Lipatov's EFT at one loop

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"Towards accuracy at amall x" workshop,
Edinburgh,
10-13 September 2019.

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Outline

The talk is based on [M. N., Nucl.Phys., B946, 114715 (2019); M.N., V.A. Saleev, Mod. Phys. Lett., A32, 1750207 (2017)].

1. Introduction to Lipatov's EFT
2. One-loop rapidity-divergent integrals
3. Application: Computation of $Q\gamma^\star q$ and $R\mathcal{O}g$ vertices
4. Comparison with QCD

Some motivation

- The gauge-invariant EFT for Multi-Regge processes in QCD, which includes *Reggeized gluons* [Lipatov; 1995] and *Reggeized quarks* [Lipatov, Vyazovsky; 2001] has been introduced as a systematic tool to *compute and resum* the higher-order corrections in QCD, enhanced by $\log(s/(-t))$, with the arbitrary N^kLL accuracy.
- Another motivation is the *unitarization program* for high-energy scattering. The BFKL equation at the fixed logarithmic accuracy predicts *power-like* growth of the cross-section with s, which violates Froissart bound ($\Leftarrow$ Unitarity). The basic idea is to write-down the *Hermitian* effective Lagrangian for QCD at high energies, so that Unitarity will hold automatically.
- In the talk I would like to describe the one-loop structiure of Lipatov's EFT. The complete picture, similar to one in ordinary QCD, emerges.

Introduction to Lipatov's EFT

Sudakov (light-cone) decomposition of momenta.

It is convenient to relate the basis vectors of Sudakov decomposition with (almost) light-like momenta of colliding highly energetic particles ($P_{1,2}^2 = 0$):

$$n_-^\mu = \frac{2P_1^\mu}{\sqrt{S}},\ n_+^\mu = \frac{2P_2^\mu}{\sqrt{S}},\ S = 2P_1P_2 \Rightarrow n_+n_- = 2.$$

Then for any four-vector k^μ one has:

$$\boxed{k^\mu = \frac{1}{2}\left(k_+n_-^\mu + k_-n_+^\mu\right) + k_T^\mu,}$$

where $k_\pm = k^\pm = n_\pm k$, $n_\pm k_T = 0$. For the dot-product one has:

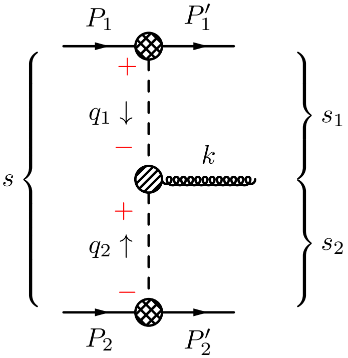
$$kq = \frac{1}{2}\left(k_+q_- + k_-q_+\right) - \mathbf{k}_T\mathbf{q}_T,\ \boxed{k^2 = k_+k_- - \mathbf{k}_T^2.}$$

Rapidity:

$$\boxed{\boxed{y = \frac{1}{2}\log\left(\frac{q^+}{q^-}\right).}}$$

Multi-Regge Kinematics.

Consider the $2 \to 2 + n$ scattering in Multi-Regge(MRK) or Quasi-Multi Regge(QMRK) kinematics.



Double Regge limit (MRK):

$$s_1 \gg -q_1^2 \simeq \mathbf{q}_{T1}^2, \; s_2 \gg -q_2^2 \simeq \mathbf{q}_{T2}^2,$$

momentum fractions $z_1 = q_1^+ / P_1^+$, $z_2 = q_2^- / P_2^-$.

Intuition: t-channel diagrams dominate.

Properties of MRK:

- $y(P_1') \to +\infty$, $y(P_2') \to -\infty$, $y(k)$ – finite,
- $\boxed{z_1 \sim z_2 \sim z \ll 1}$, $|\mathbf{k}_T| \ll \sqrt{s}$,
- $q_1^+ \sim |\mathbf{q}_{T1}| \sim O(z) \gg q_1^- \sim O(z^2)$, $q_2^- \sim |\mathbf{q}_{T2}| \sim O(z) \gg q_2^+ \sim O(z^2)$.

Reggeon fields

Let's introduce **gauge-invariant** Reggeon fields $R_\pm(x) = T^a R^a_\pm(x)$ subjet to kinematic constraints ($\Leftrightarrow$ (Q)MRK, $\partial_\pm = n^\mu_\pm \partial_\mu = 2\frac{\partial}{\partial x^\mp}$):

$$\partial_- R_+ = \partial_+ R_- = 0 \Rightarrow$$

R_+ carries $(k_+, \mathbf{k}_T)$ and R_- carries $(k_-, \mathbf{k}_T)$.

Effective action [Lipatov, 1995]:

$$S = \int d^4x(-2R^a_+ \partial^2_\perp R^a_-) + \sum_{\text{rap. ints.}} \int d^2\mathbf{x}_T \left\{ \int \frac{dx_+ dx_-}{2} L_{\text{QCD}}(x, A_\mu, \psi) \right.$$

$$\left. + \int \frac{dx_+}{2} \operatorname{tr}\left[R^a_-(x_+, \mathbf{x}_T)\mathcal{T}_+[x, A_\mu]\right] + \int \frac{dx_-}{2} \operatorname{tr}\left[R^a_+(x_-, \mathbf{x}_T)\mathcal{T}_-[x, A_\mu]\right] \right\},$$

what are the interaction operators $\mathcal{T}_\pm$?

Infinite light-like Wilson lines

Constraints we have:

- At leading power in energy, partons highly separated in rapidity perceive each-other as infinite light-like Wilson lines [Mueller, Nikolaev, Zakharov, 1990s; ...; Caron-Huot, 2013; ...],
- Hermiticity [Lipatov, 1997; Bondarenko, Zubkov, 2018]
- $R_\pm \to g$ transition is given by "non-sense" polarization $n^\mu_\mp$.

$$\Rightarrow \mathcal{T}_\pm[x, A_\mu] = \frac{i}{g_s}\partial_\perp^2 \left(W_\infty[x_\pm, \mathbf{x}_T, A_\mu] - W^\dagger_\infty[x_\pm, \mathbf{x}_T, A_\mu] \right),$$

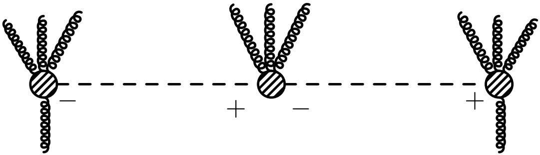
Where:

$$\begin{aligned} W_{x_\mp}[x_\pm, \mathbf{x}_T, A_\pm] &= P\exp\left[\frac{-ig_s}{2} \int\limits_{-\infty}^{x_\mp} dx'_\mp A_\pm(x_\pm, x'_\mp, \mathbf{x}_T) \right] \\ &= \left(1 + ig_s \partial_\pm^{-1} A_\pm\right)^{-1}, \end{aligned}$$

and $\partial_\pm^{-1} \to -i/(k^\pm + i\varepsilon)$ in the Feynman rules.
After IBP trick (see later, page 14):

$$S_{\text{int.}} = \int dx \frac{i}{g_s} \operatorname{tr}\left[R_+(x) \partial_\perp^2 \partial_- \left(W_{x_+}[A_-] - W^\dagger_{x_+}[A_-] \right) + (+ \leftrightarrow -) \right],$$

Structure of Induced interactions



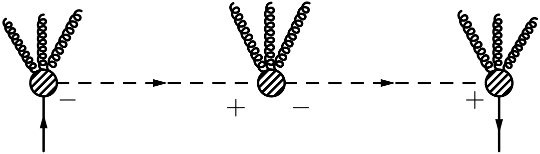
Induced interactions of particles and Reggeons:

$$\frac{i}{g_s}\,\mathrm{tr}\left[R_+\partial_\perp^2\partial_- W\left[A_-\right] + R_-\partial_\perp^2\partial_+ W\left[A_+\right]\right],$$

expansion of P-exponent generaties induced vertices:

$$\begin{aligned}&\mathrm{tr}\left[\left(R_+\partial_\perp^2 A_- + R_-\partial_\perp^2 A_+\right) + \right.\\ &(-ig_s)(\partial_\perp^2 R_+)(A_-\partial_-^{-1}A_-) + (-ig_s)^2(\partial_\perp^2 R_+)(A_-\partial_-^{-1}A_-\partial_-^{-1}A_-) + \\ &(-ig_s)(\partial_\perp^2 R_-)(A_+\partial_+^{-1}A_+) + (-ig_s)^2(\partial_\perp^2 R_-)(A_+\partial_+^{-1}A_+\partial_+^{-1}A_+) \\ &\left. + O(g_s^3)\right].\end{aligned}$$

EFT for QMRK-processes with quark exchange



EFT for Reggeized quarks [Lipatov, Vyazovsky, 2001]:

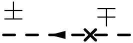
$$L_Q = \bar{Q}_- i\hat{\partial}\left(Q_+ - W^\dagger\left[A_+\right]\psi\right) + \bar{Q}_+ i\hat{\partial}\left(Q_- - W^\dagger\left[A_-\right]\psi\right) + \text{h.c.},$$

where $\hat{p} = p_\mu \gamma^\mu$, QMRK kinematic constraints:

$$\partial_\pm Q_\mp = \partial_\pm \bar{Q}_\mp = 0,$$
$$\hat{n}^\pm Q_\mp = 0,\ \bar{Q}_\mp \hat{n}^\pm = 0. \Rightarrow$$

Reggeized quark propagator ($\hat{P}_\pm = \hat{n}_\mp \hat{n}_\pm / 4$):

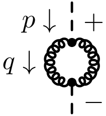
$$= \hat{P}_\pm \frac{i\hat{q}}{q^2},$$



$$= \frac{i\hat{q}}{q^2}\hat{P}_\pm .$$

Rapidity divergences and regularization.

Due to the presence of the $1/q^{\pm}$-factors in the induced vertices, loop integrals in EFT contain the light-cone (Rapidity) divergences:



$$\Pi^{(1)}_{ab} = g_s^2 C_A \delta_{ab} \int \frac{d^d q}{(2\pi)^D} \frac{\left(\mathbf{p}_T^2 (n_+ n_-)\right)^2}{q^2 (p-q)^2 q^+ q^-}$$

The regularization by explicit cutoff in rapidity was proposed by Lipatov [Lipatov, 1995] ($q^{\pm} = \sqrt{q^2 + \mathbf{q}_T^2} e^{\pm y}$, $p^+ = p^- = 0$):

$$\int \frac{dq^+ dq^-}{q^+ q^-} = \int\limits_{y_1}^{y_2} dy \int \frac{dq^2}{q^2 + \mathbf{q}_T^2},$$

then

$$\Pi^{(1)}_{ab} \sim \delta_{ab} \mathbf{p}_T^2 \times \underbrace{C_A g_s^2 \int \frac{\mathbf{p}_T^2 d^{D-2} \mathbf{q}_T}{\mathbf{q}_T^2 (\mathbf{p}_T - \mathbf{q}_T)^2}}_{\omega^{(1)}(\mathbf{p}_T^2)} \times (y_2 - y_1) + \text{finite terms}$$

Rapidity divergent one-loop integrals

Covariant regularization.

The regularization and **pole prescription** was introduced in a series of papers [Hentschinski, Sabio Vera, Chachamis *et. al.*, 2012-2013], also known in TMD factorization as "tilted Wilson lines" [Collins, 2011].
Regularization of the light-cone divergences is achieved by shifting $n^{\pm}$ vectors from the light-cone:

$$\boxed{\tilde{n}^{\pm} = n^{\pm} + r \cdot n^{\mp},\ \tilde{k}^{\pm} = k^{\pm} + r \cdot k^{\mp},\ r \to 0,}$$

and for the lowest-order(Rgg, Qgg) induced vertices the PV prescription is at work ($\Leftarrow$ Hermitian effective action):

$$I^{[\pm]}:\ \frac{1}{[\tilde{k}^{\pm}]} = \frac{1}{2}\left(\frac{1}{\tilde{k}^{\pm} + i\varepsilon} + \frac{1}{\tilde{k}^{\pm} - i\varepsilon}\right),$$

Regularization and gauge-invariance (IBP trick)

Regularization should preserve the gauge-invariance of Reggeon-gluon interactions ($\underline{x}_\pm = x_\pm - r x_\mp$):

$$S_{Rg}^{(-)} = \int d^2\mathbf{x}_T \int\limits_{-\infty}^{+\infty} \frac{dx_+ dx_-}{2} \, \mathrm{tr}\left[R^- \tilde{\partial}_+ \partial_\perp^2 W_{\tilde{x}_-}[\tilde{A}_+] \right]$$

$$= \int d^2\mathbf{x}_T \int\limits_{-\infty}^{+\infty} \frac{d\underline{x}_+ d\underline{x}_-}{1-r^2} \, \mathrm{tr}\left[R^- \frac{\partial}{\partial \underline{x}_-} \partial_\perp^2 W_{\underline{x}_-}[\tilde{A}_+] \right] =$$

$$= \int d^2\mathbf{x}_T \int\limits_{-\infty}^{+\infty} \frac{d\underline{x}_+ d\underline{x}_-}{1-r^2} \left\{ \frac{\partial}{\partial \underline{x}_-} \mathrm{tr}\left[R^- \partial_\perp^2 W_{\tilde{x}_-}[\tilde{A}_+] \right] - \frac{1}{2} \mathrm{tr}\left[\left(\tilde{\partial}_+ R_- \right) \partial_\perp^2 W_{\underline{x}_-}[\tilde{A}_+] \right] \right\}.$$

First term – *infinite Wilson line* is gauge invariant (w.r.t. gauge transformations trivial at ∞) $\Rightarrow$ **new kinematic constraint** [M.N., 2019]:

$$\boxed{\tilde{\partial}_+ R_- = \tilde{\partial}_- R_+ = 0,}$$

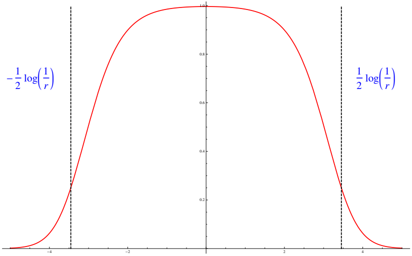
or $\tilde{p}^+ = 0$ for R_- and $\tilde{p}^- = 0$ for R_+.

Rapidity divergences in real corrections

New constraint allows to use same regularization for RDs in *virtual and real corrections*. Lipatov's vertex ($k = q_1 - q_2$, $k^2 = 0$):

$$\Gamma_{+\mu-} = -(\tilde{n}_+\tilde{n}_-)\left((q_1+q_2)_\mu + q_1^2\frac{\tilde{n}_\mu^-}{\tilde{q}_2^-} + q_2^2\frac{\tilde{n}_\mu^+}{\tilde{q}_1^+}\right) + 2\left(\tilde{q}_1^+\tilde{n}_\mu^- + \tilde{q}_2^-\tilde{n}_\mu^+\right),$$

without modified constraint, the Slavnov-Taylor identity $\boxed{k^\mu\Gamma_{+\mu-} = 0}$ is violated by terms $O(r)$.



The square of regularized LV:

$$\Gamma_{+\mu-}\Gamma_{+\nu-}P^{\mu\nu} = \frac{16\mathbf{q}_{T1}^2\mathbf{q}_{T2}^2}{\mathbf{k}_T^2}f(y),$$

$$\longleftarrow f(y) = \frac{1}{(re^{-y}+e^{y})^2(re^{y}+e^{-y})^2},$$

$$\int\limits_{-\infty}^{+\infty} dy\; f(y) = -1 - \log r + O(r)$$

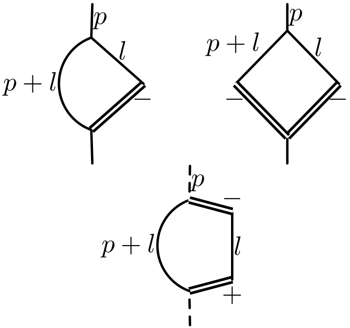
“Tadpoles” and “Bubbles”.

“Tadpoles” (one quadratic propagator):

$$A_{[-]}(p) = \int \frac{[d^d l]}{(p+l)^2[\tilde{l}^-]},\ \ A_{[--]}(p) = \int \frac{[d^d l]}{l^2[\tilde{l}^-][\tilde{l} - \tilde{p}]}$$

where $[d^D l] = \frac{(\mu^2)^\epsilon d^d l}{i\pi^{D/2} r_\Gamma}$, $r_\Gamma = \Gamma^2(1-\epsilon)\Gamma(1+\epsilon)/\Gamma(1-2\epsilon)$.

“Bubbles” (two quadratic propagators):



$$B_{[-]}(p) = \int \frac{[d^d l]}{l^2(p+l)^2[\tilde{l}^-]},$$

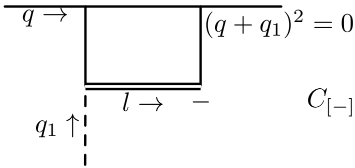
$$B_{[--]}(p) = \int \frac{[d^d l]}{l^2(p+l)^2[\tilde{l}^-][\tilde{l}^- + \tilde{p}^-]}$$

$$B_{[+-]}(p) = \int \frac{[d^d l]}{l^2(p+l)^2[\tilde{l}^+][\tilde{l}^-]},$$

where $p^+ = p^- = 0$ for the last integral.

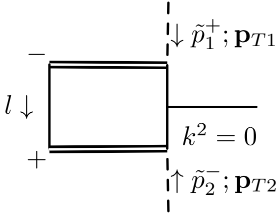
“Triangle” integrals

One light-cone propagator:



$$C_{[-]}(-q_1^2, q^2, q^-) = \int \frac{[d^d l]}{l^2(q_1+l)^2(q_1+q+l)^2[\tilde{l}^-]}.$$

Two light-cone propagators:



$$C_0^{[+-]} = \int \frac{[d^D l]}{q^2(p_1-l)^2(p_2+l)^2[\tilde{l}^+][\tilde{l}^-]}.$$

Rapidity divergences at one loop

Only log-divergence $\sim \log r$ (Blue cells in the table) is related with Reggeization of particles in t-channel.
Integrals which **do not** have log-divergence may still contain the power-dependence on r:

- $r^{-\epsilon} \to 0$ for $r \to 0$ and $\epsilon < 0$.
- $r^{+\epsilon} \to \infty$ for $r \to 0$ and $\epsilon < 0$ – **weak-power divergence** (Pink cells in the table)
- $r^{-1+\epsilon} \to \infty$ – **power divergence.** (Red)

(# LC prop.) \ (# quadr. prop.)	1	2	3	4
1	$A_{[-]}$	$B_{[-]}$	$C_{[-]}$	...
2	$A_{[+-]}$	$B_{[+-]}$	$C_{[+-]}$	...
3	...	...	...	...

The **weak-power** and **power-divergences** cancel between Feynman diagrams describing one region in rapidity, so only log-divergences are left.

Scalar integrals with power RDs.

Notation: $\left\{\frac{\mu}{k}\right\}^{2\epsilon} = \frac{1}{2}\left[\left(\frac{\mu}{k-i\varepsilon}\right)^{2\epsilon} + \left(\frac{\mu}{-k-i\varepsilon}\right)^{2\epsilon}\right]$.

Tadpoles:

$$
\begin{aligned}
A_{[-]}(p) &= -\frac{\tilde{p}^{-}\ r^{-1+\epsilon}}{\cos(\pi\epsilon)} \frac{1}{2\epsilon(1-2\epsilon)} \left\{\frac{\mu}{\tilde{p}^{-}}\right\}^{2\epsilon}, \\
A_{[--]}(p) &= \frac{1}{\tilde{p}_{-}} A_{[-]}(p).
\end{aligned}
$$

Bubbles:

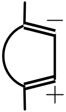
$$
\begin{aligned}
B_{[-]}(p) &= \frac{1}{p^{-}\epsilon^{2}} \left(\frac{\mu^{2}}{-p^{2}}\right)^{\epsilon} + \frac{1-2\epsilon}{\epsilon} \frac{r \cdot A_{[-]}(p)}{\tilde{p}_{-}^{2}} + \Delta B_{[-]}(-p^{2}, p_{-}) + O(r), \\
B_{[--]}(p) &= \frac{2}{\tilde{p}_{-}} B_{[-]}(p),
\end{aligned}
$$

where:

$$
\Delta B_{[-]}(-p^{2}, p_{-}) = -\frac{1}{p_{-}} \left(\frac{p_{-}^{2}\mu^{2}}{(-p^{2})^{2}}\right)^{\epsilon} \frac{\Gamma^{2}(1-2\epsilon)\Gamma(1+2\epsilon) \cdot r^{-\epsilon}}{2\epsilon^{2}\Gamma^{2}(1-\epsilon)}.
$$

Logarithmic RDs

- $[+-]$-bubble in transverse kinematics $p^- = p^+ = 0$:



$$B_{[+-]}(\mathbf{p}_T) = \frac{1}{\mathbf{p}_T^2}\left(\frac{\mu^2}{\mathbf{p}_T^2}\right)^{\epsilon}\frac{i\pi + 2\log r}{\epsilon},$$

- $[+-]$-bubble in $p^- = 0$ kinematics:

$$\begin{aligned} B_{[+-]}(\mathbf{p}_T, p^+) &= \frac{1}{\mathbf{p}_T^2}\left(\frac{\mu^2}{\mathbf{p}_T^2}\right)^{\epsilon}\frac{\Gamma^2(1+\epsilon)\Gamma(2+\epsilon)\sin(\pi\epsilon)}{\pi\epsilon^2} \\ &\times \left[i\pi + \log r - \log\frac{p_+^2}{\mathbf{p}_T^2} - \psi(1+\epsilon) + \psi(1)\right] + O(r^{1/2}) \end{aligned}$$

- $[+-]$-bubble in light-like kinematics $p^2 = 0$:

$$B_{[+-]}(\mathbf{p}_T^2, p^2 = 0) = \int \frac{[d^d l]}{l^2 (l+p)^2 [l^+][l^- + p^-]} = \frac{-2\Gamma(1-\epsilon)\Gamma(1+\epsilon)}{\mathbf{p}_T^2\epsilon^2}\left(\frac{\mu^2}{\mathbf{p}_T^2}\right)^{\epsilon}.$$

Triangle integrals, logarithmic RD

Result for $Q^2 = 0$:

$$C_{[-]}(t_1, 0, q^-) = \frac{1}{q^- t_1}\left(\frac{\mu^2}{t_1}\right)^{\epsilon}\frac{1}{\epsilon}\left[\log r + i\pi - \log\frac{|q_-|^2}{t_1} - \psi(1+\epsilon) - \psi(1) + 2\psi(-\epsilon)\right] + O(r^{1/2}),$$

coincides with the result of [G. Chachamis, *et. al.*, 2012].

Result for $Q^2 \neq 0$:

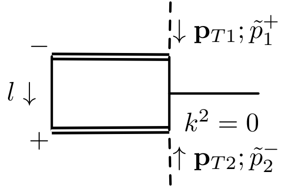
$$C_{[-]}(t_1, Q^2, q_-) = C_{[-]}(t_1, 0, q_-) + \left(\frac{\mu^2}{t_1}\right)^{\epsilon}\frac{I(Q^2/t_1)}{q_- t_1} - \frac{1}{t_1}\Delta B_{[-]}(Q^2, q_-),$$

where

$$\begin{aligned} I(X) &= -\frac{2X^{-\epsilon}}{\epsilon^2} - \frac{2}{\epsilon}\int\limits_0^X \frac{(1-x^{-\epsilon})dx}{1-x} \\ &= -\frac{2X^{-\epsilon}}{\epsilon^2} + 2\left[-\mathrm{Li}_2(1-X) + \frac{\pi^2}{6}\right] + O(\epsilon). \end{aligned}$$

Triangle with two light-cone propagators

Usual one-loop Feynman integrals with more than 4 propagators are reducible to more simple integrals up to terms $O(\epsilon)$.



We apply method of [Bern, Dixon, Kosower, 1992]. The $O(\epsilon)$ remnant is proportional to $(d-4)I^{(d+2)}$ and integral $I^{(6)}$ is finite. The resilt in Euclidean region ($p_1^+ > 0$, $-p_2^- > 0$, $\mathbf{p}_{T1,2}^2 > 0$):

$$C_{[+-]}(\mathbf{p}_{T1}^2, \mathbf{p}_{T2}^2, p_1^+, -p_2^-) = \frac{(-1)}{2\mathbf{p}_{T1}^2 \mathbf{p}_{T2}^2 \mathbf{k}_T^2} \times$$

$$\Big\{ \mathbf{p}_{T1}^2(\mathbf{p}_{T2}^2 - \mathbf{p}_{T1}^2 + \mathbf{k}_T^2)\left[B_{[+-]}(\mathbf{p}_{T1}^2, p_1^+) + (-p_2^-) C_{[-]}(\mathbf{p}_{T1}^2, \mathbf{p}_{T2}^2, -p_2^-)\right]$$
$$+ \mathbf{p}_{T2}^2(\mathbf{p}_{T1}^2 - \mathbf{p}_{T2}^2 + \mathbf{k}_T^2)\left[B_{[+-]}(\mathbf{p}_{T2}^2, -p_2^-) + p_1^+ C_{[+]}(\mathbf{p}_{T2}^2, \mathbf{p}_{T1}^2, p_1^+)\right]$$
$$- \mathbf{k}_T^2(\mathbf{p}_{T1}^2 + \mathbf{p}_{T2}^2 - \mathbf{k}_T^2) B_{[+-]}(\mathbf{k}_T^2, k^2 = 0) \Big\},$$

where $\mathbf{k}_T^2 = p_1^+(-p_2^-)$.

The $\log r$-divergence cancels within square brackets, as expected.

One-loop effective vertices

Advantages of the EFT formalism

- Gauge invariance (even at finite r!).
- **Possibility to work in covariant gauges.**
- *Provides foundation for k_T-factorization:* one-Reggeon exchange contribution is well-defined and gauge-invariant to all orders in α_s.

Parton Reggeization Approach (PRA): *gauge-invariant amplitudes with off-shell(Reggeized) initial-state partons from Lipatov's EFT should be used as short-distance parts in k_T-factorization calculations.*

Phenomenological applications to Dijet production [M.N., Saleev, Shipilova, 2012], $B\bar{B}$-correlations [Karpishkov, M.N., Saleev, 2017], J/ψ pair production [He, Kniehl, M.N., Saleev, 2019] and many more...

Thanks to Andreas and his `KaTie` code, the problem of tree-level calculations is solved for most of the practical purposes.

Next step: **go to NLO in PRA.**

Forward scattering vertices at one loop

We will consider two examples of Particle-Particle-Reggeon vertices:

$$\gamma^\star(q) + Q_+(q_1) \to q(q+q_1), \qquad (1)$$
$$\mathcal{O}(q) + R_+(q_1) \to g(q+q_1), \qquad (2)$$

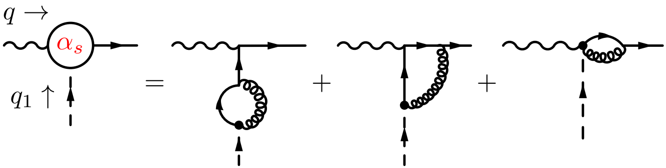
where $q^2 = Q^2 \neq 0$ and $\boxed{\tilde{q}_1^- = 0}$ to ensure GI at $r \neq 0$.

These examples are needed for the matching calculation to define unintegrated PDFs in PRA at NLO. The gluon-driven "DIS" process (1) is mediated by the operator:

$$\mathcal{O}(x) = \mathrm{tr}[G_{\mu\nu} G^{\mu\nu}].$$

$Q\gamma^\star q$-vertex

$\Gamma^{(1)}_{+\mu} =$



$$= C[\Gamma] \cdot \Gamma^{(0)}_{+\mu}(q_1, q) + C[\Delta^{(1)}] \cdot \Delta^{(1)}_{+\mu}(q_1, q) + C[\Delta^{(2)}] \cdot \Delta^{(2)}_{+\mu}(q_1, q)$$

Lorentz structures:

$$\Gamma^{(0)}_{+\mu}(q_1, k, q_2) = \gamma_\mu + \frac{\hat{q}_1 n^-_\mu}{[\tilde{k}^-]}, \qquad \longleftarrow \text{[Fadin, Sherman, 1976]}$$

$$\Delta^{(1)}_{+\mu}(q_1, q) = \frac{\hat{q}}{q_-}\left(n^-_\mu - \frac{2(q_1)_\mu}{q_1^+}\right), \; \Delta^{(2)}_{+\mu}(q_1, q) = \frac{\hat{q}}{q_-}\left(n^-_\mu - \frac{q_\mu}{q^+}\right)$$

Cancellation of RDs:

- $A_{[-]} \sim r^{-1+\epsilon}$ – cancels between diagrams
- $O(r^\epsilon)$-terms cancel between $B_{[-]}(q)$ and $B_{[-]}(q + q_1)$
- $O(r^{-\epsilon})$-terms cancel between $B_{[-]}(q)$ and $C_{[-]}$.
- **only $O(\log r)$-divergence from $C_{[-]}$ is left**

Expressions for the coefficients

$$C[\Gamma] = -\frac{\bar{\alpha}_s C_F}{4\pi}\frac{1}{2}\left\{\frac{[(d-8)Q^2+(d-6)t_1]B(t_1)-2(d-7)Q^2B(Q^2)}{Q^2-t_1}\right.$$

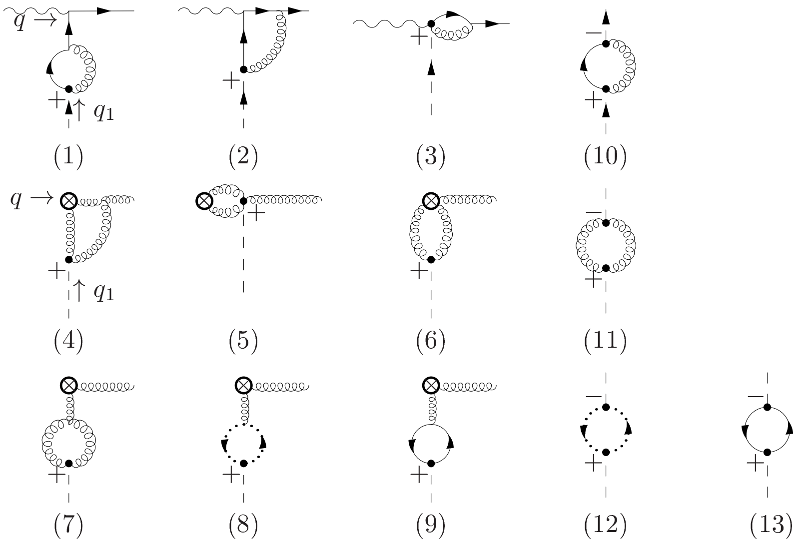
$$\left. -2\left[(Q^2-t_1)C(t_1,Q^2)-q_-\left(t_1C_{[-]}(t_1,Q^2,q_-)+(B_{[-]}(q)-B_{[-]}(q+q_1))\right)\right]\right\},$$

$$C[\Delta^{(1)}] = -\frac{\bar{\alpha}_s C_F}{4\pi}\frac{(Q^2+t_1)}{2(Q^2-t_1)^2}\left[\left((d-2)Q^2-(d-4)t_1\right)B(t_1)-2Q^2B(Q^2)\right],$$

$$C[\Delta^{(2)}] = -\frac{\bar{\alpha}_s C_F}{4\pi}\frac{Q^2}{(Q^2-t_1)^2}\left[\left((d-6)t_1-(d-8)Q^2\right)B(Q^2)+2(t_1-2Q^2)B(t_1)\right],$$

were $\bar{\alpha}_s = \frac{\mu^{-2\epsilon}g_s^2}{(4\pi)^{1-\epsilon}}r_\Gamma$, $t_1 = -q_1^2$.

$RO g$-vertex



$RO g$-vertex

The one-loop correction is proportional to the Born vertex:

$$G^{(0)}_{+\mu} = \frac{i}{2}\left((Q^2 - t_1)n^-_\mu - 2q_-(q_1)_\mu\right),$$

with the coefficient

$$\begin{aligned}
&C\left[G^{(0)}_+\right] = \frac{\bar{\alpha}_s}{4\pi}\frac{1}{2}\left\{\frac{B(t_1)}{(d-2)(d-1)(Q^2-t_1)^2}\right.\\
&\times\left[C_A\left((d-2)(5d-4)Q^4 - 2(d(7d-24)+16)Q^2 t_1\right.\right.\\
&\left.\left.+(d-2)(5d-4){t_1}^2\right) - 2(d-2)^2 n_F (Q^2-t_1)^2\right]\\
&-\frac{2C_A(d-4)Q^2 B(Q^2)}{(d-2)(Q^2-t_1)^2}\left[(d-4)Q^2 - (d-2)t_1\right]\\
&\left.-2C_A\left[q_-\left(t_1 C_{[-]}(t_1, Q^2, q_-) + B_{[-]}(q) - B_{[-]}(q+q_1)\right) + (t_1 - Q^2)C(t_1, Q^2)\right]\right\}.
\end{aligned}$$

Comparison with QCD

Test process: DIS on the on-shell partonic target

To perform comparison with QCD we consider the processes:

$$\gamma^*(q) + \gamma(P) \to q(k_1) + \bar{q}(k_2),$$
$$\mathcal{O}(q) + g(P) \to g(k_1) + g(k_2),$$

we introduce the usual variables: $Q^2 = -q^2,\ x_B = \frac{Q^2}{2(qP)}$, and work in the (q, P) center of mass frame, where $q^+ = -x_B P^+$, $q^- = Q^2/(x_B P^+)$, $\mathbf{q}_T = 0$.
We parametrize final-state momenta as:

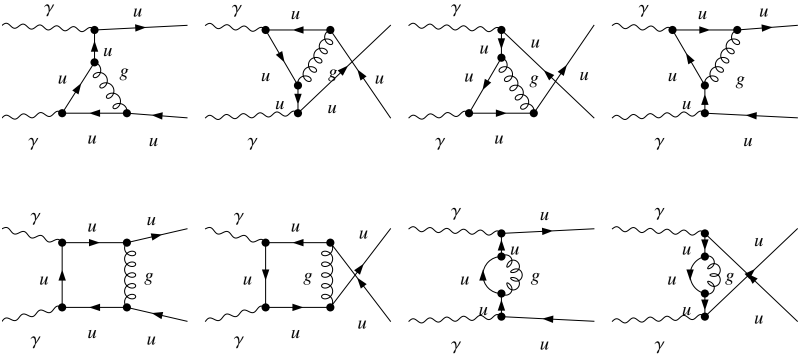
$$k_1 = q + q_1,\ k_2 = P - q_1,$$
$$t_1 = -\mathbf{q}_{T1}^2,\ x_1 = \frac{q_1^+}{P^+} = x_B \frac{Q^2 + t_1}{Q^2} \text{ for } x_B \ll 1.$$

And will study the interference of one-loop and Born amplitudes (projected on F_2 structure function in the photon case):

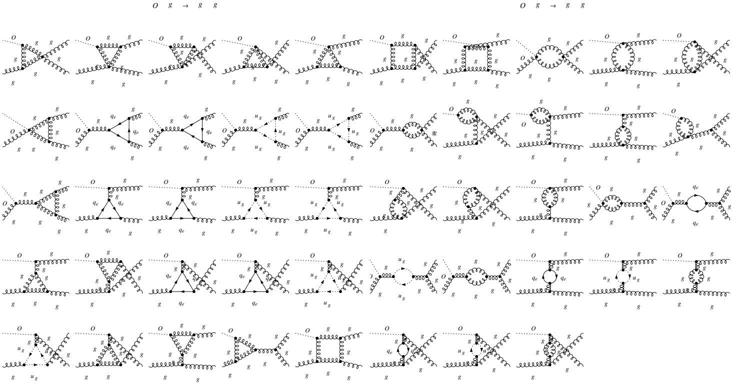
$$F_{\mathcal{O}} \text{ or } F_2(x_B, Q^2, t_1) \text{ in the limit } x_B \ll 1.$$

QCD diagrams at one loop, photon case

$\gamma \quad \gamma \quad \to \quad u \quad u$



QCD diagrams at one loop, gluon case



QCD result at one loop, photon case

The QCD result (**leading power for $x_B \ll 1$, but exact in Q^2 and** t_1):

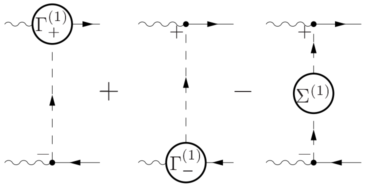
$$F_2^{(1,\mathrm{QCD})}(Q^2, t_1, x_B) = \frac{\bar{\alpha}_s C_F}{4\pi} \left\{ -\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - \frac{2L_1}{\epsilon} + \left(\frac{2\pi^2}{3} - 7 - L_1^2 - 3L_1 \right) + \right.$$
$$+ \left(\frac{1}{\epsilon} + \log \frac{\mu^2}{t_1} \right) \left(\log \frac{1}{x_B^2} - 2\pi i \right) + L_2^2 + 2\mathrm{Li}_2 \left(1 - \frac{Q^2}{t_1} \right)$$
$$\left. - \frac{1}{(Q^2 - t_1)^2} \left[Q^2 (Q^2 - t_1) + (3t_1^2 - 4Q^2 t_1) L_2 \right] \right\} + O(\epsilon, x_B),$$

where $L_1 = \log(\mu^2/Q^2)$, $L_2 = \log(Q^2/t_1)$, contains:

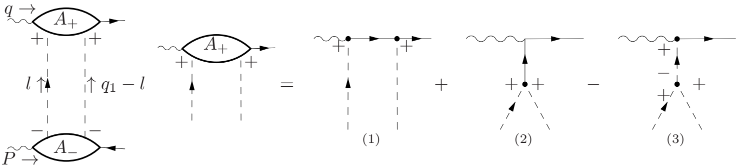
- The $1/\epsilon^2$ and $1/\epsilon$ IR-divergences,
- Single-log part in $\log x_B^{-1}$ and *imaginary part*,
- The complicated dependence on Q^2/t_1.

Contributions in the EFT, photon case

One-Reggeon contribution (*positive signature*, Re+Im parts @ 1 loop):



Two-Reggeon contribution (*negative signature*, Im part @ 1 loop):



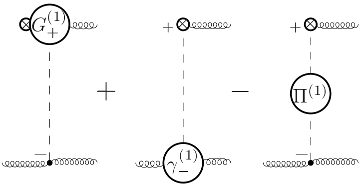
Sum of one and two-Reggeon contributions reproduces QCD result exactly.

QCD result at one loop, gluon case

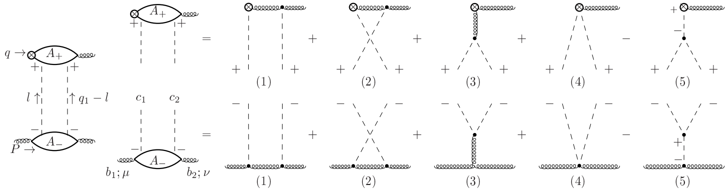
$$
\begin{aligned}
&F_{\mathcal{O}}^{(1,\mathrm{QCD})}(Q^2, t_1, x_B) = \\
&\frac{\bar{\alpha}_s C_A}{4\pi}\left\{-\frac{3}{\epsilon^2} - \frac{1}{\epsilon}(3L_1 + L_2) + \left(\frac{1}{\epsilon} + \log\frac{\mu^2}{t_1}\right)\left(\log\frac{1}{x_B^2} - i\pi\right)\right. \\
&\left. + 2\mathrm{Li}_2\left(1 - \frac{Q^2}{t_1}\right) + \frac{1}{2}(L_2 - 3L_1)(L_1 + L_2) + \frac{2\pi^2}{3}\right\} + O(\epsilon, x_B),
\end{aligned}
$$

Contributions in the EFT, gluon case

One-Reggeon contribution (*negative signature*, Re+Im parts @ 1 loop):



Two-Reggeon contribution (*positive signature*, does not contribute due to color):



The one-Reggeon contribution reproduces QCD result exactly.

Conclusions

- The consistent procedure of rapidity regularization is proposed. One should modify not only Wilson lines, but also kinematic constraints.
- One-loop integrals with log-RDs are identified. The power-RDs are contained just in a few simplest integrals.
- Triangle integrals with one and two scales are calculated.
- Reduction of one-loop integrals with more than four propagators (quadratic or light-cone) seems to work similar to the case of ordinary loop integrals.
- The one-loop corrections to the two-scale $Q\gamma^\star q$ and $R\mathcal{O}g$-vertices are computed. Comparison with QCD works!
- The triple-Reggeon vertex is needed for the subtraction terms in the two-Reggeon contributions.
- The power-RDs cancel in amplitudes with Reggeized gluons and quarks. *Is there a simple explanation?*

Thank you for your attention!