Lattice QCD form factors for radiative leptonic decays of heavy mesons

Teseo San Jose

University of Edinburgh

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THE UNIVERSITY of EDINBURGH

UK Research and Innovation

In collaboration with...

JLQCD

KEK

- Shoji Hashimoto
- Takashi Kaneko

R-CCS

- Yasumichi Aoki

UKQCD

CERN

- Matteo Di Carlo
- Felix Erben

DESY Zeuthen

- Raoul Hodgson

University of Edinburgh

- Vera Gülpers
- Maxwell T. Hansen
- Nils Hermansson-Truedsson
- Ryan Hill
- Antonin Portelli
- Ali Shakir

University of Liverpool

- J. Tobias Tsang

ETH Zurich

- Tim Harris

First, consider D-meson leptonic decay

$$D \to \mu\nu_\mu$$

The branching fraction is linked to the CKM matrix

$$\mathcal{B} \equiv \frac{\Gamma(D \to \mu\nu_\mu)}{\Gamma_{\text{total}}} = \tau_D \frac{G_F^2}{8\pi} |V_{\text{cd}}|^2 f_D^2 m_\mu^2 M_D \left(1 - \frac{m_\mu^2}{M_D^2}\right)^2$$

Use

$$\mathcal{B} = 3.71(20) \times 10^{-4} \qquad \text{BESIII [1]}$$
$$f_D = 212.0(7)\,\text{MeV} \qquad N_f = 2+1+1 \text{ [10]}$$
$$\tau_D = 1.033(5) \times 10^{-12}\text{s} \qquad \text{Lifetime [7]}$$

The CKM matrix element is then [7]

$$|V_{\text{cd}}| = 0.2181(50)$$

Lattice input at 0.3% precision!

Introduction: QED corrections

In experiment, one really measures the radiation-inclusive rate

$$\Gamma(\Delta E_\gamma) \equiv \Gamma(D^+ \to \mu^+ \nu_\mu[\gamma])|_{E_\gamma < \Delta E_\gamma}$$

with ΔE_γ the experimental sensitivity.

$$\Gamma(\Delta E_\gamma) = \Gamma_0 + \Gamma_1(\Delta E_\gamma)$$

We've added two types of QED corrections,

- Loop (virtual) corrections to $\Gamma_0 = \Gamma(D^+ \to \mu\nu_\mu)$
- Real radiative decay $\Gamma_1(\Delta E_\gamma) = \Gamma(D^+ \to \mu\nu_\mu\gamma)|_{E_\gamma < \Delta E_\gamma}$

QED corrections are expected to be 1% [1] at vanishing ΔE_γ, and dominant for large E_γ [5]

For the real radiative decay

$$D^+(p) \to \mu^+(\ell)\nu_\mu(n)\gamma(k,\lambda)$$

the amplitude $\mathcal{M}_1$ can be split into hadronic and leptonic parts

$$\mathcal{M}_1^{(\lambda)} = \epsilon_\mu^{(\lambda)*}(k)\left(\mathcal{M}_{\text{had}}^\mu + \mathcal{M}_{\text{lep}}^\mu\right)$$

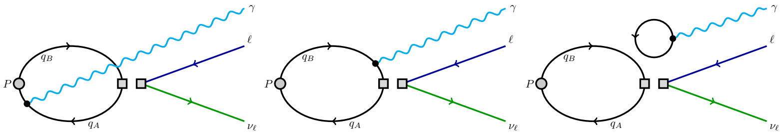
The hadronic part is

$$\mathcal{M}_{\text{had}}^\mu = \frac{G_F}{\sqrt{2}} V_{\text{cd}}^* e H^{\mu\nu}(p,k)\bar{u}(\ell)\gamma_\beta(1-\gamma_5)v(n)$$

The non-perturbative physics appears in the hadronic tensor

$$H^{\mu\nu}(p,k) = i\int \mathrm{d}^4x\, e^{ik\cdot x}\left\langle 0\middle|\mathrm{T}\left\{j_{\text{em}}^\mu(x) j_W^\nu(y)\right\}\middle|D^+(p)\right\rangle$$

There are three Wick contractions of $H^{\alpha\beta}(p,k)$



The hadronic tensor can be computed in Euclidean space [3]

$$H^{\mu\nu}(\mathbf{k},\mathbf{p}) = \int \mathrm{d}t_\gamma\, e^{t_\gamma E_\gamma}\, \mathrm{d}^3 z\, e^{i\mathbf{p}\cdot\mathbf{z}}\, \mathrm{d}^3 x\, e^{-i\mathbf{k}\cdot\mathbf{x}} \underbrace{\langle 0 | j^\mu_{\mathrm{em}}(x) j^\nu_W(y) \phi^\dagger(z) | 0 \rangle}_{C^{\mu\nu}(x,y,z)}$$

where

$$j^\mu_{\mathrm{em}}(x) = \sum_q e_q \bar{q}(x)\gamma_\mu q(x)$$
$$j^\nu_W(y) = \bar{q}_A(y)\gamma^\nu(1-\gamma_5)q_B(y)$$
$$\phi = \bar{q}_A(z)\gamma_5 q_B(z)$$

No need for explicit photon field!

Objective

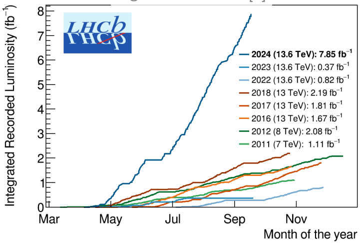
Study the 1st and 2nd rows of CKM with leptonic decays of

$$D \quad D_s \quad B \quad B_c$$

PDG [7]	Leptonic	Semi-leptonic
$\lvert V_{ub}\rvert$	$4.11(39) \times 10^{-3}$	$3.82(20) \times 10^{-3}$
$\lvert V_{cd}\rvert$	$0.2181(50)$	$0.2330(136)$
$\lvert V_{cs}\rvert$	$0.984(12)$	$0.972(7)$
$\lvert V_{cb}\rvert$	Not seen	$41.1(12) \times 10^{-3}$

Experiments get more precise [9] and look at new channels [8]

Figure taken from [6]



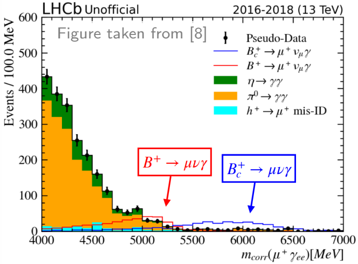


Figure taken from [8]

Setup: JLQCD DWFs

Action

→ All quarks are Möbius $b + c = 2,\ b - c = 1$

→ Tree-level improved Symanzik gauge + stout smearing

DWFs preserve chiral symmetry

→ Our observable is chiral

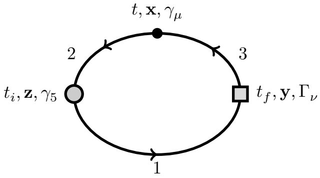
→ Simpler renormalization

We employ one **JLQCD** $N_f = 2 + 1$ ensemble [4]

Label	β	a [fm]	a^{-1} [GeV]	$L^3 \times T \times L_s$	m_π	m_K	$m_\pi L$
F-ud3-sa	4.47	**0.0439(1)**	4.494(2)	$64^3 \times 128 \times 8$	284(1)	486(1)	4.0

29 configs, 32 measurements/cnfg. Simulate at six heavy-quarks

am_ℓ	am_s			am_h			
0.003	0.015	0.210476	0.263095	0.328869	0.411086	0.513857	0.642322



Solve propagators $Q^{(1)}$, $Q^{(2)}$

→ $Z_2 \otimes Z_2$ wall source at $(t_i, \mathbf{z})$

→ Jacobi smearing

Solve propagator $Q^{(3)}$

→ Point source at $(t_f, \mathbf{y} = \mathbf{0})$

→ Use twist $\Theta = (0, 0, \pm\theta)$

Assemble Wick contraction

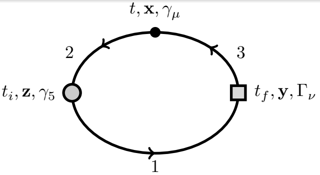
$$C^{\mu\nu} = E_\xi\{\mathrm{tr}[\Gamma^\nu Q^{(1)}(t_f, \mathbf{y}; t_i) Q^{(2)}(t, \mathbf{x}; t_i)^\dagger \gamma_5 \gamma^\mu \tilde{Q}^{(3)}(t, \mathbf{x}; t_f, \mathbf{y}; \theta)]\}$$

Kinematics

→ Meson at rest $\mathbf{p} = \mathbf{0}$

→ Photon $\mathbf{k} = \frac{2\pi}{L}(\mathbf{n} - \Theta)$ with Fourier mode $\mathbf{n}$

→ Lepton-neutrino pair $\mathbf{q} = -\mathbf{k}$

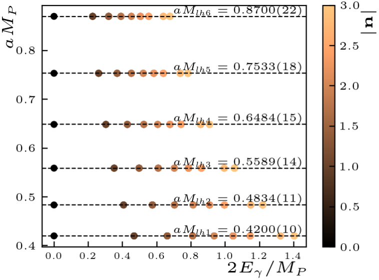


Our setup employs

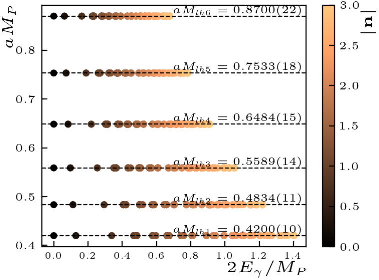
$$N_q = 8 \quad N_{t_i} = 128 \quad N_{t_f} = 32 \quad N_\theta = 2$$

\# of propagators N_{prop} grows linearly with # of sources

$$N_{\text{prop}} = N_q \cdot (N_{\text{src}} + N_{\text{snk}} \times (N_\theta + 1)) = 1792$$



Fourier $|\boldsymbol{n}| \leq 3$

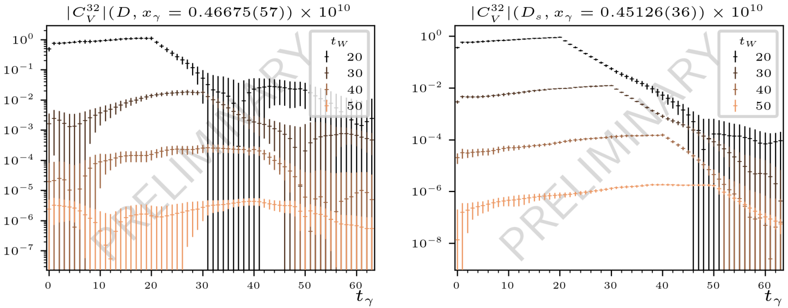


Fourier $|\boldsymbol{n}| \leq 3$ and $\pm\theta$

Example data for $\mathbf{k} = \frac{2\pi}{L}(1,0,0)$, $\mathbf{p} = (0,0,0)$

$$C_V^{\mu\nu}(t_\gamma, \mathbf{k}, \mathbf{p}) = \langle 0 | j_{\text{em}}^{\mu}(t_\gamma, \mathbf{k}) \bar{q}_A(t_W, \mathbf{0}) \gamma^\nu q_B(t_W, \mathbf{0}) \phi^\dagger(\mathbf{z}) | 0 \rangle$$

where $t_W = t_f - t_i$ and $t_\gamma = t - t_i$



We give the correlator in terms of the photon energy

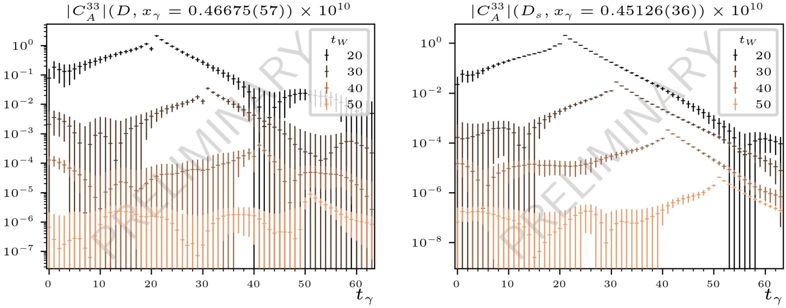
$$x_\gamma = 2E_\gamma / M_P$$

Three-point functions

Example data for $\mathbf{k} = \frac{2\pi}{L}(1,0,0)$, $\mathbf{p} = (0,0,0)$

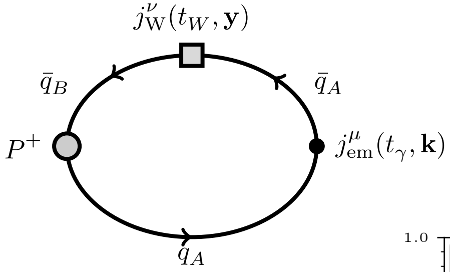
$$C_A^{\mu\nu}(t_\gamma, \mathbf{k}, \mathbf{p}) = \langle 0 | j^\mu_{\text{em}}(t_\gamma, \mathbf{k}) \bar{q}_A(t_W, \mathbf{0}) \gamma^\nu \gamma_5 q_B(t_W, \mathbf{0}) \phi^\dagger(\mathbf{z}) | 0 \rangle$$

where $t_W = t_f - t_i$ and $t_\gamma = t - t_i$



Second half of the lattice has unphysical time-ordering and vanishes

Exponential fall off

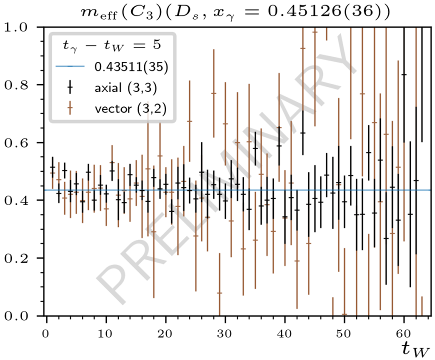


For $0 < t_W < t_\gamma$:

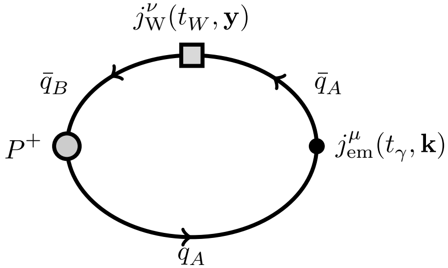
- Fix $t_\gamma - t_W > 0$
- Vary t_W
- Initial meson at rest

Compare spectrum obtained from 3pt and 2pt

$$x_\gamma = \frac{2E_\gamma}{M_P}$$



Exponential fall off



1^{--} state depends on q_A

q	s	Q_0	Q_i
$\rho(770)$	$\phi(1020)$	J/ψ	$\Upsilon(1s)$

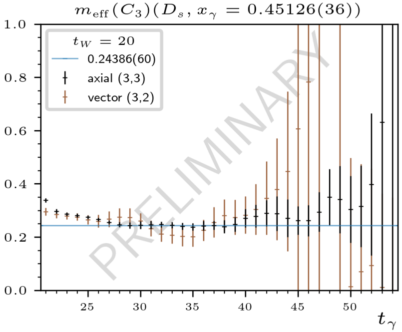
Same for $C_V^{\mu\nu}$ and $C_A^{\mu\nu}$

For $0 < t_W < t_\gamma$:

Fix t_W

Vary t_γ

1^{--} meson momentum $\mathbf{k}$



D_s vector form factor

$H^{\mu\nu}$ has a form-factor decomposition

$$H^{\mu\nu} = H_V^{\mu\nu} + H_A^{\mu\nu}$$

The vector component depends on F_V

$$H_V^{\mu\nu} = -i\epsilon^{\mu\nu\rho\sigma} k_\rho p_\sigma \frac{F_V(x_\gamma)}{M_P} \quad \rightarrow \quad iF_V(x_\gamma) \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & k_3 & -k_2 \\ 0 & -k_3 & 0 & k_1 \\ 0 & k_2 & -k_1 & 0 \end{pmatrix}$$

F_V depends only on one Lorentz scalar

$$x_\gamma = \frac{2p \cdot k}{M_P^2} \underset{\text{CM}}{=} \frac{2E_\gamma}{M_P}$$

Physical decays occur in

$$0 \leq x_\gamma \leq 1 - \left(\frac{m_\ell}{M_P}\right)^2 \sim 1$$

Use the spectral decomposition of C^{32},

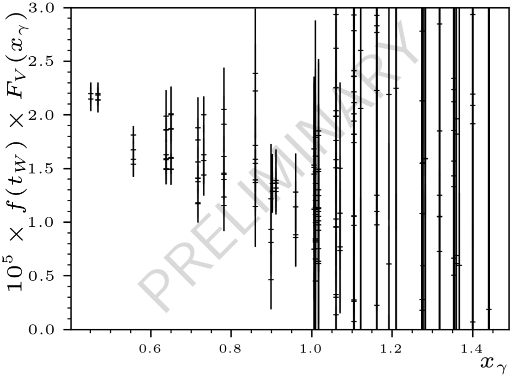
$$f(t_W)F_V(x_\gamma) = i\int_0^\infty \mathrm{d}t_\gamma\, e^{(t_\gamma - t_w)E_\gamma} R^{32}/k_1$$

where

$$R^{\mu\nu} \equiv \frac{C_V^{\mu\nu}(t_\gamma, \mathbf{k}, t_W, \mathbf{p})}{\sqrt{C_2(t_W)}}$$

and

$$f(t_W) \equiv \frac{e^{-M_P t_W/2}}{\sqrt{2M_P}}$$



Signal improvement

Use Lorentz covariance

1. Select a reference vector k, for example $k = 2\pi(1,1,0,0)/L$
2. Find all k' so that

$$k^\mu = \Lambda(R)^\mu_\alpha k'^\alpha \qquad \text{where} \qquad \Lambda(R) = \begin{pmatrix} 1 & 0 \\ 0 & R \end{pmatrix}$$

   R is one of 24 rotations in the cubic group

$$Lk'/2\pi = (1, \pm 1, 0, 0),\ (1, 0, \pm 1, 0),\ (1, 0, 0, \pm 1)$$

3. The hadronic tensor $H^{\mu\nu}$ transforms as

$$H^{\mu\nu}(k) = \Lambda(R)^\mu_\alpha \Lambda(R)^\nu_\beta H^{\alpha\beta}(k')$$

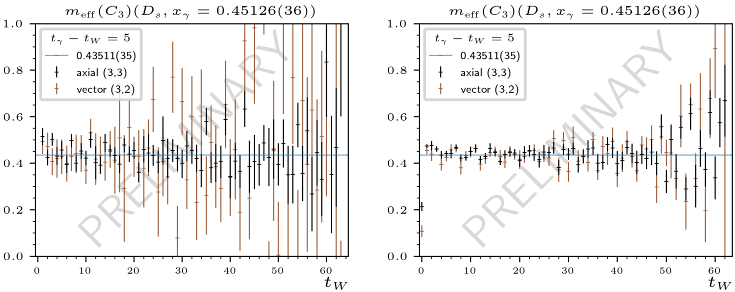
4. Average all $H^{\mu\nu}$ related in this fashion

Use T-symmetry

$$C_V^{ij}(t_W, t_\gamma, \mathbf{k}) = -C_V^{ij}(T - t_W, T - t_\gamma, \boldsymbol{k})$$

3pt effective mass vs 2pt energy fit

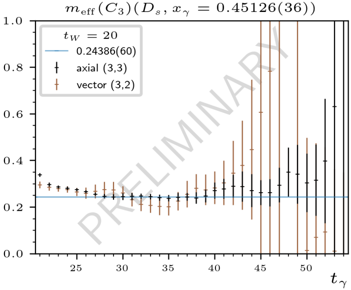
→ Initial meson at rest

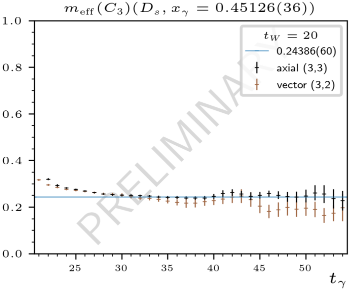


Signal improvement

3pt effective mass vs 2pt energy fit

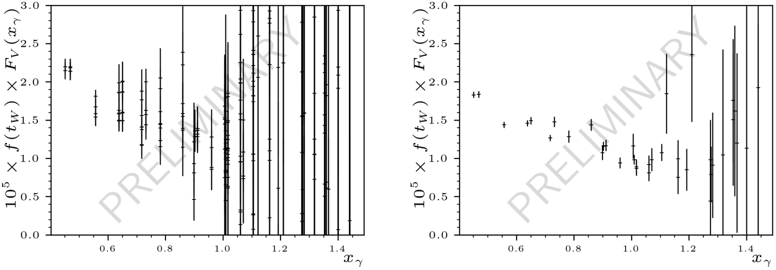
$\rightarrow$ 1^{--} meson with momentum $\mathbf{k} = 2\pi(1,0,0)/L$





D_s vector form factor

Vector form factor from C^{32}



There are two main unexplored systematics:

- $\rightarrow$ Truncation of integral in t_γ
- $\rightarrow$ Excited states/dependence on t_W

Combine fit of 2pt and 3pt functions to extract F_V

D_s axial form factor

$H^{\mu\nu}$ has a form-factor decomposition

$$H^{\mu\nu} = H_V^{\mu\nu} + H_A^{\mu\nu}$$

The axial component depends on f_P, F_A, H_1, H_2

$$\begin{aligned} H_A^{\mu\nu} =& f_P \left(\eta^{\mu\nu} + \frac{(2p-k)^\mu (p-k)^\nu}{2p\cdot k - k^2} \right) \\ &+ \frac{F_A(x_\gamma)}{M_P} \left[\eta^{\mu\nu}(p\cdot k - k^2) - (p-k)^\mu k^\nu \right] \\ &+ \frac{H_1(x_\gamma)}{M_P} \left(\eta^{\mu\nu} k^2 - k^\mu k^\nu \right) \\ &+ \frac{H_2(x_\gamma)}{M_P} \frac{k^\mu (p\cdot k - k^2) - k^2 (p-k)^\mu}{(p-k)^2 - M_P^2} (p-k)^\nu \end{aligned}$$

The same relation applies in Minkowski and Euclidean space ($\eta^{\mu\nu} = \delta^{\mu\nu}$)

Conclusions and outlook

→ QED corrections to leptonic decays are relevant for CKM unitarity tests

→ Radiative decays are given by two form factors

$$F_V(2E_\gamma/M_P) \qquad F_A(2E_\gamma/M_P)$$

→ Thanks to very fine lattice spacings, we can study

$$D \qquad D_s \qquad B \qquad B_c$$

with the same quark action

Next steps

→ Finalize main analysis

→ Study systematics:

1. Excited states t_W
2. Truncation of integral in t_γ

→ Reach the physical point with more ensembles

Appendix

D_s axial form factor

Here we need all μ for a fix ν

$$f'(t_W)F_A(x_\gamma) = \frac{1}{x_\gamma \epsilon_3^{(\lambda)}} \epsilon_\mu^{(\lambda)} \int_0^\infty \mathrm{d}t_\gamma \, e^{(t_\gamma - t_W)E_\gamma} \tilde{R}_A^{\mu 3}$$

where ν is not summed over and

$$f'(t_W) = \sqrt{M_P/8} e^{-M_P t_W/2} \qquad R_A^{\mu\nu} \equiv C_A^{\mu\nu}/\sqrt{C_2}$$

To remove the f_P term, subtract $\mathbf{k} = \mathbf{0}$ point [3]

$$\tilde{R}_A^{\mu\nu}(t_\gamma, t_W, \mathbf{k}) = R_A^{\mu\nu}(t_\gamma, t_W, \mathbf{k}) - R_A^{\mu\nu}(t_\gamma, t_W, \mathbf{k} = \mathbf{0})$$

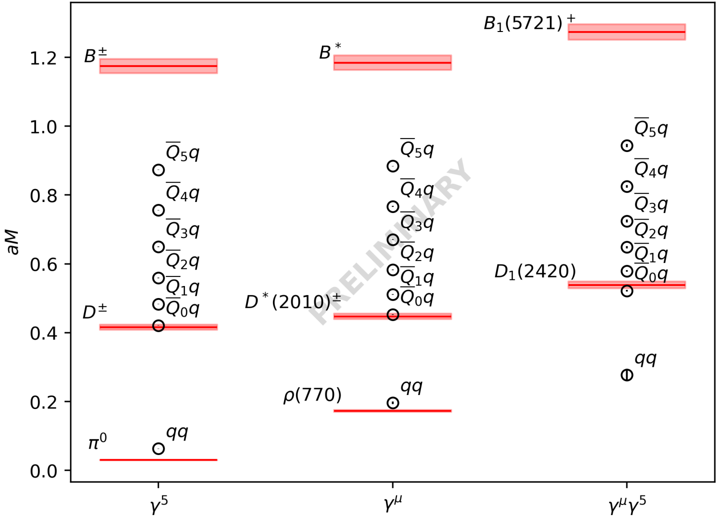
To remove the H_1 and H_2 terms, use polarization vectors such

$$\epsilon_\mu^{(\lambda)}(k) \cdot k = 0 \quad \epsilon_\mu^{(\lambda)} \epsilon_\mu^{(\lambda')} = \delta_{\lambda\lambda'}$$

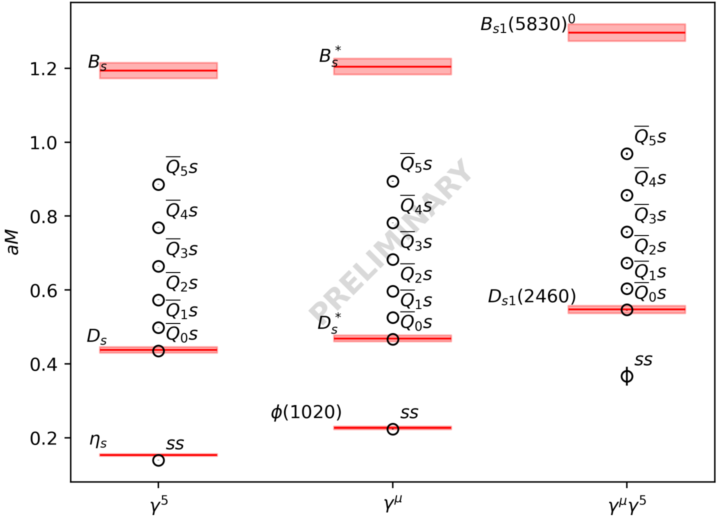
In a physical decay, $k^2 = 0$ (given by the Laplace transform)

Heavy-light spectrum on F-ud3-sa

Values at finite lattice spacing compared to PDG values [7]



Values at finite lattice spacing compared to PDG values [2, 7]



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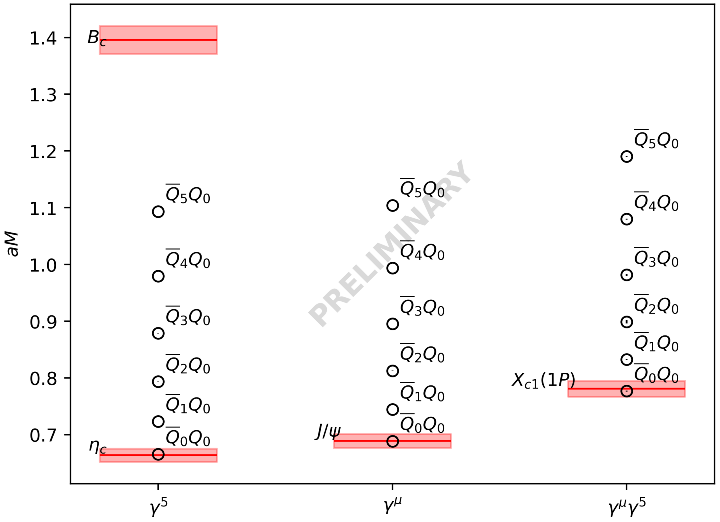

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