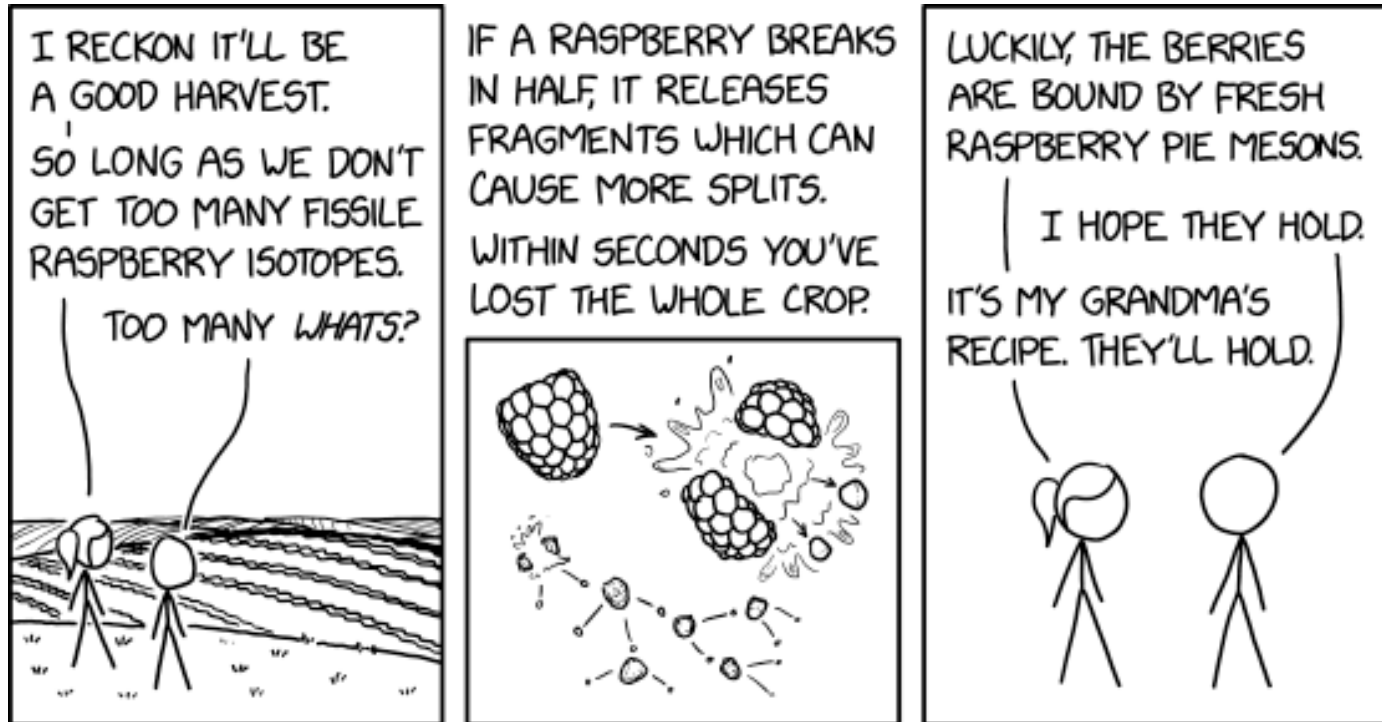


Nuclear Fission

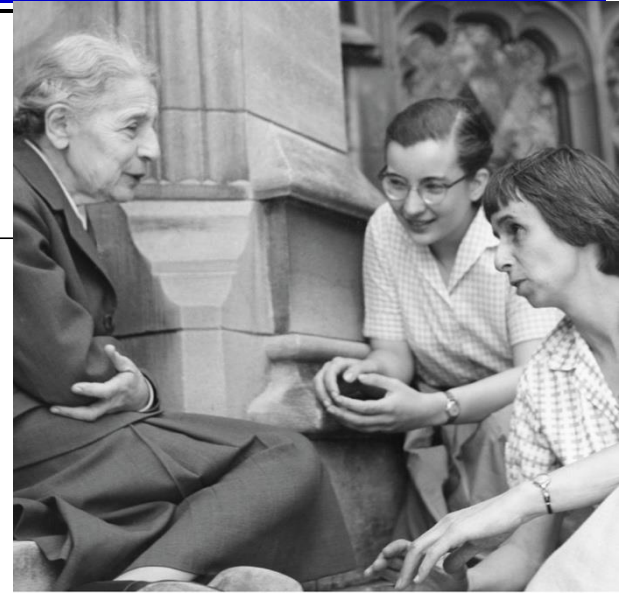
Lecture 1

Andreas Heinz
Chalmers University of Technology



<https://xkcd.com2504/>

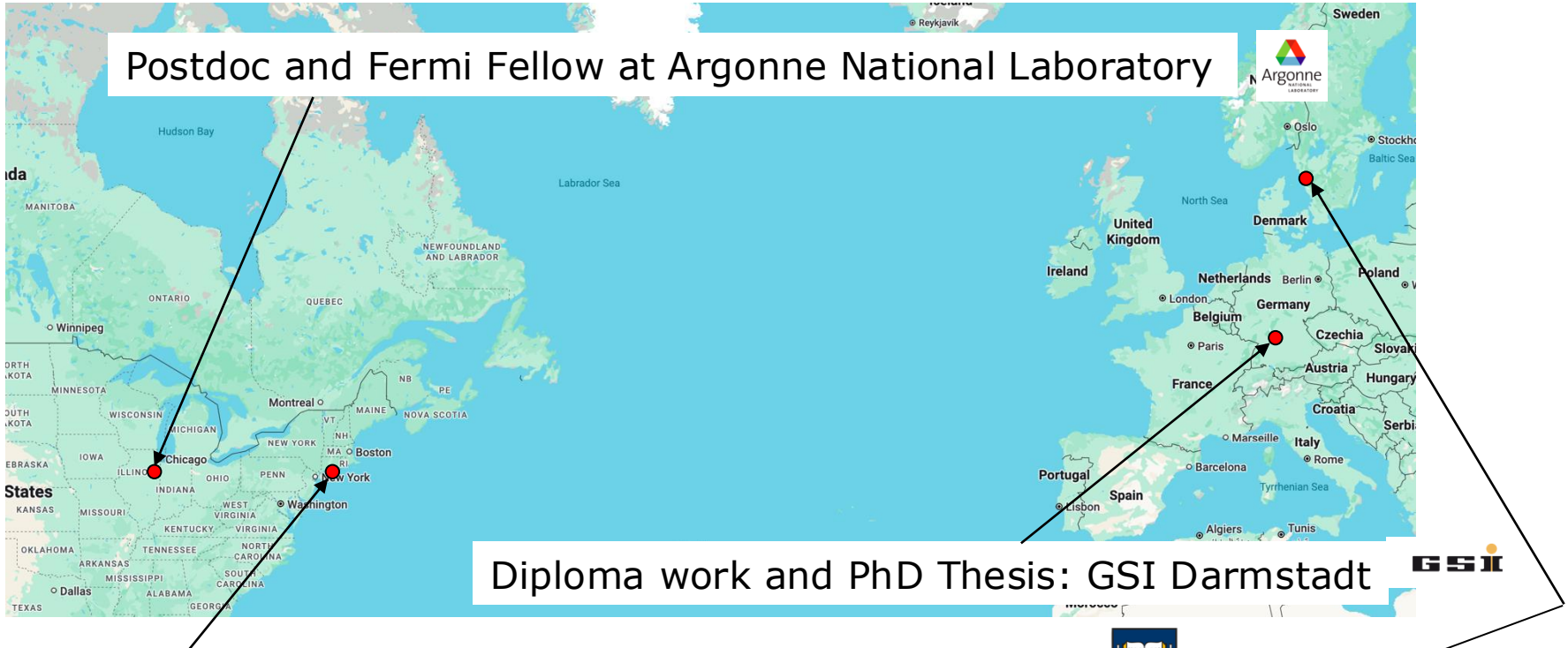
Summary



I believe all young people think about how they would like their lives to develop. When I did so I always arrived at the conclusion that **life need not be easy, provided only that it was not empty.** And that wish I have been granted.

Lise Meitner

Who am I?



Assistant professorship: Yale University, New Haven, CT



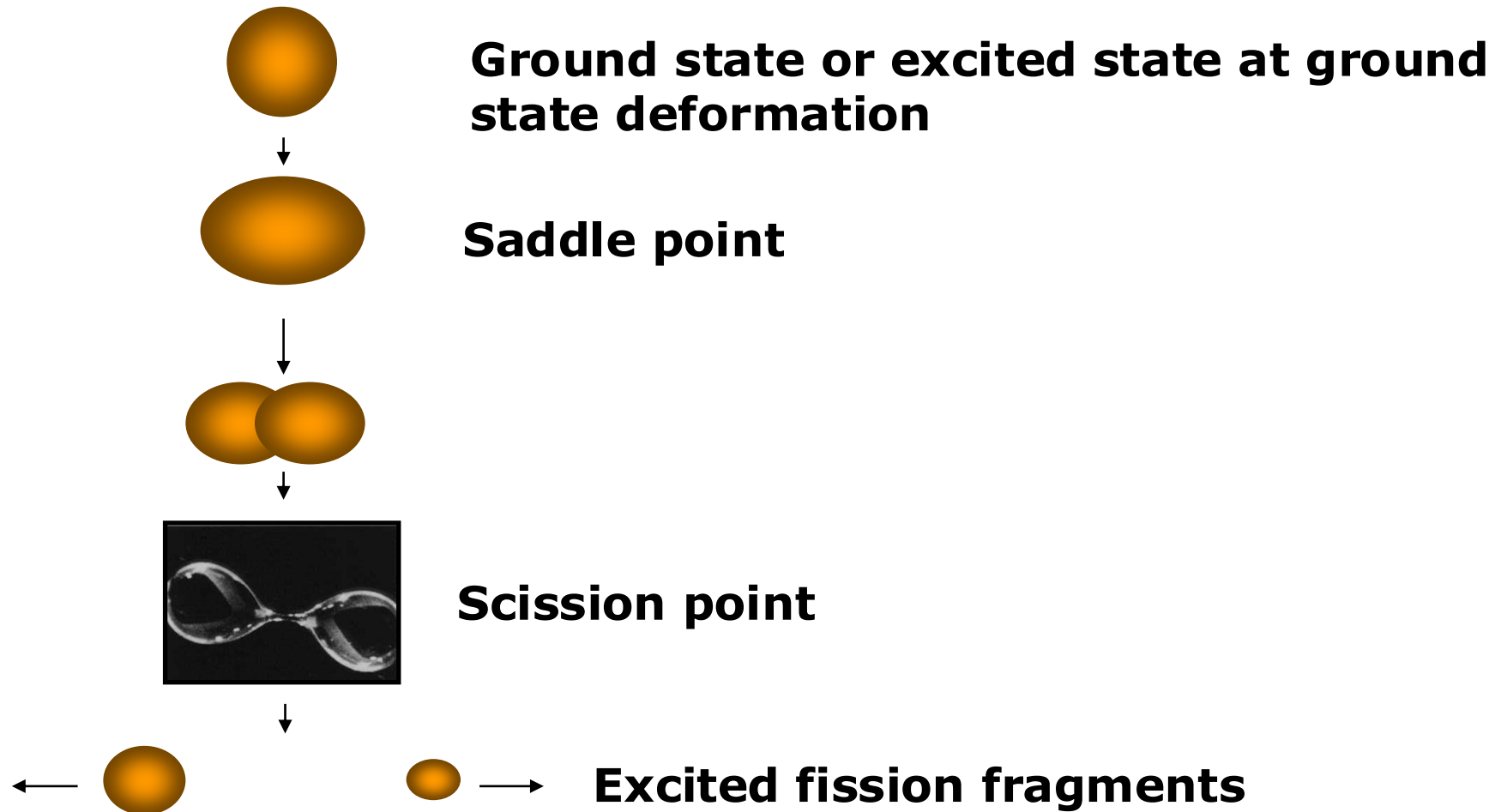
Associate Professor: Chalmers University of Technology, Gothenburg, Sweden



Warning

I often reference **books or **review articles**, rather than original works. This is **not good practice**, but makes it for you, hopefully, easier to find material.**

An intuitive picture of nuclear fission



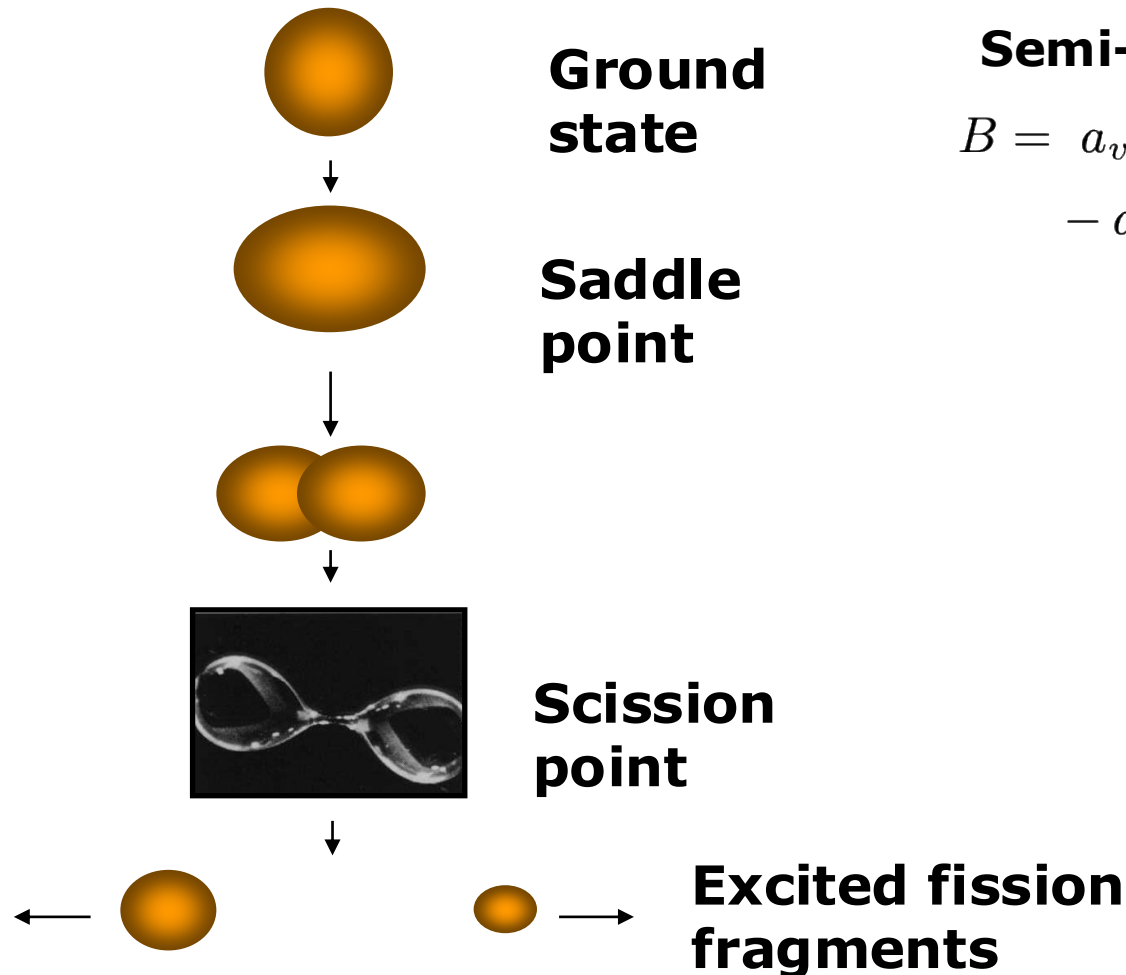
Why do we care?



<https://www.redbubble.com/i/t-shirt/Care-O-Meter-by-RetroWorks/32355504/z5wf>

- **Why do we care about nuclear fission?**
 - **Applications** (the obvious ones).
 - Production of **unstable** (neutron-rich) nuclei (or beams of nuclei).
 - Nuclear astrophysics (**r-process**).
 - Transport properties of **cold** and **hot** nuclear matter.
 - Nuclei at the extremes of **deformation**.
 - Interplay of **microscopic and macroscopic** nuclear properties.
 - Production of **super-heavy elements**.
 - ...
- **(Semi-)classical approaches go a long way in fission, but not all the way.**

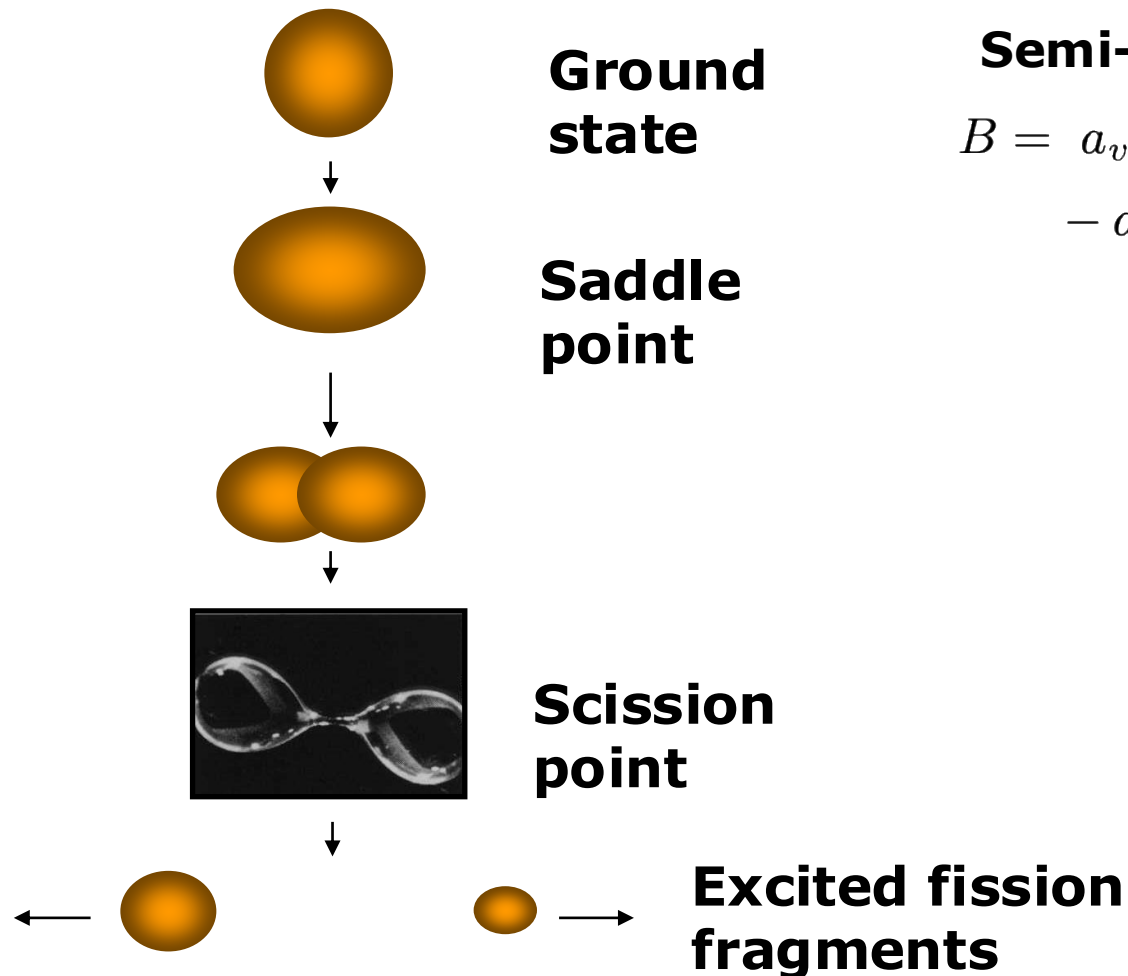
An intuitive picture of nuclear fission



Semi-empirical mass formula:

$$B = a_v A - a_s A^{2/3} - a_c Z(Z - 1) A^{-1/3} - a_{sym} (A - 2Z)^2 A^{-1} + \delta$$

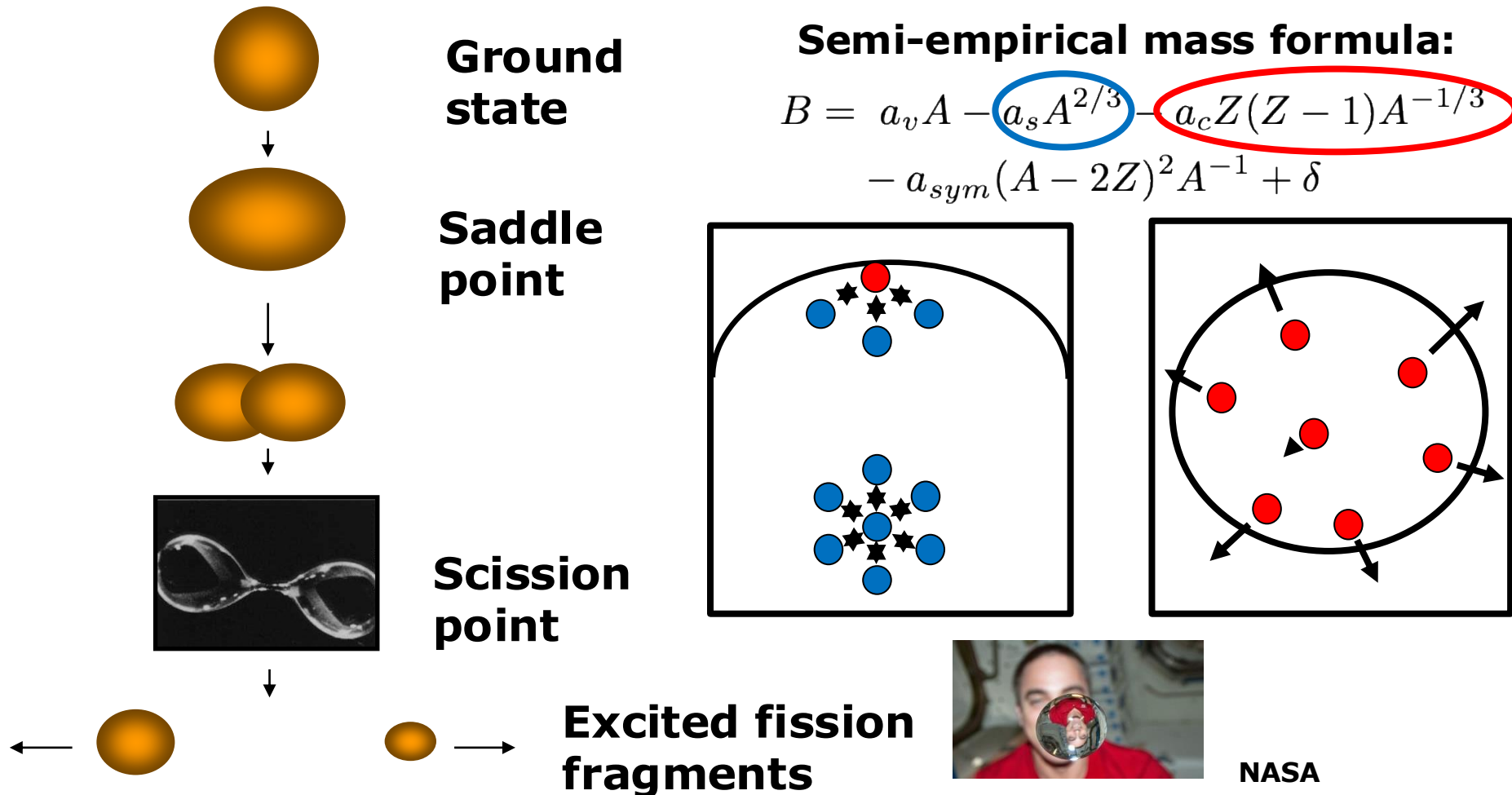
An intuitive picture of nuclear fission



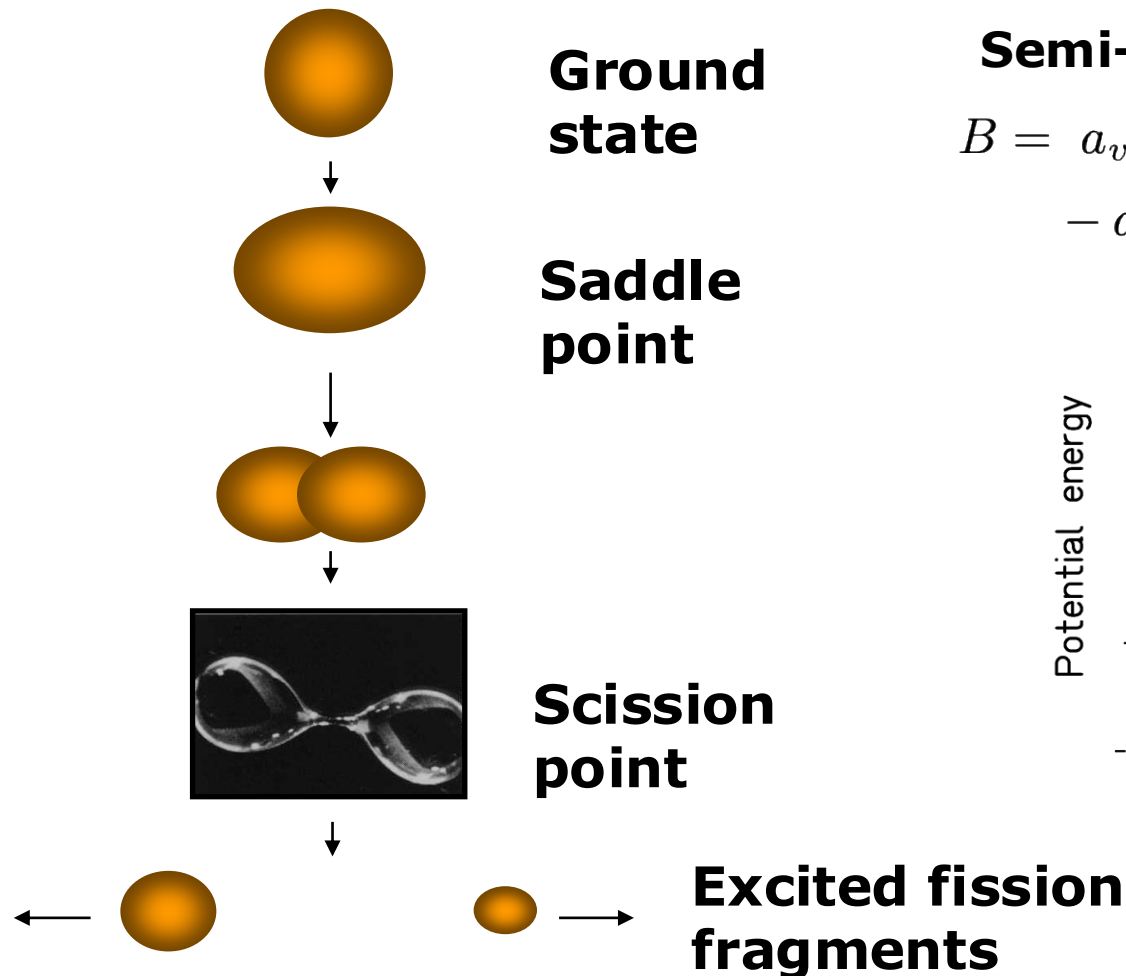
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An intuitive picture of nuclear fission

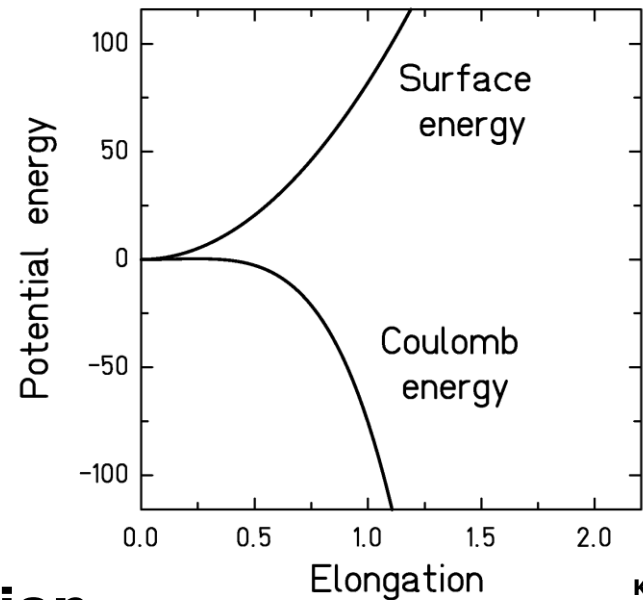


An intuitive picture of nuclear fission



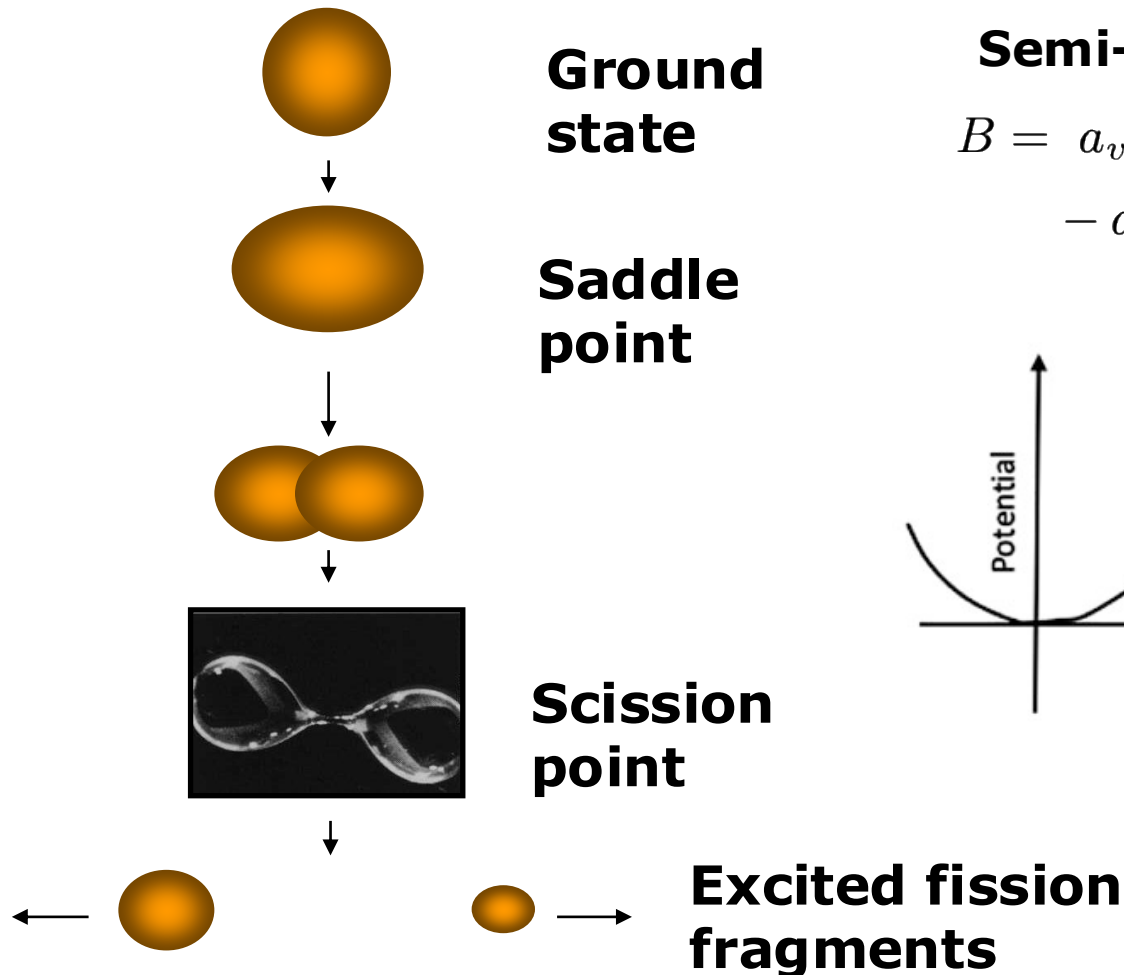
Semi-empirical mass formula:

$$B = a_v A - a_s A^{2/3} - a_c Z(Z-1)A^{-1/3} - a_{sym}(A-2Z)^2 A^{-1} + \delta$$



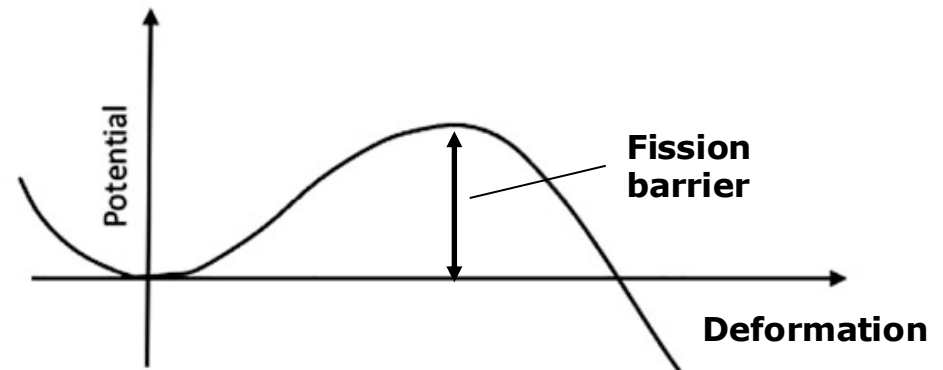
K.-H. Schmidt

An intuitive picture of nuclear fission



Semi-empirical mass formula:

$$B = a_v A - a_s A^{2/3} - a_c Z(Z-1)A^{-1/3} - a_{sym}(A-2Z)^2 A^{-1} + \delta$$



P. Talou, R. Vogt, *Nuclear Fission*, Springer (2023)

Discovery of nuclear fission



* W. NOLL, Naturwiss. 20, 283 (1932).
* W. NOLL, Naturwiss. 20, 360 (1932).
* W. NOLL, Naturwiss. 23, 197 (1935); vgl. auch
W. NOLL, Ber. dtsh. keram. Ges. 19, H. 5 (1938).

Größe und Bindungsvermögen der Atome in Betracht kommen und in Deutschland als Rohstoffe vorhanden sind.

Über den Nachweis und das Verhalten der bei der Bestrahlung des Urans mittels Neutronen entstehenden Erdalkalimetalle¹.

Von O. HAHN und F. STRASSMANN, Berlin-Dahlem.

In einer vor kurzem an dieser Stelle erschienenen vorläufigen Mitteilung² wurde angegeben, daß bei der Bestrahlung des Urans mittels Neutronen außer den von MEITNER, HAHN und STRASSMANN im einzelnen beschriebenen Trans-Uranen — den Elementen 93 bis 96 — noch eine ganze Anzahl anderer Umwandlungsprodukte entstehen, die ihre Bildung offensichtlich einem sukzessiven zweimaligen α -Strahlenzerfall des vorübergehend entstandenen Urans 239 verdanken. Durch einen solchen Zerfall muß aus dem Element mit der Kernladung 92 ein solches mit der Kernladung 88 entstehen, also ein Radium. In der genannten Mitteilung wurden in einem noch als vorläufig bezeichneten Zerfallsschema 3 derartiger isomerer Radiumisotope mit ungefähr geschätzten Halbwertszeiten und ihren Umwandlungsprodukten, nämlich drei isomeren Actiniumisotopen, angegeben, die ihrerseits offensichtlich in Thorisotope übergehen.

Zugleich wurde auf die zunächst unerwartete Beobachtung hingewiesen, daß diese unter α -Strahlenabspaltung über ein Thorium sich bildenden Radiumisotope nicht nur mit schnellen, sondern auch mit verlangsamt Neutronen entstehen.

Glieder beschrieben werden. Aus dem Aktivitätsverlauf der einzelnen Isotope ergibt sich ihre Halbwertszeit und lassen sich die daraus entstehenden Folgeprodukte ermitteln. Die letzteren werden in dieser Mitteilung aber im einzelnen noch nicht beschrieben, weil wegen der sehr komplexen Vorgänge — es handelt sich um mindestens 3, wahrscheinlich 4 Reihen mit je 3 Substanzen — die Halbwertszeiten aller Folgeprodukte bisher noch nicht erschöpfend festgestellt werden konnten.

Als Trägersubstanz für die „Radiumisotope“ diente naturgemäß immer das Barium. Am nächstliegenden war die Fällung des Bariums als Bariumsulfat, das neben dem Chromat schwerstlösliche Bariumsalz. Nach früheren Erfahrungen und einigen Vorversuchen wurde aber von der Abscheidung der „Radiumisotope“ mit Bariumsulfat abgesehen; denn diese Niederschläge reißen neben geringen Mengen Uran nicht unbeträchtliche Mengen von Actinium- und Thoriumisotopen mit, also auch die mutmaßlichen Umwandlungsprodukte der Radiumisotope, und erlauben daher keine Reindarstellung der Ausgangsglieder. Statt der quantitativen, sehr oberflächenreichen Sulfatfällung wurde da-

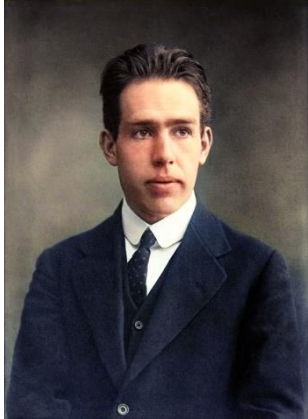
Disintegration of Uranium by Neutrons: a New Type of Nuclear Reaction

ON bombarding uranium with neutrons, Fermi and collaborators¹ found that at least four radioactive substances were produced, to two of which atomic numbers larger than 92 were ascribed. Further investigations² demonstrated the existence of at least

L. Meitner, O. Frisch, Nature 143, 239 (1939)

O. Hahn, F. Strassmann, Naturwissenschaften
27, 11 (1939)

The fission barrier



SEPTEMBER 1, 1939

PHYSICAL REVIEW

VOLUME 56

The Mechanism of Nuclear Fission

NIELS BOHR

University of Copenhagen, Copenhagen, Denmark, and The Institute for Advanced Study, Princeton, New Jersey

AND

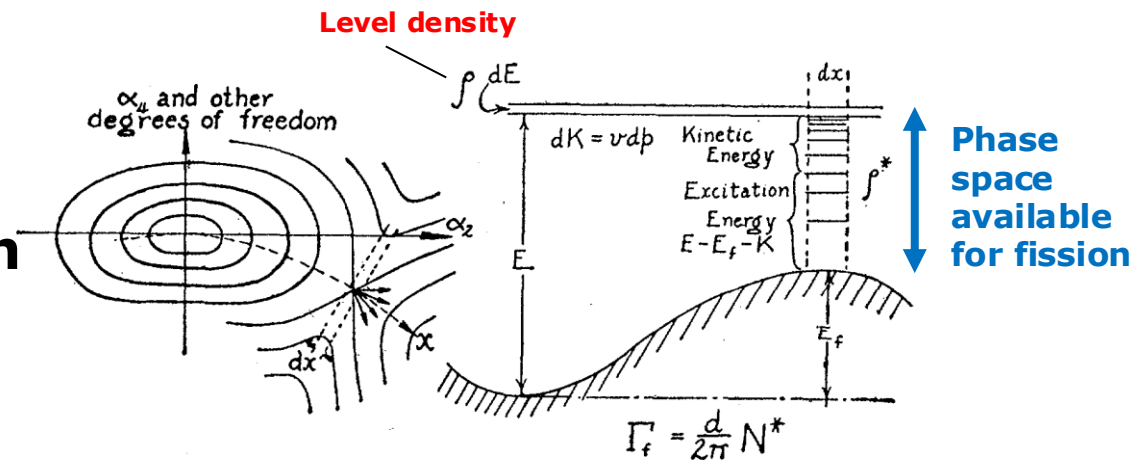
JOHN ARCHIBALD WHEELER

Princeton University, Princeton, New Jersey

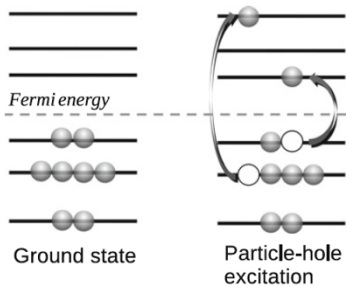
(Received June 28, 1939)

On the basis of the liquid drop model of atomic nuclei, an account is given of the mechanism of nuclear fission. In particular, conclusions are drawn regarding the variation from nucleus to nucleus of the critical energy required for fission, and regarding the dependence of fission cross section for a given nucleus on energy of the exciting agency. A detailed discussion of the observations is presented on the basis of the theoretical considerations. Theory and experiment fit together in a reasonable way to give a satisfactory picture of nuclear fission.

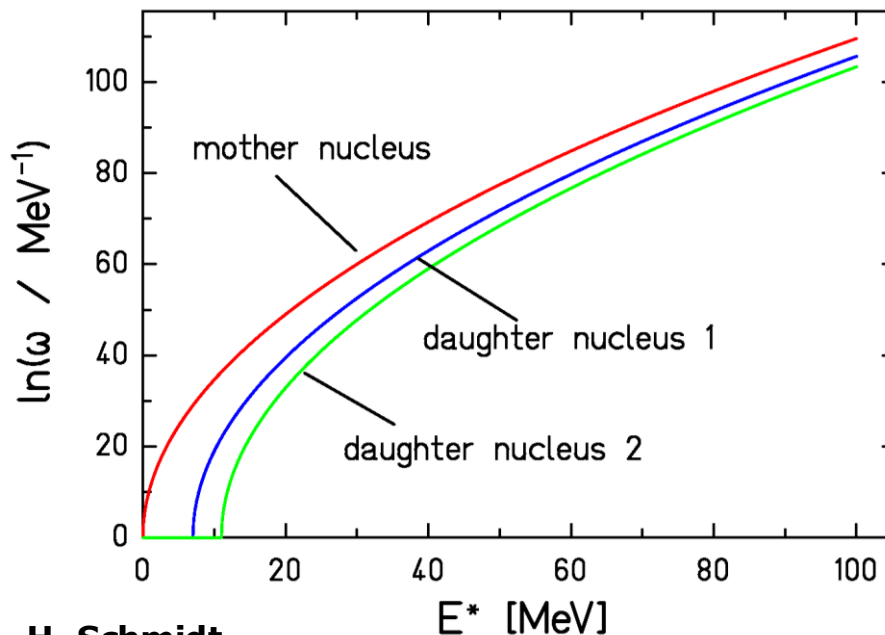
The **potential energy surface** and the relation to the **level density** above it.



Level densities: a key ingredient



W. Younes, W.D. Loveland, *An Introduction to Nuclear Fission*, Springer (2021)



K.-H. Schmidt

Nuclear level density:

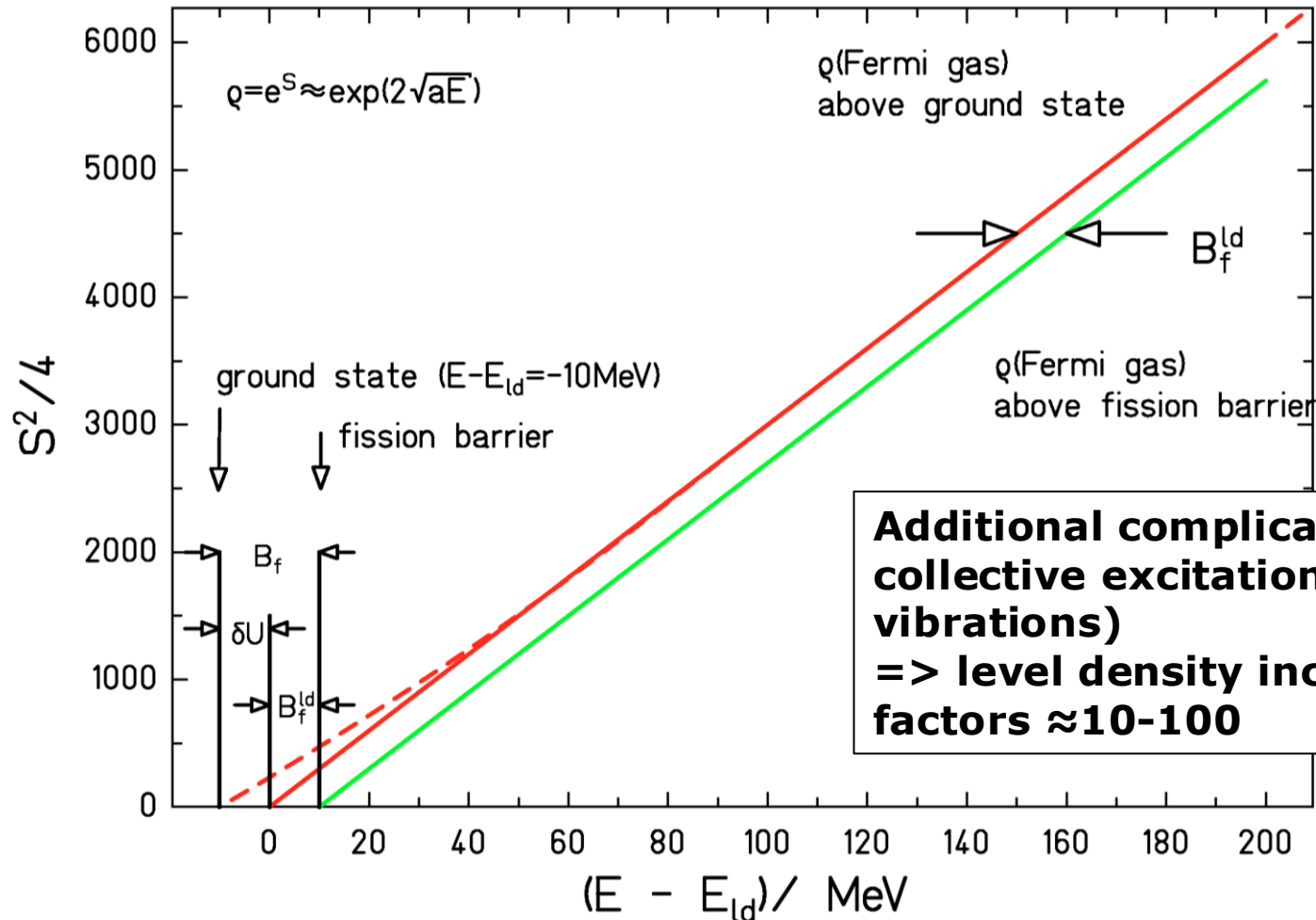
$$\rho(E^*) = \frac{dN}{dE^*}$$

Number of nuclear states for a given excitation energy E^* :

$$\rho(E^*) \propto e^{2\sqrt{aE^*}}$$

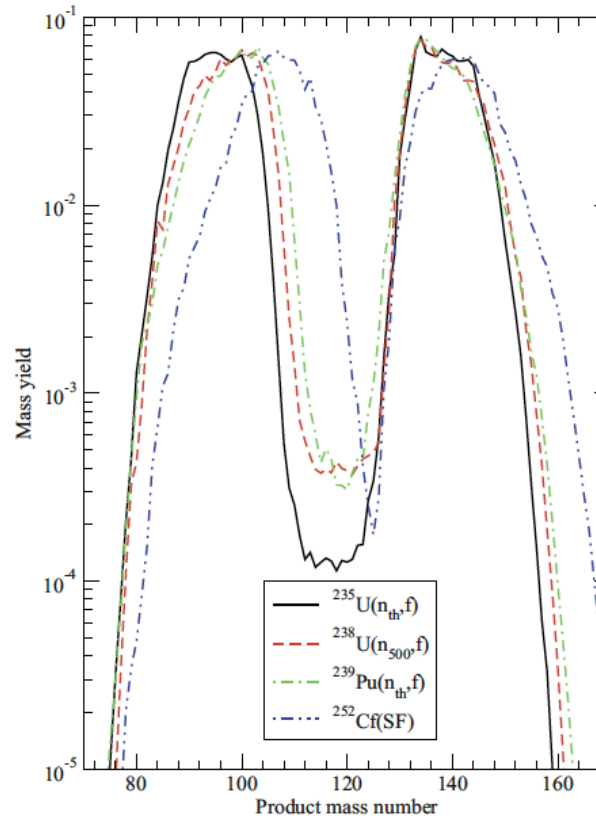
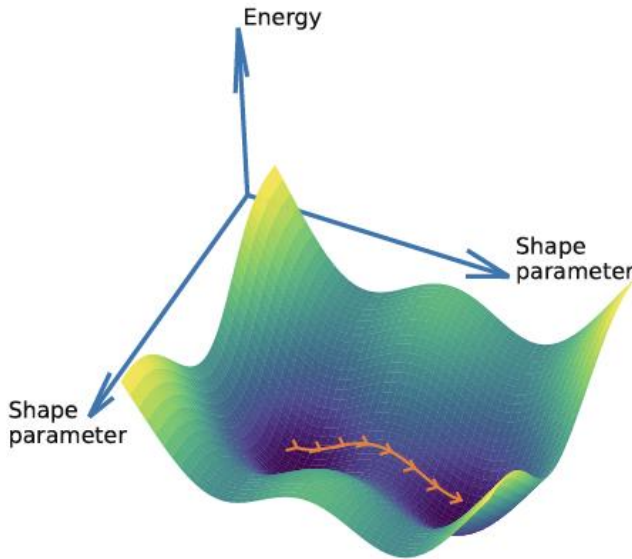
from the Fermi-gas model (a is the level density parameter).

Level densities and nuclear shell structure



K.-H. Schmidt

The potential energy surface (PES)



What the ...?

**Fission fragments
do not have the
same mass!**

**The mass
distribution
changes with
excitation
energy!**

W. Younes, W.D. Loveland, *An Introduction to Nuclear Fission*, Springer (2021)

Scission point models

PHYSICAL REVIEW C

VOLUME 14, NUMBER 5

NOVEMBER 1976

Scission-point model of nuclear fission based on deformed-shell effects*

B. D. Wilkins, E. P. Steinberg, and R. R. Chasman

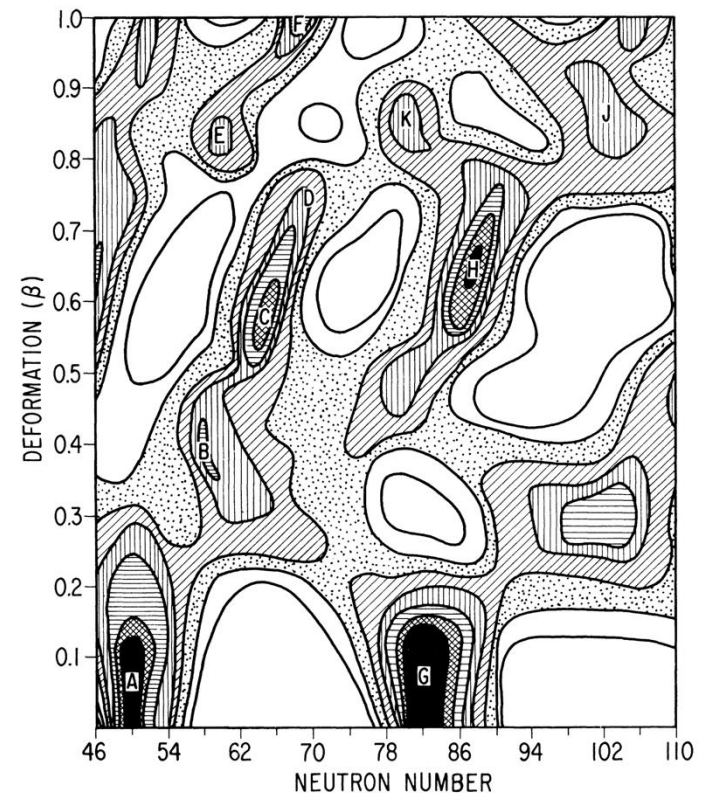
Chemistry Division, Argonne National Laboratory, Argonne, Illinois 60439

(Received 1 June 1976)

A static model of nuclear fission is proposed based on the assumption of statistical equilibrium among collective degrees of freedom at the scission point. The relative probabilities of formation of complementary fission fragment pairs are determined from the relative potential energies of a system of two nearly touching, coaxial spheroids with quadrupole deformations. The total potential energy of the system at the scission point is calculated as the sum of liquid-drop and shell- and pairing-correction terms for each spheroid, and Coulomb and nuclear potential terms describing the interaction between them. The fissioning system at the scission point is characterized by three parameters—the distance between the tips of the spheroids (d), the intrinsic excitation energy of the fragments (τ_{int}), and a collective temperature (T_{coll}). No attempt is made to adjust these parameters to give optimum fits to experimental data, but rather, a single choice of values for d , τ_{int} , and T_{coll} is used in the calculations for all fissioning systems. The general trends of the distributions of mass, nuclear charge, and kinetic energy in the fission of a wide range of nuclides from Po to Fm are well reproduced in the calculations. The major influence of the deformed-shell corrections for neutrons is indicated and provides a convenient framework for the interpretation of observed trends in the data and for the prediction of new results. The scission-point configurations derived from the model provide an interpretation of the “saw-tooth” neutron emission curve as well as previously unexplained observations on the variation of $\overline{\text{TKE}}$ for isotopes of U, Pu, Cm, and Cf; structure in the width of total kinetic energy release as a function of fragment mass ratio; and a difference in threshold energies for symmetric and asymmetric mass splits in the fission of Ra and Ac isotopes. In spite of a number of recognized simplifications in the model, quantitative fits to the data are generally within expected errors of the shell corrections determined by the Strutinski prescription.

Idea:

Describe fission fragment mass distributions based on the shell structure of fission fragments at scission.



The Strutinsky method – nuclear shell structure and fission

1.D.2

Nuclear Physics A122 (1968) 1—33; © North-Holland Publishing Co., Amsterdam

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“SHELLS” IN DEFORMED NUCLEI

V. M. STRUTINSKY

*I.V. Kurchatov Institute of Atomic Energy, Moscow, USSR
and*

The Niels Bohr Institute, University of Copenhagen, Denmark

Received 17 July 1967

Revised 17 May 1968

Abstract: New calculation results based on the shell correction method are described. The results are presented in such a way as to illustrate more clearly many effects of nucleon shells in deformed and spherical nuclei. Some complementary theoretical arguments related to the shell correction method and details of the calculation are given.

How can we include
shell effects in the
potential energy
surface?

-> microscopic-
macroscopic models.

Funny hills

REVIEWS OF MODERN PHYSICS

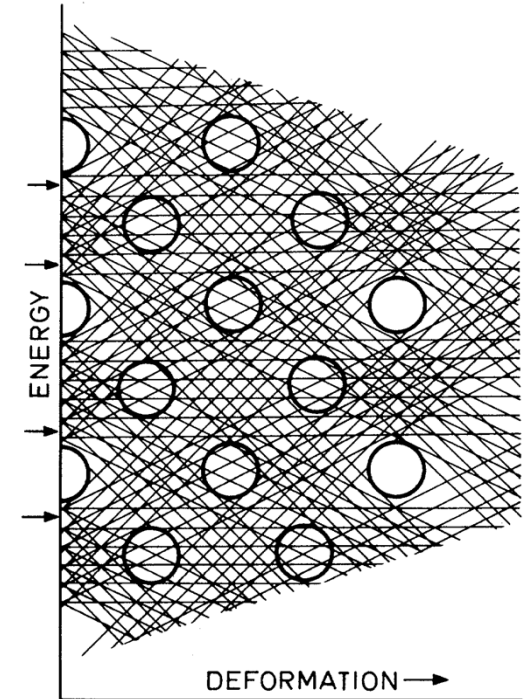
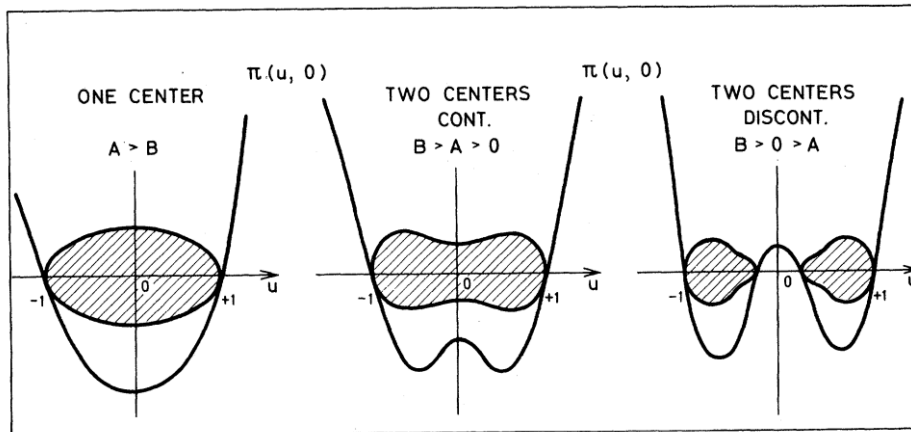
VOLUME 44, NUMBER 2

APRIL 1972

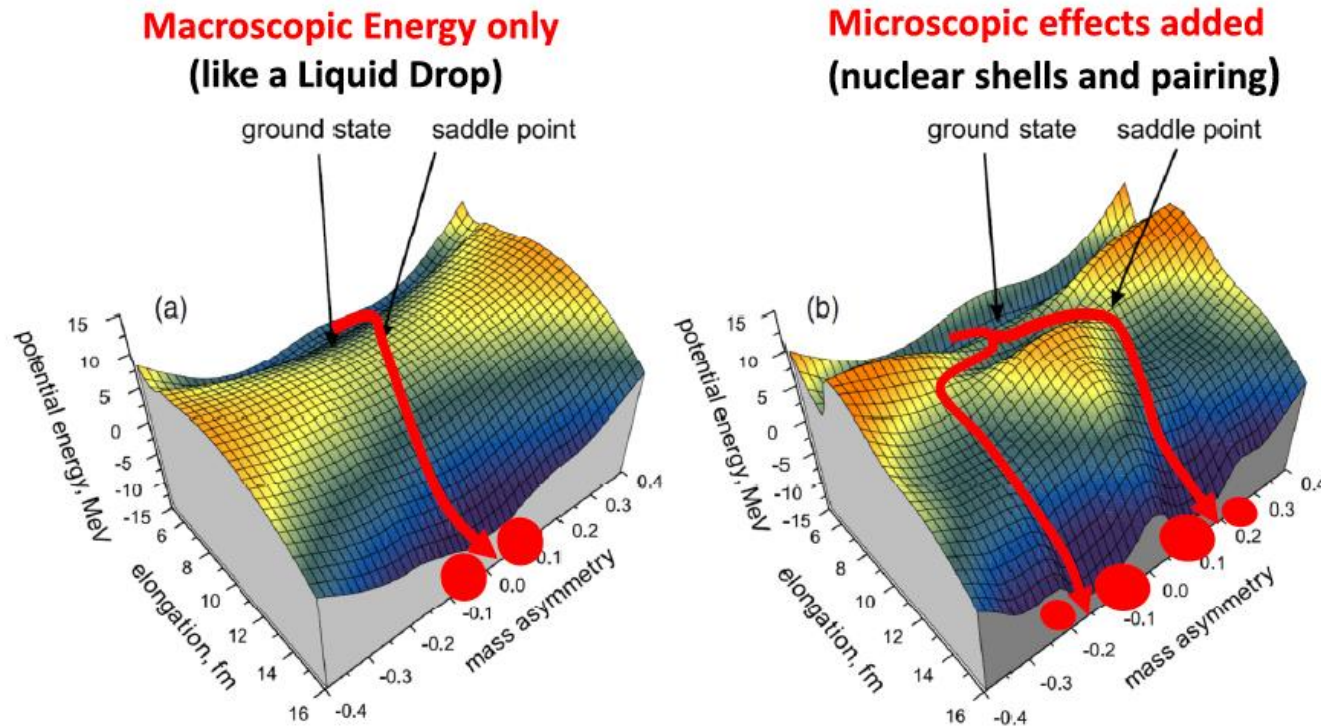
Funny Hills: The Shell-Correction Approach to Nuclear Shell Effects and Its Applications to the Fission Process

M. BRACK,* JENS DAMGAARD, A. S. JENSEN, H. C. PAULI,* V. M. STRUTINSKY,† C. Y. WONG‡
The Niels Bohr Institute, University of Copenhagen, Denmark

This paper reviews various results related to the single-particle structure in spherical and deformed nuclei, discussed from the viewpoint of the so-called shell-correction method. This method stresses the importance of large-scale non-uniformities in the energy distribution of the individual particles especially near the Fermi energy. The way in which these nonuniformities affect in an essential way many nuclear properties, such as the shape stiffness, the spatial density distribution, the total mass of the nucleus, the mass and inertia of the nuclear shape variations, etc. is also discussed. Against this background, the behavior of the nuclear deformation energy is described, in particular for larger distortions relevant to the fission process. In this connection, some qualitative singularities of the phenomenological liquid-drop deformation energy at large shape distortions are pointed out, and their possible implications for fission are discussed. As the problems considered cover a wide range of nuclear properties, this paper is not a review in the narrow sense of the word. Comparison with other approaches as well as historic references are given mainly to clarify specific points, because a complete review would be a monumental undertaking.



The potential energy surface (PES)

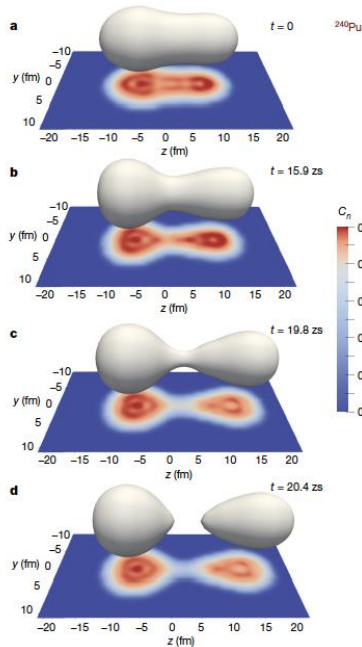


Nuclear shell effects modulate the potential energy surface -> for many nuclei **asymmetric fission** dominates (at **low excitation energies**).

Key concept:
We need the **available phase space** above each point of the PES!

A.N. Andreyev et al., Rep. Prog. Phys. 81, 016301 (2018)

The parametrization problem



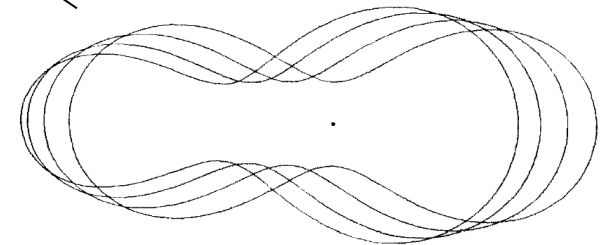
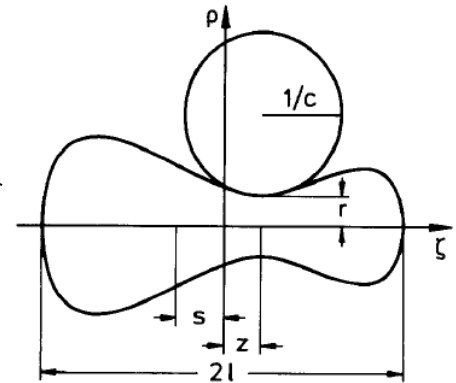
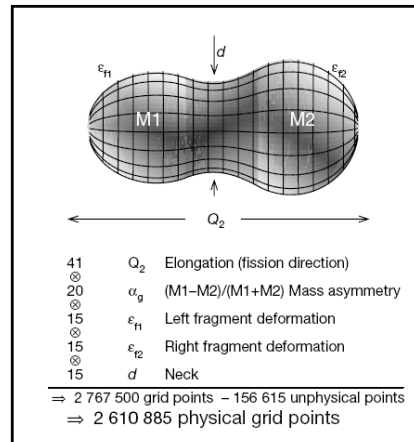
G. Scamps, C. Simenel, Nature 564, 382 (2018)

How many **parameters** are needed to describe the **evolution towards scission**?

Multimodal model (Brosa et al. Phys. Rep. 197 (1990) 167)

Cassini ovaloids (Pashkevich, NPA 169 (1971) 275)

P. Möller et al., Nature, 409 (2001) 785



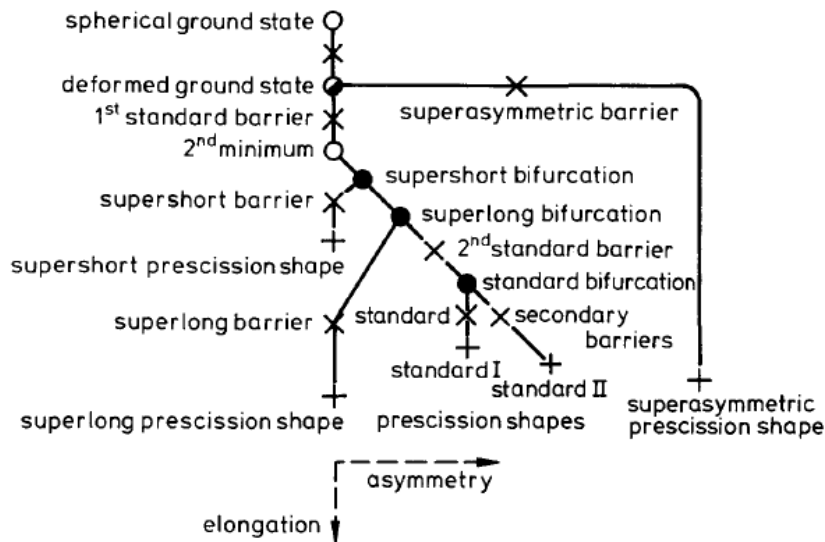
The potential energy surfaces in this talk are projections on two parameters!

Multi-modal random neck rupture

Seminal paper:

U. Brosa, S. Grossmann, A. Müller, Phys. Rep. 197, 167 (1990)

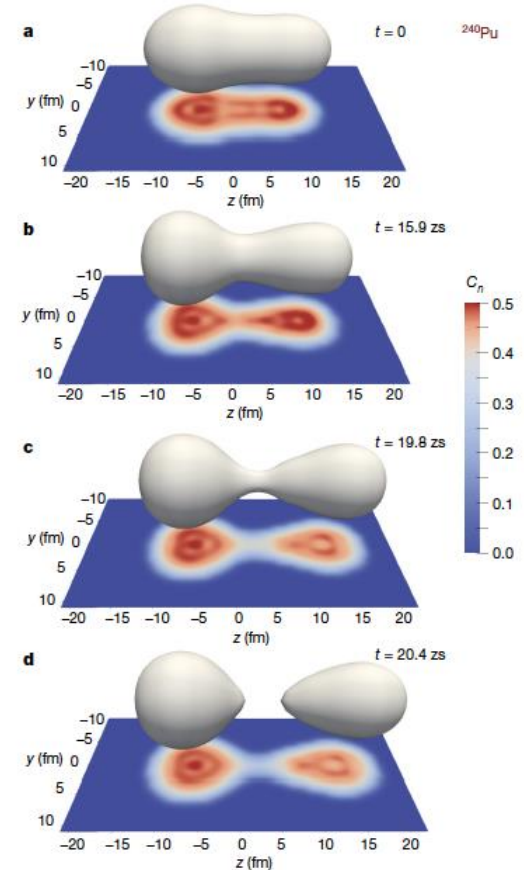
Multiple fission modes:



Random neck rupture:

The **neck** ruptures due to **Rayleigh instability** (known from hydrodynamics).

Influence on fission fragment mass charge and total kinetic energy distributions.



G. Scamps, C. Simenel, Nature 564, 382 (2018)

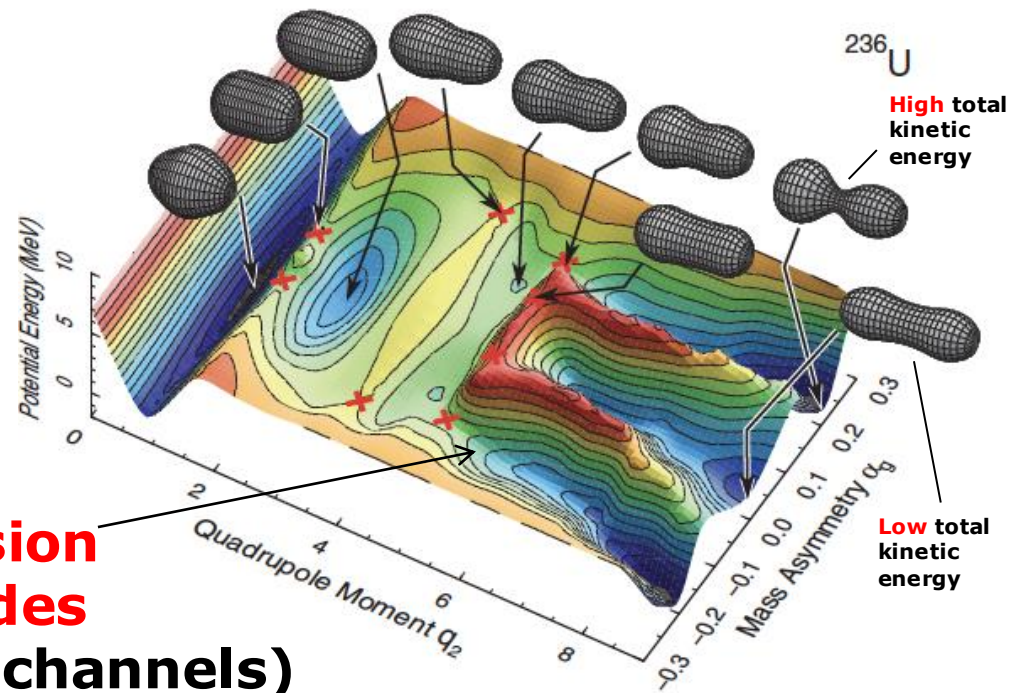
Fission modes

Valleys and passes in the potential energy surface.

- ⇒ **Fission modes** characterized by:
 - ⇒ **asymmetry** and
 - ⇒ **deformation** at scission.
- ⇒ **Experimental signatures:**
 - ⇒ **Fission fragment mass and charge distributions.**
 - ⇒ **Total kinetic energy release.**

⇒ **Goal: Mapping of the potential energy surface with data.**

Fission modes
(or channels)



T. Ichikawa et al., Phys. Rev. C 86, 024610 (2012)

Three asymmetric fission modes: S1, S2 and S3 (S = "standard").
One symmetric fission modes: SL ("superlong").

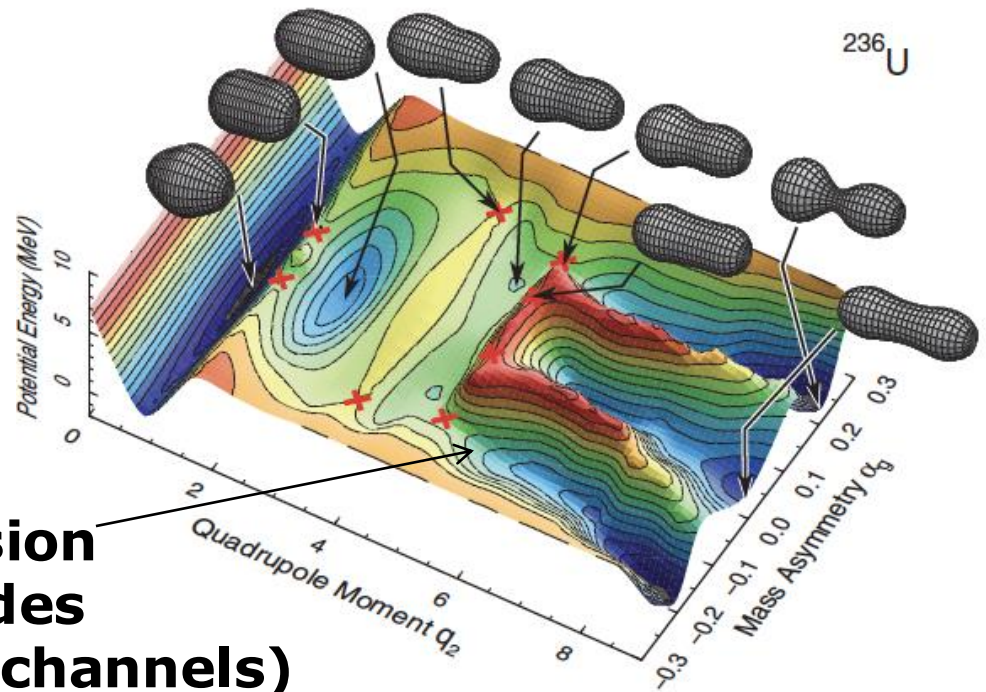
Fission modes

Valleys and passes in the potential energy surface.

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Fission modes (or channels)



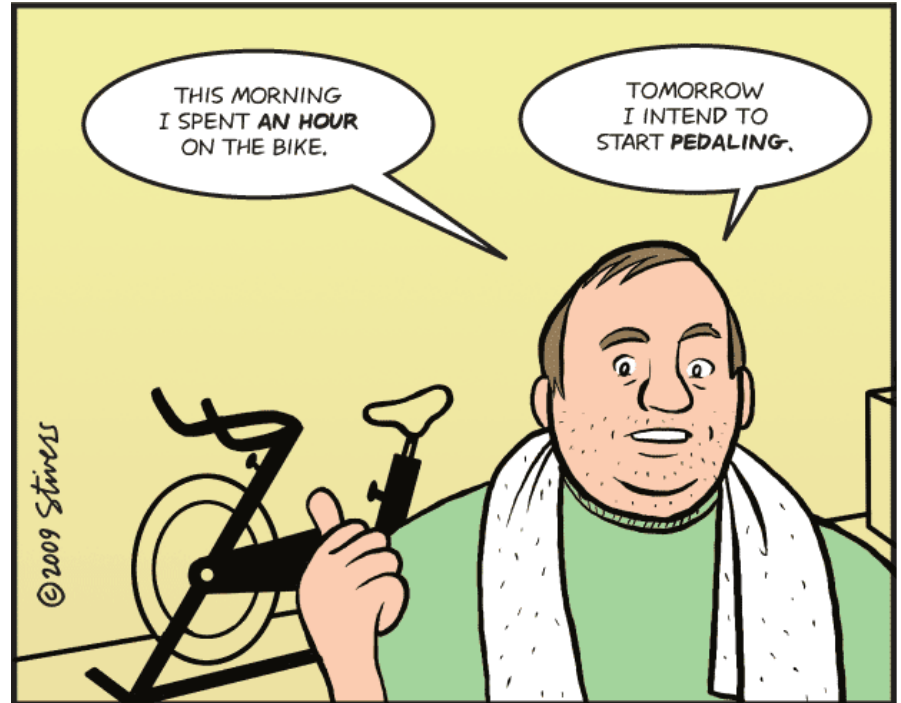
T. Ichikawa et al., Phys. Rev. C 86, 024610 (2012)

The available phase space is the key to understand the fission fragment yields.

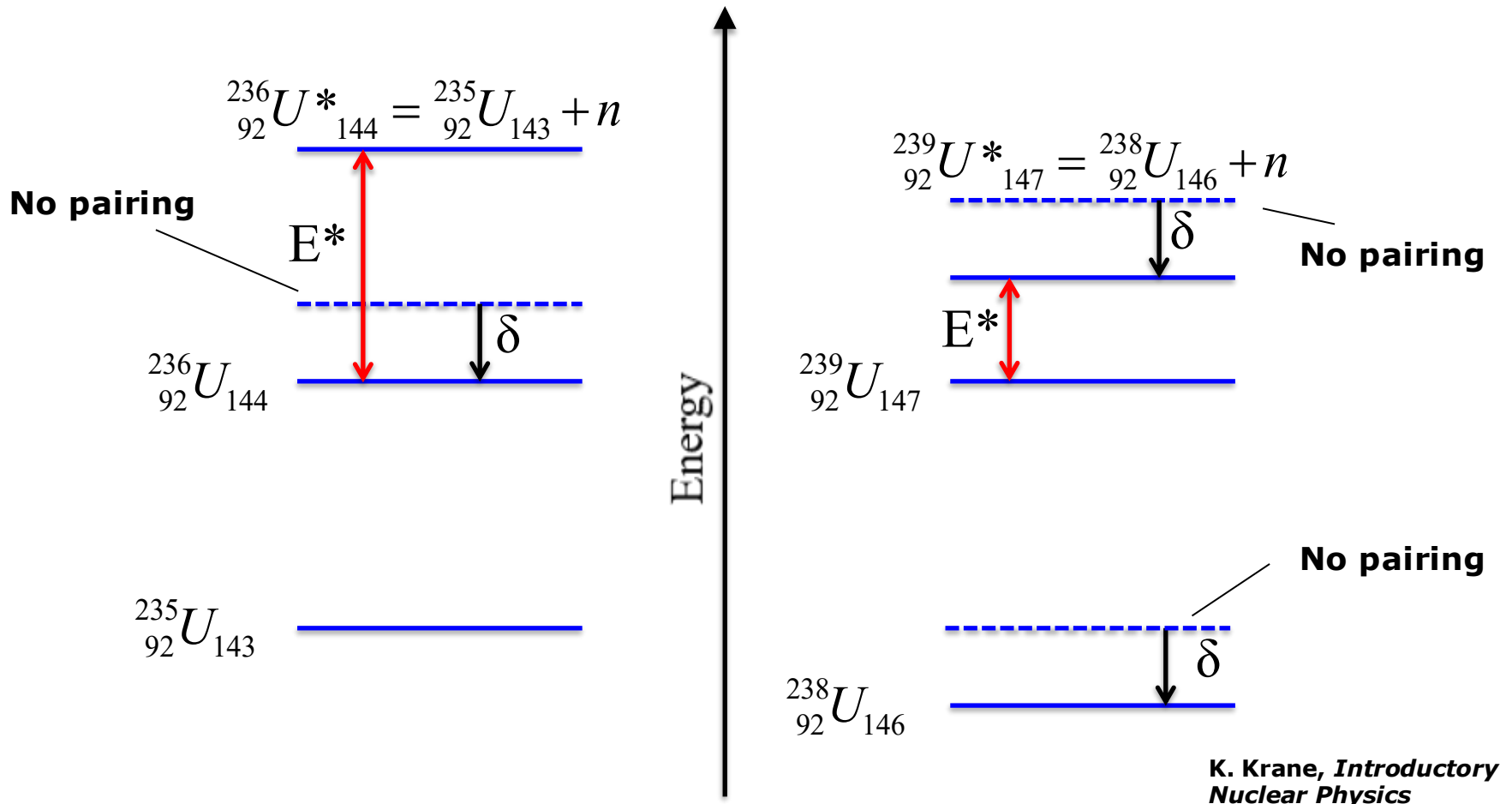
A little exercise

**A nuclear reactor,
based on uranium,
needs enriched ^{235}U
fuel.**

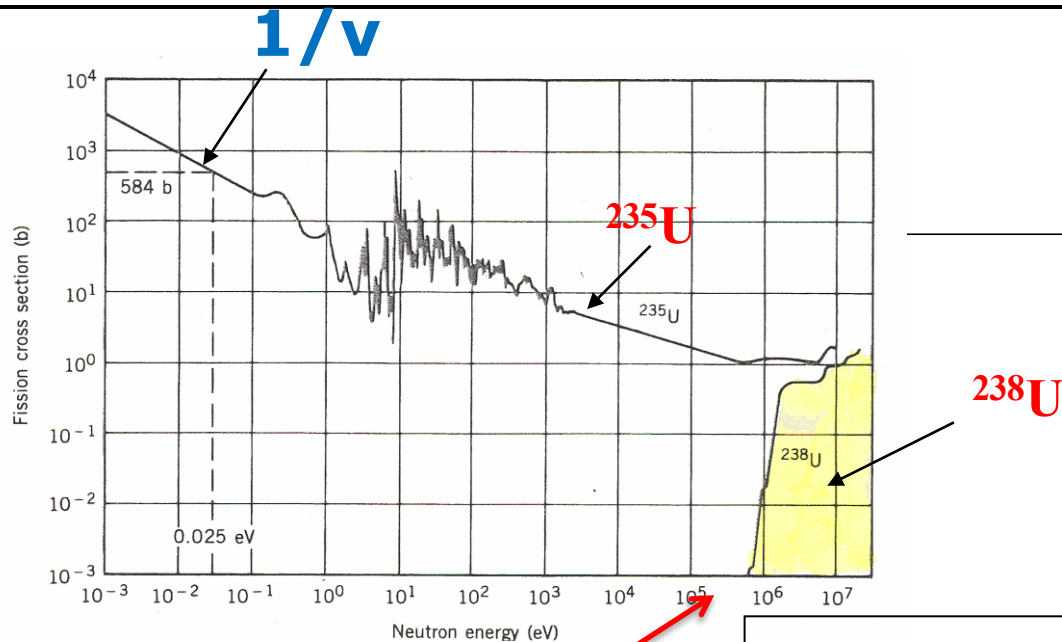
Why?



Fission and pairing



Fission and pairing



Fission Barrier

	^{236}U	^{239}U
Fission barrier	6.2 MeV	6.6 MeV
Excitation energy	6.5 MeV	4.8 MeV

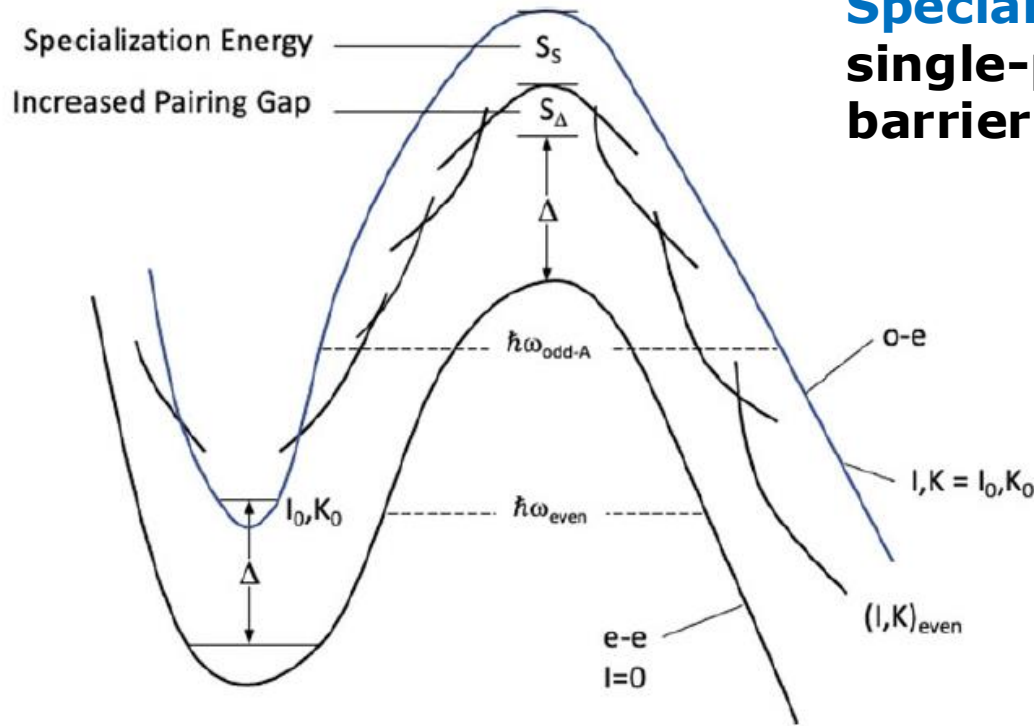
$$E^*(^{236}\text{U}) = [M(^{235}\text{U}) + m_n - M(^{236}\text{U})]c^2$$

$$E^*(^{239}\text{U}) = [M(^{238}\text{U}) + m_n - M(^{239}\text{U})]c^2$$

$$\begin{aligned} \Delta E_x &= 2\delta = 2a_p A^{-3/4} \\ &= 68/238^{3/4} = 1.1 \text{ MeV} \end{aligned}$$

K. Krane, *Introductory Nuclear Physics*

The fission barrier and nuclear structure



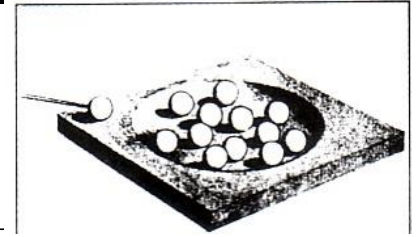
Specialization energy: impact of single-particle effects on the barrier height

Especially ground-state shell effects influence the height of the fission barrier:

-> Stability of super-heavy elements

P. Talou, R. Vogt, *Nuclear Fission*, Springer, 2023

How do we get a nucleus to fission?



N. Bohr, Nature 137 (1936) 351

We **wait**...

→ **Spontaneous fission.**

→ **Tunneling** through the barrier from the ground state.

→ Calculation similar to alpha decay.

We **add energy** (and/or angular momentum) ...

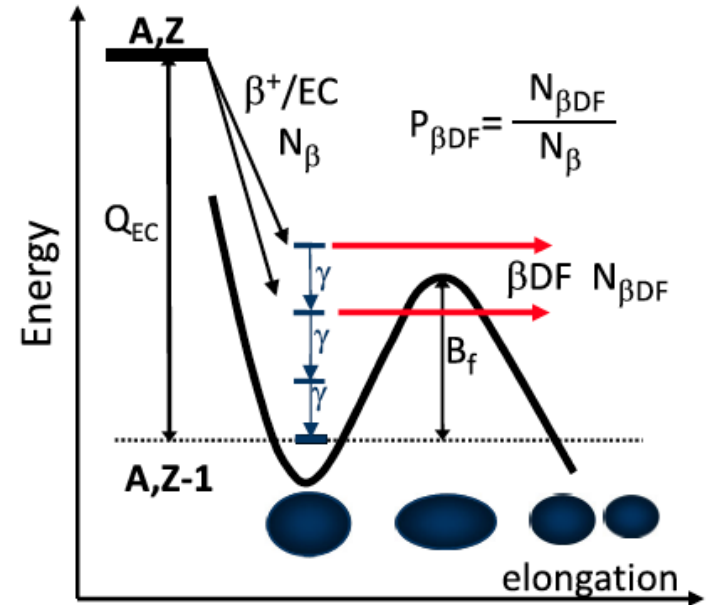
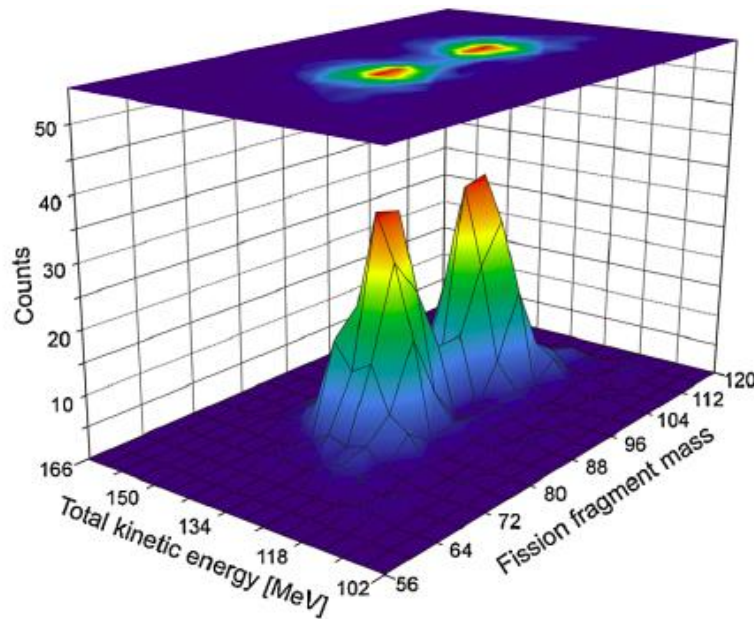
→ **Induced fission.**

→ Select the mechanism depending on your experimental goals

→ Wide range of reactions with **n, p, light charged particles, heavy ions, photons, ...**

...and: β -delayed fission

^{180}Hg



That ^{180}Hg fissions asymmetrically came as a surprise.

A.N. Andreyev et al., **PRL** **105**, 252502 (2010)

A.N. Andreyev et al., **RMP** **85**, 1541 (2013)

What do we measure?

Goal: map the potential energy surface

Observables:

- Fission **probability** as function of the excitation energy.
- Fission **cross section**.
- Fission fragment **mass distributions**.
- **Total kinetic energy (TKE)** released (for a given fission fragment pair).
- **Neutrons** released.
- Fission fragment **charge** distributions.
- Fission fragment **angular** distributions.
- **Gamma-rays** emitted by fission fragments.
- Spontaneous fission half-lives.
- ...

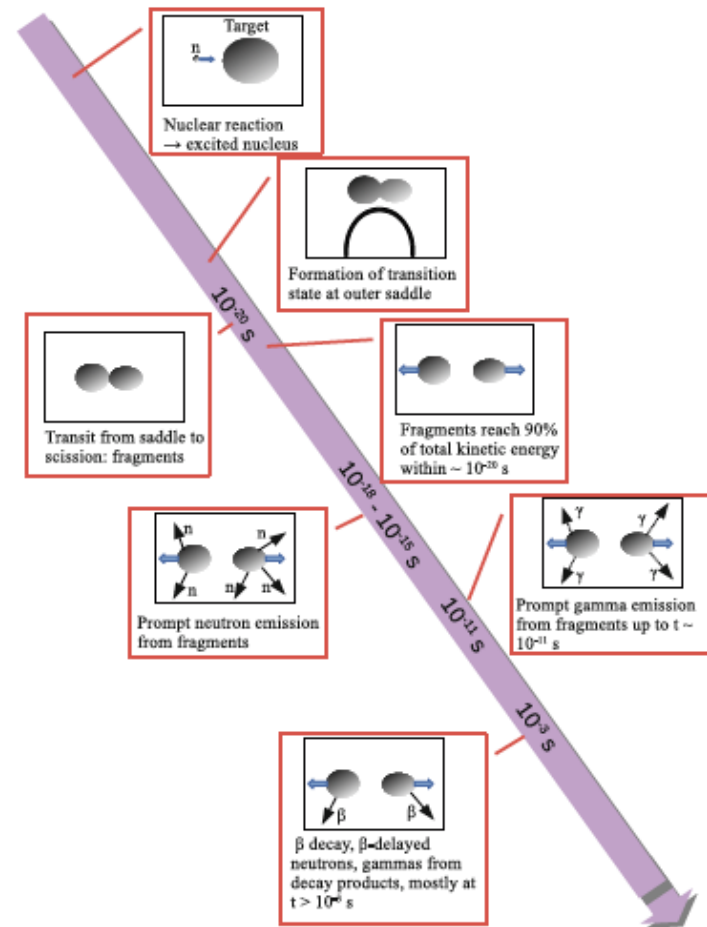
Ideally: **correlations between observables!**

Timescales

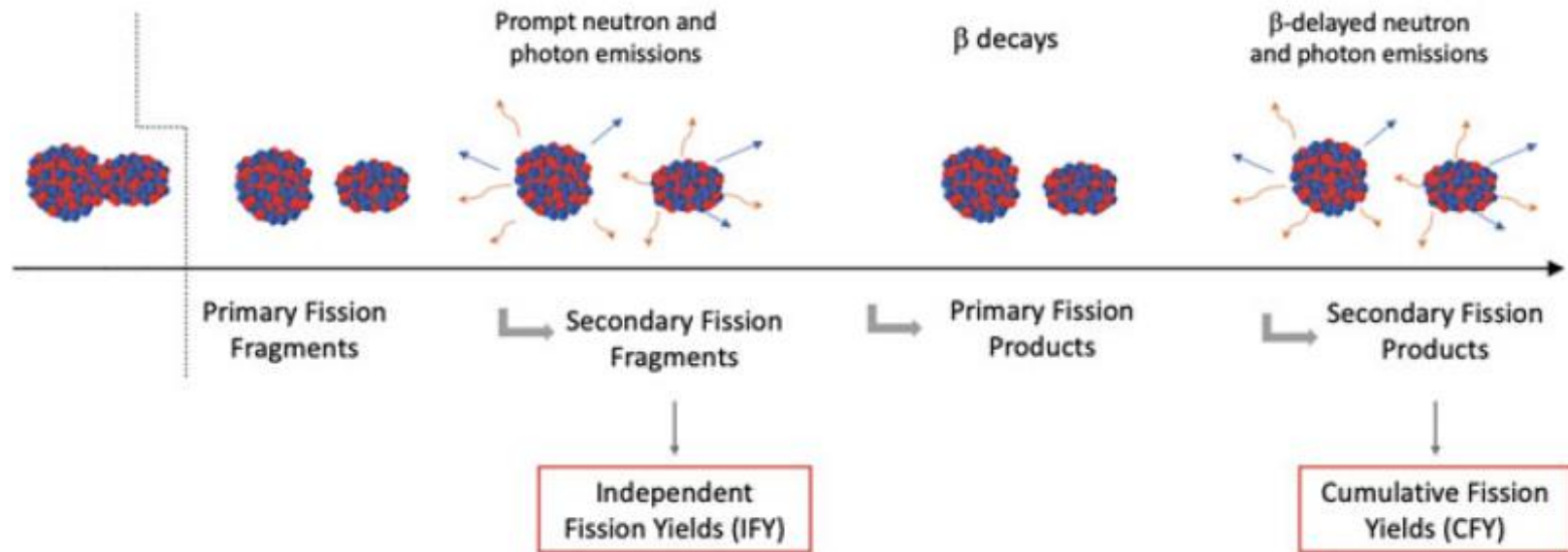
The time it takes to reach the scission point is rather short, but the process does not stop there...

Note that the time fission takes reflects **transport properties of nuclear matter**.

W. Younes, W.D. Loveland, *An Introduction to Nuclear Fission*, Springer (2021)



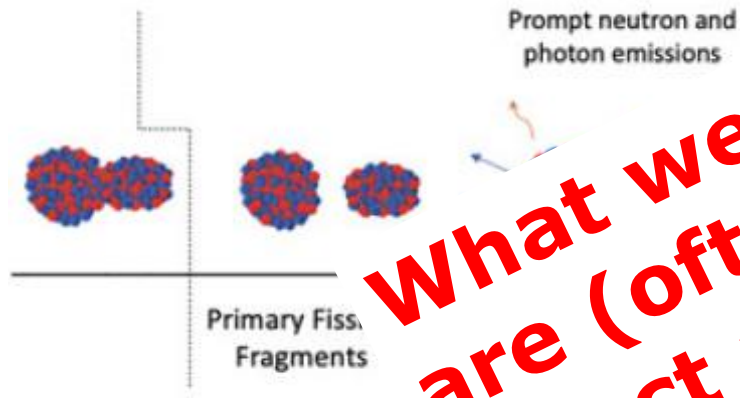
What happens after scission matters



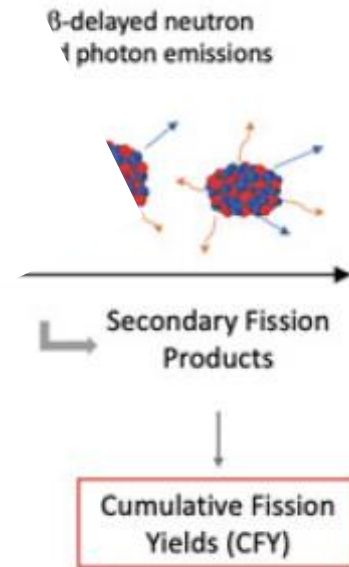
How fission fragments evolve with time:
Fission fragments are (typically) highly excited
→ Neutron evaporation
→ Gamma emission
→ Beta decay (beta-delayed neutrons)

P. Talou, R. Vogt,
Nuclear Fission, Springer (2023)

What happens after scission matters



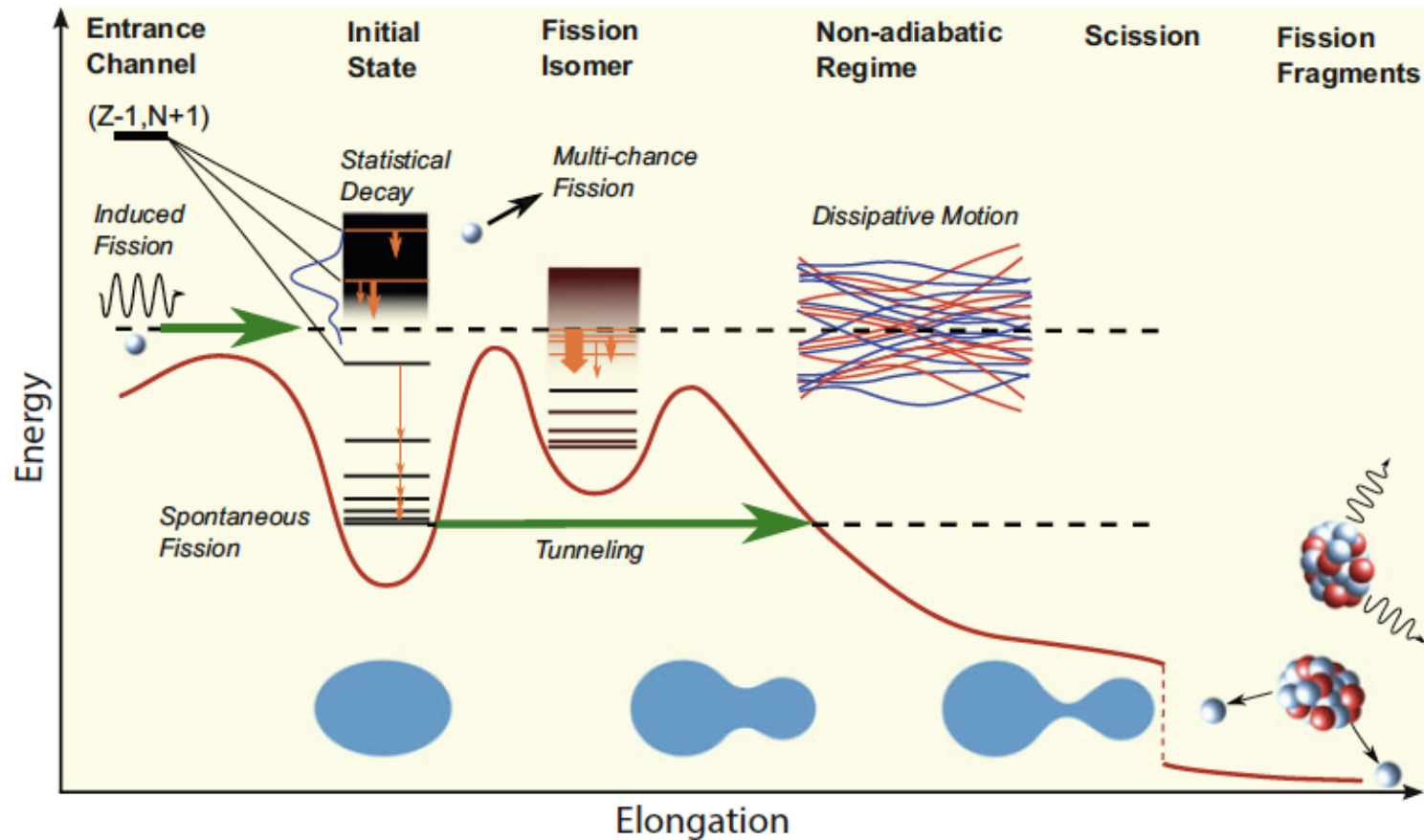
What we measure are (often) not the direct reaction products!



How **fission fragments** evolve with time:
Fission fragments are (typically) highly excited
→ Neutron evaporation
→ Gamma emission
→ Beta decay (beta-delayed neutrons)

P. Talou, R. Vogt,
Nuclear Fission, Springer (2023)

An overview



N. Schunck, *Microscopic Theory of Nuclear Fission* in *Handbook of Nuclear Physics*, Springer Nature Singapore (2023)