Forward particle production in proton-nucleus collisions at next-to-leading order

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Towards accuracy at small x

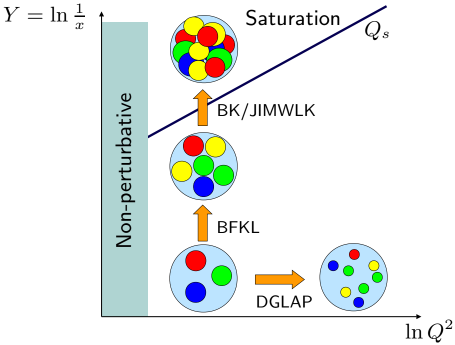
Edinburgh, 11/09/2019

B. D., T. Lappi, Y. Zhu, PRD 93 (2016) 114016 [arXiv:1604.00225]

B. D., T. Lappi, Y. Zhu, PRD 95 (2017) 114007 [arXiv:1703.04962]

B. D., E. Iancu, T. Lappi, A.H. Mueller, G. Soyez, D.N. Triantafyllopoulos, Y. Zhu, PRD 97 (2018) 054020 [arXiv:1712.07480]

Our goal is to study QCD in the saturation regime

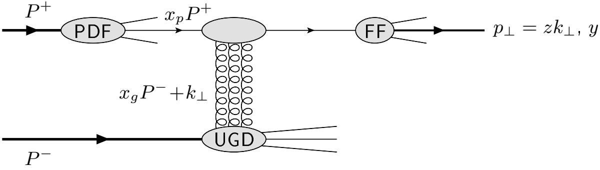


The production of forward particles is a crucial tool to probe small x values

Saturation effects stronger in pA collisions ($Q_s^2 \sim A^{1/3}$)

Here we study the inclusive production of forward hadrons in proton-nucleus collisions: $pA \to hX$

Single inclusive forward hadron production at LO in the $q \to q$ channel:



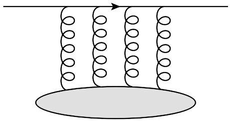
Dilute projectile: $x_p = \dfrac{k_\perp}{\sqrt{s}} e^{y}$, described by a collinear PDF

Dense target: $x_g = \dfrac{k_\perp}{\sqrt{s}} e^{-y} \ll 1$, described by unintegrated gluon distribution $\mathcal{S}$

LO quark multiplicity: $\dfrac{\mathrm{d}N}{\mathrm{d}^2\mathbf{p}\,\mathrm{d}y} \propto \mathrm{PDF} \otimes \mathcal{S} \otimes \mathrm{FF}$ $\qquad \mathcal{S}(k_\perp) = \int \mathrm{d}^2\mathbf{r} e^{-i\mathbf{k}\cdot\mathbf{r}} S(\mathbf{r})$

At high densities the target can be described as a classical color field

The eikonal interaction of the dilute probe (quark) with the dense target (nucleus) is described by a Wilson line V

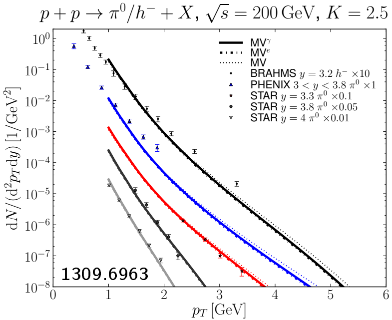


The x evolution of the dipole correlator $S(\mathbf{r} = \mathbf{x} - \mathbf{y}) = \left\langle \frac{1}{N_c} \mathrm{Tr}\, V(\mathbf{x}) V^\dagger(\mathbf{y}) \right\rangle$ is governed by the Balitsky-Kovchegov (BK) equation:

$$\frac{\partial S(\mathbf{r}, x)}{\partial \ln x} = 2\alpha_s N_c \int \frac{\mathrm{d}^2\mathbf{x}}{(2\pi)^2} \frac{\mathbf{r}^2}{\mathbf{x}^2(\mathbf{r}-\mathbf{x})^2} \left[S(\mathbf{r}, x) - S(\mathbf{x}, x) S(\mathbf{r}-\mathbf{x}, x) \right]$$

The initial condition $S(\mathbf{r}, x_0)$ cannot be computed perturbatively. It can be extracted e.g. by a fit to HERA DIS data (typically $x_0 \sim 0.01$)

Using these LO expressions together with dipole correlators constrained by HERA data (Lappi, Mäntysaari):

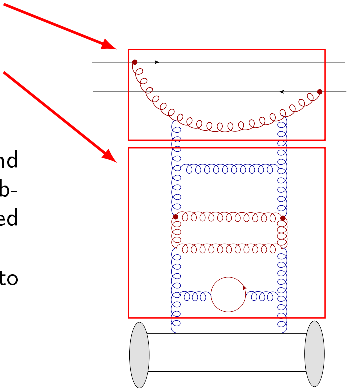


Reasonable description of the trend of the data (but large K factor needed)

This is only leading order. What about NLO corrections?

This process receives NLO corrections from two sources:

- Corrections to the process-dependent hard part (impact factor)
  Chirilli, Xiao, Yuan
- Corrections to the BK evolution
  Balitsky, Chirilli
  (see Edmond's talk on Tuesday)

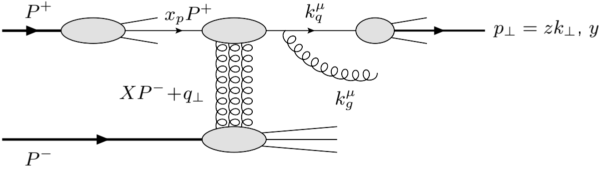


At first both sources of corrections were found to lead to unphysical results. But these problems were progressively understood and solved

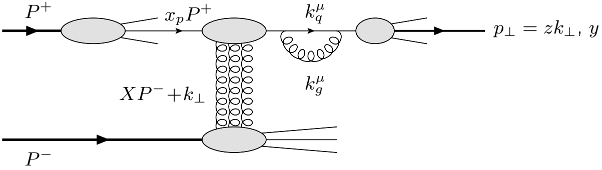
This talk will focus on the NLO corrections to the impact factor

NLO corrections to the impact factor for this process:
(Chirilli, Xiao, Yuan)

Example of real $q \to q$ contribution:

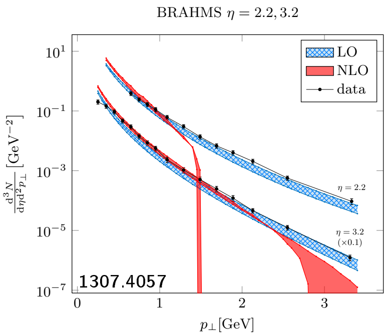


Example of virtual $q \to q$ contribution:



$1 - \xi = \frac{k_g^+}{x_p P^+}$ is the momentum fraction of the incoming quark carried by the gluon

First numerical implementation of these expressions (Staśto, Xiao, Zaslavsky):



Negative cross section for $p_\perp \gtrsim Q_s$

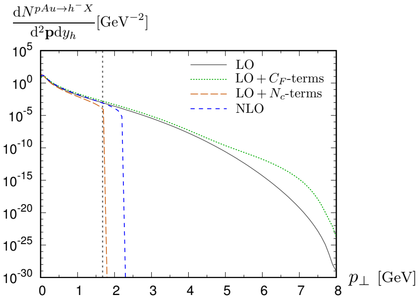
Many works devoted to solving this issue, using for example the kinematical constraint / Ioffe time cutoff (Altinoluk, Armesto, Beuf, Kovner, Lublinsky). Numerical implementation: Watanabe, Xiao, Yuan, Zaslavsky. Can extend the positivity range but doesn't solve the problem completely.

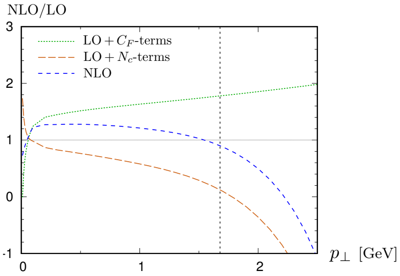
In both real and virtual corrections, there are terms proportional to the C_F color factor and terms proportional to N_c:

$$\frac{\mathrm{d}N^{\mathsf{NLO}}}{\mathrm{d}^2\mathbf{k}\mathrm{d}y} = \frac{\mathrm{d}N^{\mathsf{LO}}}{\mathrm{d}^2\mathbf{k}\mathrm{d}y} + \frac{\mathrm{d}N^{C_\mathsf{F}}}{\mathrm{d}^2\mathbf{k}\mathrm{d}y} + \frac{\mathrm{d}N^{N_\mathsf{c}}}{\mathrm{d}^2\mathbf{k}\mathrm{d}y}$$

- The terms proportional to C_F are divergent when the additional gluon at NLO is collinear to the initial or final state quark
  These divergences are absorbed in the DGLAP evolution of the PDFs and fragmentation functions
- The terms proportional to N_c are related to the high energy evolution (recall BK: $\partial_Y S = 2\alpha_s N_\mathsf{c} \int \mathrm{d}^2\mathbf{x}\,[...]$)

At high $p_\perp$ the C_F NLO corrections are positive while the N_c ones are negative:



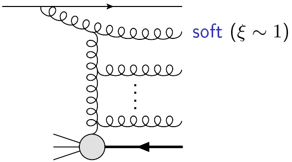


The negativity issue comes from the N_c terms, and more precisely the subtraction of the LO evolution from the NLO terms

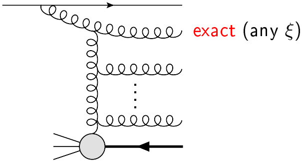
LO vs. NLO

Balitsky-Kovchegov (BK) evolution: resummation of $(\alpha_s \ln 1/x)^n$, corresponding to any number of soft gluons already at LO

LO: all gluons are soft:



NLO impact factor: the first gluon can be hard:

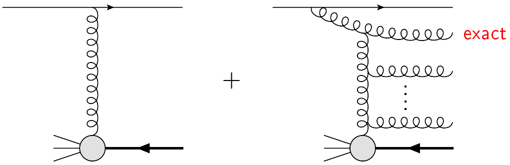


The case where the first gluon is soft is already included in the leading order
$\Rightarrow$ Need to avoid double counting between LO and NLO

Two possible solutions to avoid double counting:

1) Subtract the case where the gluon in the NLO impact factor is soft
Chirilli, Xiao, Yuan ('CXY')

2) Rearrange the terms to avoid doing a subtraction
Iancu, Mueller, Triantafyllopoulos



These two choices should be equivalent

The expression for the (quark production) multiplicity at NLO reads

$$\frac{\mathrm{d}N^{pA\to qX}}{\mathrm{d}^2\mathbf{k}\mathrm{d}y} = x_p q(x_p)\frac{\mathcal{S}(k_\perp, x_0)}{(2\pi)^2} \qquad \leftarrow \text{Lowest order}$$

$$+\frac{\alpha_s}{2\pi^2}\int_{x_p}^{\xi_{\max}}\mathrm{d}\xi\,\frac{1+\xi^2}{1-\xi}\frac{x_p}{\xi}q\left(\frac{x_p}{\xi}\right)\left\{C_\mathrm{F}\mathcal{I}(k_\perp,\xi,X(\xi)) + \frac{N_\mathrm{c}}{2}\mathcal{J}(k_\perp,\xi,X(\xi))\right\} \qquad \leftarrow \text{real}$$

$$-\frac{\alpha_s}{2\pi^2}\int_{0}^{\xi_{\max}}\mathrm{d}\xi\,\frac{1+\xi^2}{1-\xi}x_p q(x_p)\left\{C_\mathrm{F}\mathcal{I}_v(k_\perp,\xi,X(\xi)) + \frac{N_\mathrm{c}}{2}\mathcal{J}_v(k_\perp,\xi,X(\xi))\right\} \qquad \leftarrow \text{virt.}$$

with e.g.

$$\mathcal{J}(k_\perp,\xi,X(\xi)) = \int\frac{\mathrm{d}^2\mathbf{q}}{(2\pi)^2}\frac{2(\mathbf{k}-\xi\mathbf{q})\cdot(\mathbf{k}-\mathbf{q})}{(\mathbf{k}-\xi\mathbf{q})^2(\mathbf{k}-\mathbf{q})^2}\mathcal{S}(q_\perp, X(\xi))$$

$$-\int\frac{\mathrm{d}^2\mathbf{q}}{(2\pi)^2}\frac{\mathrm{d}^2\mathbf{l}}{(2\pi)^2}\frac{2(\mathbf{k}-\xi\mathbf{q})\cdot(\mathbf{k}-\mathbf{l})}{(\mathbf{k}-\xi\mathbf{q})^2(\mathbf{k}-\mathbf{l})^2}\mathcal{S}(q_\perp, X(\xi))\mathcal{S}(l_\perp, X(\xi))$$

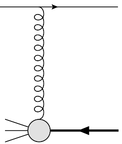
$$\mathcal{J}_v(k_\perp,\xi,X(\xi)) = \int\frac{\mathrm{d}^2\mathbf{q}}{(2\pi)^2}\frac{2(\xi\mathbf{k}-\mathbf{q})\cdot(\mathbf{k}-\mathbf{q})}{(\xi\mathbf{k}-\mathbf{q})^2(\mathbf{k}-\mathbf{q})^2}\mathcal{S}(k_\perp, X(\xi))$$

$$-\int\frac{\mathrm{d}^2\mathbf{q}}{(2\pi)^2}\frac{\mathrm{d}^2\mathbf{l}}{(2\pi)^2}\frac{2(\xi\mathbf{k}-\mathbf{q})\cdot(\mathbf{l}-\mathbf{q})}{(\xi\mathbf{k}-\mathbf{q})^2(\mathbf{l}-\mathbf{q})^2}\mathcal{S}(k_\perp, X(\xi))\mathcal{S}(l_\perp, X(\xi))$$

In the previous expressions:

- $x_p q(x_p) \frac{\mathcal{S}(k_\perp, x_0)}{(2\pi)^2}$ represents the lowest order contribution

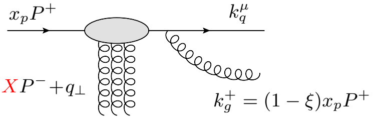
  (no BK evolution. x_0: initial condition)



- $X(\xi)$ is the rapidity scale at which the dipole correlators are evaluated

  At LO: the P^- fraction needed from the target is $\frac{k_\perp}{\sqrt{s}} e^{-y} \equiv x_g$

  At NLO:



$$X = \frac{k_\perp}{\sqrt{s}} e^{-y} \left(1 + \frac{\xi}{1-\xi} \frac{(q_\perp - k_\perp)^2}{k_\perp^2} \right)$$

$$\approx \frac{x_g}{1-\xi} \equiv X(\xi) \text{ when } k_\perp \gtrsim Q_s$$

The limit $\xi < 1 - \frac{x_g}{x_0} \equiv \xi_{\text{max}}$ enforces $X(\xi) < x_0$

We can write the sum of the LO and N_c terms as

$$\frac{dN^{\text{LO}+N_c}}{d^2\mathbf{k}dy} = x_p q(x_p)\frac{\mathcal{S}(k_\perp, x_0)}{(2\pi)^2} + \alpha_s \int_0^{1-x_g/x_0} \frac{d\xi}{1-\xi}\mathcal{K}(k_\perp, \xi, X(\xi)) \equiv \frac{dN^{\text{LO}+N_c,unsub}}{d^2\mathbf{k}dy},$$

$$\mathcal{K}(k_\perp, \xi, X) = \frac{N_c}{(2\pi)^2}(1+\xi^2)\left[\theta(\xi - x_p)\frac{x_p}{\xi}q\left(\frac{x_p}{\xi}\right)\mathcal{J}(k_\perp, \xi, X) - x_p q(x_p)\mathcal{J}_v(k_\perp, \xi, X)\right].$$

At large $k_\perp$ the function $\mathcal{K}(k_\perp, \xi, X)$ is positive and so is the cross section.

Using the integral BK equation in momentum space,

$$\mathcal{S}(k_\perp, x_g) = \mathcal{S}(k_\perp, x_0) + 2\alpha_s N_c \int_0^{1-x_g/x_0} \frac{d\xi}{1-\xi}\left[\mathcal{J}(k_\perp, 1, X(\xi)) - \mathcal{J}_v(k_\perp, 1, X(\xi))\right],$$

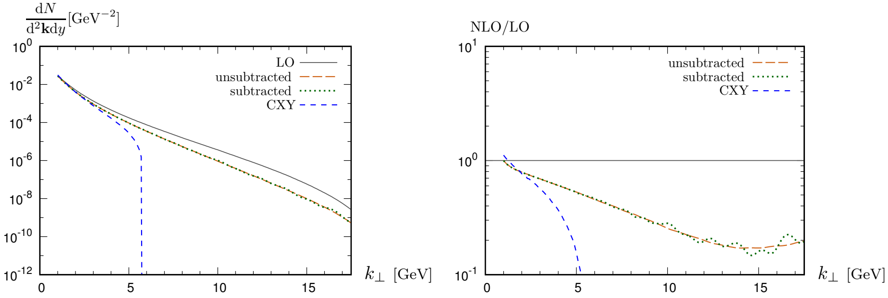
the LO+N_c terms can be rewritten as

$$\frac{dN^{\text{LO}+N_c,sub}}{d^2\mathbf{k}dy} = x_p q(x_p)\frac{\mathcal{S}(k_\perp, x_g)}{(2\pi)^2} + \alpha_s \int_0^{1-x_g/x_0} \frac{d\xi}{1-\xi}\left[\mathcal{K}(k_\perp, \xi, X(\xi)) - \mathcal{K}(k_\perp, 1, X(\xi))\right].$$

Up to NLO accuracy, one can take $X(\xi) \to x_g$ and $1 - \frac{x_g}{x_0} \to 1$ in this subtracted version. Local in rapidity: k_T-factorization as presented by CXY

N_c terms

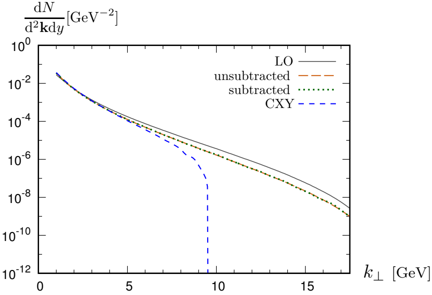
Results for the LO+N_c NLO corrections at fixed coupling ($\alpha_s = 0.2$):

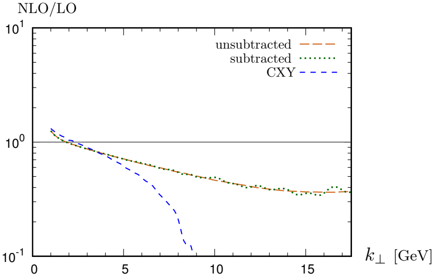


The 'subtracted' and 'unsubtracted' expressions give the same (positive) results

The 'CXY' approximation leads to negative results for $k_\perp \gtrsim 5$ GeV.

Total (LO+C_F+N_c) multiplicity ($\alpha_s = 0.2$):





Similar conclusions (the C_F terms are positive at large $k_\perp$)

Conclusions at fixed coupling

The negativity issue observed in the first implementation of the NLO impact factor can be attributed to approximations made in the LO subtraction

In the 'subtracted' formulation, we add and subtract a large contribution. If we use the 'CXY' approximation what we add and subtract is no longer the same which can make the final result negative

Without these approximations the cross section has a physical behavior

But so far we discussed only the fixed coupling case. The running of the coupling is an important effect that has to be taken into account in realistic calculations

Running coupling

The equivalence between the 'subtracted' and 'unsubtracted' formulations holds only if one uses the same coupling α_s when computing the cross section and when solving the BK equation

At semi-hard transverse momenta $k_\perp \gtrsim Q_s$ the natural choice for the scale is $\alpha_s(k_\perp)$. But the BK equation is usually solved in coordinate space

Fixed coupling BK equation:

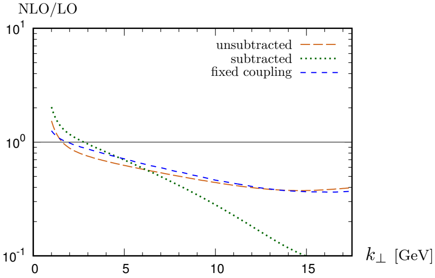
$$\frac{\partial S(\mathbf{r}, X)}{\partial \ln X} = 2\alpha_s N_c \int \frac{d^2\mathbf{x}}{(2\pi)^2} \frac{\mathbf{r}^2}{\mathbf{x}^2(\mathbf{r}-\mathbf{x})^2} [S(\mathbf{r}, X) - S(\mathbf{x}, X) S(\mathbf{r}-\mathbf{x}, X)]$$

rcBK with the simple parent dipole running coupling prescription:

$$\frac{\partial S(\mathbf{r}, X)}{\partial \ln X} = 2\alpha_s(r_\perp) N_c \int \frac{d^2\mathbf{x}}{(2\pi)^2} \frac{\mathbf{r}^2}{\mathbf{x}^2(\mathbf{r}-\mathbf{x})^2} [S(\mathbf{r}, X) - S(\mathbf{x}, X) S(\mathbf{r}-\mathbf{x}, X)]$$

The choice $\alpha_s(r_\perp)$ seems to be reasonable since $\mathbf{r}$ is Fourier-conjugate to $\mathbf{k}$

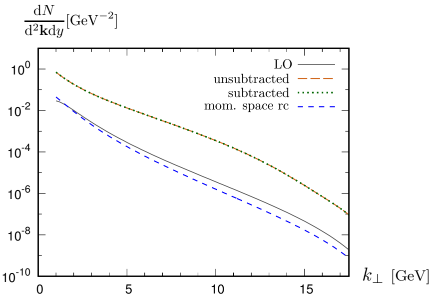
Using $\alpha_s(r_\perp)$ when solving BK and $\alpha_s(k_\perp)$ for the explicit α_s factors in the cross section:



The 'subtracted' and 'unsubtracted' expressions are no longer equivalent since we don't use exactly the same α_s in the cross section and when solving BK

- 'Subtracted' version: can become negative again at large $k_\perp$
- 'Unsubtracted' version: does not reduce to the correct LO result when $\xi \to 1$

Possible way to use consistently a coordinate-space running coupling: rewrite the cross section expression in coordinate space by Fourier transform. Then we can use the same coupling $\alpha_s(r_\perp)$ everywhere



The 'subtracted' expression gives the same results as the 'unsubtracted' one

But completely different results compared to fixed coupling or $\alpha_s(k_\perp)$, absurdly large NLO corrections

Similar situation with other prescriptions $\rightarrow$ the problem is more general

Running coupling

To illustrate the problem, let's look at the following simple quantities:

$$\mathcal{N}_k \equiv \bar{\alpha}_s(k_\perp)\,\mathcal{S}(k_\perp) = \bar{\alpha}_s(k_\perp)\int \mathrm{d}^2\mathbf{r}\, e^{-i\mathbf{k}\cdot\mathbf{r}} S(\mathbf{r})$$

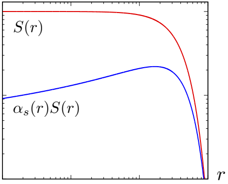
$$\mathcal{N}_r \equiv \int \mathrm{d}^2\mathbf{r}\, \bar{\alpha}_s(r_\perp) e^{-i\mathbf{k}\cdot\mathbf{r}} S(\mathbf{r})$$

These two quantities do not differ by only a small factor. Indeed, using the McLerran-Venugopalan model $S(r_\perp) = \exp\left(-\frac{r_\perp^2 Q_s^2}{4}\ln\frac{1}{r_\perp^2\Lambda^2}\right)$, we find at large $k_\perp$

$$\mathcal{N}_k \sim \frac{4\pi\bar{\alpha}_s(k_\perp)Q_s^2}{k_\perp^4} \qquad \text{while} \qquad \mathcal{N}_r \sim -\frac{4\pi}{\bar{b}[\ln(k_\perp^2/\Lambda^2)]^2}\,\frac{1}{k_\perp^2}$$

which are opposite in sign and have different tails: the choice of the running coupling prescription and the Fourier transform do not 'commute'

Note that the problem comes from the perturbative region $\mathbf{r} \to 0$, not the IR region



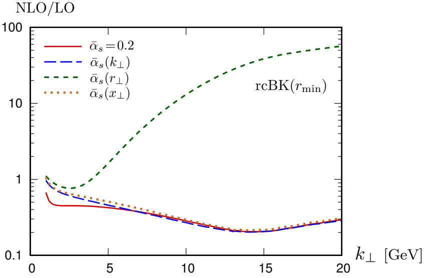
Running coupling

The problem comes from large daughter dipoles contributions $x_\perp \gg r_\perp$.
Indeed, in this limit we have for example

$$\mathcal{J}(\mathbf{k},\xi) \sim \frac{\bar{\alpha}_s}{2\pi^2}\int \mathrm{d}^2\mathbf{r}\, e^{-i\mathbf{k}\cdot\mathbf{r}} \underbrace{\int \frac{\mathrm{d}^2\mathbf{x}}{\mathbf{x}^2}\left[S((1-\xi)\mathbf{x}) - S(-\xi\mathbf{x})S(\mathbf{x})\right]}_{\mathbf{r}-\text{independent}} = 0 \text{ for } k_\perp \neq 0$$

On the contrary, if we move the coupling under the integral and replace $\alpha_s \to \alpha_s(r_\perp)$, we get a large contribution from the F.T. of the coupling

To avoid this problem we propose to use the daughter dipole prescription $\alpha_s(x_\perp)$:



The cross section has a physical behavior, similar to $\alpha_s(k_\perp)$

Conclusions

The saturation formalism is being pushed to NLO accuracy. The issues met in the first implementation of the NLO forward hadron production impact factor are now understood:

- Fixed coupling: negativity due to approximations made in the original implementation of the NLO impact factor
- Running coupling: new complications (mismatch between coordinate and momentum space couplings, spurious contributions due to F.T.)

  Can be avoided by using the daughter dipole prescription $\alpha_s(x_\perp)$

Outlook:

- Add the $q \to g$, $g \to q$ and $g \to g$ channels + fragmentation functions
- Use (resummed) NLO BK for high energy evolution
- The initial condition for the BK evolution of the target must be obtained by a fit (e.g. to HERA DIS data) also performed at NLO accuracy