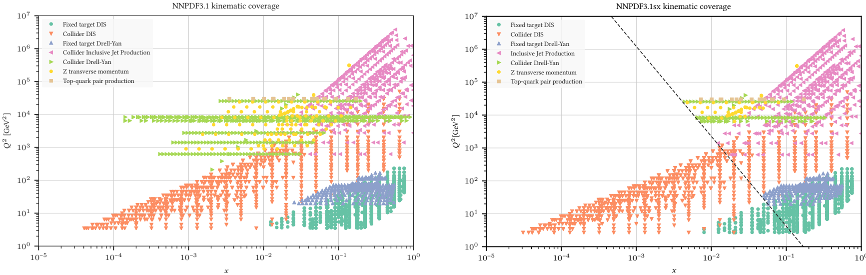
Drell-Yan at small x

Marco Bonvini, Rhorry Gauld,
Tommaso Giani and Simone Marzani

September 10, 2019

Parton distributions with small-x resummation: PDFs determination with small-x resummation of DGLAP evolution and DIS coefficient functions.
[R.D.Ball, V.Bertone, M.Bonvini, S.Marzani, J.Rojo, L.Rottoli 2017]

DY fixed-target and hadron collider cross sections obtained using fixed-order partonic results: additional kinematic cuts.



Most of W and Z Drell-Yan data are removed. In particular all the LHCb points are not considered.

Current situation

- Resummed inclusive Drell-Yan
  - ☑ general formalism and analytical results [Marzani, Ball 2009]
  - ☐ implementation of the results for pheno studies
- Resummed Drell-Yan differential distributions
  - ☐ general formalism and analytical results
  - ☐ implementation of the results for pheno studies

Resummation of differential distributions

Differential distributions

- rapidity single differential distributions for Higgs production [Caola, Forte, Marzani 2011],[Forte, Muselli 2016]
- transverse momentum and rapidity double differential distribution for Higgs production [Muselli 2017]

Generic hadronic process

$$h_1\left(P_1\right)+h_2\left(P_2\right)\to\mathcal{S}\left(p_{\mathcal{S}}\right)+X$$

Rapidity of $\mathcal{S}$

$$Y\equiv\frac{1}{2}\log\frac{E_{\mathcal{S}}+p_{\mathcal{S}_z}}{E_{\mathcal{S}}-p_{\mathcal{S}_z}}$$

Distribution differential in the rapidity of the final states $\mathcal{S}$

$$\frac{d\sigma}{dY}\left(x_h,Y,Q^2,\alpha_s\right)=\sum_{a,b}\int_{\sqrt{x_h}e^{Y}}^{1}dx_1\int_{\sqrt{x_h}e^{-Y}}^{1}dx_2$$

$$\underbrace{\boxed{\frac{d\sigma_{ab}}{dy}\left(\frac{x_h}{x_1x_2},\frac{\mu^2}{Q^2},Y-\frac{1}{2}\log\frac{x_1}{x_2},\alpha_s\right)}}_{\text{partonic cross section}}f_a^{h_1}\left(x_1,\mu^2\right)f_b^{h_2}\left(x_2,\mu^2\right)$$

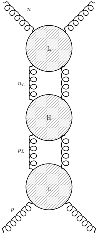
In an axial gauge high energy enhanced terms only come from cut diagrams which are two-gluon reducible in the t-channel: $d\sigma/dy$ can then be split in a process-dependent 2GI and a generally two-gluon reducible part.

Working in the partonic centre of mass frame:

$$\frac{d\sigma}{dy}\left(x, \frac{\mu^2}{Q^2}, y, \alpha_s\right) = \int \frac{|\mathcal{M}|^2}{2s}\, d\Pi_f\, \delta\left(y - \frac{1}{2}\log\frac{E_{\mathcal{S}} + p_{\mathcal{S}_z}}{E_{\mathcal{S}} - p_{\mathcal{S}_z}}\right)$$

$$= \int \frac{Q^2}{2s} \underbrace{\boxed{H^{\mu\nu\alpha\beta}\left(p_L, n_L, p_F, \mu_R, \mu_F, \alpha_s\right)}}_{\text{Hard part (2GI)}}$$

$$\underbrace{\boxed{L_{\mu\nu}\left(p_L, p, \mu_R, \mu_F, \alpha_s\right)}}_{\text{Ladder part 1}} \underbrace{\boxed{L_{\alpha\beta}\left(n_L, n, \mu_R, \mu_F, \alpha_s\right)}}_{\text{Ladder part 2}} [dp_L]\,[dn_L]$$



$$p_L = zp - k - \frac{\vec{k}_T^2}{s(1-z)}, \quad k = \left(0, \vec{k}_T, 0\right)$$

$$n_L = \bar{z}n - \bar{k} - \frac{\vec{\bar{k}}_T^2}{s(1-z)}, \quad \bar{k} = \left(0, \vec{\bar{k}}_T, 0\right)$$

Hard part

High-energy projector $\mathcal{P}_{\mu\nu} \equiv \frac{k^\mu k^\nu}{\vec{k}^2}$ selects the leading behaviour in the small-x region $\left(z \ll 1,\ \vec{k}_T^2/s \ll 1\right)$ [Catani, Hautman 1994]

$$C_y\left(x, z, \bar{z}, k, \bar{k}, y, \alpha_s\right) =$$
$$\int_0^{2\pi} \frac{d\theta}{2\pi} \int_0^{2\pi} \frac{d\bar{\theta}}{2\pi} \frac{Q^2}{2s\bar{z}z} \left[\mathcal{P}_{\mu\nu}\mathcal{P}_{\alpha\beta} H^{\mu\nu\alpha\beta}\left(n_L, p_L, p_{\mathcal{F}}, \alpha_s\right)\right] \times \delta\left(y - \frac{1}{2}\log\frac{E_{\mathcal{S}} + P_{\mathcal{S}}}{E_{\mathcal{S}} - P_{\mathcal{S}}}\right)$$

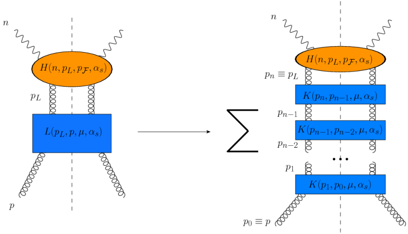
Higgs production

hard part C_y not only 2GI, but also 2PI
$\rightarrow$ no collinear singularities in the hard part

$$\lim_{k,\bar{k}\to 0} C_y = \sigma_y^{\text{on-shell}}\left(g\left(\bar{z}n\right) + g\left(zp\right) \to \mathcal{F}\right)$$

This will not be the case for DY.

Ladder part



The ladder part can be decomposed in a series of 2GI kernels. Computing the cross section for n insertion of the kernel one has to subtract the first $n-1$ poles according to $\overline{\text{MS}}$ scheme.
[Fig. from 1010.2743]

Taking the Fourier-Mellin transformed of the partonic rapidity distribution

$$\frac{d\sigma}{dy}(N,b) = \int dx\, x^{N-1} \int dy\, e^{iby} \frac{d\sigma}{dy}(x,y)\,.$$

the result for n kernels insertions reads

$$\frac{d\sigma^n}{dy}(N,b,\alpha_s) = \left[\gamma\left(N + i\frac{b}{2}\right)\right]^n \int_{\mu^2}^{\infty} \frac{dk_T^2}{k_T^2}\, C_y\left(N, \frac{k_T^2}{Q^2}, b, \alpha_s\right) \frac{\log^{n-1}\frac{k_T^2}{\mu^2}}{(n-1)!}.$$

and summing over all the emissions we get

$$\frac{d\sigma}{dy}(N,b,\alpha_s) = \sum_{n=1}^{\infty} \frac{d\sigma^n}{dy} = \gamma\left(\frac{\alpha_s}{N + i\frac{b}{2}}\right) \int_{\mu^2}^{\infty} dk_T^2\, \left(k_T^2\right)^{\gamma\left(\frac{\alpha_s}{N+i\frac{b}{2}}\right)-1} C_y\left(N, \frac{k_T^2}{Q^2}, b, \alpha_s\right)$$

The full result in $Q_0\overline{\mathsf{MS}}$ scheme reads

$$\frac{d\sigma}{dy}\left(N,\frac{\mu^2}{Q^2},b,\alpha_s\right)=\gamma\left(\frac{\alpha_s}{N+i\frac{b}{2}}\right)\int_0^\infty d\xi\,\xi^{\gamma\left(\frac{\alpha_s}{N+i\frac{b}{2}}\right)-1}$$
$$\gamma\left(\frac{\alpha_s}{N-i\frac{b}{2}}\right)\int_0^\infty d\bar{\xi}\,\bar{\xi}^{\gamma\left(\frac{\alpha_s}{N-i\frac{b}{2}}\right)-1}C_y\left(N,\xi,\bar{\xi},b,\alpha_s\right),\qquad \text{with}\quad \xi\equiv\vec{k}_T^2/Q^2.$$

To account for running coupling effects [M. Bonvini, S. Marzani, T. Peraro 2016]

$$\xi^\gamma\to U$$

with

$$U\left(N,k_T^2,\mu_F^2\right)=\exp\int_{\mu_F^2}^{k_T^2}\frac{d\mu^2}{\mu^2}\gamma_+\left(N,\alpha_s\left(\mu^2\right)\right)$$

getting

$$\frac{d\sigma}{dy}\left(N,b,\alpha_s\right)=\int_0^\infty d\xi\,\frac{d}{d\xi}U\left(N-\frac{ib}{2},Q^2\xi,\mu_F^2\right)$$
$$\int_0^\infty d\bar{\xi}\,\frac{d}{d\bar{\xi}}U\left(N+\frac{ib}{2},Q^2\bar{\xi},\mu_F^2\right)\mathcal{C}_y\left(N,\xi,\bar{\xi},b,\alpha_s\right).$$

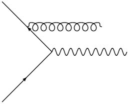
The result can be extended to double differential distribution [Muselli 2017]

$$\mathcal{C}_y\to\mathcal{C}_{y,p_T}$$

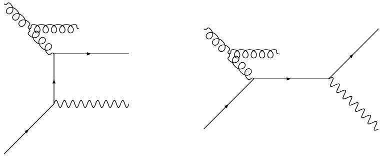
Drell-Yan

Only some of the higher order corrections contain high energy logarithms, and these are the ones we wish to isolate and compute: we have to identify the leading channels at high-energy ($N \to 0$ in Mellin space).

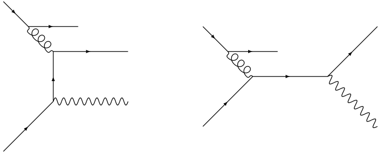
$$\gamma_{gg} \propto \frac{\alpha_s}{N}, \qquad \gamma_{qq} \propto \alpha_s, \qquad \gamma_{gq} \propto \frac{C_F}{C_A}\gamma_{gg}$$



$$C_{q\bar{q}} \propto \mathcal{O}\left(\alpha_s^k\right)$$



$$C_{qg} \propto \mathcal{O}\left(\alpha_s \left(\frac{\alpha_s}{N}\right)^k\right) \to \text{NLL}$$

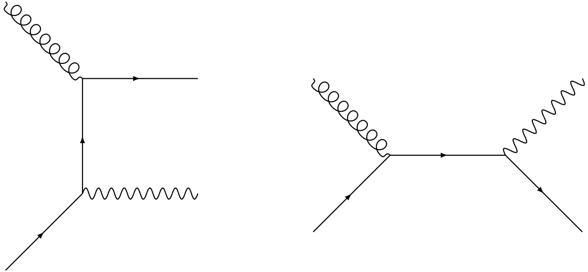


$$C_{qq} \propto \mathcal{O}\left(\alpha_s \left(\frac{\alpha_s}{N}\right)^k\right) \to \text{NLL}$$

NLLx channels: C_{gq}, C_{qq} [Marzani, Ball 2009]

The NLLx singularities are determined by the off-shell processes

$$g^* q \to W^{\pm}\, q' \quad \text{[Andersen, Del Duca, Maltoni, Stirling 2001]} \tag{1}$$
$$g^* q \to Z\, q \tag{2}$$
$$g^* q \to \gamma^*\, q \tag{3}$$



- when the transverse momentum of the vector boson p_T is 0, these diagrams do contain collinear singularities: unlike the case of Higgs production, for DY we need to subtract them
- just one off-shell leg

Consider the $q\,g$ channel, so let one of the initial legs go on-shell

$$U(N+\frac{ib}{2},Q^2\bar{\xi},\mu_F^2)\to\theta(\bar{\xi}),\quad\Leftrightarrow\quad\frac{d}{d\bar{\xi}}U(N+\frac{ib}{2},Q^2\bar{\xi},\mu_F^2)\to\delta(\bar{\xi}),$$

get

$$\sigma_{p_T,y}\left(N,p_T,b,\alpha_s\right)=-\int_0^\infty d\xi\,U\left(N-\frac{ib}{2},Q^2\xi,\mu_F^2\right)\frac{d}{d\xi}C_{p_T,y}\left(0,\xi,p_T,b,\alpha_s\right)$$

Taking the inverse Fourier-Mellin transform get

$$\begin{aligned}\sigma_{p_T,y}\left(z,p_T,y,\alpha_s\right)&\equiv\int\frac{dN}{2\pi i}\,z^{-N}\int\frac{db}{2\pi}\,e^{iby}\,\sigma_{p_T,y}\left(N,p_T,b,\alpha_s\right)\\&=-\int d\xi\,\frac{d}{d\xi}\,C_{p_T,y}\left(0,\xi,p_T,y+\frac{1}{2}\log\frac{1}{z},\alpha_s\right)U\left(z,Q^2\xi,\mu_F^2\right)\end{aligned}$$

with

$$U\left(z,Q^2\xi,\mu_F^2\right)=\int\frac{dN}{2\pi i}\,z^{-N}U\left(N,Q^2\xi,\mu_F^2\right)$$

Subtraction of collinear singularities

- The result of the prevoius slide is valid for $p_T > 0$, i.e. when the vector boson is recoiling against QCD radiation of non-vanishing transverse momentum.
- It is interesting to consider also distributions that are sensitive to the zero transverse-momentum bin

$$\int_0^{\bar{p}_T} dp_T \; \sigma_{p_T,y} = \sigma_y - \int_{\bar{p}_T}^{\infty} dp_T \; \sigma_{p_T,y}$$

σ_y contains collinear singularities which have been subtracted.

Work in the Fourier-Mellin space

$$\boxed{\sigma_y^{(0)} \propto h_{qg}\left(N, \gamma_{gg}, \alpha_s\right)\Gamma_{gg}\left(N-\frac{ib}{2}, \alpha_s, \epsilon\right)+\mathcal{O}\left(\alpha_s^2\left(\frac{\alpha_s}{N}\right)^k\right)}$$

High energy factorization

$$\boxed{\sigma_y^{(0)} = \Gamma_{qg}\left(N-\frac{ib}{2}, \alpha_s, \epsilon\right)+\sigma_y\left(N, b, \alpha_s\right)\,\Gamma_{gg}\left(N-\frac{ib}{2}, \alpha_s, \epsilon\right)+\mathcal{O}\left(\alpha_s^2\left(\frac{\alpha_s}{N}\right)^k\right)}$$

Collinear factorization

Comparing the derivatives w.r.t. Q^2 of the previous two Eqs. get

$$\begin{aligned}\sigma_y\left(N, b, \alpha_s\right) = \int_0^\infty d\xi \frac{d}{d\xi} U\left(N-\frac{ib}{2}, Q^2\xi, \mu_F^2\right) C_y\left(0, \xi, b, \alpha_s\right) \\ -S_{qg}\left(N-\frac{ib}{2}, \mu_F^2\right)\end{aligned}$$

with the subtraction term given by

$$\begin{aligned}S_{qg}\left(N, \mu_F^2\right) = \int_{Q_0^2}^{\mu_F^2} \frac{dq^2}{q^2}\gamma_{gg}\left(N, \alpha_s\left(q^2\right)\right) \\ \times\ \exp\left[\int_{\mu_F^2}^{q^2}\frac{d\mu^2}{\mu^2}\gamma_+\left(N, \alpha_s\left(\mu^2\right)\right)\right]\end{aligned}$$

Work in progress

- ◯ Computation of the resummed DY differential distribution
  - ☑ analytical computation of the off-shell corss sections
  - ☑ in-house numerical implementation of fixed order results in leptons kinematic
  - ☐ Hell implementation of resummed coefficients
  - ☐ matching to fixed order results

Experimental data and resummed PDFs

- NNPDF31sx : 821 hadronic points over 1187
- Having small-x data at high Q^2 would be a strong consistency check of the results of the previous small-x PDFs determination.
- LHCb measurements
  1. Inclusive W and Z production in the forward region at $\sqrt{s} = 7$ TeV [1204.1620]
  2. Measurement of the cross-section for $Z \to e^+e^-$ production in pp collisions at $\sqrt{s} = 7$ TeV [1212.4620]
  3. Measurement of the forward Z boson production cross-section in pp collisions at $\sqrt{s} = 7$ TeV [1505.07024]
  4. Measurement of forward W and Z boson production in pp collisions at $\sqrt{s} = 8$ TeV [1511.08039]
  5. Ongoing analysis of the off-shell process $p\,p \to l_+\,l_-$ for $m_{ll} \sim 10$ GeV, not published yet

Thanks!