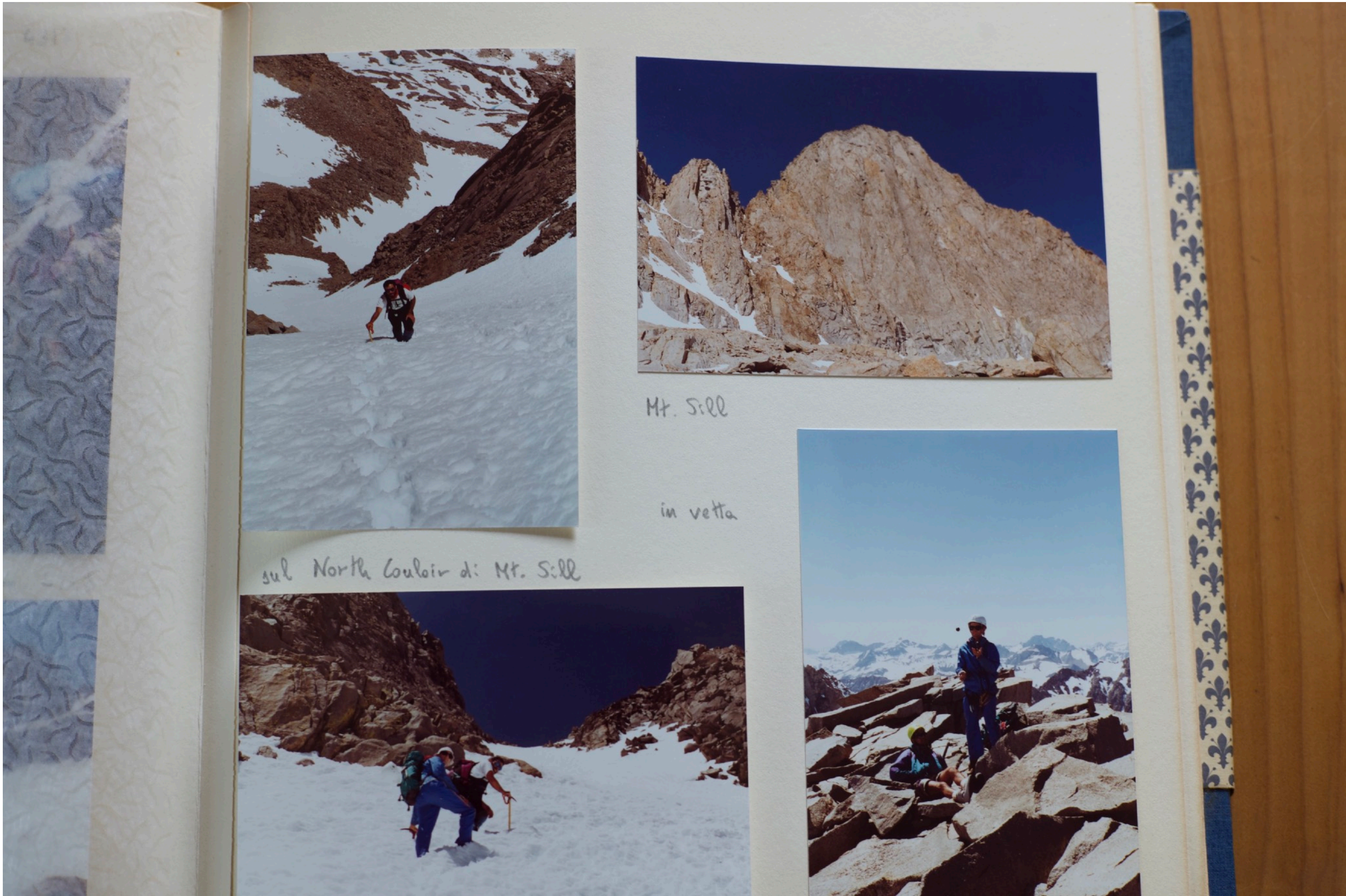
con Lance
e David

Sierra Nevada Clyde Minaret 4 July 1992

Sierra Nevada Mt Sill 29-30 June 1993



Sierra Nevada Mt Sill 29-30 June 1993



Sierra Nevada Mt Sill 29-30 June 1993

MIAPP September 2017

It has been a privilege to know Lance

thanks Mayling & Lance for a great friendship

Happy birthday Lance

Happy birthday Lance

in Italian we say: cento di questi giorni!
(we wish you 100 more (birth)days!)