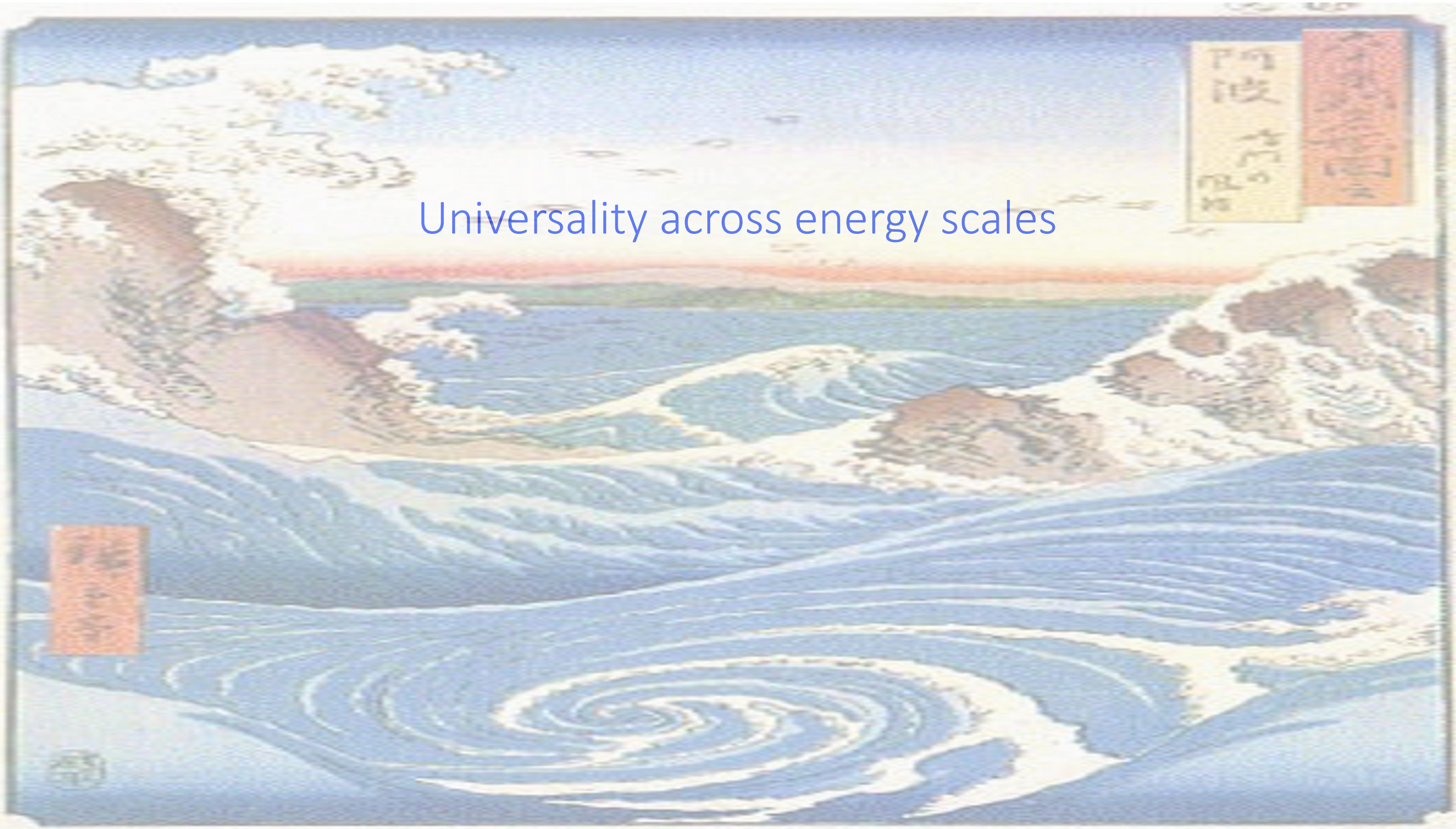
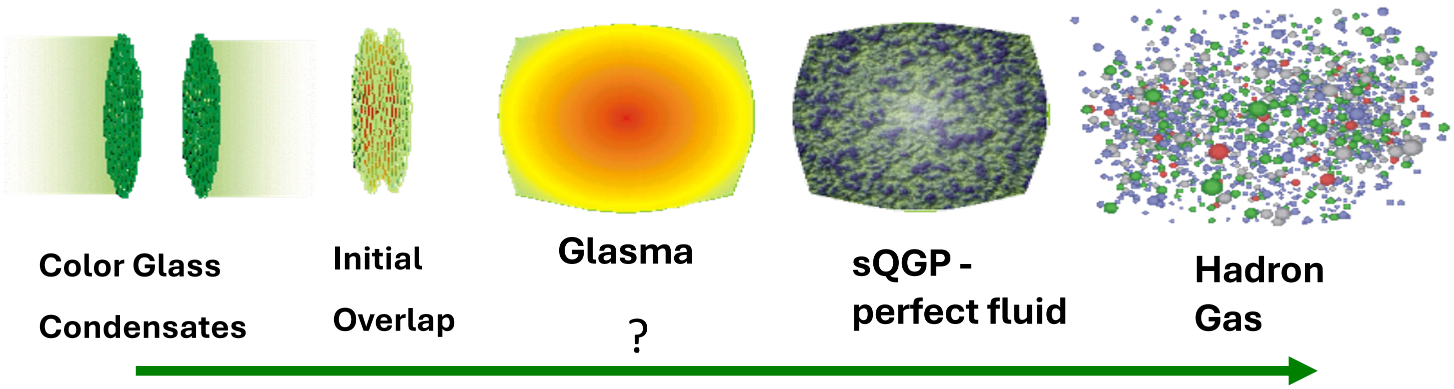
Universality across energy scales



Early thermalization from first principles in QCD?



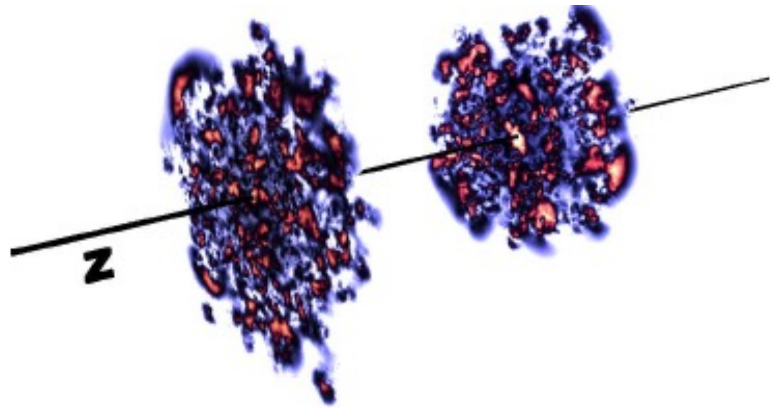
Remarkably, key features of this rich and complex space-time evolution can be described from first principles in the Regge asymptotics of QCD

The challenge is a deeper understanding of the early-time Glasma dynamics

Glasma (\Glahs-maa\): *Noun:* non-equilibrium matter between Color Glass Condensate (CGC) & Quark Gluon Plasma (QGP)

Heavy ion phenomenology in weak coupling

Collisions of lumpy gluon ``shock” waves



Leading order solution: Solution of QCD Yang-Mills eqns

Krasnitz, RV (1999)
Lappi (2003)

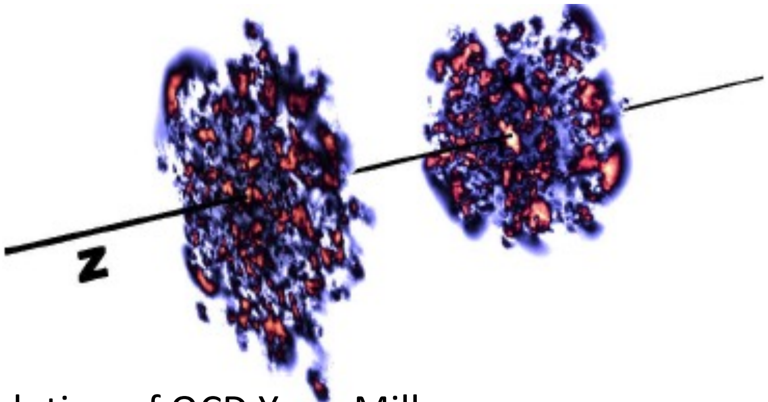
$$D_\mu F^{\mu\nu,a} = \delta^{\nu+}\rho_A^a(x_\perp)\delta(x^-) + \delta^{\nu-}\rho_B^a(x_\perp)\delta(x^+)$$

$$x^\pm = t \pm z$$

$$F^{\mu\nu,a} = \partial_\mu A^{\nu,a} - \partial_\nu A^{\nu,a} + g f^{abc} A^{\mu,b} A^{\nu,c}$$

Heavy ion phenomenology in weak coupling

Collisions of lumpy gluon ``shock" waves



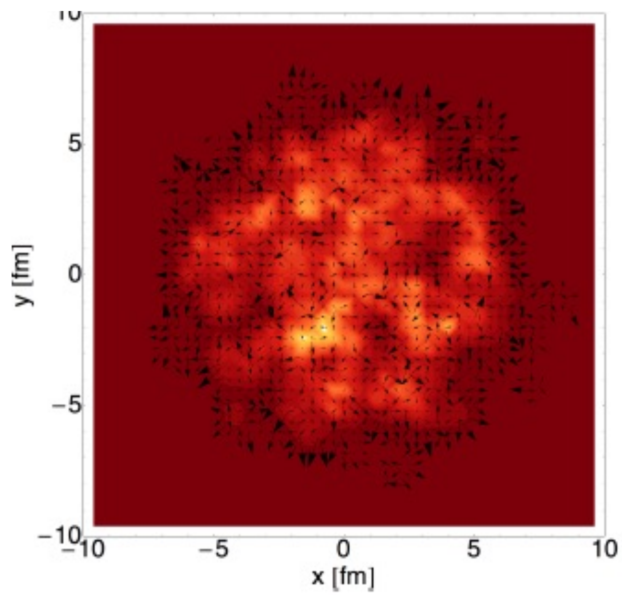
Leading order solution: Solution of QCD Yang-Mills eqns

$$D_\mu F^{\mu\nu,a} = \delta^{\nu+}\rho_A^a(x_\perp)\delta(x^-) + \delta^{\nu-}\rho_B^a(x_\perp)\delta(x^+)$$

$$\langle\rho^a_{A(B)}(x_\perp)\rho^a_{A(B)}(y_\perp)\rangle = Q^2_{S,A(B)}\delta^{(2)}(x_\perp - y_\perp)$$

Q_S (x,b_T) determined from saturation model fits to HERA inclusive and diffractive DIS data

Matching boost invariant Yang-Mills to hydrodynamics



Energy density and (u_x, u_y) at $\tau = 0.4\,\mathrm{fm}/c$

Matching to viscous hydro is “brutal” : assume “instantaneous” isotropization

The heart of the matter: how does isotropization/thermalization occur in ~ 1 fm/c ?

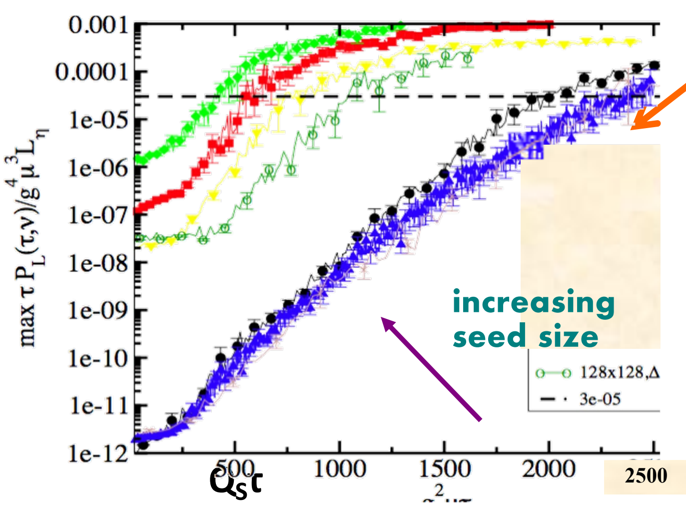
The Glasma at NLO: plasma instabilities

Romatschke,Venugopalan
Dusling,Gelis,Venugopalan
Gelis, Epelbaum

At LO: boost invariant gauge fields $A_{cl}^{\mu,a}(x_T,\tau) \sim 1/g$

NLO: $A^{\mu,a}(xT,\tau,\eta) = A_{cl}^{\mu,a}(x_T,\tau) + a^{\mu,a}(\eta)$

$a^{\mu,a}(\eta) = O(1)$

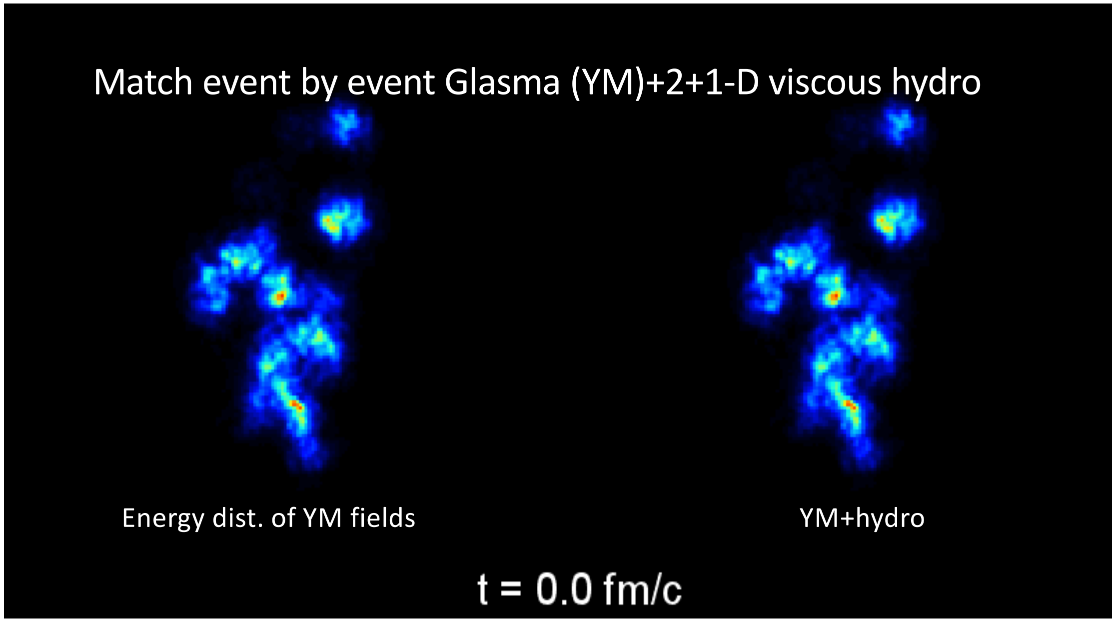


- Small fluctuations grow exponentially as ~ $e^{\sqrt{Q_S\tau}}$
- Same order of classical field at $\tau = \frac{1}{Q_S}\ln^2\frac{1}{\alpha_S}$
- Resum such contributions to all orders $(g\ e^{\sqrt{Q_S\tau}})^n$

Leading quantum corrections can be expressed as average over a classical-statistical ensemble of initial conditions

Heavy ion phenomenology in weak coupling

State of the art phenomenology: Solve relativistic viscous hydrodynamic equations with Glasma (Yang-Mills) initial conditions



Schenke,Tribedy,RV, PRL 108 (2012) 252301, PRC 86 (2012) 034908
Gale, Jeon, Schenke, Tribedy, RV, PRL 110 (2013) 1, 012302

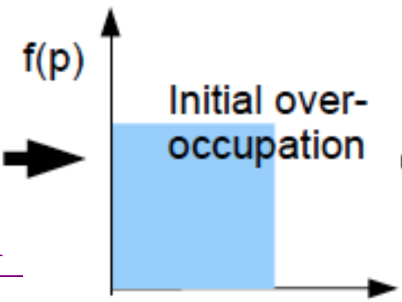
Initial conditions in the Glasma

Glasma + vacuum fluctuations

$$\tau = \frac{1}{Q_S}$$

Plasma Instabilities

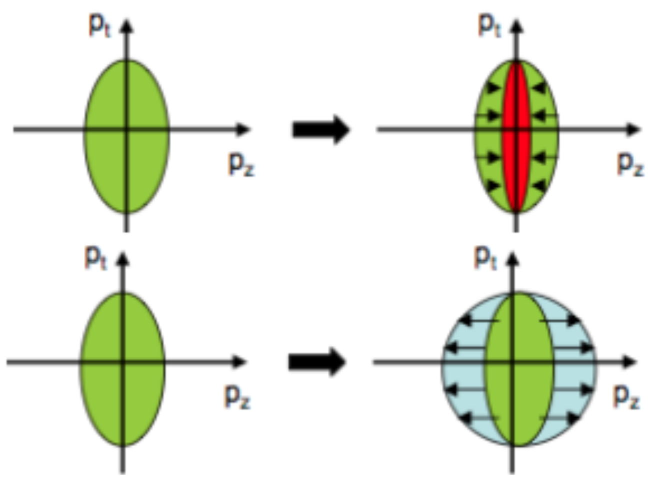
$$\tau = \frac{1}{Q_S} \ln^2 \frac{1}{\alpha_S}$$



???

$$\tau >> \frac{1}{Q_S} \ln^2 \frac{1}{\alpha_S}$$

- There is a natural ***competition*** between ***interactions*** and the ***longitudinal expansion*** which renders the system ***anisotropic*** on large time scales



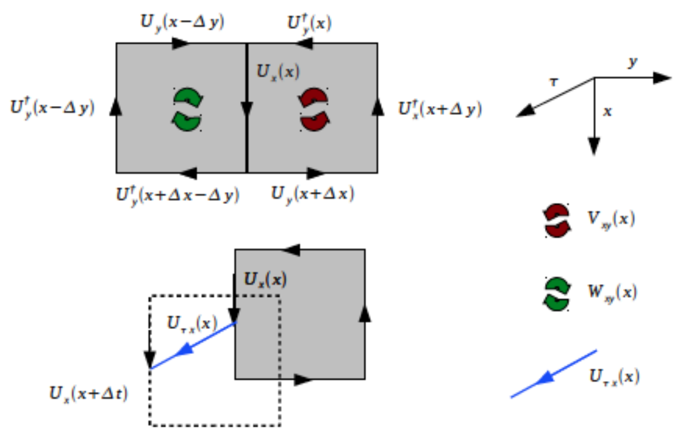
Longitudinal Expansion:
- Red-shift of longitudinal momenta p_z $\rightarrow$ increase of anisotropy
- Dilution of the system

Interactions:
- Isotropize the system

Temporal evolution in the overpopulated QGP

Berges,Boguslavski,Schlichting,Venugopalan
arXiv: 1303.5650, 1311.3005

Solve Hamilton's equation for 3+1-D SU(2) gauge theory in Fock-Schwinger gauge



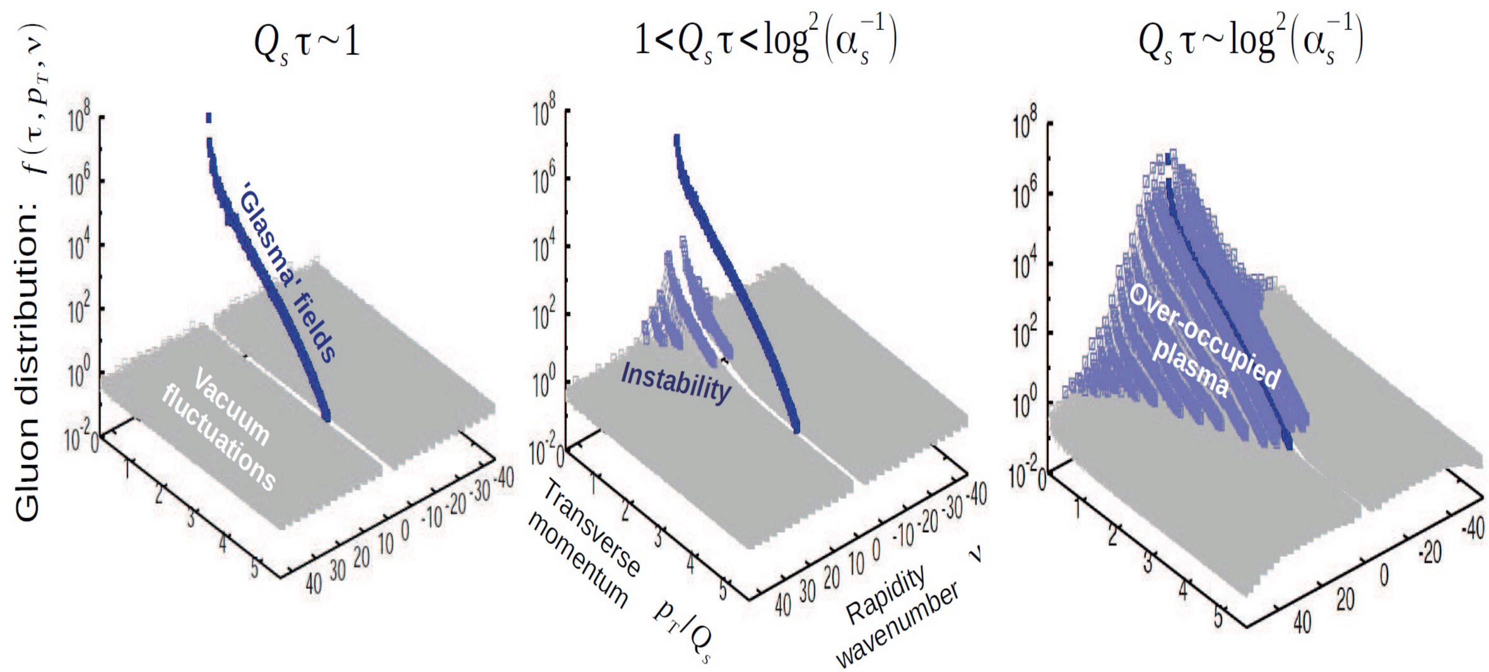
Fix residual gauge freedom imposing Coloumb gauge at each readout time

$$\partial_i A_i + t^{-2}\partial_\eta A_\eta = 0$$

Large classical-statistical numerical simulations of expanding Yang-Mills to date: $256^2 \times 4096$ lattices

Initial conditions in the overpopulated QGP

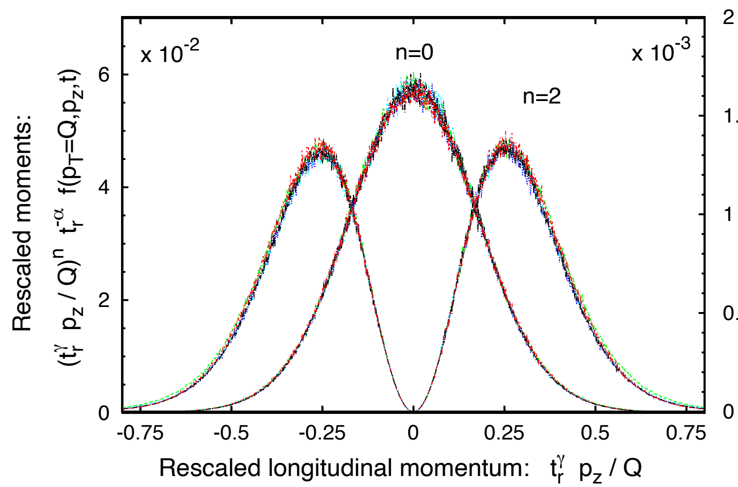
Overpopulation occurs even starting from the “first principles CGC” initial conditions

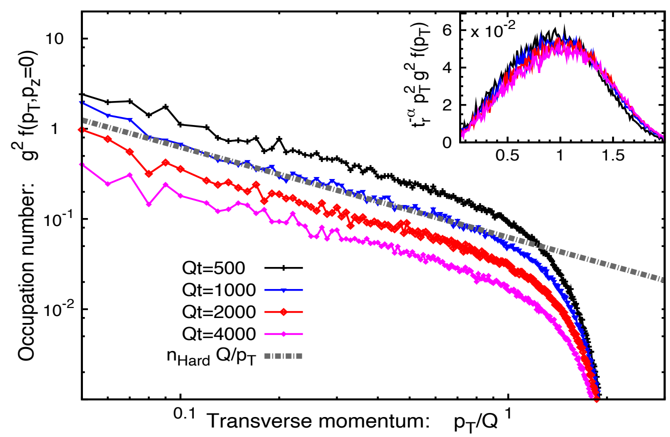


Berges,Schenke,Schlichting,RV, NPA 931 (2014) 348

Result: universal non-thermal fixed point

Conjecture: $f(p_\perp, p_z, t) = t^\alpha \ f_S(t^\beta p_T, t^\gamma p_z)$



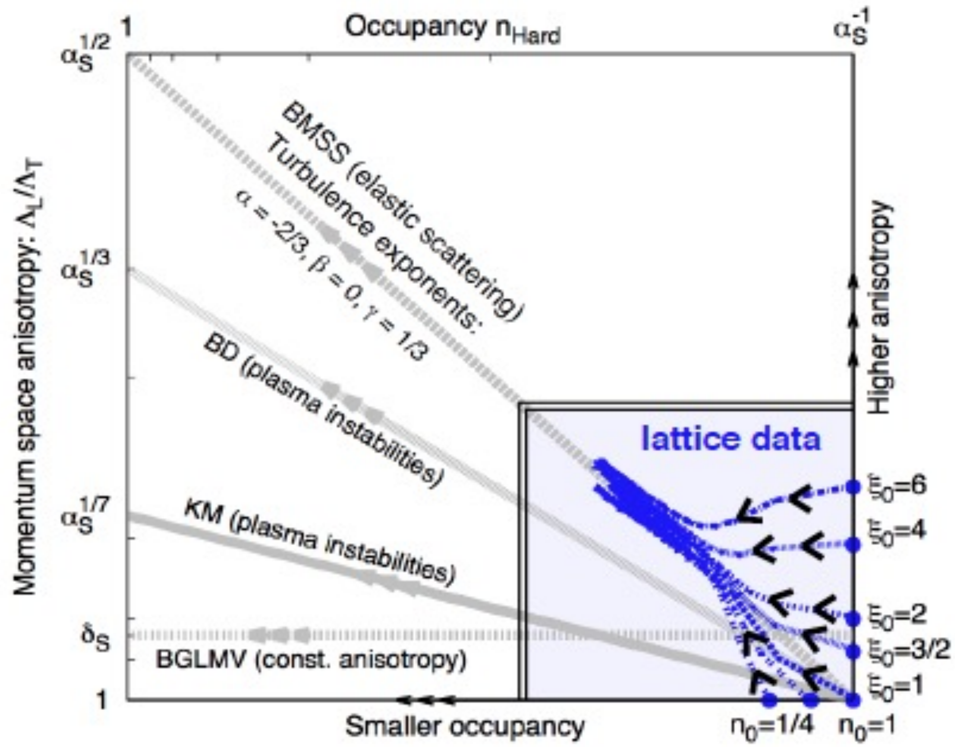


Moments of distribution extracted over range of time slices lie on universal curves

Distribution as function of p_T displays 2-D thermal behavior

Non-thermal fixed point in overpopulated QGP

Berges,Boguslavski,Schlichting,Venugopalan. arXiv: 1303.5650

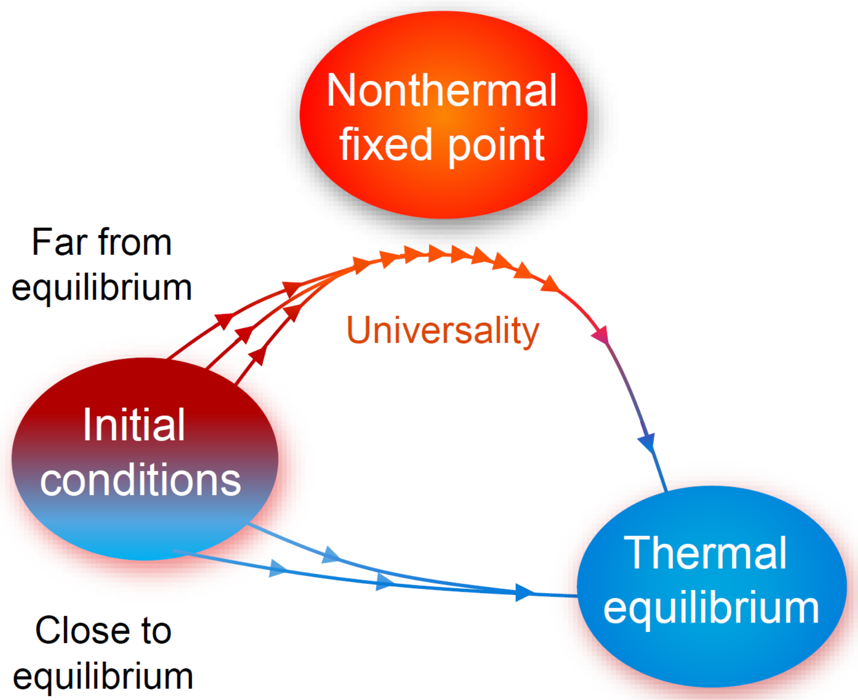


BMSS: Baier,Mueller,Schiff,Son
BD: Bodeker
KM: Kurkela, Moore
BGLMV: Blaizot,Gelis,Liao,McLerran,Venugopalan

Kinetic theory in the overoccupied regime

For $1 < f < 1/\alpha_S$ dual description feasible either in terms of kinetic theory or classical-statistical dynamics

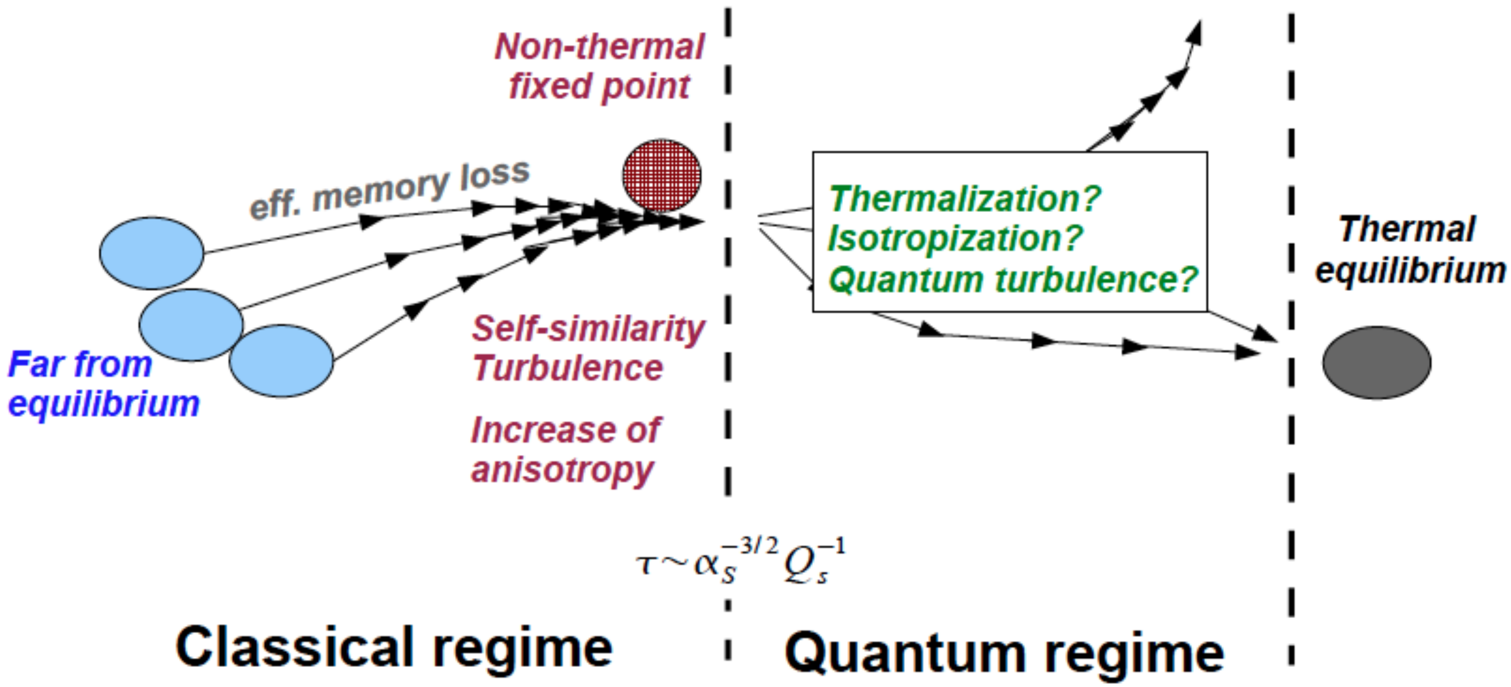
Mueller,Son (2002)
Jeon (2005)



Properties independent of initial conditions

Self-similar evolution characterized by universal scaling exponents

Quo vadis, thermal QGP?



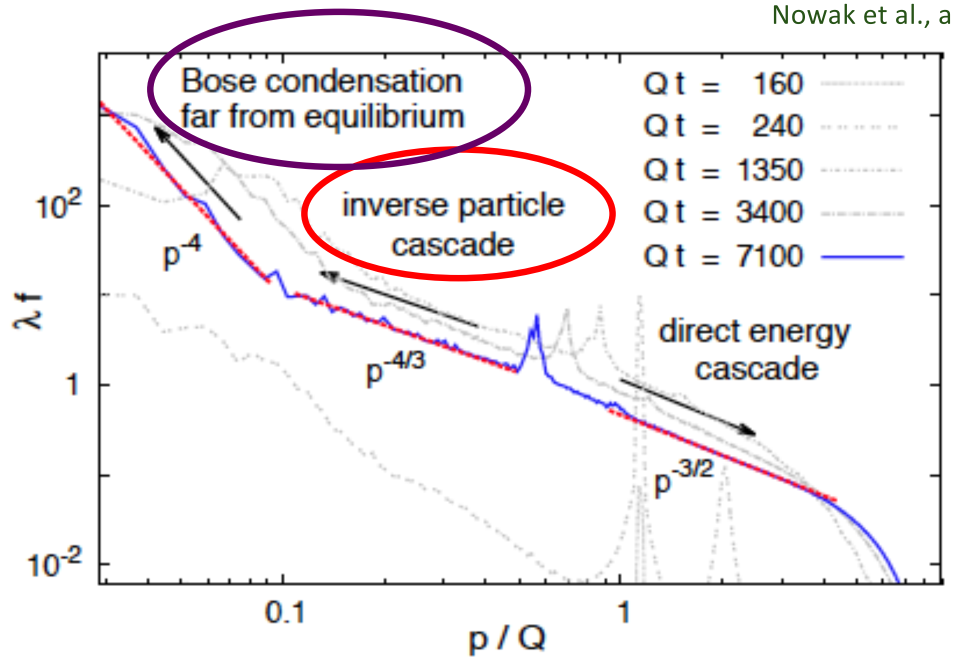
Universality: hotness is also cool

Over-occupied self-interacting scalars

$$S = \int d\tau d^2 x_T d\eta\ \tau \left(\frac{g^{\mu\nu}}{2} (\partial_\mu \varphi_a)(\partial_\nu \varphi_a) - \frac{\lambda}{4!N} (\varphi_a \varphi_a)^2 \right)$$

In a non-relativistic limit, can be used to model cold atomic gases

Scheppach,Berges,Gasenzer, PRA 81 (2010) 033611
Nowak, Schole, Sexty, Gasenzer, PRA85 (2012) 043627
Nowak et al., arXiv 1302.1448

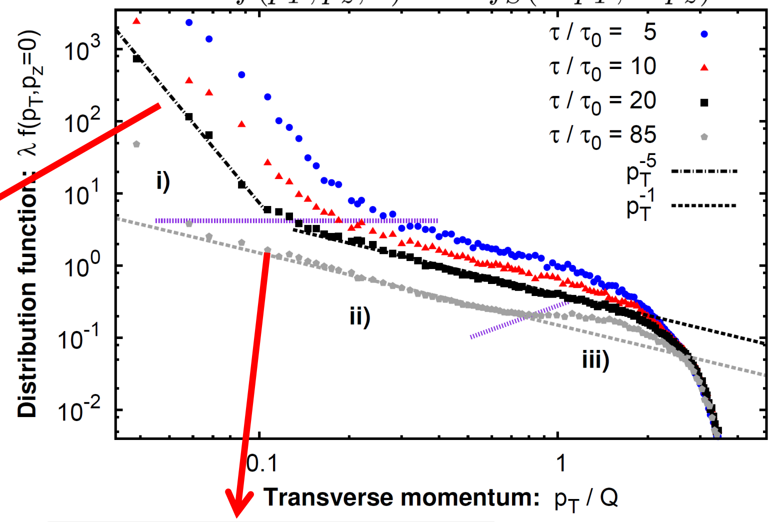


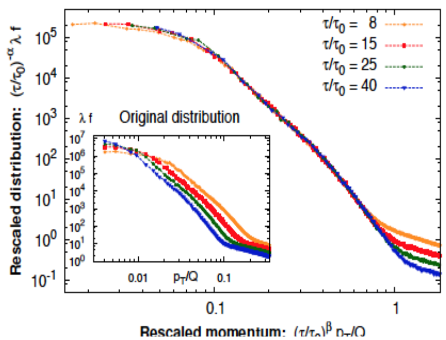
Berges, Sexty PRL 108 (2012) 161601
Berges,Boguslavski,Orioli, PRD 92, 025041 (2015)
Berges,Boguslavskii,Schlichting,Venugopalan, JHEP 1405 (2014) 054

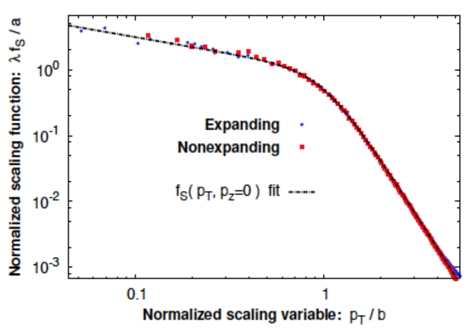
What about longitudinally expanding scalars?

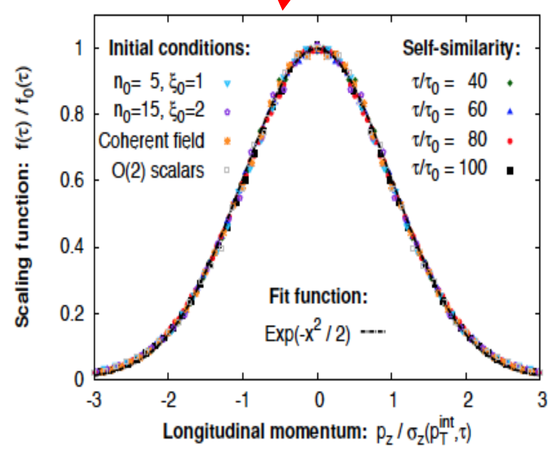
Three distinct inertial regimes with self-similar behavior

$$f(p_T, p_z, \tau) = \tau^{\alpha} f_S(\tau^{\beta} p_T, \tau^{\gamma} p_z)$$



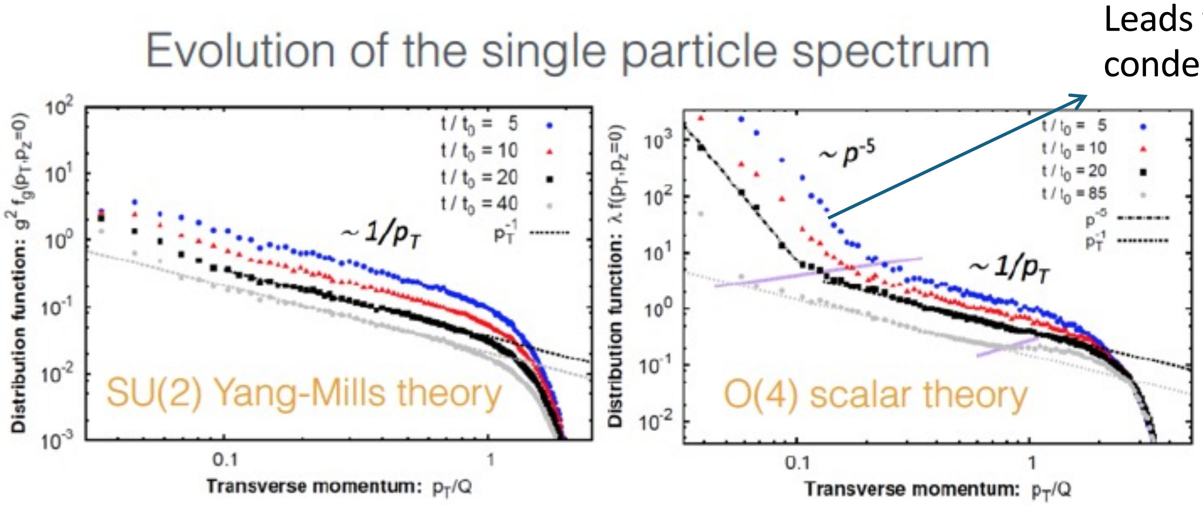




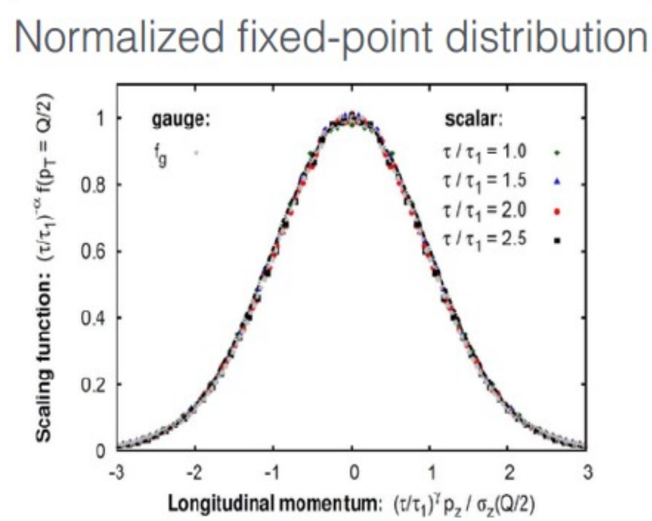


Berges,Boguslavski,Schlichting,RV, 1508.03073

A remarkable universality



Leads to Bose-Einstein condensation



Berges,Boguslavski,Schenke,Venugopalan, PRL 114 (2015) 061601,

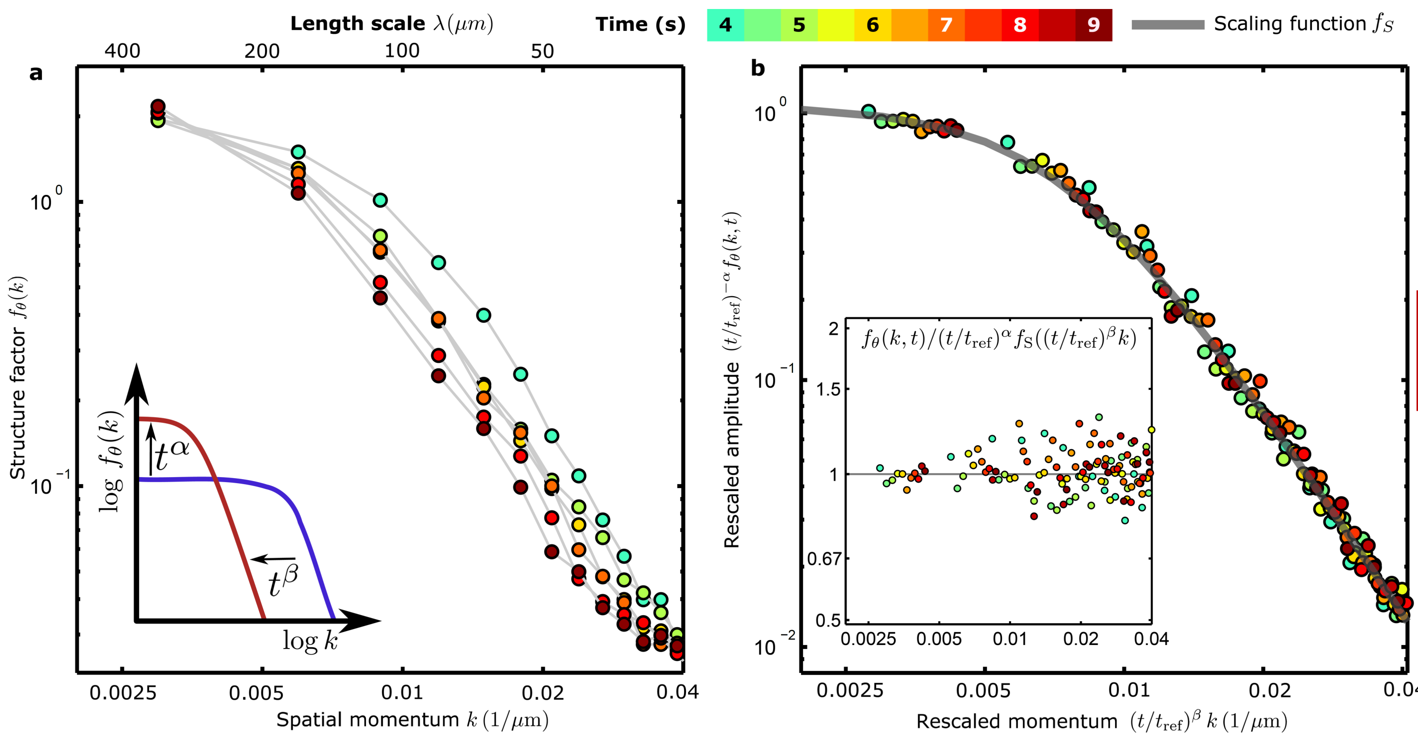
In a wide inertial range, scalars and gauge fields have identical scaling exponents and scaling functions

Very surprising from a kinetic theory perspective

The Glasma and over-occupied quantum gases

Similar non-thermal fixed points discovered in cold atom experiments - albeit only for static geometry so far

^{87}Rb BEC in a quasi 1D optical trap



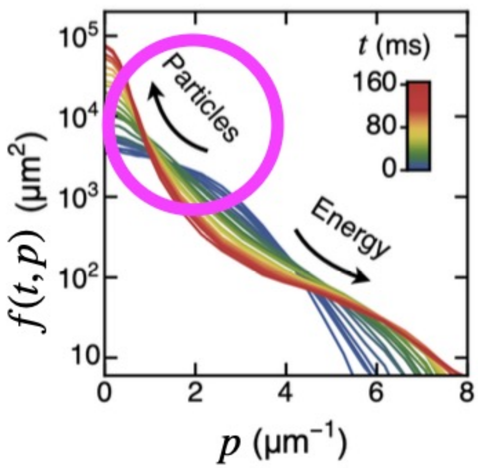
$$f_\theta(k,t) = t^\alpha f_S(t^\beta k)$$

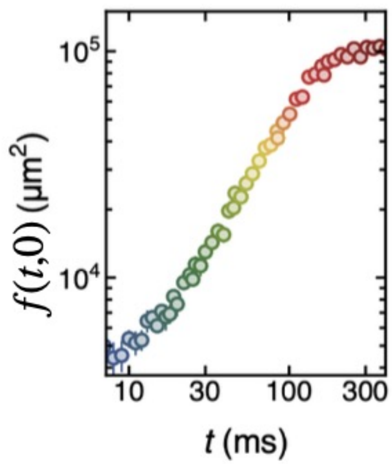
$\alpha = 0.33 \pm 0.08$ $\beta = 0.54 \pm 0.06$

Oberthaler BEC Labs

Prüfer et al, arXiv:1805.11881, *Nature* (2018)

Further experimental confirmation of non-thermal fixed points

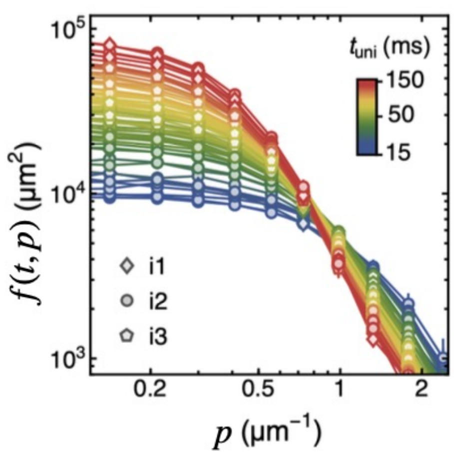




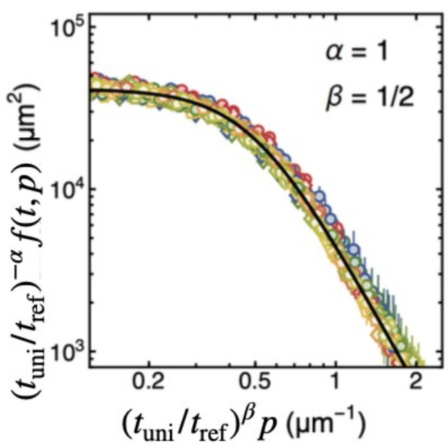
“Universal coarsening in homogeneous 2-D Bose Gas”

Gazo et al., Hadzibabic group, arXiv:2312.09248

Here, we study coarsening in a homogeneous two-dimensional (2D) Bose gas [34, 35], by engineering different far-from-equilibrium initial states and measuring the momentum distributions $n_k(k)$ by matter-wave focusing [36, 37]. Crucially, by elucidating and accounting for the non-universal effects of initial conditions, we reveal the universal long-time scaling in finite-range experimental data, introducing methods applicable to any quantitative study of universality far from equilibrium. We find that the low-k (IR) coarsening is characterized by the theoretically predicted dynamical exponent $z = 2$ [19, 22] (see also [5, 7, 18, 21, 38]), and the form of the self-similarly evolving n_k matches an analytical field-theory prediction [19], while the high-k (UV) energy dynamics corresponds to weak four-wave turbulence [39].

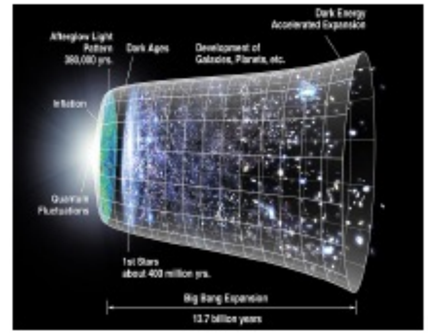


$t_{\mathrm{uni}} \equiv t - t_*$ →



Early universe cosmology: turbulent thermalization

Micha,Tkachev, PRD 70 (2004) 043538



Model for early universe thermalization:

Weakly coupled scalar field theory ($\lambda\Phi^4$) ($\lambda=10^{-8}$)

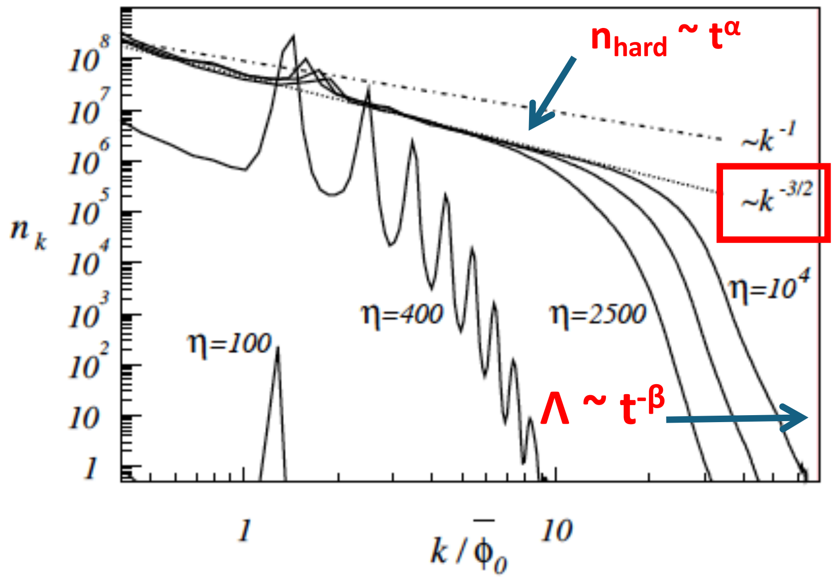
In homogeneous background field
+ vacuum fluctuations

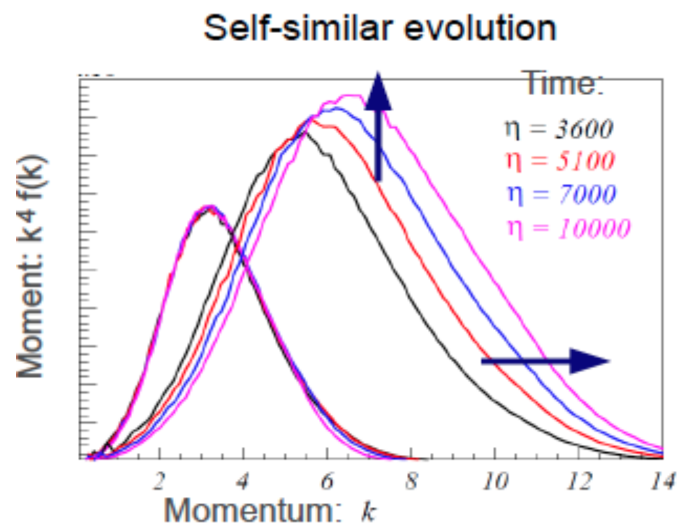
$$\Phi_0 \sim \frac{1}{\sqrt{\lambda}}$$

Growth of instabilities via parametric resonance of classical field and vacuum fluctuations

Turbulent thermalization in Cosmology

Micha,Tkachev, PRD 70 (2004)043538





Thermalization process characterized by quasi-stationary evolution with scaling exponents.

Dynamic: $\alpha=-4/5$, $\beta=-1/5$; Spectral: $\kappa= -3/2$

Kinetic interpretation of self-similar behavior

Follow wave turbulence kinetic picture of Zakharov, as developed by Micha & Tkachev

$$\left[\partial_t - \frac{p_z}{t}\partial_{p_z}\right] f(p_T, p_z, t) = C[p_T, p_z, t; f]$$

$$C[p_T, p_z, t; f] = t^{\mu}\, C[t^{\beta} p_T, t^{\gamma} p_z; f_S]$$

Fixed point solution satisfies

$$\alpha f_S(p_T, p_z) + \beta p_T \partial_{p_T} f_S(p_T, p_z) + (\gamma - 1)\, p_z \partial_{p_z} f_S(p_T, p_z) = C[p_T, p_z; f_S]$$

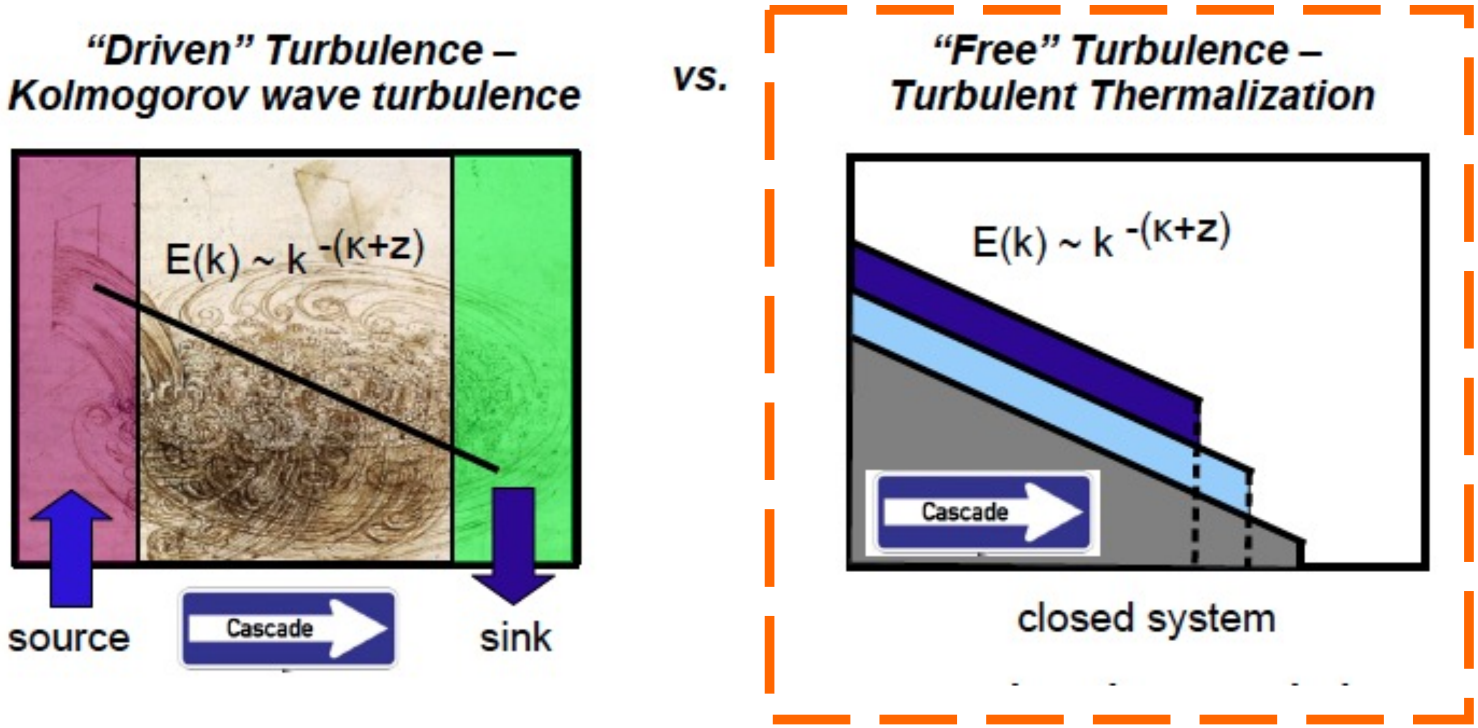
$\mu = \alpha - 1$

If we assume that small angle elastic scattering dominates

$$C^{(\mathrm{elast})}[p_T, p_z; f] = \hat{q}\ \partial^2_{p_z} f(p_T, p_z, t)$$

$$\hat{q} \sim \alpha_S^2 \int \frac{d^2 p_T}{(2\pi)^2} \int \frac{dp_z}{2\pi}\ f^2(p_T, p_z, t)$$

Turbulent thermalization in Cosmology

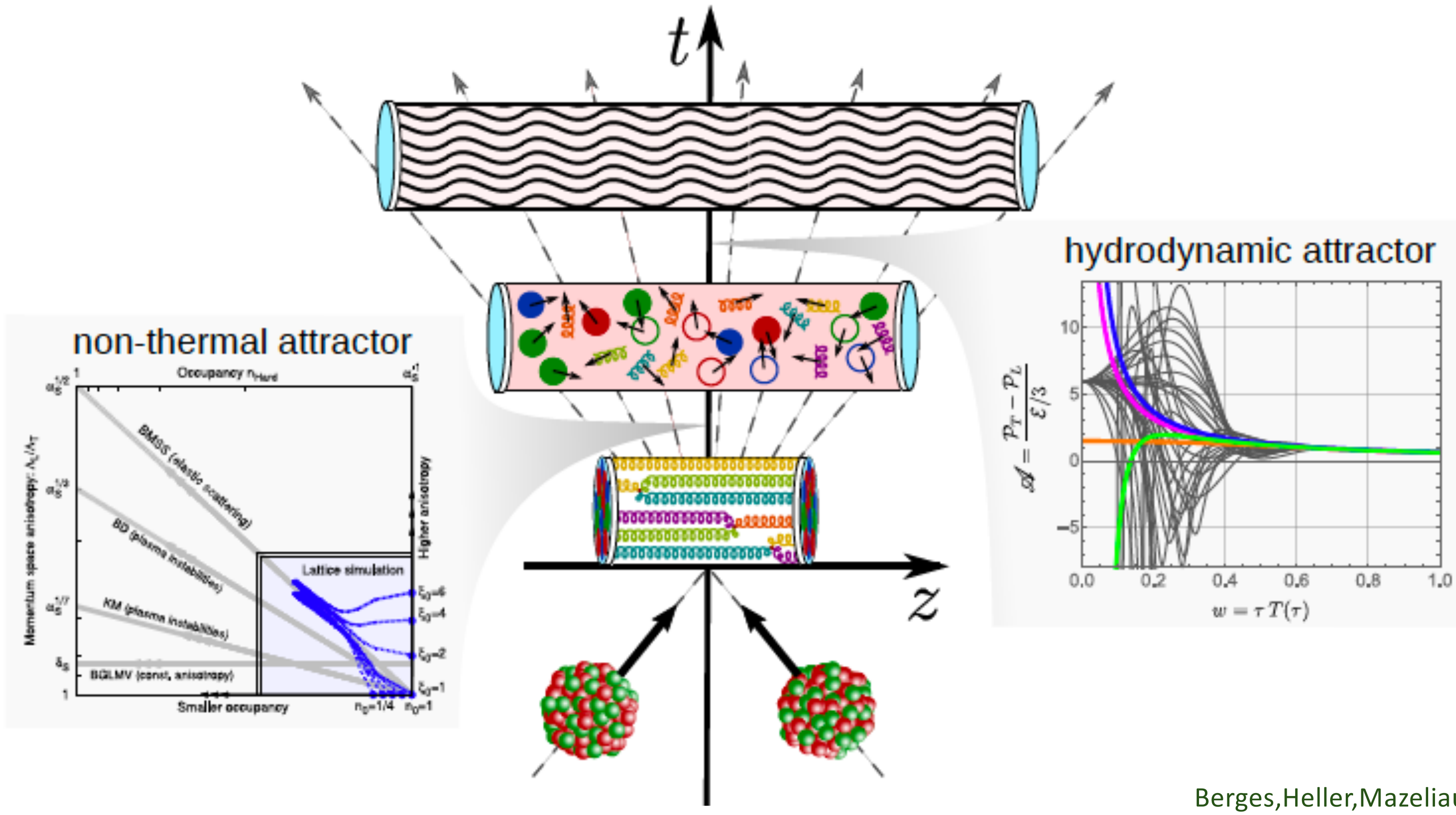


Free Turbulence:

Quasi-stationary solution: universal, non-thermal spectral exponents

Self-similar evolution with universal dynamical scaling exponents

Color Glass to Color Fluid: Thermalization in Heavy-Ion Collisions



Berges,Heller,Mazeliauskas,RV
Rev. Mod. Phys. 93, 035003 (2021)