

Conformal invariance of TMD rapidity evolution

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Towards Accuracy at small- x
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communications

- Motivation.
- Reminder of high-energy OPE for DIS
 - BK equation and $SL(2, C)$ Möbius group.
- Rapidity factorization for particle production in hadron-hadron collisions.
- Conformal invariance of TMD Operators
- Conformal TMD Rapidity evolution equation in Sudakov region.
- Conclusions and outlook.

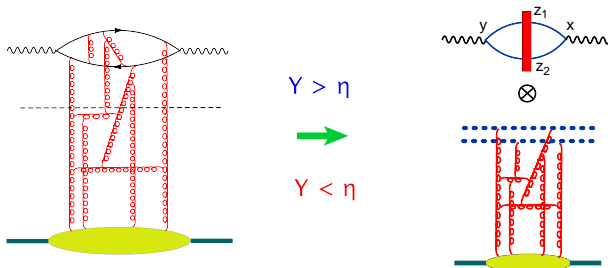
- TMD evolution equations are analyzed by different methods at moderate x_B , CSS and SCET, and at small- x_B resulting in different evolution equations.
- At the future Electron Ion Collider, TMD will be probed from low to high x_B . It is then necessary to develop a formalism which is valid in both limits.
- In the region of $x_B \sim 1$ TMD analysis is performed with a combination of UV and rapidity cutoff which gives two evolution equations in μ^2 and ζ (related to rapidity). Such evolution equations are known at two and three-loop, but their relation to the conformal properties of TMD is not known.

- **I. Balitsky and A. Tarasov (2016)**: Evolution equation for gluon TMD valid for all x_B and all k_\perp . Result is complicated and not unique. **Conformal Invariance may help.**

$$\begin{aligned}
 \frac{d}{d \ln \sigma} \tilde{\mathcal{F}}_i^a(\beta_B, x_\perp) \mathcal{F}_j^a(\beta_B, y_\perp) = & -\alpha_s \text{Tr} \left\{ \int d^2 k_\perp (x_\perp | \left\{ U^\dagger \frac{1}{\sigma \beta_{BS} + p_\perp^2} (U k_k + p_k U) \frac{\sigma \beta_{BS} g_{\mu i} - 2k_\perp^\perp k_i}{\sigma \beta_{BS} + k_\perp^2} \right. \right. \\
 & - 2k_\perp^\perp g_{ik} U^\dagger \frac{1}{\sigma \beta_{BS} + p_\perp^2} U - 2g_{\mu k} U^\dagger \frac{p_i}{\sigma \beta_{BS} + p_\perp^2} U + \frac{2k_\perp^\perp}{k_\perp^2} g_{ik} \left. \right\} \tilde{\mathcal{F}}^k \left(\beta_B + \frac{k_\perp^\perp}{\sigma s} \right) | k_\perp \rangle \\
 & \times (k_\perp | \mathcal{F}^l \left(\beta_B + \frac{k_\perp^\perp}{\sigma s} \right) \left\{ \frac{\sigma \beta_{BS} \delta_j^\mu - 2k_\perp^\perp k_j}{\sigma \beta_{BS} + k_\perp^2} (k_l U^\dagger + U^\dagger p_l) \frac{1}{\sigma \beta_{BS} + p_\perp^2} U \right. \\
 & \quad \left. - 2k_\perp^\perp g_{jl} U^\dagger \frac{1}{\sigma \beta_{BS} + p_\perp^2} U - 2\delta_i^\mu U^\dagger \frac{p_j}{\sigma \beta_{BS} + p_\perp^2} U + 2g_{jl} \frac{k_\perp^\perp}{k_\perp^2} \right\} | y_\perp \rangle \\
 & + 2\tilde{\mathcal{F}}_i(\beta_B, x_\perp) (y_\perp | \frac{p^m}{p_\perp^2} \mathcal{F}_k(\beta_B) (i \overleftarrow{\partial}_l + U_l) (2\delta_m^k \delta_j^l - g_{jm} g^{kl}) U^\dagger \frac{1}{\sigma \beta_{BS} - p_\perp^2 + i\epsilon} U \\
 & \quad + \mathcal{F}_j(\beta_B) \frac{\sigma \beta_{BS}}{p_\perp^2 (\sigma \beta_{BS} - p_\perp^2 + i\epsilon)} | y_\perp \rangle \\
 & + 2(x_\perp | - U^\dagger \frac{1}{\sigma \beta_{BS} - p_\perp^2 - i\epsilon} U (2\delta_i^k \delta_m^l - g_{im} g^{kl}) (i \partial_k - U_k) \tilde{\mathcal{F}}_l(\beta_B) \frac{p^m}{p_\perp^2} \\
 & \quad \left. + \tilde{\mathcal{F}}_i(\beta_B) \frac{\sigma \beta_{BS}}{p_\perp^2 (\sigma \beta_{BS} - p_\perp^2 - i\epsilon)} | x_\perp \rangle \mathcal{F}_j(\beta_B, y_\perp) \right\} + O(\alpha_s^2)
 \end{aligned}$$

High-Energy Operator Product Expansion

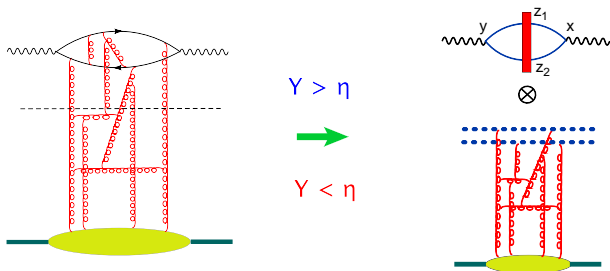
DIS amplitude is factorized in rapidity: η



$|B\rangle$ is the target state.

$$\langle B|T\{\hat{j}_\mu(x)\hat{j}_\nu(y)\}|B\rangle = \int \frac{d^2z_1 d^2z_2}{z_{12}^4} I_{\mu\nu}^{\text{LO}}(x, y; z_1, z_2) \langle B|\text{tr}\{\hat{U}_{z_1}^\eta \hat{U}_{z_2}^{\dagger\eta}\}|B\rangle + \dots$$

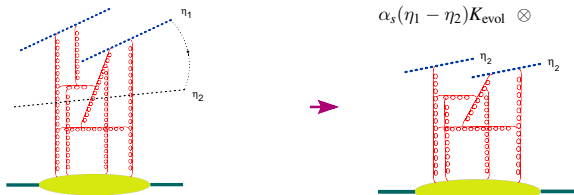
High-Energy Operator Product Expansion



$$\langle B | T \{ \hat{j}_\mu(x) \hat{j}_\nu(y) \} | B \rangle = \int \frac{d^2 z_1 d^2 z_2}{z_{12}^4} I_{\mu\nu}^{\text{LO}}(x, y; z_1, z_2) \langle B | \text{tr} \{ \hat{U}_{z_1}^\eta \hat{U}_{z_2}^{\dagger\eta} \} | B \rangle + \dots$$

- If we use a model to evaluate $\langle B | \text{tr} \{ \hat{U}_{z_1}^\eta \hat{U}_{z_2}^{\dagger\eta} \} | B \rangle$ we can calculate the DIS cross-section.
- If we want to include energy dependence to the DIS cross section, we need to find the evolution of $\langle B | \text{tr} \{ \hat{U}_{z_1}^\eta \hat{U}_{z_2}^{\dagger\eta} \} | B \rangle$ with respect to the rapidity parameter η .

Evolution Equation



- Separate fields in quantum and classical according to low and large rapidity. Formally we may write:

$$\langle B | \mathcal{O}^{\eta_1} | B \rangle \rightarrow \langle \mathcal{O}^{\eta_1} \rangle_A \rightarrow \langle \mathcal{O}^{\eta_2} \otimes \mathcal{O}'^{\eta_1} \rangle_A$$

- Integrate over the quantum fields and get one-loop rapidity evolution of the operator \mathcal{O}

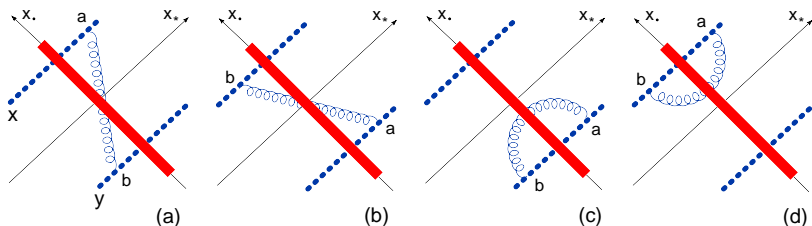
$$\langle \mathcal{O}^{\eta_1} \rangle_A = \alpha_s(\eta_1 - \eta_2)K_{\text{evol}} \otimes \langle \mathcal{O}'^{\eta_2} \rangle_A$$

- Where in principle \mathcal{O} and \mathcal{O}' are different operators.

Leading order evolution equation

$$\frac{d}{d\eta} \text{tr}\{\hat{U}_x \hat{U}_y^\dagger\} = K_{\text{LO}} \text{tr}\{\hat{U}_x \hat{U}_y^\dagger\} + \dots \Rightarrow$$

$$\frac{d}{d\eta} \langle \text{tr}\{\hat{U}_x \hat{U}_y^\dagger\} \rangle_{\text{shockwave}} = \langle K_{\text{LO}} \text{tr}\{\hat{U}_x \hat{U}_y^\dagger\} \rangle_{\text{shockwave}}$$



$$x_\bullet = \sqrt{\frac{s}{2}} x^-$$

$$x_* = \sqrt{\frac{s}{2}} x^+$$

$$\hat{U}(x, y) \equiv 1 - \frac{1}{N_c} \text{tr}\{\hat{U}(x_\perp)\hat{U}^\dagger(y_\perp)\}$$

$$\frac{d}{d\eta}\hat{U}(x, y) = \frac{\alpha_s N_c}{2\pi^2} \int \frac{d^2z}{(x-z)^2(y-z)^2} \left\{ \hat{U}(x, z) + \hat{U}(z, y) - \hat{U}(x, y) - \hat{U}(x, z)\hat{U}(z, y) \right\}$$

- LLA for DIS in pQCD \Rightarrow BFKL
 - (LLA: $\alpha_s \ll 1, \alpha_s \eta \sim 1$): describes proliferation of gluons.
- LLA for DIS in semi-classical-QCD \Rightarrow BK eqn
 - background field method: describes recombination process.

Formally, a light-like Wilson line

$$[\infty p_1 + x_\perp, -\infty p_1 + x_\perp] = \text{Pexp} \left\{ ig \int_{-\infty}^{\infty} dx^+ A_+(x^+, x_\perp) \right\}$$

is invariant under inversion (with respect to the point with $x^- = 0$).

Conformal invariance of the BK equation

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$$[\infty p_1 + x_\perp, -\infty p_1 + x_\perp] \rightarrow \text{Pexp} \left\{ ig \int_{-\infty}^{\infty} d\frac{x^+}{x_\perp^2} A_+\left(\frac{x^+}{x_\perp^2}, \frac{x_\perp}{x_\perp^2}\right) \right\} = [\infty p_1 + \frac{x_\perp}{x_\perp^2}, -\infty p_1 + \frac{x_\perp}{x_\perp^2}]$$

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\Rightarrow The dipole kernel is invariant under the inversion $V(x_\perp) = U(x_\perp/x_\perp^2)$

$$\frac{d}{d\eta} \text{Tr}\{V_x V_y^\dagger\} = \frac{\alpha_s}{2\pi^2} \int \frac{d^2 z}{z^4} \frac{(x-y)^2 z^4}{(x-z)^2 (z-y)^2} [\text{Tr}\{V_x V_z^\dagger\} \text{Tr}\{V_z V_y^\dagger\} - N_c \text{Tr}\{V_x V_y^\dagger\}]$$

SL(2,C) for Wilson lines

$$\hat{S}_- \equiv \frac{i}{2}(K^1 + iK^2), \quad \hat{S}_0 \equiv \frac{i}{2}(D + iM^{12}), \quad \hat{S}_+ \equiv \frac{i}{2}(P^1 - iP^2)$$

$$[\hat{S}_0, \hat{S}_\pm] = \pm \hat{S}_\pm, \quad \frac{1}{2}[\hat{S}_+, \hat{S}_-] = \hat{S}_0,$$

$$[\hat{S}_-, \hat{U}(z, \bar{z})] = z^2 \partial_z \hat{U}(z, \bar{z}), \quad [\hat{S}_0, \hat{U}(z, \bar{z})] = z \partial_z \hat{U}(z, \bar{z}), \quad [\hat{S}_+, \hat{U}(z, \bar{z})] = -\partial_z \hat{U}(z, \bar{z})$$

$$z \equiv z^1 + iz^2, \quad \bar{z} \equiv z^1 - iz^2, \quad U(z_\perp) = U(z, \bar{z})$$

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Conformal invariance of the evolution kernel

$$\frac{d}{d\eta} [\hat{S}_-, \text{Tr}\{U_x U_y^\dagger\}] = \frac{\alpha_s N_c}{2\pi^2} \int dz K(x, y, z) [\hat{S}_-, \text{Tr}\{U_x U_y^\dagger\} \text{Tr}\{U_x U_y^\dagger\}]$$
$$\Rightarrow \left[x^2 \frac{\partial}{\partial x} + y^2 \frac{\partial}{\partial y} + z^2 \frac{\partial}{\partial z} \right] K(x, y, z) = 0$$

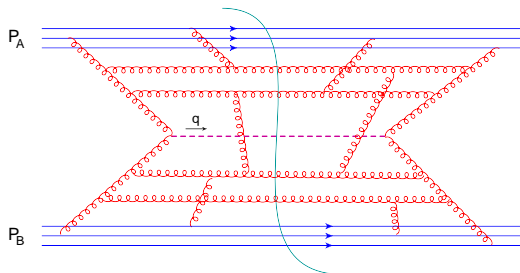
Particle production in hadron-hadron collisions

Light cone vectors n, n' and Sudakov variables:

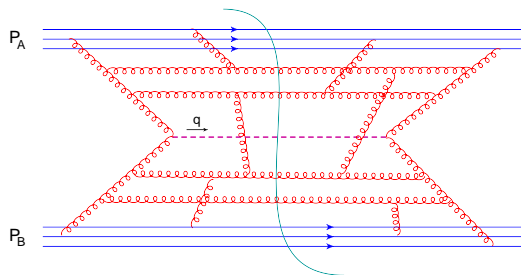
$$n^2 = n'^2 = 0, \quad n \cdot n' = 1, \quad P_A = \sqrt{\frac{s}{2}} n + \frac{P_A^2}{\sqrt{2s}} n', \quad P_B = \sqrt{\frac{s}{2}} n' + \frac{P_B^2}{\sqrt{2s}} n$$

$$x^+ = \sqrt{\frac{2}{s}} p_2 \cdot x, \quad x^- = \sqrt{\frac{2}{s}} p_1 \cdot x$$

$$k^\mu = \alpha p_1^\mu + \beta p_2^\mu + k_\perp = \sqrt{\frac{2}{s}} k^+ p_1^\mu + \sqrt{\frac{2}{s}} k^- p_2^\mu + k_\perp^\mu$$

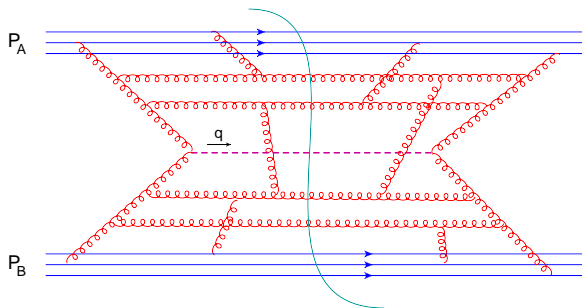


Particle production in hadron-hadron collisions



- typical TMD region: $s \sim q^2 = m_X^2 \gg q_{\perp}^2 \sim 1\text{Gev}$
- Sudakov region: $s \sim q^2 = m_X^2 \gg q_{\perp}^2 \gg 1\text{Gev}$
- small-x region : $s \gg q^2 \sim q_{\perp}^2 \gg m_N^2$

TMD factorization



$$\frac{d\sigma}{d\eta d^2q_\perp} = \sum_f \int d^2b_\perp e^{i(q,b)_\perp} \mathcal{D}_{f/A}(x_A, b_\perp, \eta) \mathcal{D}_{f/B}(x_B, b_\perp, \eta) \sigma(ff \rightarrow H) \\ + \text{power corrections} + \text{Y-terms}$$

See Collins' book for references

$$\frac{d\sigma}{d\eta d^2q_\perp} = \sum_f \int d^2b_\perp e^{i(q,b)_\perp} \mathcal{D}_{f/A}(x_A, b_\perp, \eta) \mathcal{D}_{f/B}(x_B, b_\perp, \eta) \sigma(ff \rightarrow H) \\ + \text{power corrections} + \text{Y-terms}$$

- rapidity: $\eta = \frac{1}{2} \ln \frac{q^+}{q^-}$.
- $\mathcal{D}_{f/A}$: TMD density of parton f in hadron A .
- $\sigma(ff \rightarrow H)$: cross section of production of particle H of invariant mass $m_H^2 = Q^2$ from two partons scattering.
- Power corrections: $\frac{q_\perp^2}{Q^2}$.
- Y-terms: for $q_\perp^2 \sim q^2$ allow transition to collinear factorization formula.

$$\begin{aligned}
 & \langle p_A, p_B | g^2 F_{\mu\nu}^a F^{a\mu\nu}(z_1) g^2 F_{\lambda\rho}^b F^{b\lambda\rho}(z_2) | p_A, p_B \rangle \\
 &= \frac{1}{N_c^2 - 1} \langle p_A | \tilde{\mathcal{O}}_{ij}(z_1^-, z_{1\perp}; z_2^-, z_{2\perp}) | p_A \rangle^{\sigma_A} \langle p_B | \mathcal{O}^{ij}(z_1^+, z_{1\perp}; z_2^+, z_{2\perp}) | p_B \rangle^{\sigma_B} + \dots
 \end{aligned}$$

Gluon TMD operator

$$\mathcal{O}_{ij}(z_1^+, z_{1\perp}; z_2^+, z_{2\perp}) = \mathcal{F}_i^a(z_1) [z_1 - \infty n, z_2 - \infty n]^{ab} \mathcal{F}_j^b(z_2) \Big|_{z_1^- = z_2^- = 0},$$

$$\tilde{\mathcal{O}}_{ij}(z_1^-, z_{1\perp}; z_2^-, z_{2\perp}) = \mathcal{F}_i^a(z_1) [z_1 - \infty n', z_2 - \infty n']^{ab} \mathcal{F}_j^b(z_2) \Big|_{z_1^+ = z_2^+ = 0},$$

$$(\tilde{\mathcal{F}}^{i,a}(z_\perp, z^+))^\sigma \equiv g F^{-i,m}(z) [\mathbf{P} e^{ig \int_{-\infty}^{z^+} dz^+ A^{-,\sigma}(up_1 + x_\perp)}]^{ma},$$

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$$A_\mu^\sigma(x) = \int \frac{d^4 k}{16\pi^4} \theta\left(\frac{\sigma\sqrt{2}}{z_{12\perp}} - |k^+|\right) e^{-ik \cdot x} A_\mu(k)$$

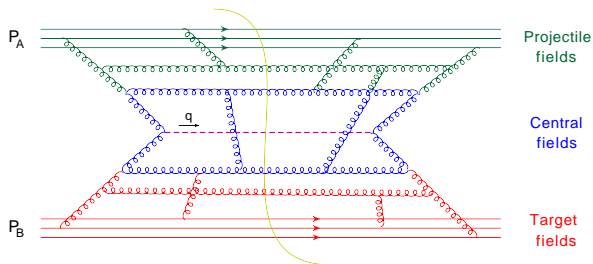
$\frac{\sigma\sqrt{2}}{z_{12\perp}}$ **cutoff preserving conformal invariance**

$$[x, y] \equiv \mathbf{P} e^{ig \int du (x-y)^\mu A_\mu(ux + (1-u)y)}$$

Particle production in hadron-hadron collisions

Rapidity Factorization

I. Balitsky and G.A.C. (2008),
I. Balitsky and A. Tarasov (2015)



- Projectile fields: $k^- < \sigma_a \Rightarrow$ Projectile TMD
- Central fields: coefficient functions
- Target fields: $k^+ < \sigma_b \Rightarrow$ Target TMD

Conformal group has 15 generators: Poincare + Dilatation + Special conformal transformation (inversion + shift + inversion)

$$i[M_{\mu\nu}, M_{\alpha\beta}] = g_{\mu\alpha}M_{\nu\beta} + g_{\nu\beta}M_{\mu\alpha} - g_{\mu\beta}M_{\nu\alpha} - g_{\nu\alpha}M_{\mu\beta}$$

$$i[M_{\alpha\beta}, P_{\mu}] = g_{\alpha\mu}P_{\beta} - g_{\beta\mu}P_{\alpha}$$

$$i[M_{\alpha\beta}, K_{\mu}] = g_{\alpha\mu}K_{\beta} - g_{\beta\mu}K_{\alpha}$$

$$i[D, P_{\mu}] = P_{\mu}, \quad i[D, K_{\mu}] = -K_{\mu}, \quad i[K_{\mu}, P_{\nu}] = 2(g_{\mu\nu}D + M_{\mu\nu})$$

Action on scalar field of canonical dimension Δ ;

$$i[D, \Phi(x)] = (x^{\alpha}\partial_{\alpha} + \Delta)\Phi(x)$$

$$i[K^{\mu}, \Phi(x)] = (2x^{\mu}x^{\alpha}\partial_{\alpha} - x^2\partial^{\mu} + 2\Delta x^{\mu})\Phi(x)$$

quantum correction: $\Delta \rightarrow \Delta + \text{anomalous}$

Conformal invariance of TMD operator

Conformal $SO(2,4)$ group has 15 generators

TMD operator transform covariantly under 11 generators:

TMD Conformal group

$$P^i, P^-, M^{12}, M^{-i}, D, K^i, K^-, M^{-+}$$

Generators P^+, K^+, M^{+i} do not preserve the form of \mathcal{F}^{-j} .

Conformal invariance of TMD operator

Action of conformal generators on TMD operators

$$-i\mathbf{P}^i \mathcal{F}^{-j}(x^+, x_\perp) = \partial^i \mathcal{F}^{-j}(x^+, x_\perp), \quad -i\mathbf{P}^- \mathcal{F}^{-j}(x^+, x_\perp) = \frac{\partial}{\partial x^+} \mathcal{F}^{-j}(x^+, x_\perp),$$

$$-i\mathbf{M}^{-i} \mathcal{F}^{-j}(x^+, x_\perp) = -x^i \frac{\partial}{\partial x^+} \mathcal{F}^{-j}(x^+, x_\perp),$$

$$-i\mathbf{D} \mathcal{F}^{-i}(x^+, x_\perp) = (x^+ \frac{\partial}{\partial x^+} + x^k \frac{\partial}{\partial x^k} + 2) \mathcal{F}^{-i}(x^+, x_\perp),$$

$$-i\mathbf{M}^{ij} \mathcal{F}^{-k}(x^+, x_\perp) = (x^i \partial^j - x^j \partial^i) \mathcal{F}^{-k}(x^+, x_\perp) + g^{ik} \mathcal{F}^{-j}(x^+, x_\perp) - g^{jk} \mathcal{F}^{-i}(x^+, x_\perp),$$

$$\begin{aligned} -\mathbf{K}^i \mathcal{F}^{-j}(x^+, x_\perp) &= 2x^i (x^+ \frac{\partial}{\partial x^+} + x^k \frac{\partial}{\partial x^k} + 2) \mathcal{F}^{-j}(x^+, x_\perp) + x_\perp^2 \frac{\partial}{\partial x^i} \mathcal{F}^{-j}(x^+, x_\perp) \\ &\quad - 2x^j \mathcal{F}^{-i}(x^+, x_\perp) + 2g^{ij} x_i \mathcal{F}^{-i}(x^+, x_\perp), \end{aligned}$$

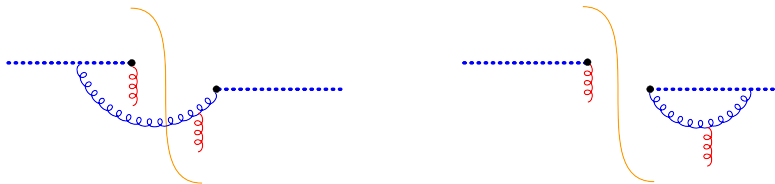
$$-i\mathbf{K}^- \mathcal{F}^{-j}(x^+, x_\perp) = x_\perp^2 \frac{\partial}{\partial x^+} \mathcal{F}^{-j}(x^+, x_\perp),$$

$$-i\mathbf{M}^{+-} \mathcal{F}^{-i}(x^+, x_\perp) = (x^+ \frac{\partial}{\partial x^+} + 1) \mathcal{F}^{-i}(x^+, x_\perp)$$

Rapidity evolution at leading order in Sudakov region

From low to moderate x_B in momentum space: I. Balitsky and A. Tarasov (2016)

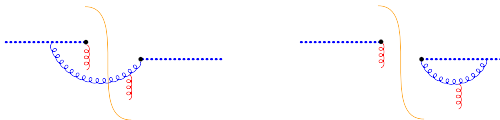
Typical LO diagrams



Rapidity evolution at leading order in Sudakov region

Coordinate space in Sudakov region: $s \sim q^2 = m_X^2 \gg q_{\perp}^2 \gg 1\text{GeV}$

Typical LO diagrams



Result ($\bar{\alpha}_s = \frac{\alpha_s N_c}{4\pi}$)

I. Balitsky and G.A.C (2019)

$$\begin{aligned} \mathcal{O}^{\sigma_2}(z_1^+, z_2^+) &= e^{-2\bar{\alpha}_s \ln \frac{\sigma_2}{\sigma_1} [\ln \sigma_1 \sigma_2 + 4\gamma_E - \ln 2]} \int dz_1'^+ dz_2'^+ \mathcal{O}^{\sigma_1}(z_1'^+, z_2'^+) z_{12\perp}^{-2\bar{\alpha}_s \ln \frac{\sigma_2}{\sigma_1}} \\ &\times \frac{1}{4\pi^2} \left[\frac{i\Gamma(1 - 2\bar{\alpha}_s \ln \frac{\sigma_2}{\sigma_1})}{(z_1^+ - z_1'^+ + i\epsilon)^{1-2\bar{\alpha}_s \ln \frac{\sigma_2}{\sigma_1}}} + c.c. \right] \left[\frac{i\Gamma(1 - 2\bar{\alpha}_s \ln \frac{\sigma_2}{\sigma_1})}{(z_2^+ - z_2'^+ + i\epsilon)^{1-2\bar{\alpha}_s \ln \frac{\sigma_2}{\sigma_1}}} + c.c. \right] \end{aligned}$$

Result transform covariantly under the TMD-conformal group generators except the Lorentz boost M^{+-} which is the generator of the evolution equation: **The Lorentz boost in z direction changes the cutoffs for the evolution.**

Conformal invariance of the TMD matrix element

Sudakov-region result is applicable in the region between:

$$\sigma_1 = \sigma_B = \frac{z_{12\perp}}{z_{12}^- \sqrt{2}} \quad \text{and} \quad \sigma_1 = \frac{z_{12}^+ \sqrt{2}}{z_{12\perp}}$$

Lorentz boost: $z^+ \rightarrow \lambda z^+$, $z^- \rightarrow \frac{1}{\lambda} z^-$

- $\langle p_A | \mathcal{O} | p_B \rangle \rightarrow \langle p_A | \mathcal{O} | p_B \rangle \exp\left\{4\lambda \bar{\alpha}_s \ln \frac{z_{12\parallel}^2}{z_{12\perp}^2}\right\}$
- $\langle p_A | \tilde{\mathcal{O}} | p_B \rangle \rightarrow \langle p_A | \tilde{\mathcal{O}} | p_B \rangle \exp\left\{-4\lambda \bar{\alpha}_s \ln \frac{z_{12\parallel}^2}{z_{12\perp}^2}\right\}$

So,

$$\begin{aligned} & \langle p_A, p_B | g^2 F_{\mu\nu}^a F^{a\mu\nu}(z_1) g^2 F_{\lambda\rho}^b F^{b\lambda\rho}(z_2) | p_A, p_B \rangle \\ &= \frac{1}{N_c^2 - 1} \langle p_A | \tilde{\mathcal{O}}_{ij}(z_1^-, z_{1\perp}; z_2^-, z_{2\perp}) | p_A \rangle^{\sigma_A} \langle p_B | \mathcal{O}^{ij}(z_1^+, z_{1\perp}; z_2^+, z_{2\perp}) | p_B \rangle^{\sigma_B} \end{aligned}$$

is invariant

- The conformal TMD group has been obtained: it is made out of 11 generators of the full conformal group.
- We obtained conformal evolution of gluon and quark ($N_c \rightarrow C_F$) TMDs in the Sudakov region.

Outlook

- Conformal properties of TMD evolution in the small- x region.
- Conformal evolution for all x_B .
- The plan is to perform the calculation for 6 point function similarly to the one done for the 4 point function in $\mathcal{N}=4$ and in in QCD.