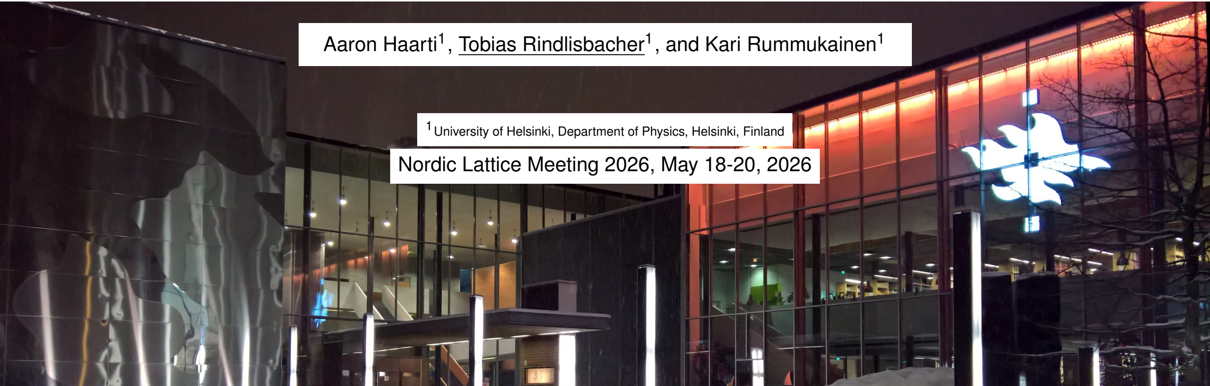
Measuring d-d interface tensions in SU(N) gauge theory with the capillary wave method

Aaron Haarti[1], Tobias Rindlisbacher[1], and Kari Rummukainen[1]

[1] University of Helsinki, Department of Physics, Helsinki, Finland

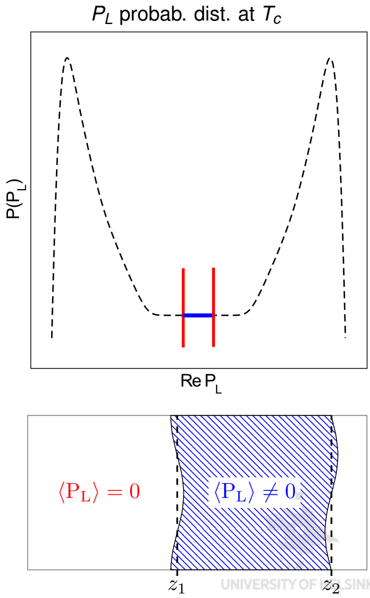
Nordic Lattice Meeting 2026, May 18-20, 2026



Capillary wave method for confined-deconfined interface tension **[Moore & Turok (1997)]**

→ avoid supercritical slowing down by sampling mixed phase ensemble.

- z-elongated lattice: $V = \underbrace{N_{x,y}^2 \times N_z}_{V_s} \times N_t$ with $N_t \ll N_{x,y} \ll N_z$
- pure SU(N) gauge theory: $S_G[U] = \frac{\beta}{N} \sum_{x,\mu<\nu} \mathrm{Re}\,\mathrm{tr}(\mathbb{1} - U_{\mu\nu}(x))$
- Monte Carlo (heatbath + overrelaxtion)



UNIVERSITY OF HELSINKI

Capillary wave method for confined-deconfined interface tension

[Moore & Turok (1997)]

➔ avoid supercritical slowing down by sampling mixed phase ensemble.

- z-elongated lattice: $V = \underbrace{N_{x,y}^2}_{V_s} \times N_z \times N_t$ with $N_t \ll N_{x,y} \ll N_z$

- pure SU(N) gauge theory: $S_G[U] = \frac{\beta}{N} \sum_{x,\mu<\nu} \mathrm{Re\,tr}(\mathbb{1} - U_{\mu\nu}(x))$

- Monte Carlo (heatbath + overrelaxtion)

- Initialize simulation:

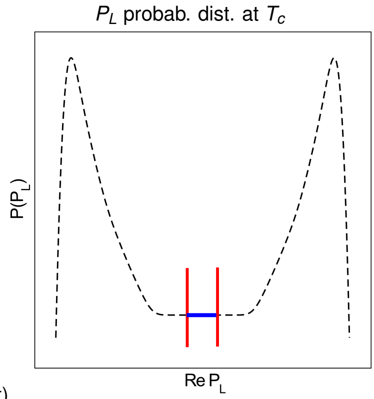
  1) induce mixed phase state: $\beta = \begin{cases} \tilde{\beta}_c - \Delta\beta \text{ if } z < N_z/2 \\ \tilde{\beta}_c + \Delta\beta \text{ if } z \geq N_z/2 \end{cases}$, $\tilde{\beta}_c \approx \beta_c$-estimate

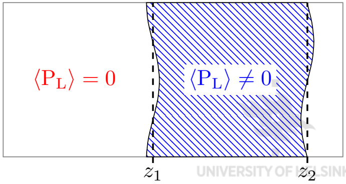
  2) set $\beta = \tilde{\beta}_c \; \forall z$ and impose constraint $\langle P_L \rangle \in [(1/2 - c)\, P_0, (1/2 + c)\, P_0]$

  where $P_0 = |\langle P_L \rangle_{\text{deconf.}}|$, $c \sim 10^{-2}$. (constr. implemented via accept/reject)

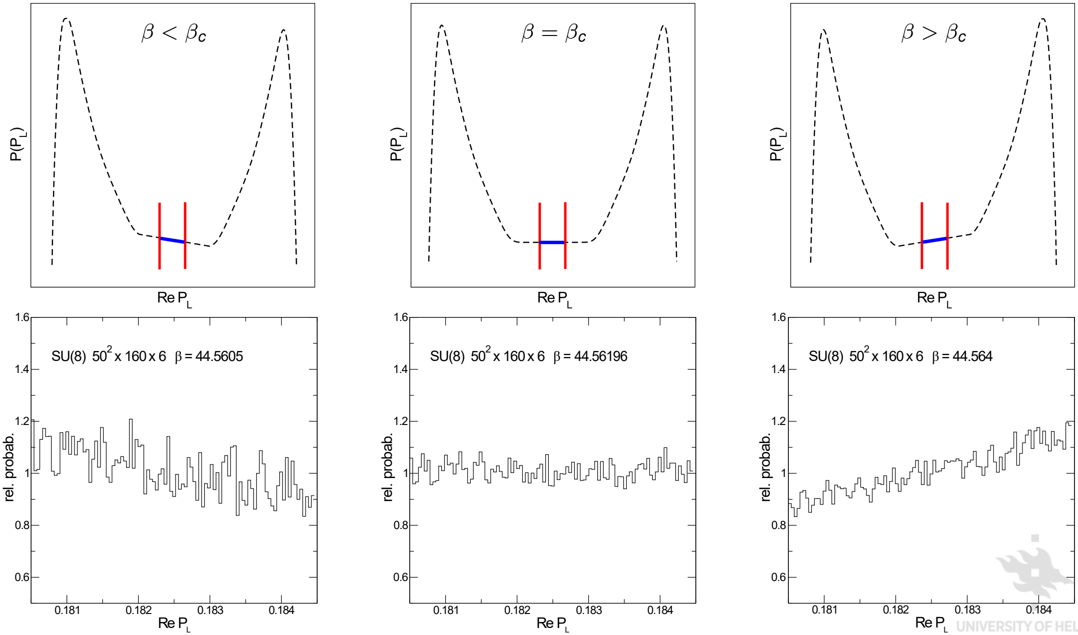
  3) thermalize (with constraint)

- Generate mixed phased configurations for measurements (with constraint)



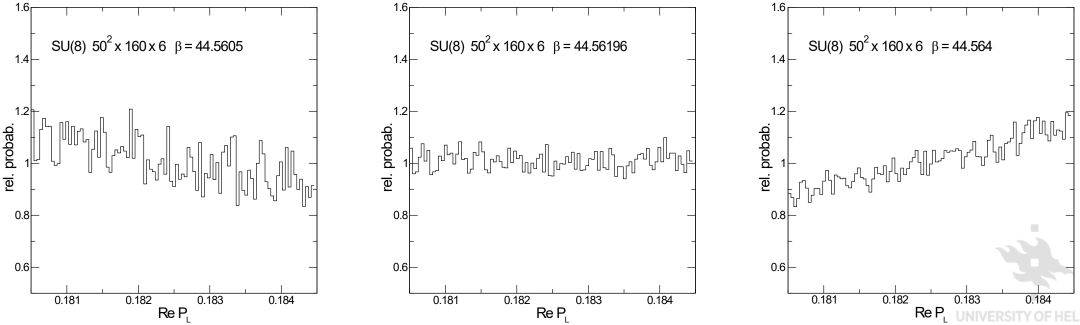


Determining the critical inverse coupling β_c



Determining the critical inverse coupling β_c

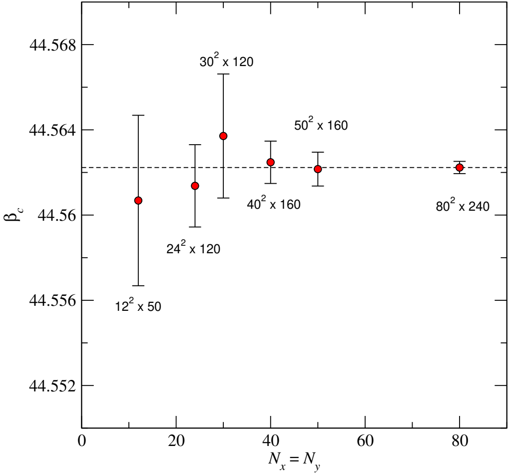
➔ can use reweighting to find precise $\beta = \beta_c$ for which Polyakov loop histogram is flat.



Determining the critical inverse coupling β_c

➔ can use reweighting to find precise $\beta = \beta_c$ for which Polyakov loop histogram is flat.

➔ exponentially suppressed finite volume effects



UNIVERSITY OF HELSINKI

Surface tension from surface fluctuation spectrum **[Moore & Turok (1997)]**

Fluctuating thin surface of size L^2, parametrized over x-y-plane.

Surface fluctuations $z(x, y)$ at finite temp. T $\rightarrow$ two-dimensional field theory: $Z = \int \prod_{x,y} \mathrm{d}z(x, y) e^{-H/T}$

with Hamiltonian:

$$H = \sigma \int_0^L \mathrm{d}x\, \mathrm{d}y \sqrt{1 + |\nabla z|^2} = \sigma \int_0^L \mathrm{d}x\, \mathrm{d}y \left[1 + \frac{1}{2}|\nabla z|^2 - \frac{1}{8}|\nabla z|^4 + \ldots\right].$$

Surface tension from surface fluctuation spectrum **[Moore & Turok (1997)]**

Fluctuating thin surface of size L^2, parametrized over x-y-plane.

Surface fluctuations $z(x,y)$ at finite temp. T $\rightarrow$ two-dimensional field theory: $Z = \int \prod_{x,y} \mathrm{d}z(x,y) e^{-H/T}$

with Hamiltonian:

$$H = \sigma \int_0^L \mathrm{d}x\,\mathrm{d}y\,\sqrt{1+|\nabla z|^2} = \sigma \int_0^L \mathrm{d}x\,\mathrm{d}y\left[1+\frac{1}{2}|\nabla z|^2 - \frac{1}{8}|\nabla z|^4 + \ldots\right] \approx \sigma \int_0^L \mathrm{d}x\,\mathrm{d}y\left[1+\frac{1}{2}|\nabla z|^2\right].$$

➔ Higher-order terms non-renormalizable $\rightarrow$ vanish in IR

Surface tension from surface fluctuation spectrum **[Moore & Turok (1997)]**

Fluctuating thin surface of size L^2, parametrized over x-y-plane.

Surface fluctuations $z(x,y)$ at finite temp. T $\rightarrow$ two-dimensional field theory: $Z = \int \prod_{x,y} \mathrm{d}z(x,y) e^{-H/T}$

with Hamiltonian:

$$H = \sigma \int_0^L \mathrm{d}x\,\mathrm{d}y \sqrt{1+|\nabla z|^2} = \sigma \int_0^L \mathrm{d}x\,\mathrm{d}y \left[1 + \frac{1}{2}|\nabla z|^2 - \frac{1}{8}|\nabla z|^4 + \dots\right] \approx \sigma \int_0^L \mathrm{d}x\,\mathrm{d}y \left[1 + \frac{1}{2}|\nabla z|^2\right].$$

➔ Higher-order terms non-renormalizable → vanish in IR

Fourier rep.: $z(x,y) = \sum_{n_x,n_y} \widehat{z}(n_x,n_y) e^{-2\pi i(n_x x + n_y y)/L}$, $\widehat{z}(n_x,n_y) = \frac{1}{L^2}\int_0^L \mathrm{d}x\mathrm{d}y\, z(x,y) e^{2\pi i(n_x x + n_y y)/L}$

➔ $Z \propto \prod_{\substack{n_x,n_y \\ n_x^2+n_y^2 \neq 0}} \int \mathrm{d}\widehat{z}(n_x,n_y)\, \mathrm{e}^{-|\widehat{z}(n_x,n_y)|^2\, 4\,\pi^2\, n^2\, \sigma/T}$ s.t. $\left\langle |\widehat{z}(n_x,n_y)|^2 \right\rangle = \frac{T}{4\,\pi^2\, n^2\,\sigma}$, $n^2 \equiv n_x^2 + n_y^2 \neq 0$

Surface tension from surface fluctuation spectrum **[Moore & Turok (1997)]**

Fluctuating thin surface of size L^2, parametrized over x-y-plane.

Surface fluctuations $z(x,y)$ at finite temp. T $\rightarrow$ two-dimensional field theory: $Z = \int \prod_{x,y} \mathrm{d}z(x,y) e^{-H/T}$

with Hamiltonian:

$$H = \sigma \int_0^L \mathrm{d}x\,\mathrm{d}y \sqrt{1+|\nabla z|^2} = \sigma \int_0^L \mathrm{d}x\,\mathrm{d}y \left[1 + \frac{1}{2}|\nabla z|^2 - \frac{1}{8}|\nabla z|^4 + \ldots\right] \approx \sigma \int_0^L \mathrm{d}x\,\mathrm{d}y \left[1 + \frac{1}{2}|\nabla z|^2\right] .$$

➔ Higher-order terms non-renormalizable $\rightarrow$ vanish in IR

Fourier rep.: $z(x,y) = \sum_{n_x,n_y} \widehat{z}(n_x,n_y) e^{-2\pi i(n_x x + n_y y)/L}$, $\widehat{z}(n_x,n_y) = \frac{1}{L^2} \int_0^L \mathrm{d}x\mathrm{d}y\, z(x,y) e^{2\pi i(n_x x + n_y y)/L}$

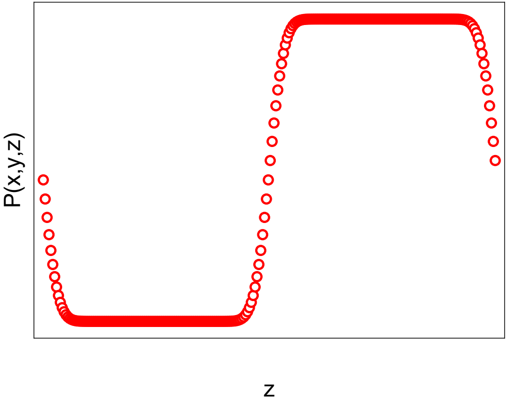
➔ $Z \propto \prod_{\substack{n_x,n_y \\ n_x^2+n_y^2\neq 0}} \int \mathrm{d}\widehat{z}(n_x,n_y)\, \mathrm{e}^{-|\widehat{z}(n_x,n_y)|^2\, 4\,\pi^2\, n^2\, \sigma/T}$ s.t. $\left\langle |\widehat{z}(n_x,n_y)|^2 \right\rangle = \frac{T}{4\,\pi^2\, n^2\, \sigma}$, $n^2 \equiv n_x^2 + n_y^2 \neq 0$

➔ Surface tension from surface fluctuation spectrum: $\sigma = \frac{T}{4\,\pi^2\, n^2 \left\langle |\widehat{z}(n_x,n_y)|^2 \right\rangle}$, $n^2 \equiv n_x^2 + n_y^2 \neq 0$, $n^2 \rightarrow 0$

➔ Effects from non-renormalizeable terms in H expected to vanish as $O(n^2/L^2)$.

Interface detection and smearing

For each (x, y) detect interface from Polyakov loop profile as function of z

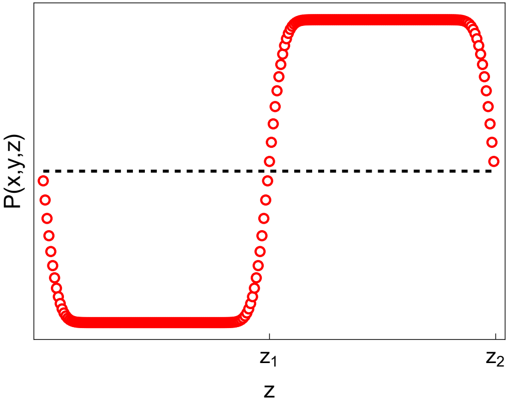


UNIVERSITY OF HELSINKI

Interface detection and smearing

For each (x, y) detect interface from Polyakov loop profile as function of z

Expect to find $z_i(x, y)$, $i = 1, 2$ so that $P(x, y, z_i(x, y)) = P_0$ (with $P_0 = \left|\langle P \rangle_{\text{deconf.}}\right|/2$)



Interface detection and smearing

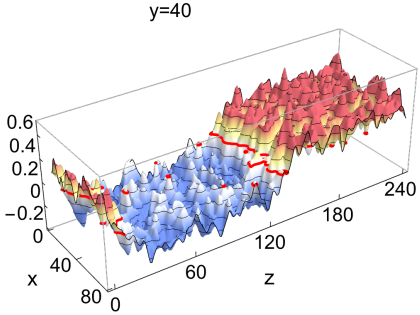
For each (x, y) detect interface from Polyakov loop profile as function of z

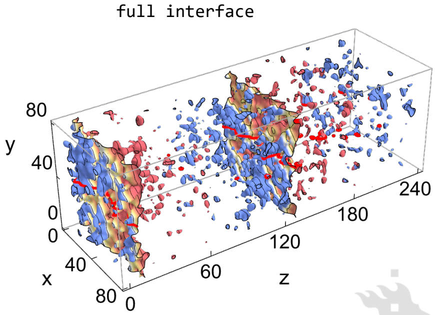
Expect to find $z_i(x, y)$, $i = 1, 2$ so that $P(x, y, z_i(x, y)) = P_0$ (with $P_0 = \left|\langle P\rangle_{\text{deconf.}}\right|/2$)

Polyakov loop field very noisy $\Rightarrow$ perform n_s iter. of recursive smearing $P_L \to P_L^{(n_s)}$

Smearing iter.: $P_L^{(n+1)}(\bar{x}) = \sum_{\bar{y}} S(\bar{x}, \bar{y}) P_L^{(n)}(\bar{y})$, kernel: $S(\bar{x}, \bar{y}) = \frac{1}{1 + 6\,\rho}\left(\delta_{\bar{x},\bar{y}} + \rho \sum_{i=1}^{3} (\delta_{\bar{x}+\hat{i},\bar{y}} + \delta_{\bar{x}-\hat{i},\bar{y}})\right)$.

$n_s = 10$





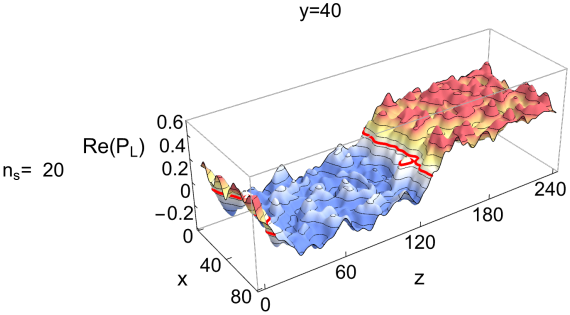
Interface detection and smearing

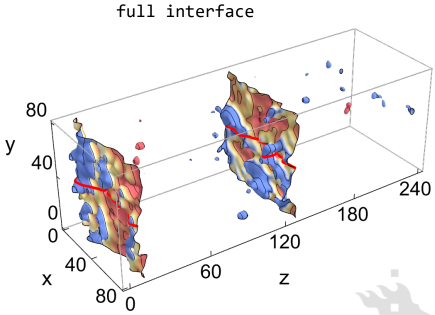
For each (x, y) detect interface from Polyakov loop profile as function of z

Expect to find $z_i(x, y)$, $i = 1, 2$ so that $P(x, y, z_i(x, y)) = P_0$ (with $P_0 = \left|\langle P\rangle_{\text{deconf.}}\right|/2$)

Polyakov loop field very noisy $\Rightarrow$ perform n_s iter. of recursive smearing $P_L \rightarrow P_L^{(n_s)}$

Smearing iter.: $P_L^{(n+1)}(\bar{x}) = \sum_{\bar{y}} S(\bar{x}, \bar{y}) P_L^{(n)}(\bar{y})$, kernel: $S(\bar{x}, \bar{y}) = \frac{1}{1 + 6\rho}\left(\delta_{\bar{x},\bar{y}} + \rho \sum_{i=1}^{3}(\delta_{\bar{x}+\hat{i},\bar{y}} + \delta_{\bar{x}-\hat{i},\bar{y}})\right)$.





UNIVERSITY OF HELSINKI

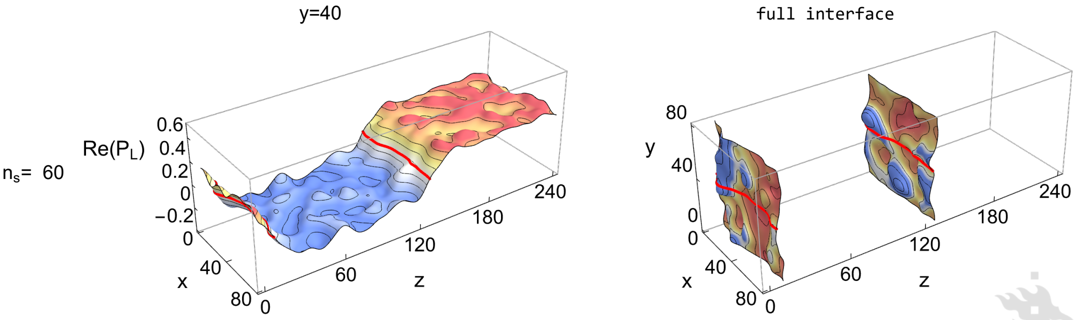
Interface detection and smearing

For each (x, y) detect interface from Polyakov loop profile as function of z

Expect to find $z_i(x, y)$, $i = 1, 2$ so that $P(x, y, z_i(x, y)) = P_0$ (with $P_0 = \left|\langle P\rangle_{\text{deconf.}}\right|/2$)

Polyakov loop field very noisy $\Rightarrow$ perform n_s iter. of recursive smearing $P_L \to P_L^{(n_s)}$

Smearing iter.: $P_L^{(n+1)}(\bar{x}) = \sum_{\bar{y}} S(\bar{x}, \bar{y}) P_L^{(n)}(\bar{y})$, kernel: $S(\bar{x}, \bar{y}) = \frac{1}{1+6\rho}\left(\delta_{\bar{x},\bar{y}} + \rho \sum_{i=1}^{3}(\delta_{\bar{x}+\hat{i},\bar{y}} + \delta_{\bar{x}-\hat{i},\bar{y}})\right)$.



UNIVERSITY OF HELSINKI

Confined-deconfined interface tension

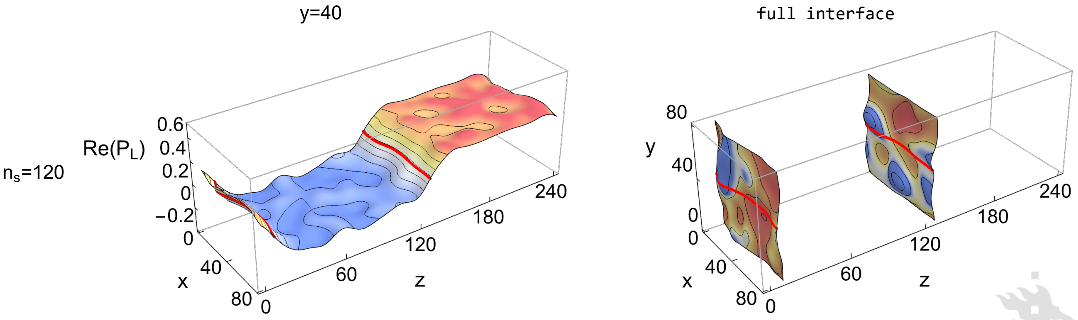
Interface detection and smearing

For each (x, y) detect interface from Polyakov loop profile as function of z

Expect to find $z_i(x, y)$, $i = 1, 2$ so that $P(x, y, z_i(x, y)) = P_0$ (with $P_0 = \left|\langle P \rangle_{\text{deconf.}}\right|/2$)

Polyakov loop field very noisy $\quad \Rightarrow \quad$ perform n_s iter. of recursive smearing $P_L \rightarrow P_L^{(n_s)}$

Smearing iter.: $P_L^{(n+1)}(\bar{x}) = \sum_{\bar{y}} S(\bar{x}, \bar{y}) P_L^{(n)}(\bar{y})$, kernel: $S(\bar{x}, \bar{y}) = \frac{1}{1 + 6\,\rho} \left(\delta_{\bar{x},\bar{y}} + \rho \sum_{i=1}^{3} (\delta_{\bar{x}+\hat{i},\bar{y}} + \delta_{\bar{x}-\hat{i},\bar{y}}) \right)$.



UNIVERSITY OF HELSINKI

Smearing kernel correction

Effect of smearing on surface fluctuation spectrum?

Smearing kernel correction

Effect of smearing on surface fluctuation spectrum?

➔ idealized situation: Polyakov loop field $P(x, y, z)$ given by: $$A(x, y, z) = \begin{cases} A_0 & \text{if} \quad z_1(x, y) \leq z < z_2(x, y) \\ 0 & \text{else} \end{cases}$$

with interface parametrizations $z_i(x, y)$, $i = 1, 2$, $A_0 = \left|\langle P_L \rangle_{\text{deconf.}}\right|$ (neglecting bulk fluctuations)

Smearing kernel correction

Effect of smearing on surface fluctuation spectrum?

➔ idealized situation: Polyakov loop field $P(x, y, z)$ given by: $A(x, y, z) = \begin{cases} A_0 & \text{if} \quad z_1(x, y) \leq z < z_2(x, y) \\ 0 & \text{else} \end{cases}$

with interface parametrizations $z_i(x, y)$, $i = 1, 2$, $A_0 = \left|\langle P_L \rangle_{\text{deconf.}}\right|$ (neglecting bulk fluctuations)

Fourier transform in z: $\widehat{A}(x, y, k_z) = \frac{i\,A_0}{2\,\pi\,k_z}\left(e^{i\,k_z z_1(x,y)} - e^{i\,k_z z_2(x,y)}\right)$

Smearing kernel correction

Effect of smearing on surface fluctuation spectrum?

➔ idealized situation: Polyakov loop field $P(x,y,z)$ given by: $A(x,y,z)=\begin{cases} A_0 & \text{if} \quad z_1(x,y)\leq z < z_2(x,y) \\ 0 & \text{else} \end{cases}$

with interface parametrizations $z_i(x,y)$, $i=1,2$, $A_0=\left|\langle P_L\rangle_{\text{deconf.}}\right|$ (neglecting bulk fluctuations)

Fourier transform in z: $\widehat{A}(x,y,k_z)=\frac{i\,A_0}{2\,\pi\,k_z}\left(e^{i\,k_z z_1(x,y)}-e^{i\,k_z z_2(x,y)}\right)$

Assume: $z_i(x,y)=z_{0,i}+\Delta z_i(x,y)$ with $|\Delta z_i\,k_z|\ll 1$ $\forall\, i=1,2$ for relevant k_z

➔ $\widehat{A}(x,y,k_z)=\frac{A_0}{2\,\pi}\left(-\frac{e^{i\,k_z z_{0,1}}-e^{i\,k_z z_{0,2}}}{i\,k_z}-e^{i\,k_z z_{0,1}}\,\Delta z_1(x,y)+e^{i\,k_z z_{0,2}}\,\Delta z_2(x,y)+\mathcal{O}\big(k_z\,\Delta z_i^2(x,y)\big)\right)$

Smearing kernel correction

Effect of smearing on surface fluctuation spectrum?

➔ idealized situation: Polyakov loop field $P(x,y,z)$ given by: $A(x,y,z) = \begin{cases} A_0 & \text{if} \quad z_1(x,y) \leq z < z_2(x,y) \\ 0 & \text{else} \end{cases}$

with interface parametrizations $z_i(x,y)$, $i = 1,2$, $A_0 = \left|\langle P_L \rangle_{\text{deconf.}}\right|$ (neglecting bulk fluctuations)

Fourier transform in z: $\widehat{A}(x,y,k_z) = \frac{i\,A_0}{2\,\pi\,k_z}\left(e^{i\,k_z z_1(x,y)} - e^{i\,k_z z_2(x,y)}\right)$

Assume: $z_i(x,y) = z_{0,i} + \Delta z_i(x,y)$ with $|\Delta z_i\,k_z| \ll 1 \quad \forall\, i = 1,2$ for relevant k_z

➔ $\widehat{A}(x,y,k_z) = \frac{A_0}{2\,\pi}\left(-\frac{e^{i\,k_z z_{0,1}} - e^{i\,k_z z_{0,2}}}{i\,k_z} - e^{i\,k_z z_{0,1}}\,\Delta z_1(x,y) + e^{i\,k_z z_{0,2}}\,\Delta z_2(x,y) + \mathcal{O}\big(k_z\,\Delta z_i^2(x,y)\big)\right)$

Fourier transform also in (x,y):

➔ $\widehat{A}(k_x,k_y,k_z) = \frac{A_0}{2\,\pi}\left(-e^{i\,k_z z_{0,1}}\,\Delta\widehat{z}_1(k_x,k_y) + e^{i\,k_z z_{0,2}}\,\Delta\widehat{z}_2(k_x,k_y) + \mathcal{O}\big(k_z\,\Delta z_i^2\big)\right)$ for $\bar{k}^2 \equiv k_x^2 + k_y^2 > 0$.

➔ $\widehat{z}_i(k_x,k_y) \overset{(\bar{k}^2>0)}{=} \Delta\widehat{z}_i(k_x,k_y) \propto \widehat{A}(k_x,k_y,k_z)$, $i = 1,2$

➔ $\widehat{A}^{(n_s)}(k_x,k_y,k_z) = \widehat{S}^{n_s}(k_x,k_y,k_z)\,\widehat{A}(k_x,k_y,k_z) \quad \Rightarrow \quad \widehat{z}_i^{(n_s)}(k_x,k_y) = \widehat{S}^{n_s}(k_x,k_y,k_z \approx 0)\,\widehat{z}_i(k_x,k_y)$

Smearing kernel correction

Effect of smearing on surface fluctuation spectrum?

➔ idealized situation: Polyakov loop field $P(x,y,z)$ given by: $A(x,y,z) = \begin{cases} A_0 & \text{if} \quad z_1(x,y) \leq z < z_2(x,y) \\ 0 & \text{else} \end{cases}$

with interface parametrizations $z_i(x,y)$, $i=1,2$, $A_0 = \left|\langle P_L \rangle_{\text{deconf.}}\right|$ (neglecting bulk fluctuations)

Fourier transform in z: $\widehat{A}(x,y,k_z) = \frac{i\,A_0}{2\,\pi\,k_z}\left(e^{i\,k_z z_1(x,y)} - e^{i\,k_z z_2(x,y)}\right)$

Assume: $z_i(x,y) = z_{0,i} + \Delta z_i(x,y)$ with $|\Delta z_i\,k_z| \ll 1$ $\forall\, i=1,2$ for relevant k_z

➔ $\widehat{A}(x,y,k_z) = \frac{A_0}{2\,\pi}\left(-\frac{e^{i\,k_z z_{0,1}} - e^{i\,k_z z_{0,2}}}{i\,k_z} - e^{i\,k_z z_{0,1}}\,\Delta z_1(x,y) + e^{i\,k_z z_{0,2}}\,\Delta z_2(x,y) + \mathcal{O}\big(k_z\,\Delta z_i^2(x,y)\big)\right)$

Fourier transform also in (x,y):

➔ $\widehat{A}(k_x,k_y,k_z) = \frac{A_0}{2\,\pi}\left(-e^{i\,k_z z_{0,1}}\,\Delta\widehat{z}_1(k_x,k_y) + e^{i\,k_z z_{0,2}}\,\Delta\widehat{z}_2(k_x,k_y) + \mathcal{O}\big(k_z\,\Delta z_i^2\big)\right)$ for $\bar{k}^2 \equiv k_x^2 + k_y^2 > 0$.

➔ $\widehat{z}_i(k_x,k_y) \overset{(\bar{k}^2>0)}{=} \Delta\widehat{z}_i(k_x,k_y) \propto \widehat{A}(k_x,k_y,k_z)$, $i=1,2$

➔ $\widehat{A}^{(n_s)}(k_x,k_y,k_z) = \widehat{S}^{n_s}(k_x,k_y,k_z)\,\widehat{A}(k_x,k_y,k_z) \quad \Rightarrow \quad \widehat{z}_i^{(n_s)}(k_x,k_y) = \widehat{S}^{n_s}(k_x,k_y,k_z \approx 0)\,\widehat{z}_i(k_x,k_y)$

➔ Kernel corrected estimator for $\left\langle\left|\widehat{z}(k_x,k_y)\right|^2\right\rangle = \frac{1}{\widehat{S}^{2\,n_s}(k_x,k_y,k_z \approx 0)}\left\langle\left|\widehat{z}^{(n_s)}(k_x,k_y)\right|^2\right\rangle$

Zero-mode extrapolation

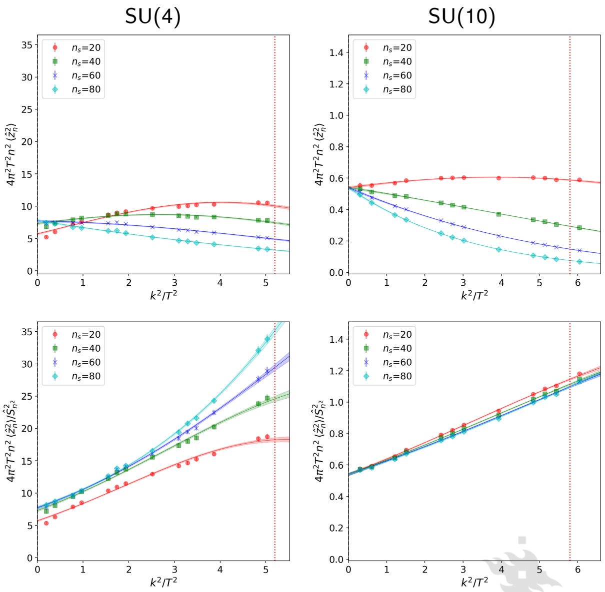
Fit ansatz: $f(x) = c_1 \exp(c_2\, x + c_3\, x^2)$ with $x = k^2/T^2$

UNIVERSITY OF HELSINKI

Zero-mode extrapolation

Fit ansatz: $f(x) = c_1 \exp(c_2\, x + c_3\, x^2)$ with $x = k^2/T^2$

N	k.-corr.	n_s	c_1	c_2	c_3	χ^2/#dof
4	no	20	5.71(11)	0.308(16)	-0.0383(28)	3.792
		40	7.31(15)	0.132(17)	-0.0247(30)	1.597
		60	7.71(16)	-0.014(17)	-0.0140(30)	1.047
		80	7.79(18)	-0.146(18)	-0.0046(31)	0.737
	yes	20	5.71(11)	0.423(16)	-0.0384(28)	3.787
		40	7.30(15)	0.362(17)	-0.0247(30)	1.573
		60	7.71(16)	0.331(17)	-0.0141(30)	1.007
		80	7.78(17)	0.315(18)	-0.0048(31)	0.695
10	no	20	0.5397(75)	0.0607(84)	-0.0079(11)	1.070
		40	0.5404(76)	-0.0682(85)	-0.0064(11)	1.462
		60	0.5397(77)	-0.1913(86)	-0.0053(11)	1.809
		80	0.5397(77)	-0.3110(86)	-0.0045(11)	2.155
	yes	20	0.5400(75)	0.1751(84)	-0.0078(11)	0.850
		40	0.5409(76)	0.1607(85)	-0.0062(11)	0.961
		60	0.5404(77)	0.1522(86)	-0.0052(11)	0.984
		80	0.5407(77)	0.1469(86)	-0.0043(11)	0.985

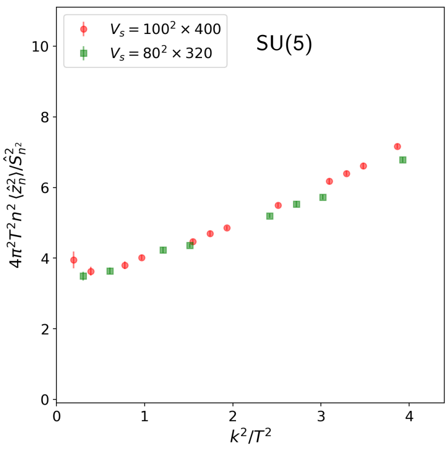


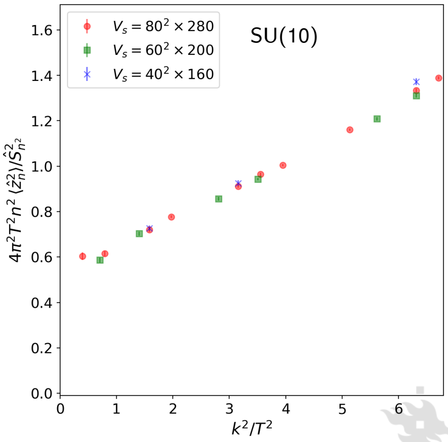
UNIVERSITY OF HELSINKI

Zero-mode extrapolation

Fit ansatz: $f(x) = c_1 \exp(c_2\, x + c_3\, x^2)$ with $x = k^2/T^2$

finite-volume effects?

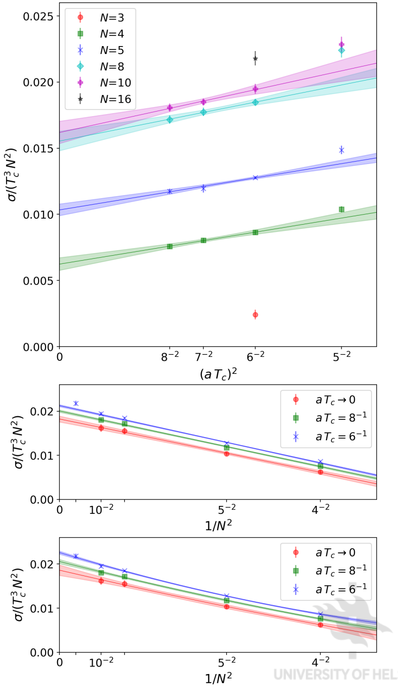




Continuum limit

Fit ansatz: $f(x) = c_1 + c_2\, x$ with $x = (a\, T_c)^2 = 1/N_t^2$, $N_t > 5$

(error bands: $\delta f(x) = \sqrt{\sum_{i,j} \sigma^{ij} \frac{\partial f(x)}{\partial c_i} \frac{\partial f(x)}{\partial c_j}}$, with covariance matrix σ^{ij})

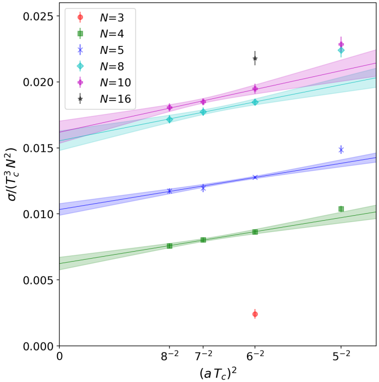


UNIVERSITY OF HELSINKI

Continuum limit

Fit ansatz: $f(x) = c_1 + c_2\, x$ with $x = (a\,T_c)^2 = 1/N_t^2$, $N_t > 5$

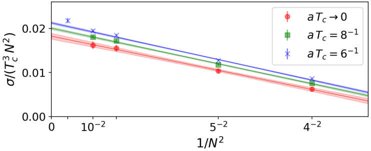
(error bands: $\delta f(x) = \sqrt{\sum_{i,j} \sigma^{ij} \frac{\partial f(x)}{\partial c_i} \frac{\partial f(x)}{\partial c_j}}$, with covariance matrix σ^{ij})



Large-N limit

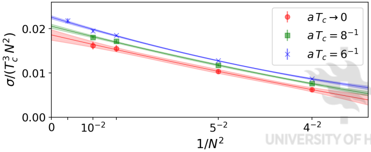
Fit ansatz (linear): $f(x) = c_1 + c_2\, x$ with $x = 1/N^2$,

➔ $\frac{\sigma}{T^3} = 0.0182(7)N^2 - 0.194(15)$ (continuum)
$(\chi^2/\#\text{dof} = 0.2)$



Fit ansatz (quadratic): $f(x) = c_1 + c_2\, x + c_3\, x^2$ with $x = 1/N^2$

➔ $\frac{\sigma}{T^3} = 0.0186(12)N^2 - 0.220(71) + 0.35(92)/N^2$ (continuum)
$(\chi^2/\#\text{dof} = 0.25)$



UNIVERSITY OF HELSINKI

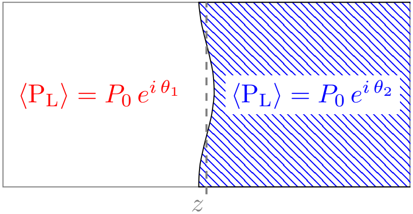
Capillary wave method for deconfined-deconfined interface tension

- $\beta > \beta_c$ ➔ Interface between two deconfined Z_N-vacua, corresponding to $\langle P_L \rangle = P_0\ \mathrm{e}^{\mathrm{i}\,\theta_i}$, $i = 1, 2$

  $\theta_i = 2\,\pi\, n_i / N \quad , \quad n_i \in \{\, 0, \ldots, N-1 \,\} \quad , \quad n_1 \neq n_2.$

- periodic boundary conditions with Z_N-twist Δn in z-direction.

  ➔ interface always across x-y-plane

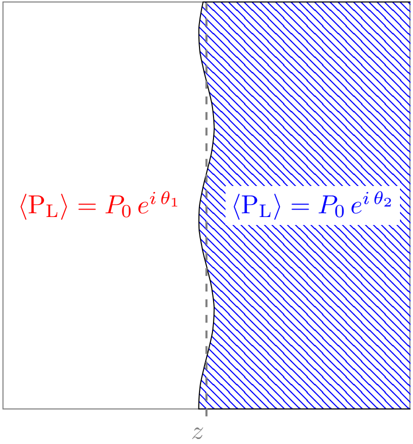


"Capillary" wave method for deconfined-deconfined interface tension

- $\beta > \beta_c$ ➔ Interface between two deconfined Z_N-vacua, corresponding to $\langle P_L \rangle = P_0 \, e^{i\,\theta_i}$, $i = 1, 2$

  $\theta_i = 2\,\pi\, n_i/N$, $n_i \in \{0, \ldots, N-1\}$, $n_1 \neq n_2$.
- periodic boundary conditions with Z_N-twist Δn in z-direction.

  ➔ interface always across x-y-plane $\Rightarrow$ no need for z-elongated system



"Capillary" wave method for deconfined-deconfined interface tension

- $\beta > \beta_c$ ➔ Interface between two deconfined Z_N-vacua, corresponding to $\langle P_L \rangle = P_0\, e^{i\,\theta_i}$, $i = 1, 2$

  $\theta_i = 2\,\pi\, n_i/N$, $n_i \in \{0, \ldots, N-1\}$, $n_1 \neq n_2$.

- periodic boundary conditions with Z_N-twist Δn in z-direction.

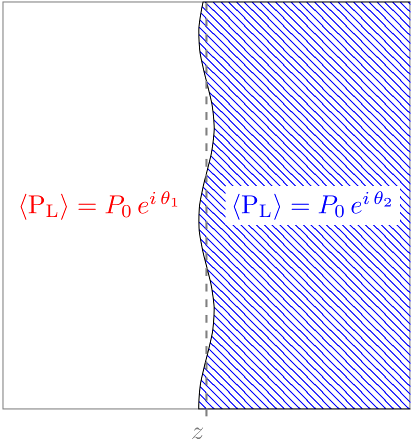
  ➔ interface always across x-y-plane $\Rightarrow$ no need for z-elongated system

- Monte Carlo (heatbath + overrelaxtion)
- Initialize simulation:

  1) start with "mixed phase" state: $U_4(x_1, x_2, x_3, 0) = \begin{cases} \mathbb{1}\ e^{i\,\theta_1} & \text{if } z < N_z/2 \\ \mathbb{1}\ e^{i\,\theta_2} & \text{if } z \geq N_z/2 \end{cases}$

  2) thermalize

- Sample configurations for measurements



UNIVERSITY OF HELSINKI

"Capillary" wave method for deconfined-deconfined interface tension

- $\beta > \beta_c$ ➔ Interface between two deconfined Z_N-vacua, corresponding to $\langle P_L \rangle = P_0 \, e^{i\,\theta_i}$, $i = 1, 2$

  $\theta_i = 2\,\pi\, n_i / N \quad , \quad n_i \in \{ 0, \ldots, N-1 \} \quad , \quad n_1 \neq n_2 .$

- periodic boundary conditions with Z_N-twist Δn in z-direction.

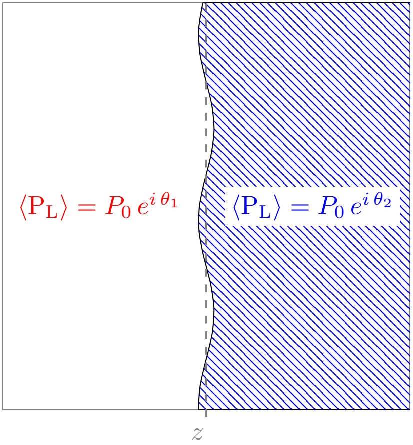
  ➔ interface always across x-y-plane $\Rightarrow$ no need for z-elongated system

- Monte Carlo (heatbath + overrelaxtion)
- Initialize simulation:

  1) start with "mixed phase" state: $U_4(x_1, x_2, x_3, 0) = \begin{cases} \mathbb{1}\; e^{i\,\theta_1} & \text{if } z < N_z/2 \\ \mathbb{1}\; e^{i\,\theta_2} & \text{if } z \geq N_z/2 \end{cases}$

  2) thermalize

- Sample configurations for measurements
- no constraint sampling needed



UNIVERSITY OF HELSINKI

"Capillary" wave method for deconfined-deconfined interface tension

- $\beta > \beta_c$ ➔ Interface between two deconfined Z_N-vacua, corresponding to $\langle P_L \rangle = P_0 \; e^{i\,\theta_i}$, $i = 1, 2$

  $\theta_i = 2\,\pi\, n_i/N \quad , \quad n_i \in \{\, 0, \ldots, N-1 \,\} \quad , \quad n_1 \neq n_2$.

- periodic boundary conditions with Z_N-twist Δn in z-direction.

  ➔ interface always across x-y-plane $\Rightarrow$ no need for z-elongated system

- Monte Carlo (heatbath + overrelaxtion)

- Initialize simulation:

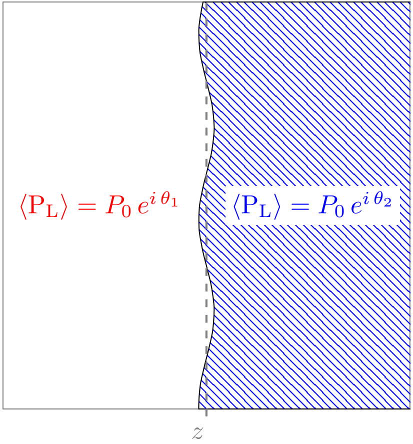
  1) start with "mixed phase" state: $U_4(x_1, x_2, x_3, 0) = \begin{cases} \mathbb{1}\; e^{i\,\theta_1} & \text{if } z < N_z/2 \\ \mathbb{1}\; e^{i\,\theta_2} & \text{if } z \geq N_z/2 \end{cases}$

  2) thermalize

- Sample configurations for measurements

- no constraint sampling needed

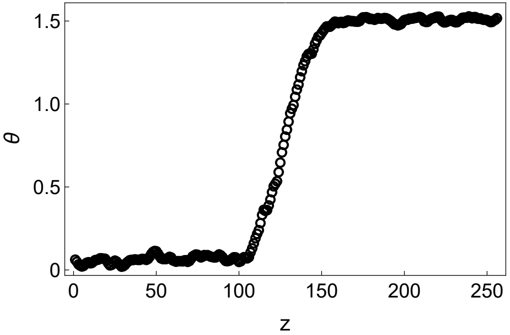
- Can periodically recenter interface in z-direction to simplify detection.



UNIVERSITY OF HELSINKI

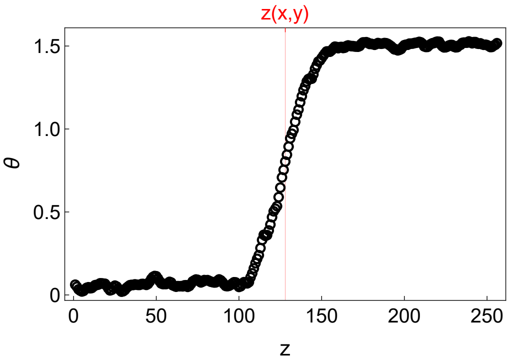
(Naive) interface detection and smearing

- For each (x, y) detect interface from profile of Polyakov loop phase $\theta(z) = \mathrm{arg}(P(x, y, z))$ as function of z



(Naive) interface detection and smearing

- For each (x, y) detect interface from profile of Polyakov loop phase $\theta(z) = \arg(P(x, y, z))$ as function of z
- Expect single interface location $z(x, y)$ so that $\theta(z(x, y)) = \theta_0$ (with $\theta_0 = (\theta_1 + \theta_2)/2$)



(Naive) interface detection and smearing

- For each (x, y) detect interface from profile of Polyakov loop phase $\theta(z) = \arg(P(x, y, z))$ as function of z
- Expect single interface location $z(x, y)$ so that $\theta(z(x, y)) = \theta_0$ (with $\theta_0 = (\theta_1 + \theta_2)/2$)

Polyakov loop field (and its phase) very noisy $\Rightarrow$ perform $n_s = n_{pre} + n$ iter. of smearing $P_L \to P_L^{(n_s)}$

Kernel: $$S(\bar{x}, \bar{y}) = \frac{1}{1 + 6\,\rho}\left(\delta_{\bar{x},\bar{y}} + \rho \sum_{i=1}^{3} (\delta_{\bar{x}+\hat{i},\bar{y}} + \delta_{\bar{x}-\hat{i},\bar{y}})\right)$$

n_{pre} pre-smearing steps:

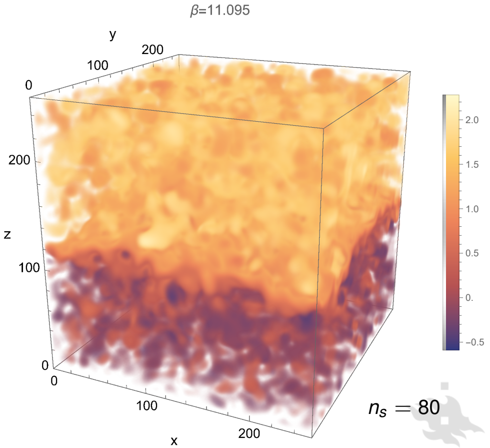
$$P_L^{(n+1)}(\bar{x}) = \sum_{\bar{y}} S(\bar{x}, \bar{y}) P_L^{(n)}(\bar{y})$$

n log-smearing steps:

$$\log\left(P_L^{(n+1)}(\bar{x})\right) = \sum_{\bar{y}} S(\bar{x}, \bar{y}) \log\left(P_L^{(n)}(\bar{y})\right)$$

with $\log\left(P_L\right) = \log\left(|P_L|\right) + \mathrm{i}\,\theta$

➔ enables easy kernel correction of surf. fluct. spectrum



UNIVERSITY OF HELSINKI

(Naive) interface detection and smearing

- For each (x, y) detect interface from profile of Polyakov loop phase $\theta(z) = \arg(P(x, y, z))$ as function of z
- Expect single interface location $z(x, y)$ so that $\theta(z(x, y)) = \theta_0$ (with $\theta_0 = (\theta_1 + \theta_2)/2$)

Polyakov loop field (and its phase) very noisy $\Rightarrow$ perform $n_s = n_{pre} + n$ iter. of smearing $P_L \to P_L^{(n_s)}$

Kernel: $S(\bar{x}, \bar{y}) = \frac{1}{1 + 6\,\rho}\left(\delta_{\bar{x},\bar{y}} + \rho \sum_{i=1}^{3}(\delta_{\bar{x}+\hat{i},\bar{y}} + \delta_{\bar{x}-\hat{i},\bar{y}})\right)$

n_{pre} pre-smearing steps:

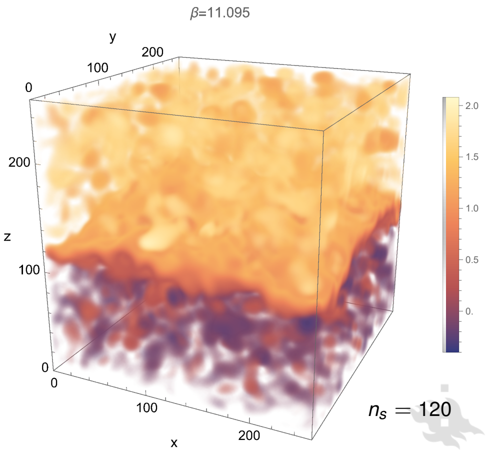
$$P_L^{(n+1)}(\bar{x}) = \sum_{\bar{y}} S(\bar{x}, \bar{y}) P_L^{(n)}(\bar{y})$$

n log-smearing steps:

$$\log\left(P_L^{(n+1)}(\bar{x})\right) = \sum_{\bar{y}} S(\bar{x}, \bar{y}) \log\left(P_L^{(n)}(\bar{y})\right)$$

with $\log\left(P_L\right) = \log\left(|P_L|\right) + \mathrm{i}\,\theta$

➔ enables easy kernel correction of surf. fluct. spectrum



$n_s = 120$

(Naive) interface detection and smearing

- For each (x, y) detect interface from profile of Polyakov loop phase $\theta(z) = \arg(P(x, y, z))$ as function of z
- Expect single interface location $z(x, y)$ so that $\theta(z(x, y)) = \theta_0$ (with $\theta_0 = (\theta_1 + \theta_2)/2$)

Polyakov loop field (and its phase) very noisy $\Rightarrow$ perform $n_s = n_{\text{pre}} + n$ iter. of smearing $P_L \to P_L^{(n_s)}$

Kernel: $S(\bar{x}, \bar{y}) = \frac{1}{1 + 6\,\rho}\left(\delta_{\bar{x},\bar{y}} + \rho \sum_{i=1}^{3}(\delta_{\bar{x}+\hat{i},\bar{y}} + \delta_{\bar{x}-\hat{i},\bar{y}})\right)$

n_{pre} pre-smearing steps:

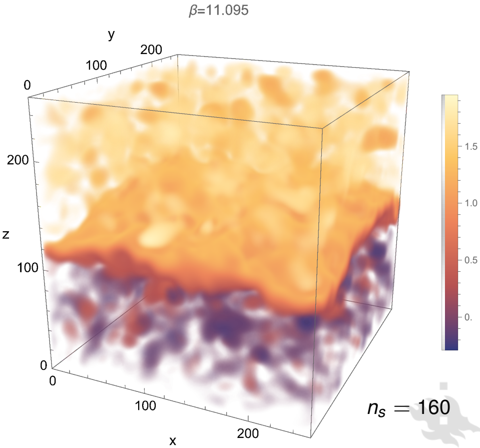
$$P_L^{(n+1)}(\bar{x}) = \sum_{\bar{y}} S(\bar{x}, \bar{y}) P_L^{(n)}(\bar{y})$$

n log-smearing steps:

$$\log\left(P_L^{(n+1)}(\bar{x})\right) = \sum_{\bar{y}} S(\bar{x}, \bar{y}) \log\left(P_L^{(n)}(\bar{y})\right)$$

with $\log\left(P_L\right) = \log\left(|P_L|\right) + \mathrm{i}\,\theta$

➔ enables easy kernel correction of surf. fluct. spectrum



$n_s = 160$

(Naive) interface detection and smearing

- For each (x, y) detect interface from profile of Polyakov loop phase $\theta(z) = \arg(P(x, y, z))$ as function of z
- Expect single interface location $z(x, y)$ so that $\theta(z(x, y)) = \theta_0$ (with $\theta_0 = (\theta_1 + \theta_2)/2$)

Polyakov loop field (and its phase) very noisy $\Rightarrow$ perform $n_s = n_{\text{pre}} + n$ iter. of smearing $P_L \to P_L^{(n_s)}$

Kernel: $S(\bar{x}, \bar{y}) = \frac{1}{1 + 6\,\rho}\left(\delta_{\bar{x},\bar{y}} + \rho \sum_{i=1}^{3}(\delta_{\bar{x}+\hat{i},\bar{y}} + \delta_{\bar{x}-\hat{i},\bar{y}})\right)$

n_{pre} pre-smearing steps:

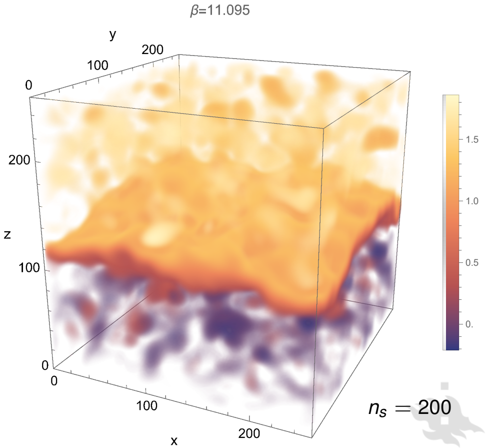
$$P_L^{(n+1)}(\bar{x}) = \sum_{\bar{y}} S(\bar{x}, \bar{y}) P_L^{(n)}(\bar{y})$$

n log-smearing steps:

$$\log\left(P_L^{(n+1)}(\bar{x})\right) = \sum_{\bar{y}} S(\bar{x}, \bar{y}) \log\left(P_L^{(n)}(\bar{y})\right)$$

with $\log\left(P_L\right) = \log\left(|P_L|\right) + \mathrm{i}\,\theta$

➔ enables easy kernel correction of surf. fluct. spectrum



(Naive) interface detection and smearing

- For each (x, y) detect interface from profile of Polyakov loop phase $\theta(z) = \arg(P(x, y, z))$ as function of z
- Expect single interface location $z(x, y)$ so that $\theta(z(x, y)) = \theta_0$ (with $\theta_0 = (\theta_1 + \theta_2)/2$)

Polyakov loop field (and its phase) very noisy $\Rightarrow$ perform $n_s = n_{\text{pre}} + n$ iter. of smearing $P_L \to P_L^{(n_s)}$

Kernel: $S(\bar{x}, \bar{y}) = \frac{1}{1 + 6\rho}\left(\delta_{\bar{x},\bar{y}} + \rho \sum_{i=1}^{3}(\delta_{\bar{x}+\hat{i},\bar{y}} + \delta_{\bar{x}-\hat{i},\bar{y}})\right)$

n_{pre} pre-smearing steps:

$$P_L^{(n+1)}(\bar{x}) = \sum_{\bar{y}} S(\bar{x}, \bar{y}) P_L^{(n)}(\bar{y})$$

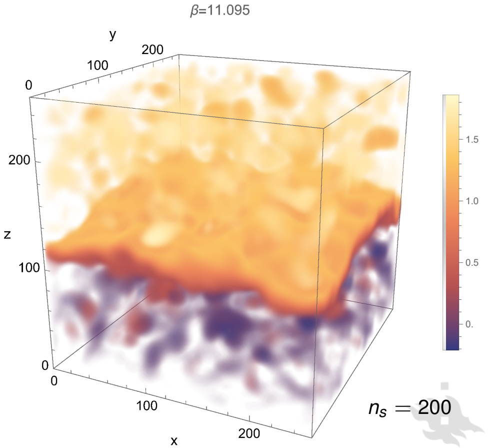
n log-smearing steps:

$$\log\left(P_L^{(n+1)}(\bar{x})\right) = \sum_{\bar{y}} S(\bar{x}, \bar{y}) \log\left(P_L^{(n)}(\bar{y})\right)$$

with $\log(P_L) = \log(|P_L|) + \mathrm{i}\,\theta$

➔ enables easy kernel correction of surf. fluct. spectrum

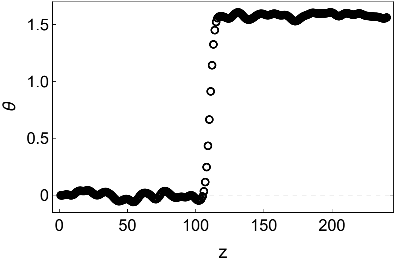
➔ Naive interface detection requires lots of smearing!



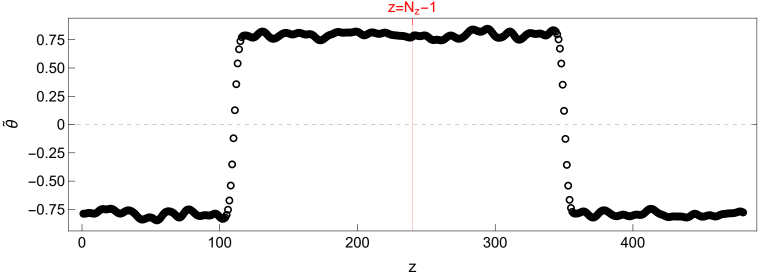
Interface detection from Fourier mode phase shift

➔ For each (x, y) transform profile of Polyakov loop phase $\theta(z) = \arg(P(x, y, z))$

from



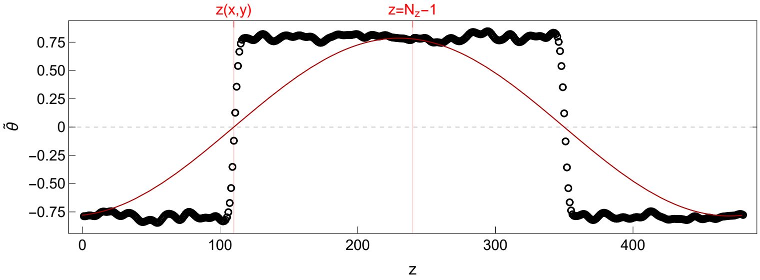
to



Interface detection from Fourier mode phase shift

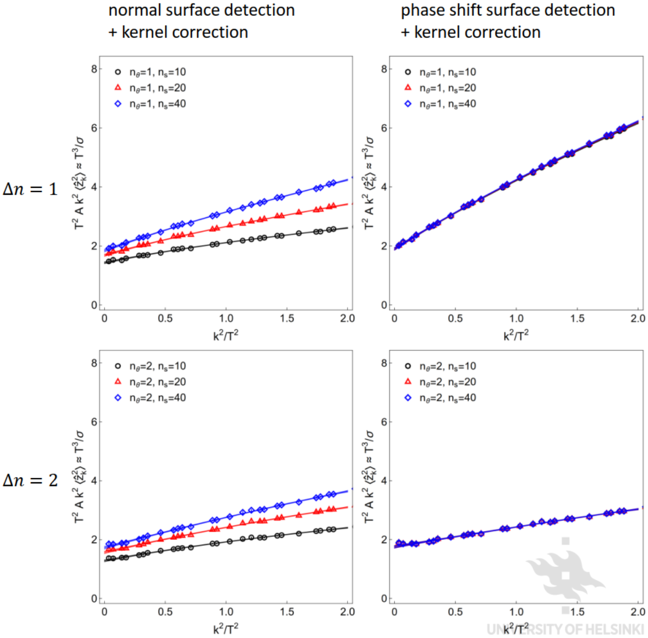
➔ For each (x, y) transform profile of Polyakov loop phase $\theta(z) = \arg(P(x, y, z))$

➔ extract $z(x, y)$ from phase shift of lowest non-trivial Fourier mode:



Comparison of normal and phase-shift interface detection method

- interface fluctuation spectra obtained from normally detected interface would require more smearing to collapse after kernel correction.
- interface fluctuation spectra obtained from interface located by phase shift method collapse perfectly after kernel correction.

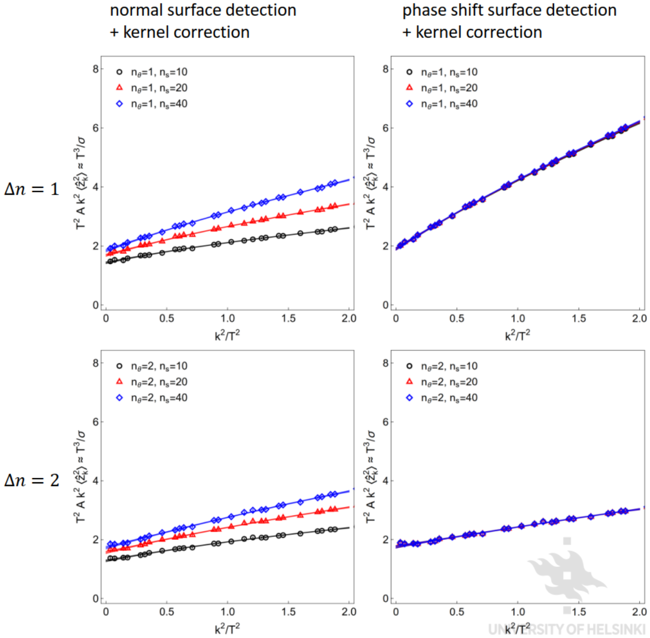


UNIVERSITY OF HELSINKI

Comparison of normal and phase-shift interface detection method

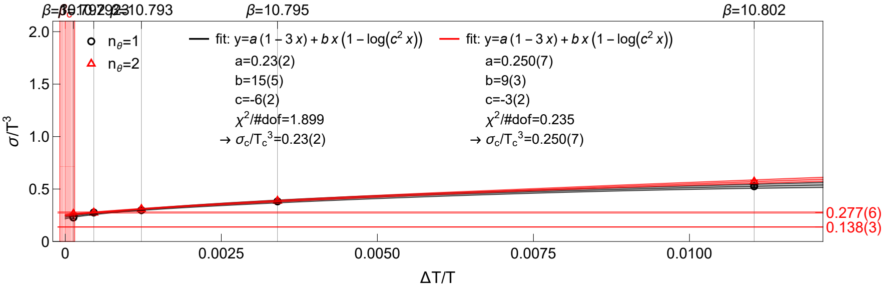
- interface fluctuation spectra obtained from normally detected interface would require more smearing to collapse after kernel correction.
- interface fluctuation spectra obtained from interface located by phase shift method collapse perfectly after kernel correction.

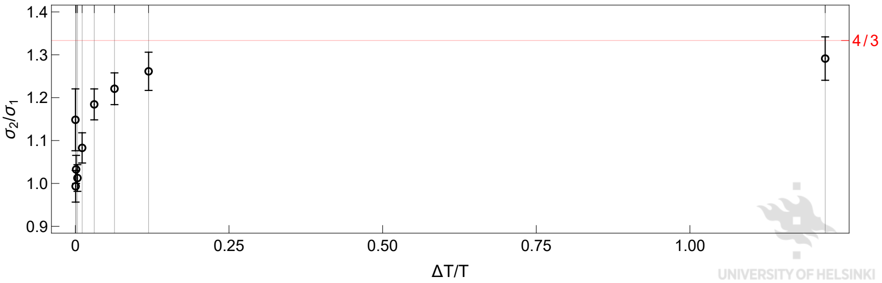
➔ With sufficient smearing (and in sufficiently big boxes) the two methods give the same results.



UNIVERSITY OF HELSINKI

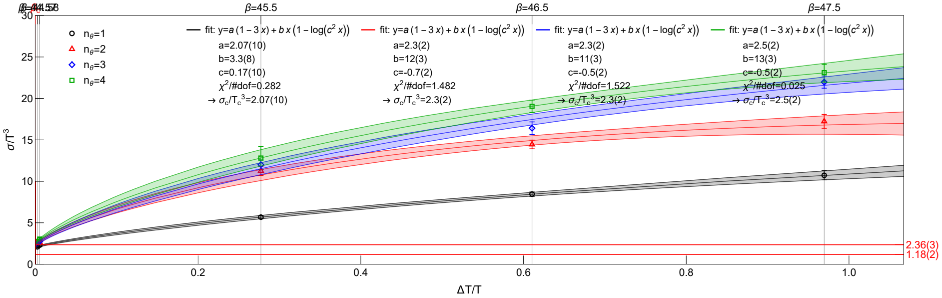
Results for SU(4)

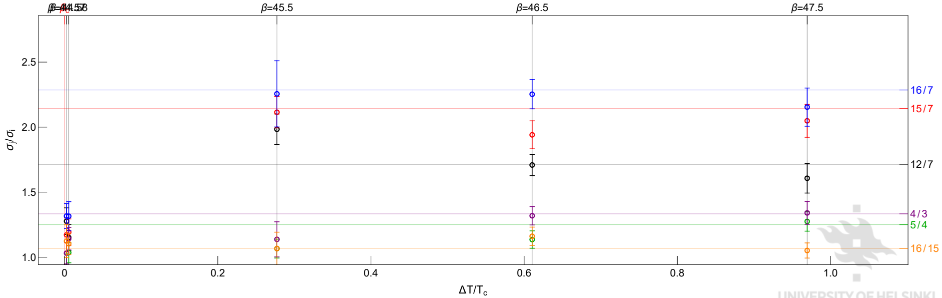




UNIVERSITY OF HELSINKI

Results for SU(8)





UNIVERSITY OF HELSINKI