Dynamical nucleation rates

for early universe phase transitions and analog systems

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Aims of the talk

- Thermal nucleation in phase transitions
- Rate computed with an assumption of equilibrium
- Observed to be time dependent

★ Numerical and theoretical approach

★ An asymptotic rate?

★ Deviations important?

Nucleation in phase transitions

- System trapped in a metastable state (ϕ_1)
- The escape
  - Fluctuates locally over the barrier, *nucleates*
  - Bubbles of ϕ_2 grow

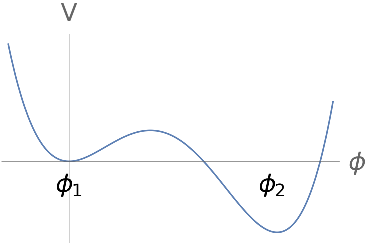


Figure: Potential with a transition

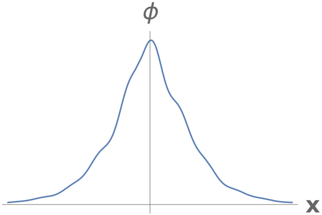


Figure: Cross section of a bubble

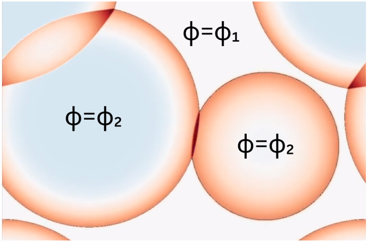


Figure: Growing bubbles [Cutting et al. '20]

Systems of interest

- Early universe
  - Gravitational waves
  - BSM physics
- Lab experiments like analog systems
  - Testing our understanding
  - Modeling early universe

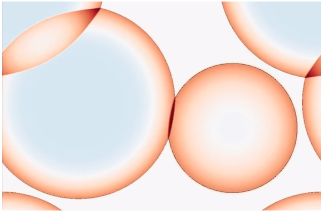


Figure: Growing, colliding bubbles (arXiv:1906.00480)

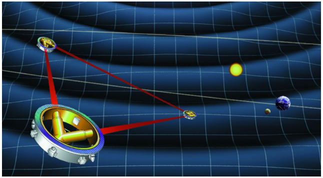


Figure: LISA, instrument for gravitational waves (ESA)

Time-dependent rate

- Rate observed to be time dependent
- Current methods: single value
  - Lattice [Moore & Rummukainen '00]
  - Perturbative [Langer '69]

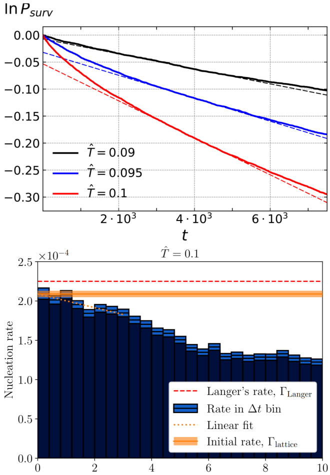


Figure: Time-dependent rate upper: [Pîrvu, Shkerin, Sibiryakov '24] lower: [Hirvonen, Gould '25]

Qualitative results

- Lab experiments like analog systems
  - Can be very different
  - Dissipation can become defining factor
- Early universe
  - Likely no strong effects

Classical simulations for high-T QFTs

- Long wavelenght, $\gg T^{-1}$, low energy, $\ll T$
- Large occupation numbers
- Classicalization

$$f_{\text{eq,bos}} = \frac{1}{e^{\beta E} - 1} \sim \frac{T}{m_{\text{bos}}} \gg 1$$

Model

- 1D scalar field
- Unbounded, $\mathbb{Z}_2$ symmetric potential
- Only one parameter, $\hat{T}$
  - Dimensionless temperature
- Hamiltonian evolution

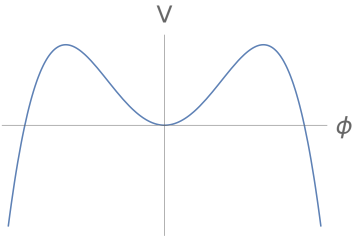


Figure: Model potential

$$H = \int \mathrm{d}x \left(\frac{\pi^2}{2} + \frac{(\partial_x \phi)^2}{2} + \frac{m^2 \phi^2}{2} - \frac{\lambda \phi^4}{4} \right)$$

$$\hat{T} \equiv \frac{\lambda T}{m^3} \ll 1$$

Classical nucleation theory

- Setup
- Current approaches

Setup for classical nucleation theory [Kramers '40]

- Two phases
  - Metastable: thermal population
  - Stable: empty
- Nucleation rate
  - Leak from meta to stable

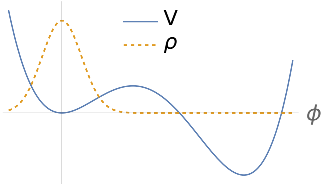


Figure: Schematic figure of an equilibrium distribution in the metastable state

$$\Gamma(t) = -\frac{1}{P_{\text{meta}}(t)}\frac{\mathrm{d}P_{\text{meta}}}{\mathrm{d}t}(t)\,,$$

$$P_{\text{meta}} = \int_{\text{meta}} \mathcal{D}\phi\mathcal{D}\pi\,\rho[\phi,\pi]$$

Direct, wait-and-see method

- Sample the metastable phase
- Evolve the configurations
- Compute the rate

$$\Gamma(t_i) \approx -\frac{1}{P(t_i)}\frac{\Delta P_i}{\Delta t_i}$$

- Exponentially expensive

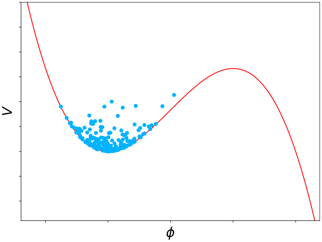
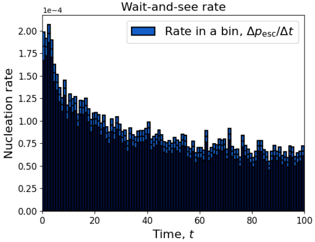


Figure: Sampling metastable state



Equilibrium assumption [Kramers '40]

- Rate sourced by equilibrium in meta

$$\rho_{\text{eq}} = \frac{1}{Z_{\text{meta}}} e^{-\beta H}$$

- Bypasses exponential suppression [Moore & Rummukainen '00]
  - Sampling from the barrier in equilibrium
  - Evolve to determine the barrier crossings
- Removing the equilibrium assumption

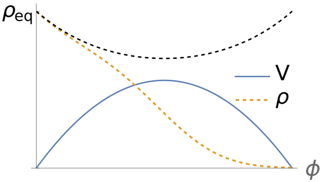


Figure: Boundary conditions for the distribution

Combining the strengths

- Sample equilibrium flux on the surface
  - Avoids exponential suppression
- Time evolve
  - ★ Rate in time directly
  - (Up to a multiplicative constant)

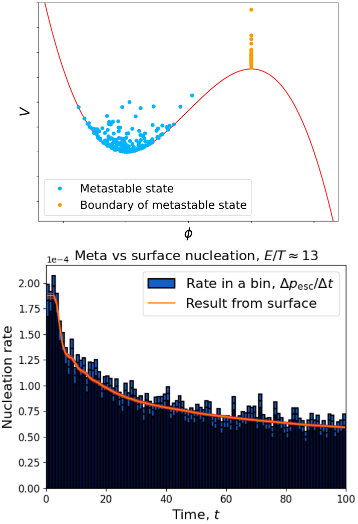


Figure: Different approaches

Modeling time dependence

- Understanding through flux out of metastable phase
- Numerically practical formula for time-dependent flux

Particle in one dimension

- Thermalized metastable phase ($\mathcal{R}$)
- Rate from the flux, I, through the boundary, $\partial\mathcal{R}$:

$$\Gamma = \frac{1}{P_{\text{meta}}} \underbrace{\left(-\frac{\mathrm{d}P_{\text{meta}}}{\mathrm{d}t}\right)}_{=I} = \frac{I}{P_{\text{meta}}}$$

- Time evolution
  - Flux goes down, $I \approx 0$ quickly
  - Probability: $P_{\text{meta}} \approx 1$

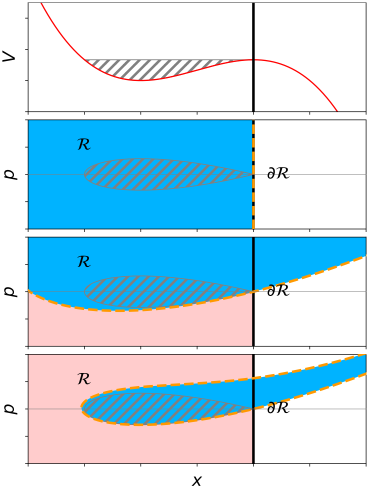


Figure: Thermal distribution for 1D particle escaping meta

Examining flux further

- Initial flux:

$$I_{\text{eq}} \equiv I(t = 0^+)$$
$$= \int_{\partial\mathcal{R}} \underbrace{\mathrm{d}S\, Z_{\text{meta}}^{-1} e^{-\beta H}\, \pi_\perp}_{\mathrm{d}I_{\text{eq}}} \theta(\pi_\perp)$$
$$= \int_{\partial\mathcal{R}^{(+)}} \mathrm{d}I_{\text{eq}}$$

  - [M&R '00] w/o time evolution part

- Some parts do not contribute

$$I(t) = I_{\text{eq}} - \int_{\partial\mathcal{R}^{(+)}} \mathrm{d}I_{\text{eq}}\, \theta(\text{not contr.})$$

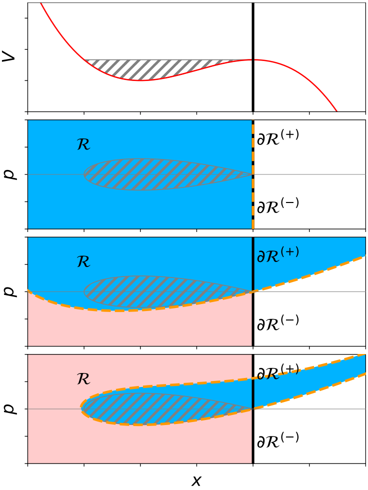


Figure: Thermal distribution for 1D particle escaping meta

Flux formula with trajectories

- Focusing on trajectories
  - Either $\mathrm{d}I_{\mathrm{eq}}$ or 0
- End points of thermalized pieces determine the contribution

$$I(t) = I_{\mathrm{eq}} - \int_{\partial\mathcal{R}^{(-)}} \mathrm{d}I_{\mathrm{eq}}\,\theta(\mathbf{z}(t) \notin \mathcal{R}) - \int_{\partial\mathcal{R}^{(+)}} \mathrm{d}I_{\mathrm{eq}}\,\theta(\mathbf{z}(t) \in \mathcal{R})$$

- Holds for dissipative case

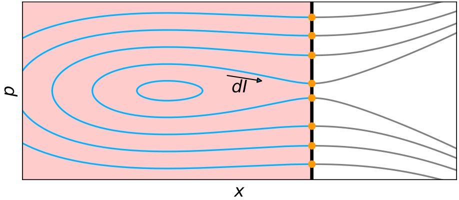


Figure: Phase space trajectories

Surface flux ensemble

- Translating into ensemble averages
- Weights proportional to equilibrium flux (or probability current)

$$\int_{\partial\mathcal{R}^{(\pm)}} \mathrm{d}I_{\mathrm{eq}} \bullet = I_{\mathrm{eq}} \times \int_{\partial\mathcal{R}^{(\pm)}} \frac{\mathrm{d}I_{\mathrm{eq}}}{I_{\mathrm{eq}}} \bullet$$

$$= I_{\mathrm{eq}} \times \left\langle \bullet \right\rangle_{\partial\mathcal{R}^{(\pm)}}$$

$$I(t) = I_{\mathrm{eq}} \times \Big(1 - \left\langle \theta(\mathbf{z}(t) \notin \mathcal{R}) \right\rangle_{\partial\mathcal{R}^{(-)}} - \left\langle \theta(\mathbf{z}(t) \in \mathcal{R}) \right\rangle_{\partial\mathcal{R}^{(+)}} \Big)$$

Sampling surface flux ensemble

- Lower-dimensional surface difficult
- Physical interpretation
  - Evolve with short Δt
  - Probability mass through matches the weight
- Method
  - Sample around $\partial\mathcal{R}$
  - Representatives within Δt of $\partial\mathcal{R}$
- (Or reweighting MR)

$$\mathrm{d}P = \overbrace{\mathrm{d}S\, Z_{\text{meta}}^{-1} e^{-\beta H}\, \pi_\perp}^{=\mathrm{d}I_{\text{eq}}} \Delta t$$
$$\propto \frac{\mathrm{d}I_{\text{eq}}}{I_{\text{eq}}}$$

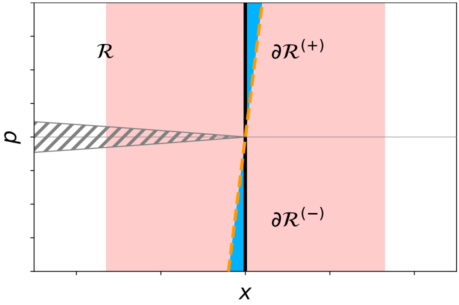
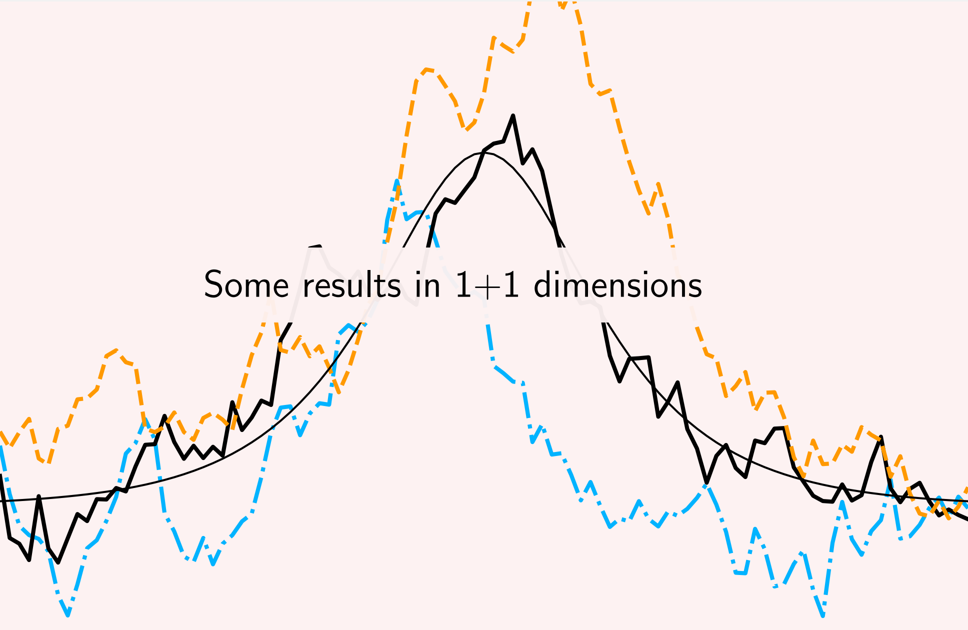


Figure: Red: thermal distribution
Blue: representatives

Results



Some results in 1+1 dimensions

Finite-volume effects

- Smaller volume
  - Stronger decrease
- Thermodynamic limit
  - Strong suppression: stationary rate

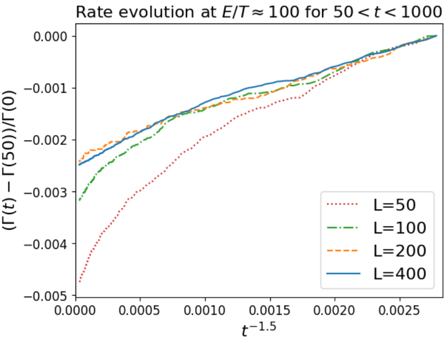


Figure: Rate evolving in high suppressions

Oscillons

- Long-lived, local, oscillating configurations
- Explain the late-time behavior

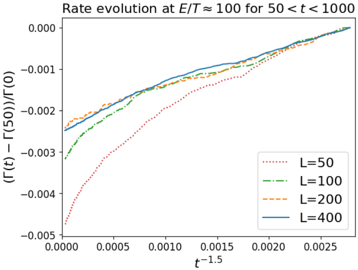


Figure: Late-time evolution is $C + at^{-n}$

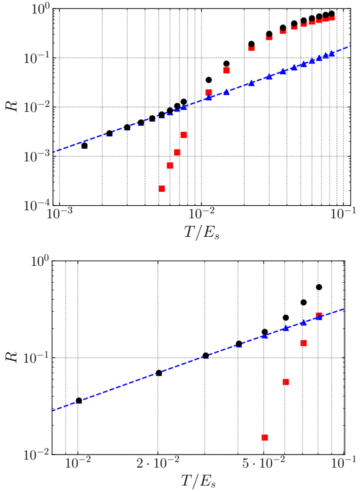


Figure: Absent sloshing contributions in the absense of oscillons (Bottom figure in a different theory with no oscillons)

Effect strength in suppression

- Strong effects in moderate suppressions!
- Strong suppression: mild effects
  - MR simulations takes into account the blue corrections

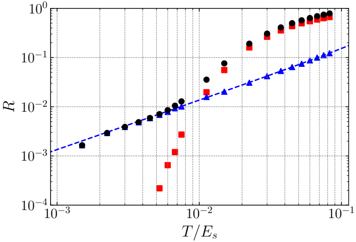


Figure: Multiplicative correction to the rate at late times is $1 - R$. Split into sloshing (red) and prompt (blue)

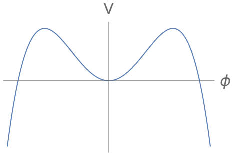
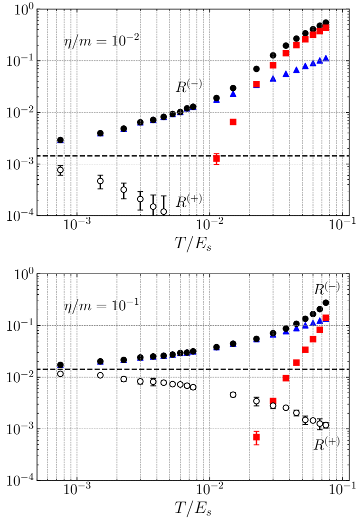


Figure: Model potential

Dissipative case



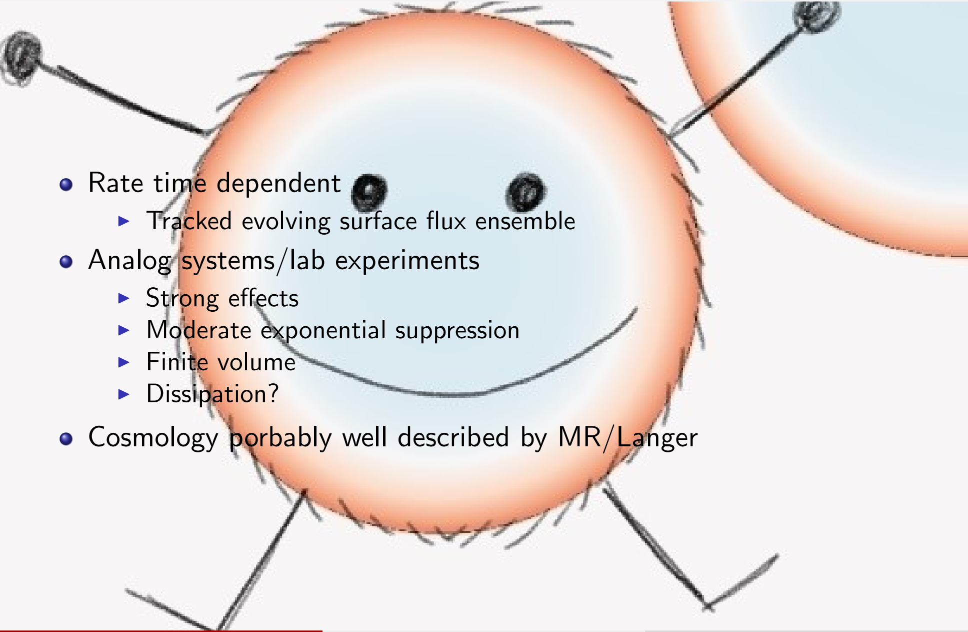
- Thermalization
  - Effects milder
  - Surface flux ensemble $\rightarrow$ meta ensemble
- Crucial for analog systems

Figure: Sloshing contributions (red) weakened by dissipation and noise

Summary

- Rate time dependent
  - Tracked evolving surface flux ensemble
- Analog systems/lab experiments
  - Strong effects
  - Moderate exponential suppression
  - Finite volume
  - Dissipation?
- Cosmology probably well described by MR/Langer

Thanks for listening!



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