Rare s to d transitions from Lattice QCD

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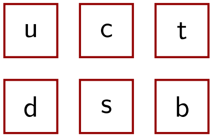
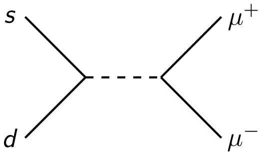
THE UNIVERSITY of EDINBURGH
School of Physics & Astronomy

Centre for Theoretical Physics · Higgs

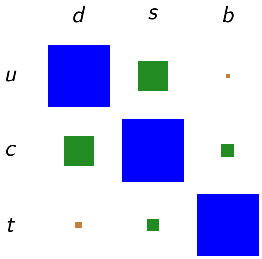
Weak transition of quarks

- three generations of quarks in the Standard Model
- weak transitions of quarks in the Standard Model
- charged weak interaction ($W^{\pm}$):
  $\rightarrow$ mixes quark generations (CKM matrix)
- neutral weak interaction (Z):
  $\rightarrow$ no flavour changing neutral currents (FCNC) at tree-level





- CKM matrix



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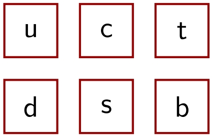
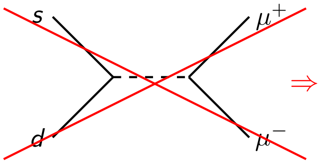
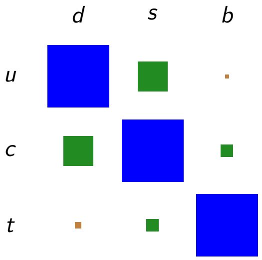




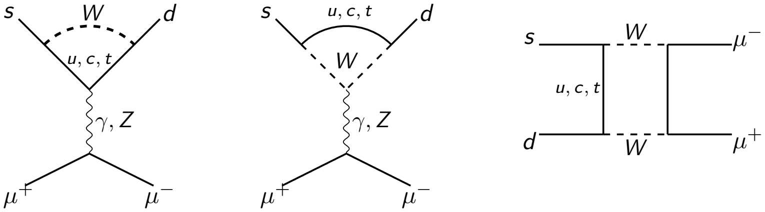
$\Rightarrow$ no Standard Model process

- CKM matrix



FCNC in the Standard Model

- s to d transition in Standard Model only via loop-processes, e.g.



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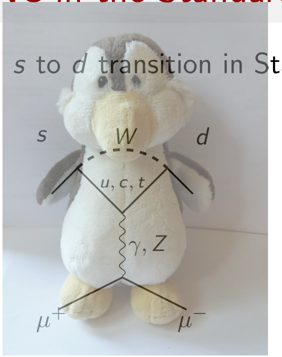

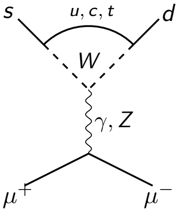

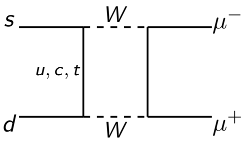

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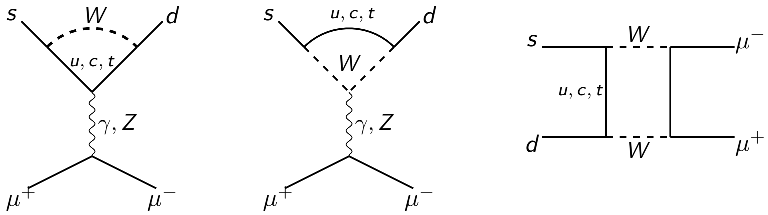

$\rightarrow$ rare in the Standard Model

$\rightarrow$ sensitive to new physics including a FCNC at tree-level

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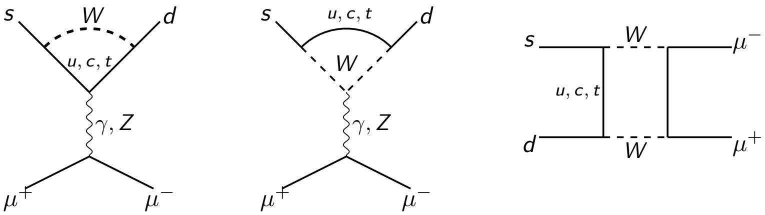

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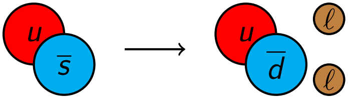
Outline

- rare Kaon decay $K^+ \rightarrow \pi^+ \ell^+ \ell^-$ [VG *et al* Phys.Rev.D 107 (2023) 1, L011503]
- rare Hyperon decay $\Sigma^+ \rightarrow p \ell^+ \ell^-$ [VG *et al* JHEP 04 (2023) 108, JHEP 07 (2025) 038]

The rare Kaon Decay $K^+ \to \pi^+ \ell^+ \ell^-$

The rare Kaon decay

- rare decay of a charged kaon $K^+ \to \pi^+ \ell^+ \ell^-$
- measured experimentally by NA48, NA62 @CERN [PDG]



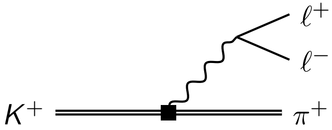
$$\mathrm{Br}\left(K^+ \to \pi^+ e^+ e^-\right) = (3.00 \pm 0.09) \times 10^{-7}$$

$$\mathrm{Br}\left(K^+ \to \pi^+ \mu^+ \mu^-\right) = (9.17 \pm 0.14) \times 10^{-8}$$

- long-distance dominated [D'Ambrosio *et al* 1998]

→ decay via virtual photon $K^+ \to \pi^+ \gamma^* \to \pi^+ \ell^+ \ell^-$

$$\mathcal{A}_\mu(q^2) = \int \mathrm{d}^4x \langle \pi(\vec{p}) \big| T\left[J_\mu(0) H_W(x)\right] \big| K(\vec{k}) \rangle$$



- em current J_μ
- $\Delta S = 1$ effective weak Hamiltonian H_W

The rare Kaon decay - hadronic amplitude

- H_W given by four-quark operators

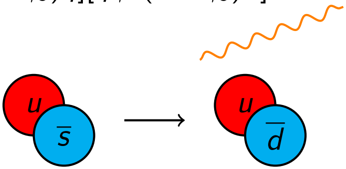
$$H_W = \frac{G_F}{\sqrt{2}} V_{us}^* V_{ud} \sum_{j=1}^{2} C_j \left(Q_j^u - Q_j^c\right)$$

[Buchalla *et al* 1996]

- four quark operators

$$Q_1^q = [\bar{s}\gamma_\mu(1-\gamma_5)d][\bar{q}\gamma^\mu(1-\gamma_5)q]$$
$$Q_2^q = [\bar{s}\gamma_\mu(1-\gamma_5)q][\bar{q}\gamma^\mu(1-\gamma_5)d]$$

- Wilson coefficients C_j



- form factor decomposition for $K^+ \to \pi^+\gamma^*$ amplitude

$$\mathcal{A}_\mu(q^2) = -i\frac{G_F}{(4\pi)^2} V(q^2/M_K^2)\left[q^2(k+p)_\mu - (M_K^2 - M_\pi^2)q_\mu\right]$$

- em form factor $V(z)$ parameterised as
[D'Ambrosio et al 1998]

$$V(z) = a_+ + b_+ z + V^{\pi\pi}(z)$$

	a_+	b_+
NA48 $\ell = e$	$-0.578(16)$	$-0.779(66)$
NA64 $\ell = \mu$	$-0.592(15)$	$-0.699(58)$

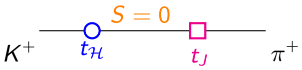
The rare Kaon decay from Euclidean space-time

- strategy developed in [N. Christ et al, PRD 92, 094512 (2015)]
- interested in the **Minkowski** amplitude

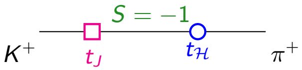
$$\mathcal{A}_\mu(q^2) = \int \mathrm{d}^4x \langle \pi(\vec{p}) \big| T\left[J_\mu(0) H_W(x)\right] \big| K(\vec{k}) \rangle$$

- spectral representation

$$\mathcal{A}_\mu(q^2) = i \int_0^\infty \mathrm{d}E \, \frac{\rho(E)}{2E} \frac{\langle \pi(p) | J_\mu | E, k \rangle \langle E, k | H_W | K(\vec{k}) \rangle}{E_K(\vec{k}) - E + i\epsilon}$$



$$- i \int_0^\infty \mathrm{d}E \, \frac{\sigma(E)}{2E} \frac{\langle \pi(p) | H_W | E, p \rangle \langle E, p | J_\mu | K(\vec{k}) \rangle}{E - E_\pi(\vec{p}) - i\epsilon}$$



- spectral densities ρ (strangeness $S = 0$), σ (strangeness $S = 1$)

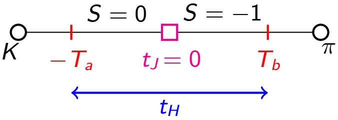
The rare Kaon decay from Euclidean space-time

- Euclidean 4pt function

$$\Gamma^{(4)}_{\mu}(t_H, t_\pi, t_K; \vec{p}, \vec{k}) = \int \mathrm{d}\vec{x} \left\langle \psi^{\pi}(t_\pi, \vec{p})\, \mathcal{H}_W(t_H, \vec{x}) J_\mu(0)\, \overline{\psi}^K(t_K, \vec{k}) \right\rangle$$

- extract amplitude $\mathcal{A}_\mu$ by integrating over t_H

$$I_\mu(T_a, T_b; \vec{p}, \vec{k}) = \int_{-T_a}^{T_b} \mathrm{d}t_H\, \Gamma^{(4)}_{\mu}(t_H, t_\pi, t_K; \vec{p}, \vec{k})$$



- spectral representation

$$I_\mu(T_a, T_b; \vec{p}, \vec{k}) \propto -\int_0^\infty \mathrm{d}E\, \frac{\rho(E)}{2E} \frac{\langle \pi(p)|J_\mu|E, k\rangle\langle E, k|H_W|K(\vec{k})\rangle}{E_K(\vec{k}) - E} \left[1 - e^{(E_K(\vec{k}) - E)T_a}\right]$$

$$+ \int_0^\infty \mathrm{d}E\, \frac{\sigma(E)}{2E} \frac{\langle \pi(p)|H_W|E, p\rangle\langle E, p|J_\mu|K(\vec{k})\rangle}{E_K(\vec{k}) - E} \left[1 - e^{-(E - E_\pi(\vec{p}))T_b}\right]$$

The rare Kaon decay from Euclidean space-time

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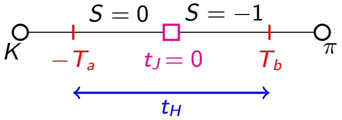

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$$+ \int_0^\infty \mathrm{d}E\, \frac{\sigma(E)}{2E} \frac{\langle \pi(p)|H_W|E, p\rangle\langle E, p|J_\mu|K(\vec{k})\rangle}{E_K(\vec{k}) - E}\left[1 - e^{-(E-E_\pi(\vec{p}))T_b}\right]$$

desired $\mathcal{A}_\mu$

Growing exponentials

- all intermediate states $|E, p\rangle$ with $S = 1$ have $E > E_\pi(\vec{p})$

$$\int_0^\infty \mathrm{d}E\, \frac{\sigma(E)}{2E} \frac{\langle \pi(p)|H_W|E,p\rangle\langle E,p|J_\mu|K(\vec{k})\rangle}{E_K(\vec{k}) - E}\left[1 - \cancel{e^{-(E-E_\pi(\vec{p}))T_b}}\right] \quad \text{for } T_b \to \infty$$

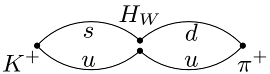
- $S = 0$ intermediate states $|E, k\rangle$ with $E < E_K(\vec{k})$ $\rightarrow$ growing exponentials for $T_a \to \infty$

$$\int_0^\infty \mathrm{d}E\, \frac{\rho(E)}{2E} \frac{\langle \pi(p)|J_\mu|E,k\rangle\langle E,k|H_W|K(\vec{k})\rangle}{E_K(\vec{k}) - E}\left[1 - e^{(E_K(\vec{k})-E)T_a}\right]$$

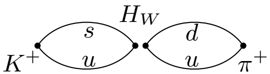
  - single pion: needs to be removed (e.g. by explicit reconstruction)
  - two pions: $\langle \pi|J_\mu|\pi\pi\rangle$ vanishes for $a \to 0$
  - three pions: phase-space suppressed

Wick contractions

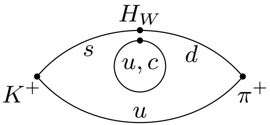
- Wick contractions of 3pt function $C_{H_W}^{3\mathrm{pt}}(t_H, \vec{p}) = \sum_{\vec{x}} \left\langle \phi_\pi(t_\pi, \vec{p})\, H_W(t_H, \vec{x})\, \phi_K^\dagger(0, \vec{p}) \right\rangle$



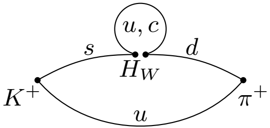
connected



Wing



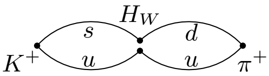
Eye



Saucer

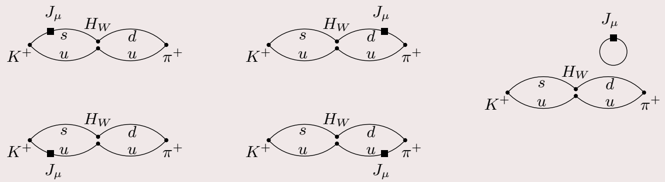
Wick contractions

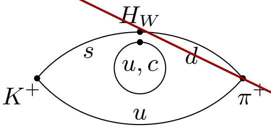
- Wick contractions of 3pt function $C^{\mathrm{3pt}}_{H_W}(t_H, \vec{p}) = \sum_{\vec{x}} \left\langle \phi_\pi(t_\pi, \vec{p})\, H_W(t_H, \vec{x})\, \phi_K^\dagger(0, \vec{p}) \right\rangle$
- Wick contractions of 4pt function
  $C^{\mathrm{4pt}}_{\mu}(t_H, t_J, \vec{k}, \vec{p}) = \sum_{\vec{x}} \sum_{\vec{y}} e^{-i\vec{q}\cdot\vec{x}} \left\langle \phi_\pi(t_\pi, \vec{p})\, J_\mu(t_J, \vec{x})\, H_W(t_H, \vec{y})\, \phi_K^\dagger(t_K, \vec{k}) \right\rangle$



connected

4pt Connected



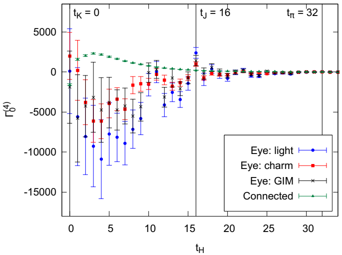


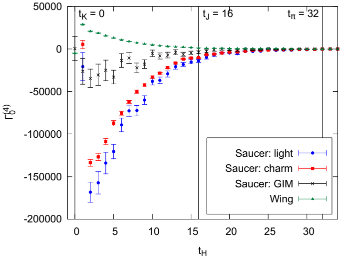
Eye

Saucer

$K^+ \to \pi^+ \ell^+ \ell^-$ physical quark-mass result [VG et al, Phys.Rev.D 107 (2023) 1, L011503]

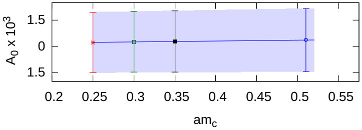
- $N_f = 2+1$ Möbius Domain Wall Fermions
- $48^3 \times 96$ lattice with $a^{-1} = 1.73$ GeV
- physical light and strange mass
- three lighter-than-physical charm masses
- kinematics: $\vec{k} = 0$, $\vec{p} = (1,0,0)\ 2\pi/L$





- error dominated by eye-type diagrams with GIM

- extrapolation of $\mathcal{A}_0$ to physical charm mass



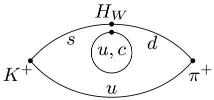
- form factor $V(q^2/M_K^2)$

$$V(0.013) = 0.87(4.44)$$

- reduction of ≈ 10 in error needed to be sensitive to experiment

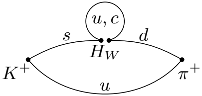
Split-even estimators for eye-type diagrams

- Eye-type diagrams (GIM) need quark-loop differences



$$\Delta L(x_H) = S^\ell(x_H|x_H) - S^c(x_H|x_H) = (m_c - m_\ell)\sum_x S^\ell(x_H|x)S^c(x|x_H)$$

- standard noise estimator

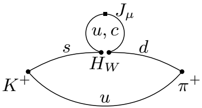


$$S^q(x_H|x_H) = \frac{1}{N}\sum_{i=1}^{N}\phi_i^q(x_H)\eta_i^\dagger(x_H) \qquad \text{with} \qquad \sum_{x_H} D^q(x|x_H)\phi_i^q(x_H) = \eta_i(x)$$

- split-even estimator can greatly improve noise-to-signal [L. Giusti et al, Eur. Phys. J. C 79 (2019)]

$$\Delta L(x_H) = (m_c - m_\ell)\sum_{i=1}^{N}\sum_{x,y} S^\ell(x_H|x)\eta_i(x)\eta_i^\dagger(y)S^c(y|x_H)$$

- current insertion on loop:

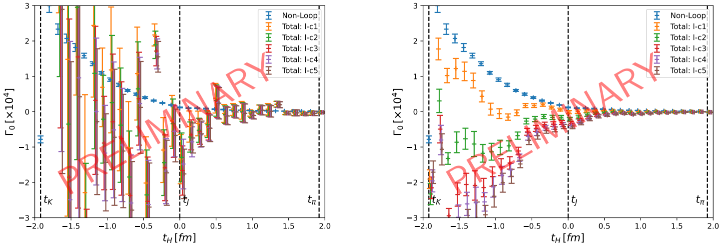


$$S^u(x_H|x)\gamma_\mu S^u(x|x_H) - S^c(x_H|x)\gamma_\mu S^c(x|x_H)$$

$$\left[S^u(x_H|x)\gamma_\mu S^u(x|x_H) - S^c(x_H|x)\gamma_\mu S^u(x|x_H)\right] + \left[S^c(x_H|x)\gamma_\mu S^u(x|x_H) - S^c(x_H|x)\gamma_\mu S^c(x|x_H)\right]$$

Preliminary results using split-even

- Preliminary results with split-even estimator [R. Hodgson, VG et al, PoS Lattice 2024]
- small statistics: 10 gauge configurations, 6 time translations, 32 noise sources
- five charm masses ($m_s^{\text{phys}} \leq m_{c_i} < m_c^{\text{phys}}$)
  $\rightarrow$ frequency splitting: $l - c = (l - c_1) + (c_1 - c_2) + \cdots + (c_N - c)$
- left: standard noise estimator, right: split-even estimator

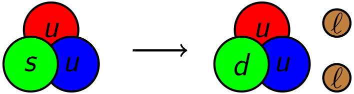


- $\mathcal{O}(10)\times$ reduction in statistical error with split-even

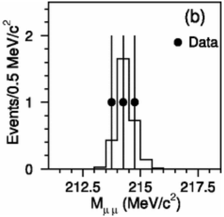
The rare Hyperon Decay $\Sigma^+ \to p\ell^+\ell^-$

The rare Hyperon decay - Experiment

- rare Hyperon decay $\Sigma^+ \to p\ell^+\ell^-$

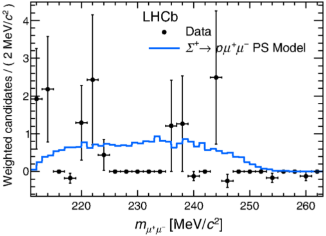


- HyperCP @Fermilab (2005)
- 3 events
- $\mathcal{B}(\Sigma^+ \to p\mu^+\mu^-) = (8.6^{+6.6}_{-5.4} \pm 5.5) \times 10^{-8}$
- all at $M_{\mu\mu} \approx 214$ MeV
  "HyperCP-anomaly"



HyperCP
[PRL 94 (2005) 021801]

- LHCb @CERN (2018)
- 10 events
- $\mathcal{B}(\Sigma^+ \to p\mu^+\mu^-) = (2.2^{+1.8}_{-1.3}) \times 10^{-8}$

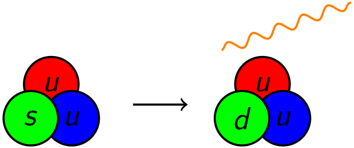


LHCb [PRL 120 (2018) 22, 221803]

The rare Hyperon decay - Standard Model Prediction

- long distance dominated [He et al, Phys.Rev.D72 (2005) 074003]

$$\mathcal{B}^{\text{short}} \sim 10^{-12}$$



- long distance given by hadronic amplitude for $\Sigma^+ \to p\gamma^*$

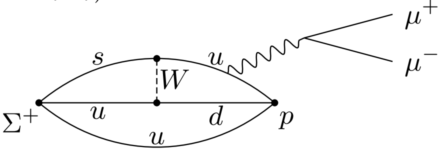
$$\mathcal{A}_\mu^{rs}(q^2) = \int \mathrm{d}^4x \left\langle p(\vec{p}_N), r \middle| \mathrm{T}\left[\mathcal{H}_W(x)\, J_\mu(0)\right] \middle| \Sigma^+(\vec{p}_\Sigma), s \right\rangle \equiv \bar{u}_N^r\, \tilde{\mathcal{A}}_\mu(q^2)\, u_\Sigma^s$$

- form factor decomposition

$$\tilde{\mathcal{A}}_\mu(q^2) = i\sigma_{\nu\mu}q^\nu \left(a(q^2) + \gamma_5 b(q^2)\right) + \left(q^2\gamma_\mu - q_\mu \not{q}\right)\left(c(q^2) + \gamma_5 d(q^2)\right)$$

- positive parity a, c, negative parity b, d
- SM prediction (Models & experimental input)

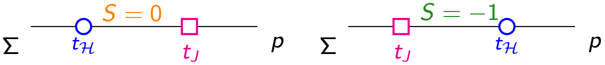
$$1.6 \times 10^{-8} < \mathcal{B}(\Sigma^+ \to p\mu^+\mu^-) < 9.0 \times 10^{-8}$$



The rare Hyperon decay from Lattice QCD

- formalism developed in [VG et al, JHEP 04 (2023) 108]
- rare Hyperon decay 4pt function

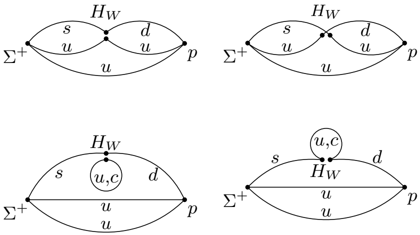
$$\Gamma^{(4)}_{\mu}(t_H, t_p, t_\Sigma; \vec{p}, \vec{k})_L = \int \mathrm{d}\vec{x} \left\langle \psi^p(t_p, \vec{p})\, \mathcal{H}_W(t_H, \vec{x}) J_\mu(0)\, \overline{\psi}^{\Sigma^+}(t_\Sigma, \vec{k}) \right\rangle_L$$



- intermediate states with $S = 0$: single proton $P(k)$; $N\pi$ state with $E_{N\pi}(k) < E_\Sigma(k)$

→ growing exponentials when integrating over t_H

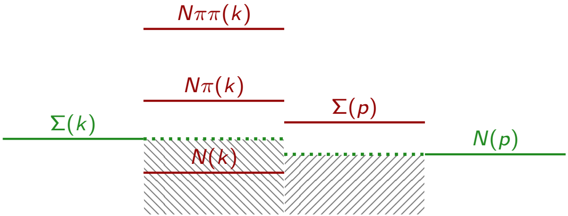
→ FV corrections from $N\pi$ state



- Wick contractions: four topologies

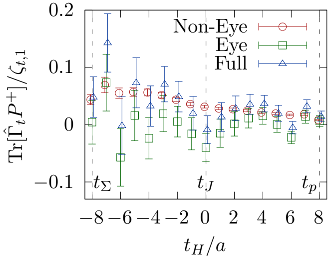
The rare Hyperon decay - exploratory calculation [VG et al, JHEP 07 (2025) 038]

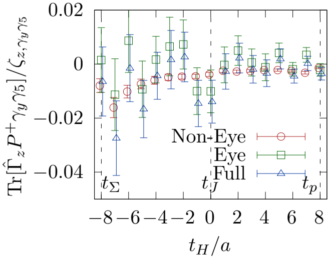
- $24^3 \times 64$ lattice, $a^{-1} = 1.78$ GeV
- $m_\pi \approx 340$ MeV
- $\vec{k} = 0$ and $\vec{p} = (1, 0, 0)\frac{2\pi}{L}$



- 4pt functions (Baryons) spin-matrices

$\rightarrow$ traces with Γ-structures to access form factors





- positive parity form factors

	Re a [MeV]	Re c [10^{-2}]
NE	−6.2(8.3)	−1.6(1.7)
Full	−27.0(39.0)	−2.7(9.4)

- $\mathcal{O}$ of pheno results:

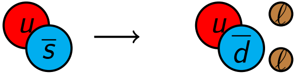
$$\mathrm{Re}\ a \sim 10\,\mathrm{MeV}$$

$$\mathrm{Re}\ c \sim 10^{-2}\,.$$

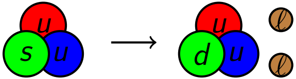
Conclusions

Summary

- weak decays with $s \to d$ quark transitions require FCNC $\to$ sensitive to new physics
  - rare Kaon decay $K^+ \to \pi^+ \ell^+ \ell^-$
  - rare Hyperon decay $\Sigma^+ \to p \ell^+ \ell^-$



- physical quark-mass results for $K^+ \to \pi^+ \ell^+ \ell^-$
  - error cominated by eye-type diagrams with GIM subtraction
  - preliminary results using split-even approach promising
- exploratory calculation for $\Sigma^+ \to p \ell^+ \ell^-$ with m_π 340 MeV
  - positive parity form factors same $\mathcal{O}$ as phenomenology
  - physical mass calculation very challenging



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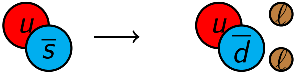

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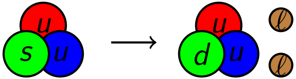


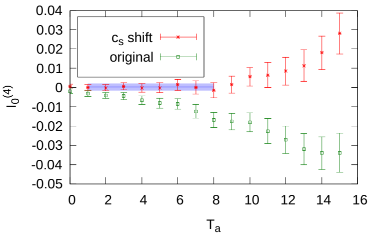
Thank you

appendix

Subtraction of single pion growing exponential

- single pion intermediate state with growing exponential

$$\sim \left\langle K(\vec{k}) \,|H_W|\, \pi(\vec{p}) \right\rangle \langle \pi(\vec{p}) \,|J_\mu|\, \pi(\vec{p}) \rangle$$



1. reconstruct the single-pion state from 3pt functions

$$\Gamma_H^3 = \left\langle \psi^\pi(\vec{p}) \; H_W \; \overline{\psi}^K(\vec{k}) \right\rangle \qquad \Gamma_J^3 = \left\langle \psi^\pi(\vec{p}) \; J_\mu \; \overline{\psi}^\pi(\vec{p}) \right\rangle$$

  - direct fit of growing term to 4pt function
  - reconstruct from Γ^3 and subtract
  - approximation: 0 mom transfer
  - aproximation: $SU(3)$ symmetry

2. additive scalar shift to the weak Hamiltonian

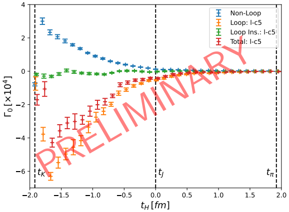
$$H'_W = H_W - c_s \bar{s} d$$

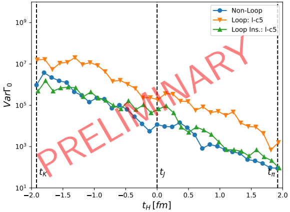
leaves amplitude $\mathcal{A}_\mu$ unchanged
[Christ et al, Phys.Rev.D 92 (2015) 9, 094512]

Analysis	$\mathcal{A}_0 @ m_{c_1}$
Method 1	
Direct fit	-0.00052(208)
2pt/3pt recon	-0.00036(162)
0 mom transfer	-0.00087(165)
$SU(3)$ symm lim	0.00055(165)
Method 2	
c_s shift	0.00022(172)

Split-even estimators: current insertion at loop

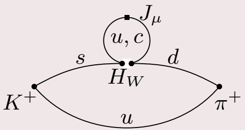
- comparison of noise of eye-type diagrams with current at loop (loop insertion LI) or not at loop
- LI computationally more expensive than non non-LI
- correlators on left, variance on the right



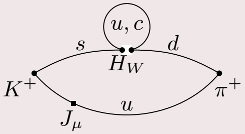


- noise dominated by eye-type diagrams with current not inserted at the loop

example: eye-type LI



example: eye-type non-LI



rare Hyperon decay - phenomenological estimate

- Standard Model prediction
  [He *et al*, Phys.Rev.D 72 (2005) 074003, JHEP 10 (2018) 040]
  - Baryon ChiPT & experimental input $\Sigma^+ \to N\pi$
  - vector meson dominance
  - experimental input from $\Sigma^+ \to p\gamma$

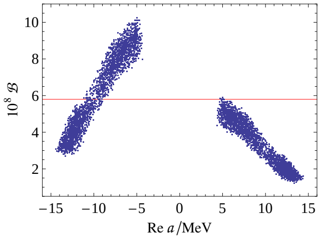
$$\Gamma(\Sigma \to p\gamma) \propto (|a(0)|^2 + |b(0)|^2)$$

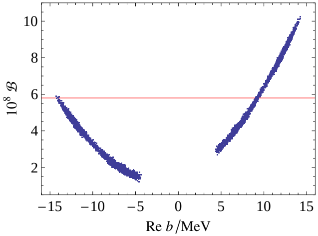
$$\frac{d\Gamma(\Sigma \to p\gamma)}{d\cos\theta} \to 2\,\mathrm{Re}(a(0)b(0)^*)$$

- arrive at a large range of results

$$1.6 \times 10^{-8} < \mathcal{B}(\Sigma^+ \to p\mu^+\mu^-)_{\mathrm{SM}} < 9.0 \times 10^{-8}$$

[He *et al*, JHEP 10 (2018) 040]

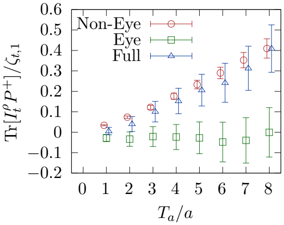


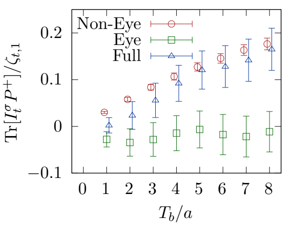


rare Hyperon decay: from integrated 4pt to form factors

- integrated 4pt function for both time orderings

$$I_\mu^\rho = -i \int_{-T}^{0} \mathrm{d}t_H \, \hat{\Gamma}_\mu^{(4)} \qquad I_\mu^\sigma = -i \int_{0}^{T} \mathrm{d}t_H \, \hat{\Gamma}_\mu^{(4)}$$





exp growing

$$\mathcal{A}_t^\rho - c_t^\rho \, e^{-(m_p - m_\Sigma)T}$$

exp slowly decaying

$$\mathcal{A}_t^\sigma - c_t^\sigma \, e^{-(E_\Sigma - E_p)T}$$

- construct appropriate traces

$$\mathrm{Tr}\left[\mathcal{A}_\mu \Gamma\right] = \zeta_\mu^\Gamma \, f_\mu$$

- ζ_μ^Γ: kinematic factors
- f_μ: linear combinations of form factors

$$\begin{pmatrix} f_t \\ f_z \end{pmatrix} = \begin{pmatrix} 1 & m_\Sigma + m_p \\ m_\Sigma + m_p & q^2 \end{pmatrix} \begin{pmatrix} a \\ c \end{pmatrix}$$

- fits to the data give

	$f_t^\rho[10^{-1}]$	$f_t^\sigma[10^{-1}]$
NE	2.81(0.33)	−3.07(0.26)
Full	3.41(1.14)	−3.94(0.93)