# Hadrons at $O(\alpha_s{}^0)$ in QCD

ECT* Seminar

Trento 3 May 2013

Paul Hoyer
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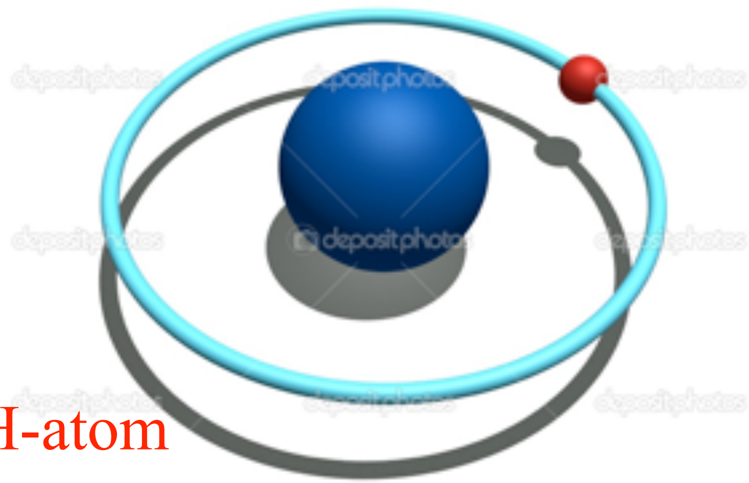
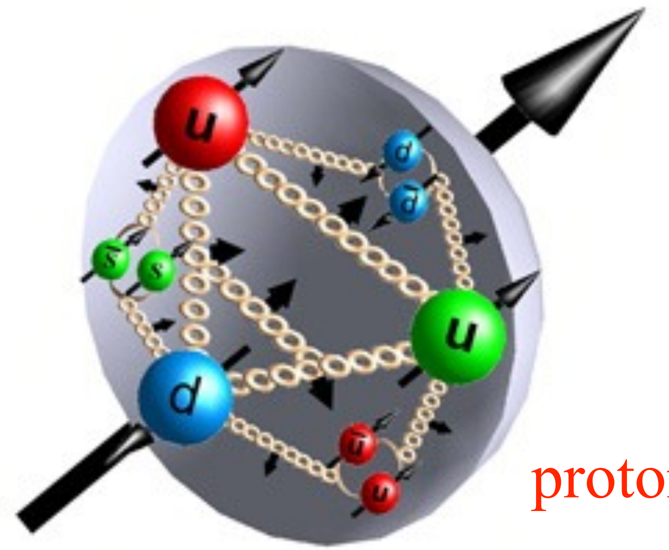



H-atom

Bound states




proton

QED

Non-relativistic

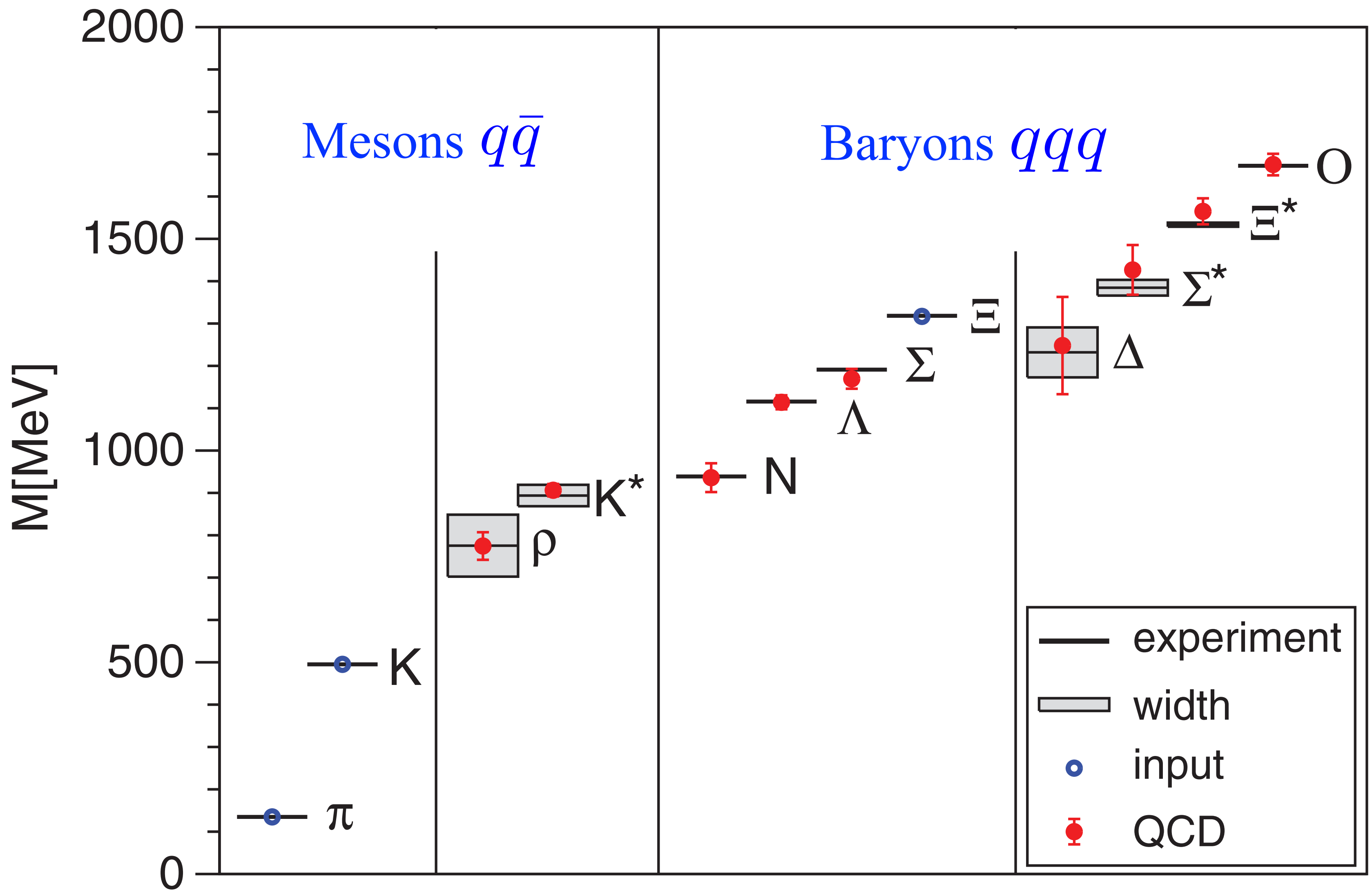
QCD

Relativistic

Work done with

Dennis Dietrich and Matti Järvinen 1212.4747

# Hadron spectrum from QCD Lattice

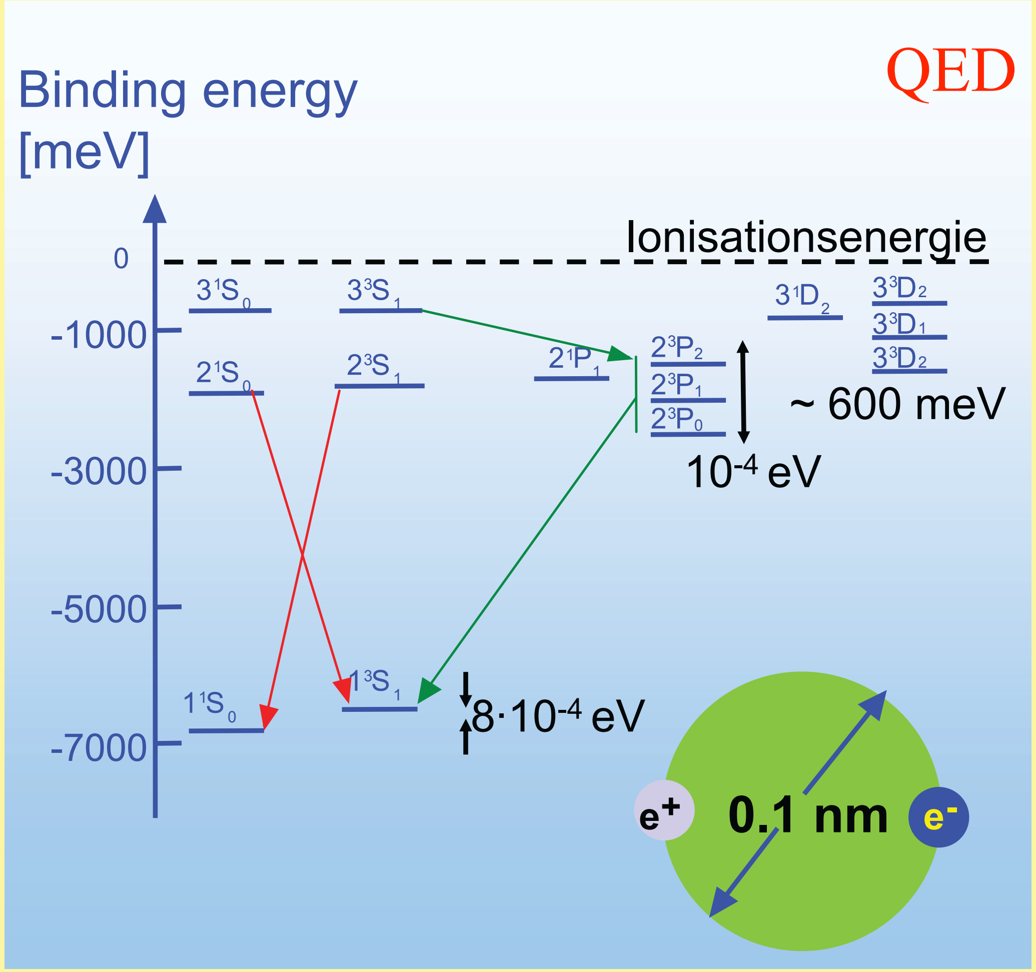


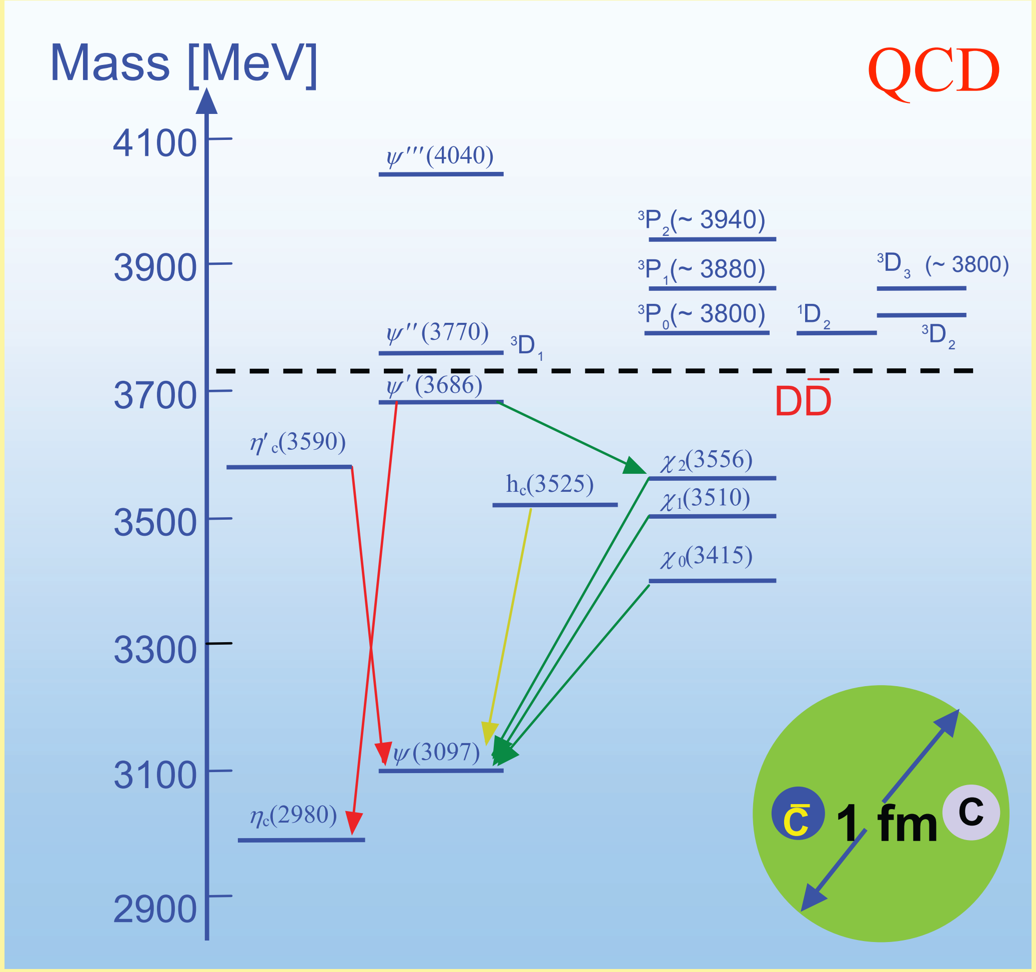

Z. Fodor, BMW Collab. **PTEP 2012 (2012) 01A108**

**Fig. 2.** The light hadron spectrum of QCD. Horizontal lines and bands are the experimental values with their decay widths. Our results are shown by solid circles. Vertical error bars represent our combined statistical and systematic error estimates. $\pi$, $K$, and $\Xi$ have no error bars, because they are used to set the light quark mass, the strange quark mass, and the overall scale, respectively.

# Similarity of atomic and hadronic spectra

- Positronium
- Charmonium




$$V(r) = -\frac{\alpha}{r}$$

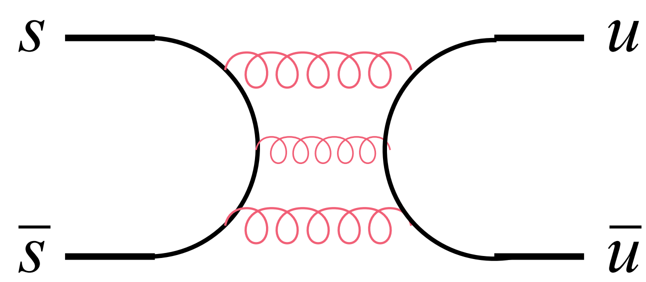
$$V(r) = c\,r - \frac{4}{3}\frac{\alpha_s}{r}$$

# PQCD even in soft phenomena?

$$\phi(1020) \to K\bar{K} \quad 83\% \qquad m(\phi) - 2\, m(K) = 30 \text{ MeV}$$

$$\phi(1020) \to \pi\pi\pi \quad 15\% \qquad m(\phi) - 3\, m(\pi) = 610 \text{ MeV}$$

$\Rightarrow$ OZI suppression of:




Are soft gluons weakly coupled ?

Single gluon exchange explains the spin dependence of baryons, in particular the Λ - Σ mass difference.

A.De Rujula, H. Georgi, S. L. Glashow, PRD 12 (1975) 147

Quarkonium models:

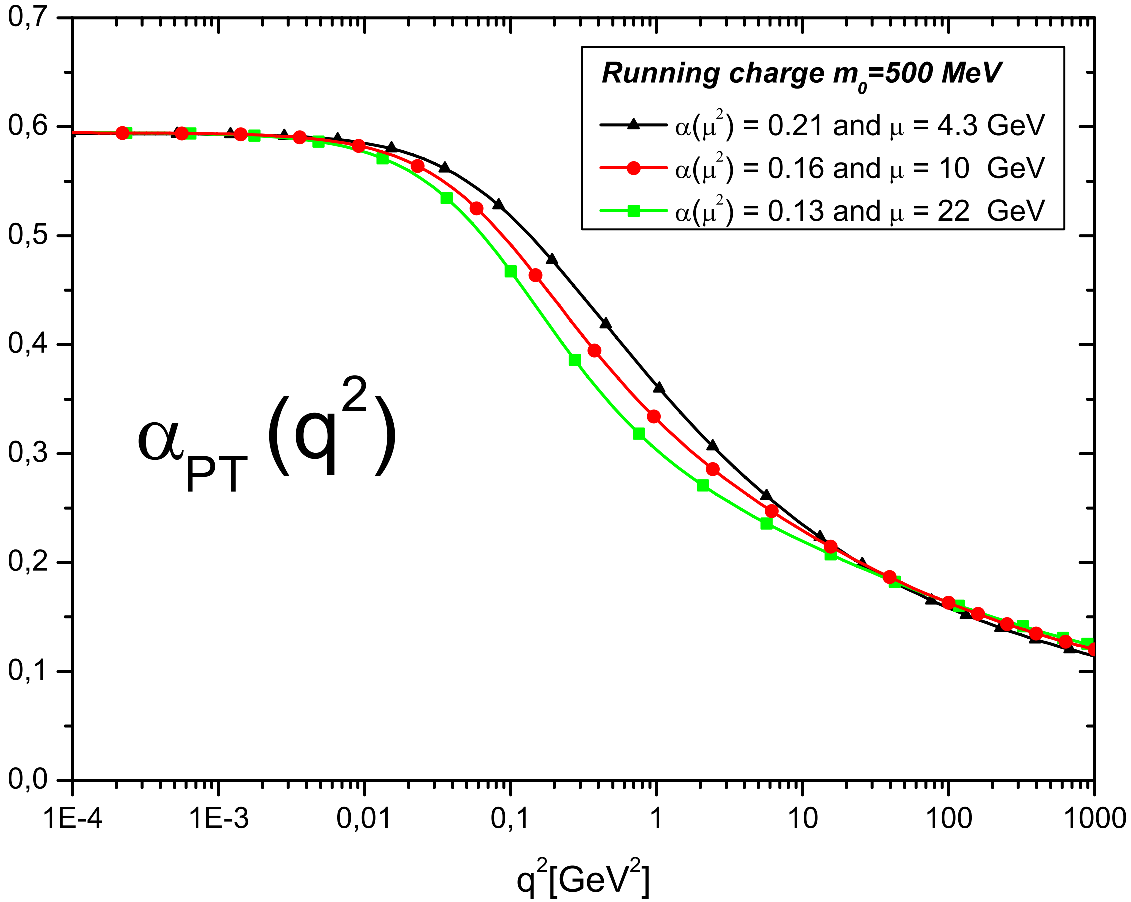
$$V(r) = c\, r - \frac{4}{3}\frac{\alpha_s}{r}$$

$\mathcal{O}\left(\alpha_s{}^0\right)$ $\qquad$ $\mathcal{O}\left(\alpha_s\right)$

Is it conceivable that $\alpha_s(0)$ is small enough to make PQCD meaningful?

# Freezing of $\alpha_s$ in the infrared

0.6




Pinch Technique

A. C. Aguilar, D. Binosi,
J. Papavassiliou,
J. Rodriguez-Quintero,
PRD 80 (2009) 085018

Event shapes

$$\frac{1}{\mu_I}\int_0^{\mu_I} \mathrm{d}Q\,\alpha_{\mathrm{eff}}(Q^2) = \alpha_0(\mu_I)$$

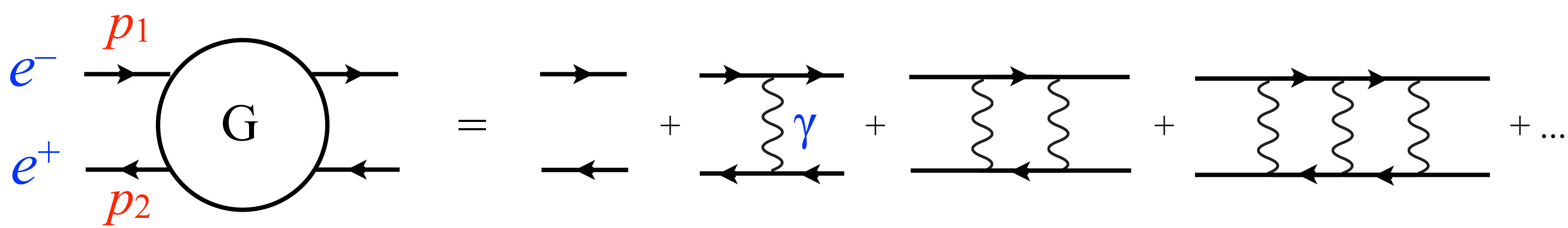
$$\alpha_0(2\ \mathrm{GeV}) = 0.5132 \pm 0.0115(\mathrm{exp}) \pm 0.0381(\mathrm{th})$$

Yu.L. Dokshitzer, G. Marchesini,
G.P. Salam, EPJdirect **C3** (1999) 1

T. Gehrmann, M. Jaquier,
G. Luisoni,
Eur. Phys. J. C **67** (2010) 57

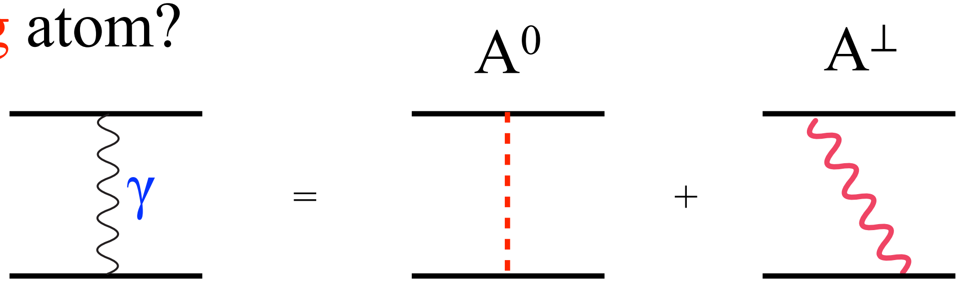
# Atoms from QED: (Should be) Textbook stuff

Bound state poles do not appear in any single Feynman diagram
– they are generated by the divergence of the perturbative sum



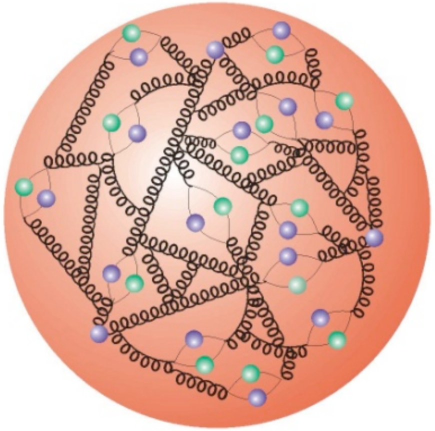

$$= \frac{R}{(p_1+p_2)^2 - M^2} + \ldots$$

- Which diagrams should be included in the infinite sum?
- How come the QED series diverges for arbitrarily small α ?
- What is the wave function of moving atom?



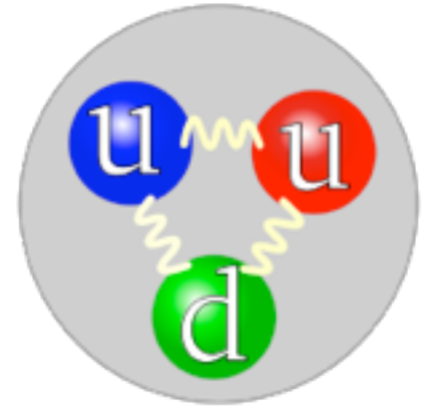

M. Järvinen, arXiv:hep-ph/0411208

# Dichotomy of Proton structure



## Parton Picture

DIS and QFT require an infinite # of constituents




## Quark Model

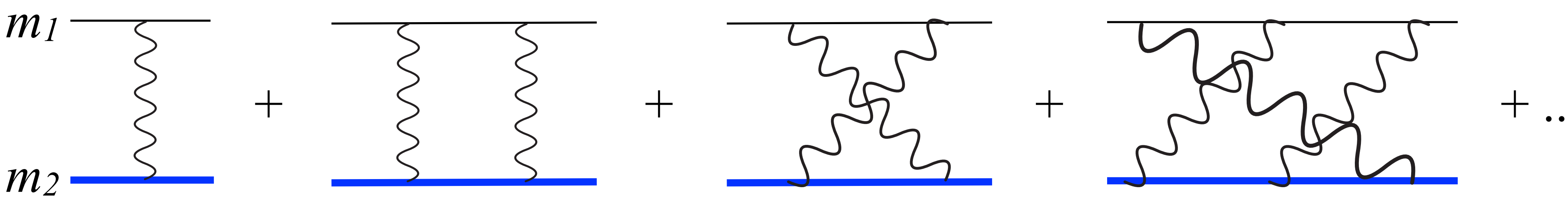
The hadron spectrum reflects only $q\bar{q}$ or $qqq$ degrees of freedom

| |
|---|
| How can QCD combine multiparton, relativistic Fock states with a valence quark spectrum? |

$\Rightarrow$ Consider the Dirac equation

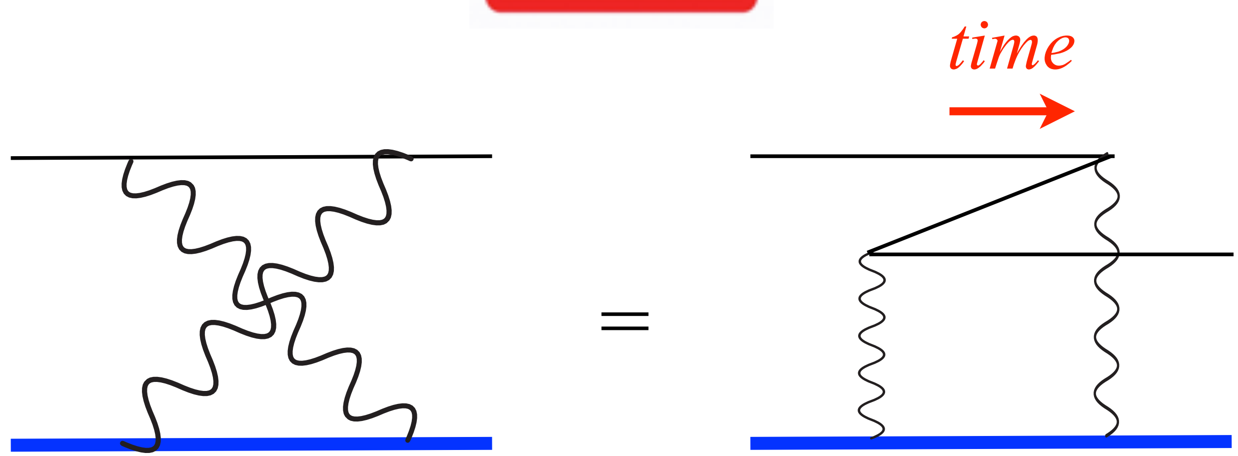
# The Dirac equation from Feynman diagrams

As $m_2 \to \infty$ the Dirac equation for particle 1 emerges from the sum of all uncrossed + crossed ladder diagrams:




Note: The kernel of a Bethe-Salpeter equation would have to be of infinite order!

Since Coulomb exchange is instantaneous, crossed diagrams correspond to intermediate states with particle pairs.




$\Rightarrow$ Dirac bound states have an infinite number of pairs, but the spectrum reflects a single particle dof.

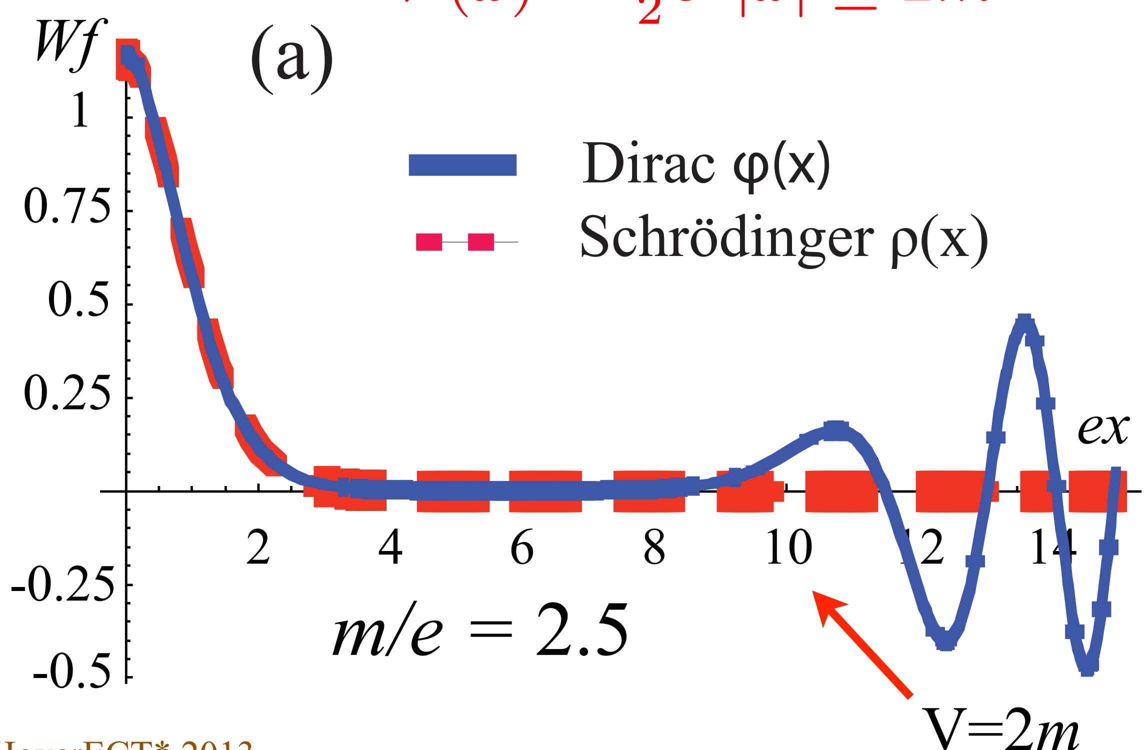
# Features of the Dirac wf in D=1+1

In D=1+1 the Dirac matrices may be represented as 2x2 Pauli matrices:

$$\left[-i\sigma_1\partial_x + \tfrac{1}{2}e^2|x| + m\sigma_3\right]\left[\begin{array}{c}\varphi(x)\\ \chi(x)\end{array}\right] = M\left[\begin{array}{c}\varphi(x)\\ \chi(x)\end{array}\right]$$

The wf's φ($x$), χ($x$) are given by ${}_1F_1$-functions. For large $m$, they approach the Schrödinger wf's when V($x$) << $m$.

Pair contributions are manifest for $V(x) = \frac{1}{2}e^2|x| \geq 2m$




We were not taught that:

For polynomial potentials the Dirac wave function is not normalizable, and the mass spectrum $M$ is continuous.

Its normalizability for the V($r$) = 1/$r$ potential in D=3+1 is an exception.

Plesset, Phys. Rev. **41** (1932) 278

AUGUST 1, 1932 PHYSICAL REVIEW VOLUME 41

# The Dirac Electron in Simple Fields*

*By* MILTON S. PLESSET
*Sloane Physics Laboratory, Yale University*

(Received June 6, 1932)

The relativity wave equations for the Dirac electron are transformed in a simple manner into a symmetric canonical form. This canonical form makes readily possible the investigation of the characteristics of the solutions of these relativity equations for simple potential fields. If the potential is a polynomial of any degree in $x$, a continuous energy spectrum characterizes the solutions. If the potential is a polynomial of any degree in $1/x$, the solutions possess a continuous energy spectrum when the energy is numerically greater than the rest-energy of the electron; values of the energy numerically less than the rest-energy are barred. When the potential is a polynomial of any degree in $r$, all values of the energy are allowed. For potentials which are polynomials in $1/r$ of degree higher than the first, the energy spectrum is again continuous. The quantization arising for the Coulomb potential is an exceptional case.

See also: E. C. Titchmarsh, Proc. London Math. Soc. (3) 11 (1961) 159 and 169; Quart. J. Math. Oxford (2), 12 (1961), 227.

# Oscillations as $x \to \infty$ : The Klein-Gordon case



$$\left[(i\partial_\mu - eA_\mu)(i\partial^\mu - eA^\mu) - m^2\right]\varphi(x)e^{-iMt} = 0$$

$$eA^0(x) \equiv V(x) = \tfrac{1}{2}e^2|x| \qquad A^1 = 0$$

$$\left[M^2 - m^2 - 2V(x)M + V^2(x)\right]\varphi(x) + \varphi''(x) = 0$$

$$x \to \infty : \qquad \varphi(x) \propto \exp(\pm i e^2 x^2/4)$$

NR reduction: $V(x) \ll m$ and $M = m + \varepsilon$

$$\left[2m\varepsilon - 2mV(x) + \partial_x^2\right]\varphi_{NR}(x) = 0$$

$\Rightarrow$ Normalizable (Airy function) solution of the Schrödinger equation $\quad \varphi(x) \sim \exp\left(-\dfrac{2e}{3}m^{1/2}x^{3/2}\right)$

# Hamiltonian approach to the Dirac equation

Consider the QED Hamiltonian with a fixed external field $A^0(\boldsymbol{x})$

$$H(t) = \int d^3\boldsymbol{x}\,\bar{\psi}(t,\boldsymbol{x})\left[-i\boldsymbol{\nabla}\cdot\boldsymbol{\gamma} + m + e\gamma^0 A^0(\boldsymbol{x})\right]\psi(t,\boldsymbol{x})$$

In terms of its vacuum eigenstate $H|0\rangle_A = 0$

field operator

construct the Dirac state $|M,t\rangle \equiv \int d^3\boldsymbol{x}\psi^\dagger_\alpha(t,\boldsymbol{x})\varphi_\alpha(\boldsymbol{x})|0\rangle_A$

c-numbered spinor

This is an eigenstate of $H$,

$$H|M,t\rangle = \left[H, \int d^3\boldsymbol{x}\psi^\dagger(t,\boldsymbol{x})\right]\varphi(\boldsymbol{x})|0\rangle_A = M|M,t\rangle$$

provided $\varphi(\boldsymbol{x})$ satisfies the Dirac equation:

$$\left[-i\boldsymbol{\nabla}\cdot\gamma^0\boldsymbol{\gamma} + m\gamma^0 + eA^0(\boldsymbol{x})\right]\varphi(\boldsymbol{x}) = M\varphi(\boldsymbol{x})$$

# Field theory: QED in D=1+1

Action of $QED_2$ in $A^1 = 0$ gauge:

$$S = \int d^2x \Big[ -\tfrac{1}{2}\big(\partial_1 A^0\big)\big(\partial^1 A^0\big) + \psi^\dagger(x)\gamma^0\big(i\partial\!\!\!/ - m - e\gamma^0 A^0\big)\psi(x)\Big]$$

Equation of motion for $A^0$ (Gauss' law): $-\partial_1^2 A^0(x) = e\psi^\dagger\psi(x)$

allows to express $A^0$ in terms of the fermion field:

$$A^0(x) = -\frac{e}{2}\int dy^1 |x^1 - y^1| \psi^\dagger\psi(x^0, y^1)$$

Eliminating $A^0$ in the $QED_2$ action gives

$$S = \int d^2x\, \psi^\dagger(x)\gamma^0\big(i\partial\!\!\!/ - m\big)\psi(x) + \frac{e^2}{4}\int d^2x\, d^2y\, \delta(x^0 - y^0)\psi^\dagger\psi(x)|x^1 - y^1|\psi^\dagger\psi(y)$$

From this we may determine the Poincare generators of $QED_2$ :

# Poincaré generators of QED in D=1+1

$$P^\mu(x^0) = \int dx^1 \mathcal{P}^\mu(x^0, x^1)$$

Generators of time (μ=0) and space (μ=1) translations

$$M^{01}(x^0) = \int dx^1 \mathcal{M}^{01}(x^0, x^1)$$

Boost generator

$$\mathcal{P}^0 = \bar{\psi}\big(-\tfrac{1}{2}i\gamma^1 \overleftrightarrow{\partial}_1 + m\big)\psi - \frac{e^2}{4}\int dy^1\ \psi^\dagger\psi(x^0, x^1)|x^1 - y^1|\psi^\dagger\psi(x^0, y^1)$$

$$\mathcal{P}^1 = \bar{\psi}\big(-\tfrac{1}{2}i\gamma^0 \overleftrightarrow{\partial}_1\big)\psi$$

The boost density has the expected form: $\mathcal{M}^{01} = x^0\mathcal{P}^1 - x^1\mathcal{P}^0$

The Lie algebra is satisfied (only for the linear potential): $\big[P^0, P^1\big] = 0$

$$\big[P^0, M^{01}\big] = iP^1$$

$$\big[P^1, M^{01}\big] = iP^0$$

# $f\,\bar{f}$ bound states in D=1+1

A state with two fermions of energy $E$ and 1-momentum $P^1 = P$ :

$$|E,P\rangle = \int dx_1 dx_2\, \bar\psi(t,x_1)\exp\left[\tfrac{1}{2}iP(x_1+x_2)\right]\Phi(x_1-x_2)\psi(t,x_2)|0\rangle$$

In analogy to the Dirac case take $\hat{P}^0|0\rangle \equiv H|0\rangle = 0$

This is a crucial approximation which allows a simple bound state solution.

Later I shall motivate it as being correct at $O(e)$, whereas perturbative pair production is of $O(e^2)$.

It is now easy to check that $\hat{P}^1|E,P\rangle = P|E,P\rangle$

Bound state has momentum $P$

Stationarity in time $\hat{P}^0|E,P\rangle = E|E,P\rangle$

defines the bound state equation for $\Phi(x_1 - x_2)$. With $x \equiv x_1 - x_2$ it reads:

$$i\partial_x \{\sigma_1, \Phi(x)\} + \left[-\tfrac{1}{2}P\sigma_1 + m\sigma_3, \Phi(x)\right] = \left[E - V(x)\right]\Phi(x)$$

where $V(x) = \frac{1}{2}e^2|x|$ and $\gamma^0 = \sigma_3, \quad \gamma^1 = i\sigma_2, \quad \gamma^0\gamma^1 = \sigma_1$

Here the CM momentum $P$ is a parameter, thus $E$ and $\Phi$ depend on $P$ .

It is a welcome surprise that the state is covariant under boosts:

$$|E + d\xi P, P + d\xi E\rangle = (1 - id\xi \hat{M}^{01})|E, P\rangle$$

This holds only for a linear potential and ensures that $E(P) = \sqrt{P^2 + M^2}$

The $P$-dependence of the wave function $\Phi$ can be expressed as:

$$\Phi^P(s) = e^{\sigma_1\zeta/2}\Phi^{(P=0)}(s)e^{-\sigma_1\zeta/2}$$

where $\quad dx = 2\,\dfrac{ds}{E - V(x)} \quad$ and $\quad \tanh\zeta = -\dfrac{P}{E - V}$

# Solutions of the bound state equation (D=1+1, $m_1=m_2$)

The "invariant length" can be expressed as $s = \dfrac{\varepsilon(s)}{2e^2}(M^2 - \Pi^2)$

where the "kinetic 2-momentum" is $\Pi(x) \equiv (E - V(x), P)$

and thus $\Pi^2 \equiv \sigma \equiv (E-V)^2 - P^2 = M^2 - 2EV + V^2$

Expanding the 2x2 wave function as $\Phi = \Phi_0 + \sigma_1\Phi_1 + \sigma_2\Phi_2 + \sigma_3\Phi_3$ the bound state equation reduces to two coupled, frame-independent equations:

$$-2i\partial_\sigma \Phi_1(\sigma) = \Phi_0(\sigma) \qquad -2i\partial_\sigma \Phi_0(\sigma) = \left[1 - \frac{4m^2}{\sigma}\right]\Phi_1(\sigma)$$

with the general solution

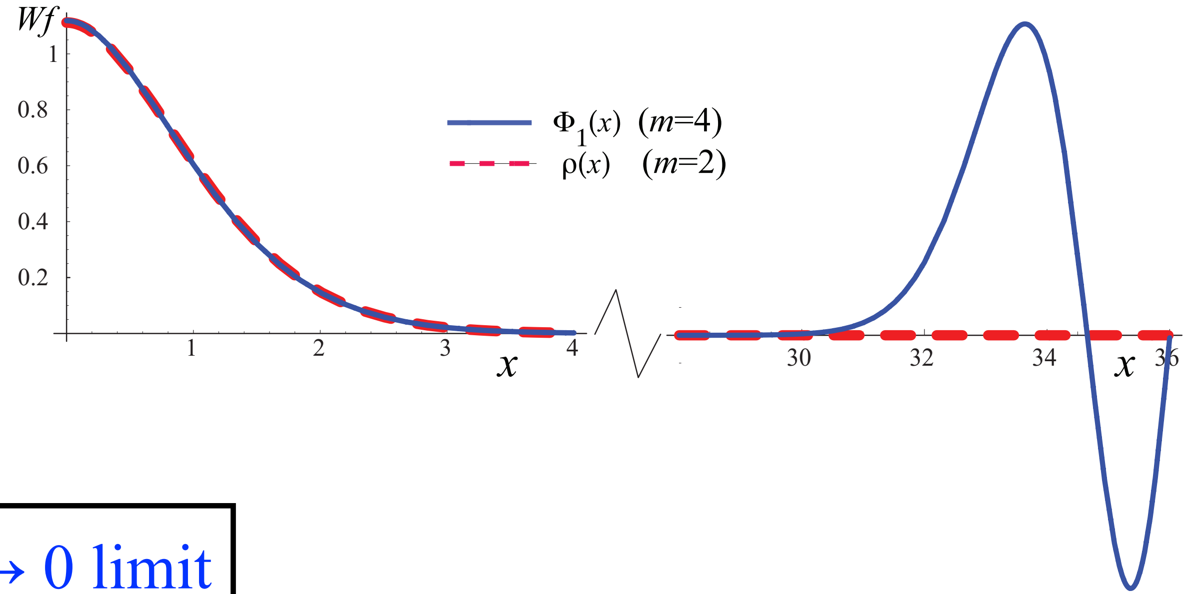
$$\Phi_1(\sigma) = \sigma\, e^{-i\sigma/2}\left[a\,{}_1F_1(1-im^2, 2, i\sigma) + b\,U(1-im^2, 2, i\sigma)\right]$$

If $b \neq 0$ the full wf $\Phi$ is singular at $\sigma = 0$. Requiring $b = 0$ the spectrum is discrete.

*C.f.* the Dirac equation: All solutions are regular, hence the continuous spectrum.

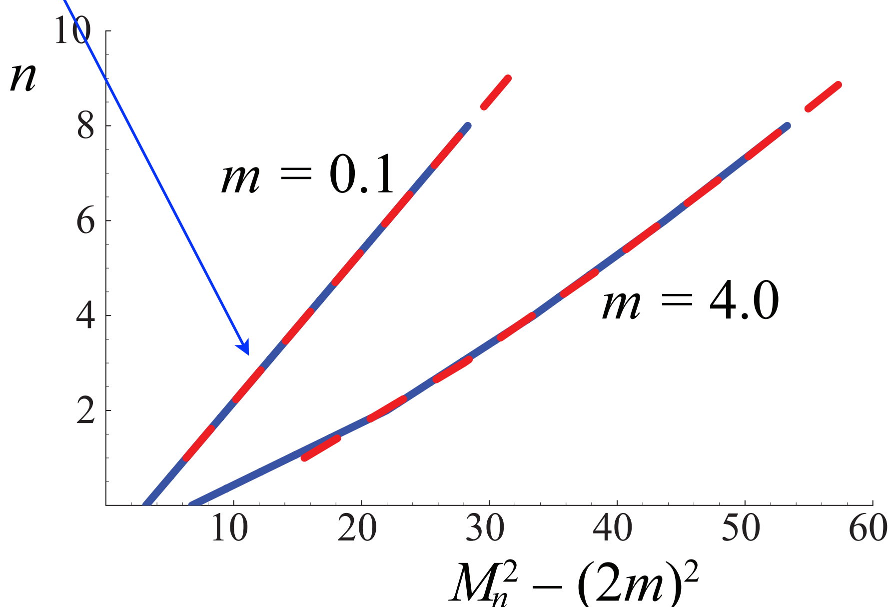
# Properties of the bound state solutions (D=1+1, $m_1=m_2$)

ffbar wf $\Phi_1(x)$ in nearly NR case, cf. Schr. wf. $\rho(x)$.

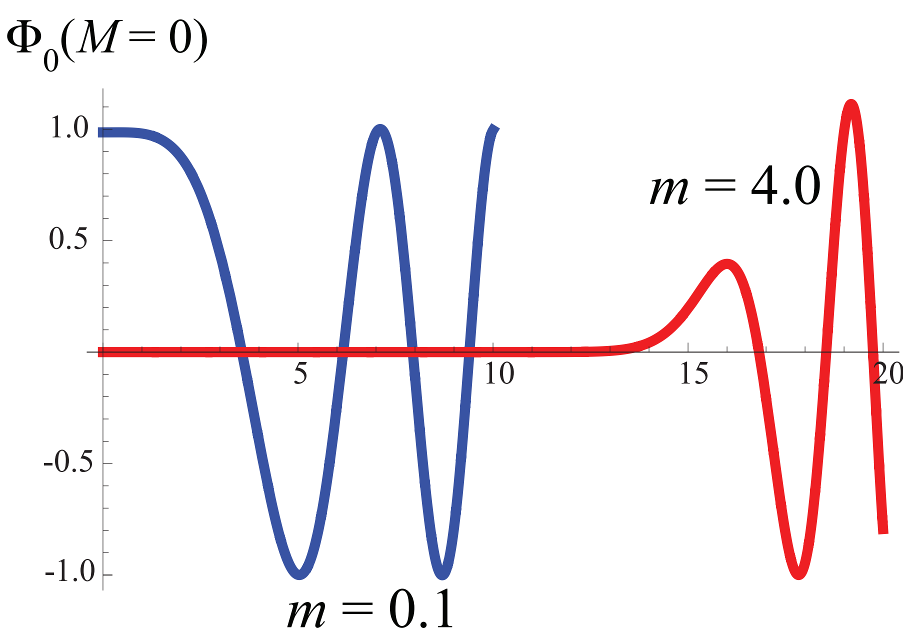



**No parity degeneracy in $m \to 0$ limit**

Mass spectra for $m = 0.1, 4.0$



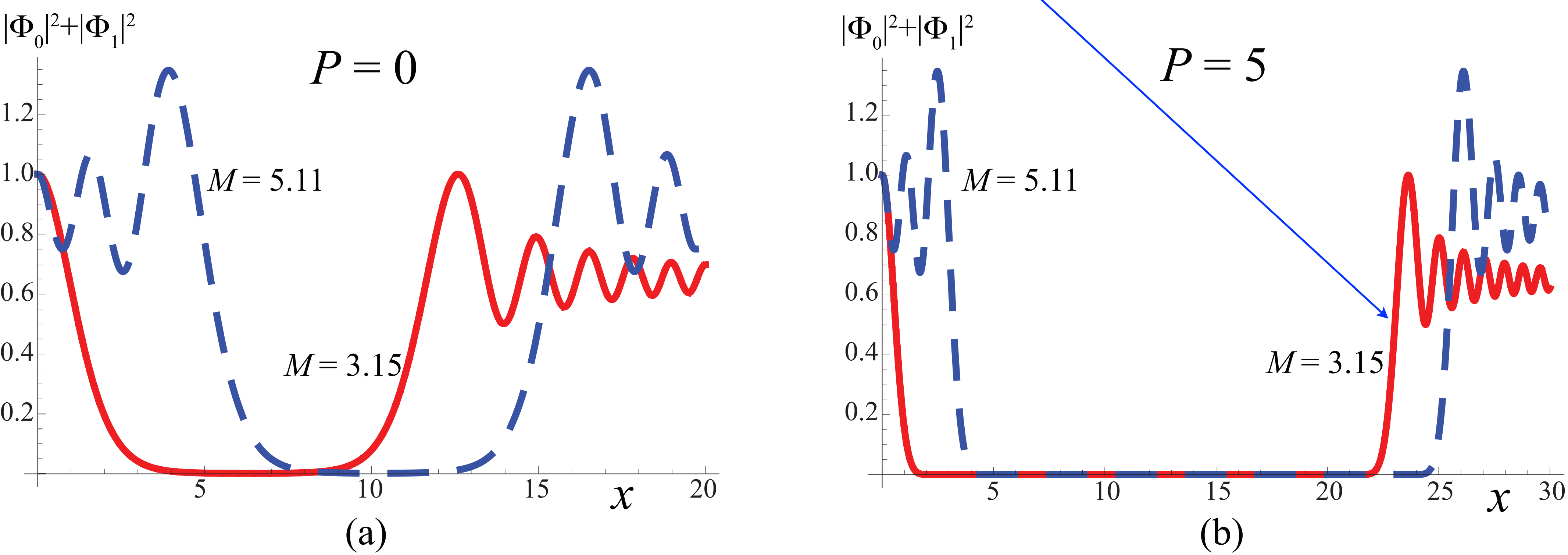

Wfs. for $M$=0 solutions



# Solutions of the bound state equation (D=1+1, $m_1 \neq m_2$)

Comparisons of ground and excited state wave functions in the CM and in a moving frame.

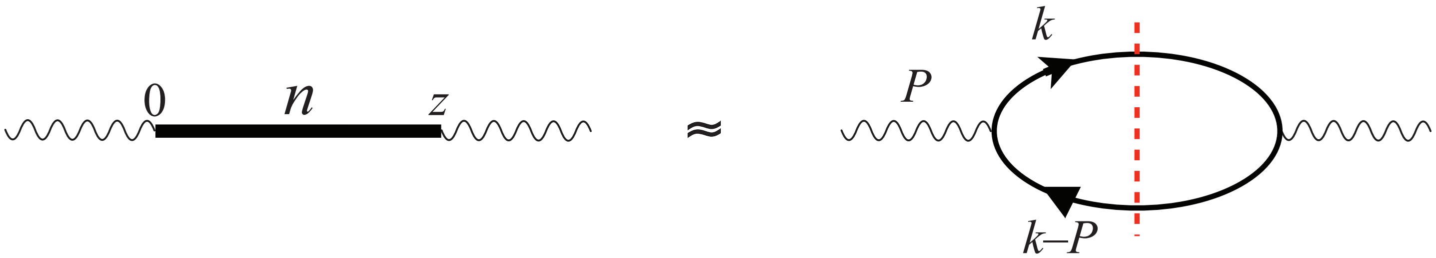
Moves away in IMF ($P \to \infty$ limit)




$m_1=1.0$ $m_2=1.5$

# Quark - Hadron duality (D=1+1, $m_1=m_2$)

The wave functions of highly excited bound states can be normalized by comparison with free parton loop contributions to current propagators. All currents give consistent results.




$$\Rightarrow \quad |\Phi_0(x=0)|^2 = |\Phi_1(x=0)|^2 = \pi/2$$

Consistency with the parton model: At large $M$, and for separations $x$ such that $V(x) << M$, the Fock states reduce to an $f\bar{f}$ pair with positive energy and momenta $k = \pm M/2$ (in the CM).

# Electromagnetic form factors

Taking the bound states as external states we may define as usual

$$F^{\mu}_{AB}(x) = \langle B(P_b), out|j^{\mu}(x)\,|A(P_a), in\rangle = e^{i(P_b-P_a)\cdot x}\langle B(P_b), out|j^{\mu}(0)\,|A(P_a), in\rangle$$

where *in* (*out*) implies $t = -\infty$ ($t = +\infty$).

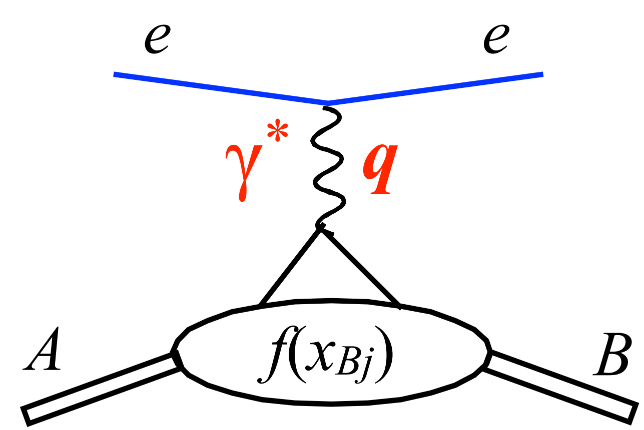
Using the BSE we may verify gauge invariance:

$$\partial_\mu F^{\mu}_{AB}(x) = 0$$

# Parton distributions (D=1+1, $m_1=m_2$)

Consider DIS in the Bj limit through transition form factor for $\gamma^* + A \to B$

$$M_b^2 = Q^2\left(\frac{1}{x_{Bj}} - 1\right) \to \infty$$




Hence can use asymptotic expression for $\Phi_B$

Parton distribution:

$$f(x_{bj}) = \frac{1}{8\pi m^2}\frac{1}{x_{bj}}|Q^2 F_{AB}(Q^2)|^2$$

$$\lim_{Q^2\to\infty} Q^2 F_{AB} =$$

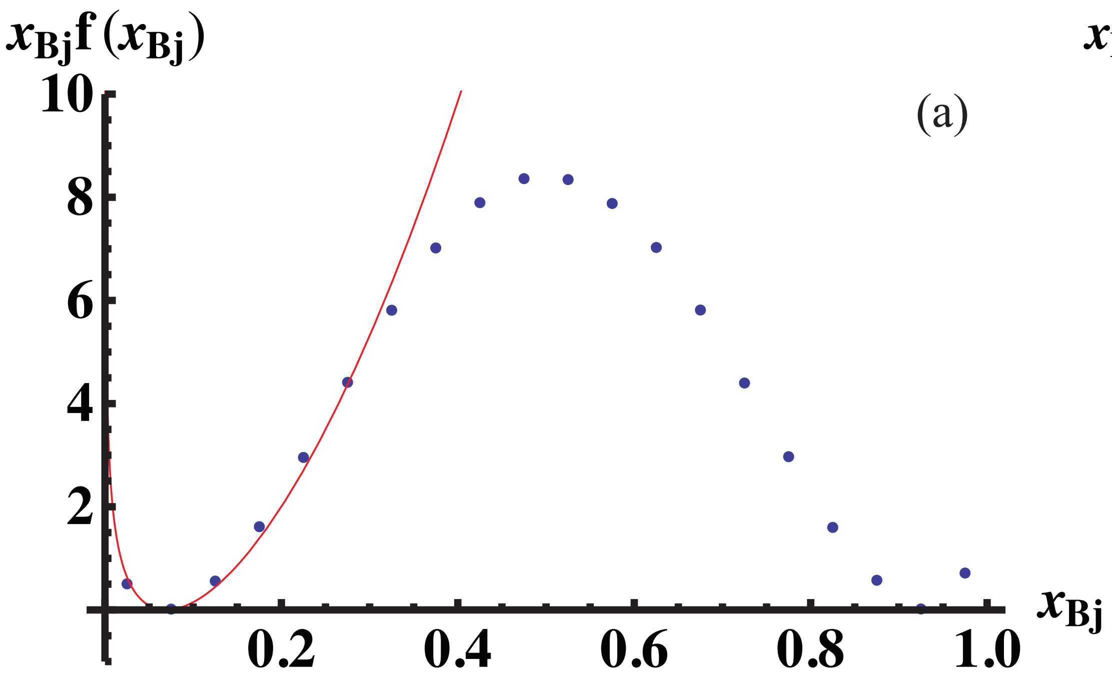
$$-8i\sqrt{2\pi}\int_0^\infty dv\,\sin v\Big[\cos\left(\frac{v}{2x_{Bj}}\right)i\Phi_{0A}(\sigma_a) - \sin\left(\frac{v}{2x_{Bj}}\right)\Phi_{1A}(\sigma_a)\left(1+\frac{2m^2}{x_{Bj}\sigma_a}\right)\Big]$$

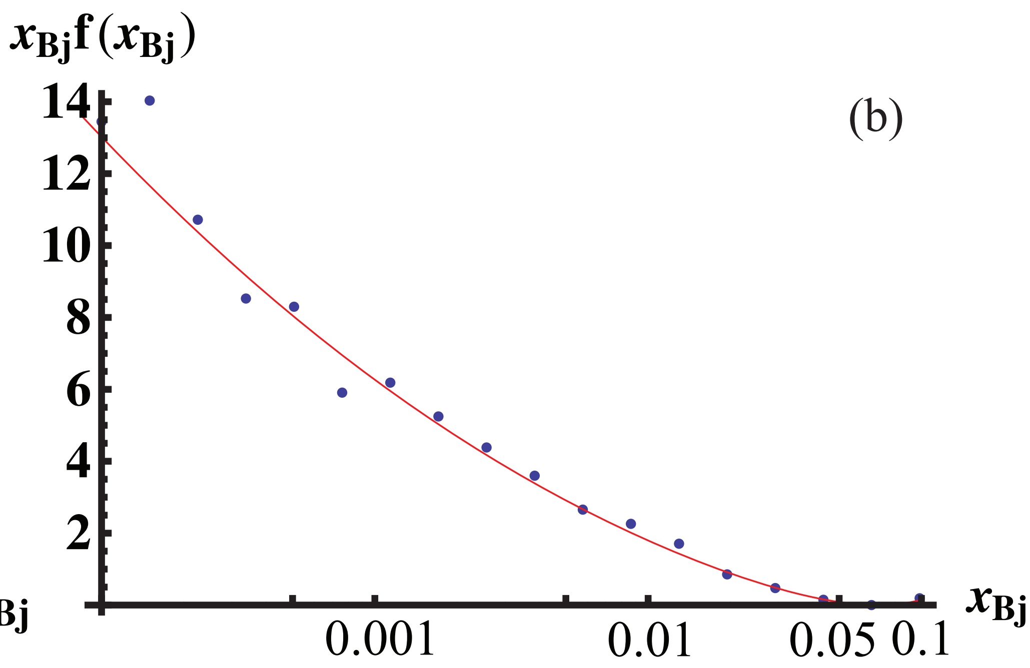
where $\sigma_a = M_a^2 - \dfrac{v}{x_{Bj}}$

# Numerical result for the parton distribution

The parton distribution of the ground state has a sea component at low *m/e* :

$$m/e = 0.1$$



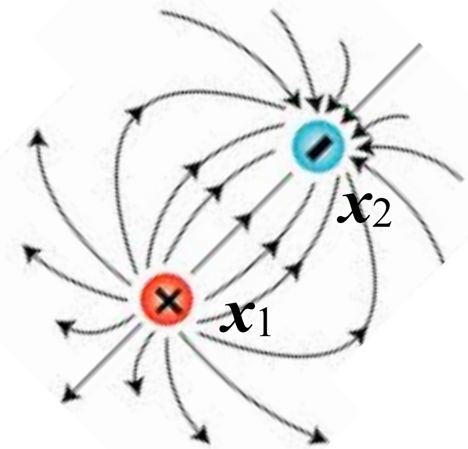




(log scale in $x_{Bj}$)

The red curve is an analytic approximation, valid in the $x_{Bj} \to 0$ limit.

# Atomic binding by the classical Coulomb field

At lowest order in $\alpha$ the Schrödinger eq. for an atom can be obtained also inserting the classical EM field in the Hamiltonian:




$$-\boldsymbol{\nabla}^2 A^0(\boldsymbol{x}) = e\left[\delta^3(\boldsymbol{x}-\boldsymbol{x}_1) - \delta^3(\boldsymbol{x}-\boldsymbol{x}_2)\right]$$

$$eA^0(\boldsymbol{x};\boldsymbol{x}_1,\boldsymbol{x}_2) = \frac{\alpha}{|\boldsymbol{x}-\boldsymbol{x}_1|} - \frac{\alpha}{|\boldsymbol{x}-\boldsymbol{x}_2|}$$

$A^0$ depends on $\boldsymbol{x}_1$, $\boldsymbol{x}_2$

The potential in the BSE = Schrödinger equation is then

$$V(\boldsymbol{x}_1-\boldsymbol{x}_2) = \tfrac{1}{2}\left[eA^0(\boldsymbol{x}=\boldsymbol{x}_1) - eA^0(\boldsymbol{x}=\boldsymbol{x}_2)\right] = -\frac{\alpha}{|\boldsymbol{x}_1-\boldsymbol{x}_2|}$$

Pair production is suppressed due to the NR limit.

# Non-vanishing boundary condition for classical field

Gauss' law $-\boldsymbol{\nabla}^2 A^0(\boldsymbol{x}) = e\left[\delta^3(\boldsymbol{x}-\boldsymbol{x}_1) - \delta^3(\boldsymbol{x}-\boldsymbol{x}_2)\right]$

has also homogeneous solutions (specified by the boundary condition)

$$eA^0(\boldsymbol{x};\boldsymbol{x}_1,\boldsymbol{x}_2) = \frac{\alpha}{|\boldsymbol{x}-\boldsymbol{x}_1|} - \frac{\alpha}{|\boldsymbol{x}-\boldsymbol{x}_2|} + e\Lambda^2 \boldsymbol{\ell}\cdot\boldsymbol{x}$$

where $\Lambda$ is a constant and the unit vector $\boldsymbol{l}$ may depend on $\boldsymbol{x}_1, \boldsymbol{x}_2$ . This adds a term to the potential

$$V_\Lambda(\boldsymbol{x}_1-\boldsymbol{x}_2) = \tfrac{1}{2}e\Lambda^2 \ell\cdot(\boldsymbol{x}_1-\boldsymbol{x}_2)$$

Choosing $\boldsymbol{l} \parallel \boldsymbol{x}_1 - \boldsymbol{x}_2$ gives the linear confining potential

$$V_\Lambda(\boldsymbol{x}_1-\boldsymbol{x}_2) = \tfrac{1}{2}e\Lambda^2|\boldsymbol{x}_1-\boldsymbol{x}_2|$$

which is of $O(e)$ and thus leading compared to the $O(\alpha)$ perturbative potential.

Note: Neutral state ($e_1 = -e_2$) required for space translation invariance!

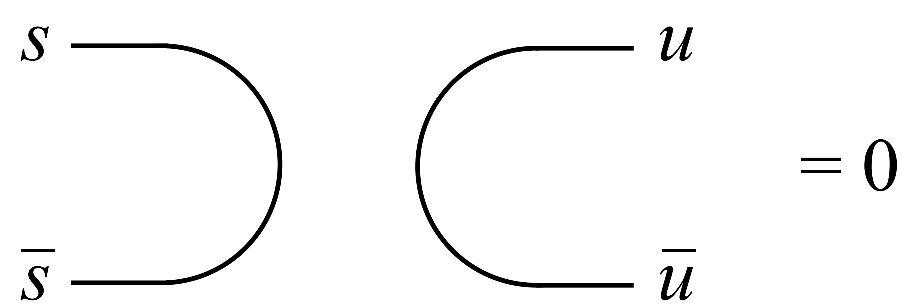
# Pair production effects



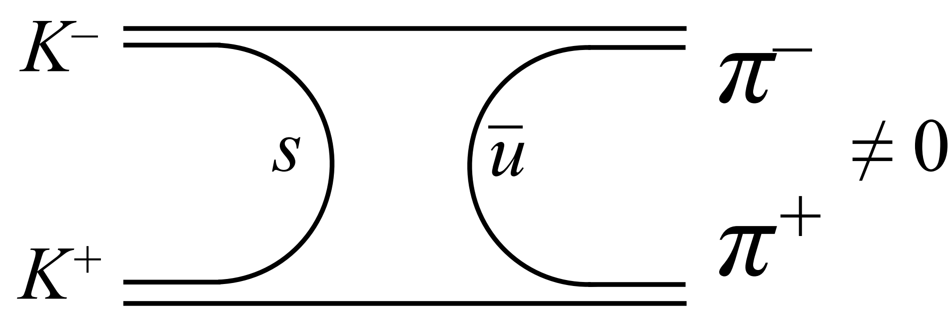
The Hamiltonian can create neutral, pointlike pairs. For these, $V_\Lambda = 0$, hence

$H\,|0\rangle = 0$ This was used in the BSE derivation above (D = 1+1).

The bound states derived similarly in D=3+1 appear to be boost covariant, again only for a purely linear potential.




The general picture seems to fit with dual diagram phenomenology:



# $u\bar{d}$ meson states in in QCD

$$\mathcal{L}_{QCD} = -\tfrac{1}{4} F_a^{\mu\nu} F_{\mu\nu}^a + \sum_f \bar{\psi}_f^A (i\partial\!\!\!/ - g A\!\!\!/_a T_{AB}^a - m_f)\psi_f^B$$

$$F_a^{\mu\nu} = \partial^\mu A_a^\nu - \partial^\nu A_a^\mu - g f_{abc} A_b^\mu A_c^\nu$$

$$|E, t=0\rangle = \int d^3\boldsymbol{y}_1 d^3\boldsymbol{y}_2\, \psi_u^{A\dagger}(t=0, \boldsymbol{y}_1)\chi^{AB}(\boldsymbol{y}_1, \boldsymbol{y}_2)\psi_d^B(t=0, \boldsymbol{y}_2)|0\rangle_R$$

Under time-independent gauge transformations $\psi(t, \boldsymbol{x}) \to U(\boldsymbol{x})\psi(t, \boldsymbol{x})$ the wave function transforms as

$$\chi(\boldsymbol{y}_1, \boldsymbol{y}_2) \to U(\boldsymbol{y}_1)\chi(\boldsymbol{y}_1, \boldsymbol{y}_2)U^\dagger(\boldsymbol{y}_2)$$

In a gauge where $\chi^{AB}(\boldsymbol{y}_1, \boldsymbol{y}_2) = \delta^{AB}\chi(\boldsymbol{y}_1, \boldsymbol{y}_2)$

only the diagonal color fields $A_a^0$ with $a = 3,8$ can be nonzero.

Since $f_{a38} = 0$ the commutator terms do not contribute at $O(g)$.

Fock states with quarks of color $C$ give the EOM for $A_a^0$

$$-\boldsymbol{\nabla}^2 A_a^0(\boldsymbol{x}) = g\, T_a^{CC}\left[\delta^3(\boldsymbol{x}-\boldsymbol{x}_1) - \delta^3(\boldsymbol{x}-\boldsymbol{x}_2)\right]$$

$$A_a^0(\boldsymbol{x};\boldsymbol{x}_1,\boldsymbol{x}_2,C) = \Lambda_a^2\,\hat{\ell}_a\cdot\boldsymbol{x} + \frac{gT_a^{CC}}{4\pi}\left(\frac{1}{|\boldsymbol{x}-\boldsymbol{x}_1|} - \frac{1}{|\boldsymbol{x}-\boldsymbol{x}_2|}\right) \qquad (a=3,8)$$

$$-\frac{1}{4}\sum_a \int d^3\boldsymbol{x} F^a_{\mu\nu}F_a^{\mu\nu} = \sum_{a=3,8}\left[\frac{1}{2}\Lambda_a^4 \int d^3\boldsymbol{x} + \frac{1}{3}g\Lambda_a^2\, T_a^{CC}\,\hat{\ell}_a\cdot(\boldsymbol{x}_1-\boldsymbol{x}_2) + \mathcal{O}\left(g^2\right)\right]$$

$\Lambda^4 \equiv \sum_{a=3,8} \Lambda_a^4$ must be independent of $\boldsymbol{x}_1, \boldsymbol{x}_2$, and $\hat{\boldsymbol{\ell}}_a \parallel \boldsymbol{x}_1 - \boldsymbol{x}_2$

Determining $\Lambda_3/\Lambda_8$ from stationarity it turns out that
the potential is independent of the quark color $C$,

$$V(\boldsymbol{x}_1,\boldsymbol{x}_2) = \frac{2g\Lambda^2}{3\sqrt{3}}|\boldsymbol{x}_1-\boldsymbol{x}_2|$$

and the bound state equation for the color singlet wave function $\chi$
has the same form as in QED.

# uds baryon states in in QCD

$$|E, t=0\rangle = \int \prod_{j=1}^{3} d^3\boldsymbol{y}_j\, \psi^{A\dagger}_{u\alpha_1}(t=0,\boldsymbol{y}_1)\psi^{B\dagger}_{d\alpha_2}(t=0,\boldsymbol{y}_2)\psi^{C\dagger}_{s\alpha_3}(t=0,\boldsymbol{y}_3)\chi^{\alpha_1\alpha_2\alpha_3}_{ABC}(\boldsymbol{y}_1,\boldsymbol{y}_2,\boldsymbol{y}_3)|0\rangle_R$$

In a gauge where

$$\chi^{\alpha_1\alpha_2\alpha_3}_{ABC}(\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{x}_3) = \epsilon_{ABC}\chi^{\alpha_1\alpha_2\alpha_3}(\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{x}_3)$$

the relevant gauge fields are, for quark colors $ABC = 123$

$$\begin{aligned} A_3^0(\boldsymbol{x};\{\boldsymbol{x}_i\},ABC=123) &= \Lambda_3^2\,\hat{\ell}_3\cdot\boldsymbol{x} + \frac{g}{4\pi}\frac{1}{2}\left(\frac{1}{|\boldsymbol{x}-\boldsymbol{x}_1|} - \frac{1}{|\boldsymbol{x}-\boldsymbol{x}_2|}\right) \\ A_8^0(\boldsymbol{x};\{\boldsymbol{x}_i\},ABC=123) &= \Lambda_8^2\,\hat{\ell}_8\cdot\boldsymbol{x} + \frac{g}{4\pi}\frac{1}{2\sqrt{3}}\left(\frac{1}{|\boldsymbol{x}-\boldsymbol{x}_1|} + \frac{1}{|\boldsymbol{x}-\boldsymbol{x}_2|} - 2\frac{1}{|\boldsymbol{x}-\boldsymbol{x}_3|}\right) \end{aligned}$$

and the interference term of $O(g)$ in the action is

$$S^{123}_{int} = \frac{g\Lambda_3^2}{6}\,\hat{\boldsymbol{\ell}}_3\cdot(\boldsymbol{x}_1-\boldsymbol{x}_2) + \frac{g\Lambda_8^2}{6\sqrt{3}}\,\hat{\boldsymbol{\ell}}_8\cdot(\boldsymbol{x}_1+\boldsymbol{x}_2-2\boldsymbol{x}_3)$$

and is stationary for

$$\hat{\ell}_3 \parallel \boldsymbol{x}_1 - \boldsymbol{x}_2, \qquad \hat{\ell}_8 \parallel \boldsymbol{x}_1 + \boldsymbol{x}_2 - 2\boldsymbol{x}_3$$

$$\frac{\Lambda_3^2}{\Lambda_8^2} = \sqrt{3}\,\frac{|\boldsymbol{x}_1 - \boldsymbol{x}_2|}{|\boldsymbol{x}_1 + \boldsymbol{x}_2 - 2\boldsymbol{x}_3|}$$

For different colors $ABC = 213$, *etc*., the result is given by $\boldsymbol{x}_1 \leftrightarrow \boldsymbol{x}_2$ , *etc.*

When expressed in terms of the universal strength the potential obtained for stationary action is the same for all color choices $ABC$,

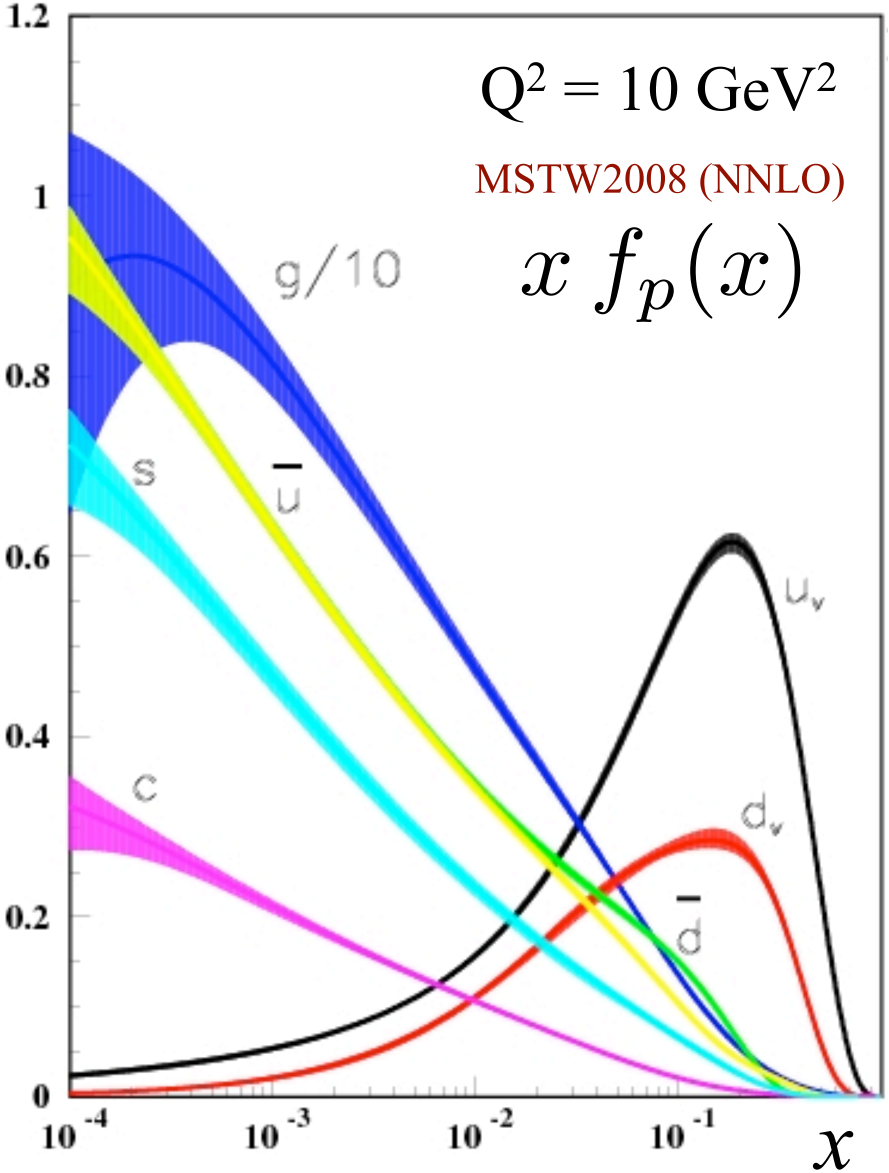
$$\Lambda^4 \equiv \sum_{a=3,8} \Lambda_a^4$$

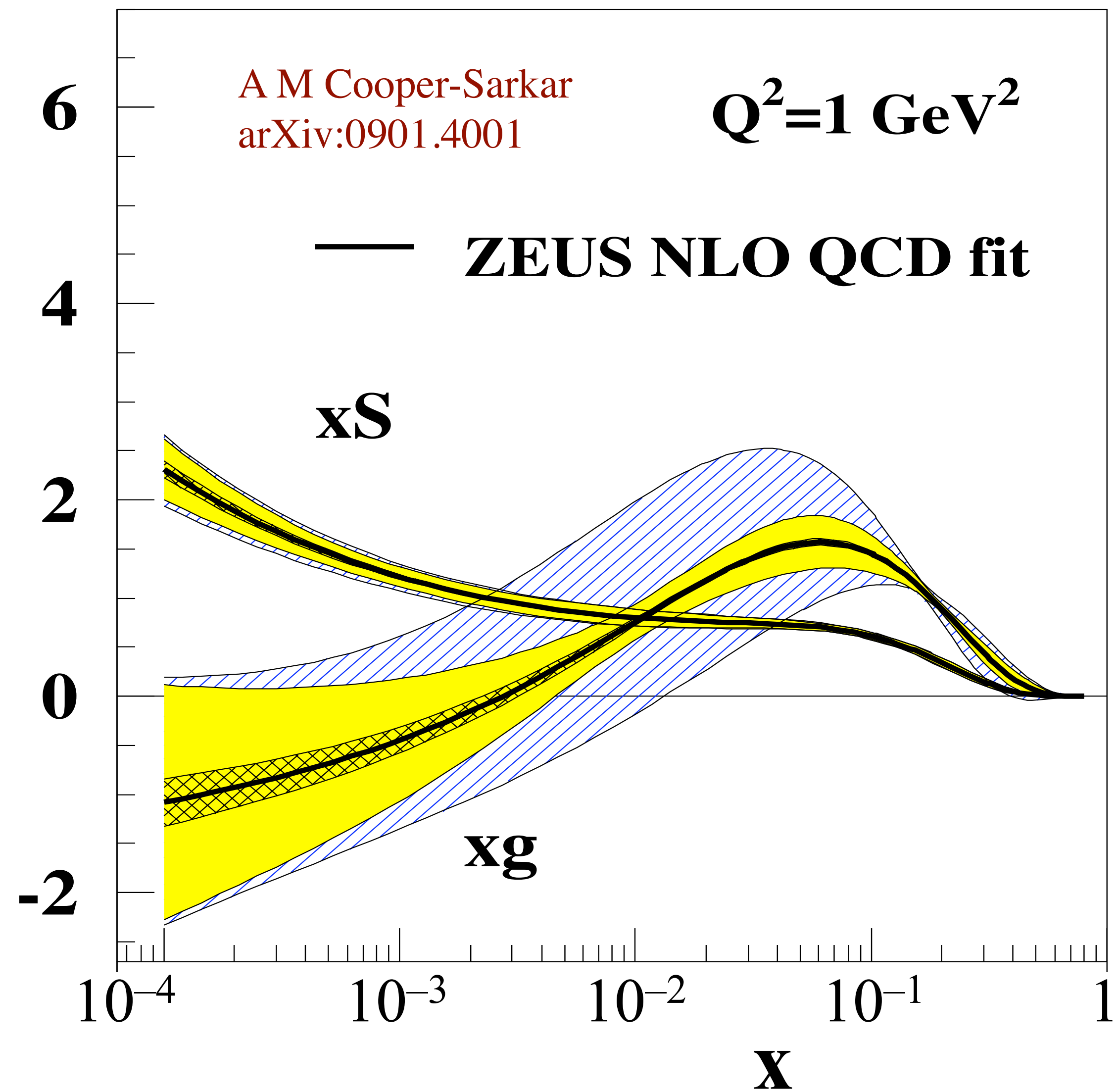
$$V(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3) = \frac{\sqrt{2}g\Lambda^2}{3\sqrt{3}}\sqrt{(\boldsymbol{x}_1 - \boldsymbol{x}_2)^2 + (\boldsymbol{x}_2 - \boldsymbol{x}_3)^2 + (\boldsymbol{x}_3 - \boldsymbol{x}_1)^2}$$

and the bound state equation for the color singlet wave function is

$$\sum_{j=1}^{3}\left[\gamma^0(-i\boldsymbol{\nabla}_j\cdot\boldsymbol{\gamma}_j + m_j)\right]\chi = (E - V)\chi$$

# Absence of gluon distribution at low $Q^2$ ?



# Summary

- Hadron phenomenology encourages the search for an analytic approach.
- Relativistic states with an ∞ number of constituents can be described by inclusive, "valence" wave functions. *Cf:* Dirac wave function.
- Need a perturbative expansion: $\alpha_s \approx 0.5$ should freeze in the infrared.
- A non-vanishing boundary condition in Gauss' law for $A^0$ provides an $O(\alpha_s^0)$ linear potential.
- The $O(\alpha_s^0)$ states are Poincaré covariant (probably also in D = 3+1).
- Pertubatively expand around the $O(\alpha_s^0)$ qqbar, qqq "in" and "out" states.
- Form factors are gauge invariant, and duality is OK.
- Parton distributions have a sea component.
- Standard color neutral mesons and baryons emerge.

Much exploring remains!