Bubble nucleation
in SU(8) (de)confinement transition

Riikka Seppä
University of Helsinki

Based on [arXiv:2603.22088]
with Kari Rummukainen and David Weir

Nordic Lattice 2026
18.05.2026

Bubble nucleation rate from the lattice

[arXiv:hep-ph/0009132, hep-lat/0103036]

$$\Gamma = A_{\mathrm{dyn}} \exp(-F_c/T)$$

Bubble nucleation rate from the lattice

[arXiv:hep-ph/0009132, hep-lat/0103036]

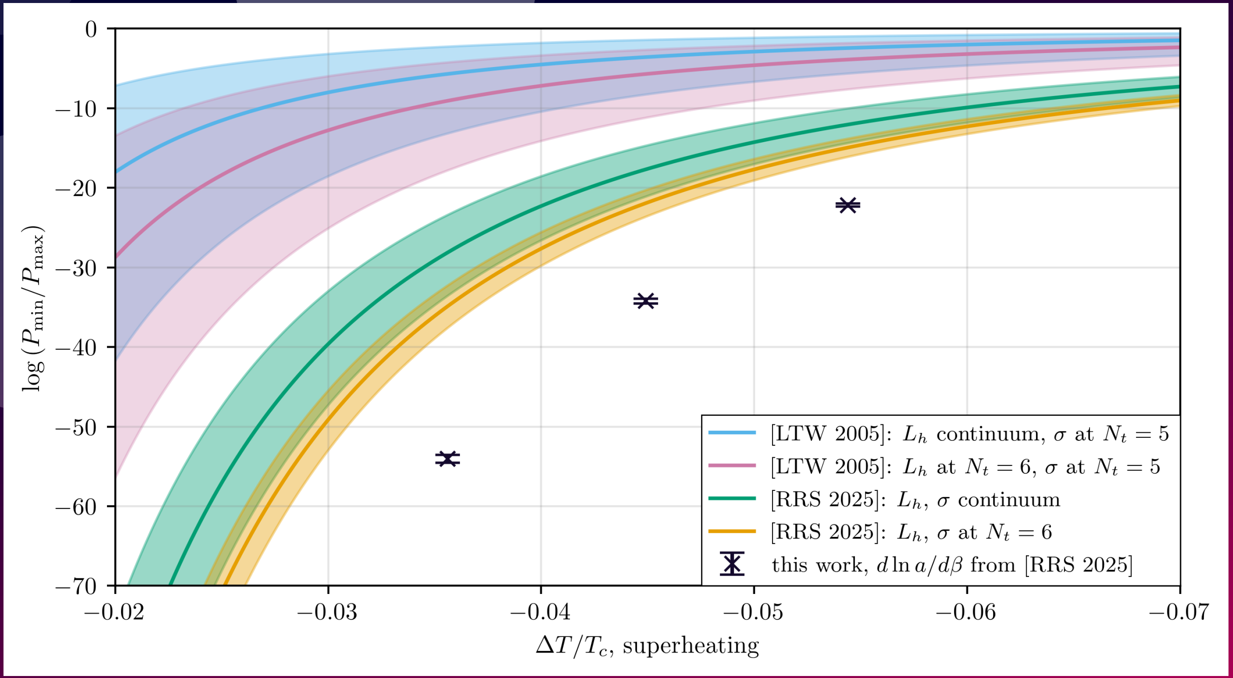
$$\Gamma = A_{\text{dyn}} \exp(-F_c/T)$$

1) **Multicanonical Monte Carlo**:
Boltzmann suppression of the rate $\exp(-S_b)$ or $\exp(-F_c/T)$

2) **Real time evolution of the critical bubble:**
Dynamical prefactors of the rate A_{dyn}

Does this work for strong coupling?

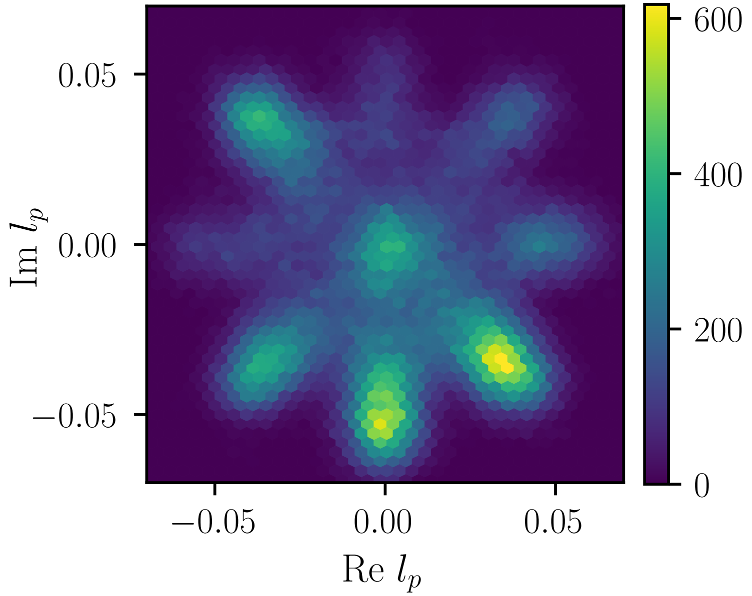
We use SU(8) deconfinement transition to find out!



- The exponential suppression can be calculated
- Dynamical prefactors: not as of yet

SU(N_c) confinement transition

- First order for $N_c \geq 3$
- Nonperturbative near transition
- Usual order parameter:
  the Polyakov loop
  - Confined phase: vanishing
  - Deconfined phase: N_c values, differing by a complex phase



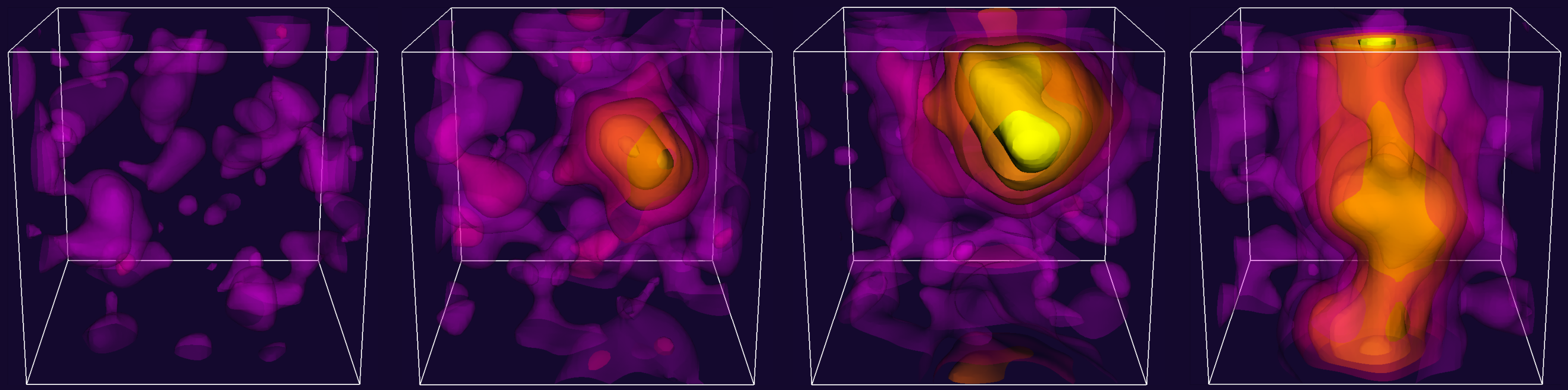
Why SU(8) **de**confinement?

Pure gauge SU(N_c):

Various DM models with differing number of colors
e.g [arXiv:2006.16429] [arXiv:2012.11614]

N_c = 8:

Can access large superheating → smaller critical bubbles → easier to fit on the lattice

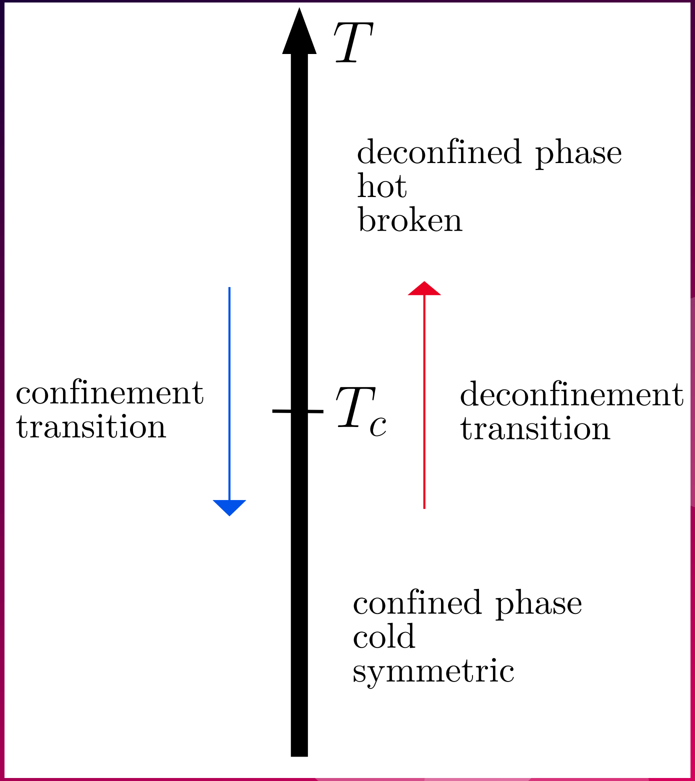


Why SU(8) **deconfinement**?

Deconfinement:

We thought this would be easier due to the vanishing Polyakov loop value

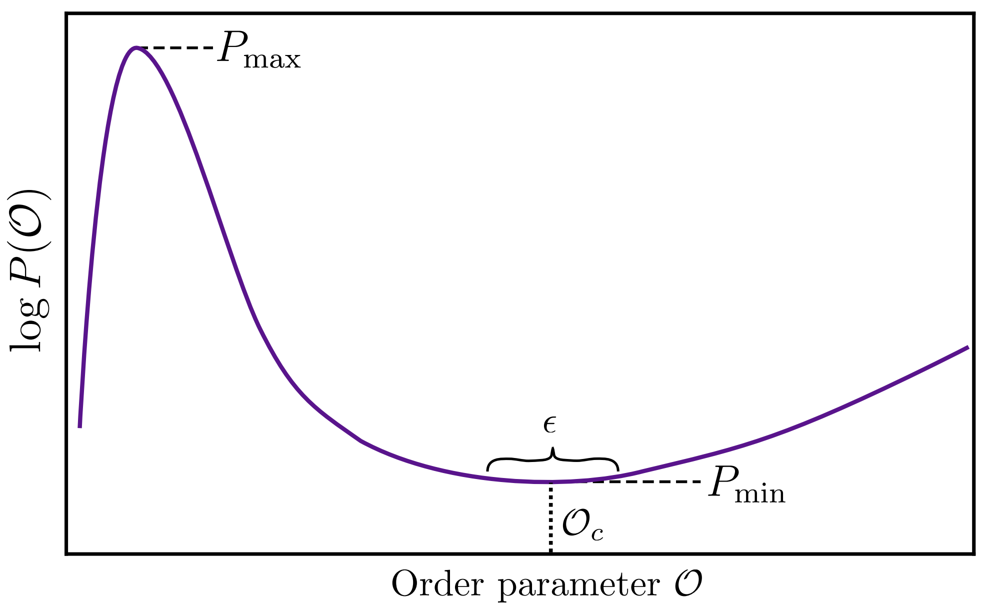
Work in progress: confinement transition



Statistical part of the rate from lattice

Sample configurations with Monte Carlo and measure order parameter $\mathcal{O}$ from each

→ Obtain the probability distribution of the order parameter, $P(\mathcal{O})$

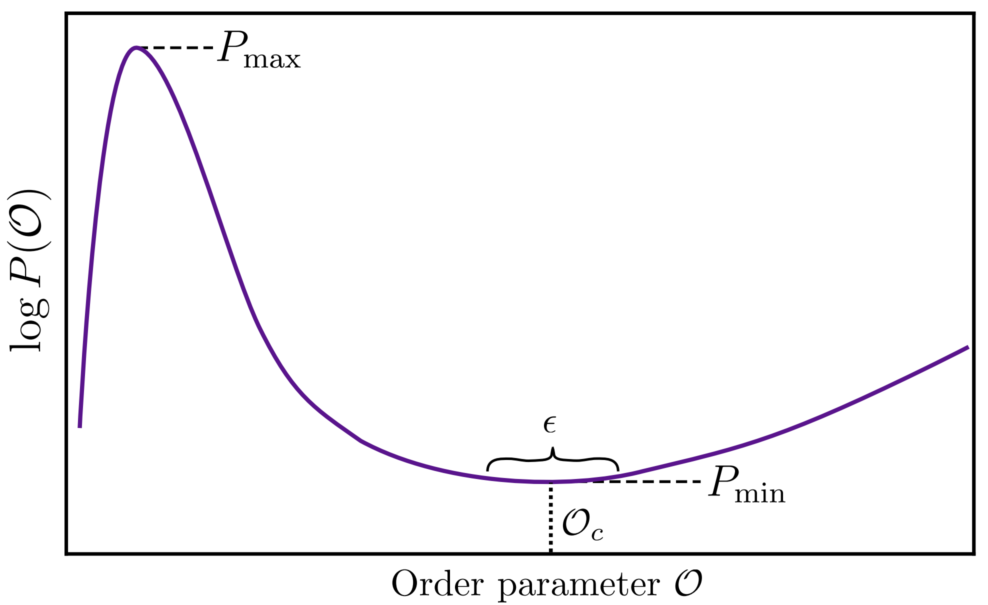


Statistical part of the rate from lattice

We want:

The probability of a critical bubble configuration, normalized by the metastable phase configurations

$$\frac{P(|\mathcal{O} - \mathcal{O}_c| < \epsilon/2)}{\epsilon P(\mathcal{O} < \mathcal{O}_c)} \sim \frac{P_{\text{min}}}{P_{\text{max}}}$$



Statistical part of the rate from lattice

Some simulation detail on 4D SU(8) simus

- **Finite temperature simulation**
- **Periodic lattice in all 4 dim**
- **Time extent N_t related to temperature** $T=1/(N_t a(\beta))$
- $S = \beta \sum_{x,\mu>\nu} \left[1 - \frac{1}{8}\mathrm{ReTr}U_{\mu\nu}(x)\right]$

For SU(N_c):
β_c, σ, L_h
well known
[arxiv:hep-lat/0502003]
[arxiv:2506.15509]

Order parameter:
Polakov loop vol average

$$l_p(\vec{x}) = \prod_{t=0}^{N_t-1} U_4(\vec{x}, t)$$

$$|\bar{l}_p| = \left| \frac{1}{N_s^3} \sum_{\vec{x}} l_p(\vec{x}) \right|$$

Statistical part of the rate from lattice

Some simulation detail on 4D SU(8) simus

- **Finite temperature simulation**
- **Periodic lattice in all 4 dim**
- **Time extent N_t related to temperature** $T = 1/(N_t a(\beta))$
- $S = \beta \sum_{x,\mu>\nu} \left[1 - \frac{1}{8} \mathrm{R}\right.$

We use only $N_t = 6$

Number spatial lattice sites $V = N_s^3$ with $N_s = 60, .., 80$

For SU(N_c):
β_c, σ, L_h
well known
[arxiv:hep-lat/0502003]
[arxiv:2506.15509]

Order parameter:
Polakov loop vol average

$$l_p(\vec{x}) = \prod_{t=0}^{N_t-1} U_4(\vec{x}, t)$$

$$|\bar{l}_p| = \left| \frac{1}{N_s^3} \sum_{\vec{x}} l_p(\vec{x}) \right|$$

Statistical part of the rate from lattice

Multicanonical

- Updates: 1 heatbath sweep per 5-6 overrelaxation sweeps
- Bubble configurations are extremely rare → use **multicanonical algorithm**

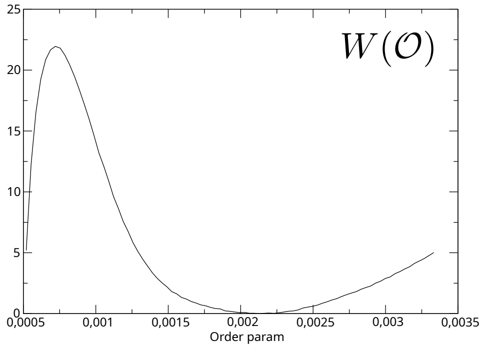
Statistical part of the rate from lattice

Multicanonical

- Updates: 1 heatbath sweep per 5-6 overrelaxation sweeps
- Bubble configurations are extremely rare → use **multicanonical algorithm**

Sample with an additional weight
$$p \propto \exp(-S + W(\mathcal{O}))$$

instead of canonical
$$p \propto \exp(-S)$$

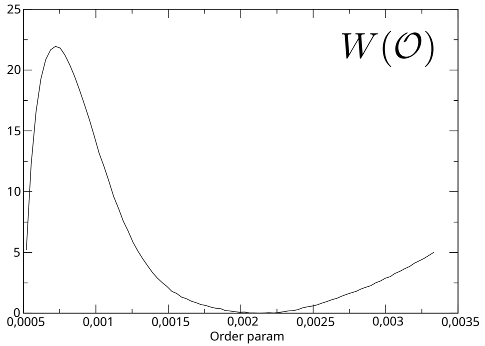


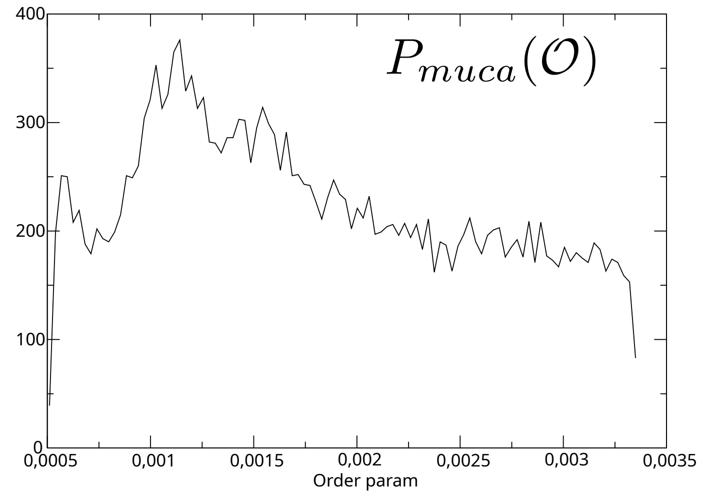
Statistical part of the rate from lattice

Multicanonical

- Updates: 1 heatbath sweep per 5-6 overrelaxation sweeps
- Bubble configurations are extremely rare → use **multicanonical algorithm**

Sample with an additional weight
$$p \propto \exp(-S + W(\mathcal{O}))$$
instead of canonical
$$p \propto \exp(-S)$$





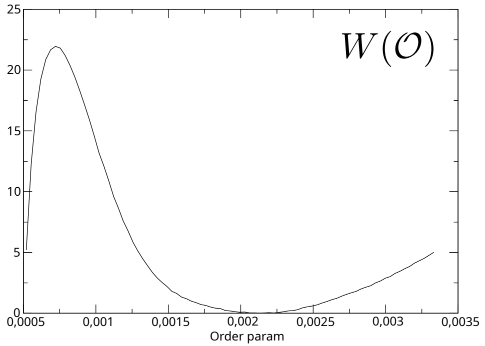
Statistical part of the rate from lattice

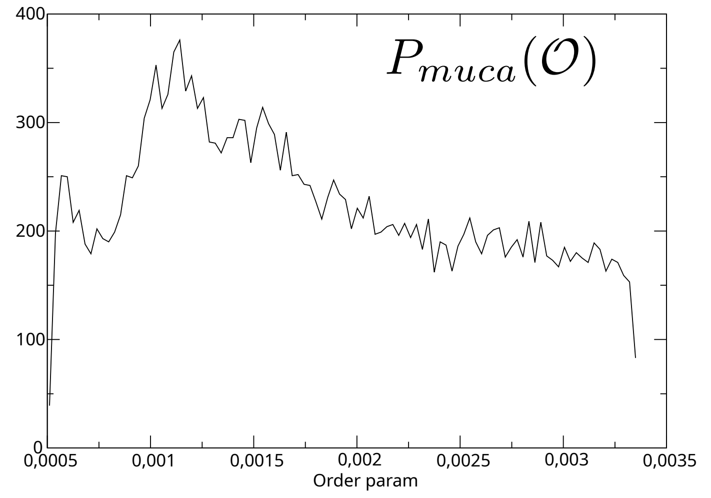
Multicanonical

- Updates: 1 heatbath sweep per 5-6 overrelaxation sweeps
- Bubble configurations are extremely rare → use **multicanonical algorithm**

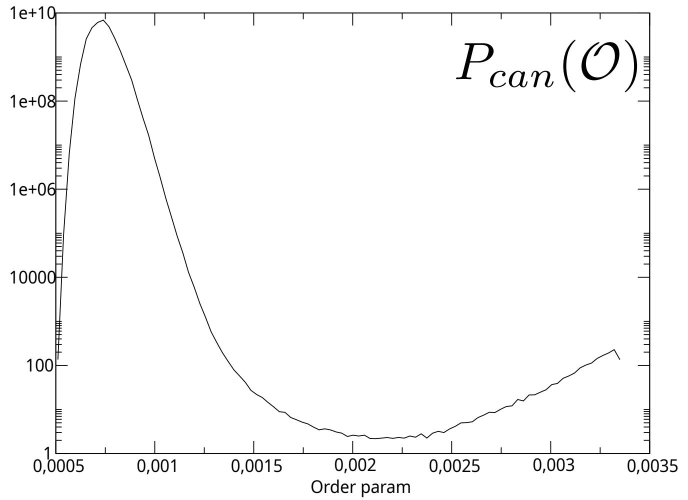
Sample with an additional weight
$p \propto \exp(-S + W(\mathcal{O}))$

instead of canonical
$p \propto \exp(-S)$





Reweight with
$\exp(-W(\mathcal{O}))$



Statistical part of the rate from lattice

Multicanonical

- Weight function needs to be constructed iteratively
- $W(\mathcal{O})$ can be constructed for any order parameter $\mathcal{O}$; generally results in different W

Iteration as in Appendix B of [arxiv:2502.14610]

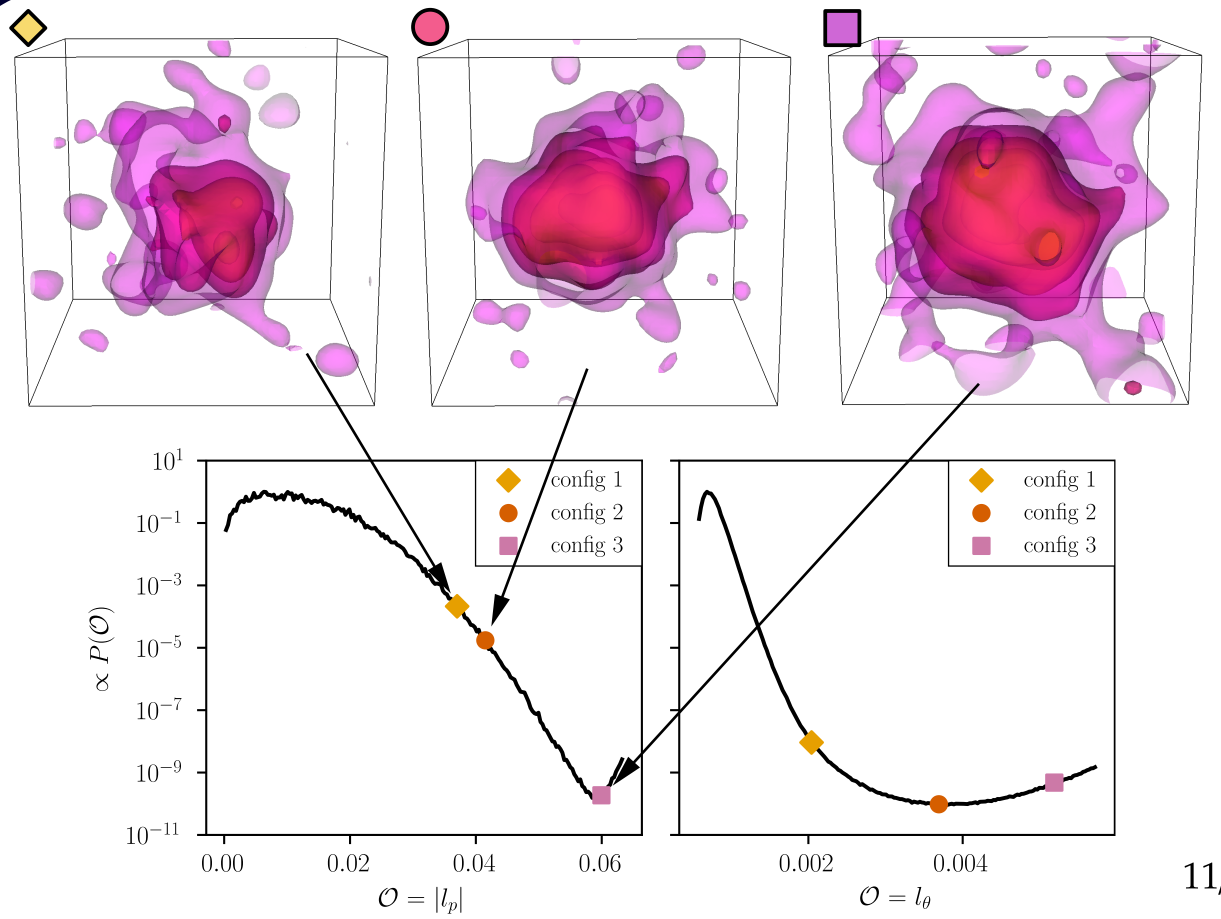
Trouble identifying bubbles:

SU(8) deconfinement transition

[arXiv:2603.22088]

Some order parameters are **better** than others at **identifying** bubbles

Bad one:
Bubble config values coincide with metastable peak values



Trouble identifying bubbles:

SU(8) deconfinement transition

[arXiv:2603.22088]

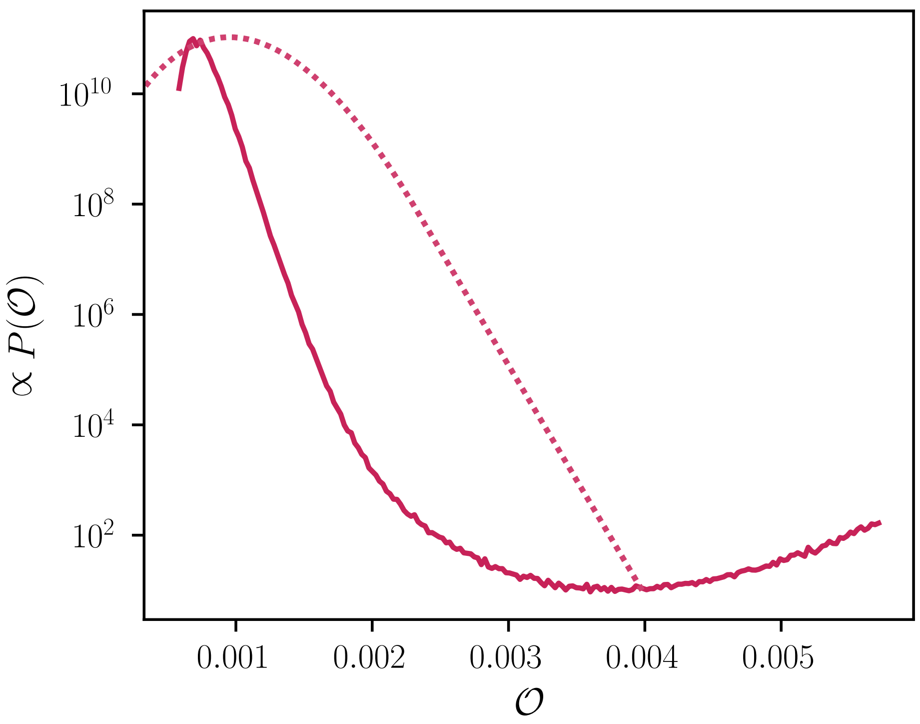
The bulk phase fluctuations drown out the bubble signal
→ reduce fluctuations

- Nearest neighbour smearing of Polyakov loop field, $l_s^{(0)} = l_p$

$$l_s^{(m)}(\vec{x}) = \frac{1}{4}\left(l_s^{(m-1)}(\vec{x}) + \frac{1}{2}\sum_{\hat{i}} l_s^{(m-1)}(\vec{x}+\hat{i}) \right)$$

- Inspired by [arxiv:2404.01876], substract bulk peak value A and square, $(l_s - A)^2$

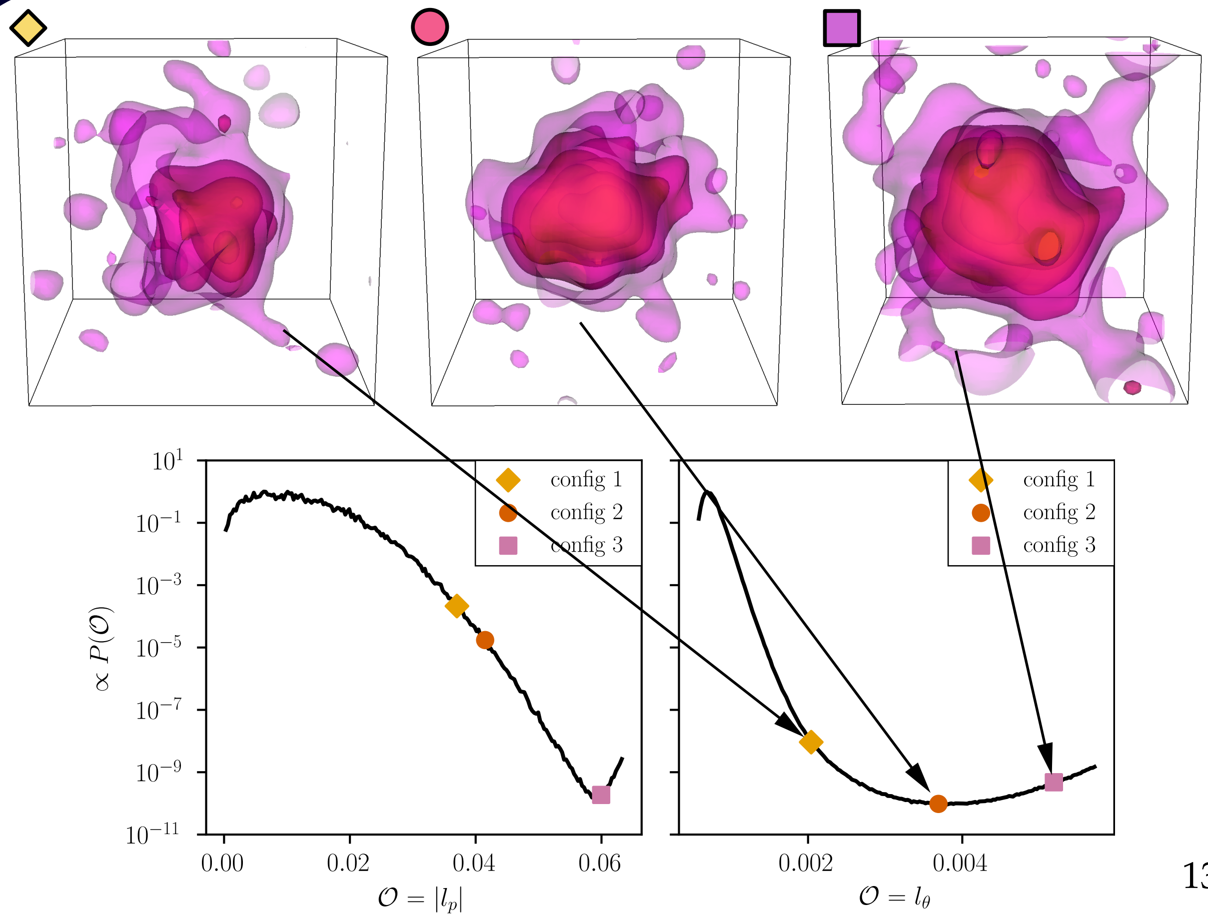
$$\bar{l}_\theta = \frac{1}{N_s^3}\left(\sum_{\vec{x}} |l_s(\vec{x})|^2 - 2A\sum_{\vec{x}} |l_s(\vec{x})| \right)$$



Trouble identifying bubbles:
SU(8) deconfinement transition
[arXiv:2603.22088]

Some order parameters are **better** than others at **identifying** bubbles

Good one:
Bubble config values are separated from metastable peak values



Thin wall approximation

Consider a sphere of stable phase within the metastable phase

The free energy at $\Delta T = T_c - T$
will be determined only by

- **Interface tension** $\sigma \propto A_r$
- **Latent heat** $L_h \propto V_r$

- Spherical shape minimizes the surface area wrt volume

$$F(r) = 4\pi r^2 \sigma - \frac{4\pi}{3} r^3 L_h \frac{\Delta T}{T}$$

- Critical bubble: sphere with maximum free energy

$$r_c = \frac{2\sigma}{L_h}\left(\frac{\Delta T}{T}\right)^{-1}, \; F_c = \frac{16\pi}{3}\frac{\sigma^3}{L_h^2}\left(\frac{\Delta T}{T}\right)^{-2}$$

TW rate is then:

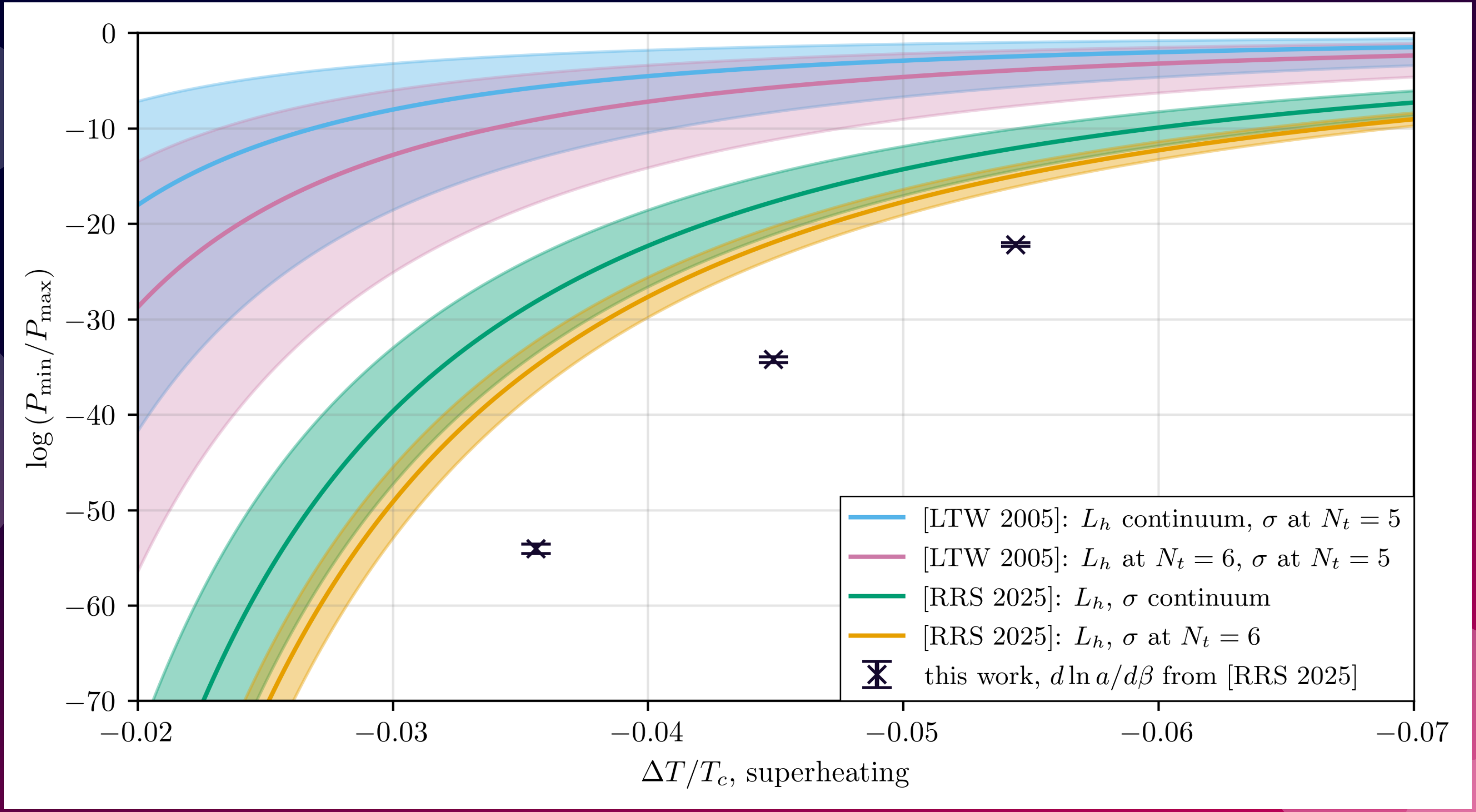
$$\Gamma \simeq A \exp(-F_c/T)$$

Same from lattice:

$$F_c \simeq -\log(P_{\min}/P_{\max})$$

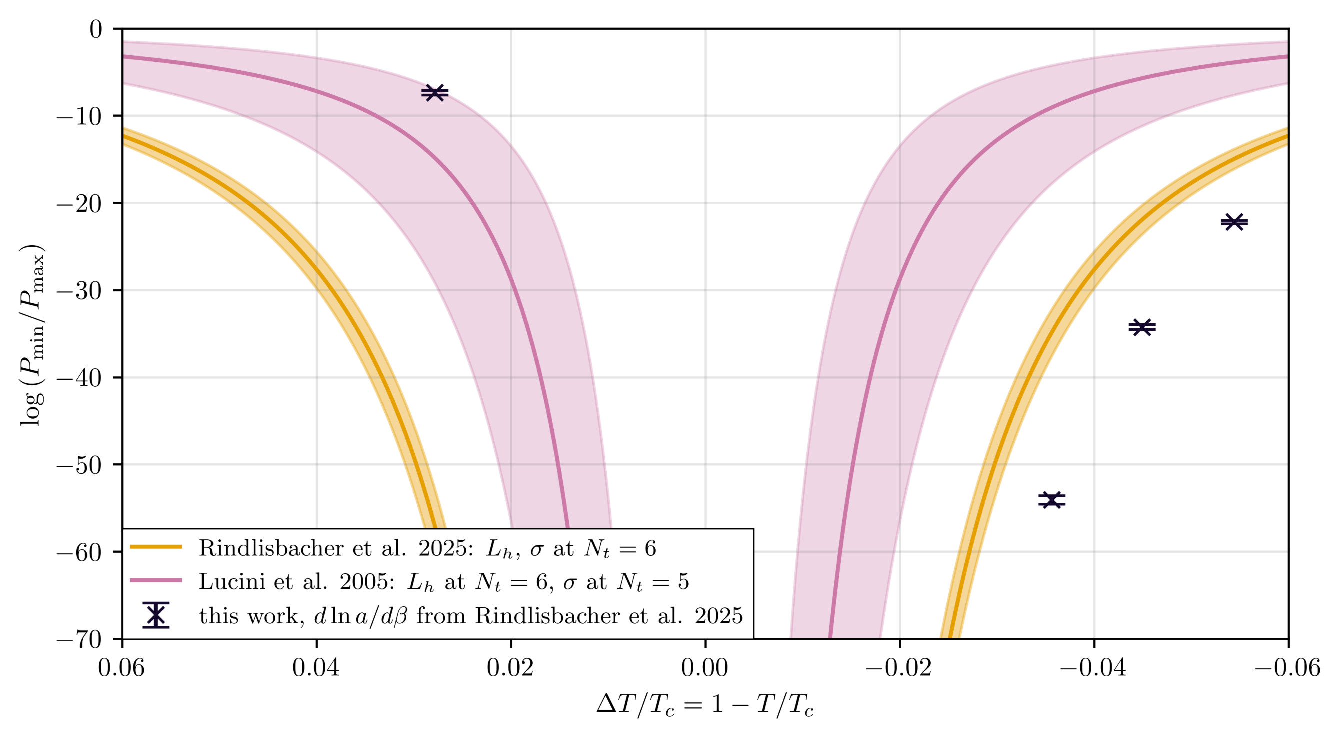
$F(r) = 4\pi r^2\sigma - \frac{4\pi}{3} r^3 L_h \frac{\Delta T}{T} \pm \gamma(T) 8\pi r$

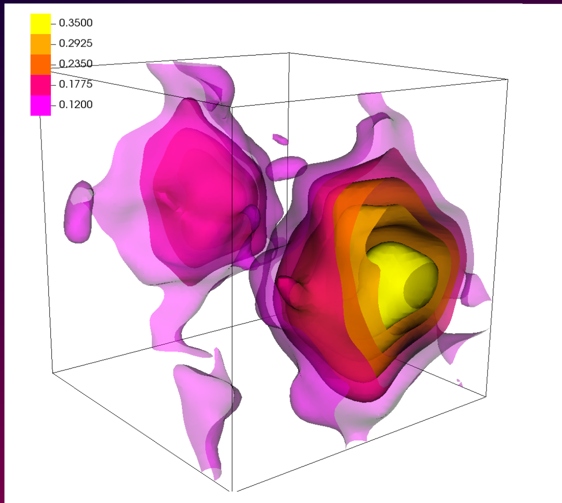
Deconfinement results

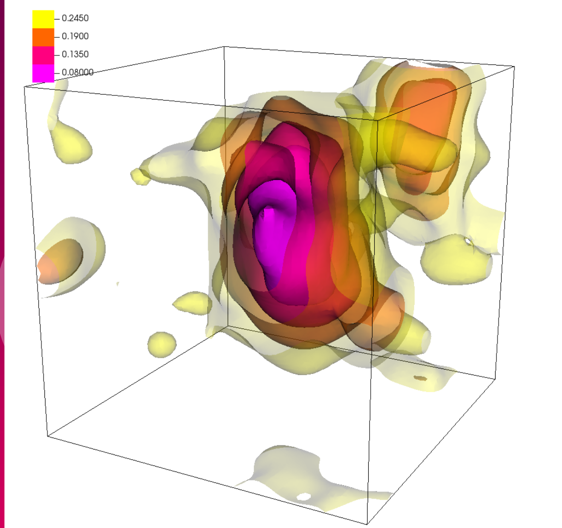


$$E(r) = 4\pi r^2\sigma - \frac{4\pi}{3} r^3 L_h \frac{\Delta T}{T} \pm \gamma(T) 8\pi r$$

Preliminary confinement vs. deconf. results

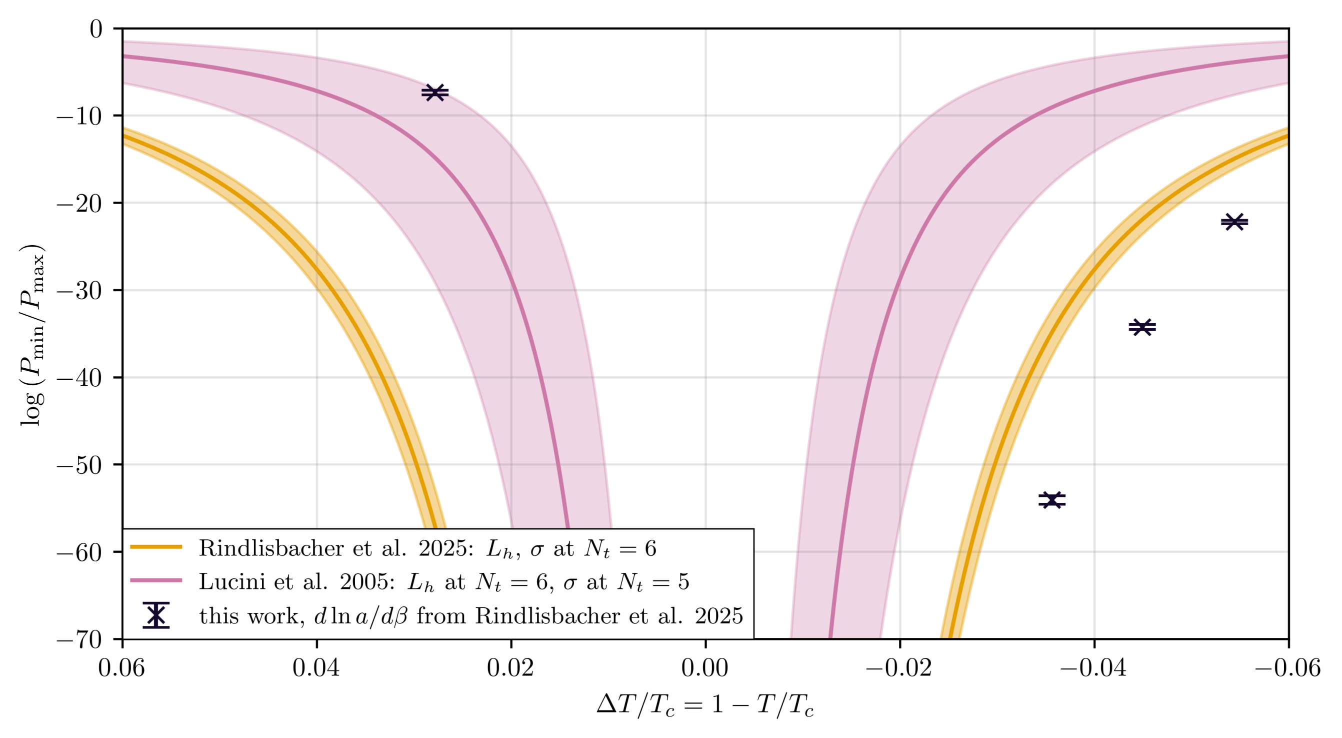


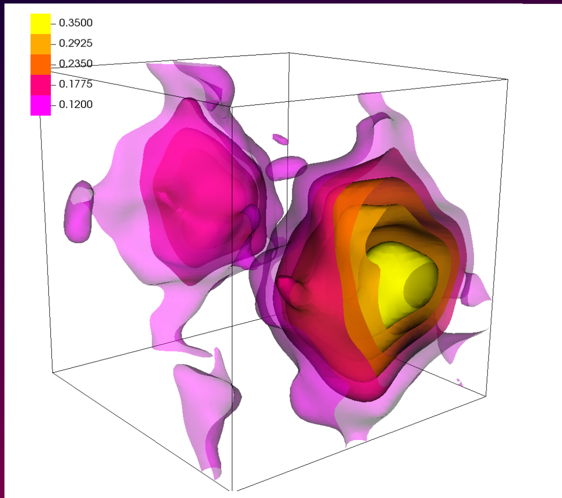


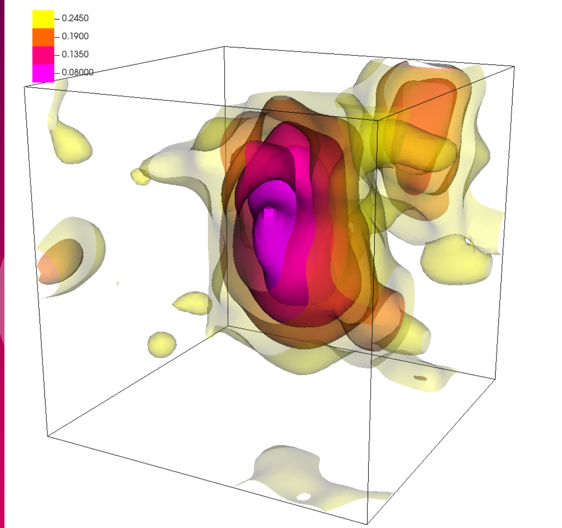


$F(r) = 4\pi r^2\sigma - \frac{4\pi}{3}r^3 L_h \frac{\Delta T}{T} \pm \gamma(T)8\pi r$

Preliminary confinement vs. deconf. results





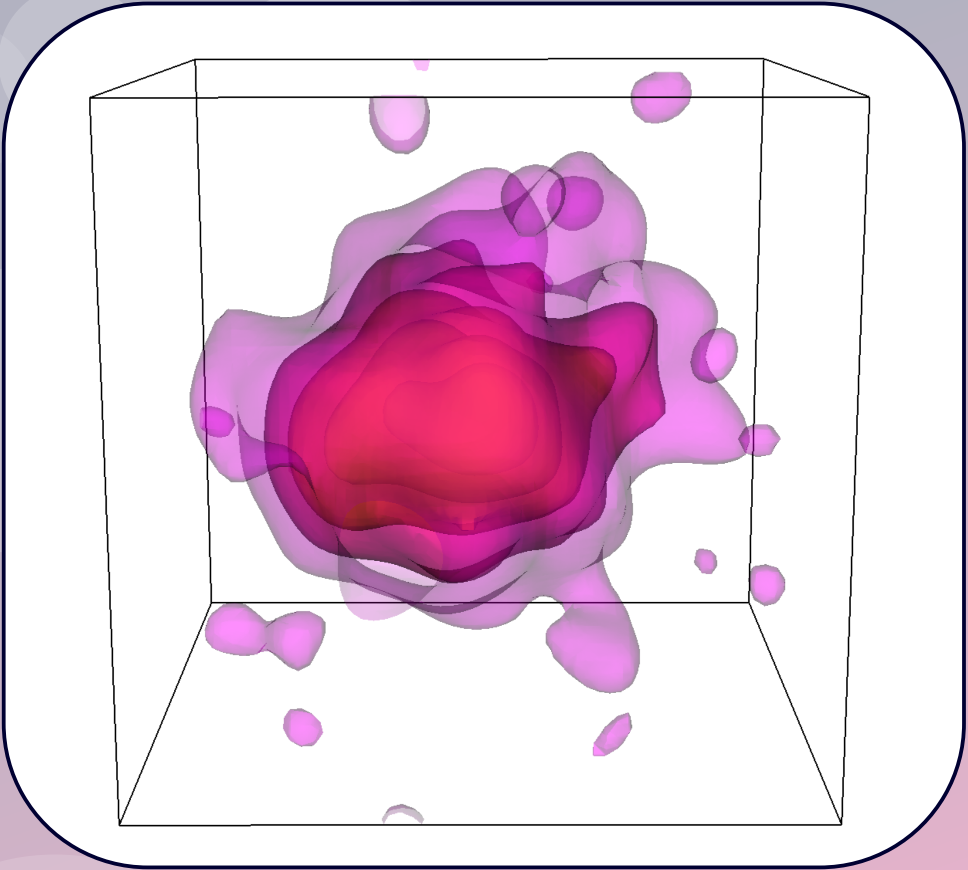


$$F(r) = 4\pi r^2\sigma - \frac{4\pi}{3}r^3 L_h \frac{\Delta T}{T} \pm \gamma(T)8\pi r$$

[Kajantie et al., 1992, Kajantie et al., 1993]

[arxiv:2508.10091]

Are these bubbles really critical?



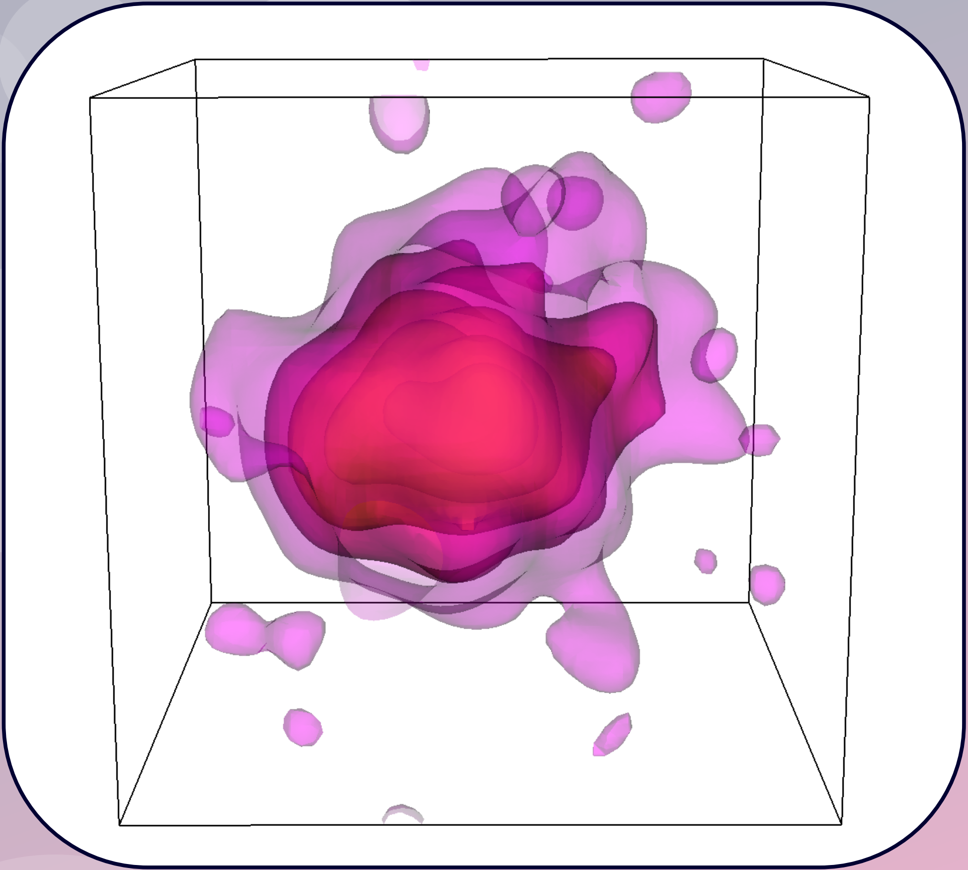
Statistical part of the rate is order parameter **dependent**

Dynamical factors would counteract this

$$A_{\mathrm{dyn}} \sim \langle \frac{1}{2}\mathbf{d} \rangle \times \langle |\Delta\mathcal{O}/\Delta t|_{\mathcal{O}_c} \rangle$$

$$\mathbf{d} = \frac{1}{N_{\mathrm{traj}}} \sum_{\mathrm{traj}} \frac{\delta_{\mathrm{tunnel}}}{N_{\mathrm{cross}}}$$

Are these bubbles really critical?

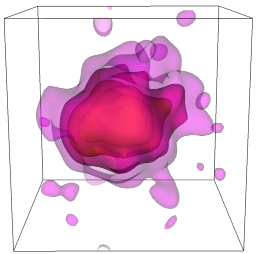


Computing dynamical prefactors:

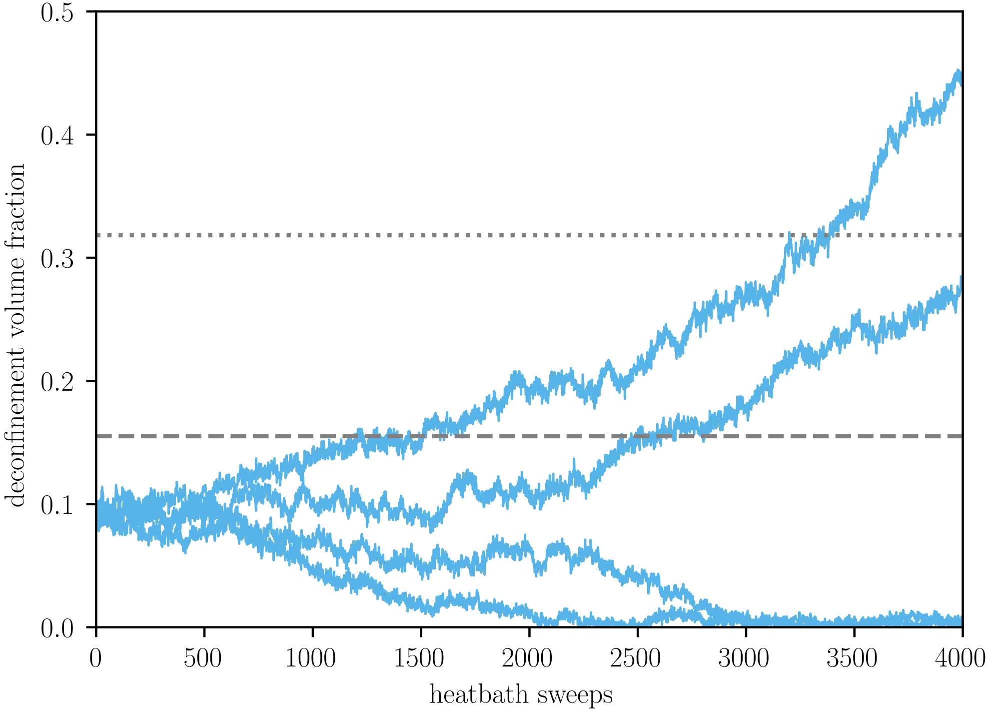
Weakly coupled non-Abelian:

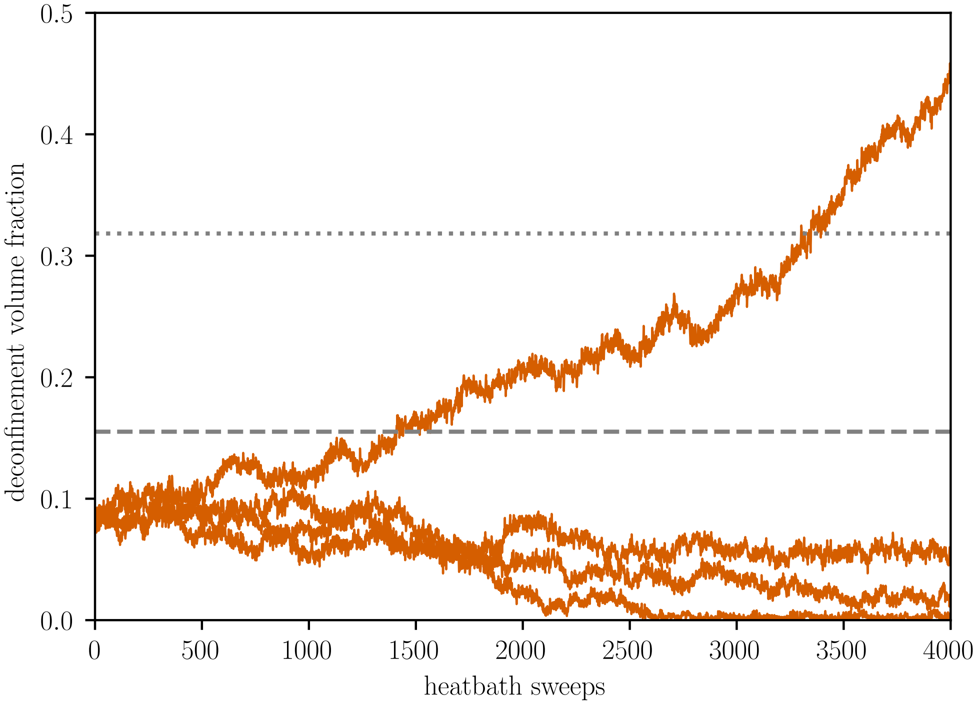
instead of real time evolution, can do Langevin updates
(ok upto 1/log(1/g), good enough)
[Bödeker, arxiv:hep-ph/9801430]

Further, heatbath updates instead of Langevin [Moore, arxiv:hep-ph/9810313]

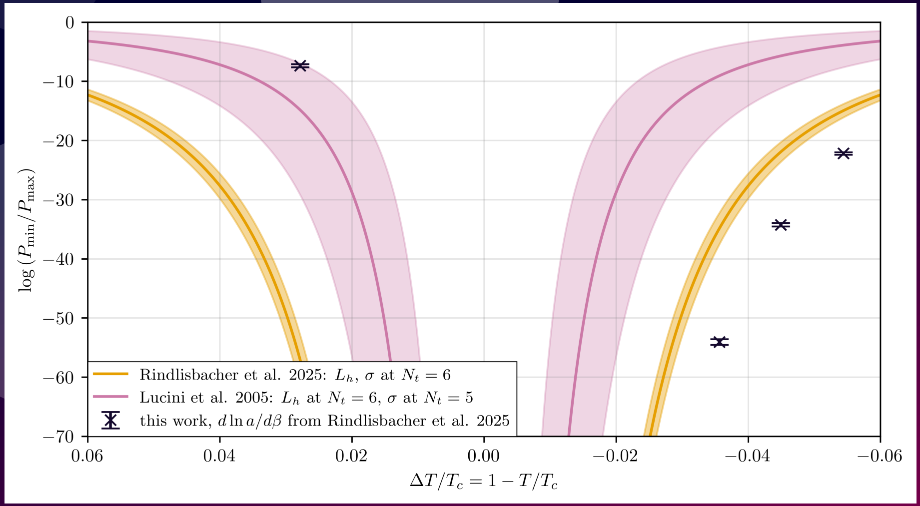


Are these bubbles really critical?





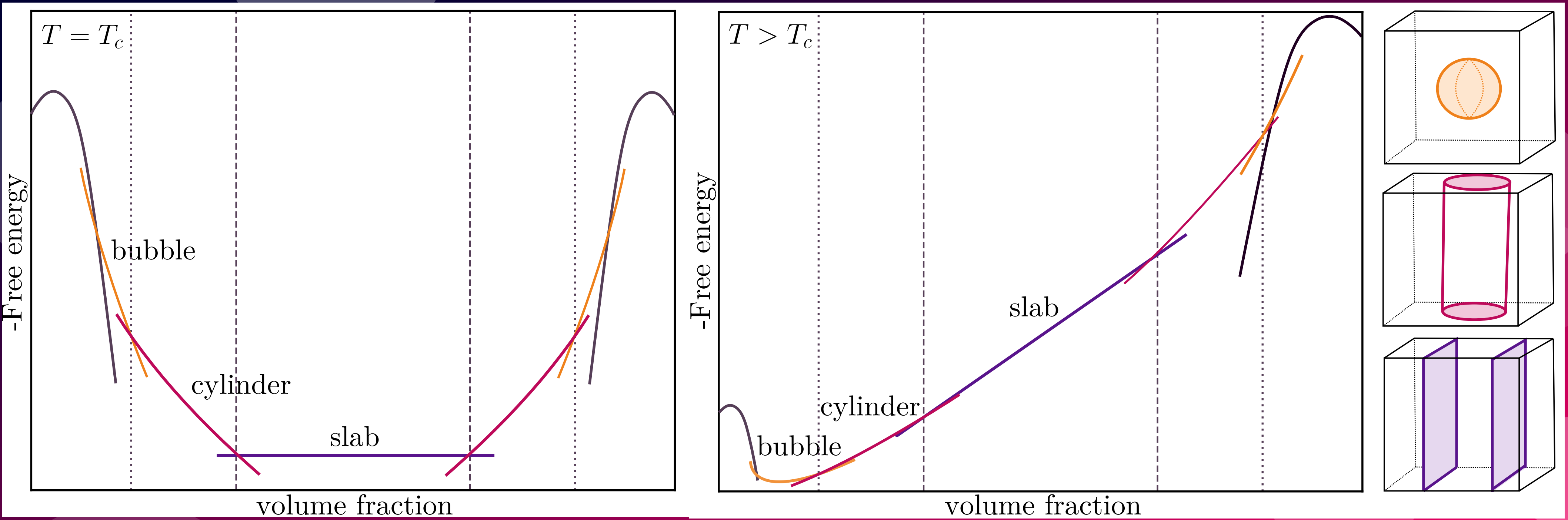
Thank you!



Some key points:

- Nucleation rate partially obtainable from lattice in confining theories
- Deconfinement $\neq$ confinement
- Order parameter choice important

Backup 1



$T = T_c$
-Free energy
bubble
cylinder
slab
volume fraction
$T > T_c$
-Free energy
slab
cylinder
bubble
volume fraction