Forward jet production in proton-nucleus collisions at high energy: from trijet to NLO dijet

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Partially based on hep-ph/1809.05526 (with E. Iancu)

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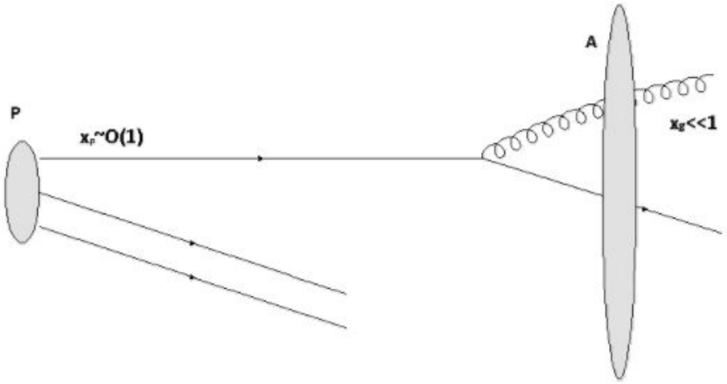
Towards accuracy at small x, Edinburgh, 10-13/Sep.

Forward Jet Production

In our approach we adopt the formalism of the light-cone wave function in perturbative QCD, together with the hybrid factorization. The derivation of the forward LO dijet cross-section was done in hep-ph/0708.0231 (C. Marquet).

The basic setup: a large-x parton from the proton scatters off the small-x gluon distribution in the target nucleus. Large-x parton is most likely a quark.



Quark fragmentation in the presence of a shockwave.

The time evolution of the initial (bare) quark state is given by:

$$\left|q_\lambda^\alpha(q^+,\,\boldsymbol{q})\right\rangle_{\rm in} \equiv U(0,\,-\infty)\,\left|q_\lambda^\alpha(q^+,\,\boldsymbol{q})\right\rangle$$

Where U denotes a unitary operator:

$$U(t,t_0) = \mathrm{T}\exp\left\{-i\int_{t_0}^{t} dt_1\, H_I(t_1)\right\}$$

The information both on the time evolution and interaction of the bare quark with the target nucleus is given by the “outgoing state”:

$$\left|q_\lambda^\alpha(q^+,\,\boldsymbol{w})\right\rangle_{out} \equiv U(\infty,\,0)\,\hat{S}\,U(0,\,-\infty)\,\left|q_\lambda^\alpha(q^+,\,\boldsymbol{w})\right\rangle$$

This state will be shown to generate all the possible insertions of the shockwave. More importantly, the outgoing state is directly related to expectation values:

$$\left\langle \hat{\mathcal{O}} \right\rangle = \left\langle \langle q|\, U^\dagger\, \hat{S}\, U\, \hat{\mathcal{O}}\, U^\dagger\, \hat{S}\, U\, |q\rangle \right\rangle_{cgc}$$

The LO Outgoing State

The production state at leading order is given by

$$\left|q^{\alpha}_{\lambda}(q^+,\,\boldsymbol{w})\right\rangle^{(g)}_{out} \equiv U(\infty,\,0)\,\hat{S}\,U(0,\,-\infty)\,\left|q^{\alpha}_{\lambda}(q^+,\,\boldsymbol{w})\right\rangle = \left|\psi^{\alpha}_{\lambda}(q^+,\,\boldsymbol{w})\right\rangle_{qg} + \left|q^{\alpha}_{\lambda}(q^+,\,\boldsymbol{w})\right\rangle$$

$$\left|\psi^{\alpha}_{\lambda}\right\rangle_{qg} = \left|q^{\gamma}g^{b}\right\rangle\left(-\left\langle q^{\gamma}g^{b}\right|\hat{S}\left|q^{\beta}g^{a}\right\rangle\frac{\left\langle q^{\beta}g^{a}\right|H_{q\to qg}\left|q^{\alpha}\right\rangle}{E_{qg}-E_{q}} + \frac{\left\langle q^{\gamma}g^{b}\right|H_{q\to qg}\left|q^{\beta}\right\rangle}{E_{qg}-E_{q}}\left\langle q^{\beta}\right|\hat{S}\left|q^{\alpha}\right\rangle\right)$$

Where only terms of order g were kept. The following result is obtained for the |qg> contribution:

$$\left|\psi^{\alpha}_{\lambda}(q^+,\,\boldsymbol{w})\right\rangle_{qg} = \int_{\boldsymbol{x},\,\boldsymbol{z}}\int_0^1 d\vartheta\,\frac{ig\phi^{ij}_{\lambda_1\lambda}(\vartheta)\sqrt{q^+}\,\boldsymbol{X}^j}{4\pi^{3/2}\sqrt{\vartheta}\,\boldsymbol{X}^2}\,\delta^{(2)}(\boldsymbol{w}-(1-\vartheta)\boldsymbol{x}-\vartheta\boldsymbol{z})$$
$$\times\left[V^{\gamma\beta}(\boldsymbol{x})\,U^{ba}(\boldsymbol{z})\,t^{a}_{\beta\alpha} - t^{b}_{\gamma\beta}\,V^{\beta\alpha}(\boldsymbol{w})\right]\left|q^{\gamma}_{\lambda_1}((1-\vartheta)q^+,\,\boldsymbol{x})\,g^{b}_{i}(\vartheta q^+,\,\boldsymbol{z})\right\rangle$$

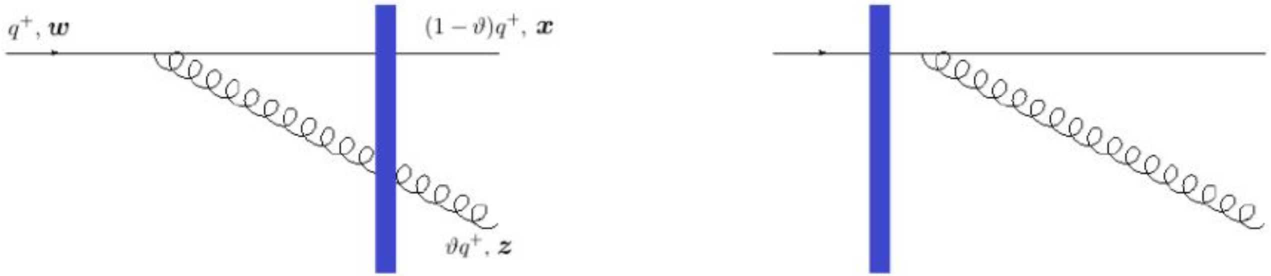
$$\boldsymbol{X} \equiv \boldsymbol{x}-\boldsymbol{z} \qquad \vartheta \equiv k^+/q^+$$

$$\phi^{ij}_{\lambda_1\lambda}(\vartheta) \equiv \chi^{\dagger}_{\lambda_1}\left[(2-\vartheta)\delta^{ij} - i\vartheta\varepsilon^{ij}\sigma^3\right]\chi_{\lambda}$$

$$U(\boldsymbol{x}) = \mathrm{T}\exp\left\{ig\int dx^+\,T^a A^-_a(x^+,\,\boldsymbol{x})\right\}$$

$$V(\boldsymbol{x}) = \mathrm{T}\exp\left\{ig\int dx^+\,t^a A^-_a(x^+,\,\boldsymbol{x})\right\}$$

Diagrammatically (blue bar denotes a shockwave = interaction with the target):



One gluon production at leading order with shockwave before and after the emission.

The LO forward dijet cross-section

From the production state we can pass easily to the quark-gluon dijet cross section:

$$\frac{d\sigma_{\rm LO}^{qA\to qg+X}}{d^3k\, d^3p} \equiv \frac{1}{2N_c\, L}\, {}^{(g)}_{out}\langle q_\lambda^\alpha(q^+,\boldsymbol{q})|\, \hat{\mathcal{N}}_q(p)\hat{\mathcal{N}}_g(k)\, |q_\lambda^\alpha(q^+,\boldsymbol{q})\rangle^{(g)}_{out}$$

$$= \frac{1}{2N_c\, L}\int_{\boldsymbol{w},\overline{\boldsymbol{w}}} e^{i(\boldsymbol{w}-\overline{\boldsymbol{w}})\cdot\boldsymbol{q}}\, {}_{qg}\langle \psi_\lambda^\alpha(q^+,\overline{\boldsymbol{w}})|\, \hat{\mathcal{N}}_q(p)\hat{\mathcal{N}}_g(k)\, |\psi_\lambda^\alpha(q^+,\boldsymbol{w})\rangle_{qg}$$

The following number density operators were introduced:

$$\hat{\mathcal{N}}_q(p) \equiv \frac{1}{(2\pi)^3}\, b_\lambda^{\alpha\dagger}(p)\, b_\lambda^\alpha(p) \qquad\qquad \hat{\mathcal{N}}_g(k) \equiv \frac{1}{(2\pi)^3}\, a_i^{a\dagger}(k)\, a_i^a(k)$$

Then the result for the leading-order dijet cross section is given by:

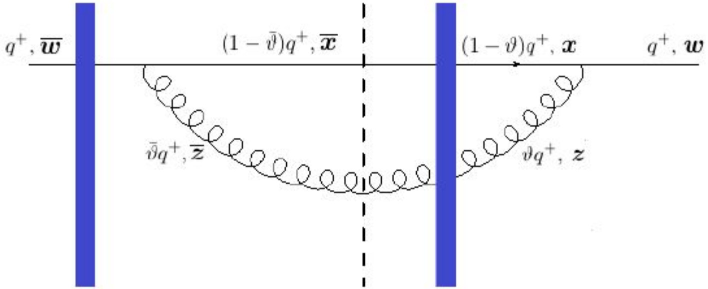
$$\frac{d\sigma_{\rm LO}^{qA\to qg+X}}{dk^+\, d^2\boldsymbol{k}\, dp^+\, d^2\boldsymbol{p}} = \frac{2\alpha_s C_F\left(1+(1-\vartheta)^2\right)}{(2\pi)^6\vartheta q^+}\,\delta(q^+-k^+-p^+)$$

$$\times\int_{\boldsymbol{x},\overline{\boldsymbol{x}},\boldsymbol{z},\overline{\boldsymbol{z}}} \frac{\boldsymbol{X}\cdot\overline{\boldsymbol{X}}}{\boldsymbol{X}^2\,\overline{\boldsymbol{X}}^2}\, \mathrm{e}^{-i\boldsymbol{p}\cdot(\boldsymbol{x}-\overline{\boldsymbol{x}})-i\boldsymbol{k}\cdot(\boldsymbol{z}-\bar{\boldsymbol{z}})}\, \mathbb{S}_{\rm LO}\left(\overline{\boldsymbol{w}},\,\overline{\boldsymbol{x}},\,\overline{\boldsymbol{z}},\,\boldsymbol{w},\,\boldsymbol{x},\,\boldsymbol{z}\right)$$

with $\boldsymbol{X}\equiv\boldsymbol{x}-\boldsymbol{z}$, $\overline{\boldsymbol{X}}\equiv\overline{\boldsymbol{x}}-\overline{\boldsymbol{z}}$, $\boldsymbol{w}=(1-\vartheta)\boldsymbol{x}+\vartheta\boldsymbol{z}$ and $\overline{\boldsymbol{w}}=(1-\vartheta)\overline{\boldsymbol{x}}+\vartheta\overline{\boldsymbol{z}}$.

$$\mathbb{S}_{\rm LO}\left(\overline{\boldsymbol{w}},\,\overline{\boldsymbol{x}},\,\overline{\boldsymbol{z}},\,\boldsymbol{w},\,\boldsymbol{x},\,\boldsymbol{z}\right) \equiv S_{qgqg}\left(\overline{\boldsymbol{x}},\,\overline{\boldsymbol{z}},\,\boldsymbol{x},\,\boldsymbol{z}\right) - S_{qgq}\left(\overline{\boldsymbol{w}},\,\boldsymbol{x},\,\boldsymbol{z}\right) - S_{qgq}\left(\overline{\boldsymbol{x}},\,\boldsymbol{w},\,\overline{\boldsymbol{z}}\right) + \mathcal{S}\left(\overline{\boldsymbol{w}},\,\boldsymbol{w}\right)$$

There are four different insertions of Wilson lines. For example, below is the relevant diagram which corresponds to $S_{q\bar{q}g}(\overline{\boldsymbol{w}},\, \boldsymbol{x},\, \boldsymbol{z})$ (the location of the measurement is denoted by a dashed line).



Where the following combinations of Wilson lines were introduced (in the large Nc limit these combinations represent the quadropole-dipole and dipole-dipole interactions):

$$S^{(1)}_{q\bar{q}gg}(\overline{x},\, \overline{z},\, x,\, z) \equiv \frac{1}{C_F N_c}\,\mathrm{tr}\left(V^\dagger(\overline{x})\, V(x)\, t^a\, t^c\right)\left[U^\dagger(\overline{z})\, U(z)\right]^{ca}$$
$$= \frac{1}{2C_F N_c}\left(N_c^2\, \mathcal{Q}(\overline{x},\, x,\, z,\, \overline{z})\, \mathcal{S}(\overline{z},\, z) - \mathcal{S}(\overline{x},\, x)\right) \simeq \mathcal{Q}(\overline{x},\, x,\, z,\, \overline{z})\, \mathcal{S}(\overline{z},\, z)$$

$$S_{q\bar{q}g}(\overline{w},\, x,\, z) \equiv \frac{1}{C_F N_c}\,\mathrm{tr}\left(V^\dagger(\overline{w})\, t^b\, V(x)\, t^a\right) U^{ba}(z)$$
$$= \frac{1}{2C_F N_c}\left(N_c^2\, \mathcal{S}(\overline{w},\, z)\, \mathcal{S}(z,\, x) - \mathcal{S}(\overline{w},\, x)\right) \simeq \mathcal{S}(\overline{w},\, z)\, \mathcal{S}(z,\, x)$$

The dipole and quadropole are defined by:

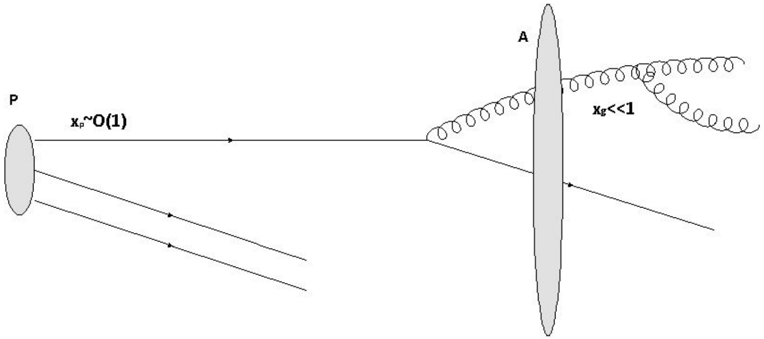
$$\mathcal{S}(\overline{\boldsymbol{w}},\, \boldsymbol{w}) \equiv \frac{1}{N_c}\,\mathrm{tr}\left[V^\dagger(\overline{\boldsymbol{w}})\, V(\boldsymbol{w})\right] \qquad \mathcal{Q}(\overline{\boldsymbol{x}},\, \boldsymbol{x},\, \boldsymbol{z},\, \overline{\boldsymbol{z}}) \equiv \frac{1}{N_c}\,\mathrm{tr}\left[V^\dagger(\overline{\boldsymbol{x}})\, V(\boldsymbol{x})\, V^\dagger(\boldsymbol{z})\, V(\overline{\boldsymbol{z}})\right]$$

The Trijet Setup

In the new setup, we have to produce three particles in the final state. There are two configurations of particles:

a) Quark, quark and anti-quark
b) Quark together with two gluons.

Due to the fact that we are using the light-cone gauge, the production of these configurations can happen both instantaneously (via one emission), or in the regular way, via two successive emissions or one emission followed by splitting process.



An example for a contribution with three particles in the final state

The Perturbative Outgoing State

The perturbative expression for the outgoing state is:

$$|out\rangle = |in\rangle + |out\rangle^{(1)} + |out\rangle^{(2)} + \cdots$$

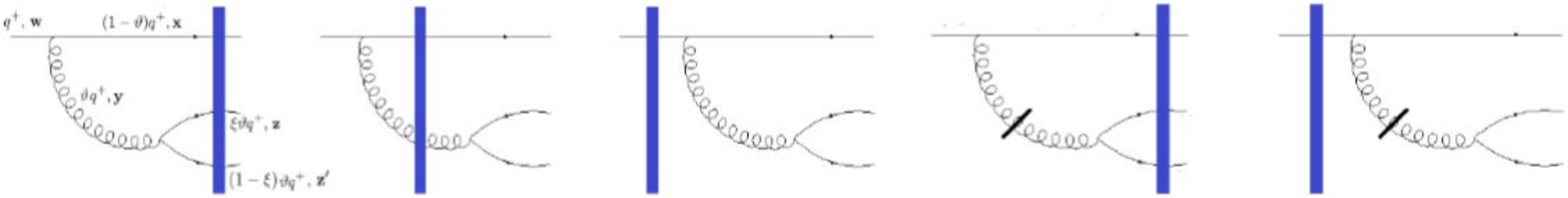
with:

$$|out\rangle^{(1)} = -\sum_{f,j} |f\rangle\langle f|S|j\rangle \frac{\langle j|H_{\mathrm{int}}|in\rangle}{E_j - E_{in}} + \sum_{f,j} |f\rangle \frac{\langle f|H_{\mathrm{int}}|j\rangle}{E_f - E_j} \langle j|S|in\rangle$$

$$|out\rangle^{(2)} = \sum_{f,j,i} |f\rangle\langle f|S|j\rangle \frac{\langle j|H_{\mathrm{int}}|i\rangle \langle i|H_{\mathrm{int}}|in\rangle}{(E_j - E_{in})(E_i - E_{in})} + \sum_{f,j,i} |f\rangle \frac{\langle f|H_{\mathrm{int}}|j\rangle \langle j|H_{\mathrm{int}}|i\rangle}{(E_f - E_j)(E_f - E_i)} \langle i|S|in\rangle$$
$$- \sum_{f,j,i} |f\rangle \frac{\langle f|H_{\mathrm{int}}|j\rangle}{E_f - E_j} \langle j|S|i\rangle \frac{\langle i|H_{\mathrm{int}}|in\rangle}{E_i - E_{in}}$$

Where i, j and k runs over the relevant bare states, and H_{int} represent the interaction part of the QCD Hamiltonian.

The Quark Quark Anti-quark Outgoing State

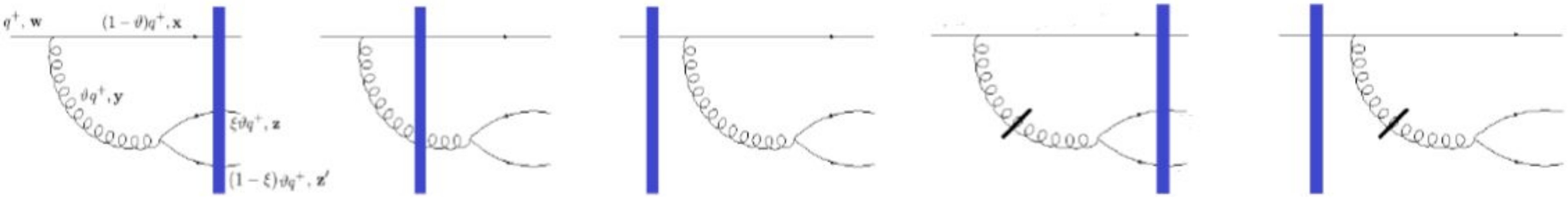


$$|\psi^\alpha\rangle_{qq\bar{q}}^{inst} \equiv |\bar{q}^\rho q^\varrho q^\sigma\rangle \left(\frac{\langle \bar{q}^\rho q^\varrho q^\sigma | \mathsf{H}_{q\to qq\bar{q}} | q^\beta\rangle \langle q^\beta | \hat{S} | q^\alpha\rangle}{E_{qq\bar{q}} - E_q} - \frac{\langle \bar{q}^\rho q^\varrho q^\sigma | \hat{S} | \bar{q}^\epsilon q^\delta q^\beta\rangle \langle \bar{q}^\epsilon q^\delta q^\beta | \mathsf{H}_{q\to qq\bar{q}} | q^\alpha\rangle}{E_{qq\bar{q}} - E_q} \right)$$

$$|\psi^\alpha\rangle_{qq\bar{q}}^{reg} \equiv |\bar{q}^\rho q^\varrho q^\sigma\rangle \left(\frac{\langle \bar{q}^\rho q^\varrho q^\sigma | \hat{S} | \bar{q}^\delta q^\epsilon q^\kappa\rangle \langle \bar{q}^\delta q^\epsilon q^\kappa | \mathsf{H}_{g\to q\bar{q}} | q^\beta g^i\rangle \langle q^\beta g^i | \mathsf{H}_{q\to qg} | q^\alpha\rangle}{(E_{qq\bar{q}} - E_q)(E_{qg} - E_q)} \right.$$
$$\left. + \frac{\langle \bar{q}^\rho q^\varrho q^\sigma | \mathsf{H}_{g\to q\bar{q}} | q^\gamma g^i\rangle \langle q^\gamma g^i | \mathsf{H}_{q\to qg} | q^\beta\rangle \langle q^\beta | \hat{S} | q^\alpha\rangle}{(E_{qq\bar{q}} - E_{qg})(E_{qq\bar{q}} - E_q)} - \frac{\langle \bar{q}^\rho q^\varrho q^\sigma | \mathsf{H}_{g\to q\bar{q}} | q^\gamma g^j\rangle \langle q^\gamma g^j | \hat{S} | q^\beta g^i\rangle \langle q^\beta g^i | \mathsf{H}_{q\to qg} | q^\alpha\rangle}{(E_{qq\bar{q}} - E_{qg})(E_{qg} - E_q)} \right)$$

C denotes the c.o.m for of the three produced particles: $\boldsymbol{C} \equiv (1-\vartheta)\boldsymbol{x} + \xi\vartheta\boldsymbol{z} + (1-\xi)\vartheta\boldsymbol{z}'$.
Note that the result above vanishes under the limit S->1. This property of the results has to be expected since the new particles are produced by the shockwave.

The Quark Quark Anti-quark Outgoing State



After insertion of the matrix elements:

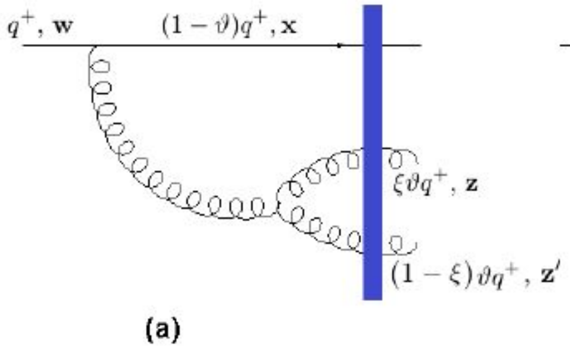
$$
\begin{aligned}
\left|q^{\alpha}_{\lambda}(q^+,\, \boldsymbol{w})\right\rangle_{qq\bar{q}} = &-\int_{\boldsymbol{x},\,\boldsymbol{z},\,\boldsymbol{z}'} \int_0^1 d\vartheta\, d\xi\, \frac{g^2\, q^+}{(2\pi)^4\, \left((1-\vartheta)\,(\boldsymbol{X}+\xi\boldsymbol{Z})^2 \,+\, \xi(1-\xi)\boldsymbol{Z}^2\right)} \\
&\times \Big\{ F^{\lambda_3\lambda_2\lambda_1}_{qq\bar{q}1}(\vartheta,\, \xi,\, \mathbf{X},\, \mathbf{Z}) \left[V^{\varrho\delta}(\boldsymbol{z}')\, t^a_{\delta\epsilon}\, V^{\dagger\epsilon\rho}(\boldsymbol{z})\, V^{\sigma\beta}(\boldsymbol{x})\, t^a_{\beta\alpha} - t^b_{\varrho\rho}\, V^{\sigma\beta}(\boldsymbol{x})\, U^{ba}(\boldsymbol{y})\, t^a_{\beta\alpha} \right] \\
&+ F^{\lambda_3\lambda_2\lambda_1}_{qq\bar{q}2}(\vartheta,\, \xi,\, \mathbf{X},\, \mathbf{Z}) \left[t^b_{\varrho\rho}\, V^{\sigma\beta}(\boldsymbol{x})\, U^{ba}(\boldsymbol{y})\, t^a_{\beta\alpha} - t^a_{\varrho\rho}\, t^a_{\sigma\beta}\, V^{\beta\alpha}(\boldsymbol{w}) \right] \Big\} \\
&\times \delta^{(2)}\left(\boldsymbol{w}-\boldsymbol{C}\right) \left|\bar{q}^{\rho}_{\lambda_3}((1-\xi)\vartheta q^+,\, \boldsymbol{z})\, q^{\varrho}_{\lambda_2}(\xi\vartheta q^+,\, \boldsymbol{z}')\, q^{\sigma}_{\lambda_1}((1-\vartheta)q^+, \boldsymbol{x})\right\rangle
\end{aligned}
$$

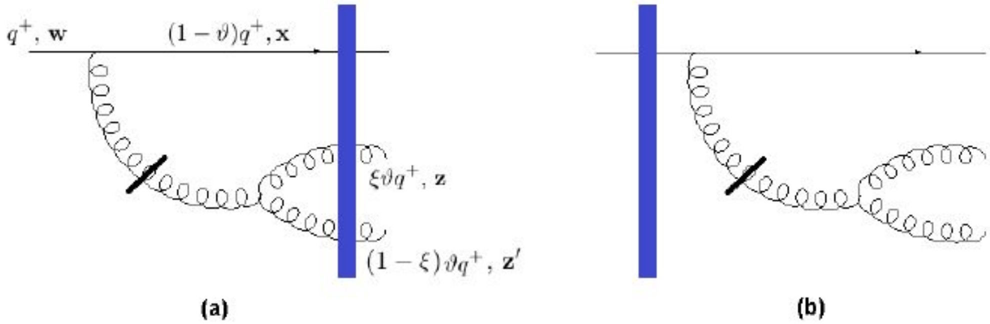
$$
F^{\lambda_3\lambda_2\lambda_1}_{qq\bar{q}1}(\vartheta,\, \xi,\, \mathbf{X},\, \mathbf{Z}) \equiv (1-\vartheta) \left(\frac{\varphi^{il}_{\lambda_2\lambda_3}(\xi)\, \phi^{ij}_{\lambda_1\lambda}(\vartheta)\, \boldsymbol{Z}^l\, \left(\boldsymbol{X}^j + \xi \boldsymbol{Z}^j\right)}{2\boldsymbol{Z}^2} + \xi(1-\xi)\delta_{\lambda_3\lambda_2}\delta_{\lambda_1\lambda} \right)
$$

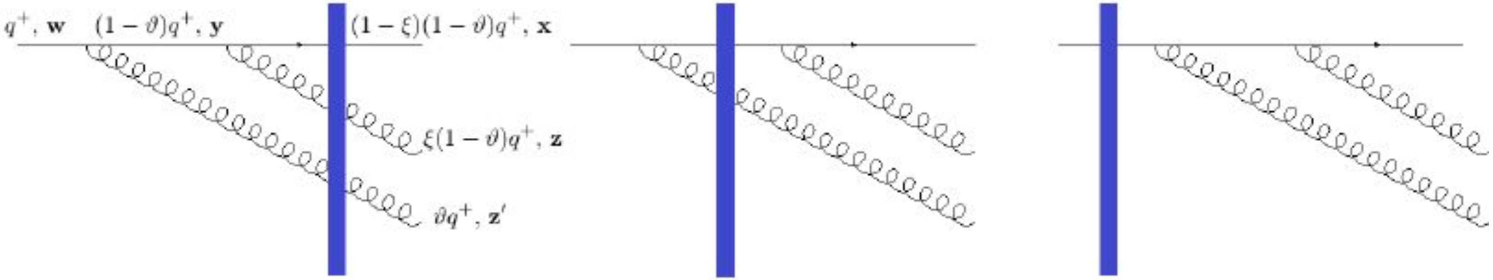
$$
F^{\lambda_3\lambda_2\lambda_1}_{qq\bar{q}2}(\vartheta,\, \xi,\, \mathbf{X},\, \mathbf{Z}) \equiv \xi(1-\xi) \left(-\frac{\varphi^{il}_{\lambda_2\lambda_3}(\xi)\, \phi^{ij}_{\lambda_1\lambda}(\vartheta)\, \boldsymbol{Z}^l\, \left(\boldsymbol{X}^j + \xi \boldsymbol{Z}^j\right)}{2\,(\boldsymbol{X}+\xi\boldsymbol{Z})^2} + (1-\vartheta)\delta_{\lambda_3\lambda_2}\delta_{\lambda_1\lambda} \right)
$$

$$
\varphi^{ij}_{\lambda_2\lambda_1}(\xi) \equiv \chi^{\dagger}_{\lambda_2} \left[(2\xi-1)\delta^{ij} + i\varepsilon^{ij}\sigma^3 \right] \chi_{\lambda_1}
$$

The Diagrams for the Quark and Two Gluons Outgoing States

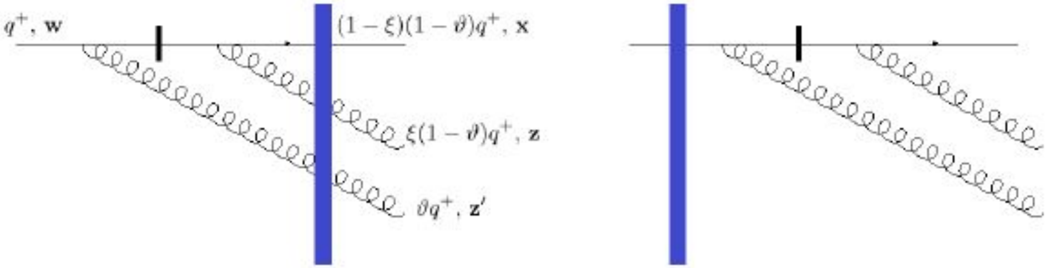






q^+, $\mathbf{w}$
$(1-\vartheta)q^+$, $\mathbf{y}$
$(1-\xi)(1-\vartheta)q^+$, $\mathbf{x}$
$\xi(1-\vartheta)q^+$, $\mathbf{z}$
ϑq^+, $\mathbf{z}'$

(a) **(b)** **(c)**



q^+, $\mathbf{w}$
$(1-\xi)(1-\vartheta)q^+$, $\mathbf{x}$
$\xi(1-\vartheta)q^+$, $\mathbf{z}$
ϑq^+, $\mathbf{z}'$

(a) **(b)**

From the outgoing state to the trijet cross section

The expression for the forward trijet cross section is composed by two contributions:

$$\frac{d\sigma^{pA\to 3jet+X}}{d^3q_1\, d^3q_2\, d^3q_3} = \int dx_p\, q(x_p,\mu^2)\left(\frac{d\sigma^{qA\to qgg+X}}{d^3q_1\, d^3q_2\, d^3q_3} + \frac{d\sigma^{qA\to qq\bar{q}+X}}{d^3q_1\, d^3q_2\, d^3q_3}\right)$$

The two contributions to the two final partonic state:

$$\frac{d\sigma^{qA\to qq\bar{q}+X}}{d^3q_1\, d^3q_2\, d^3q_3} \equiv \frac{1}{2N_c\, L}\, {}^{(g^2)}_{out}\langle q^{\alpha}_{\lambda}(q^+, \boldsymbol{q}=0_\perp)|\, \hat{\mathcal{N}}_q(q_1)\, \hat{\mathcal{N}}_q(q_2)\, \hat{\mathcal{N}}_{\bar{q}}(q_3)\, |q^{\alpha}_{\lambda}(q^+, \boldsymbol{q}=0_\perp)\rangle^{(g^2)}_{out}$$

$$\frac{d\sigma^{qA\to qgg+X}}{d^3q_1\, d^3q_2\, d^3q_3} \equiv \frac{1}{2N_c\, L}\, {}^{(g^2)}_{out}\langle q^{\alpha}_{\lambda}(q^+, \boldsymbol{q}=0_\perp)|\, \hat{\mathcal{N}}_q(q_1)\, \hat{\mathcal{N}}_g(q_2)\, \hat{\mathcal{N}}_g(q_3)\, |q^{\alpha}_{\lambda}(q^+, \boldsymbol{q}=0_\perp)\rangle^{(g^2)}_{out}$$

The results for the forward trijet cross section

The contribution to the quark quark anti-quark trijet cross section:

$$\frac{d\sigma^{qA\to qq\bar{q}+X}}{d^3q_1\, d^3q_2\, d^3q_3} \equiv \frac{\alpha_s^2 C_F N_f}{2(2\pi)^{10}(q^+)^2}\,\delta(q^+ - q_1^+ - q_2^+ - q_3^+) \int_{\overline{\boldsymbol{x}},\,\overline{\boldsymbol{z}},\,\overline{\boldsymbol{z}}',\,\boldsymbol{x},\,\boldsymbol{z},\,\boldsymbol{z}'} \mathrm{e}^{-i\boldsymbol{q}_1\cdot(\boldsymbol{x}-\overline{\boldsymbol{x}})-i\boldsymbol{q}_2\cdot(\boldsymbol{z}-\overline{\boldsymbol{z}})-i\boldsymbol{q}_3\cdot(\boldsymbol{z}'-\overline{\boldsymbol{z}}')}$$
$$\times \left[K^1_{qq\bar{q}}\left(\vartheta,\,\xi,\,\overline{\mathbf{X}},\,\overline{\mathbf{Z}},\,\mathbf{X},\,\mathbf{Z}\right)\mathbb{S}^1_{qq\bar{q}}\left(\overline{\mathbf{x}},\,\overline{\mathbf{z}},\,\overline{\mathbf{z}}',\,\mathbf{x},\,\mathbf{z},\,\mathbf{z}'\right) + K^2_{qq\bar{q}}\left(\vartheta,\,\xi,\,\overline{\mathbf{X}},\,\overline{\mathbf{Z}},\,\mathbf{X},\,\mathbf{Z}\right)\mathbb{S}^2_{qq\bar{q}}\left(\overline{\mathbf{x}},\,\overline{\mathbf{z}},\,\overline{\mathbf{z}}',\,\mathbf{x},\,\mathbf{y}\right)\right.$$
$$\left. +\, h.c. + K^3_{qq\bar{q}}\left(\vartheta,\,\xi,\,\overline{\mathbf{X}},\,\overline{\mathbf{Z}},\,\mathbf{X},\,\mathbf{Z}\right)\mathbb{S}_{\rm LO}\left(\overline{\boldsymbol{w}},\,\overline{\boldsymbol{x}},\,\overline{\boldsymbol{z}},\,\boldsymbol{w},\,\boldsymbol{x},\,\boldsymbol{z}\right)\right] + \left(q_1^+ \leftrightarrow q_2^+,\, \boldsymbol{q}_1 \leftrightarrow \boldsymbol{q}_2\right)$$

Where we introduced the following structures:

$$\mathbb{S}^1_{qq\bar{q}}\left(\overline{\boldsymbol{x}},\,\overline{\boldsymbol{y}},\,\overline{\boldsymbol{z}},\,\overline{\boldsymbol{z}}',\,\boldsymbol{x},\,\boldsymbol{y},\,\boldsymbol{z},\,\boldsymbol{z}'\right) \equiv S_{q\bar{q}qq\bar{q}q}\left(\overline{\boldsymbol{x}},\,\overline{\boldsymbol{z}},\,\overline{\boldsymbol{z}}',\,\boldsymbol{x},\,\boldsymbol{z},\,\boldsymbol{z}'\right) - S_{q\bar{q}qqq}\left(\overline{\boldsymbol{x}},\,\overline{\boldsymbol{z}},\,\overline{\boldsymbol{z}}',\,\boldsymbol{x},\,\boldsymbol{y}\right) - S_{qqq\bar{q}q}\left(\overline{\boldsymbol{x}},\,\overline{\boldsymbol{y}},\,\boldsymbol{x},\,\boldsymbol{z},\,\boldsymbol{z}'\right) + S_{qqqq}\left(\overline{\boldsymbol{x}},\,\overline{\boldsymbol{y}},\,\boldsymbol{x},\,\boldsymbol{y}\right)$$

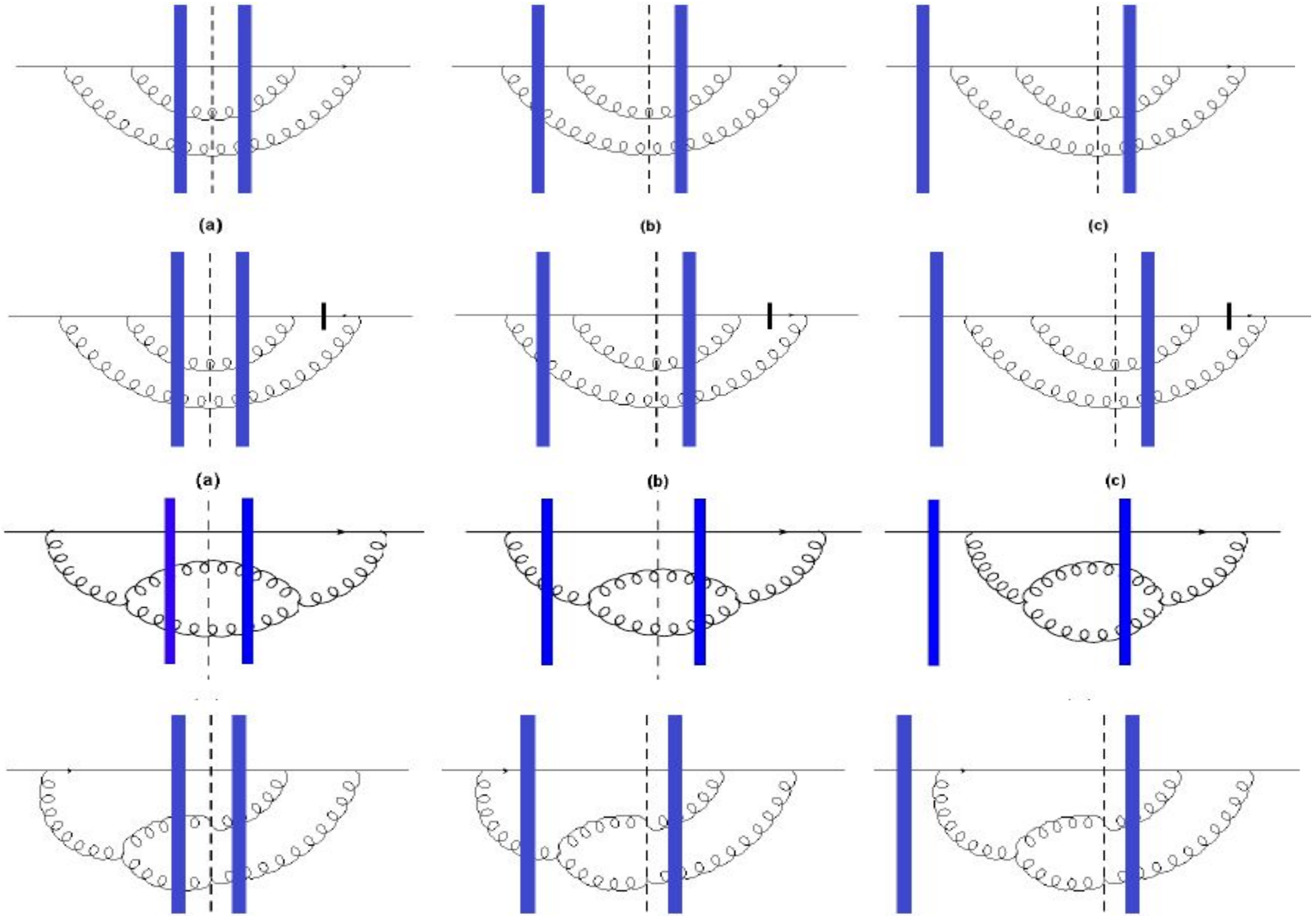
$$\mathbb{S}^2_{qq\bar{q}}\left(\overline{\boldsymbol{w}},\,\overline{\boldsymbol{x}},\,\overline{\boldsymbol{y}},\,\boldsymbol{x},\,\boldsymbol{y},\,\boldsymbol{z},\,\boldsymbol{z}'\right) \equiv S_{qqq\bar{q}q}\left(\overline{\boldsymbol{x}},\,\overline{\boldsymbol{y}},\,\boldsymbol{x},\,\boldsymbol{z},\,\boldsymbol{z}'\right) - S_{qq\bar{q}q}\left(\overline{\boldsymbol{w}},\,\boldsymbol{x},\,\boldsymbol{z},\,\boldsymbol{z}'\right) - S_{qqqq}\left(\overline{\boldsymbol{x}},\,\overline{\boldsymbol{y}},\,\boldsymbol{x},\,\boldsymbol{y}\right) + S_{qqq}\left(\overline{\boldsymbol{w}},\,\boldsymbol{y},\,\boldsymbol{x}\right)$$

Each of these structures involves contraction of Wilson lines:

$$S_{q\bar{q}qq\bar{q}q}\left(\overline{\boldsymbol{x}},\,\overline{\boldsymbol{z}},\,\overline{\boldsymbol{z}}',\,\boldsymbol{x},\,\boldsymbol{z},\,\boldsymbol{z}'\right) \equiv \frac{2}{C_F N_c}\,\mathrm{tr}\left(V^\dagger(\bar{x})\,V(x)\,t^a\,t^b\right)\,\mathrm{tr}\left(V(\bar{z}')\,t^b\,V^\dagger(\bar{z})\,V(z)\,t^a\,V^\dagger(z')\right)$$
$$= \frac{1}{2C_F N_c}\left(N_c^2\,\mathcal{Q}(\overline{x},\,x,\,z',\,\overline{z}')\,\mathcal{S}(\overline{z},\,z) - \mathcal{H}(\overline{x},\,x,\,z',\,\overline{z}',\,\overline{z},\,z) - \mathcal{H}(\overline{x},\,x,\,\overline{z},\,z,\,z',\,\overline{z}')\right.$$
$$\left. +\,\mathcal{S}(\overline{x},\,x)\,\mathcal{Q}(\overline{z},\,z,\,z',\,\overline{z}')\right) \simeq \mathcal{Q}(\overline{x},\,x,\,z',\,\overline{z}')\,\mathcal{S}(\overline{z},\,z)$$

At the large Nc limit only the dipole and quadrupole structures remain.

Contribution from the Gluons



The NLO WF

In order to allow phenomenology to be reliable, higher order corrections as dictated by pQCD must be included in the result of hep-ph/0708.0231.

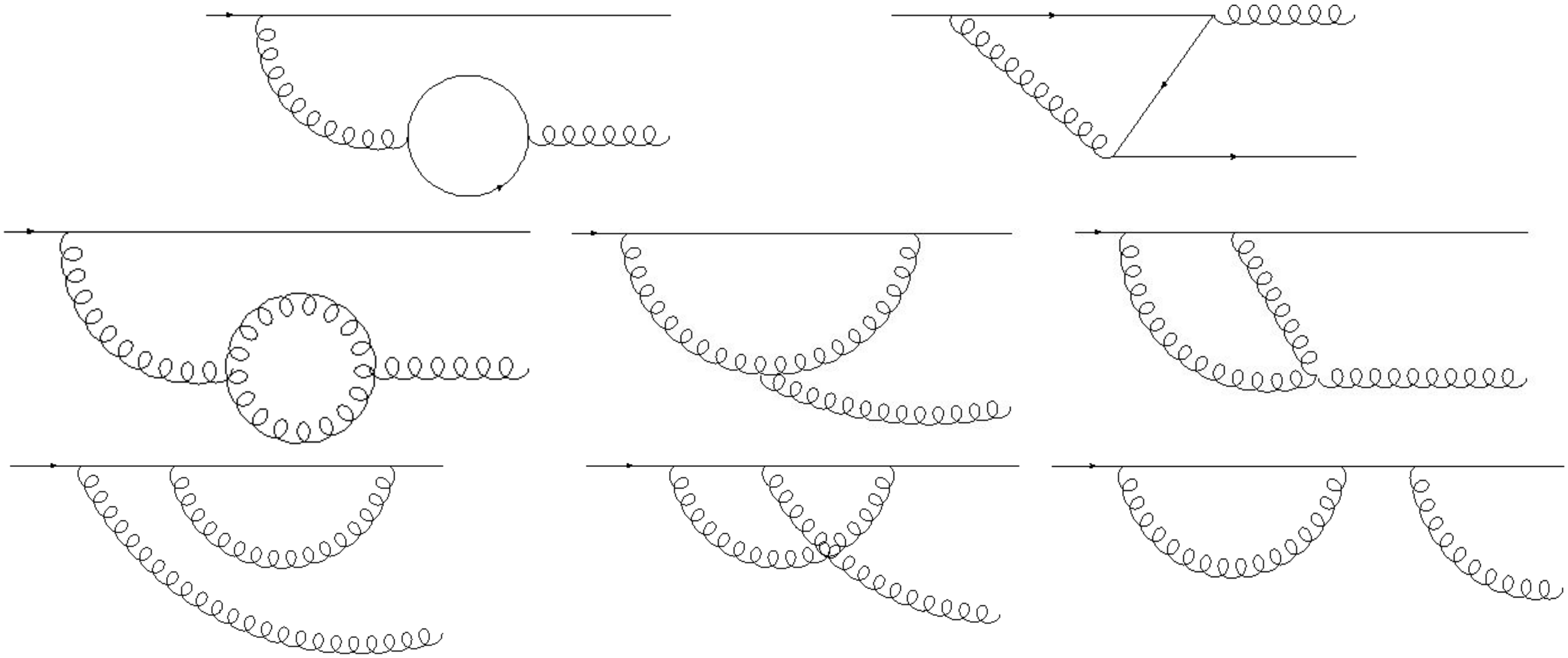
The missing part of the new outgoing state is the part which involves the production of a quark and a gluon together with a loop (virtual) correction.

Each of these diagrams has a dependence on an IR longitudinal momentum cutoff. This dependence must not be a part of the final result for the cross section.

The NLO outgoing quark state has the following structure:

$$\left|q_\lambda^\alpha(q^+,\mathrm{w})\right\rangle_{NLO} = \hat{\mathcal{Z}}_{NLO}\left|q_\lambda^\alpha(q^+,\mathrm{w})\right\rangle + \left|\Phi_\lambda^\alpha(q^+,\mathrm{w})\right\rangle_{LO} + \left|\Phi_\lambda^\alpha(q^+,\mathrm{w})\right\rangle_{qg}$$
$$+ \left|\Phi_\lambda^\alpha(q^+,\mathrm{w})\right\rangle_{qq\bar{q}} + \left|\Phi_\lambda^\alpha(q^+,\mathrm{w})\right\rangle_{qgg}.$$

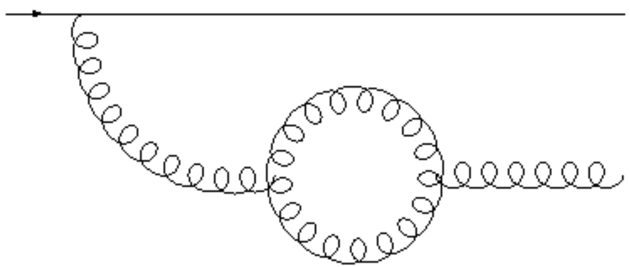
The Diagrams



First calculated also in hep-ph/1611.00497 (T. Lappi and R. Paatelainen)

The structure of virtual contributions

Example of results for the diagrams:



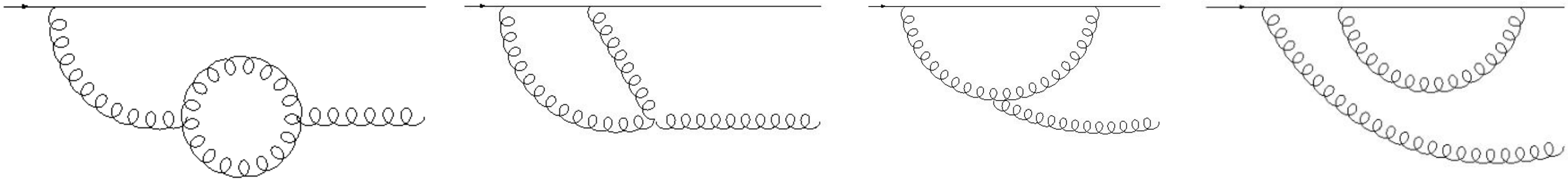
$$|\psi_\lambda^\alpha\rangle_{qg}^2 = \int_0^1 d\vartheta \int d^2\tilde{\mathbf{k}}\, \frac{g^3\, N_c\, t^a_{\beta\alpha}\, \phi^{ij}_{\lambda_1\lambda}(\vartheta)\, \tilde{\mathbf{k}}^j\, \sqrt{q^+}}{4(2\pi)^5\sqrt{2\vartheta}\,\tilde{\mathbf{k}}^2} \left(\left[\frac{11}{3} + 4\ln\left(\frac{\Lambda}{\vartheta q^+}\right)\right] \left[-\frac{2}{\epsilon} + \ln\left(\frac{\tilde{\mathbf{k}}^2}{\mu^2_{\overline{MS}}}\right)\right] \right.$$

$$\left. +2\ln^2\left(\frac{\Lambda}{\vartheta(1-\vartheta)q^+}\right) - \frac{67}{9} + \frac{2\pi^2}{3} - \frac{11}{3}\ln(1-\vartheta) - 2\ln^2(1-\vartheta)\right) \left| q^\beta_{\lambda_1}((1-\vartheta)q^+,\, (1-\vartheta)\mathbf{q} - \tilde{\mathbf{k}})\, g^a_i(\vartheta q^+,\, \vartheta\mathbf{q} + \tilde{\mathbf{k}})\right\rangle$$

Two types of IR logs are involved: $\ln\left(\frac{\Lambda}{q^+}\right)\ln\left(\frac{\tilde{\mathbf{k}}^2}{\mu^2_{\overline{MS}}}\right)$ and $\ln^2\left(\frac{\Lambda}{\vartheta(1-\vartheta)q^+}\right)$

Cancellation of the IR logs

The IR logs cancellation pattern is:



	$\vert\psi^{\alpha}_{\lambda}\rangle^{2}_{qg}$	$\vert\psi^{\alpha}_{\lambda}\rangle^{3}_{qg}$	$\vert\psi^{\alpha}_{\lambda}\rangle^{4}_{qg}$	$\vert\psi^{\alpha}_{\lambda}\rangle^{5}_{qg}$
$\ln\left(\frac{\Lambda}{q^+}\right)\ln\left(\frac{\tilde{\mathbf{k}}^2}{\mu^2_{\overline{MS}}}\right)$	4	-3	1	-2
$\ln^2\left(\frac{\Lambda}{\vartheta(1-\vartheta)q^+}\right)$	2	-2	1	-1

Results for the NLO WF

By combining all the loop contribution:

$$\left|\psi_\lambda^\alpha(q^+,\boldsymbol{w})\right\rangle_{qg} = -\int_0^1 d\vartheta \int_{\mathbf{x},\mathbf{z}} \frac{ig^3\, N_c\, t^a_{\beta\alpha}\sqrt{q^+}\,\mathbf{X}^j}{4(2\pi)^4\sqrt{2\vartheta}\,\mathbf{X}^2}\left\{\phi^{ij}_{\lambda_1\lambda}(\vartheta)\left(-\beta(\vartheta)\left[\frac{2}{\epsilon}+\ln\left(\frac{\mathbf{X}^2\mu^2_{\overline{MS}}}{4e^{-2\gamma}}\right)\right] + \gamma(\vartheta) + \mathcal{I}(\vartheta)\right) + \kappa^{ij}_{\lambda_1\lambda}(\vartheta)\right\}$$
$$\times\, \delta(\boldsymbol{w}-(1-\vartheta)\boldsymbol{x}-\vartheta\boldsymbol{z})\left|q^\beta_{\lambda_1}((1-\vartheta)q^+,\boldsymbol{x})\, g^a_i(\vartheta q^+,\boldsymbol{z})\right\rangle$$

with:

$$\beta(\vartheta) \equiv \frac{11}{3}N_c - \frac{2}{3}N_f + \Delta_\beta(\vartheta); \qquad \Delta_\beta(\vartheta) \equiv N_c \ln(1-\vartheta)$$

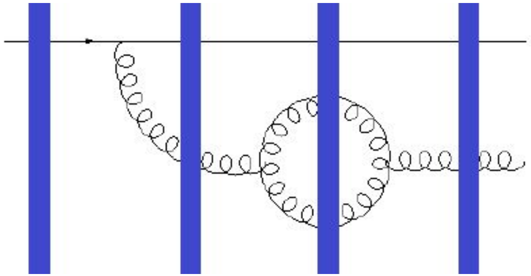
$$\gamma(\vartheta) \equiv \left(\frac{67}{9}-\frac{\pi^2}{3}\right)N_c - \frac{10}{9}N_f + \Delta_\gamma(\vartheta) \qquad \Delta_\gamma(\vartheta) \equiv \left[\frac{2}{3}N_f + \left(3Li_2(\vartheta) + \frac{1}{2}\ln\left(e^{\frac{13}{3}}\vartheta^2(1-\vartheta)\right)\right)N_c\right]\ln(1-\vartheta)$$

$$\mathcal{I}(\vartheta) \equiv \left(3 + 4\ln\left(\frac{\Lambda}{\vartheta q^+}\right)\right)N_c \int \frac{d^2\tilde{\boldsymbol{p}}}{\tilde{\boldsymbol{p}}^2}$$

$$\kappa^{ij}_{\lambda_1\lambda}(\vartheta) \equiv \chi^\dagger_{\lambda_1}\left\{\frac{\vartheta(2-\vartheta)}{2}\delta^{ij} + \left[4\vartheta(2-\vartheta) + (2-\vartheta)\ln(1-\vartheta) + \frac{3\vartheta^2}{1-\vartheta}\ln(\vartheta)\right] i\varepsilon^{ij}\sigma^3\right.$$
$$\left. +\vartheta(\vartheta-4)\left(-\frac{2}{\epsilon}+\ln\left(\frac{\tilde{\mathbf{k}}^2}{\mu^2_{\overline{MS}}}\right)\right)(\delta^{ij}-i\varepsilon^{ij}\sigma^3)\right\}\chi_\lambda.$$

Adding the interactions with a shockwave

$$
\begin{aligned}
|out\rangle^{(3)} = -\Bigg[&\frac{\langle q\, g_2|\, H\, |q\, g_3\, g_4\rangle\, \langle q\, g_3\, g_4|\, H\, |q\, g_1\rangle\, \langle q\, g_1|\, H\, |q\rangle}{(E_{qgg} - E_{qg_2})\,(E_{qgg} - E_{qg_1})\,(E_{qg_1} - E_q)}\, (S_F\, S_{A_1} - S_F) \\
+ &\frac{\langle q\, g_2|\, H\, |q\, g_3\, g_4\rangle\, \langle q\, g_3\, g_4|\, H\, |q\, g_1\rangle\, \langle q\, g_1|\, H\, |q\rangle}{(E_{qgg} - E_{qg_2})\,(E_{qgg} - E_q)\,(E_{qg_1} - E_q)}\, (S_F\, S_{A_3}\, S_{A_4} - S_F) \\
+ &\frac{\langle q g_2|\, H\, |q\, g_3\, g_4\rangle\, \langle q\, g_3\, g_4|\, H\, |q\, g_1\rangle\, \langle q\, g_1|\, H\, |q\rangle}{(E_{qg_2} - E_q)\,(E_{qgg} - E_q)\,(E_{qg_1} - E_q)}\, (S_F\, S_{A_2} - S_F)\Bigg]\, |q g_2\rangle\,.
\end{aligned}
$$



Partial results for the outgoing state can be obtained from the WF calculation:

$$
\begin{aligned}
|\psi^\alpha_\lambda(q^+, \boldsymbol{w})\rangle_{qg} = -\int_0^1 d\vartheta \int_{\mathbf{x},\mathbf{z}} \frac{i g^3\, N_c\, t^a_{\beta\alpha}\, \sqrt{q^+}\, \mathbf{X}^j}{4(2\pi)^4 \sqrt{2\vartheta}\, \mathbf{X}^2} &\left\{ \phi^{ij}_{\lambda_1\lambda}(\vartheta) \left(-\beta(\vartheta) \left[\frac{2}{\epsilon} + \ln\left(\frac{\mathbf{X}^2 \mu^2_{\overline{MS}}}{4e^{-2\gamma}} \right) \right] + \gamma(\vartheta) + \mathcal{I}(\vartheta) \right) + \kappa^{ij}_{\lambda_1\lambda}(\vartheta) \right\} \\
\times\, \delta(\boldsymbol{w} - (1-\vartheta)\boldsymbol{x} - \vartheta \boldsymbol{z}) &\left[V^{\gamma\beta}(\boldsymbol{x})\, U^{ba}(\boldsymbol{z})\, t^a_{\beta\alpha} - t^b_{\gamma\beta}\, V^{\beta\alpha}(\boldsymbol{w}) \right] \left| q^\gamma_{\lambda_1}((1-\vartheta)q^+, \boldsymbol{x})\, g^b_i(\vartheta q^+, \boldsymbol{z}) \right\rangle
\end{aligned}
$$

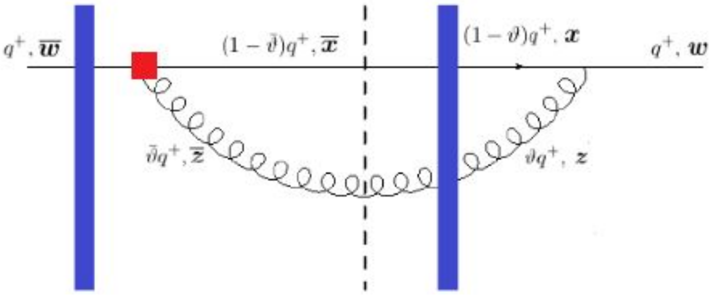
The full NLO dijet cross section will involve both g^2 x g^2 and g x g^3 contributions:

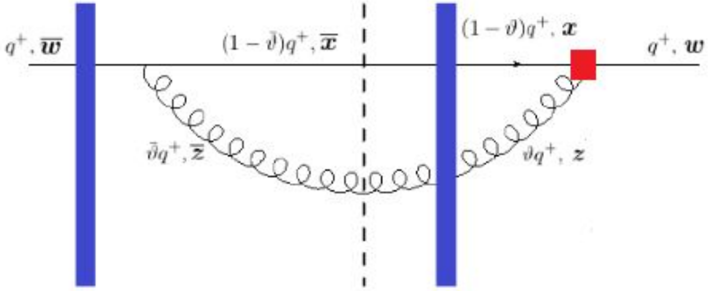
$$
\frac{d\sigma^{dijet}}{d^3k\, d^3p} \equiv \frac{d\sigma_R^{q\to qqX}}{d^3k\, d^3p} + \frac{d\sigma_R^{q\to q\bar{q}X}}{d^3k\, d^3p} + \frac{d\sigma_R^{q\to ggX}}{d^3k\, d^3p} + \frac{d\sigma_R^{q\to qgX}}{d^3k\, d^3p} + \frac{d\sigma_V^{q\to qgX}}{d^3k\, d^3p}
$$

The real contributions are directly related to the results in hep-ph/1809.05526.
Real-Virtual cancellation: analogous to calculation by Chirilli, Xiao, Yuan
hep-ph/1112.1061,hep-ph/1203.6139.

Partial Results for the NLO dijet

The corresponding diagrams for NLO dijet cross section:





Contributions to the dijet cross section when the shockwave is inserted at final - initial state:

$$\frac{d\sigma_I^{qA\to qg+X}}{dk^+\,d^2\boldsymbol{k}\,dp^+\,d^2\boldsymbol{p}} = \frac{\alpha_s^2}{(2\pi)^6 q^+}\,\delta(q^+-k^+-p^+)$$

$$\times \int_{\boldsymbol{x},\overline{\boldsymbol{x}},\boldsymbol{z},\overline{\boldsymbol{z}}} \frac{\mathbf{X}\cdot\overline{\mathbf{X}}}{\mathbf{X}^2\,\overline{\mathbf{X}}^2}\, K\left(\vartheta,\,\mathbf{X}^2,\,\overline{\mathbf{X}}^2\right)\, \mathrm{e}^{-i\boldsymbol{p}\cdot(\boldsymbol{x}-\overline{\boldsymbol{x}})-i\boldsymbol{k}\cdot(\boldsymbol{z}-\overline{\boldsymbol{z}})}\, \mathbb{S}_{\mathrm{LO}}\left(\overline{\boldsymbol{w}},\,\overline{\boldsymbol{x}},\,\overline{\boldsymbol{z}},\,\boldsymbol{w},\,\boldsymbol{x},\,\boldsymbol{z}\right)$$

$$K\left(\vartheta,\mathbf{X}^2,\overline{\mathbf{X}}^2\right) = \left\{\frac{1+(1-\vartheta)^2}{\vartheta}\left(\beta(\vartheta)\left[-\frac{2}{\epsilon}+\ln\left(\frac{\mathbf{X}^2\mu^2_{\overline{MS}}}{4e^{-2\gamma}}\right)\right]+\gamma(\vartheta)+\mathcal{I}(\vartheta)\right)+F(\vartheta)+\vartheta\beta(\vartheta)\right\}+(\mathbf{X}\longleftrightarrow\overline{\mathbf{X}})$$

$$F(\vartheta)\equiv\frac{1}{2}\left[(\vartheta-4)\left(-\frac{2}{\epsilon}+\ln\left(\frac{\mathbf{X}^2\mu^2_{\overline{MS}}}{4e^{-2\gamma}}\right)\right)+\frac{1}{2}(13\vartheta^2-36\vartheta+4)-(1-\vartheta)\ln(1-\vartheta)-\frac{3\vartheta^2}{2(1-\vartheta)}\ln(\vartheta)\right]N_c$$

Summary

1) Generalization of the method by C. Marquet (2007) to all orders, for the calculation of the forward particle production in proton-nucleus collisions at high energy, was shown to be possible by adopting the outgoing state approach.

2) We computed the full light-cone wave function of the incoming quark, and partially its corresponding outgoing state.

3) IR divergences has been shown to cancel after combining all the loop contributions (except normalization contribution).

4) Partial results for the inclusive forward NLO dijet cross section are available.