Nuclear Theory:
Lecture 1

Alex Gezerlis

UNIVERSITY OF GUELPH

23rd STFC Nuclear Physics Summer School
University of Aberdeen
August 2026

My world line

National Technical University of Athens

BEng
1998-2003

University of Illinois Urbana-Champaign

Msc + started PhD
2003-2006

Los Alamos National Laboratory

finished PhD
2006-2009

University of Washington

postdoc
2009-2012

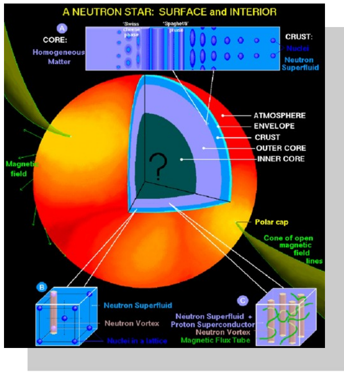
Technische Universität Darmstadt

Herzberg fellow
2012-2013

University of Guelph

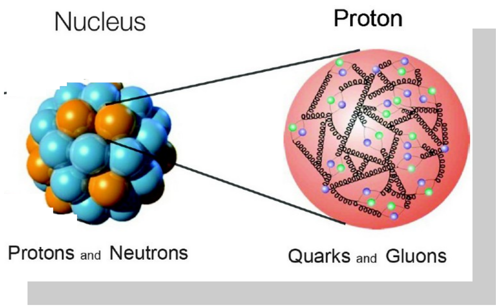
(all kinds of)
professor
2013-

Outline

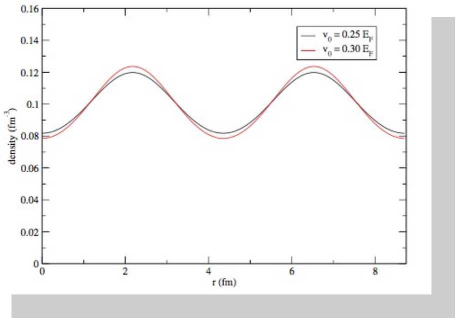


Credit: Dany Page

Motivation



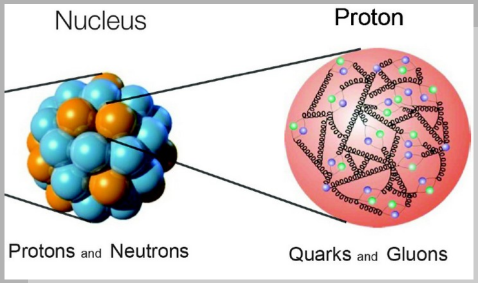
Nuclear forces



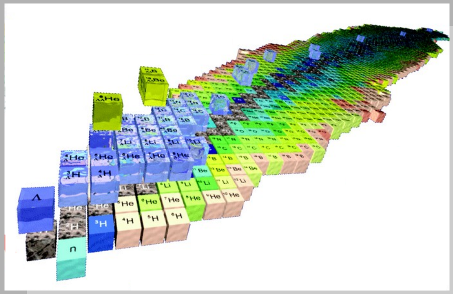
Many-body physics

Physical systems studied

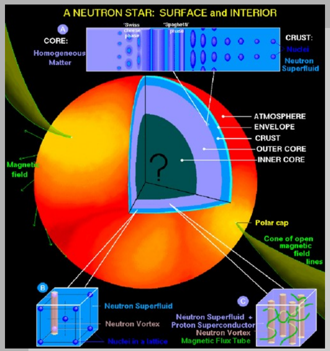
Nuclear forces

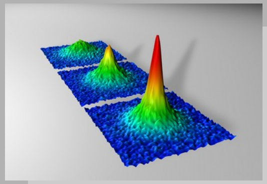


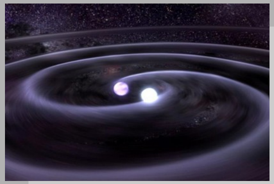
Nuclear structure



Nuclear astrophysics







Physical systems studied

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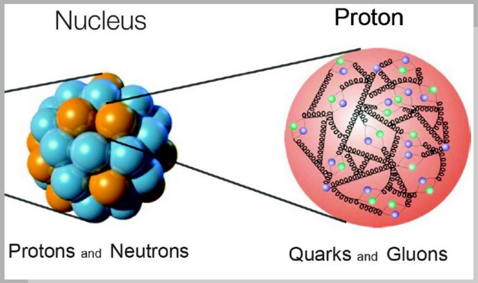
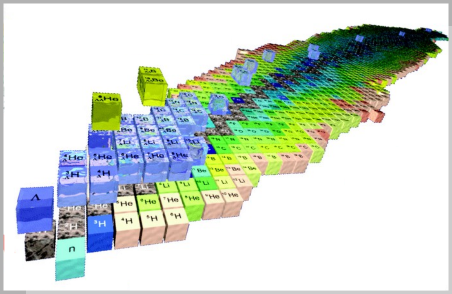
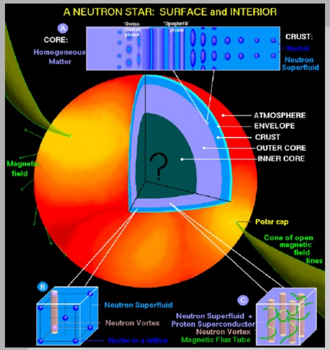
Few nucleons

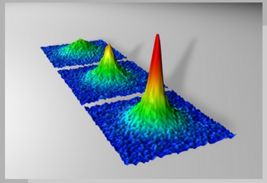


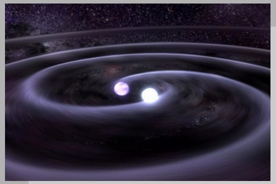
Many nucleons



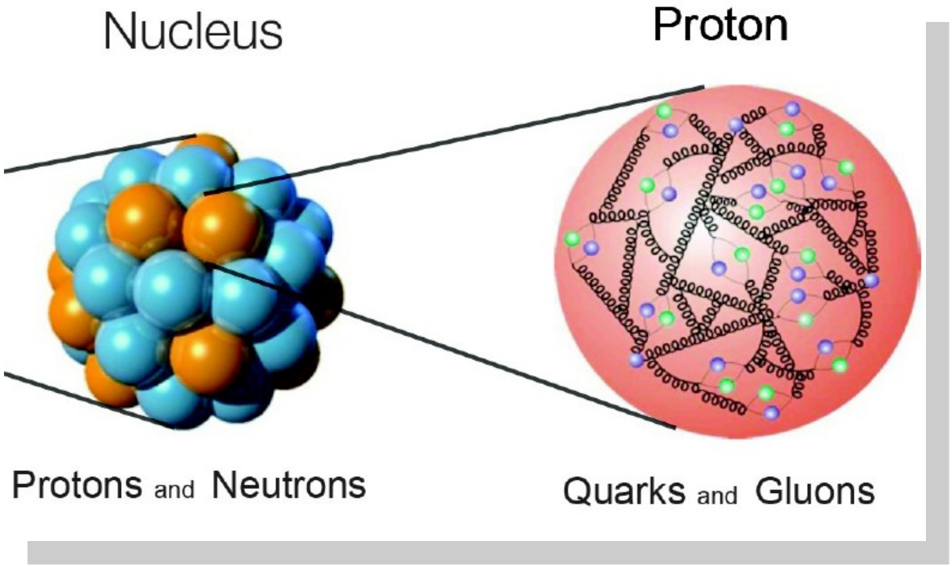
Very many nucleons





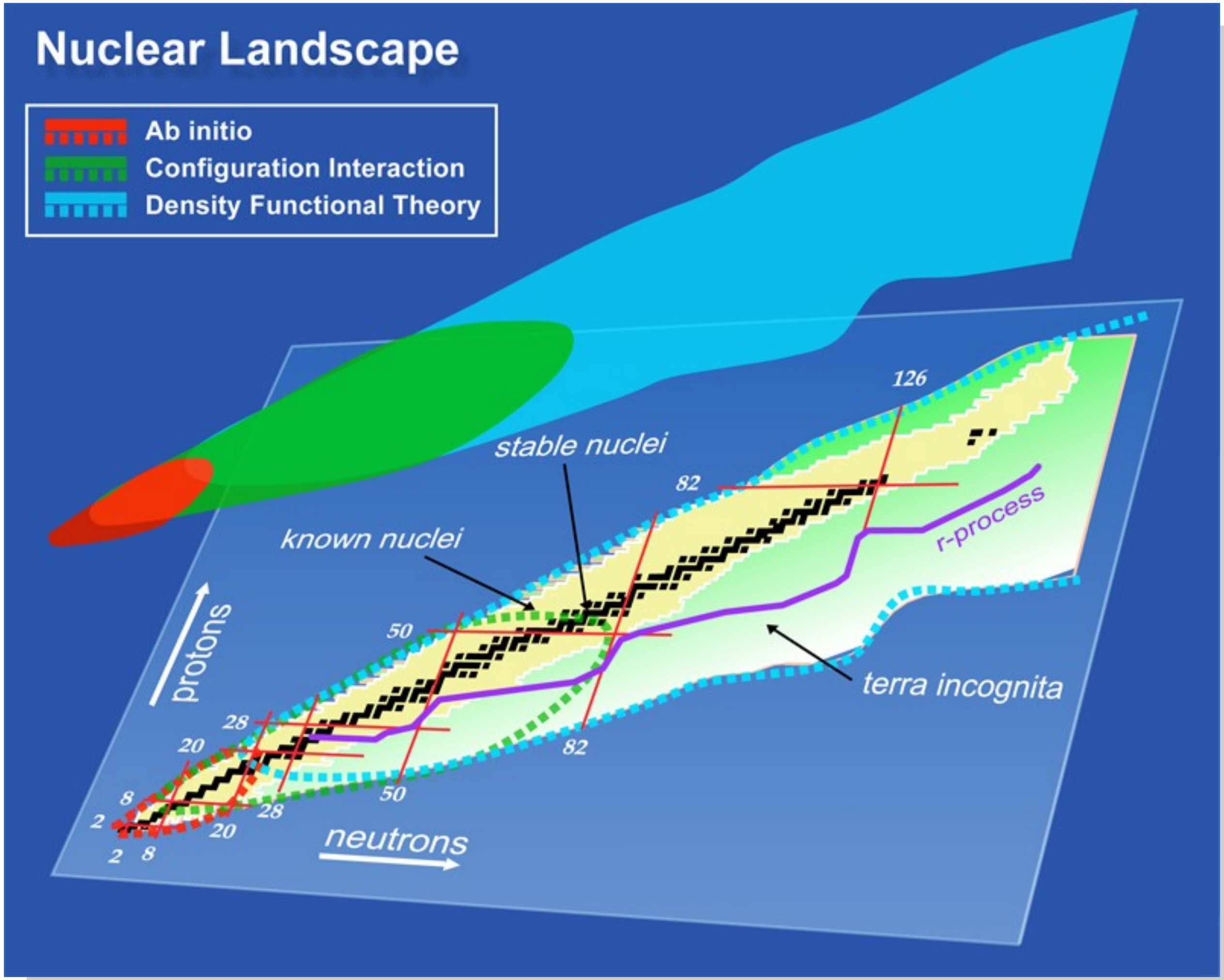


Key system: few nucleons



- No unique nuclear potential
- Preferable to use combination of phenomenological (high-quality) and more modern (conceptually clean) approach
- Desirable to make contact with underlying level
- New era, where practitioners design interactions themselves

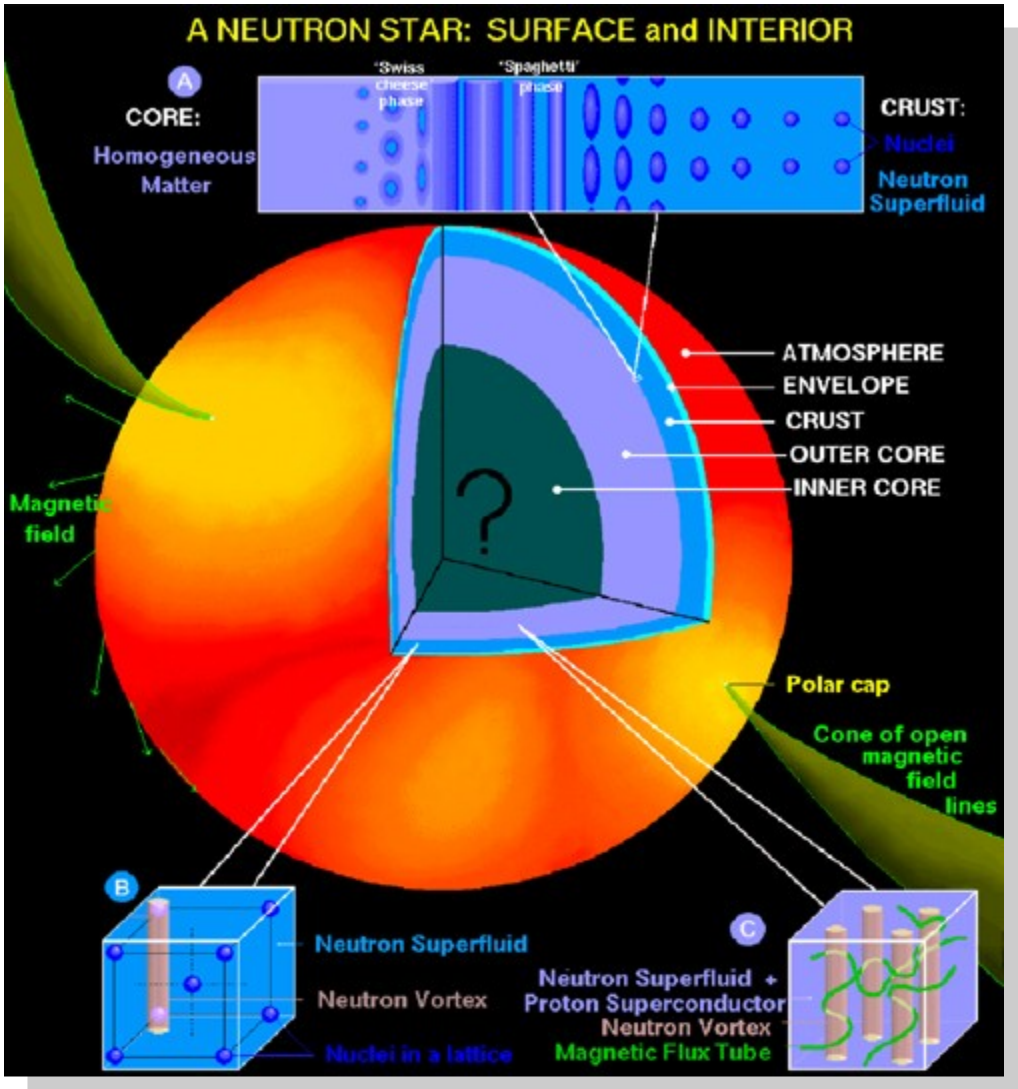
Key system: nuclei



- Experimental facilities continue to push the envelope
- Using complicated many-body methods we can try to "build nuclei from scratch"
- No universal theoretical method exists (yet?)
- Regions of overlap between different methods are crucial
- Goal is to study nuclei *from first principles* (when possible)

Key system: neutron stars

Neutron stars as ultra-dense matter laboratories

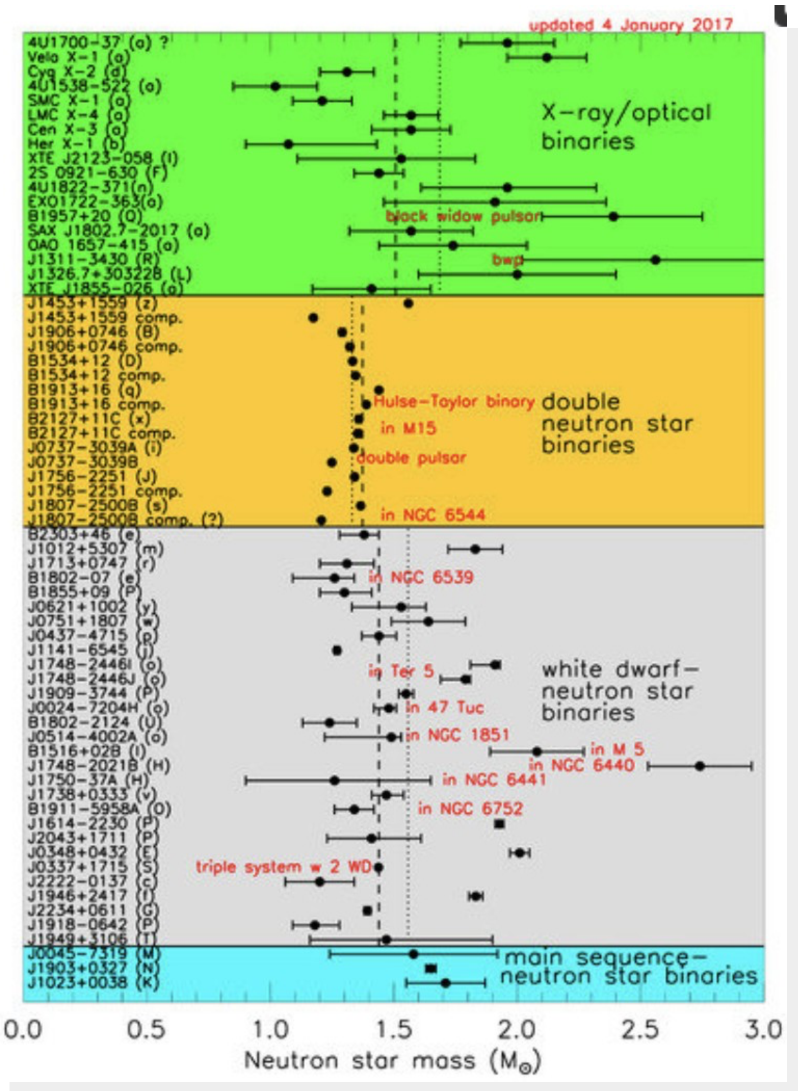


Credit: Dany Page

- Ultra-dense: 1.4 solar masses (or more) within a radius of 10 kilometres
- Terrestrial-like (outer layers) down to exotic (core) behaviour
- Observationally probed, i.e., not experimentally accessible
- Goal is to study neutron stars *from first principles* (when possible)

Key system: neutron stars

Neutron stars as ultra-dense matter laboratories



Credit: J. Lattimer Universe **2019**, 5, 159

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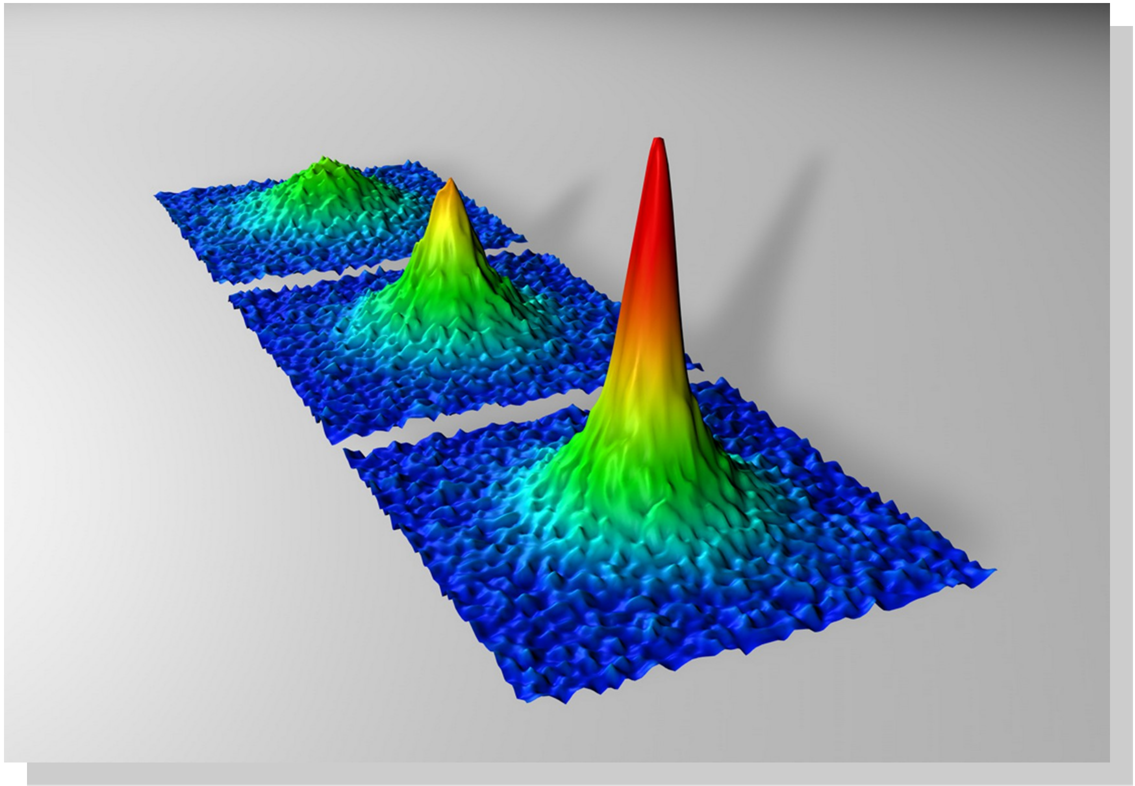
Key system: cold atoms



Credit: University of Colorado

- Starting in the 1990s, it became possible to experimentally probe degenerate bosonic atoms (beyond ^{4}He)
- Starting in the 2000s, the same happened for fermionic atoms (beyond ^{3}He)
- These are very cold and strongly interacting (as well as strongly correlated)
- Can be used to simulate other systems, investigating pairing, polarization, polaron physics, many species, reduced dimensionality

Key system: binaries

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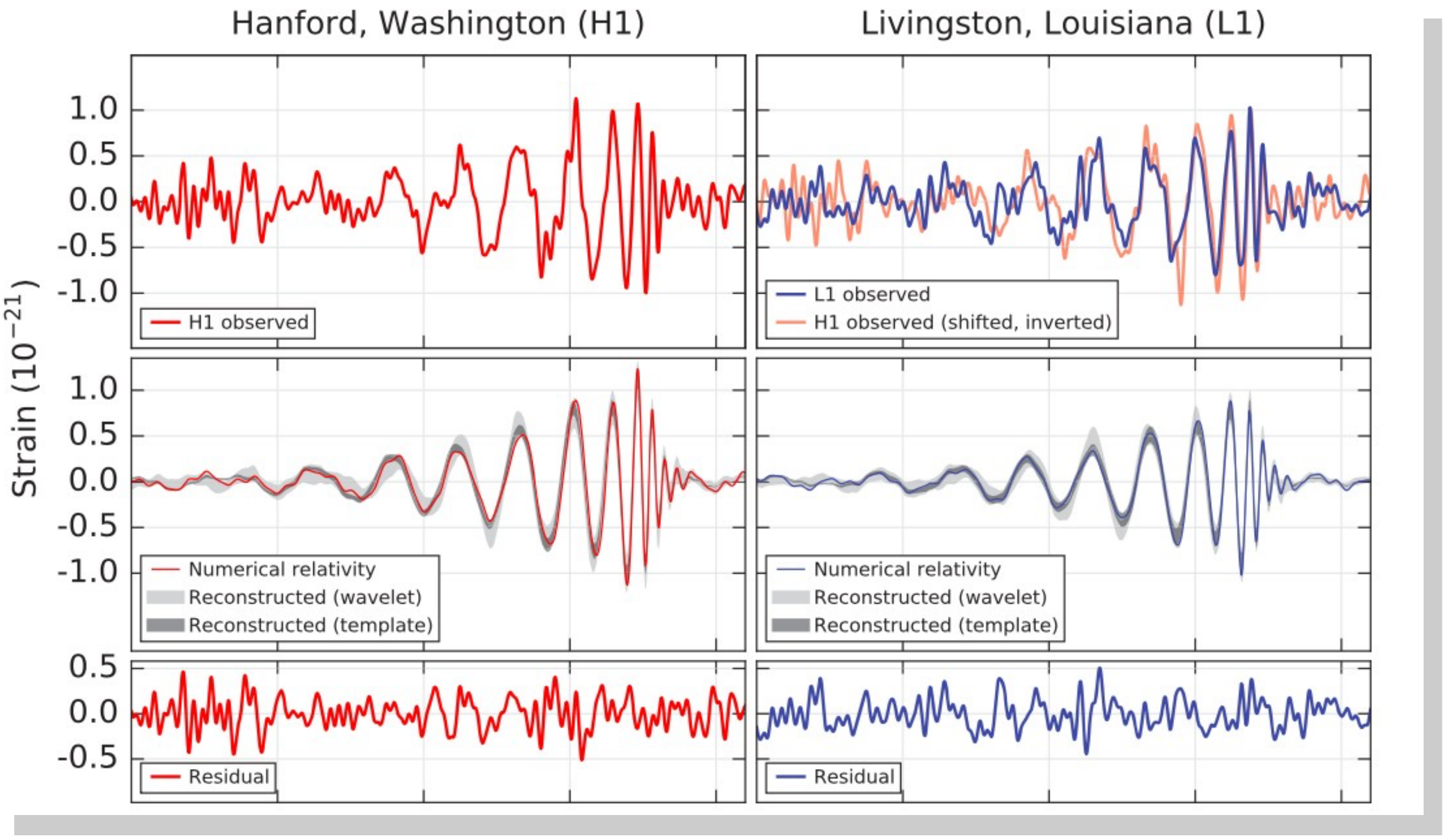
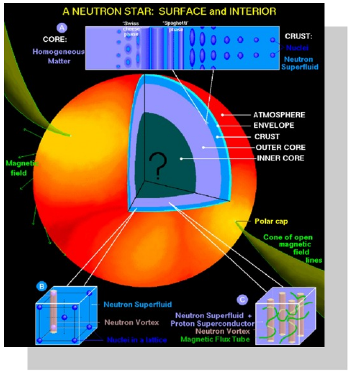
Credit: LIGO first detection PRL



- New era of gravitational wave astronomy (more like a microphone than a telescope)
- Several black-hole binary detections Also BH-NS and NS-NS events

Outline



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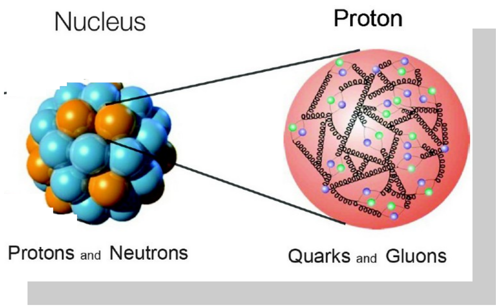
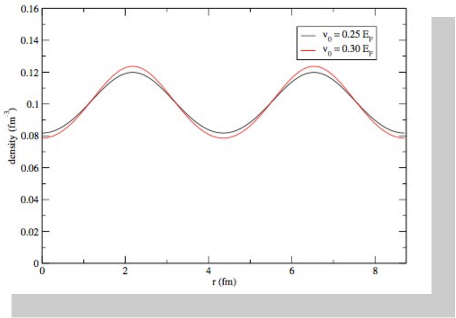
Motivation



Nuclear forces



Many-body physics

Getting the TLAs out of the way

QCD = Quantum Chromodynamics

EFT = Effective Field Theory

QMC = Quantum Monte Carlo

S-wave scattering in QM

Scattering (i.e., positive *E*) means no bound state (i.e., negative *E*)

Our task is to solve: $$-\frac{\hbar^2}{2\mu}\frac{d^2u(r)}{dr^2} + V(r)u(r) = Eu(r)$$

Where:

$$\psi(r) = u(r)/r$$ wave function

$$\mu = m_0 m_1/(m_0 + m_1)$$ reduced mass

$$E = \hbar^2 k^2/(2\mu)$$ energy

$$V(r) = -c\frac{\hbar^2}{2\mu}\frac{\beta^2}{\cosh^2(\beta r)}$$ interaction potential

S-wave scattering in QM

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Scattering (i.e., positive *E*) means no bound state (i.e., negative *E*)

At an *R* outside the range of the potential we have:

$$u(R) = A\sin(kR + \delta_0)$$

where $\delta_0 = \delta_0(k)$ is the S-wave phase shift.

Two undetermined quantities. What do we do?

Combine: $u'(R) = Ak\cos(kR + \delta_0)$

With above to get logarithmic derivative condition:

$$k\frac{u(R)}{u'(R)} = \tan(kR + \delta_0)$$

S-wave scattering in QM

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Scattering (i.e., positive *E*) means no bound state (i.e., negative *E*)

Use an ODE solver for: $$-\frac{\hbar^2}{2\mu}\frac{d^2u(r)}{dr^2}+V(r)u(r)=Eu(r)$$

Up to a large *R*. For example, with RK4 you get both the function value and the derivative value, so you can use the equation

$$k\frac{u(R)}{u'(R)}=\tan\left(kR+\delta_0\right)$$

to find the phase shift.

You'll do this during the skills session

S-wave scattering in QM

Scattering (i.e., positive *E*) means no bound state (i.e., negative *E*)

More difficult problem: $$-\frac{\hbar^2}{2\mu}\frac{d^2u(r)}{dr^2} + V(r)u(r) = Eu(r)$$

$$V(r) = -c\frac{\hbar^2}{2\mu}\frac{\beta^2}{\cosh^2(\beta r)}$$

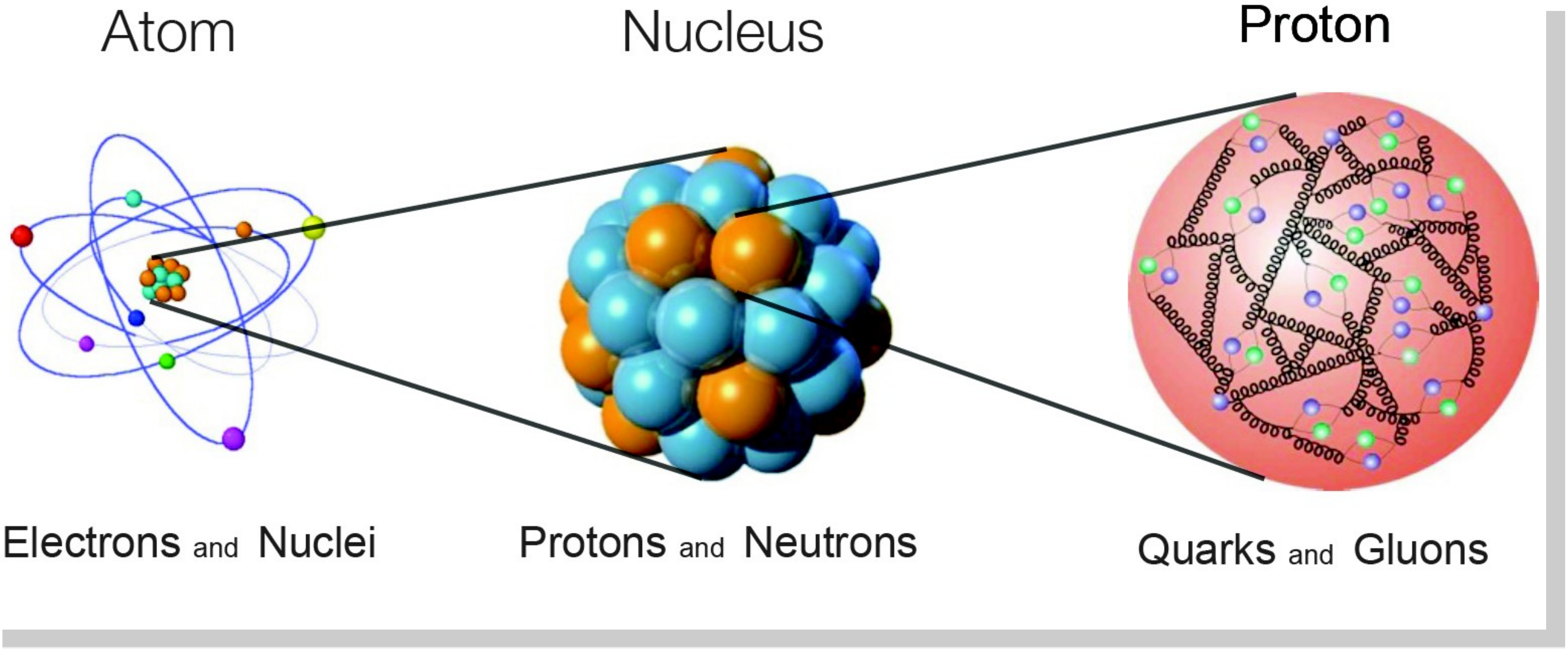
where you don't know the potential parameters but do know:

$$\delta_0(0.0245522951103708) = 0.51934174301976,$$
$$\delta_0(0.0347221887324730) = 0.67081991599790$$

Nonlinear problem (two equations with two unknowns)

Degrees of freedom

What does *from first principles* mean?



Quotes on degrees of freedom

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“The underlying physical laws necessary for the mathematical theory of a large part of physics and the whole of chemistry are thus completely known”

– Paul Dirac

“To understand macroscopic properties of matter based on understanding these microscopic laws is just unrealistic. Even though the microscopic laws are, in a strict sense, controlling what happens at the larger scale, they are not the right way to understand that.”

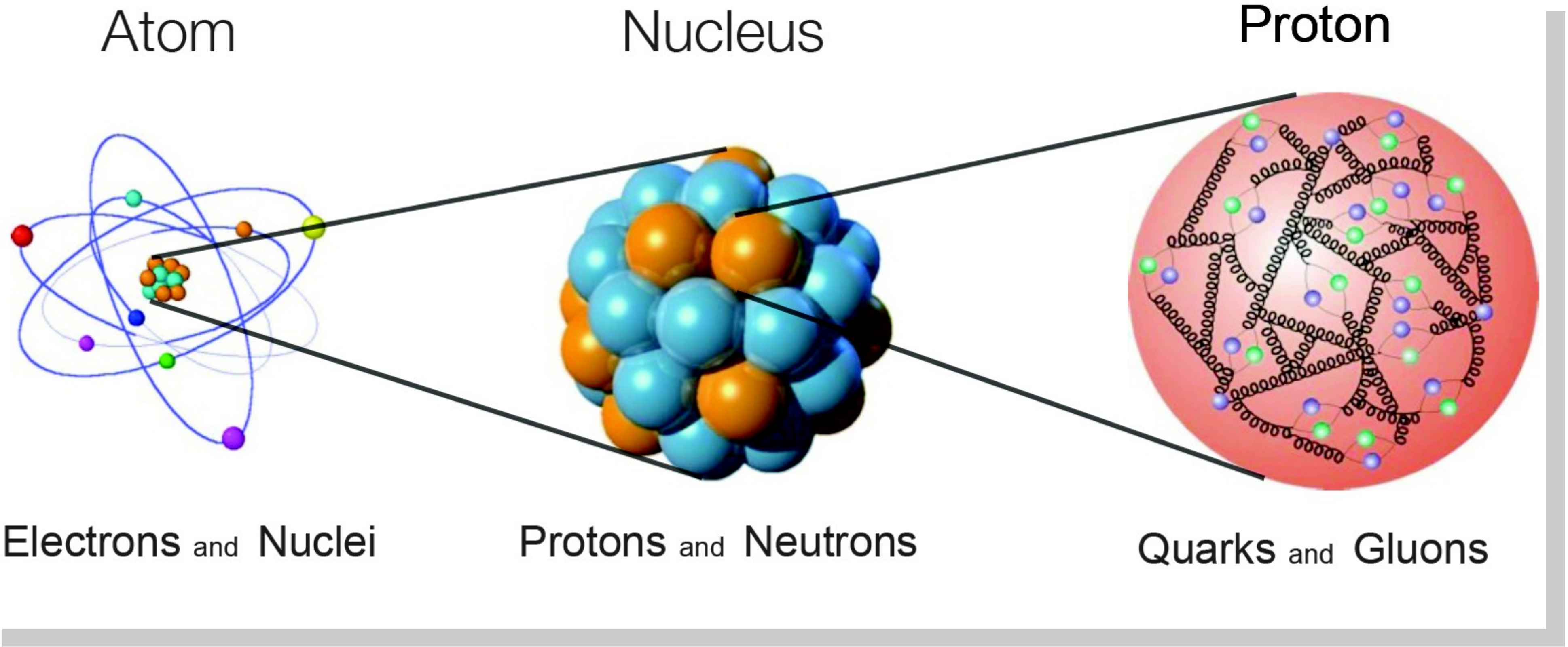
– John Schwarz

“only a fool would imagine that one should try to understand the properties of waves in the ocean in terms of Feynman-diagram calculations in the standard model, even if the latter understanding is possible 'in principle'.”

– Tom Banks

Degrees of freedom

What does *from first principles* mean?



Steven Weinberg's Third Law of Progress in Theoretical Physics:

You may use any degrees of freedom you like to describe a physical system, but if you use the wrong ones, you'll be sorry!

Nuclear interactions 1

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Historically

“Effective Interactions” were employed in the context of mean-field theory.

Phenomenological

NN interaction fit to N-body experiment

Non-microscopic

NN interaction does not claim to (and will not) describe np scattering

Nuclear physics is difficult

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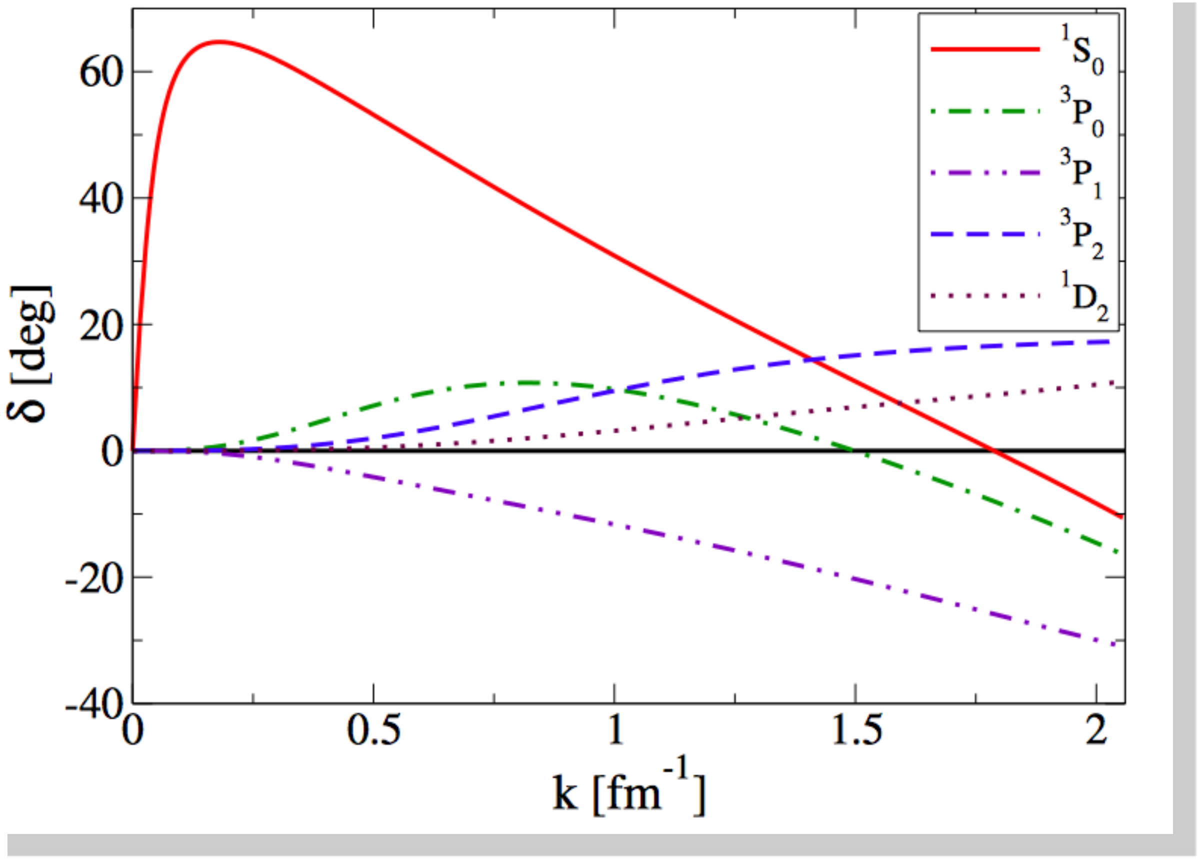
Scattering phase shifts: different "channels" have different behavior.

Nuclear physics is difficult

Scattering phase shifts: different "channels" have different behavior.



Any potential that reproduces them must be spin (and isospin) dependent

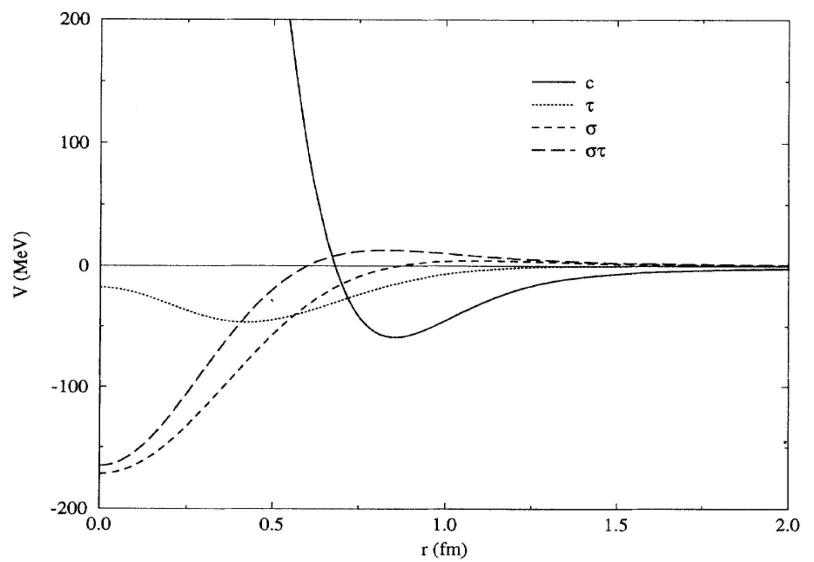
Nuclear interactions 2

Different approach: phenomenology treats NN scattering without connecting with the underlying level

$$V_2 = \sum_{j<k} v_{jk} = \sum_{j<k} \sum_{p=1}^{8} v_p(r_{jk}) O^{(p)}(j,k)$$

$$O^{p=1,8}(j,k) = (1, \sigma_j \cdot \sigma_k, S_{jk}, \mathbf{L}_{jk} \cdot \mathbf{S}_{jk}) \otimes (1, \tau_j \cdot \tau_k)$$

Such potentials are hard, making them non-perturbative at the many-body level (which is a problem for most methods on the market).



Softer, momentum-space formulations like CD-Bonn very popular

How to go beyond?

Historically, fit NN interaction to N-body experiment

Parallel approach, fit NN interaction to 2-body experiment, ignoring underlying level of quarks and gluons

Natural goal: fit NN interaction to 2-body experiment, without ignoring underlying level

Chiral effective field theory

Nuclear Hamiltonian: chiral EFT

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How to build on QCD in a systematic manner?

Exploit separation of scales:

$$a_{^1S_0} = (11 \text{ MeV})^{-1}$$

$$m_\pi = 140 \text{ MeV}$$

$$\Lambda_\chi \approx m_\rho \approx \ 800 \text{ MeV}$$

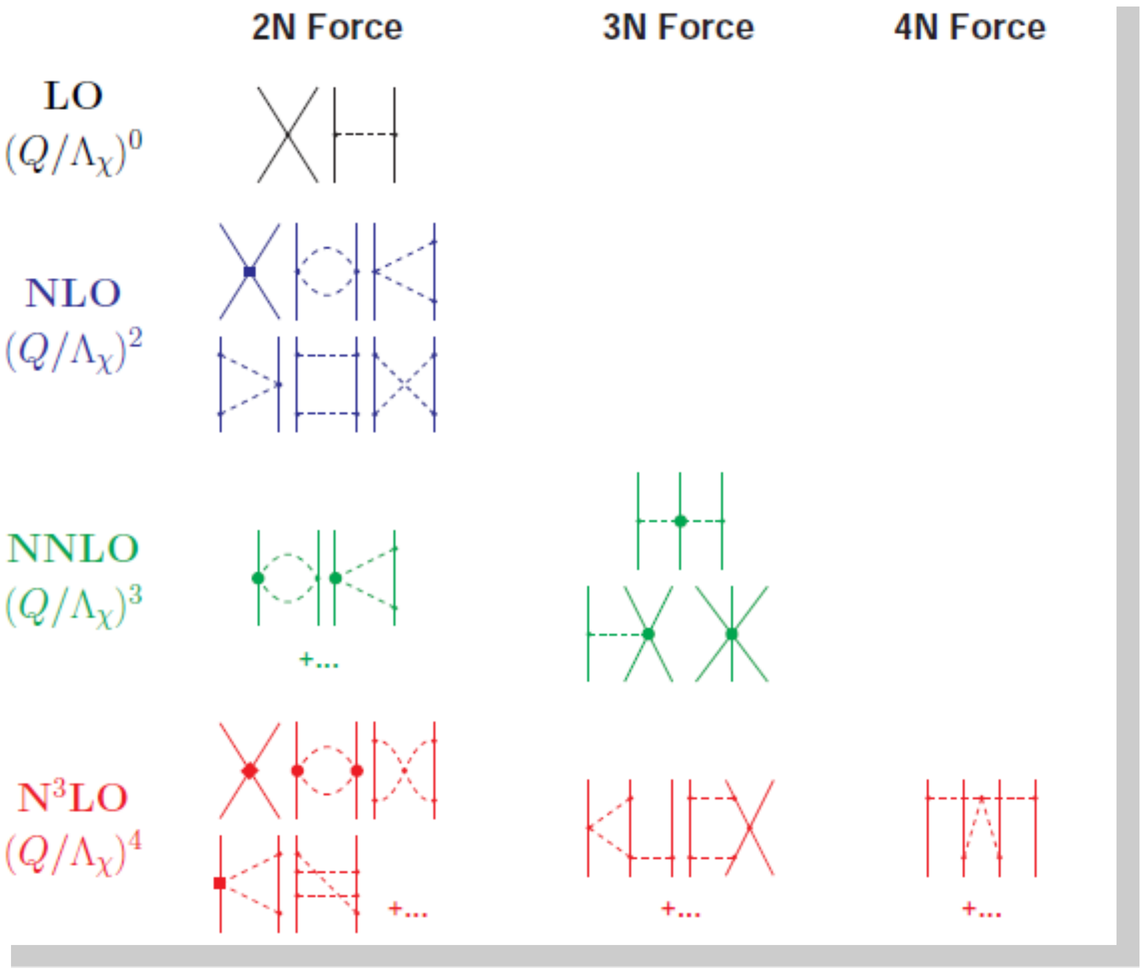
Chiral Effective Field Theory approach:

Use nucleons and pions as degrees of freedom

Systematically expand in $\dfrac{Q}{\Lambda_\chi}$

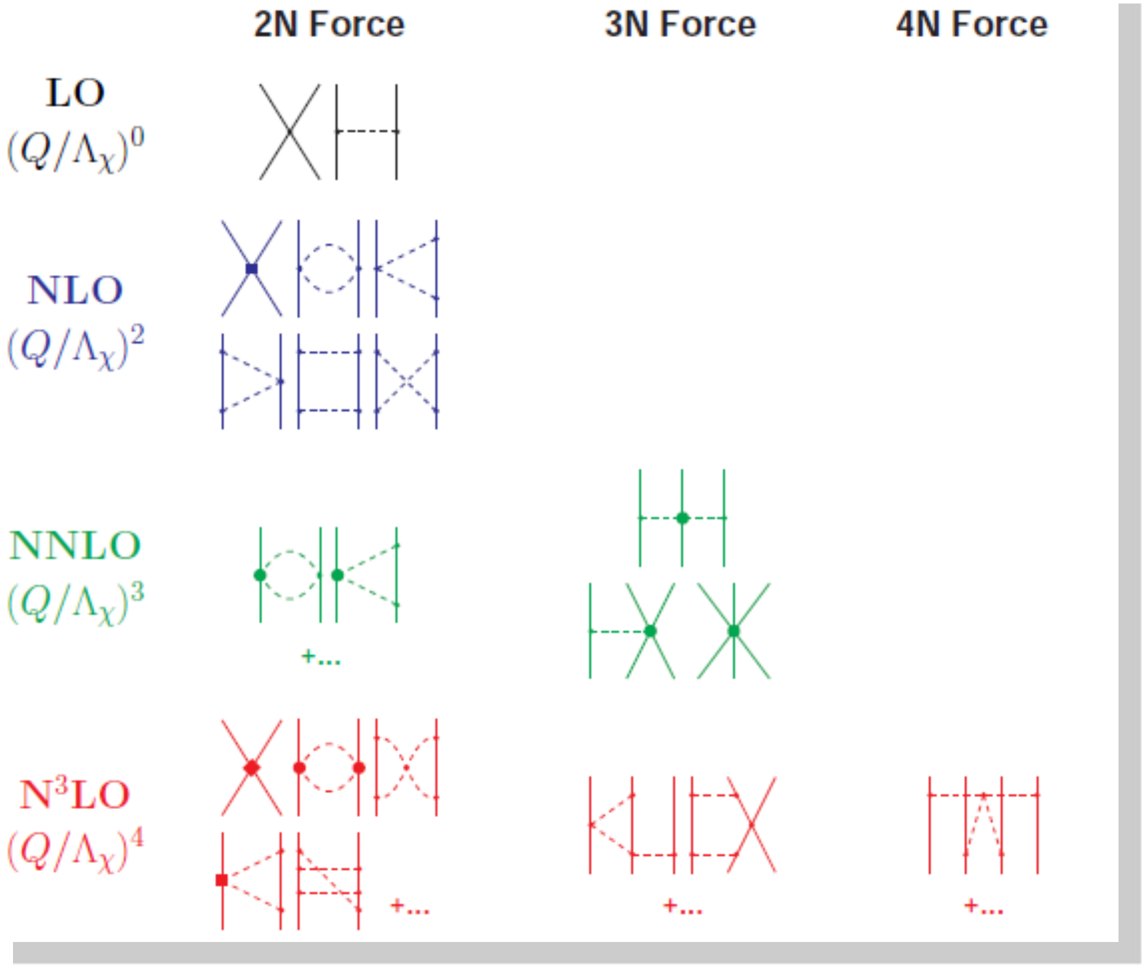
Program introduced by S. Weinberg, now taken over by the nuclear community

Nuclear Hamiltonian: chiral EFT



- Attempts to connect with underlying theory (QCD)
- Systematic low-momentum expansion
- Consistent many-body forces
- Low-energy constants from experiment or lattice QCD
- Until now non-local in coordinate space, so unused in continuum QMC
- Power counting's relation to renormalization tackled only very recently

Nuclear Hamiltonian: chiral EFT



Nuclear Hamiltonian: chiral EFT

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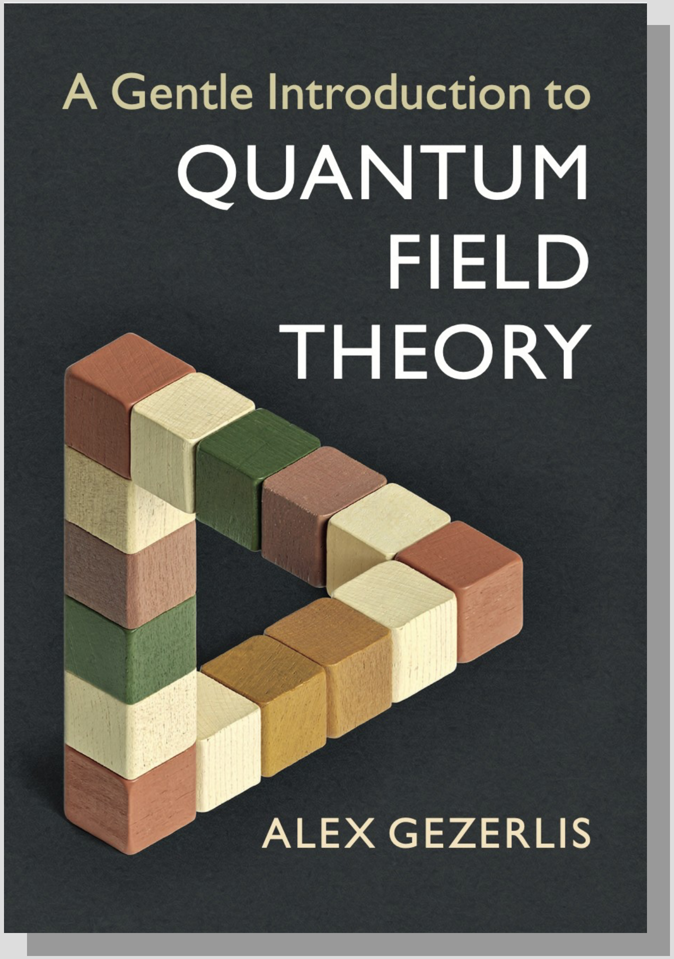
If you're scared of
Feynman diagrams, see:

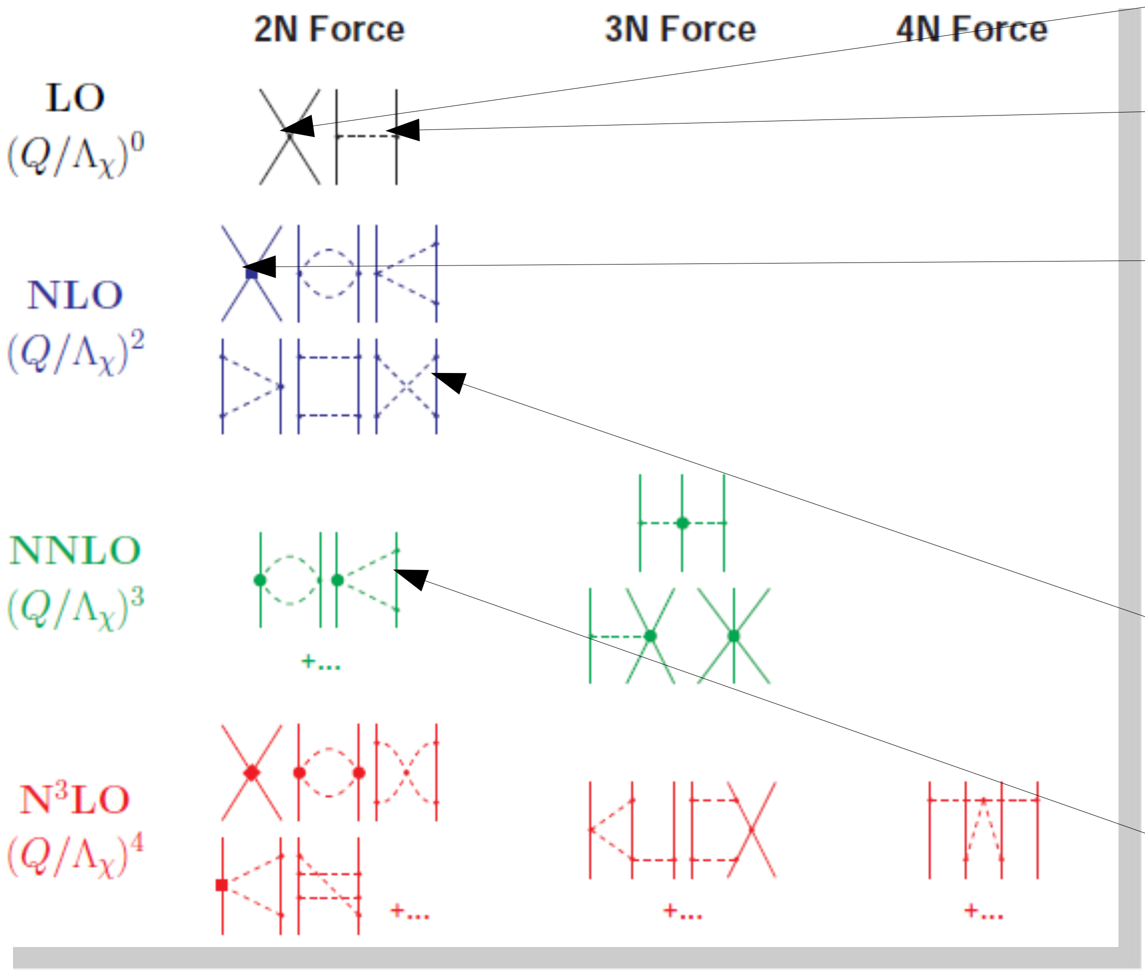
Alex Gezerlis

A Gentle Introduction to Quantum Field Theory

(Cambridge University Press, 2026)



Nuclear Hamiltonian: chiral EFT



$$V_{\rm ct}^{(0)} = C_S + C_T \, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$$

$$V_{1\pi}^{(0)} = -\left(\frac{g_A}{2f_\pi}\right)^2 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \frac{(\boldsymbol{\sigma}_1 \cdot \mathbf{q})(\boldsymbol{\sigma}_2 \cdot \mathbf{q})}{q^2 + m_\pi^2}$$

$$\begin{aligned} V_{\rm ct}^{(2)} &= C_1 \, q^2 + C_2 \, k^2 \\ &+ \left(C_3 \, q^2 + C_4 \, k^2\right) \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \\ &+ i\frac{C_5}{2}(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot (\mathbf{q} \times \mathbf{k}) \\ &+ C_6(\boldsymbol{\sigma}_1 \cdot \mathbf{q})(\boldsymbol{\sigma}_2 \cdot \mathbf{q}) \\ &+ C_7(\boldsymbol{\sigma}_1 \cdot \mathbf{k})(\boldsymbol{\sigma}_2 \cdot \mathbf{k}) \end{aligned}$$

Long-studied
two-pion exchange

Contains couplings
from πN scattering

Conclusions

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- Modern chiral nuclear forces now being routinely produced
- Unlike in past decades, attention is being paid to fine-tuning and other effects
- Rich interplay between novel nuclear interactions and modern quantum many-body theory

Acknowledgments

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Funding

NSERC
CRSNG

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