Super Yang-Mills on the lattice in the large number of colours limit

B. Jäger

based on ***Phys.Rev.D*** **112 (2025)**
with M. Della Morte, P. Butti, S. Martins, J. T. Tsang

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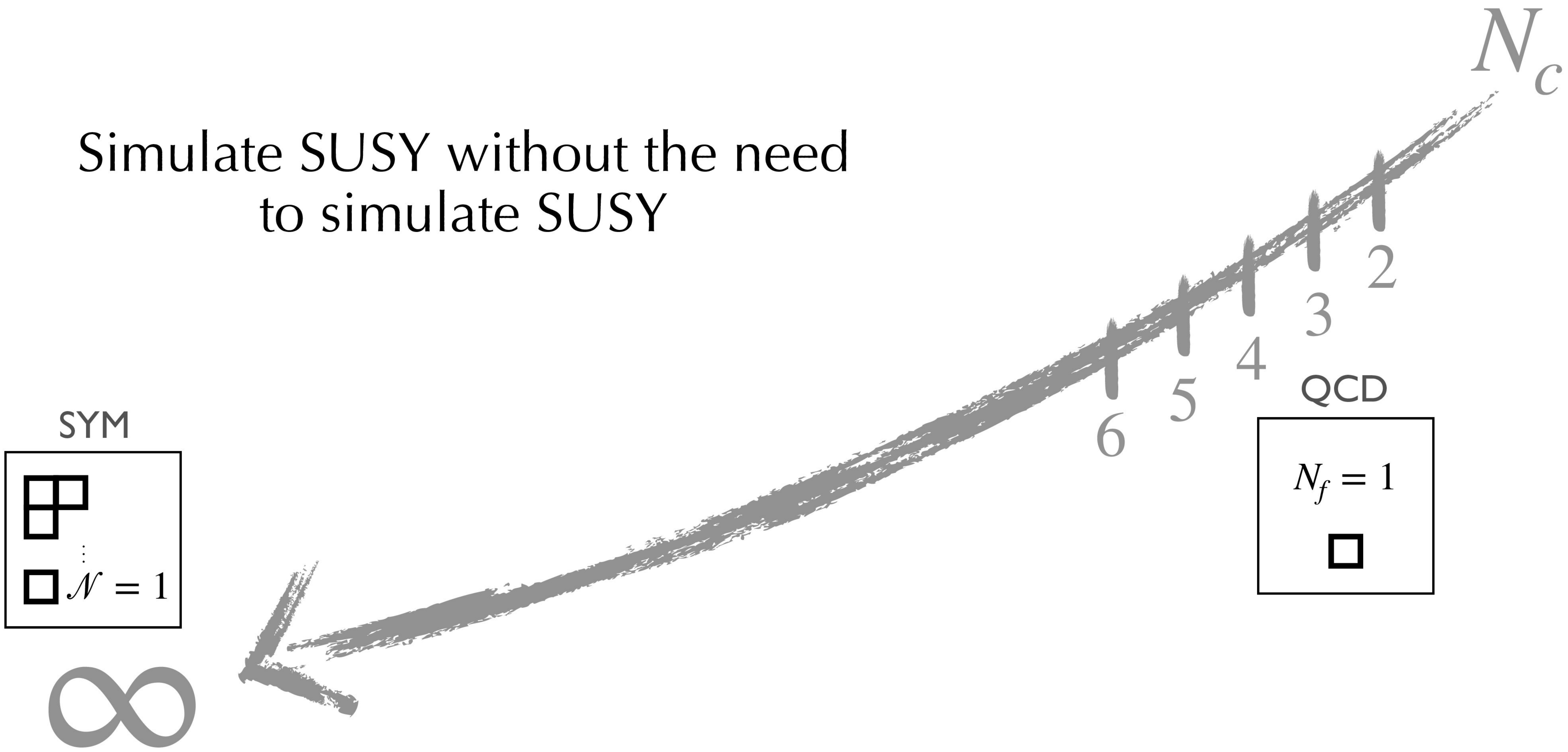
ħQTC

Nordic Lattice Meeting 2026 @ Edinburgh
May 18th, 2026

Targeting $\mathcal{N} = 1$ SUSY Yang-Mills

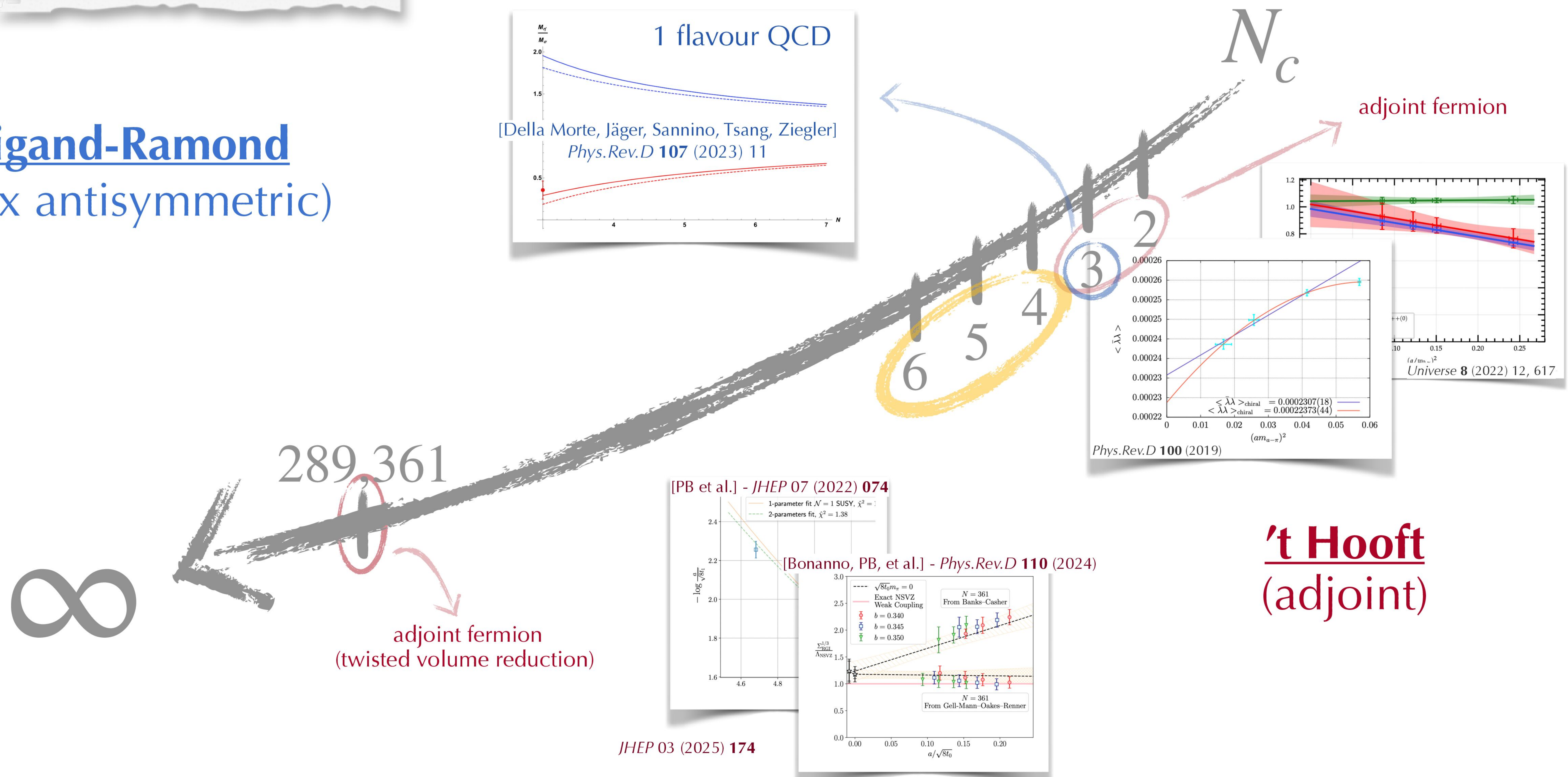
Simulate SUSY without the need
to simulate SUSY



Targeting $\mathcal{N} = 1$ SUSY Yang-Mills

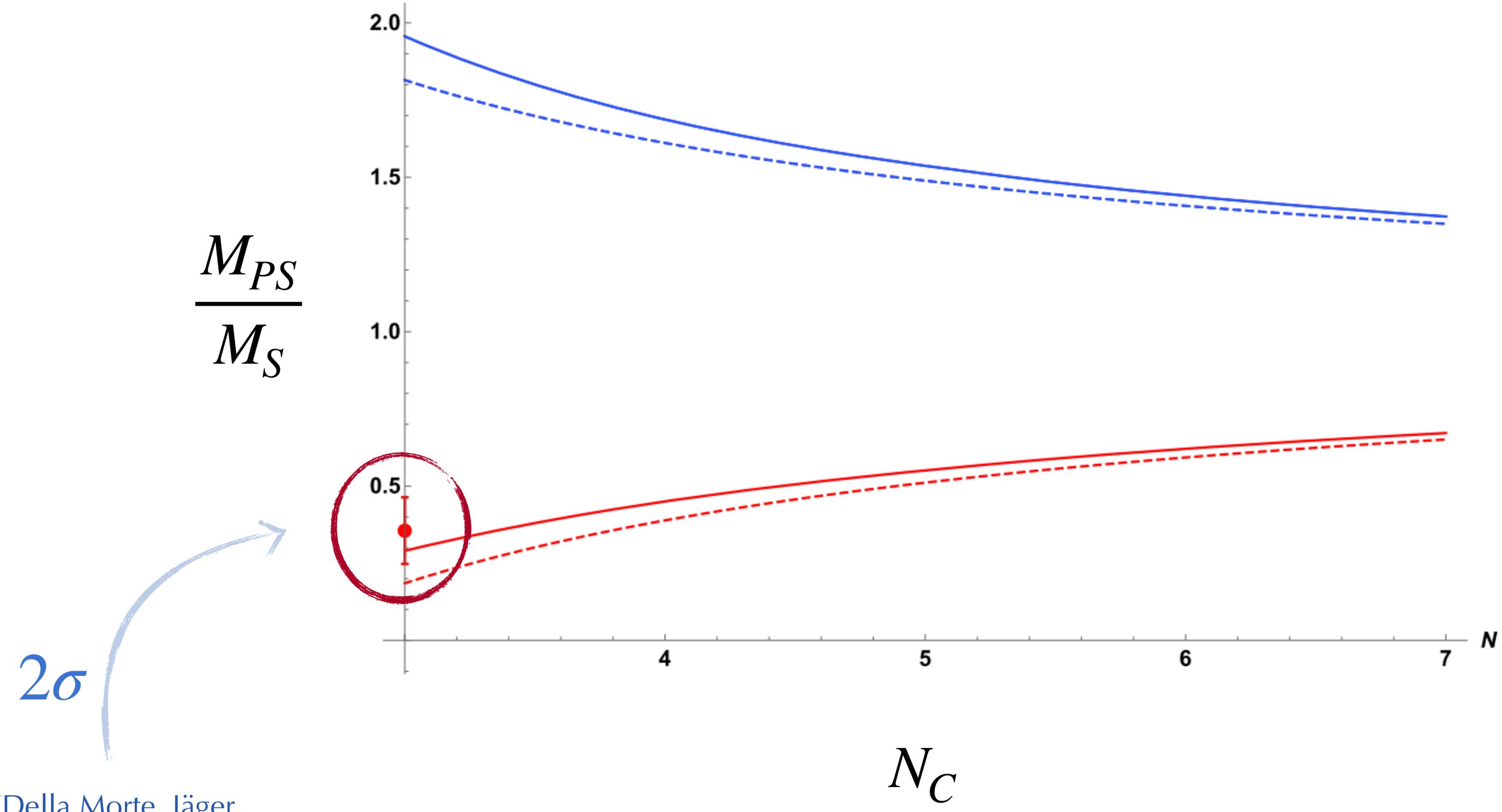
$$\mathscr{L} = -\frac{1}{2}\mathrm{tr}\,F^2 + i\bar{\lambda}D\lambda$$

Corrigand-Ramond
(2-index antisymmetric)



Result for Nc = 3

Sannino, hep-th/
2402.05850

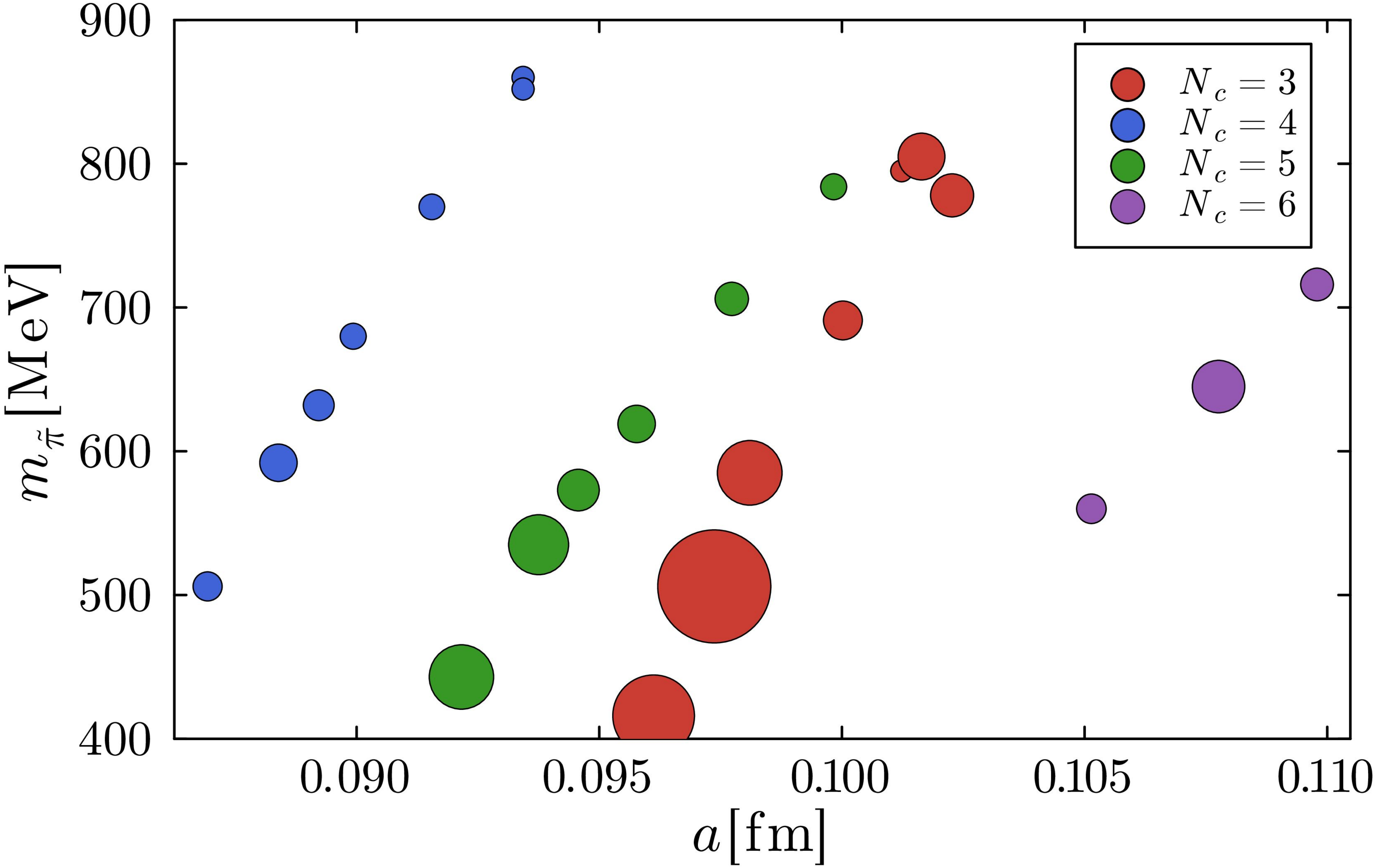


[Della Morte, Jäger,
Sannino, Tsang, Ziegler]
Phys.Rev.D **107** (2023) 11

Expectations:

$$\frac{M_{PS}}{M_S} = \frac{1 - 2/N_C}{1 + \frac{4}{9N_C}}$$

Status on large-Nc runs



$\vdash\!\dashv \sim \frac{1}{\sqrt{N_{\text{conf}}}}$

Increasing N_C by 1 leads to $\sim 3\times$ CPU

Finest Lattices that allow topological charge tunnelling (except $N_C = 3$)

While we wait ...

$$\text{Index}(D) = 2T_R Q$$

2-index antisymm.

$$\Rightarrow$$

$$T_{2A} = \frac{N_c - 2}{2}$$

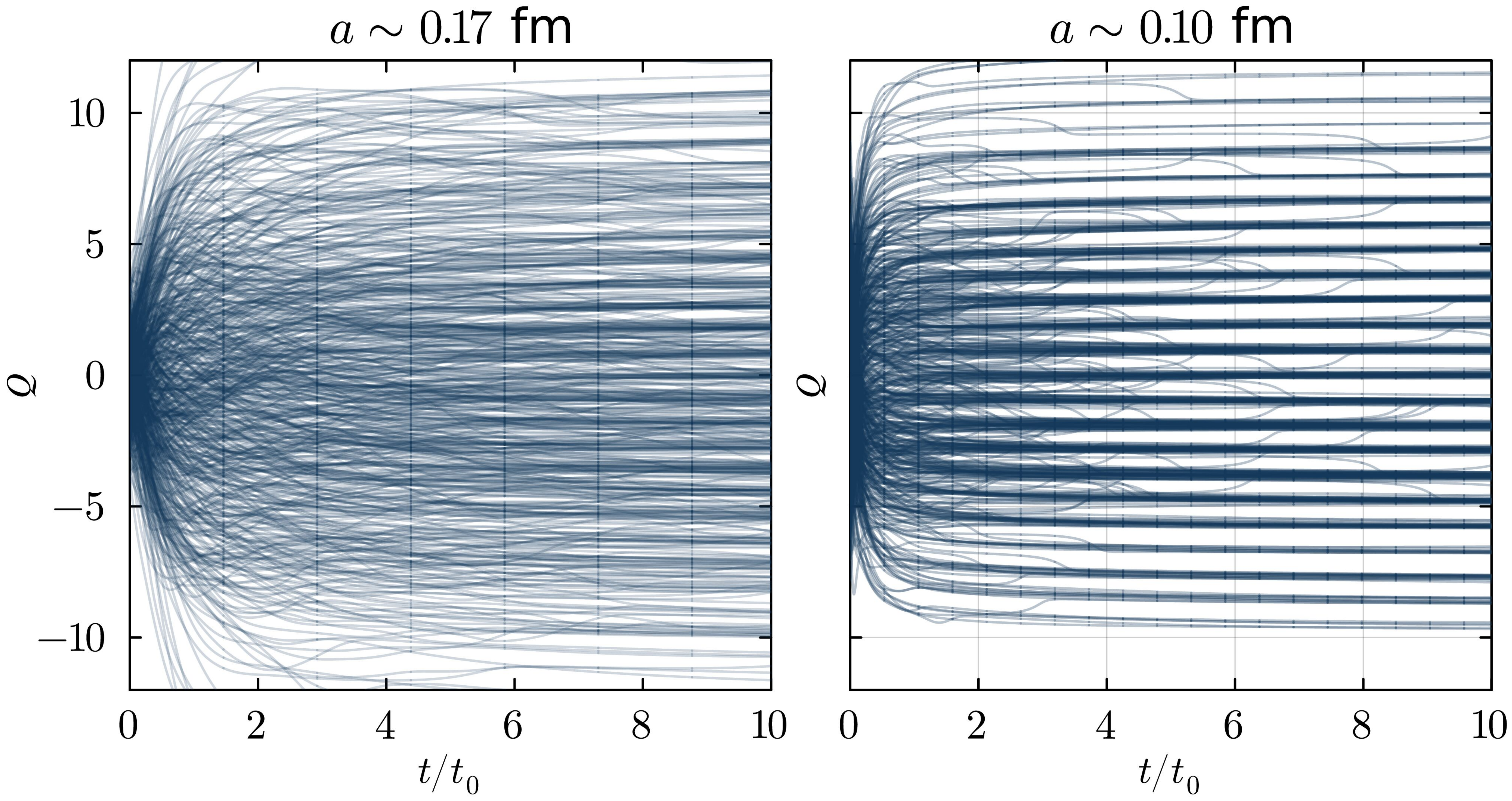
Topological Charge

$$Q = \frac{n_+ - n_-}{N_c - 2} \notin \mathbb{Z}$$

- On the lattice with periodic BC, for SU(3) w/ 2-index symmetric, fractional objects disappears as $a \to 0$ [Fodor, Holland, Kuti, Nogradi, Schroeder] - *JHEP* **08** (2009) 084
- Fractional instantons are possible (also on the lattice) with twisted BC (with non-orthogonal twist) [González-Arroyo] - *JHEP* **02** (2020) 137, [Dasilva Golan, García Pérez] - *JHEP* **12** (2022) 109

Configurations with *non-integer* Q may survive at $a \to 0$?

The topological charge - Wilson



While we wait ...

Topological Charge

…on the lattice

✓ Choose a discretisation for (bare) Q

$$Q = \sum_n \frac{1}{32\pi^2}\epsilon_{\mu\nu\rho\sigma}\mathrm{tr}\left[\hat{C}_{\mu\nu}(n)\hat{C}_{\rho\sigma}(n)\right]$$

✓ Renormalise or treat UV singularities

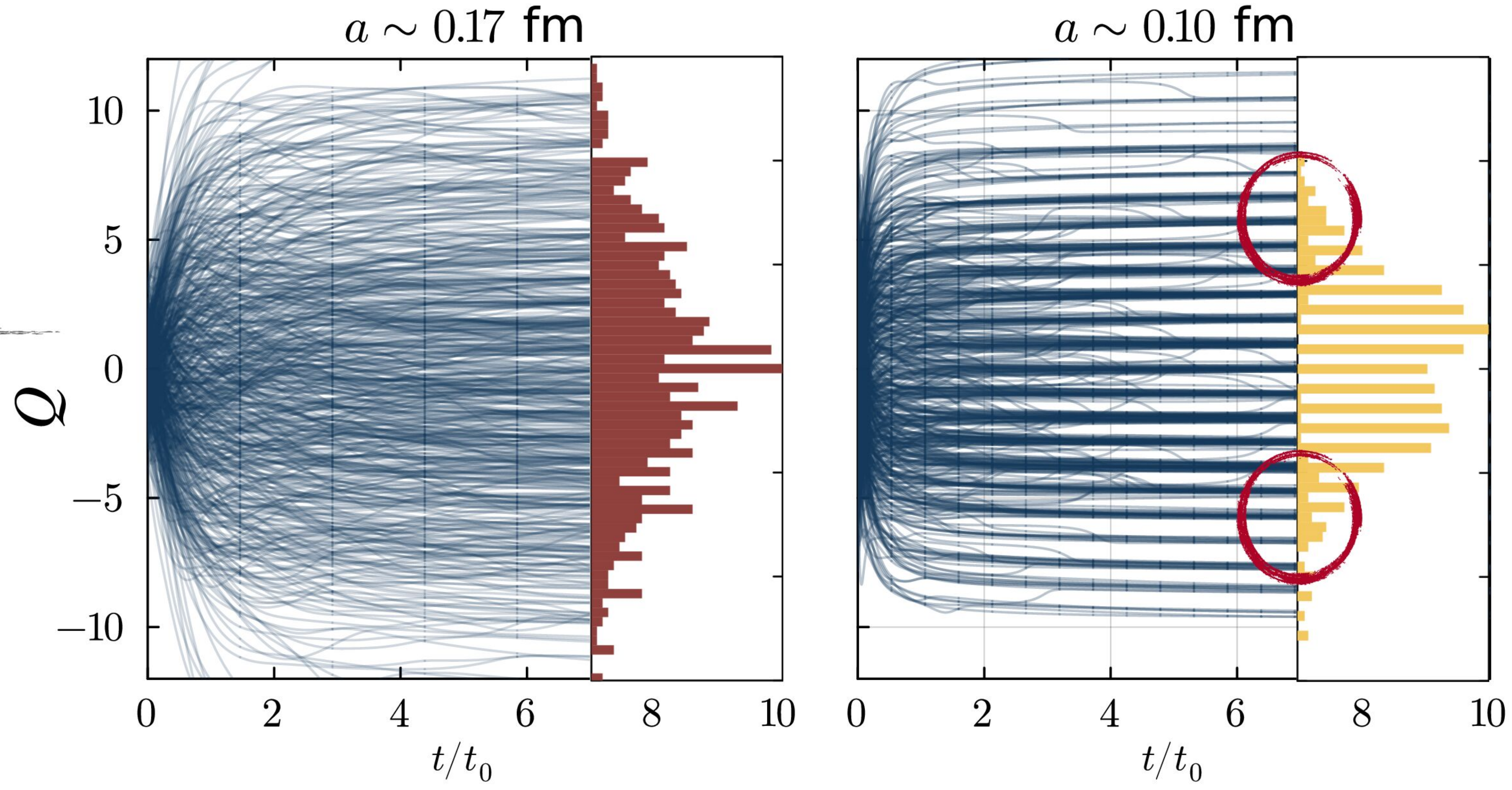
$$\hat{Q} = Z_Q Q + Z_{\mathrm{add}} \qquad \text{or} \qquad \text{gradient flow!}$$

✓ Compute observables from P_Q (also the tails)

$$\chi = \frac{\langle Q^2 \rangle}{V}$$

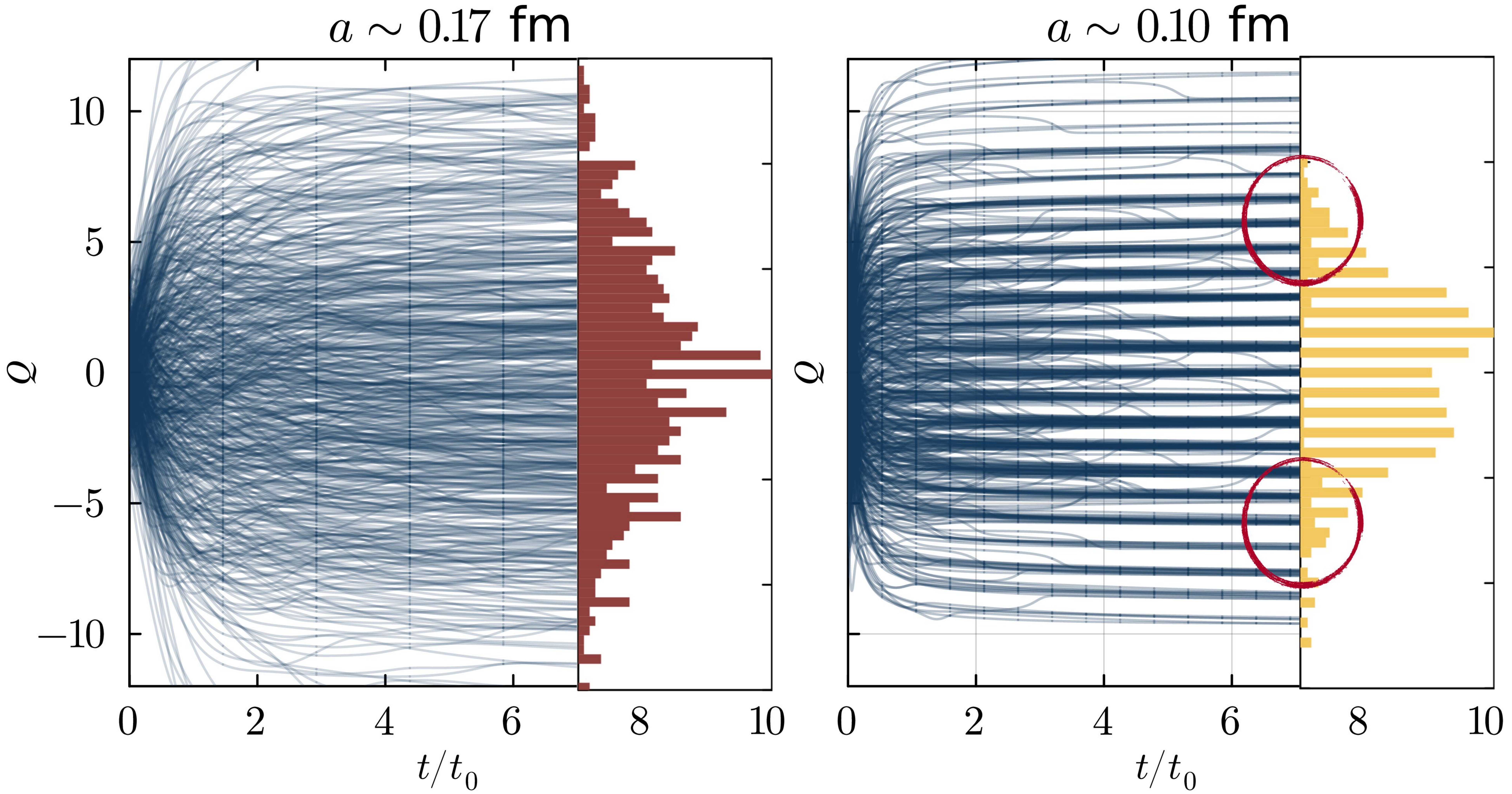
✓ Take $a \to 0$ at fixed (big) volume

$$S = c_0 \sum_n \mathrm{tr}\left(1 - \Box\right) + c_1 \sum_n \mathrm{tr}\left(1 - \Box\Box\right)$$

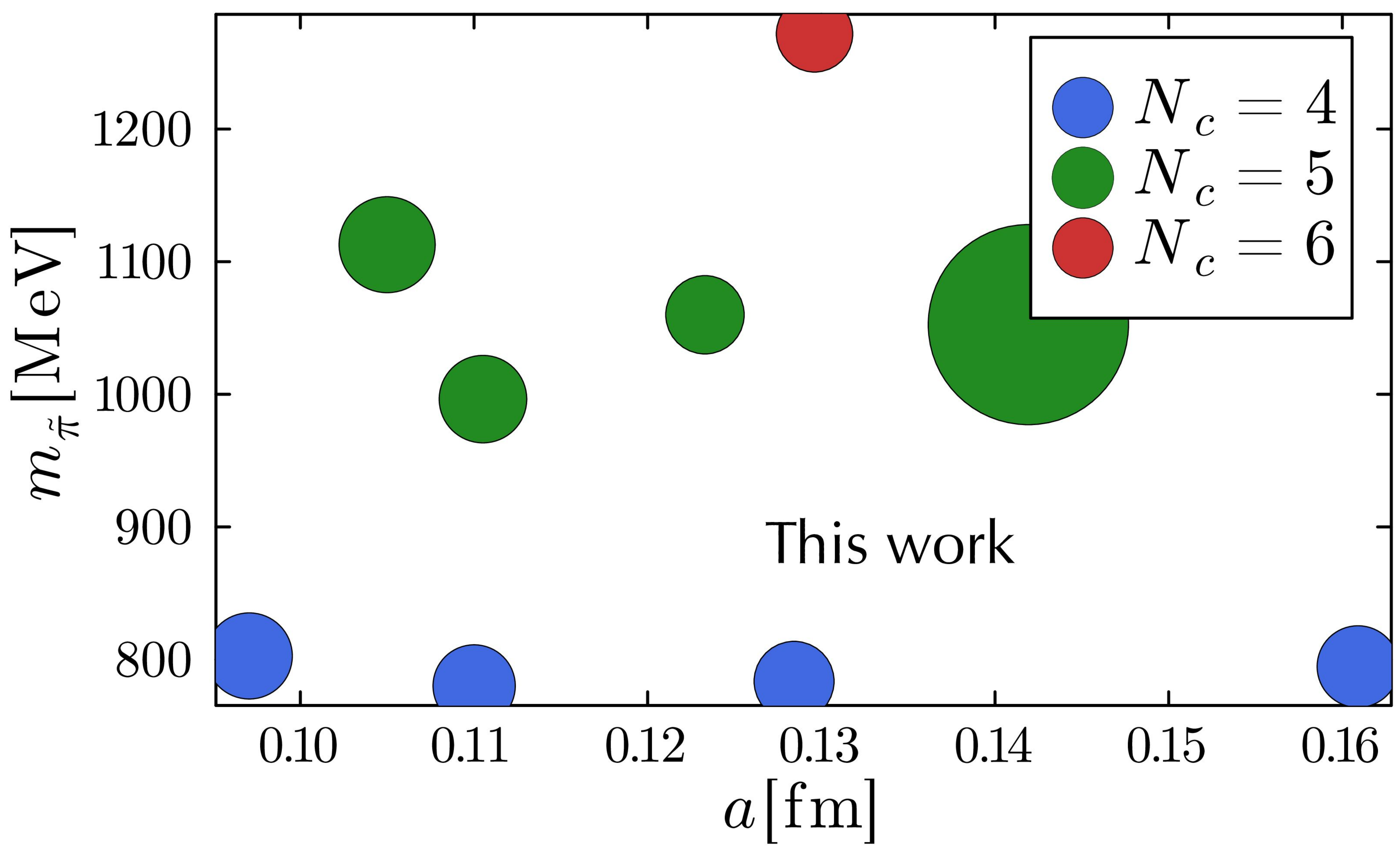


fractional Q?

The topological charge



Ensembles



$\vdash\!\dashv \sim \frac{1}{\sqrt{N_{\mathrm{conf}}}}$

Fixed mass @ N_C

Vary a

The topological charge

Topological Charge

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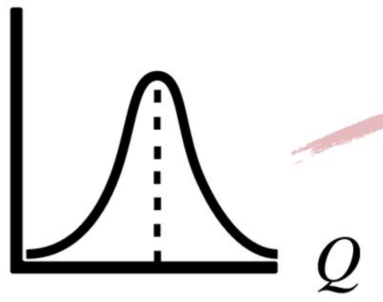

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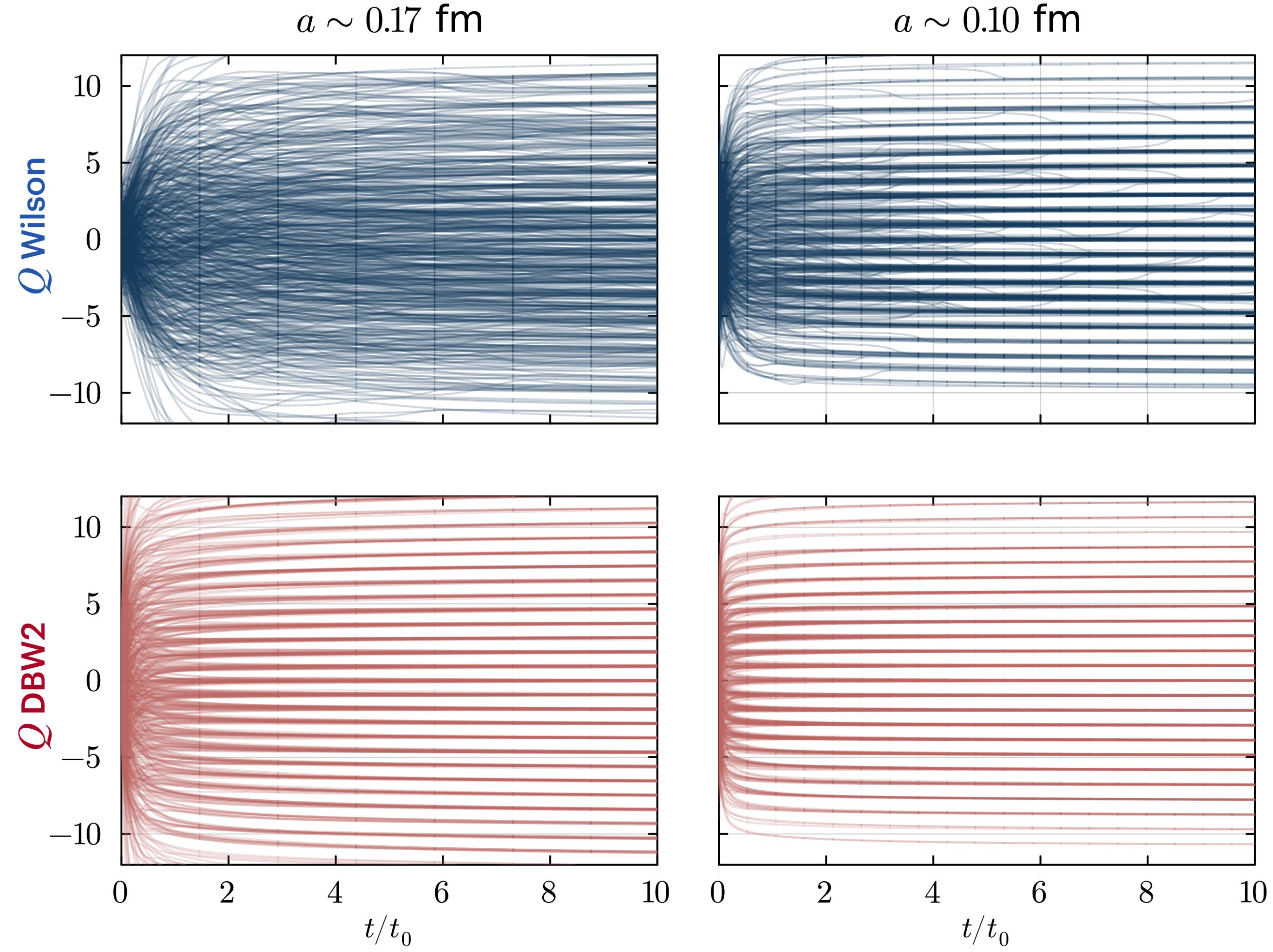
- ✓ Take $a \to 0$ at fixed (big) volume

$$S = c_0 \sum_n \mathrm{tr}\left(1 - \square\right) + c_1 \sum_n \mathrm{tr}\left(1 - \square\square\right)$$

$$a^2 \left(\frac{d}{dt} U_{\mu,x}(t) \right) U^\dagger_{\mu,x} = - g_0^2 \, \partial_{\mu,x} S_{fl}[U]$$

The topological charge



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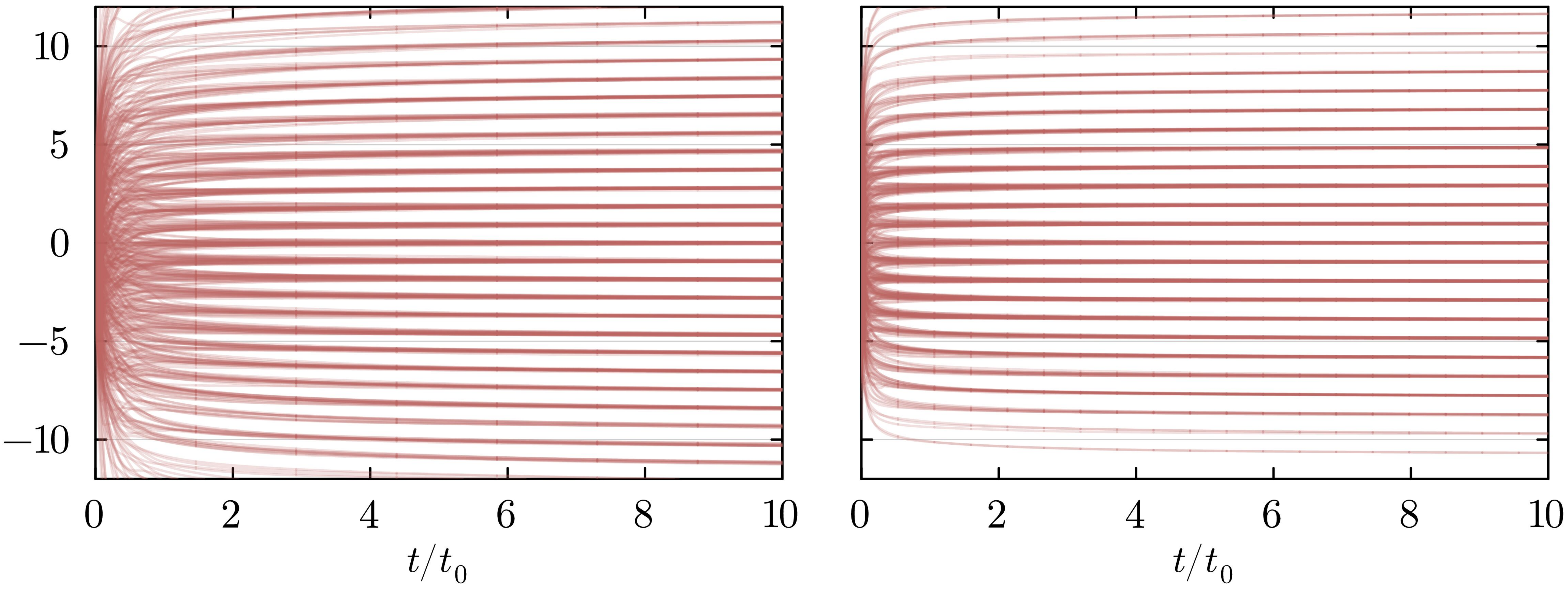
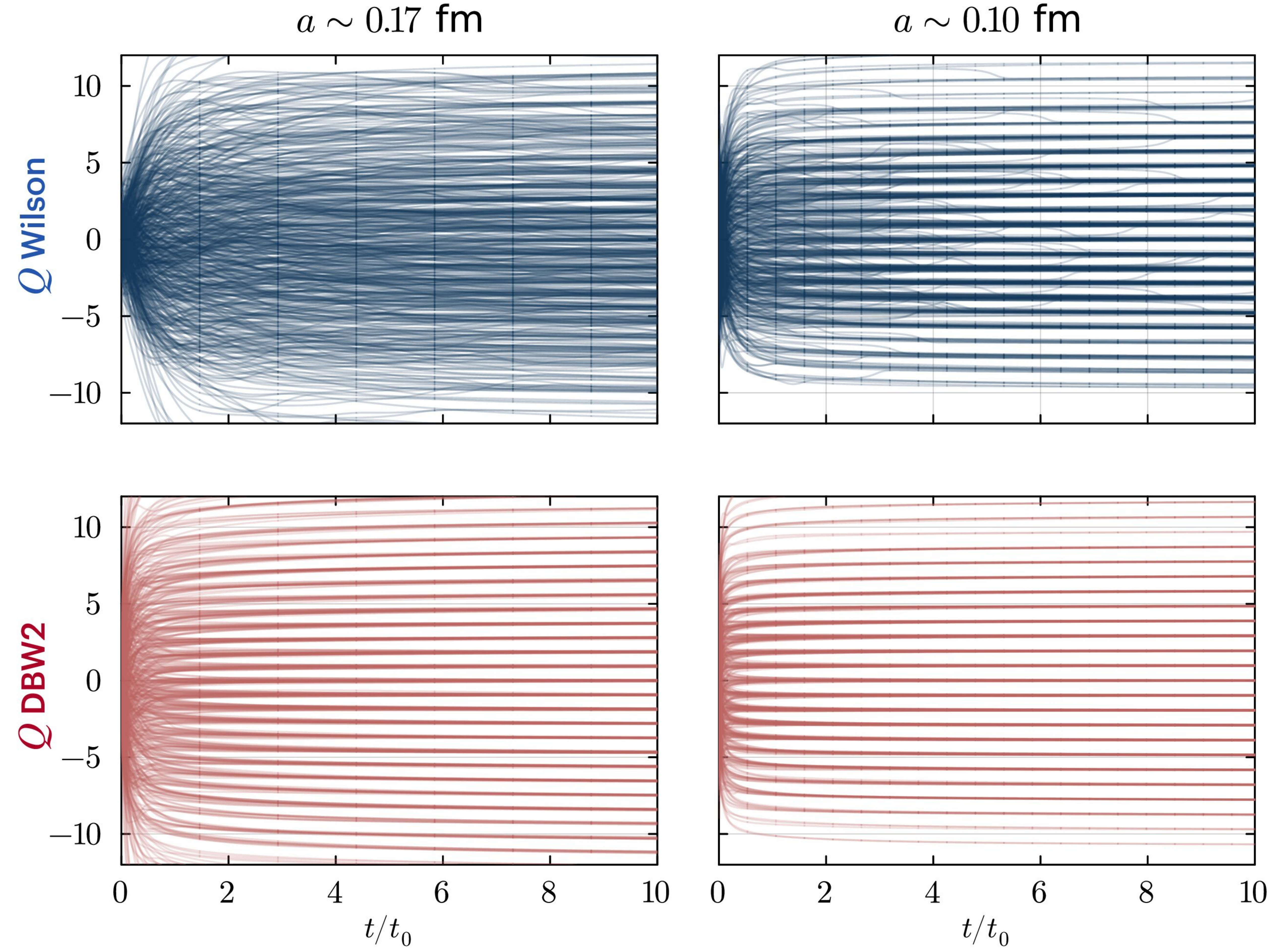
	$c_0 = 1 - 8c_1$	c_1
Wilson	**1**	**0**
Lüscher-Weisz	**5/3**	**-1/12**
Iwasaki	**3.648**	**-0.331**
DBW2	**12.2704**	**-1.4088**

The topological charge - DBW2



The topological charge



[García Pérez, González-Arroyo, Snippe, van Baal]
Instantons from over-improved cooling,
Nucl. Phys. B **413** (1994)

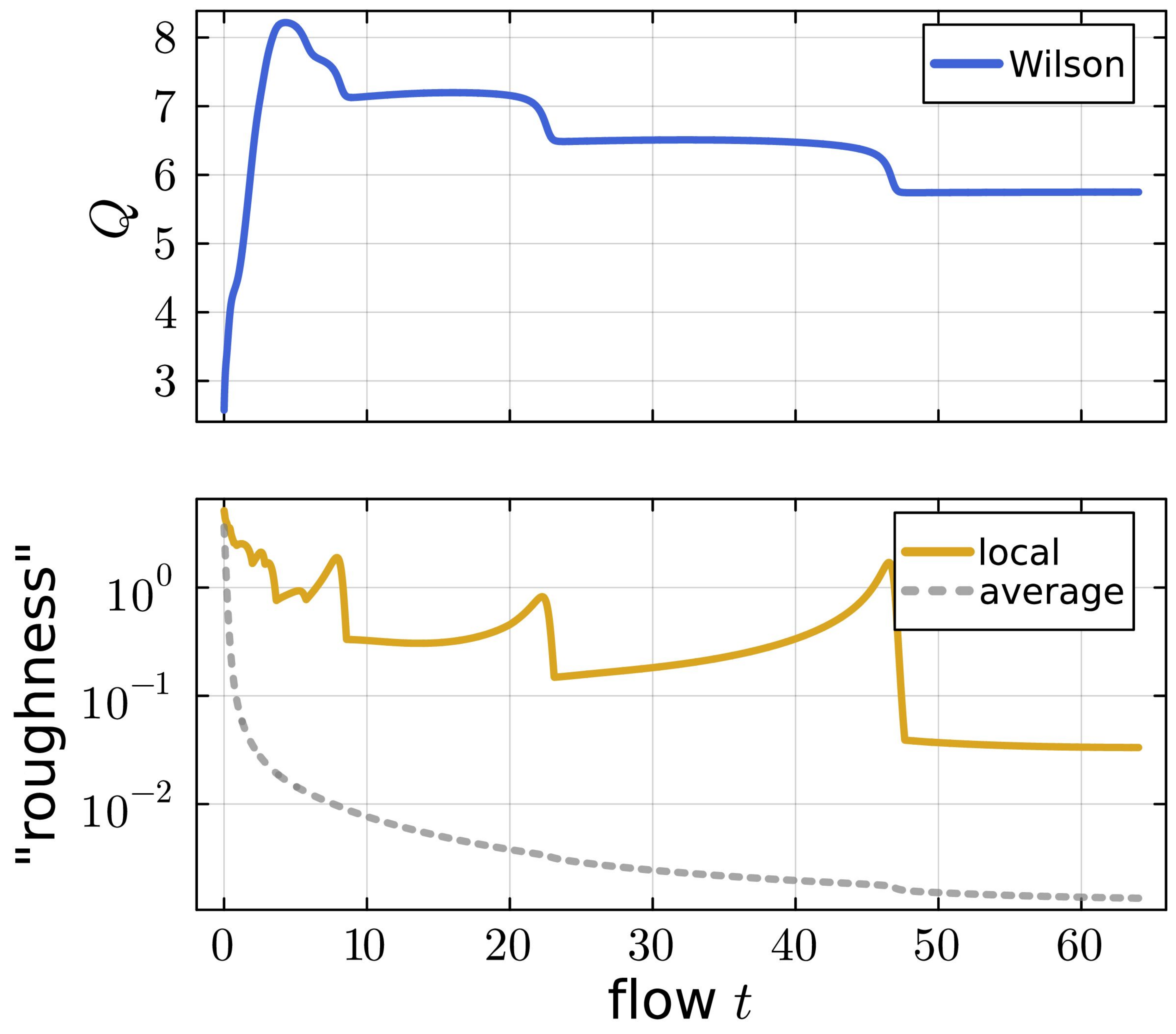
The lattice action for an instanton of typical size λ is

$$S_{\text{lat}}(a, \lambda) = S_{\text{cont}}\left(1 - \frac{1 + 12c_1}{5}\left(\frac{a}{\lambda}\right)^2 + \mathcal{O}\left(\frac{a}{\lambda}\right)^2\right)$$

	c_0	c_1	$1 + 12c_1$
Wilson	1	0	1
Lüscher-	5/3	-1/12	0
Iwasaki	3.648	-0.33	-2.972
DBW2	12.27	-1.40	-15.9

under-improved actions: small size instantons ($a/\lambda \gg 1$) destabilise the action and causes Q to jump

The topological charge



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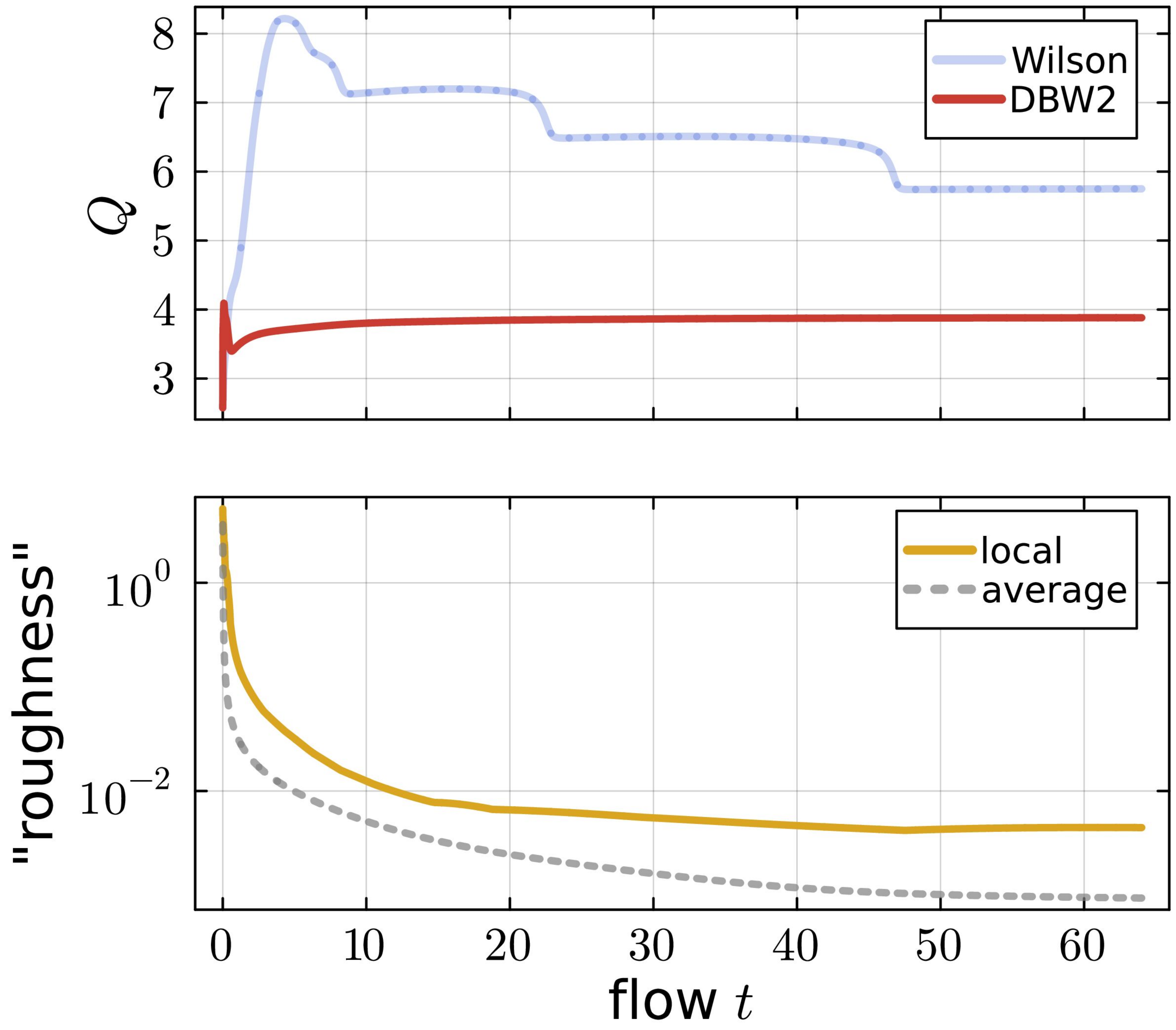
Local effects becomes evident from the "roughness"

$$h(P) \equiv \mathrm{tr}(1 - P[V_t]) \qquad h_{max} = \max_P h(P)$$

In QCD, configurations "between sectors" have $h > 0.067$

The topological charge



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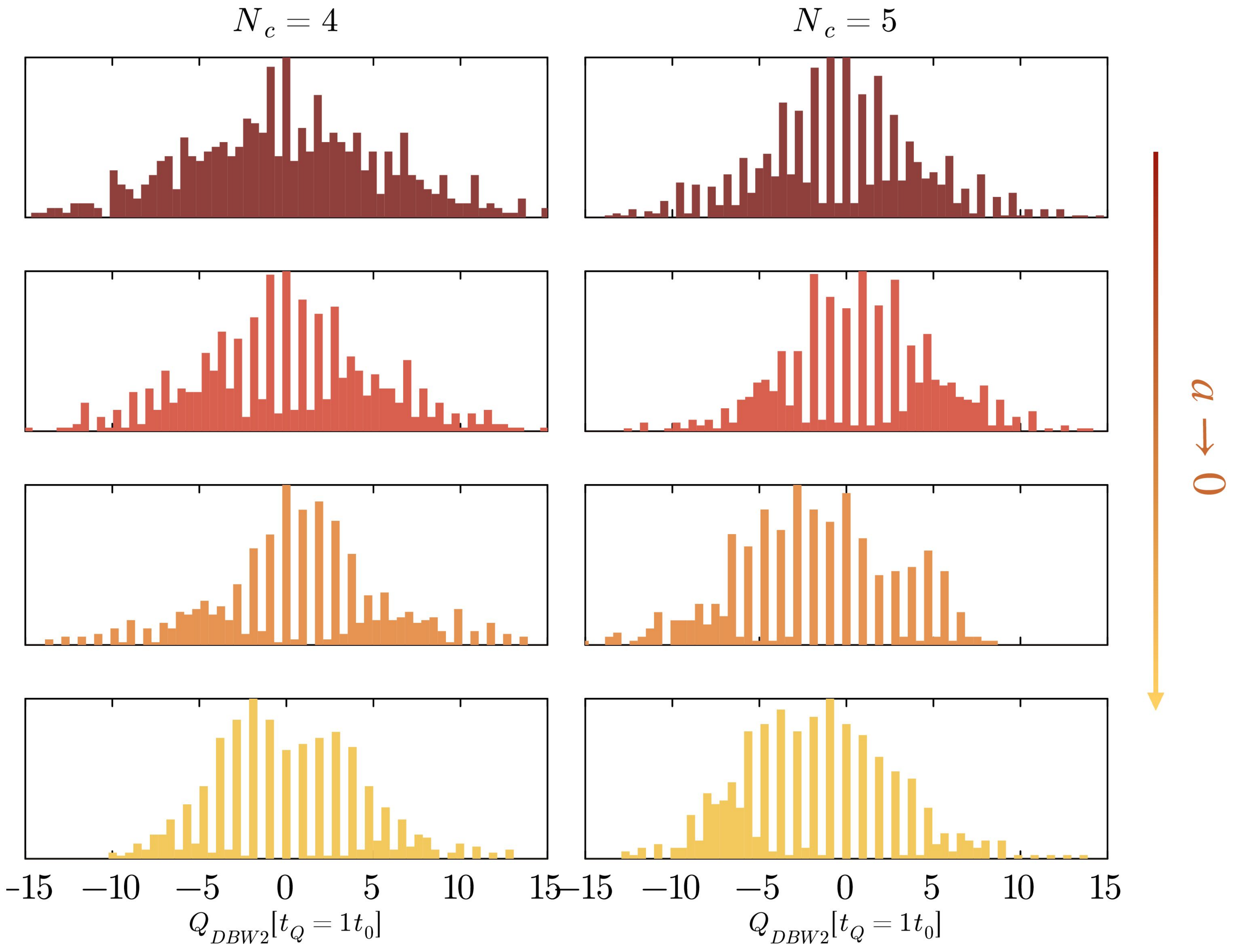
The topological charge

$$Q = \frac{1}{32\pi^2} \int d^4x \, \epsilon_{\mu\nu\rho\sigma} \, \mathrm{tr}\left[G_{\mu\nu}(x) G_{\rho\sigma}(x)\right] \stackrel{?}{=} \frac{I}{2T}$$

$t_Q = 1 \cdot t_0$



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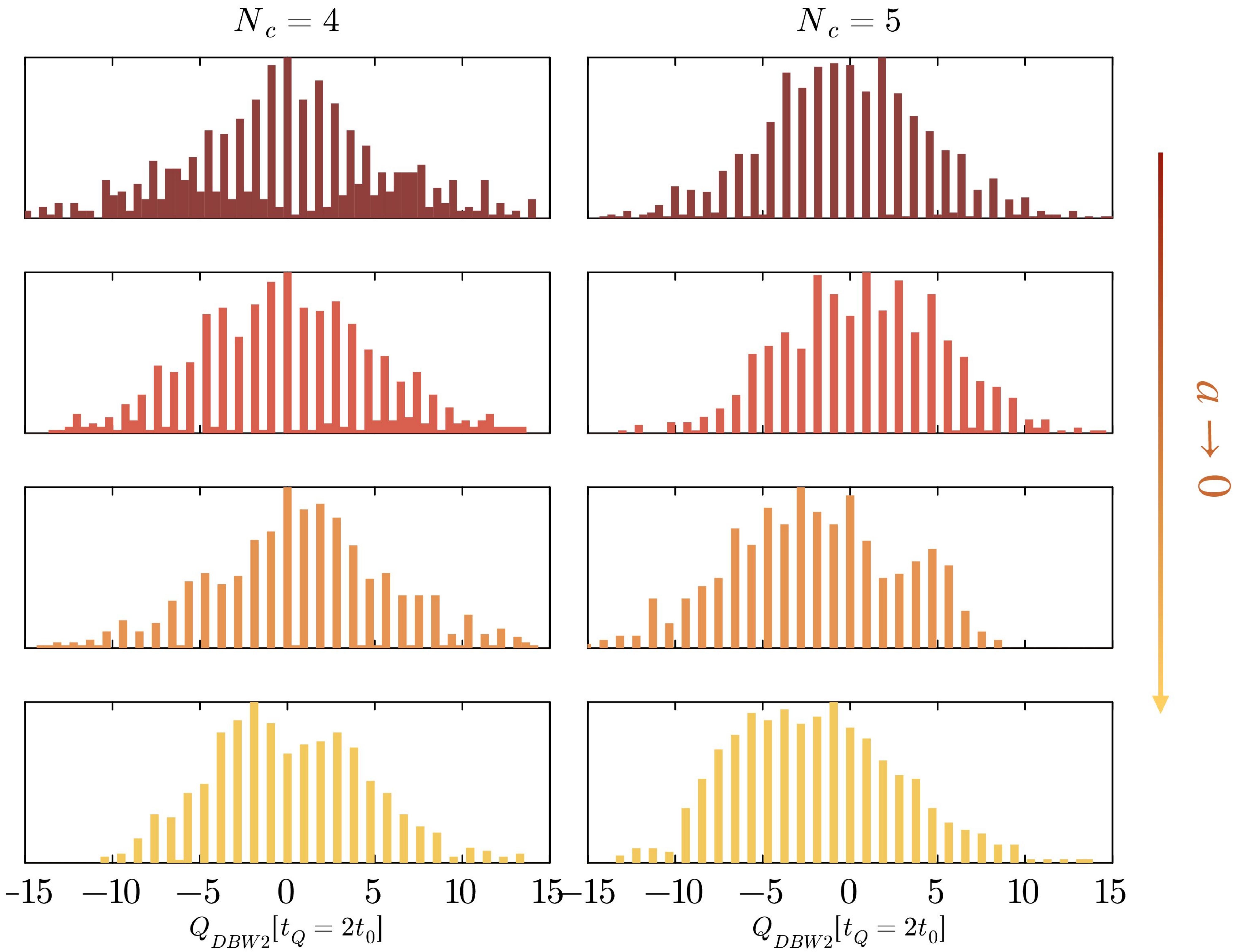

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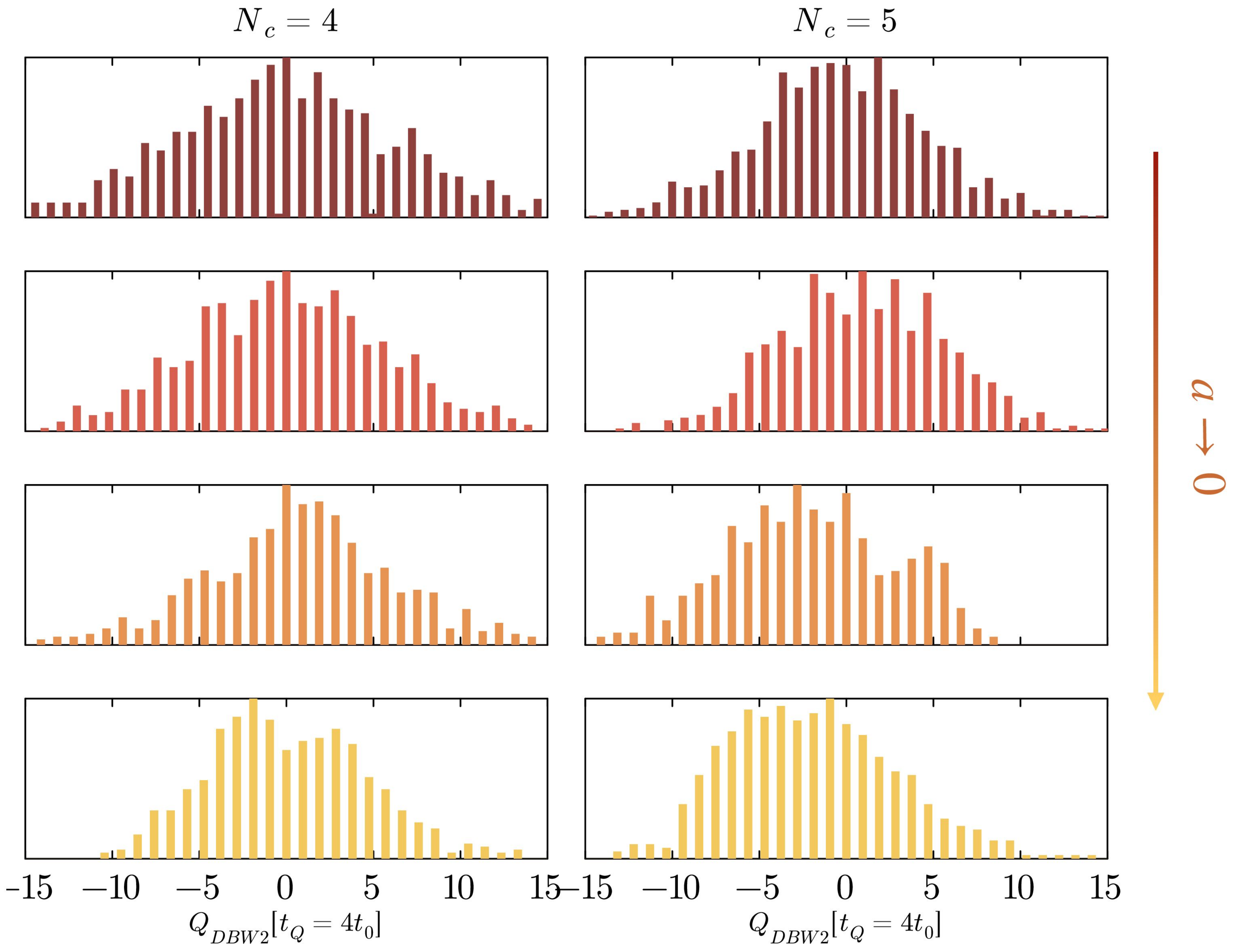
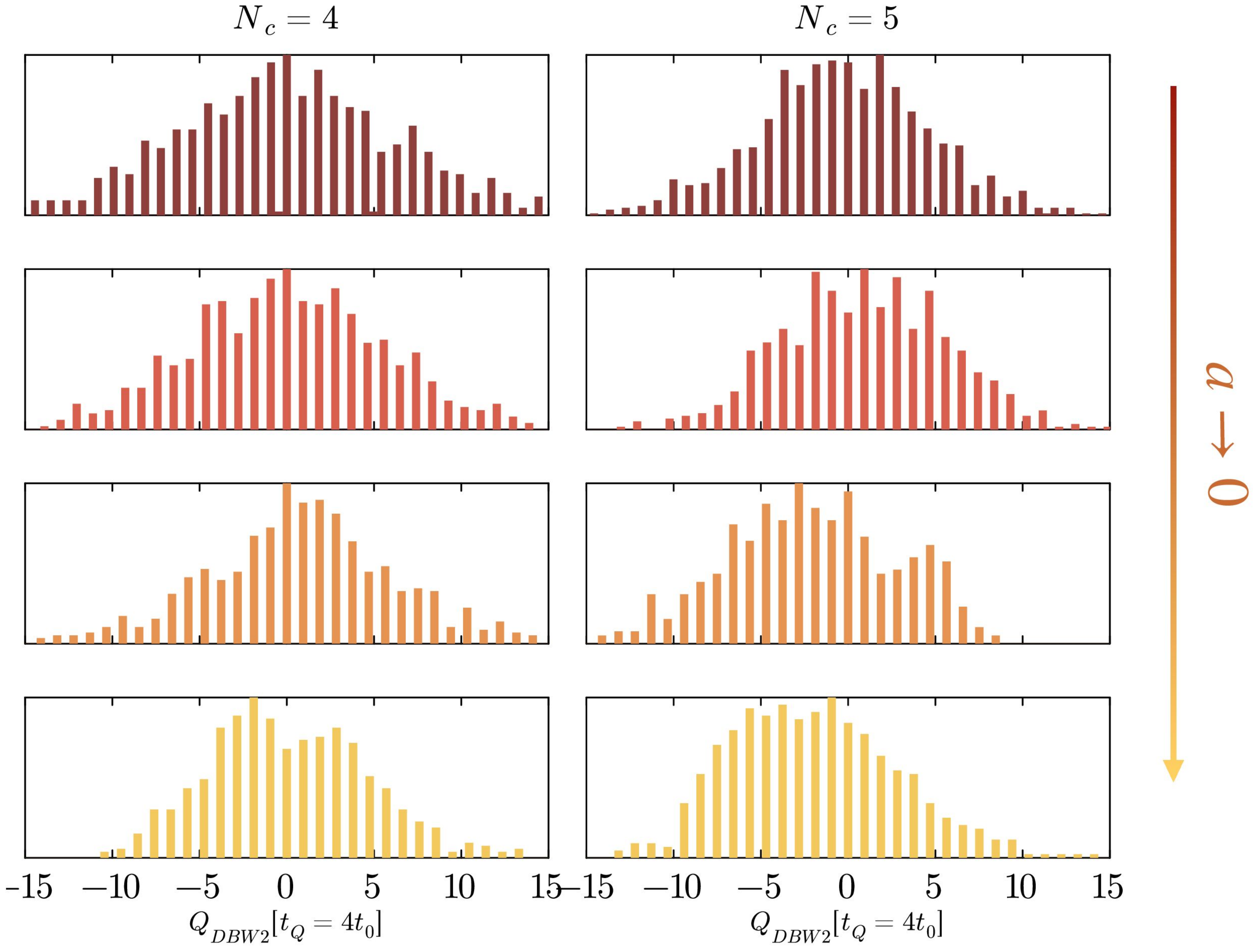


The topological charge

$t_Q = 4 \cdot t_0$

No fractional charges observed



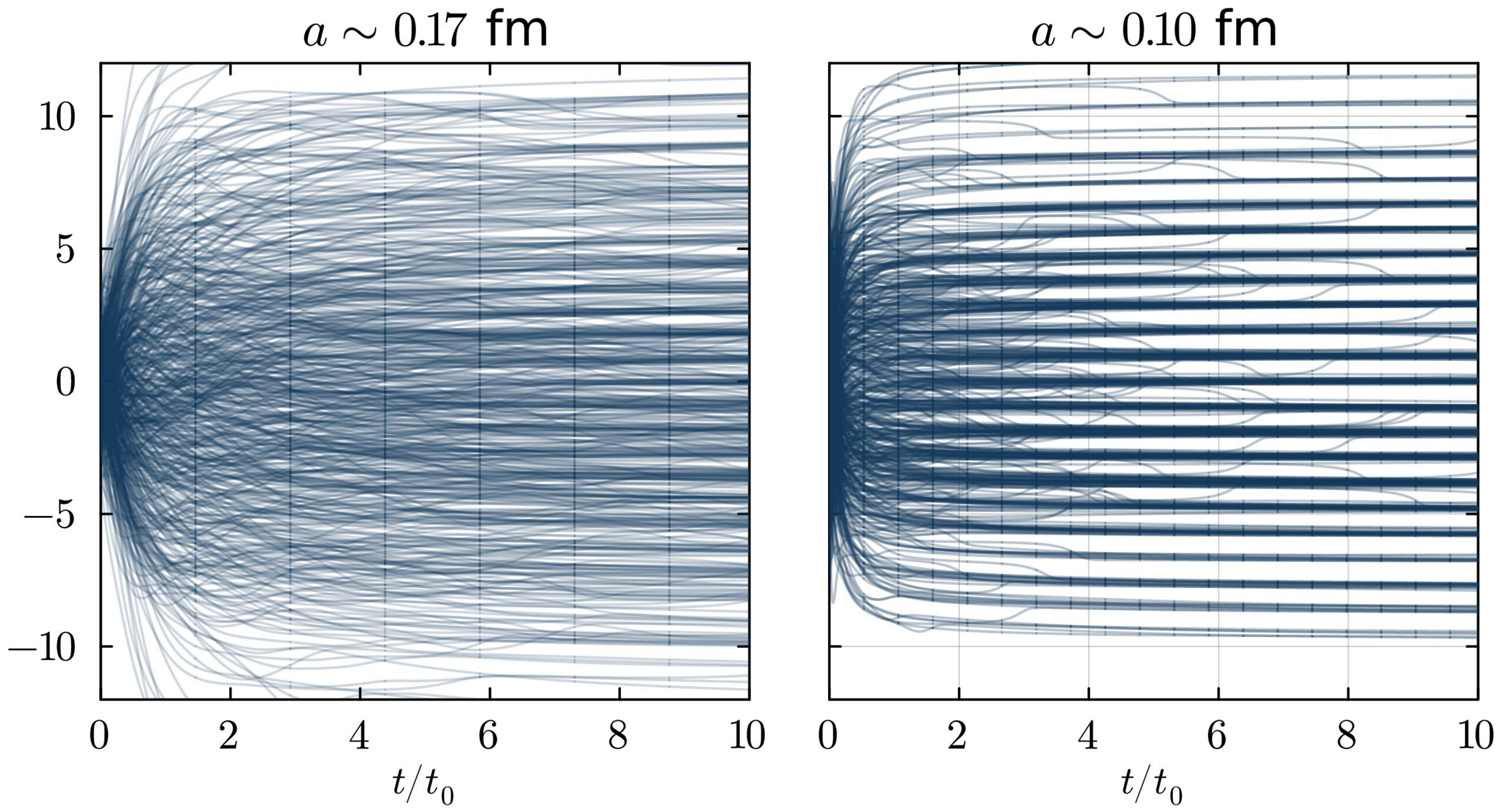
Scale dependence

Setting the scale @ $N_C = 3$

$$\langle t^2 E \rangle = 0.3$$

Setting the scale @ $N_C \neq 3$

$$\langle t^2 E \rangle = 0.3 \left(\frac{N_C^2 - 1}{N_C} \right) \frac{3}{8}$$



$$\sqrt{8\, t_0} = 0.45\,\mathrm{fm}$$

In between of $N_f = 0$ and $N_f = 2$

The topological susceptibility

$$\chi = \lim_{a\to 0} \lim_{V\to\infty} \frac{\langle Q^2 \rangle}{V}$$

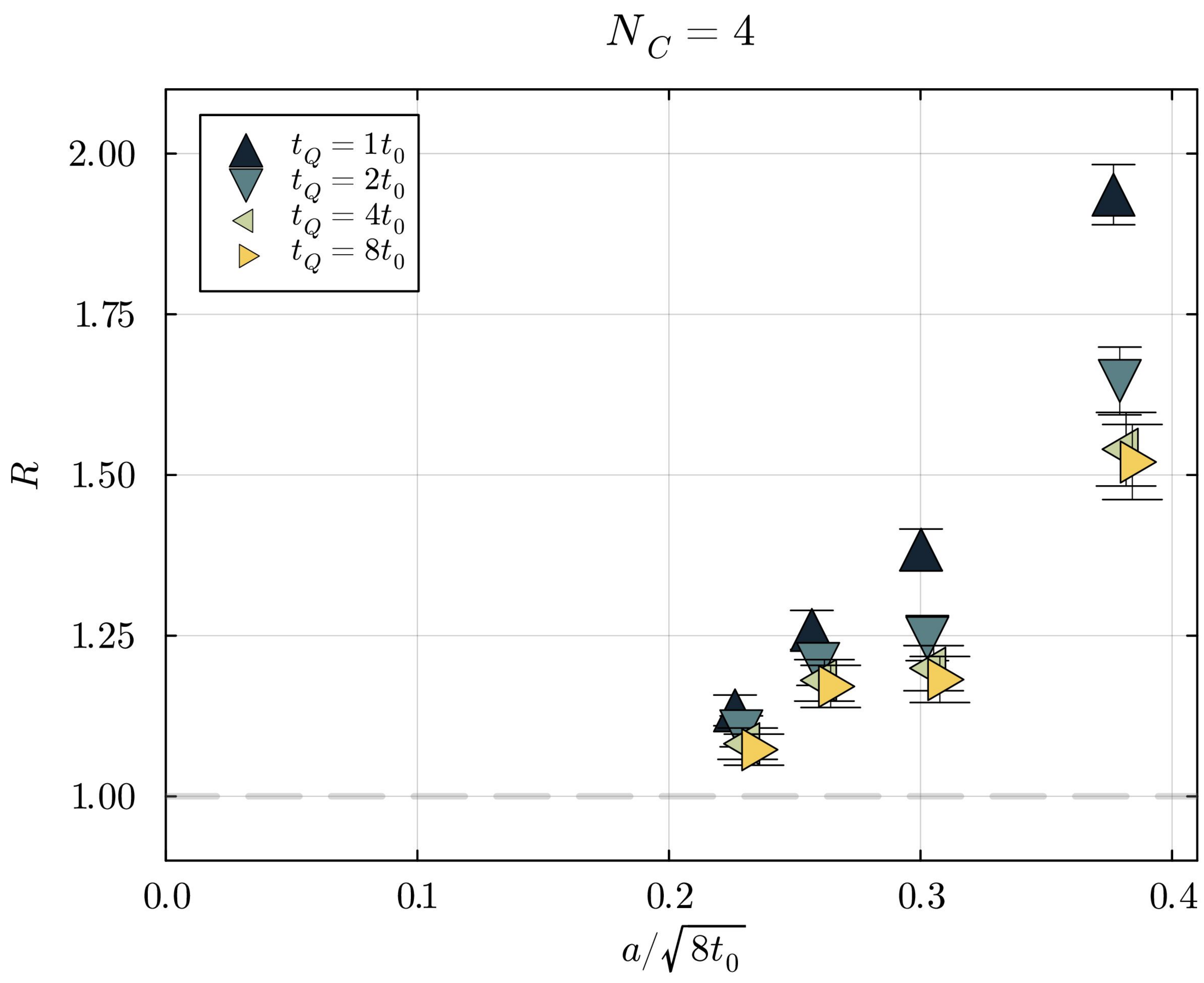
[Sint, Ramos] - Eur. Phys. J. C. **76** (2016) [Catumba, Ramos, Lang] - Lattice2024

$$\langle \chi \rangle_{\text{latt}} \sim \langle \chi \rangle + a^2 \left[\langle \chi_2 \rangle + \langle \chi S_{2,c} \rangle + \langle \chi S_{2,fl} \rangle + c_b \frac{d}{dt} \langle \chi \rangle |_{t_0} \right]$$

- Symanzik expansion of the observable $\mathcal{O} \sim \mathcal{O}_0 + a^2 \mathcal{O}_2$ ⇒ use classically improved observables
- Cutoff effects of path integral action (continuum extrap. of spectral quantities, universality,...)
- Cutoff effects from flow kernel action ⇒ use impr flow

...for the future

$$R = \frac{\chi^{\text{DBW2}}}{\chi^{\text{Wilson}}} \to 1$$



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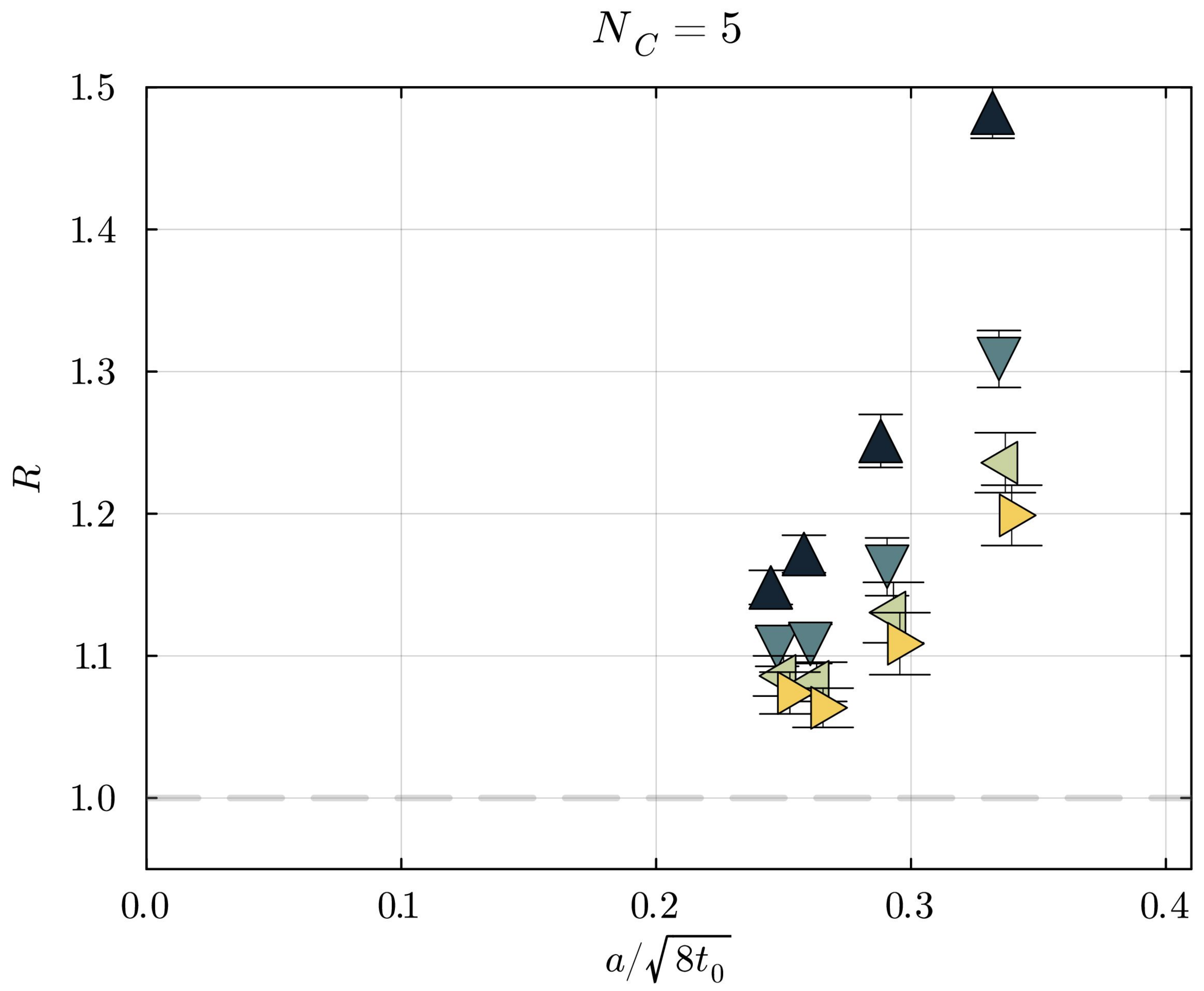
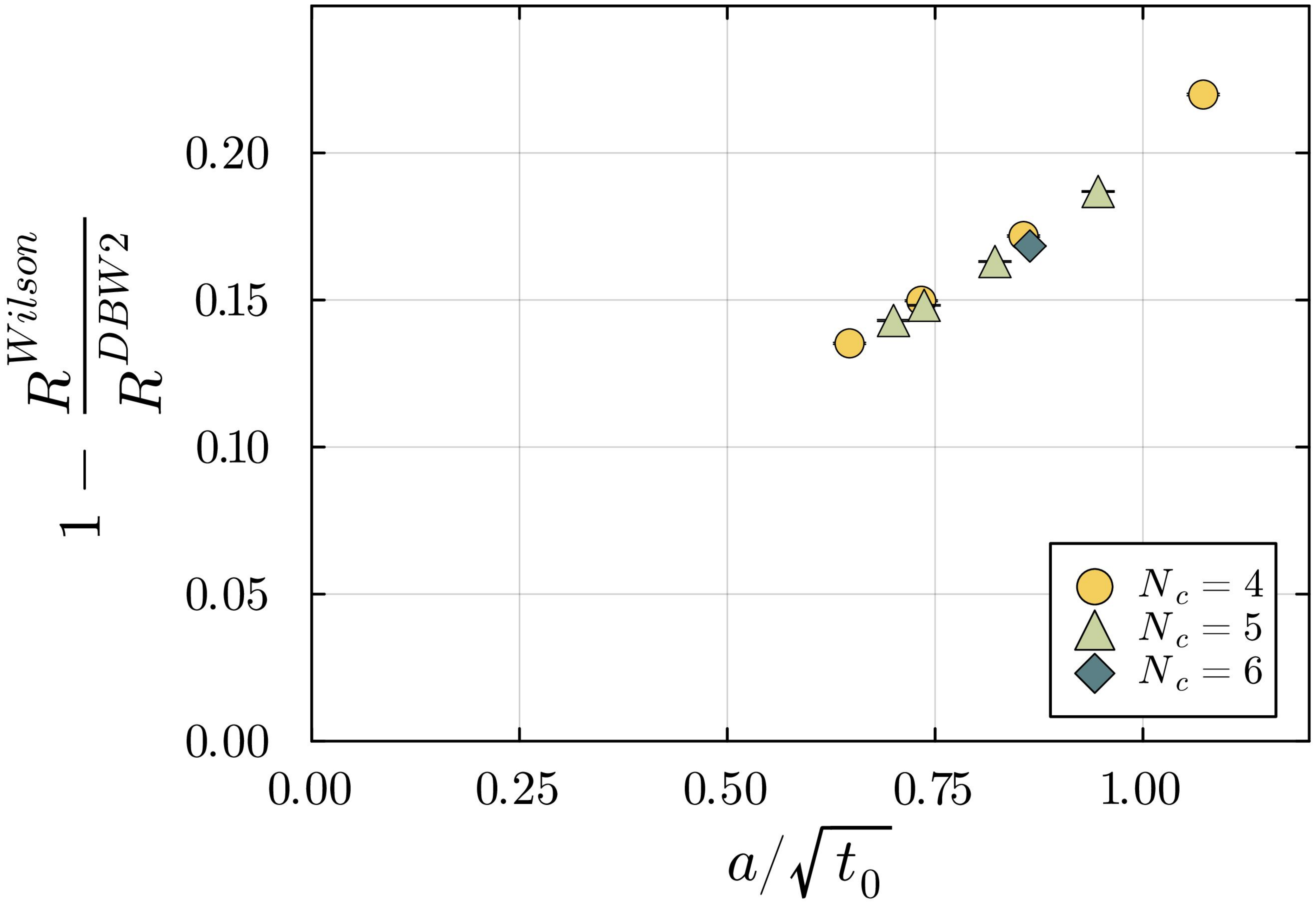


The topological susceptibility

In the continuum

$$R = \frac{\chi^{\text{DBW2}}}{\chi^{\text{Wilson}}} \to 1$$



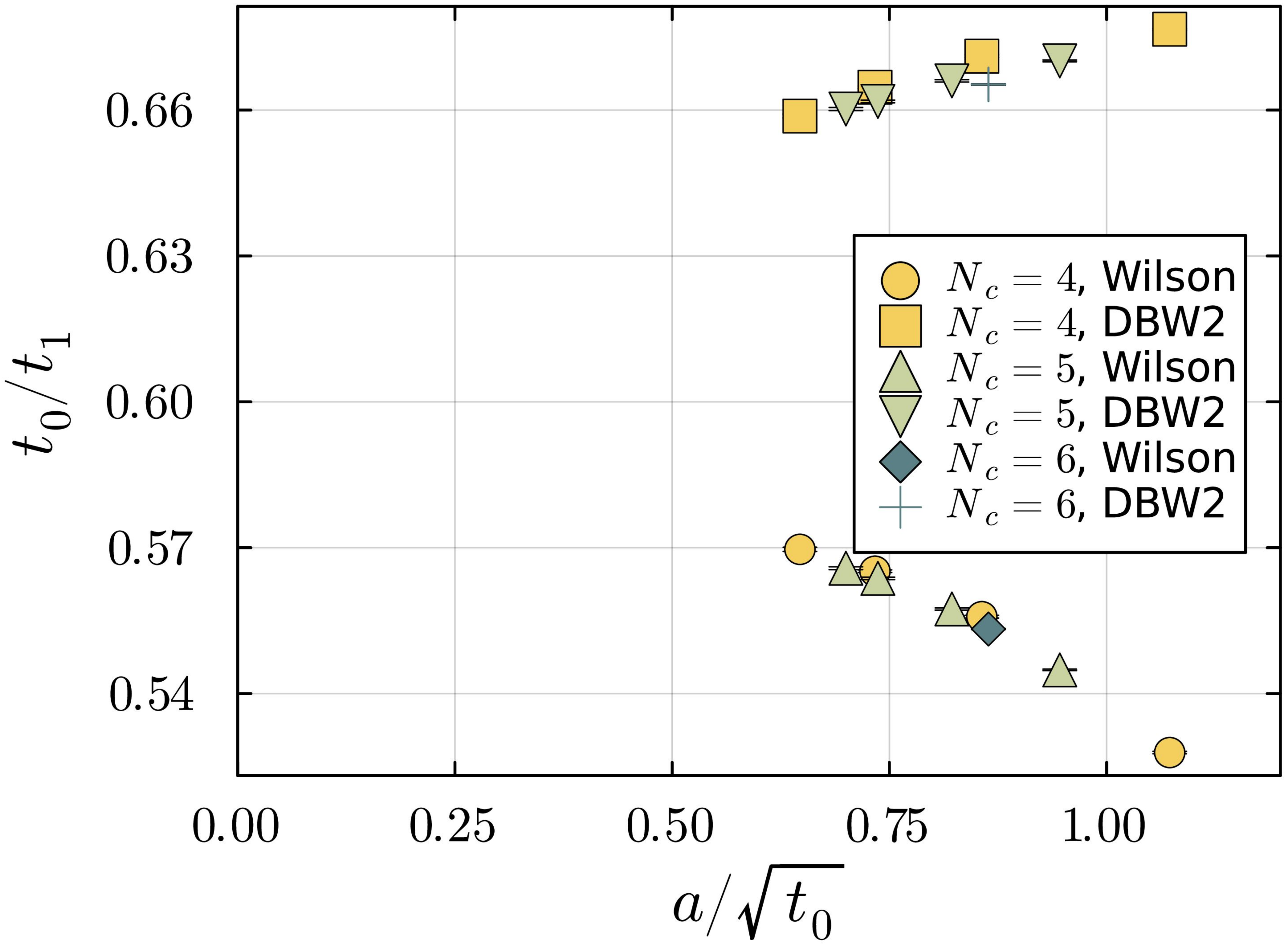
The topological susceptibility

Different reference points

$$\langle t_0^2 E \rangle = 0.3 \left(\frac{N_C^2 - 1}{N_C} \right) \frac{3}{8}$$

t_0 @ 0.3 and t_1 @ 0.5

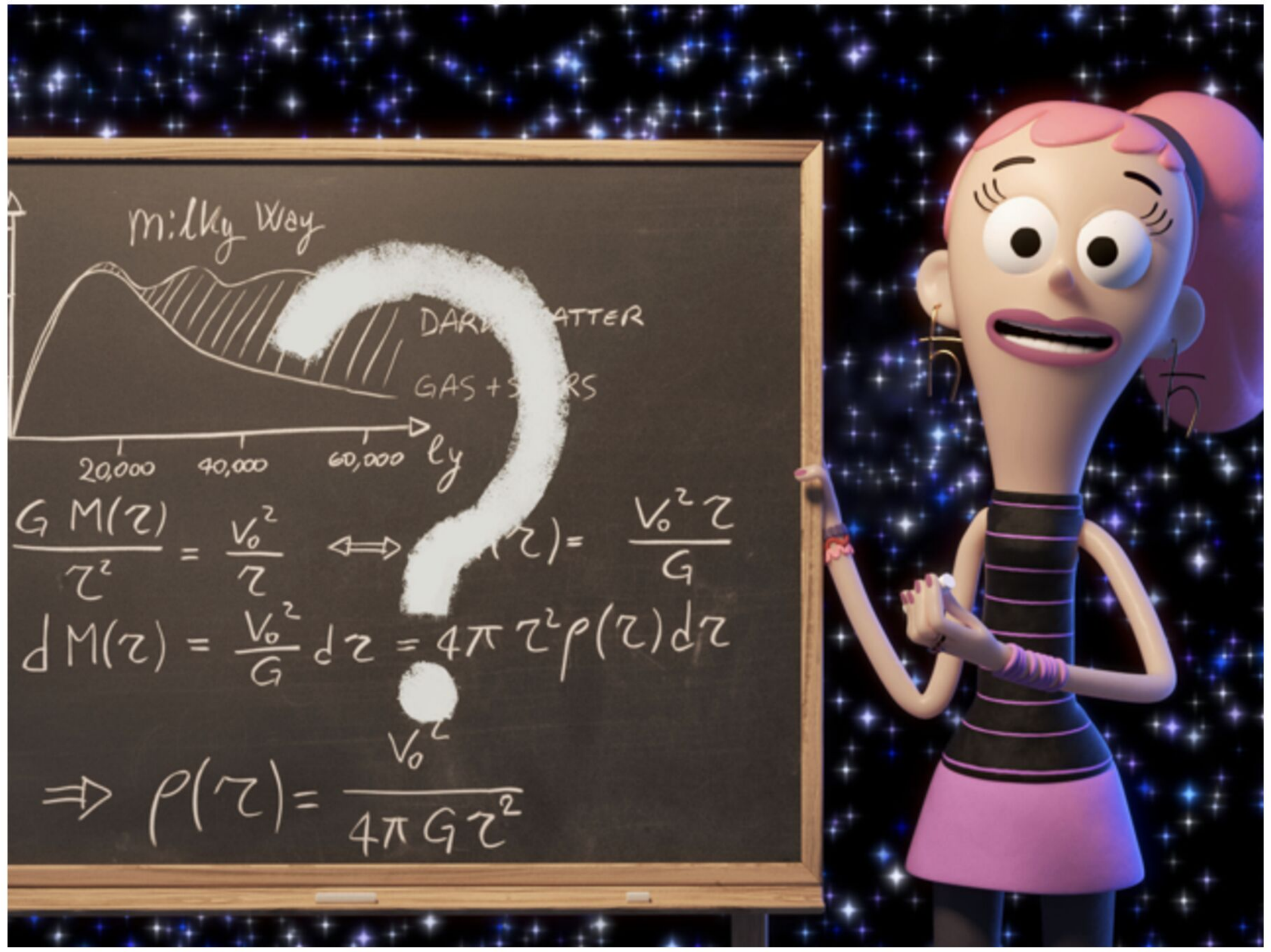
$$\langle t_1^2 E \rangle = 0.5 \left(\frac{N_C^2 - 1}{N_C} \right) \frac{3}{8}$$



Thank you for your attention!

Conclusion

- No fractional charges seen
- DBW2 provides a stable definition for Q
- Lattice artefacts $\sim 10\,\%$



Quantum Kate (orig. Kvante Karina): CP3 Outreach http://www.kvantebanditter.dk/en

Questions?