Color electric field correlators for diffusion of heavy quarkonia

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Based on:
Nora Brambilla, V.L., Julian Mayer-Steudte, Péter Petreczky
Phys.Rev.D 107 (2023) 5, 054508, [arXiv:2206.02861]
Nora Brambilla, Saumen Datta, Marc Janer, V.L., Julian Mayer-Steudte, Péter Petreczky, Antonio Vairo
Phys. Rev. D 112 (2025) 074509 [arXiv: 2505.16603]
and upcoming work

SDU Syddansk Universitet

D·IAS DANISH INSTITUTE FOR ADVANCED STUDY

Nordic lattice 26
Edinburgh,
19.05.2026

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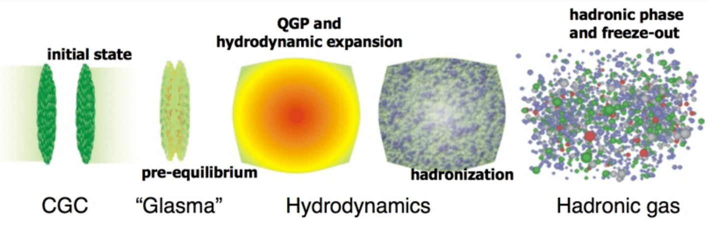
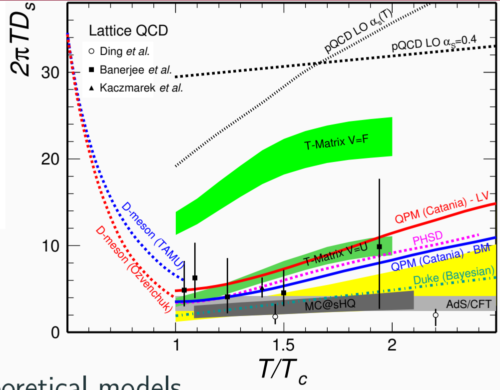




- Nuclear modification factor R_{AA} and elliptic flow v_2 described by spatial diffusion coefficient D_x
- Varying results for temperature dependence across theoretical models
- Transport coefficients are input for quarkonium production models to describe heavy-ion collision physics and quark gluon plasma

Figure: S. Bass (left) X. Dong CIPANP (2018) (right)

Heavy Quark diffusion: Langevin perspective

- Heavy quark energy changes only little when colliding with medium

$$E_k \sim T\,, \quad p \sim \sqrt{MT} \gg T$$

- HQ momentum is changed by random kicks from the medium

  $\to$ Brownian motion; Langevin dynamics can be used

$$\frac{dp_i}{dt} = -\frac{\kappa}{2MT}p_i + \xi_i(t)\,, \quad \langle \xi(t)\xi(t')\rangle = \kappa\delta(t-t')$$

Svetitsky 88; Mustafa *et.al.*97; Moore & Teaney 05; Rapp & van Hees 05

Associated Fokker-Planck equation

$$\frac{\partial f_Q(p,t)}{\partial t} = -\frac{\partial}{\partial p_i}[p_i\eta_{\rm D}(p)f_Q(p,t)] + \frac{\partial^2}{\partial p_i\partial_j}[\kappa_{ij}(p)f_Q(p,t)]$$

- Single coefficient κ gives access to multiple interesting quantities:

$D_{\rm s} = 2T^2/\kappa$	$\eta_{\rm D} = \kappa/(2MT)$	$\tau_{\rm Q} = \eta_{\rm D}^{-1}$
Spatial diffusion	Drag coefficient	Relaxation time

HQET picture

- Expand the force in $1/M$

$$\mathcal{F}^i = \phi^\dagger \left[-gE^i + \frac{[D^i, D^2 + c_b g\sigma \cdot B]}{2M} + \cdots \right] \phi$$

- Note also Lorentz force

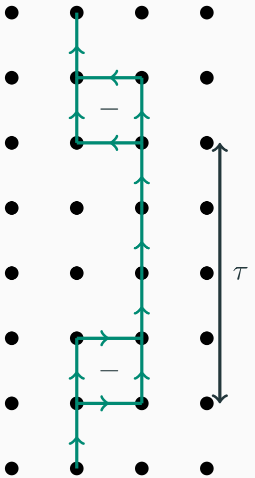
$$F(t) = \dot{p} = q\left(E + v \times B\right))(t)$$

- Switch to Euclidean space correlation function:

$$G_{\mathrm{E}}(\tau) = -\frac{1}{3}\sum_{i=1}^{3} \frac{\langle \mathrm{Re\,Tr}\,[U(\beta,\tau)gE_i(\tau,0)U(\tau,0)gE_i(0,0)]\rangle}{\langle \mathrm{Re\,Tr}\,[U(\beta,0)]\rangle},$$

$$G_{\mathrm{B}}(\tau) = \sum_{i=1}^{3} \frac{\langle \mathrm{Re\,Tr}\,[U(1/T,\tau)B_i(\tau,0)U(\tau,0)B_i(0,0)]\rangle}{3\,\langle \mathrm{Re\,Tr}\,U(1/T,0)\rangle}$$

$$\kappa_{\mathrm{E,B}} = \lim_{\omega\to 0} \frac{2T}{\omega}\rho(\omega) \qquad G_{\mathrm{E,B}}(\tau) = \int_0^\infty \frac{\mathrm{d}\omega}{\pi}\rho(\omega)\frac{\cosh\left(\frac{\omega}{T}[\tau T - \frac{1}{2}]\right)}{\sinh\frac{\omega}{2T}}$$



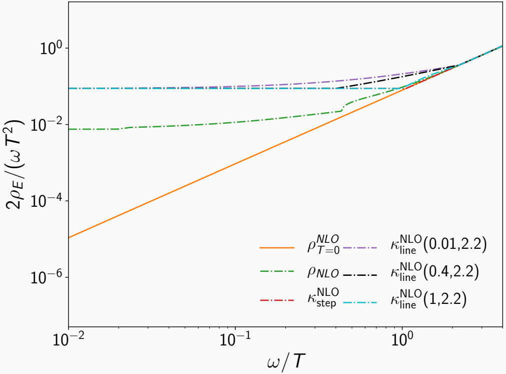
Moore and Teaney PRC71 (2005), Caron-Huot and Moore JHEP02 (2008)
A. Bouttefeux and M. Laine JHEP 12 (2020) 150, M. Laine JHEP 06 (2021) 139

Heavy Quark diffusion

- Well studied on the lattice
  - Known both quenched and unquenched
  - Temperature dependence known
  - $\frac{1}{\mathrm{M}}$ corrections measured
- No transport peak
- Known IR and UV behavior
- Model the intermediate behavior
- Choose scale so that NLO contributions vanish

$$\rho^{\mathrm{IR}}_{E,B}(\omega) = \frac{\kappa\omega}{2T}\,, \quad \rho^{\mathrm{UV}}_{\mathrm{E}}(\omega) = \frac{g^2 C_F \omega^3}{6\pi}\left\{1 + \frac{g^2}{(4\pi)^2}\left[N_{\mathrm{c}}\left(\frac{11}{3}\ln\frac{\mu^2}{4\omega^2} + \frac{149}{9} - \frac{2\pi^2}{3}\right)\right]\right\}$$

- More complicated inversions under work, but generally this observable well behaved enough for this approach

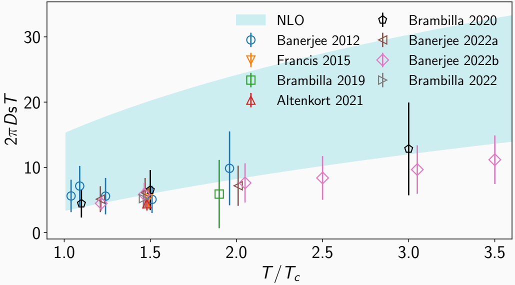


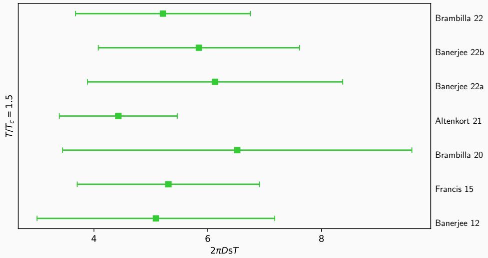
Lattice details

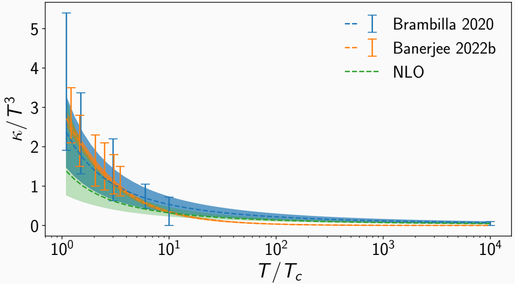
- SU(3) pure gauge Yang-Mills simulations
- Two temperatures $T = 1.5\,T_{\mathrm{c}}$ and $T = 10^4\,T_{\mathrm{c}}$
- High temperature for comparing to perturbation theory
- Measurements with both gradient flow and multilevel algorithm
- Two different discretizations of the E-field (clover and 2-plaquette)
- Tree-level improve the correlators
- At zero flow time all out operators have same LO contribution (up to color factors)

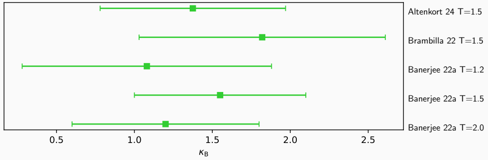
$$\frac{G^{\mathrm{LO}}_{\mathrm{E,B}}}{g^2 C_{\mathrm{F}}} \equiv f(\tau) = \pi^2 T^4 \left[\frac{\cos^2(\pi\tau T)}{\sin^4(\pi\tau T)} + \frac{1}{3\sin^2(\pi\tau T)} \right]$$

κ_{E} results



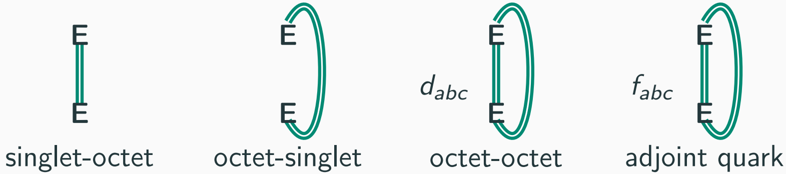






Heavy Quarkonium Diffusion

- Quarkonium in medium can be described by Limbland equation by using pNRQCD and open quantum systems
- Three possible interactions and adjoint quark Brambilla et.al. TUM-EFT 191/24, FERMILAB-PUB-24-0451-T



- Each process described by two parameters κ_{xx} and γ_{xx}
- κ_{so} is related to the thermal width and describes heavy quarkonium diffusion
- γ_{so} is related to the mass shift $\gamma = -\frac{1}{3N_c}\int_0^\infty \frac{d\omega}{2\pi}\frac{\rho(\omega)}{\omega}$

Brambilla *et.al.*PRD96 (2017), Brambilla *et.al.*PRD97 (2018), Brambilla *et.al.*PRD100 (2019), Eller *et.al.*PRD99 (2019)
Scheihing-Hitschfeld & Yao PRD108 (2023), J. Mayer-Steudte PoSLattice2024, Brambilla *et.al.*PRD112 (2025)

Heavy Quarkonium Diffusion on the lattice

- Euclidean correlators similar to HQ-case, but with adjoint Wilson line
- κ_{so} and κ_{os} given by

$$G_{\mathrm{E}}(\tau) = -\frac{1}{3}\sum_{i=1}^{3}\langle \mathrm{Re}\,\mathrm{Tr}\,[gE_i(\tau,0)\Phi(\tau,0)gE_i(0,0)]\rangle = -\frac{2}{3}\sum_{i=1}^{3}\langle \mathrm{Tr}\,[E_i(\tau)U(\tau,0)E_i(0)U(0,\tau)]\rangle\,,$$

- Separating κ_{so} and κ_{os} on lattice still work in progress
- κ_{oo} similar to HQ-diffusion

$$G_{\mathrm{EE}}^{\mathrm{oct}} \equiv \frac{1}{3\langle l_8\rangle}\sum_{i=1}^{3}\langle \Phi^{\mathrm{A}}_{xa}(N_T,t)d_{abc}E^{i,c}(t)\Phi^{\mathrm{A}}_{bz}(t,0)d_{zxg}E^{i,g}(0)\rangle$$

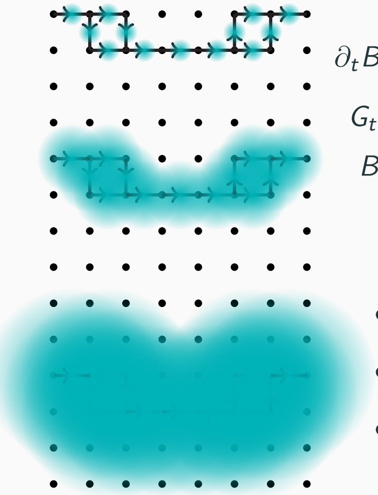
$$= \frac{-1}{3\langle l_8\rangle}\sum_{i=1}^{3}\langle \mathrm{Tr}\,[E_i(\tau)P(\tau)^{\dagger}]\mathrm{Tr}\,[E_i(0)P(0)] + \mathrm{Tr}\,[E_i(\tau)U(\tau,1/T)E_i(0)U(0,\tau)]\mathrm{Tr}\,[P(0)] - \frac{4}{3}\mathrm{Tr}\,[E_i(\tau)U(\tau,0)E_i(0)U(0,\tau)] + \mathrm{h.c.}$$

- Also related: Diffusion of an adjoint static quark

$$G_{\mathrm{EE}}^{\mathrm{symm}} \equiv \frac{1}{3\langle l_8\rangle}\sum_{i=1}^{3}\langle \Phi^{\mathrm{A}}_{xa}(N_T,t)f_{abc}E^{i,c}(t)\Phi^{\mathrm{A}}_{bz}(t,0)f_{zxg}E^{i,g}(0)\rangle$$

$$= \frac{1}{3\langle l_8\rangle}\sum_{i=1}^{3}\langle \mathrm{Tr}\,[E_i(\tau)P(\tau)^{\dagger}]\mathrm{Tr}\,[E_i(0)P(0)] - \mathrm{Tr}\,[E_i(\tau)U(\tau,1/T)E_i(0)U(0,\tau)]\mathrm{Tr}\,[P(0)] + \mathrm{h.c.}$$

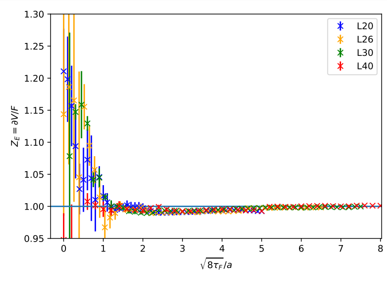
Gradient Flow



$$\partial_t B_{t,\mu} = -\frac{\delta S_{YM}}{\delta B} = D_{t,\mu} G_{t,\mu\nu} \,,$$

$$G_{t,\mu\nu} = \partial_\mu B_{t,\nu} - \partial_\nu B_{t,\mu} + [B_{t,\mu}, B_{t,\nu}] \,.$$

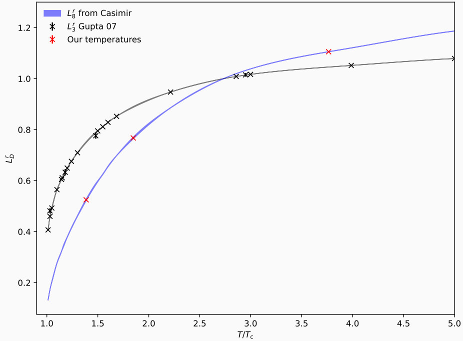
$$B_{0,\mu} = A_\mu \;\leftarrow \text{ the original gauge field}$$

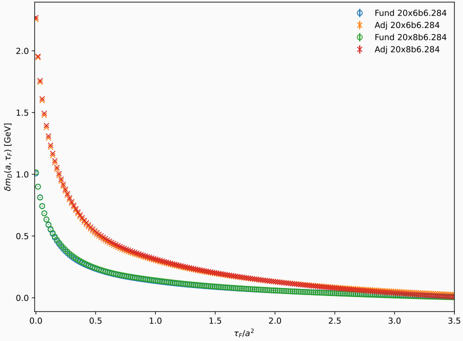


- Smearing the Wilson lines
- Avoid overlap $\sqrt{8\tau_f} < \tau/2$
- Automatically renormalizes the E-fields $\sqrt{8\tau_f} > a$

See e.g. M. Lüscher JHEP 08 (2010). Z_e figure from Brambilla *et.al.*PRD 105 (2022)

Removing Wilson line self-energy



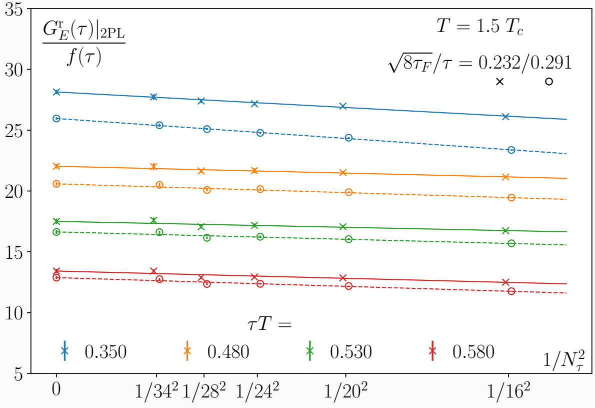


- For symmetric correlators: same procedure as fundamental $\Rightarrow$ divide by Polyakov loop
- Non-symmetric correlator:

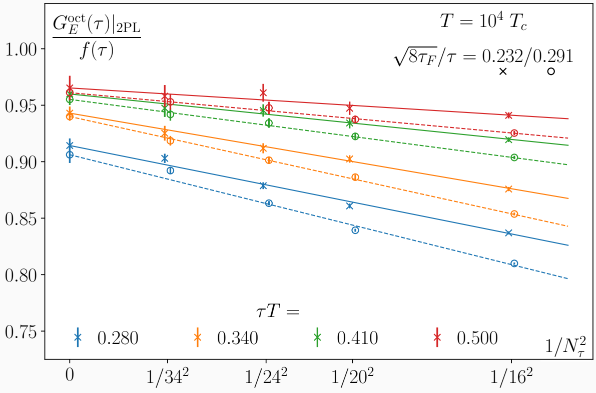
  $\Rightarrow$ Wilson line has divergence $\exp(\delta m \tau)$ with $\delta m \propto 1/\sqrt{8\tau_F}$
- Use renormalized Polyakov loops from Gupta *et.al.*PRD77 2008

$$L_8^{\mathrm{r}} = e^{\delta m(\tau_F)/T} L_8(\tau_F)\,, \quad G_E^{\mathrm{r}}(\tau, \tau_F) = e^{\delta m(\tau_F)\tau} G_E(\tau, \tau_F) = \left(\frac{L_8^{\mathrm{r}}}{L_8(\tau_F)}\right)^{\tau T} G_E(\tau, \tau_F)$$

Continuum limit



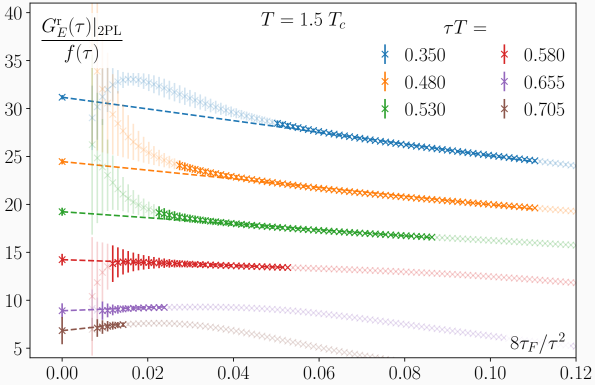
Low T, Asymmetric



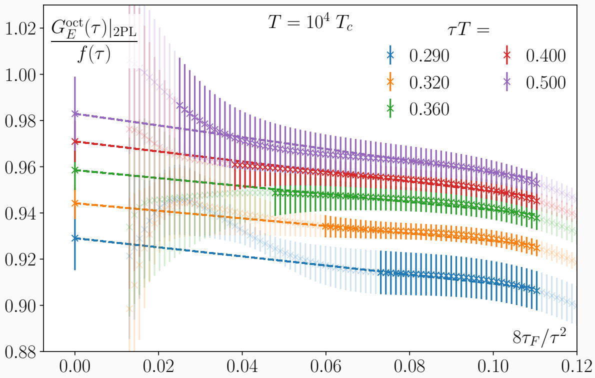
High T, Symmetric

- Good limits for valid ranges of τT and τ_{F} for both symmetric and asymmetric correlators at both temperatures
- Spatial volume scaling negligible

Flow time dependence of G_{E}



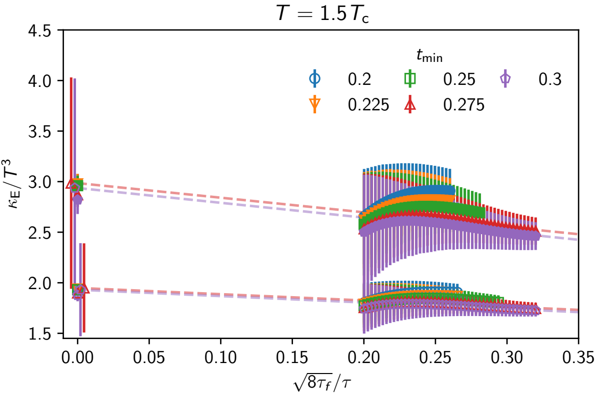
Low T, Asymmetric

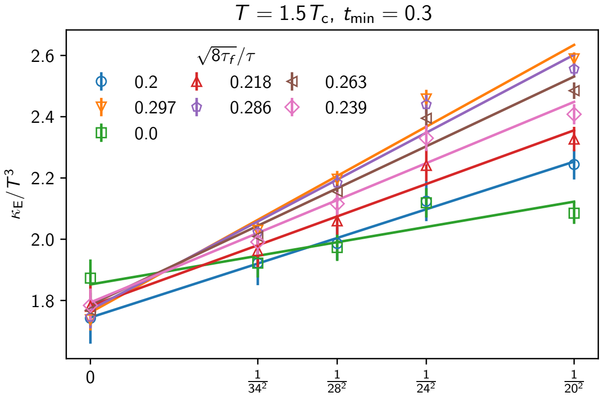


High T, Symmetric

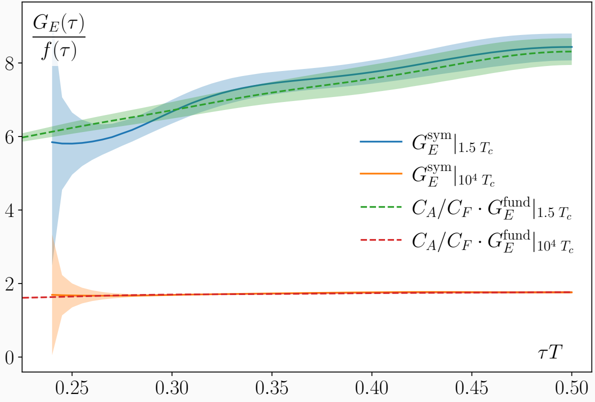
- Good linear flow time limits within valid range for both correlators

Flow time κ and order of limits

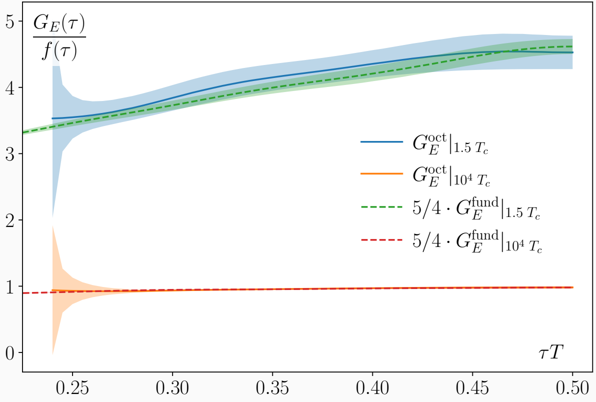




- Very little dependence on flow time
- Ordering of Continuum limit, zero flow time limit, and spectral function inversion doesn't seem to matter much



Adjoint quark

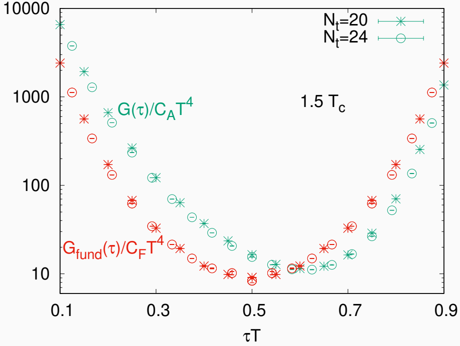


Octet-Octet

- For symmetric correlators we observe expected (Casimir) scaling nonperturbatively
- These results translate from G_E to κ trivially

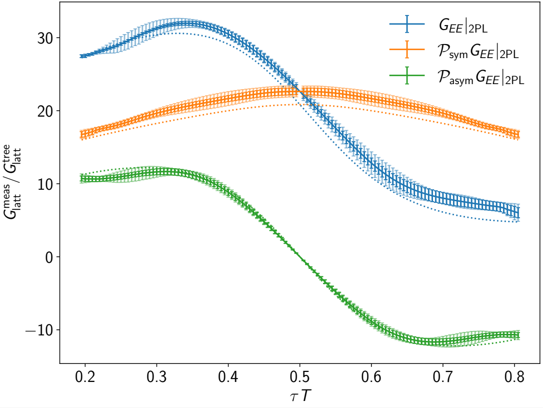
Singlet-to-Octet transition

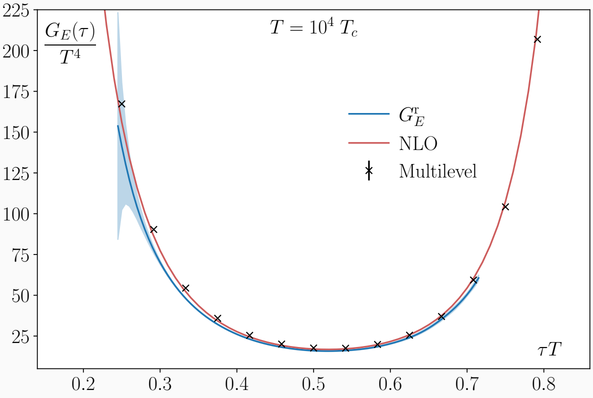
- Asymmetric correlator
- LO contribution still given by $f(\tau)$
- Starting NLO the contribution from even spectral function breaks the color scaling
- Spectral function has both even and odd contributions



$$G(\tau) = \int_0^\infty \frac{d\omega}{\pi}\, \rho_s^{EE}\, \frac{\cosh \omega(1/2T - \tau)}{\sinh \omega/2T} + \int_0^\infty \frac{d\omega}{\pi}\, \rho_a^{EE}\, \frac{\sinh \omega(1/2T - \tau)}{\sinh \omega/2T}$$

Brambilla *et.al.*JHEP 08 (2025). See also: Scheihing-Hitschfeld & Yao PRD108 (2023)



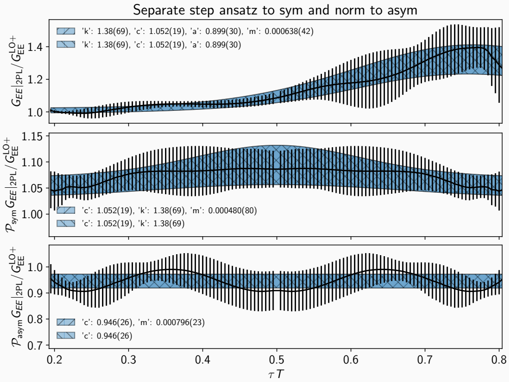


- Asymmetric correlator on lattice relates to both κ_{so} and κ_{os}
- Spectral reconstruction still pending
- At high temperatures, excellent agreement with the perturbation theory

NLO result: Brambilla *et.al.*JHEP 08 (2025)

κ_{so} and κ_{os}

- Similar fit as the heavy quark case
- Preliminary analysis, still work in progress
- Different normalizations for symmetric and asymmetric parts to gauge higher order effects
- Perturbation theory expectation $\kappa_{so} = \kappa_{\text{fund}}$ cannot be ruled out but some indication that κ_{so} could be smaller
- Still to do: extract γ



Brambilla *et.al.*JHEP 08 (2025). See also: Scheihing-Hitschfeld & Yao PRD108 (2023)

Advert about zero temperature correlators

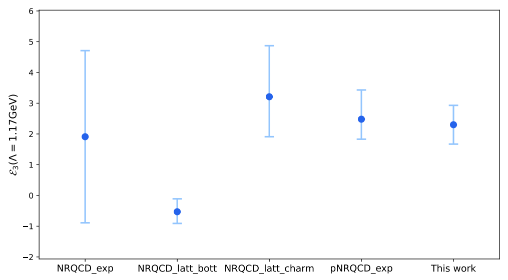
- We have also measured the zero temperature version of the correlators

TUM-EFT 204/26, Julian Mayer-Steudte's talk last week

- The 3rd moment related to quarkonia production and decays via pNRQCD

$$\mathcal{E}_3(\Lambda) = \frac{T_\mathrm{F}}{N_\mathrm{c}} \int_0^\infty \mathrm{d}t\, t^3 \langle EE \rangle(t)$$

- Fit the adjoint Wilson line divergence w.r.t. flow time



Brambilla *et.al.*JHEP 08 (2025). See also: Scheihing-Hitschfeld & Yao PRD108 (2023)

Summary

- Quarkonia in plasma can be described with transport coefficients
- We have measured the relevant correlators on the lattice
- Extraction of the relevant transport coefficients κ, γ work in progress
- The correlator for singlet-to-octet transition is asymmetric in nature
- We observe Casimir scaling with the symmetric operators, indicating Casimir scaled κ
- Magnetic version of quarkonium correlators also in progress

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Thank you for your attention!