

Hadron Structure

UK Nuclear Physics Summer School

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Hadron Structure

Introduction

The strong interaction

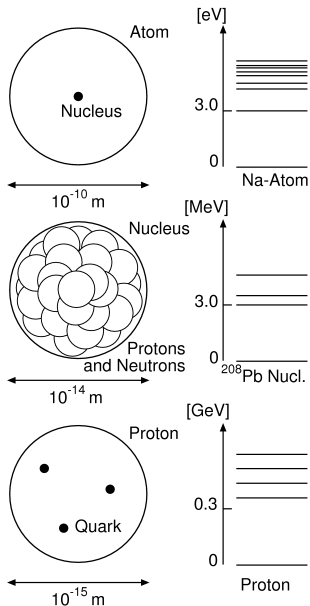
Quantum field theory
Quantum chromodynamics

Electron scattering

Kinematics
Elastic scattering
DIS

Hadron femtography

Form factors
Inclusive meson structure
Deeply virtual reactions



Hadron structure involves studying quarks and gluons at sub-fm and multi-GeV scales.



Hadron Structure

Introduction

The strong interaction

Quantum field theory
Quantum chromodynamics

Electron scattering

Kinematics
Elastic scattering
DIS

Hadron femtography

Form factors
Inclusive meson structure
Deeply virtual reactions

Introduction

The strong interaction

Quantum field theory
Quantum chromodynamics

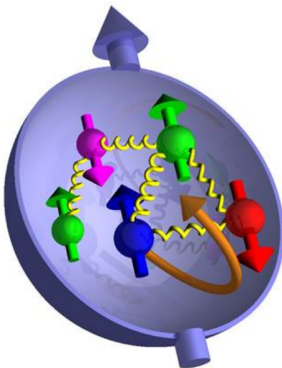
Electron scattering

Kinematics
Elastic scattering
DIS

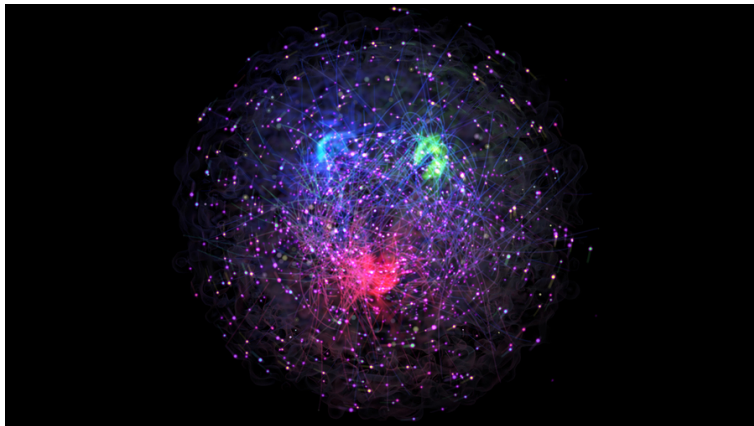
Hadron femtography

Form factors
Inclusive meson structure
Deeply virtual reactions

- The goal of hadron structure research is to understand both the origin and nature of the fundamental properties of hadrons.
- Characterising any physical system requires an answer to the following two questions:
 - What are the relevant degrees of freedom of the system?
 - How do they occupy the available phase space?
- Unlike in most other fields, answering the first question is far from trivial due to the nature of QCD.



Are these the relevant degrees of freedom?



Hadron Structure

Introduction

The strong interaction

- Quantum field theory
- Quantum chromodynamics

Electron scattering

- Kinematics
- Elastic scattering
- DIS

Hadron femtography

- Form factors
- Inclusive meson structure
- Deeply virtual reactions

Or these?



Hadron Structure

Introduction

The strong interaction

- Quantum field theory
- Quantum chromodynamics

Electron scattering

- Kinematics
- Elastic scattering
- DIS

Hadron femtography

- Form factors
- Inclusive meson structure
- Deeply virtual reactions

Or these?



Hadron Structure

Introduction

The strong interaction

- Quantum field theory
- Quantum chromodynamics

Electron scattering

- Kinematics
- Elastic scattering
- DIS

Hadron femtography

- Form factors
- Inclusive meson structure
- Deeply virtual reactions

The strong interaction

Hadron Structure

Introduction

The strong interaction

- Quantum field theory
- Quantum chromodynamics

Electron scattering

- Kinematics
- Elastic scattering
- DIS

Hadron femtography

- Form factors
- Inclusive meson structure
- Deeply virtual reactions

- QCD is the quantum field theory of the strong interaction and is based on *exact $SU(3)$ colour symmetry*.



- Colour interactions between quarks are mediated by gluons and are characterised by three distinct features:
 - confinement*
 - asymptotic freedom*
 - dynamic chiral symmetry breaking*

Classical and Quantum Fields

Our classical picture of interaction in a field is based on the concept of there being a *potential*. When a particle encounters the potential it experiences a force.

QCD is a *relativistic quantum field theory*, meaning that we have to move beyond thinking in terms of potentials.

The element of the field that is quantised (the quanta) is an excitation of the field which represents *a force carrier* (or in the Standard Model, the gauge boson).

Interactions between two particles is then described by exchange of these field quanta, which are themselves particles.

Hadron Structure

Introduction

The strong interaction

Quantum field theory

Quantum chromodynamics

Electron scattering

Kinematics

Elastic scattering

DIS

Hadron femtography

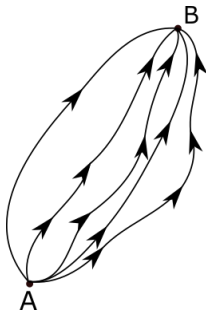
Form factors

Inclusive meson structure

Deeply virtual reactions

Feynman Diagrams

- Now we have a quantum picture of an interaction we need a way of calculating the associated probabilities (eg transition amplitudes).
- In order to calculate the amplitude for a quantum state A changing to B we have to *integrate over all possible paths*.
- Feynman proposed an approach based on:
 - *Feynman diagrams* which give a graphical representation of these complicated path integrals, and
 - *Feynman rules* which relate the features on the diagrams to an expression in the integral.



Hadron Structure

Introduction

The strong interaction

Quantum field theory

Quantum chromodynamics

Electron scattering

Kinematics

Elastic scattering

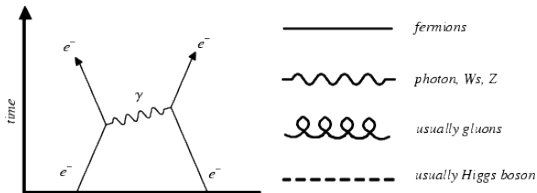
DIS

Hadron femtography

Form factors

Inclusive meson structure

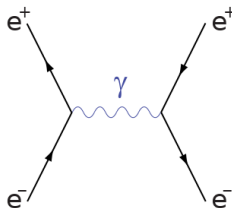
Deeply virtual reactions



- Making use of perturbation theory, Feynman diagrams represent a contribution at a given order to the overall transition amplitude.
- They are space-time diagrams in which:
 - The initial state is (usually) on the left and final state on the right.
 - Each point or vertex represents an interaction.
 - The total number of vertices corresponds to the perturbative order.
 - Fermions are represented by solid lines and move forward in time; anti-fermions move backwards in time.
 - Bosons are represented by wavy or curly lines.

QED

- The simplest quantum field theory is *Quantum Electrodynamics (QED)* which describes the electromagnetic interaction in terms of exchange of virtual photons.
- Each vertex is associated with the coupling strength of the electromagnetic interaction $\sqrt{\alpha}$.
- As more and more complicated diagrams are added they enter at higher orders (eg α^2 , α^3).
- Since $\alpha = 1/137 \ll 1$ the *perturbative series converges* and higher order diagrams do not contribute significantly to transition amplitudes.



Hadron Structure

Introduction

The strong interaction

Quantum field theory

Quantum chromodynamics

Electron scattering

Kinematics

Elastic scattering

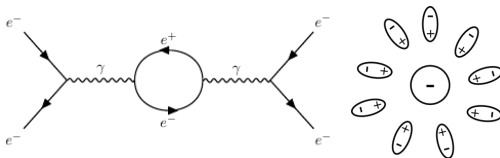
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Hadron femtography

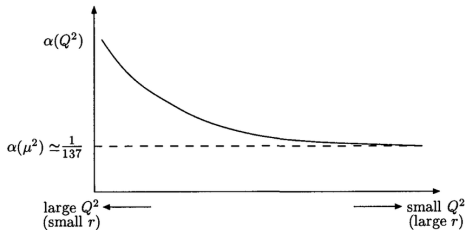
Form factors

Inclusive meson structure

Deeply virtual reactions



- The concept of the vacuum plays a crucial role in quantum field theory.
- The Feynman diagram above shows an example of a fermion loop – a *virtual* electron-positron pair produced (excited) from the vacuum.
- the presence of virtual $e^+ e^-$ pairs leads to the notion of *charge screening* and *running of the coupling constant*.



Hadron Structure

Introduction

The strong interaction

Quantum field theory

Quantum chromodynamics

Electron scattering

Kinematics

Elastic scattering

DIS

Hadron femtography

Form factors

Inclusive meson structure

Deeply virtual reactions

Key Differences between QED and QCD

- Unlike the photon which has $Q = 0$ the gluon carries colour charge.
- Gluons are therefore self-interacting.
- This leads to *colour charge anti-screening*.
- That means that the strong coupling constant α_s runs in the opposite direction to α .
- In other words the strong interaction becomes weaker at high energy (asymptotic freedom).
- Perturbation theory for QCD only works at high energy; at lower energies non-perturbative techniques are needed.

Hadron Structure

Introduction

The strong interaction

Quantum field theory

Quantum chromodynamics

Electron scattering

Kinematics

Elastic scattering

DIS

Hadron femtography

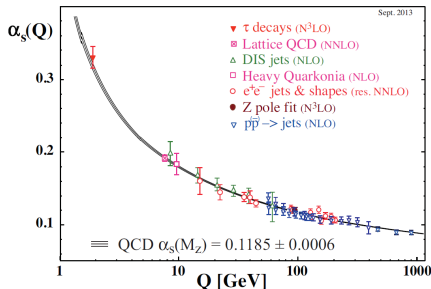
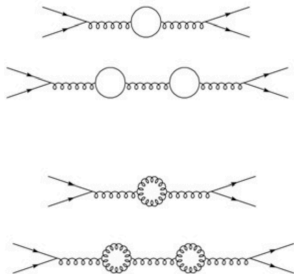
Form factors

Inclusive meson structure

Deeply virtual reactions

QCD: running of α_s

- Each vertex (interaction) between colour charges is associated with *the strong coupling constant, $\alpha_s(Q)$* .
- Q here is the four-momentum transfer between the two colour charges, which can be thought of as the inverse of distance between them.
- Note that the strength of the force does not change – just the effective colour charge.



Quark mass

QUARKS	mass charge → spin →	$\approx 2.3 \text{ MeV}/c^2$ 2/3 1/2 up	$\approx 1.275 \text{ GeV}/c^2$ 2/3 1/2 charm	$\approx 173.07 \text{ GeV}/c^2$ 2/3 1/2 top	0 1 gluon	$\approx 126 \text{ GeV}/c^2$ 0 Higgs boson
		$\approx 4.8 \text{ MeV}/c^2$ -1/3 1/2 down	$\approx 95 \text{ MeV}/c^2$ -1/3 1/2 strange	$\approx 4.18 \text{ GeV}/c^2$ -1/3 1/2 bottom	0 0 photon	
		0.511 MeV/c^2 -1 1/2 electron	105.7 MeV/c^2 -1 1/2 muon	1.777 GeV/c^2 -1 1/2 tau	0 1 Z boson	
		$< 2.2 \text{ eV}/c^2$ 0 1/2 electron neutrino	$< 0.17 \text{ MeV}/c^2$ 0 1/2 muon neutrino	$< 15.5 \text{ MeV}/c^2$ 0 1/2 tau neutrino	$\approx 80.4 \text{ GeV}/c^2$ ± 1 1 W boson	
LEPTONS					GAUGE BOSONS	

- Rest mass of the up quark
 $2.3 \text{ MeV}/c^2$
- Rest mass of the down quark
 $4.8 \text{ MeV}/c^2$
- Rest mass of the proton (uud)
 $938.3 \text{ MeV}/c^2$

Where does the other $\simeq 928 \text{ MeV}/c^2$ come from?

Hadron Structure

Introduction

The strong interaction

Quantum field theory

Quantum chromodynamics

Electron scattering

Kinematics

Elastic scattering

DIS

Hadron femtography

Form factors

Inclusive meson structure

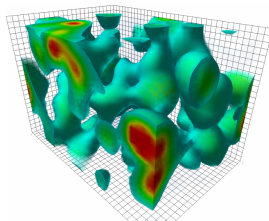
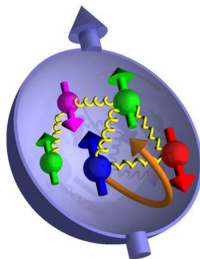
Deeply virtual reactions

Hadrons in QCD

- The QCD Lagrangian density has the form:

$$\begin{aligned}\mathcal{L}_{QCD} = & \bar{\psi}_i (i\gamma^\mu \partial_\mu - m) \psi_i \\ & - g G_\mu^a \bar{\psi}_i \gamma^\mu T_{ij}^a \psi_j \\ & - \frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu}\end{aligned}$$

- A full description of hadrons in QCD has to account for both the valence quarks and *the vacuum (gluons and sea quarks)* – and the interactions between them.



Hadron Structure

Introduction

The strong interaction

Quantum field theory

Quantum chromodynamics

Electron scattering

Kinematics

Elastic scattering

DIS

Hadron femtography

Form factors

Inclusive meson structure

Deeply virtual reactions

Much of what we have learned of QCD and hadrons has been at very high energies, in the so-called asymptotic regime.

However to understand the constituents of the atomic nucleus we need to look at lower energies, where *the relevant degrees of freedom* are different.

C.D. Roberts arxiv:nucl-th 12035341

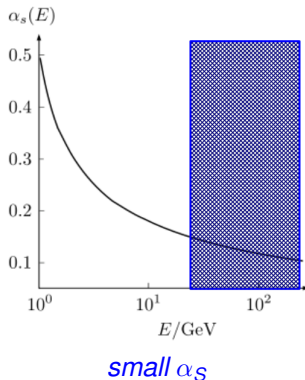
“The struggle with QCD defines the difference between nuclear and particle physics: nuclear physicists try to solve this theory, whereas particle physicists escape to a place where perturbation theory, the last refuge of a scoundrel, is all that is necessary.”

Asymptotic freedom

- The QCD Lagrangian density has the form:

$$\begin{aligned}\mathcal{L}_{QCD} = & \bar{\psi}_i (i\gamma^\mu \partial_\mu - m) \psi_i \\ & - g G_\mu^a \bar{\psi}_i \gamma^\mu T_{ij}^a \psi_j \\ & - \frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu}\end{aligned}$$

- The coupling *between the quarks and gluons at high energies (small distance scales) becomes weak*, meaning perturbation theory can be used to solve QCD.

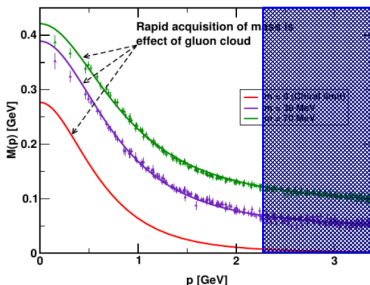


Asymptotic freedom

- The QCD Lagrangian density has the form:

$$\begin{aligned}\mathcal{L}_{QCD} = & \bar{\psi}_i (i\gamma^\mu \partial_\mu - m) \psi_i \\ & - g G_\mu^a \bar{\psi}_i \gamma^\mu T_{ij}^a \psi_j \\ & - \frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu}\end{aligned}$$

- The relevant degrees of freedom in this asymptotic regime are *bare (or current) quarks with masses close to their (Higgs-generated) rest mass*.



low quark mass

Hadron Structure

Introduction

The strong interaction

Quantum field theory

Quantum chromodynamics

Electron scattering

Kinematics

Elastic scattering

DIS

Hadron femtography

Form factors

Inclusive meson structure

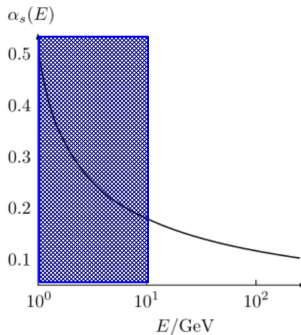
Deeply virtual reactions

Confinement

- The QCD Lagrangian density has the form:

$$\begin{aligned}\mathcal{L}_{QCD} = & \bar{\psi}_i (i\gamma^\mu \partial_\mu - m) \psi_i \\ & - g G_\mu^a \bar{\psi}_i \gamma^\mu T_{ij}^a \psi_j \\ & - \frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu}\end{aligned}$$

- The *coupling between the quarks and gluons at low energies (large distance scales) becomes very strong*, leading to the phenomenon of colour confinement.
- Confinement is inherently a non-perturbative feature of QCD.*

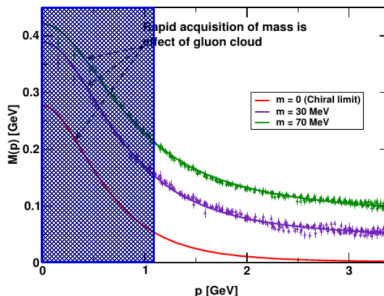


Dynamical chiral symmetry breaking

- The QCD Lagrangian density has the form:

$$\begin{aligned}\mathcal{L}_{QCD} = & \bar{\psi}_i (i\gamma^\mu \partial_\mu - m) \psi_i \\ & - g G_\mu^a \bar{\psi}_i \gamma^\mu T_{ij}^a \psi_j \\ & - \frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu}\end{aligned}$$

- As the quarks inside hadrons move apart, they *acquire mass from the vacuum* as a result of the interaction between them becoming very strong.
- This means the relevant degrees of freedom at low energies become *dressed (or constituent) quarks*.



Large quark mass

*(this is where the other
928 MeV/c² in the proton
comes from)*

- As noted, understanding the non-perturbative features of QCD is not possible with analytical methods. Instead, a numerical technique is what is needed: *lattice QCD*.
- Wilson (1974) suggested the introduction of a 4D space-time lattice or grid, with quark fields at the grid points and gluon fields linking them.
- The integration over all possible paths between two states is then performed by means of *Monte Carlo integration*.
- The closer these path integrals are calculated to the real world (by extrapolating to zero lattice spacing and physical quark masses), the more accurate the predictions.

Introduction

The strong interaction

Quantum field theory

Quantum chromodynamics

Electron scattering

Kinematics

Elastic scattering

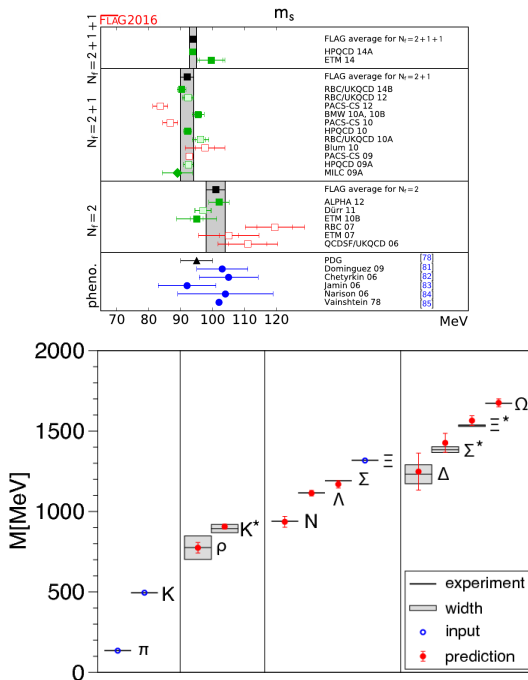
DIS

Hadron femtography

Form factors

Inclusive meson structure

Deeply virtual reactions



Electron scattering

Hadron Structure

Introduction

The strong interaction

Quantum field theory

Quantum chromodynamics

Electron scattering

Kinematics

Elastic scattering

DIS

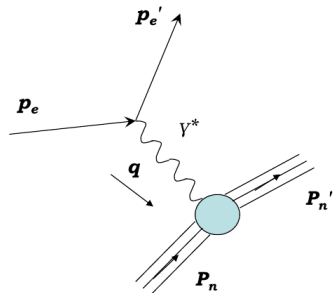
Hadron femtography

Form factors

Inclusive meson structure

Deeply virtual reactions

Electron scattering



Particle four-momenta in natural units ($\hbar = c = 1$):

$$P_e = k = (E, \vec{k})$$

$$P_e' = k' = (E', \vec{k}')$$

$$P_N = (E_N, \vec{p}_N) = (M, \vec{0})$$

$$P_N' = (E_N', \vec{p}_N')$$

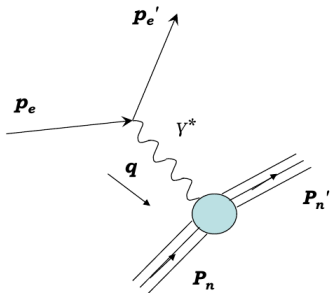
We can further define two Lorentz invariant quantities:

- The *energy transfer* associated with the virtual photon:

$$\nu = \frac{P_N \cdot q}{M} \stackrel{\text{lab}}{=} E - E'$$

- The *four-momentum transfer* associated with the virtual photon:

$$q = k - k', \quad q = (\nu, \vec{q})$$



We assume that electron scattering from a nucleon target at high energy proceeds via exchange of a virtual photon (γ^*), which transfers energy, momentum (and spin).

Recalling that the dot product of a four-momentum is just the *invariant mass* (virtuality), the *squared four-momentum transfer* for fixed target experiments is then:

$$q^2 = \mathbf{q} \cdot \mathbf{q} \leq 0, \quad Q_{\text{space-like}}^2 = -q^2.$$

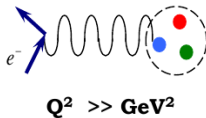
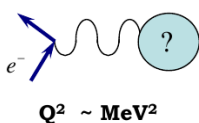
It can be calculated in the lab frame in terms of the kinematic quantities

$$Q^2 = 4EE' \sin^2 \frac{\theta}{2},$$

where θ is the electron scattering angle.

Q^2 and resolution scale

We associate Q^2 with the resolution of the virtual photon (electromagnetic probe).



- Higher Q^2 means a more virtual photon (larger mass).
- Higher virtuality means a smaller wavelength:

$$\lambda_{\gamma^*} \propto \frac{1}{\sqrt{Q^2}}.$$

- Studying hadron structure at *short distance scales* therefore requires large four-momentum transfer.

Hadron Structure

Introduction

The strong interaction

Quantum field theory
Quantum chromodynamics

Electron scattering

Kinematics

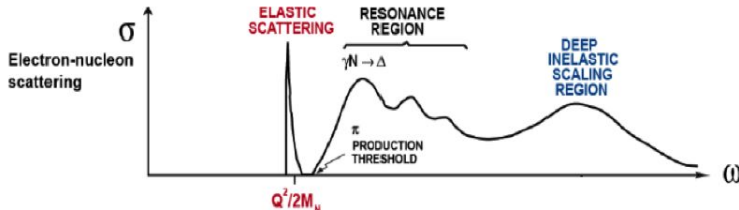
Elastic scattering
DIS

Hadron femtography

Form factors
Inclusive meson structure
Deeply virtual reactions

Scattering experiments

There are three distinct types of lepton scattering experiments.



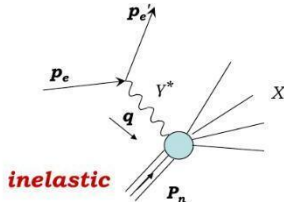
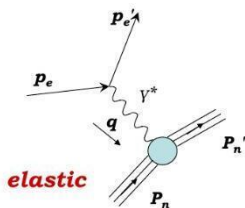
- **Exclusive**: detect all particles in the final state. The most common exclusive reaction is *elastic scattering*.
- **Inclusive**: detect only the scattered lepton. The most common inclusive reaction is *deep inelastic scattering*.
- **Semi-inclusive**: detect the scattered lepton and at least one hadron in the final state (eg *SIDIS*).

Final-state invariant mass

Kinematics can be used to characterise scattering types, specifically the *invariant mass of the hadronic final state*:

$$\begin{aligned}W^2 &= (\mathbf{P}_N + \mathbf{q})^2 \\&= M^2 + q^2 + 2\mathbf{P}_N \cdot \mathbf{q} \\&= M^2 + 2M\nu - Q^2.\end{aligned}$$

The *deep inelastic* region is characterised by a multi-hadron final state with $W_X^2 \gg M^2$.



Hadron Structure

Introduction

The strong interaction

- Quantum field theory
- Quantum chromodynamics

Electron scattering

- Kinematics

Elastic scattering

- DIS

Hadron femtography

- Form factors
- Inclusive meson structure
- Deeply virtual reactions

Whereas, by definition elastic scattering requires $W = M$:

$$\nu = \frac{Q^2}{2M},$$

which implies that the scattered energy and scattering angle in the lab frame are related and only one independent variable is needed:

$$E' = \frac{E}{1 + \frac{2E}{M} \sin^2 \frac{\theta}{2}}.$$

Rutherford Scattering

Consider elastic electron scattering in the non-relativistic limit:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{16E_K^2 \sin^4 \theta/2},$$

where

$$\alpha = \frac{e^2}{4\pi} \quad \text{Lorentz-Heaviside units } (\epsilon_0 = \mu_0 = 1).$$

This is just the *Rutherford cross section* for scattering in an electrostatic field:

- Born approximation
- no recoil of target
- scattering and target particle are spin 0
 - *no magnetic interaction*
- point-like beam and target particles

Hadron Structure

Introduction

The strong interaction

Quantum field theory
Quantum chromodynamics

Electron scattering

Kinematics

Elastic scattering

DIS

Hadron femtography

Form factors
Inclusive meson structure
Deeply virtual reactions

Mott Scattering

The relativistic form for scattering of two fermions (spin-1/2) including target recoil is the *Mott cross section*:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E^2 \sin^4 \theta/2} \left(\frac{E'}{E} \right) \left(\cos^2 \frac{\theta}{2} + 2\tau \sin^2 \frac{\theta}{2} \right),$$

where

$$\tau = \frac{Q^2}{4M^2}.$$

The only element that is missing in the above expressions is a term for the extended structure (or form) of the target nucleon, $F(\vec{q}^2)$, such that:

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega} \right)_{\text{Mott}} |F(\vec{q}^2)|^2$$

Hadron Structure

Introduction

The strong interaction

- Quantum field theory
- Quantum chromodynamics

Electron scattering

- Kinematics

Elastic scattering

- DIS

Hadron femtography

- Form factors
- Inclusive meson structure
- Deeply virtual reactions

Rosenbluth Formula

By introducing two new nucleon form factors, we arrive at the *Rosenbluth cross section*

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Rosenbluth}} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \left(\frac{G_E^2 + \tau G_M^2}{(1 + \tau)} + 2\tau G_M^2 \tan^2 \frac{\theta}{2} \right),$$

where

- $G_E(Q^2)$ is the electric form factor and
- $G_M(Q^2)$ is the magnetic form factor of the nucleon

These *Sachs form factors* represent the *distribution of charge and magnetic moment in the nucleon*.

Hadron Structure

Introduction

The strong interaction

Quantum field theory
Quantum chromodynamics

Electron scattering

Kinematics

Elastic scattering

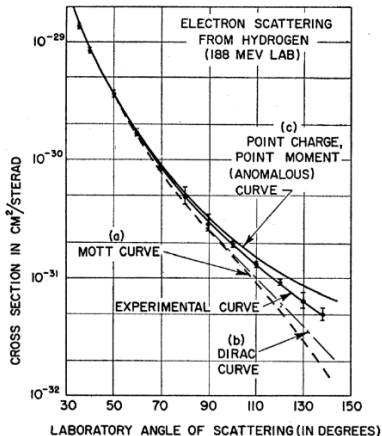
DIS

Hadron femtography

Form factors
Inclusive meson structure
Deeply virtual reactions

Early data

- SLAC 1956: scattering of 188 MeV electrons from a proton target.
- Clear deviation from the Mott curve observed at large scattering angle.
- This was the first evidence for the nucleon's extended structure.
- Hofstadter was awarded the Nobel prize for this discovery.



Hadron Structure

Introduction

The strong interaction

Quantum field theory
Quantum chromodynamics

Electron scattering

Kinematics

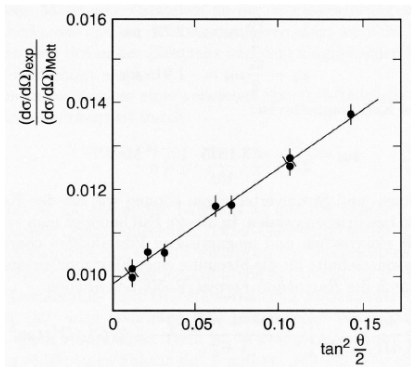
Elastic scattering

DIS

Hadron femtography

Form factors
Inclusive meson structure
Deeply virtual reactions

Rosenbluth separation



Extract G_E and G_M by measuring cross section and separating the respective contributions.

$$\frac{d\sigma/d\Omega}{(d\sigma/d\Omega)_{\text{Mott}}} = 2\tau G_M^2 \left(\tan^2 \frac{\theta}{2} \right) + \frac{G_E^2 + \tau G_M^2}{1 + \tau}.$$

Hadron Structure

Introduction

The strong interaction

- Quantum field theory
- Quantum chromodynamics

Electron scattering

- Kinematics

Elastic scattering

- DIS

Hadron femtography

- Form factors
- Inclusive meson structure
- Deeply virtual reactions

Physical Interpretation

Recall that $F(\vec{q}^2) = \int \rho(\vec{r}) e^{i\vec{q}\cdot\vec{r}} d^3\vec{r}$

$\rho(r)$	$ F(q^2) $	Example
pointlike	constant	Electron
exponential	dipole	Proton
gauss	gauss	${}^6\text{Li}$
homogeneous sphere	oscillating	-#
sphere with diffuse surface	oscillating	${}^{40}\text{Ca}$

$r \longrightarrow$
 $|q| \longrightarrow$

Hadron Structure

Introduction

The strong interaction

Quantum field theory
Quantum chromodynamics

Electron scattering

Kinematics

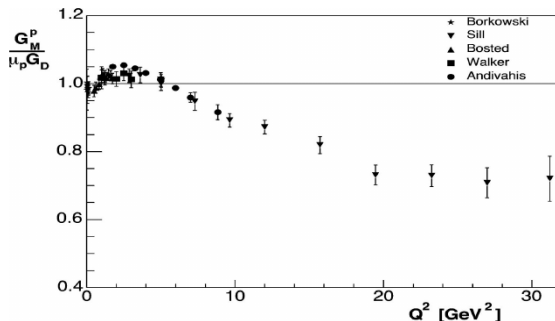
Elastic scattering

DIS

Hadron femtography

Form factors
Inclusive meson structure
Deeply virtual reactions

$$\begin{aligned}
 G_E^p(0) &= 1 & G_E^n(0) &= 0 \\
 G_M^p(0) &= 2.79 & G_M^n(0) &= -1.91
 \end{aligned}$$



Good agreement with a dipole fit up to several $(\text{GeV}/c)^2$:

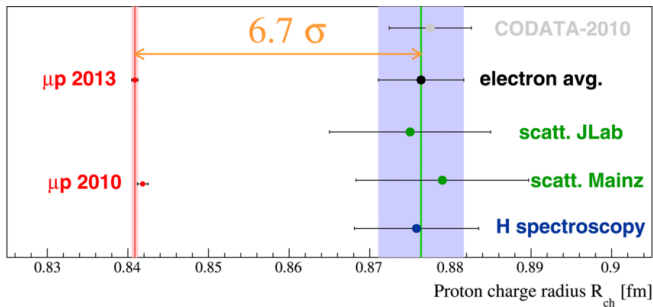
$$G_D(Q^2) = \frac{1}{(1 + Q^2/0.71)^2}$$

Low Q^2 : charge density and radius

Extrapolation of low Q^2 form factor measurements to $Q^2 = 0$ allow for the extraction of the *mean-square nucleon radius*:

$$\langle r^2 \rangle = \int d^3r r^2 \rho(r)$$

$$\langle r^2 \rangle = 6\hbar^2 \left[\frac{dG(Q^2)}{dQ^2} \right]_{Q^2=0}$$



Hadron Structure

Introduction

The strong interaction

Quantum field theory

Quantum chromodynamics

Electron scattering

Kinematics

Elastic scattering

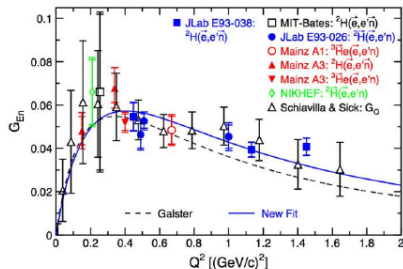
DIS

Hadron femtography

Form factors

Inclusive meson structure

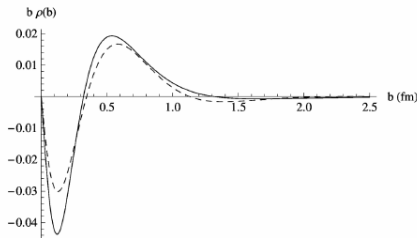
Deeply virtual reactions



G_E of the **neutron**

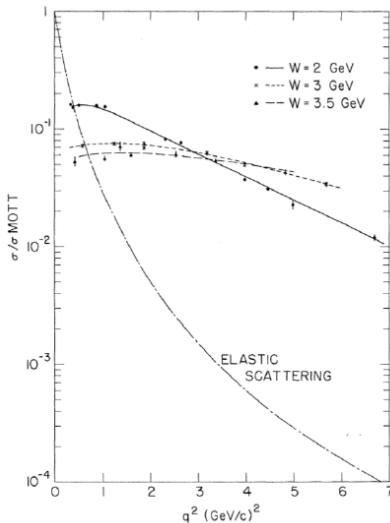
Charge radius:

Neutrons appear
to be negative at
the core,
positive around
the edges!



Exclusive vs inclusive cross section

- Experiments involving elastic scattering become increasingly difficult at higher Q^2 .
- This is mostly due to a *rapid decrease in cross section*.
- The *inclusive* $W > 2$ GeV data on the plot exhibit a *very different Q^2 evolution*.



Hadron Structure

Introduction

The strong interaction

Quantum field theory
Quantum chromodynamics

Electron scattering

Kinematics

Elastic scattering

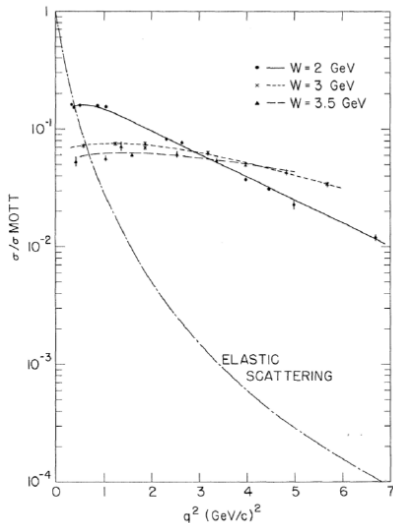
DIS

Hadron femtography

Form factors
Inclusive meson structure
Deeply virtual reactions

Deep inelastic scattering

- SLAC, 1968:
inelastic scattering
cross section is
roughly independent
of Q^2 .
- Evidence for
scattering from
point-like structure
within the nucleon
(partons).
- Nobel prize awarded
to Friedman, Kendall
and Taylor.



Hadron Structure

Introduction

The strong interaction

Quantum field theory
Quantum chromodynamics

Electron scattering

Kinematics
Elastic scattering

DIS

Hadron femtography

Form factors
Inclusive meson structure
Deeply virtual reactions

Inclusive cross section

In inelastic scattering, the scattered electron energy (E') and angle (θ) are independent, which means we need to re-evaluate the cross section:

$$\frac{d\sigma}{dE' d\Omega} = \frac{\alpha^2}{4E^2 \sin^4 \theta/2} \left[\frac{1}{\nu} F_2(x, Q^2) \cos^2 \frac{\theta}{2} + \frac{2}{M} F_1(x, Q^2) \sin^2 \frac{\theta}{2} \right]$$

where we have introduced a new variable:

$$x = \frac{Q^2}{2\mathbf{P}_N \cdot \mathbf{q}} \stackrel{\text{lab}}{=} \frac{Q^2}{2M\nu}.$$

Note that the form factors for elastic scattering have been replaced with *inclusive structure functions, F_1 and F_2* .

Measuring the cross section over a range of energies and angles allows for extraction of the structure functions as a function of x_B and Q^2 .

Hadron Structure

Introduction

The strong interaction

- Quantum field theory
- Quantum chromodynamics

Electron scattering

- Kinematics
- Elastic scattering

DIS

Hadron femtography

- Form factors
- Inclusive meson structure
- Deeply virtual reactions

Parton momentum

We normally think of DIS in the *infinite momentum frame*:
i.e. we assume that $|\vec{p}_N| \rightarrow \infty$ along the beam direction.

In this frame, a parton will carry momentum $x\vec{p}_N$ where
 $0 < x \leq 1$ is the *longitudinal momentum fraction* carried by the
parton.

If DIS proceeds via (quasi-) elastic scattering from a parton:

$$\begin{aligned} W^2 &= (x\mathbf{P}_N + \mathbf{q})^2 = x^2 M^2 + q^2 + 2x\mathbf{P}_N \cdot \mathbf{q} \\ &= m_q^2 \approx 0. \end{aligned}$$

Since $x^2 M^2 \ll q^2$, the expression can be rearranged:

$$x = \frac{Q^2}{2\mathbf{P}_N \cdot \mathbf{q}} = x_B.$$

In other words, the *fraction of the nucleon's momentum carried by the struck quark is Bjorken- x* .

Hadron Structure

Introduction

The strong interaction

- Quantum field theory
- Quantum chromodynamics

Electron scattering

- Kinematics
- Elastic scattering

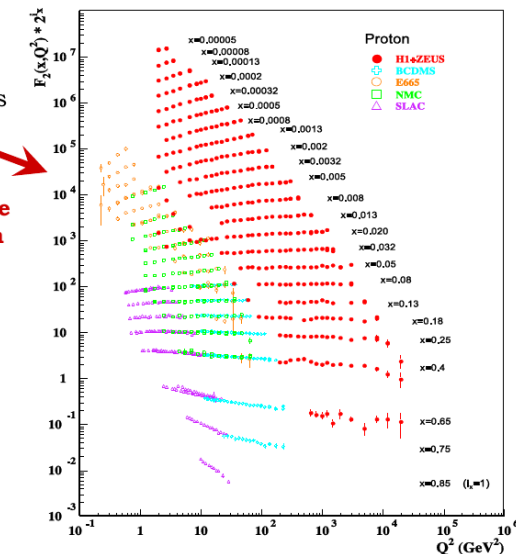
DIS

Hadron femtography

- Form factors
- Inclusive meson structure
- Deeply virtual reactions

Experimentally,
 observe scaling at
 much lower values
 of Q^2 and ν than
 the Bjorken limit

**Point-like structure
 inside the nucleon
 evident!**



James Bjorken
 (b. 1934)

Parton momentum distributions

It is now clear that DIS allows access to the *distribution of quark momentum* within the nucleon. If we consider the flavour dependence of the structure functions:

$$F_2(x) = x \sum_i e_i^2 f_i(x),$$

where e_i is the electric charge of the i th quark and $f_i(x)$ is its *parton distribution function*.

Consider the proton (uud):

$$\frac{F_{2p}(x)}{x} = \frac{4}{9} [u(x) + \bar{u}(x)] + \frac{1}{9} [d(x) + \bar{d}(x)] + \frac{1}{9} [s(x) + \bar{s}(x)] .$$

Measuring the structure functions as a function of x will give access to the contribution to the nucleon momentum from *valence and sea quarks* of different flavours.

Hadron Structure

Introduction

The strong interaction

- Quantum field theory
- Quantum chromodynamics

Electron scattering

- Kinematics
- Elastic scattering

DIS

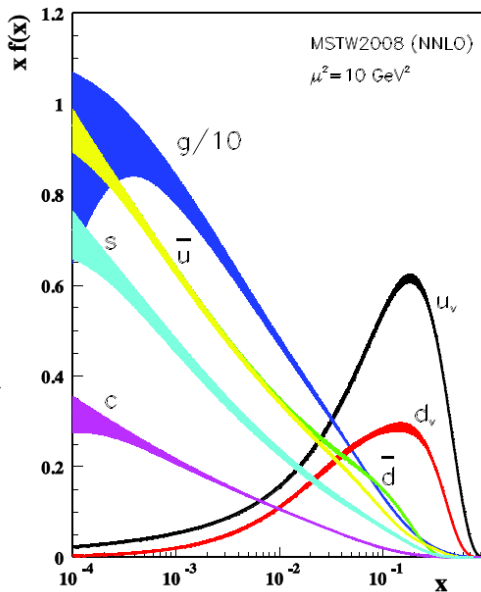
Hadron femtography

- Form factors
- Inclusive meson structure
- Deeply virtual reactions

Proton PDFs

In the proton, PDFs have smeared peak at $x \sim 0.25$ for the valence quarks (uud).

Probing partons with very low fractions of nucleon momentum reveals the quark/anti-quark and gluon sea.



Hadron Structure

Introduction

The strong interaction

- Quantum field theory
- Quantum chromodynamics

Electron scattering

- Kinematics
- Elastic scattering

DIS

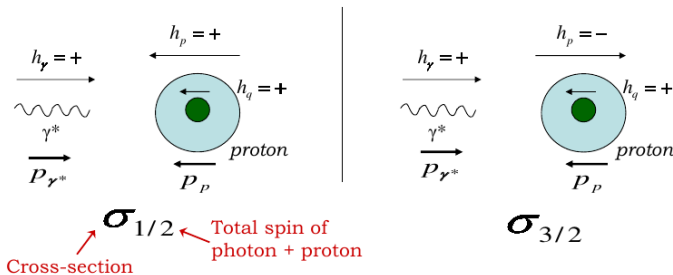
Hadron femtography

- Form factors
- Inclusive meson structure
- Deeply virtual reactions

Spin dependent structure functions

★ Helicity – projection of a particle's spin onto its momentum vector:
$$h \equiv \frac{\vec{s} \cdot \vec{p}}{|\vec{p}|}$$

★ Infinite momentum Breit Frame (where parton has equal energy before and after absorbing photon – elastic scattering):



★ The photon is absorbed by the parton in the scattering. Conservation laws allow only two different configurations of photon - nucleon - parton spin.

Hadron Structure

Introduction

The strong interaction

Quantum field theory
Quantum chromodynamics

Electron scattering

Kinematics
Elastic scattering

DIS

Hadron femtography

Form factors
Inclusive meson structure
Deeply virtual reactions

It is obvious that the spin-averaged structure functions relate to the sum of helicity-dependent cross sections:

$$F_{1,2}(x) \propto \sigma_{1/2} + \sigma_{3/2}.$$

We can now introduce new *spin dependent structure functions*:

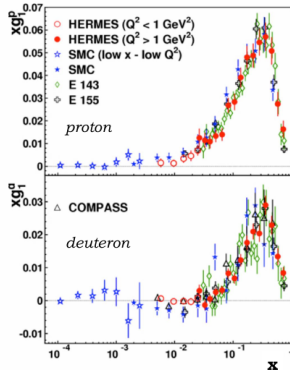
$$g_1(x) \propto \sigma_{1/2} - \sigma_{3/2} = \frac{1}{2} \sum_i e_i^2 \Delta f_i(x),$$

where $\Delta f_i(x) = f_i^\uparrow(x) - f_i^\downarrow(x)$ are the flavour dependent *polarised parton distribution function*.

Similarly, we can define the g_2 structure function in terms of the correlation between beam helicity and transverse nucleon spin.

Much has been learned from DIS over the past few decades but the picture is still incomplete:

- Valence quarks carry less than 40% of the nucleon momentum; sea quarks carry around 20%.
- Quark spin only accounts for around 1/3 of the nucleon spin.



Further investigating *the role of both gluons and quark orbital angular momentum* requires a new experimental and theoretical framework.

Hadron femtography

Hadron Structure

Introduction

The strong interaction

Quantum field theory

Quantum chromodynamics

Electron scattering

Kinematics

Elastic scattering

DIS

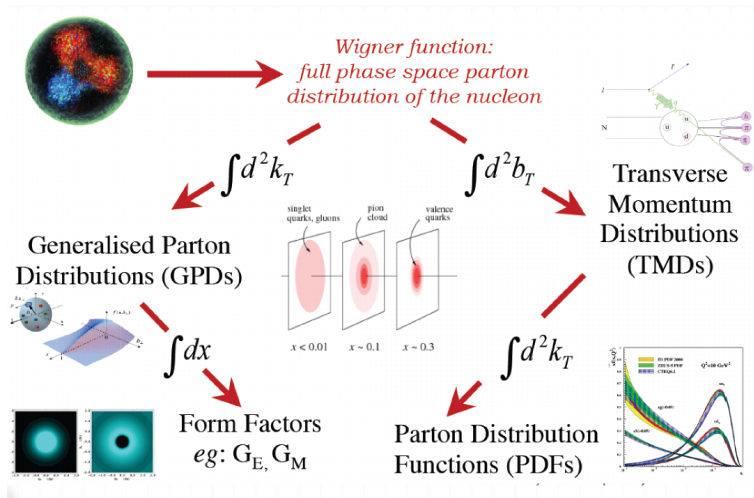
Hadron femtography

Form factors

Inclusive meson structure

Deeply virtual reactions

Parton imaging in higher dimensions



Hadron Structure

Introduction

The strong interaction

Quantum field theory
Quantum chromodynamics

Electron scattering

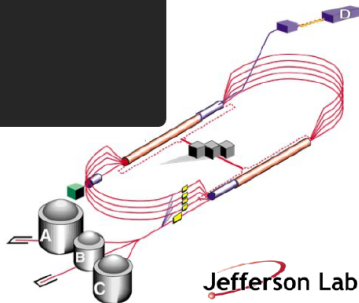
Kinematics
Elastic scattering
DIS

Hadron femtography

Form factors
Inclusive meson structure
Deeply virtual reactions

Jefferson Lab

- CEBAF at JLab is a 12 GeV, high power SRF electron accelerator which delivers high polarisation beams to four experimental halls simultaneously.
- Researchers have been doing experiments there since the late 1990s, although the 12 GeV program has only been running since 2012.
- Two STFC-funded UK NP groups involved (Glasgow and York).



Hadron Structure

Introduction

The strong interaction

Quantum field theory
Quantum chromodynamics

Electron scattering

Kinematics
Elastic scattering
DIS

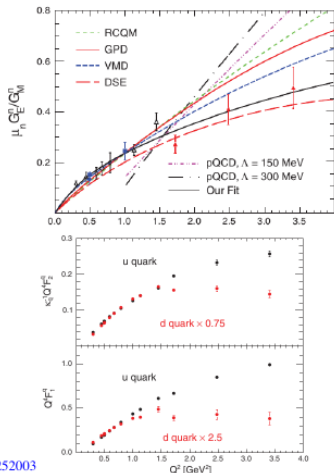
Hadron femtography

Form factors
Inclusive meson structure
Deeply virtual reactions

Electromagnetic form factors: exclusive structure of the nucleon

- The electromagnetic form factors of the nucleon are one of the most fundamental parameterisations of the nucleon's transverse structure, relating to the distribution of electric charge and magnetic moment.
- Measurements at high momentum are extremely challenging but offer the possibility of key insights into the strong interaction.
- The most recent results of the neutron electric form factor from JLab (over 10 years ago) allowed for the first ever quark flavour decomposition, and revealed differences in scaling between u and d quarks.

Cates et al
DOI: [10.1103/PhysRevLett.106.252003](https://doi.org/10.1103/PhysRevLett.106.252003)



Hadron Structure

Introduction

The strong interaction

Quantum field theory
Quantum chromodynamics

Electron scattering

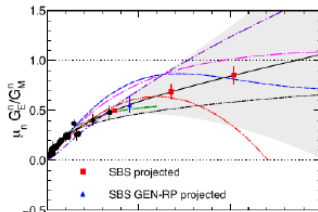
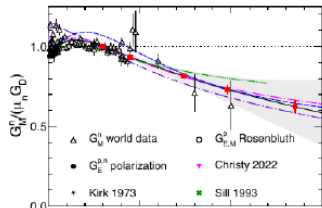
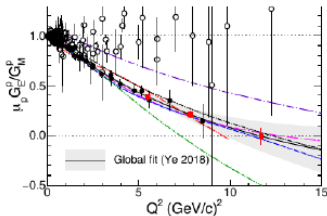
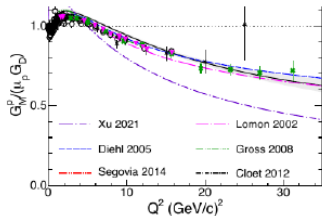
Kinematics
Elastic scattering
DIS

Hadron femtography

Form factors

Inclusive meson structure
Deeply virtual reactions

Electromagnetic form factors: SBS experiments



A.J.R. Puckett
hep-ph:2212.11107

Hadron Structure

Introduction

The strong interaction

Quantum field theory
Quantum chromodynamics

Electron scattering

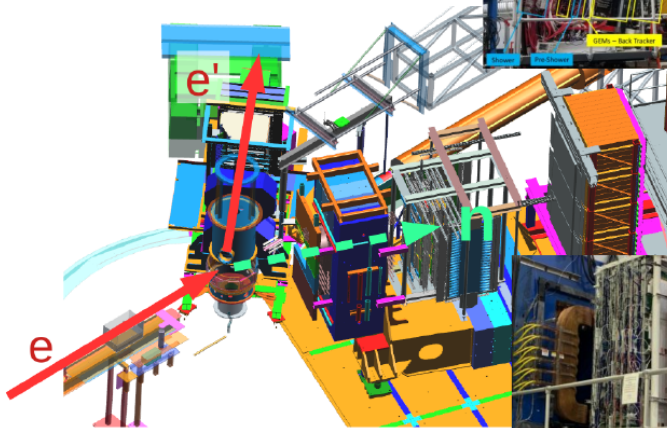
Kinematics
Elastic scattering
DIS

Hadron femtography

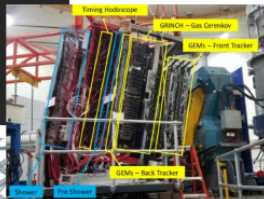
Form factors

Inclusive meson structure
Deeply virtual reactions

Electromagnetic form factors: SBS experiments



JLab experiment
E12-17-008



Hadron Structure

Introduction

The strong interaction

Quantum field theory
Quantum chromodynamics

Electron scattering

Kinematics
Elastic scattering
DIS

Hadron femtography

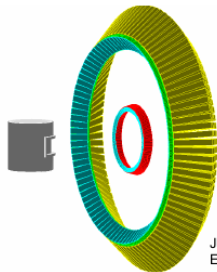
Form factors

Inclusive meson structure
Deeply virtual reactions

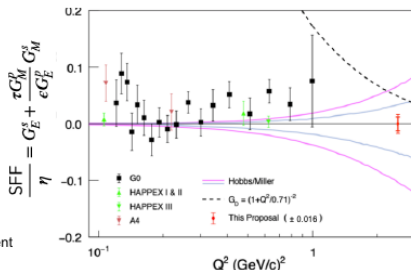


Electromagnetic form factors: strange quark contribution

- Measurement of the parity-violating asymmetry in polarised elastic electron-nucleon scattering allows access to the strange quark contribution to the form factors.
- All previous parity-violating electron scattering measurements (at SLAC, Mainz and JLab) have run in integrating mode at low four-momentum transfer;
- A new JLab experiment will be the first to make a coincidence measurement at moderately high momentum transfer.

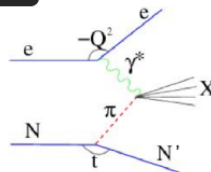


JLab experiment
E12-23-004

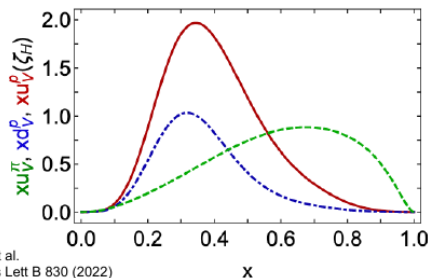


Inclusive meson structure functions: tagged DIS

- Tagged deep inelastic scattering allows access to meson structure via the Sullivan process, which is a natural complement to the more traditional Drell-Yan process.
- Extraction of PDFs can help answer questions on the mass budget of light hadrons and in particular on the difference in gluon content between the pion, kaon and proton.



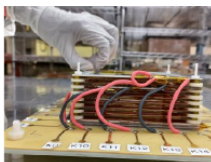
Hadron	Observed Mass (MeV)	Higgs Generated Mass (MeV)
Proton (uud)	~940	~10
Pion ($u\bar{d}$)	~140	~7
Kaon ($u\bar{s}$)	~490	~100



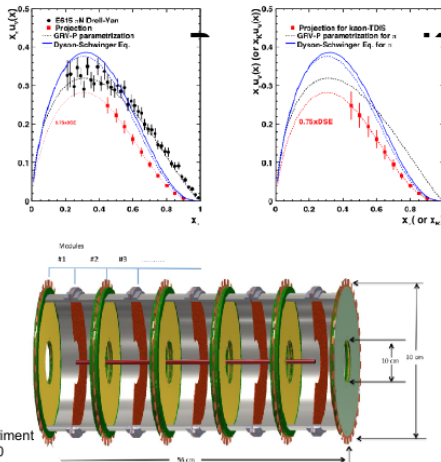
Lu et al.
Phys Lett B 830 (2022)

Inclusive meson structure functions: tagged DIS

- Detecting recoil (and spectator) nucleon provides an effective tag of the pionic content of the nucleon, while tagging the final state from hyperon decay will allow for the first ever measurement of DIS from a kaon.
- Requires detection of low energy hadrons in a high luminosity, fixed target arrangement using a new mTPC device.

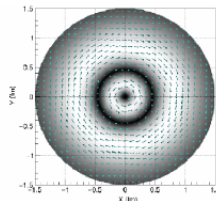
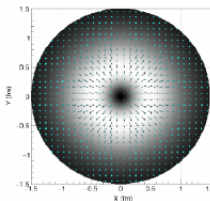
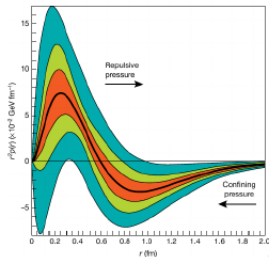
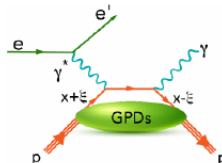


JLab experiment
E12-14-010



Gravitational structure of the nucleon: quark pressure and shear distributions

- Recent advances in the understanding of deeply virtual Compton scattering (DVCS) at JLab have allowed for the first time a determination of the nucleon's energy-momentum tensor via its gravitational form factor.
- From this, the radial pressure and shear forces on quarks can be extracted.



Burkert et al.
Nature 557 (2018)

Hadron Structure

Introduction

The strong interaction

Quantum field theory
Quantum chromodynamics

Electron scattering

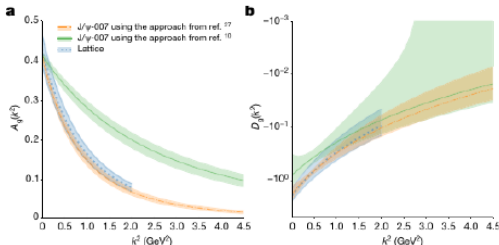
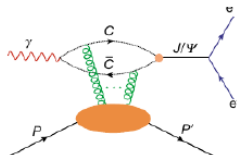
Kinematics
Elastic scattering
DIS

Hadron femtography

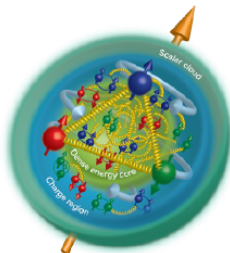
Form factors
Inclusive meson structure
Deeply virtual reactions

Gravitational structure of the nucleon: gluon mass distributions

- Similarly, access to the gluon gravitational form factors is possible via J/Psi photoproduction.
- Early, tentative results from JLab suggest the nucleon's mass radius may be significantly smaller than its charge radius.



Duran et al.
Nature 615 (202)



Hadron Structure

Introduction

The strong interaction

Quantum field theory
Quantum chromodynamics

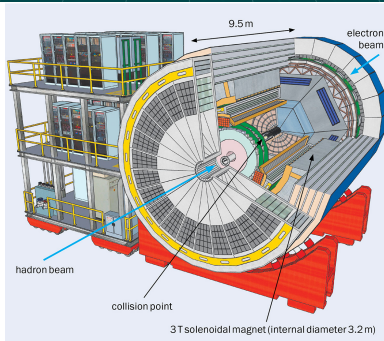
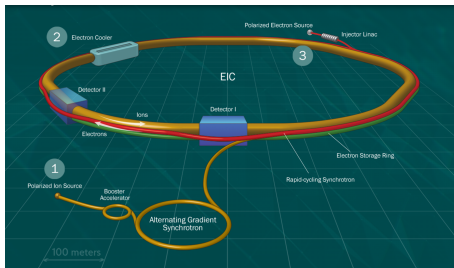
Electron scattering

Kinematics
Elastic scattering
DIS

Hadron femtography

Form factors
Inclusive meson structure
Deeply virtual reactions

Electron-Ion Collider at Brookhaven



Hadron Structure

Introduction

The strong interaction

Quantum field theory
Quantum chromodynamics

Electron scattering

Kinematics
Elastic scattering
DIS

Hadron femtography

Form factors
Inclusive meson structure

Deeply virtual reactions