Calculations in k_T-factorization

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Towards accuracy at small x

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Contents

- amplitudes with space-like initial-state partons
- some calculations
- improved transverse momentum factorization
- towards NLO

High Energy Factorization

a.k.a. k_T-factorization

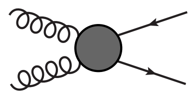
Catani, Ciafaloni, Hautmann 1991 Collins, Ellis 1991

$$\sigma_{h_1,h_2\to QQ} = \int d^2k_{1\perp}\,\frac{dx_1}{x_1}\,\mathcal{F}(x_1,k_{1\perp})\,d^2k_{2\perp}\,\frac{dx_2}{x_2}\,\mathcal{F}(x_2,k_{1\perp})\,\hat{\sigma}_{gg}\left(\frac{m^2}{x_1x_2s},\frac{k_{1\perp}}{m},\frac{k_{2\perp}}{m}\right)$$

- reduces to collinear factorization for $s \gg m^2 \gg k_\perp^2$, but holds al so for $s \gg m^2 \sim k_\perp^2$
- typically associated with small-x physics
- $k_\perp$-dependent $\mathcal{F}$ imagined to satisfy BFKL-eqn, CCFM-eqn, ...
- allows for higher-order kinematical effects at leading order
- requires matrix elements with *off-shell* initial-state partons with $k_i^2 = k_{i\perp}^2 < 0$

$k_1 = x_1p_1 + k_{1\perp}$

$k_2 = x_2p_2 + k_{2\perp}$

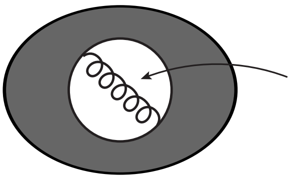


- Can this be generalized to "arbitrary" processes, with higher multiplicities in the final state?
- With well-defined gauge-invariant matrix elements?

Gauge invariance

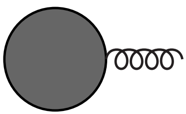
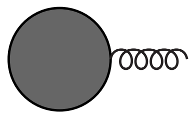
In order to be physically relevant, any scattering amplitude following the constructive definition given before must satisfy the following

Freedom in choice of gluon propagator:



$$\left\{ \begin{array}{l} \dfrac{-i}{k^2}\left[g^{\mu\nu} - (1-\xi)\dfrac{k^\mu k^\nu}{k^2}\right] \\[2ex] \dfrac{-i}{k^2}\left[g^{\mu\nu} - \dfrac{k^\mu n^\nu + n^\mu k^\nu}{k\cdot n} + (n^2 + \xi k^2)\dfrac{k^\mu k^\nu}{(k\cdot n)^2}\right] \end{array} \right.$$

Ward identity:

 $_\mu \varepsilon^\mu(k) \quad \rightarrow$  $_\mu k^\mu \; = \; 0$

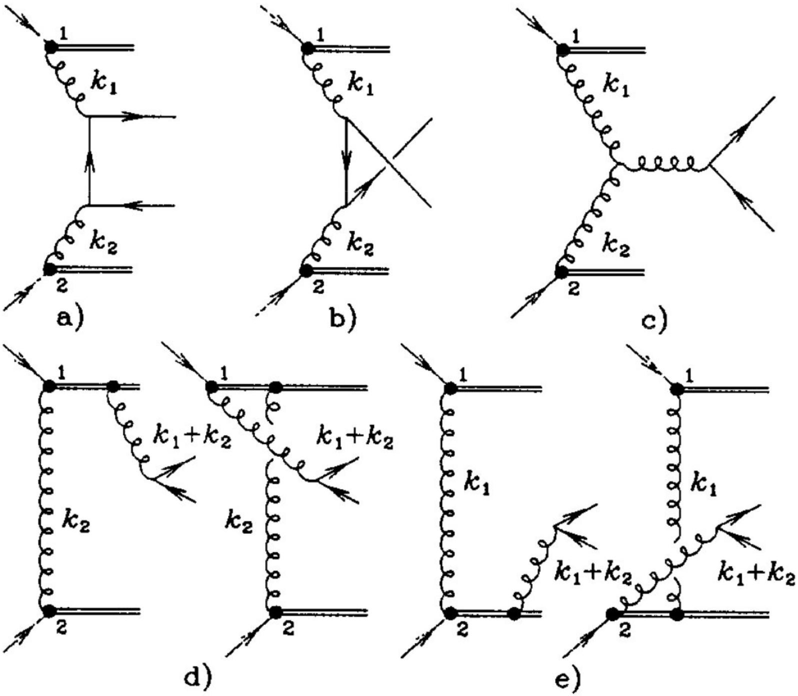
- Only holds if all external particles are on-shell.
- k_T-factorization requires off-shell initial-state momenta $k^\mu = p^\mu + k_T^\mu$.
- Keeping off-shell kinematics using "just" the usual Feynman graphs will in general not lead to a gauge invariant result.

A list of calculations

- M. Deak, F. Schwennsen 2008: *Z and $W^{\pm}$ production associated with quark-antiquark pair in kT-factorization at the LHC.* $g^*g^* \to q\bar{q} + W^{\pm}/Z$.
- S.P. Baranov, A.V. Lipatov, N.P. Zotov 2008: *Prompt photon hadroproduction at high energies in off-shell gluon-gluon fusion.* $g^*g^* \to q\bar{q} + \gamma$. *Production of electroweak gauge bosons in off-shell gluon-gluon fusion.* $g^*g^* \to q\bar{q} + W^{\pm}/Z$.
- A.V. Lipatov, N.P. Zotov 2009: *Associated production of Higgs boson and heavy quarks at the LHC: predictions with the kt-factorization.* $g^*g^* \to q\bar{q} + H$.
- M. Deak, F. Hautmann, H. Jung, K. Kutak 2009: *Forward Jet Production at the Large Hadron Collider.* $g^*q \to gq \ , \ g^*g \to q\bar{q} \ , \ g^*g \to gg$.
- L. Motyka, M. Sadzikowski, T. Stebel 2017: *Lam-Tung relation breaking in Z0 hadroproduction as a probe of parton transverse momentum.* $g^*g^* \to q\bar{q} + Z$.

Gauge invariance for $g^*g^* \to c\bar{c}$

Collins, Ellis, 1991



Using Wilson-line operators to terminate ladders leads to the 7 leading order graphs on the left.

Double-line represents eikonal line.

Gauge invariance for $g^*g^* \to c\bar{c}$

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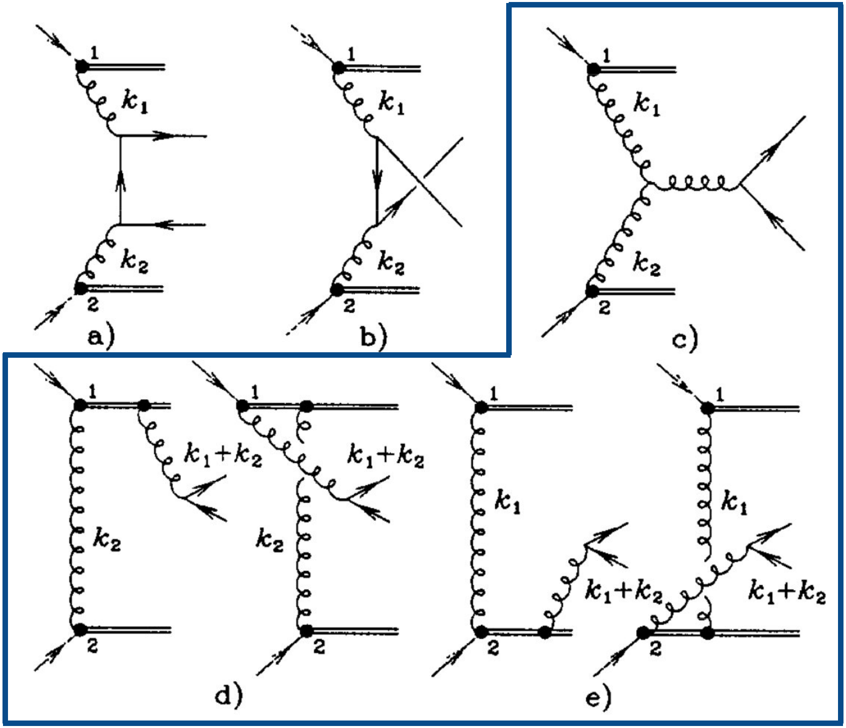


Using Wilson-line operators to terminate ladders leads to the 7 leading order graphs on the left.

Double-line represents eikonal line.

Lipatov vertex clearly recognizable.

Lipatov's effective action

Lipatov 1995

Effective action in terms of quarks $\psi, \bar{\psi}$, gluons A^μ, and reggeized gluons $R^\mu_\pm$ associated with direction $n^\mu_\pm$.
Tool to calculate effective vertices, that is hard centers, in multi-regge kinematics.

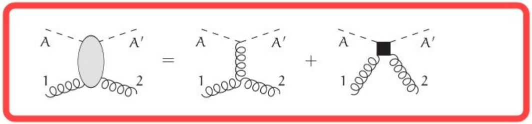
$$
\begin{aligned}
\mathcal{L} &= \mathcal{L}_{\text{QCD}} + \mathcal{L}_{\text{ind}} \\
\mathcal{L}_{\text{QCD}} &= i\bar{\psi}\not{D}\psi + \frac{1}{2}\mathrm{Tr}\, G^2_{\mu\nu} \qquad D_\mu = \partial_\mu + gA_\mu \qquad G_{\mu\nu} = \frac{1}{g}[D_\mu, D_\nu] \\
\mathcal{L}_{\text{ind}} &= -\mathrm{Tr}\left\{ \frac{1}{g}\partial_+ \left[\mathcal{P}\exp\left(-\frac{g}{2}\int_{-\infty}^{x^+} A_+(y)\,dy^+ \right) \right] \cdot \partial^2_\sigma R_-(x) \right. \\
&\qquad \left. + \frac{1}{g}\partial_- \left[\mathcal{P}\exp\left(-\frac{g}{2}\int_{-\infty}^{x^-} A_-(y)\,dy^- \right) \right] \cdot \partial^2_\sigma R_+(x) \right\}
\end{aligned}
$$

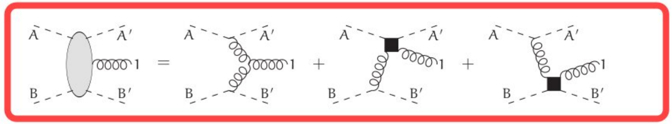
Reggeized gluons = gluons with momenta $k_1^\mu = En_+^\mu + k_T^\mu$, $k_2^\mu = En_-^\mu + k_T^\mu$.

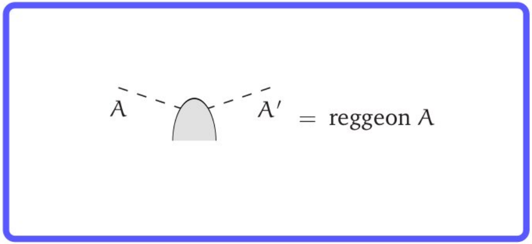
Extended to include reggeized quarks Lipatov, Vyazovsky 2000.

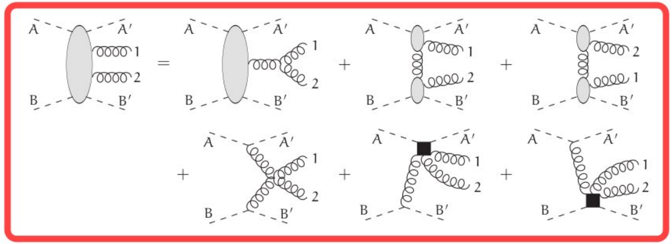
Effective vertices

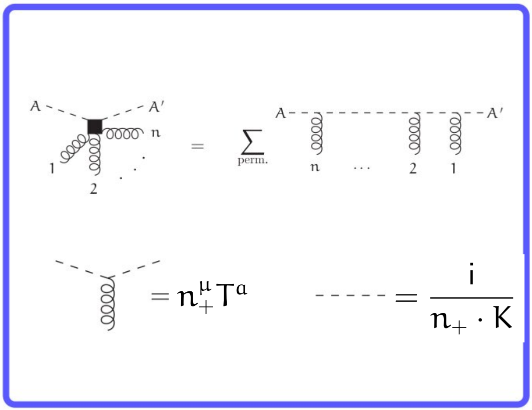
Antonov, Lipatov, Kuraev, Cherednikov 2005

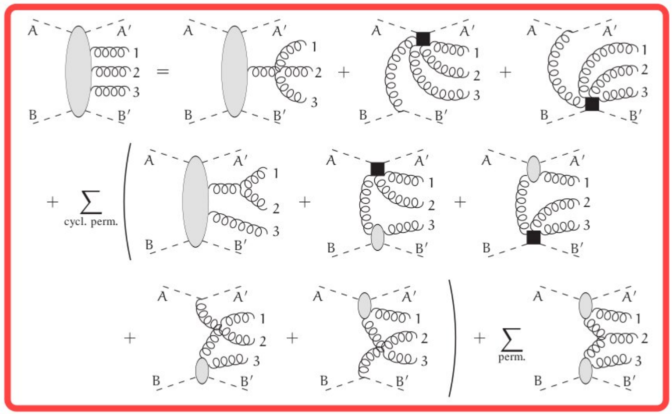












Dijet azimuthal decorrelations at the LHC

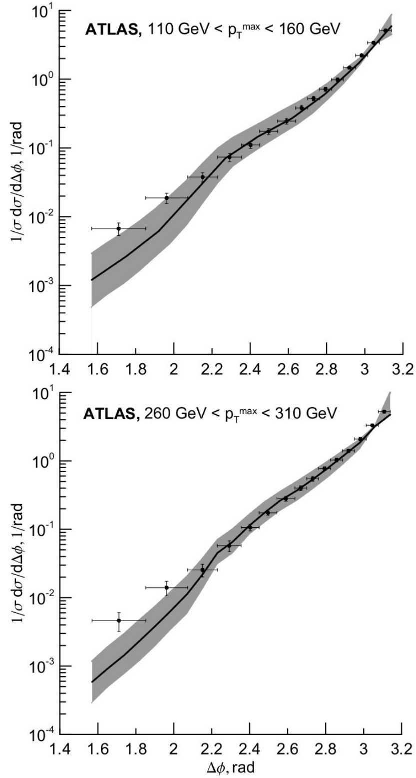
in the parton Reggeization approach

Nefedov, Saleev, Shipilova 2013

1. $RR \to gg$

$$\overline{|\mathcal{M}|^2} = \pi^2 \alpha_S^2 A \sum_{n=0}^{4} W_n S^n$$

$$
\begin{aligned}
A &= \frac{18}{a_1 a_2 b_1 b_2 s^2 t^2 u^2 t_1 t_2},\\
W_0 &= x_1 x_2 s^2 t u t_1 t_2 \big(x_1 x_2 (tu + t_1 t_2) + (a_1 b_2 + a_2 b_1) tu\big),\\
W_1 &= x_1 x_2 s t_1 t_2 \Bigg[t^2 u \Bigg(a_1 b_2 (a_2 b_2 + a_1 x_2)(t_1 + t_2) - a_2 b_1 (a_1 b_1 t_1 + a_2 b_2 t_2) + \\
&+ \big(x_2 (a_1^2 b_2 + a_2^2 b_1) + a_1 a_2 (b_1 - b_2)^2\big) u + x_1 x_2 a_1 b_2 t \Bigg) \Bigg] + \Big[a_1 \leftrightarrow a_2, b_1 \leftrightarrow b_2, t \leftrightarrow u \Big],\\
W_2 &= a_1 a_2 b_1 b_2 t u \Bigg(x_1^2 x_2^2 \big[2(t_1 + t_2)\big(t^2 u + t_1 t_2 (s + u - t)\big) + tu\big((t_1 - t_2)^2 + t(u + 2t)\big)\big] + \\
&+ x_1 x_2 t t_1 t_2 \big(4(x_1 b_1 + x_2 a_2)(s + u) - (a_1 b_1 + a_2 b_2) u\big) + \\
&+ tu \big(x_1^2 b_2 (2 x_2 t - b_1 t_1) t_1 + x_2^2 a_1 (2 x_1 t - a_2 t_2) t_2\big) \Bigg) + \Big(a_1 \leftrightarrow a_2, b_1 \leftrightarrow b_2, t \leftrightarrow u \Big),\\
W_3 &= x_1 x_2 a_1 a_2 b_1 b_2 \Bigg[t^2 u \Bigg(2 a_1 b_2 \big(x_1 x_2 (t_1 + t_2)(2t - u - s) - (x_1 b_2 t_1 + x_2 a_1 t_2)(u + s)\big) + \\
&+ \big[x_1 t_1 \big(2(a_1 b_2^2 + a_2 b_1^2) + 3 x_1 b_1 b_2\big) + x_2 t_2 \big(2(a_1^2 b_2 + a_2^2 b_1) + 3 a_1 a_2 x_2\big)\big] u + \\
&+ 4 x_1 x_2 t \big((a_1 b_2 + a_2 b_1) u + a_1 b_2 t\big) \Bigg) \Bigg] + \Big[a_1 \leftrightarrow a_2, b_1 \leftrightarrow b_2, t \leftrightarrow u \Big],\\
W_4 &= x_1^2 x_2^2 a_1 a_2 b_1 b_2 \Bigg[t \Bigg(a_1 a_2 b_1 b_2 u (t_1 + t_2)(t - u - s) + (a_1 b_2 + a_2 b_1)^2 t u^2 - \\
&- 2 a_1 b_2 t (s + u)(2 a_2 b_1 u - a_1 b_2 s) \Bigg) \Bigg] + \Big[a_1 \leftrightarrow a_2, b_1 \leftrightarrow b_2, t \leftrightarrow u \Big].
\end{aligned}
$$



All partonic channels included.

Dijet azimuthal decorrelations at the LHC

in the parton Reggeization approach

Nefedov, Saleev, Shipilova 2013

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$$\overline{|\mathcal{M}|^2} = \pi^2 \alpha_S^2 A \sum_{n=0}^{4} W_n S^n$$

$$A = \frac{18}{a_1 a_2 b_1 b_2 s^2 t^2 u^2 t_1 t_2},$$

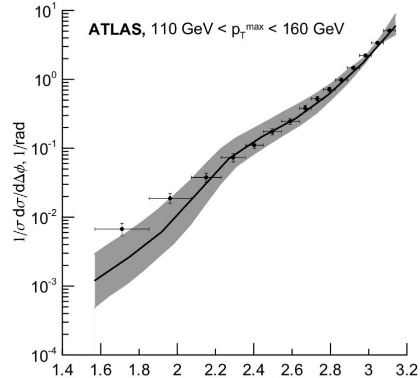
$$W_0 = x_1 x_2 s^2 t u t_1 t_2 \big(x_1 x_2 (tu + t_1 t_2) + (a_1 b_2 + a_2 b_1) tu\big),$$

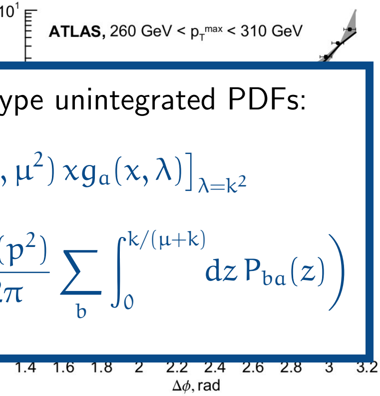
$$W_1 = x_1 x_2 s t_1 t_2 \Big[t^2 u \Big(a_1 b_2 (a_2 b_2 + a_1 x_2)(t_1 + t_2) - a_2 b_1 (a_1 b_1 t_1 + a_2 b_2 t_2) + $$
$$+ \big(x_2 (a_1^2 b_2 + a_2^2 b_1) + a_1 a_2 (b_1 - b_2)^2\big) u + x_1 x_2 a_1 b_2 t \Big) \Big] + \Big[a_1 \leftrightarrow a_2, b_1 \leftrightarrow b_2, t \leftrightarrow u \Big],$$

$$W_2 = a_1 a_2 b_1 b_2 t u \Big(x_1^2 x_2^2 \big[2(t_1 + t_2)\big(t^2 u + t_1 t_2 (s + u - t)\big) + tu\big((t_1 - t_2)^2 + t(u + 2t)\big)\big] + $$
$$+ x_1 x_2 t t_1 t_2 \big(4(x_1 b_1 + x_2 a_2)(s + u) - (a_1 b_1 + a_2 b_2) u\big) + $$
$$+ tu \big(x_1^2 b_2 (2 x_2 t - b_1 t_1) t_1 + x_2^2 a_1 (2 x_1 t \ \ldots$$

$$W_3 = x_1 x_2 a_1 a_2 b_1 b_2 \Big[t^2 u \Big(2 a_1 b_2 \big(x_1 x_2 (t_1 + \ \ldots$$
$$+ \big[x_1 t_1 \big(2(a_1 b_2^2 + a_2 b_1^2) + 3 x_1 b_1 b_2\big) + x \ \ldots$$
$$+ 4 x_1 x_2 t \big((a_1 b_2 + a_2 b_1) u + a_1 b_2 t\big) \Big) \Big] + \ \ldots$$

$$W_4 = x_1^2 x_2^2 a_1 a_2 b_1 b_2 \Big[t \Big(a_1 a_2 b_1 b_2 u (t_1 + t_2)(\ \ldots$$
$$- 2 a_1 b_2 t (s + u)(2 a_2 b_1 u - a_1 b_2 s) \Big) \Big] + \ \ldots$$





Kimber-Martin-Ryskin (KMR)-type unintegrated PDFs:

$$\mathcal{F}_a(x, k^2, \mu^2) = \partial_\lambda \left[T_a(\lambda, \mu^2)\, x g_a(x, \lambda) \right]_{\lambda = k^2}$$

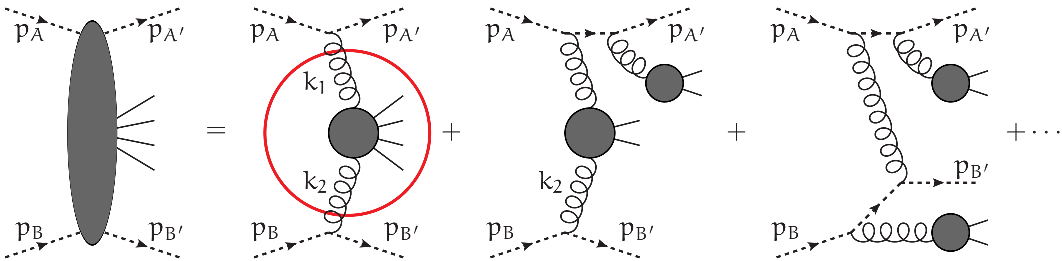
$$T_a(k^2, \mu^2) = \exp\left(- \int_{k^2}^{\mu^2} \frac{dp^2}{p^2} \frac{\alpha_S(p^2)}{2\pi} \sum_b \int_0^{k/(\mu+k)} dz\, P_{ba}(z) \right)$$

All partonic channels included.

Amplitude as embedding

AvH, Kutak, Kotko 2013

Embed the process in an on-shell process with auxiliary partons and eikonal Feynman rules.



$$p_A^\mu = \Lambda p_1^\mu - \frac{\kappa_1^*}{2}\,\varepsilon_1^{*\mu} \qquad p_A^2 = p_{A'}^2 = 0 \qquad k_{1T}^\mu = -\frac{\kappa_1}{2}\,\varepsilon_1^\mu - \frac{\kappa_1^*}{2}\,\varepsilon_1^{*\mu}$$

$$p_{A'}^\mu = -(\Lambda - x_1)p_1^\mu - \frac{\kappa_1}{2}\,\varepsilon_1^\mu \qquad p_A^\mu + p_{A'}^\mu = x_1 p_1^\mu - \frac{\kappa_1}{2}\,\varepsilon_1^\mu - \frac{\kappa_1^*}{2}\,\varepsilon_1^{*\mu} = k_1^\mu$$

Amplitude as embedding

AvH, Kutak, Kotko 2013

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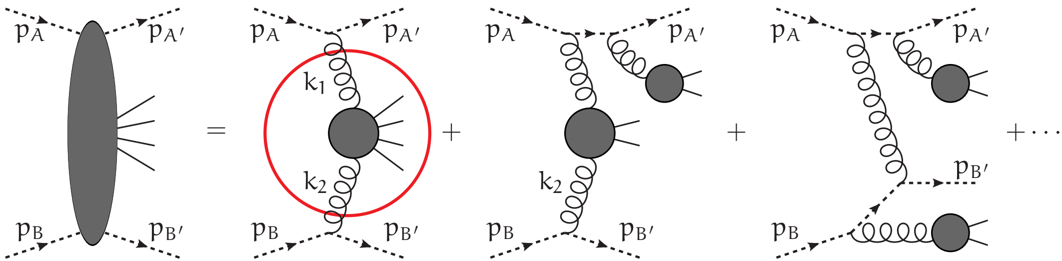
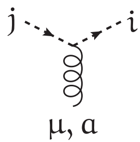
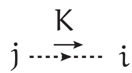


$$\left.\begin{aligned} p_A^\mu &= \Lambda p_1^\mu - \frac{\kappa_1^*}{2}\,\varepsilon_1^{*\mu} \\ p_{A'}^\mu &= -(\Lambda - x_1)p_1^\mu - \frac{\kappa_1}{2}\,\varepsilon_1^\mu \\ \Lambda &\to \infty \end{aligned}\right\} \Rightarrow$$



$$= -\mathrm{i}\, T^a_{i,j}\, p_1^\mu$$

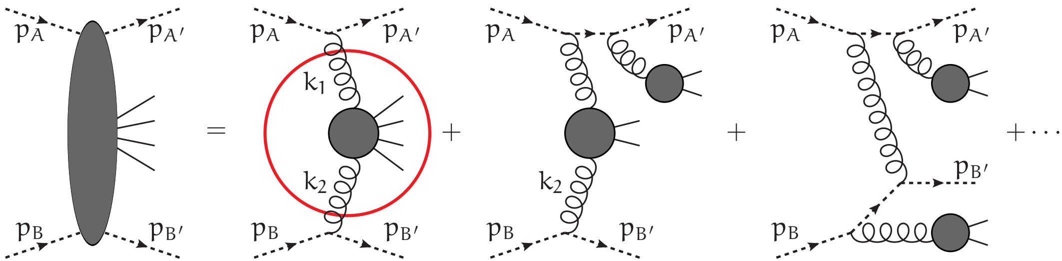


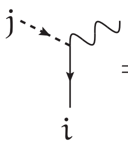
$$= \delta_{i,j}\,\frac{\mathrm{i}}{p_1\cdot K}$$

Amplitude as embedding

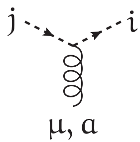
AvH, Kutak, Kotko 2013
AvH, Kutak, Salwa 2013

Embed the process in an on-shell process with auxiliary partons and eikonal Feynman rules.

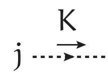




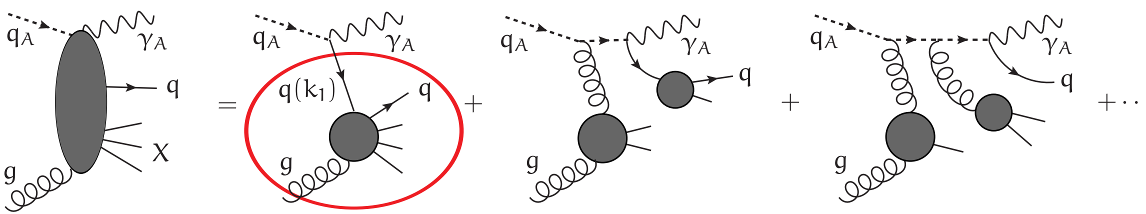
$= -i\,\delta_{i,j}\,u(p_1)$



$= -i\,T^a_{i,j}\,p_1^\mu$



$= \delta_{i,j}\,\dfrac{i}{p_1\cdot K}$



Gauge invariant amplitudes from Wilson lines

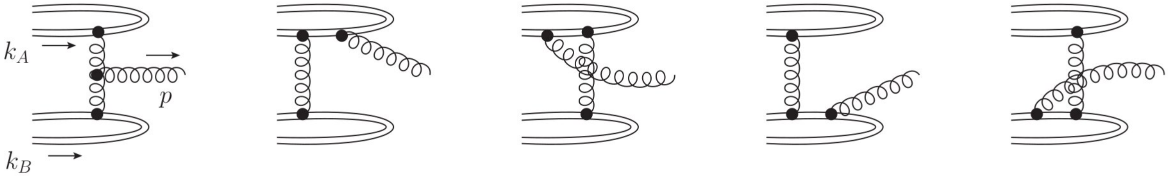
Kotko 2014

Using infinite Wilson line operator

$$\mathcal{R}^a(p,k) = \int d^4y\, e^{iy\cdot k}\, \mathrm{Tr}\left\{ \frac{1}{\pi g} T^a\, \mathcal{P} \exp\left[ig \int_{-\infty}^{\infty} ds\, p\cdot A_b(y+sp) T^b \right] \right\}$$

amplitudes with n on-shell gluons and m off-shell gluons can be defined by

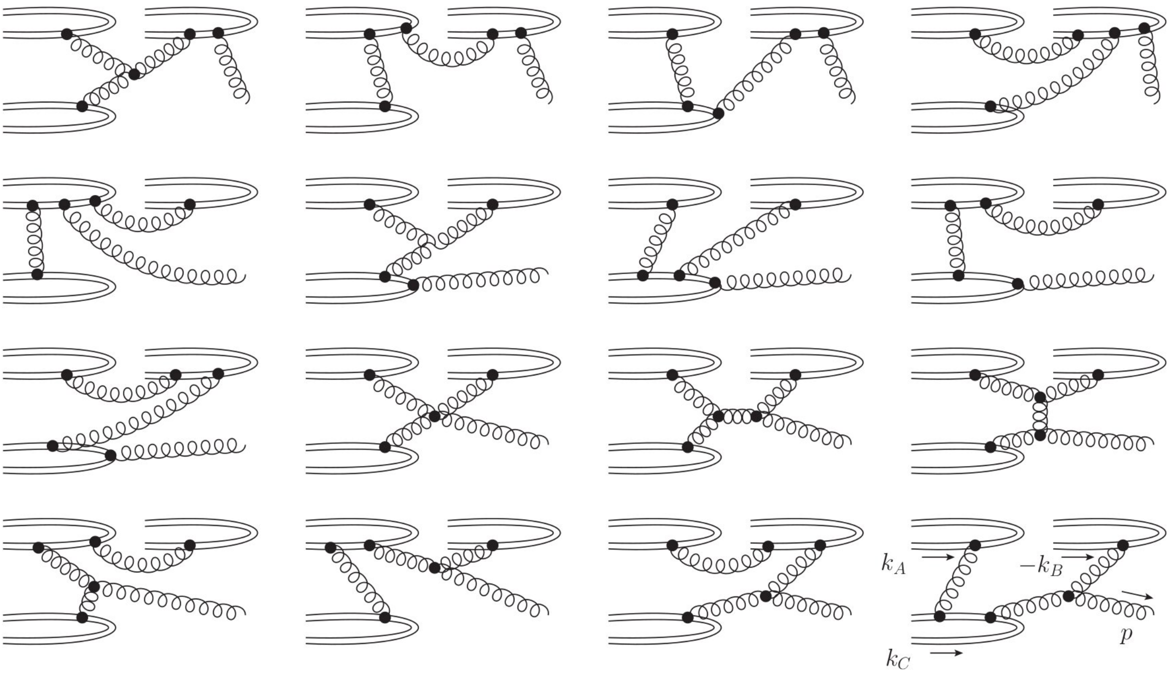
$$\begin{aligned}
&\left\langle k_1, k_2, \ldots, k_n \right| \mathcal{R}^{a_{n+1}}(p_{n+1}, k_{n+1})\, \mathcal{R}^{a_{n+2}}(p_{n+2}, k_{n+2}) \cdots \mathcal{R}^{a_{n+m}}(p_{n+m}, k_{n+m}) \left| 0 \right\rangle \\
&\qquad = \delta^4(k_1 + k_2 + \cdots + k_{n+m}) \\
&\qquad \times \delta(p_{n+1}\cdot k_{n+1})\, \delta(p_{n+2}\cdot k_{n+2}) \cdots \delta(p_{n+m}\cdot k_{n+m}) \\
&\qquad \times \mathcal{A}(k_1, k_2, \ldots, k_{n+m}\,;\, p_{n+1}, p_{n+2}, \ldots, p_{n+m}) \qquad + \cdots
\end{aligned}$$



Gauge invariant amplitudes from Wilson lines

Kotko 2014

$\emptyset \to g^* g^* g^* g$

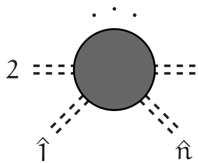


Program OGIME: `http://nz42.ifj.edu.pl/~pkotko/OGIME.html`

BCFW recursion

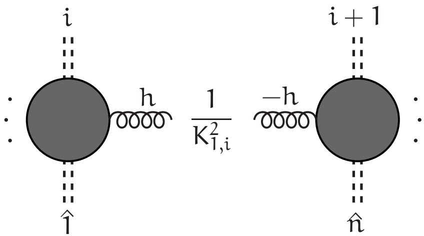
Britto, Cachazo, Feng, Witten 2005

The BCFW recursion formula



$$= \sum_{i=2}^{n-2} \sum_{h=+,-} A_{i,h}$$

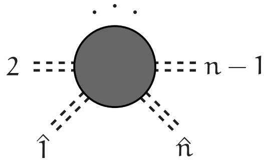
$$A_{i,h} =$$



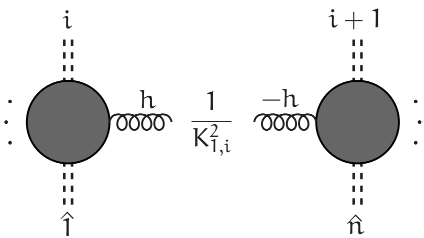
BCFW recursion for off-shell amplitudes

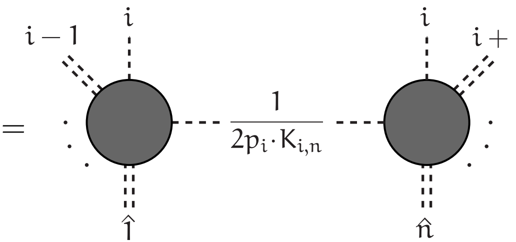
AvH 2014
AvH, Serino 2015

The BCFW recursion formula becomes



$$= \sum_{i=2}^{n-2} \sum_{h=+,-} A_{i,h} + \sum_{i=2}^{n-1} B_i$$

$A_{i,h} =$ 

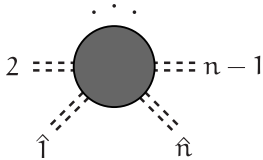
$B_i =$ 

“On-shell condition” for “off-shell” gluons: $p_i \cdot k_i = 0$

BCFW recursion for off-shell amplitudes

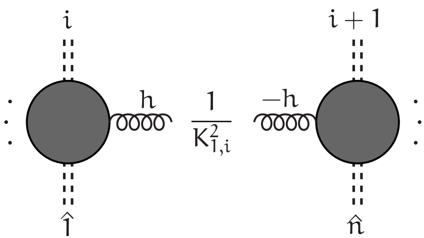
AvH 2014
AvH, Serino 2015

The BCFW recursion formula becomes

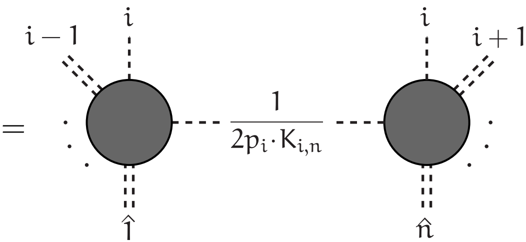


$$= \sum_{i=2}^{n-2} \sum_{h=+,-} A_{i,h} + \sum_{i=2}^{n-1} B_i + C + D \,,$$

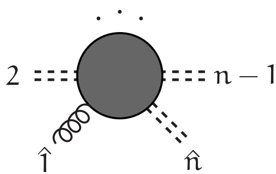
$A_{i,h} =$



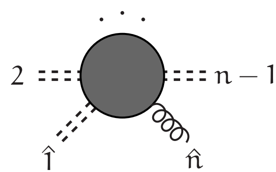
$B_i =$



$C = \frac{1}{\kappa_1}$



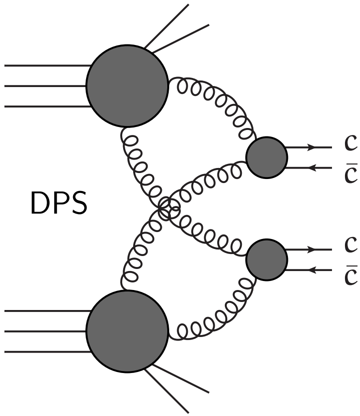
$D = \frac{1}{\kappa_1^*}$

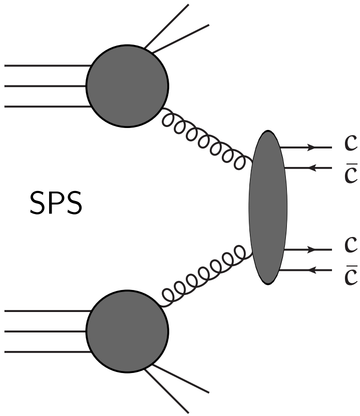


Charm with k_T-factorization

DPS vs SPS for $pp \to c\bar{c}\,c\bar{c}$

- LHCb measured a surprisingly large cross section for the production of D-meson pairs JHEP 06 141 (2012)
- production of $c\bar{c}\,c\bar{c}$ is a good place to study DPS effects Łuszczak, Maciuła, Szczurek 2012
- DPS $c\bar{c}\,c\bar{c}$ cross section approaches $c\bar{c}$ cross section for large energies
- k_T-factorization with KMR updfs gives a good description for open charm production Maciuła, Szczurek 2013
- DPS $c\bar{c}\,c\bar{c}$ cross section is orders of magnitude larger than LO SPS $c\bar{c}\,c\bar{c}$ cross section
  Schäfer, Szczurek 2012, Maciuła, Szczurek, AvH 2014
- SPS for $pp \to c\bar{c}\,c\bar{c}$ in k_T-factorization AvH, Maciuła, Szczurek 2015
- triple-parton scattering for $pp \to c\bar{c}\,c\bar{c}\,c\bar{c}$ Maciuła, Szczurek 2017
- DPS and SPS for $pp \to c\bar{c}\,b\bar{b}$ and $pp \to c\bar{c}\,jj$ Maciuła, Szczurek 2017,2018. (performed with KaTie)



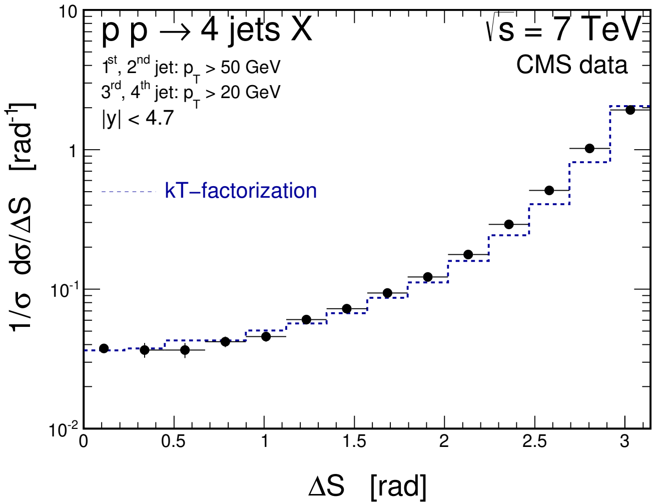


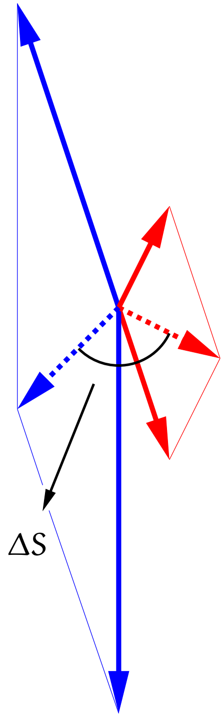
Charm with k_T-factorization

- k_T-factorization with KMR updfs gives a good description for open charm production Nefedov, Karpishkov, Saleev, Shipilova 2014
- double charmed meson production at LHCb Maciuła, Saleev, Shipilova, Szczurek 2016
- D mesons at LHCb Karpishkov, Saleev, Shipilova 2016
- B mesons at the LHC Karpishkov, Nefedov, Saleev, Shipilova 2015, Karpishkov, Nefedov, Saleev 2017
- $\psi(2S)$ and $\Upsilon(3S)$ hadroproduction Kniehl, Nefedov, Saleev 2016

Four jets with k_T-factorization

Maciuła, Szczurek, Kutak, Serino, AvH 2016



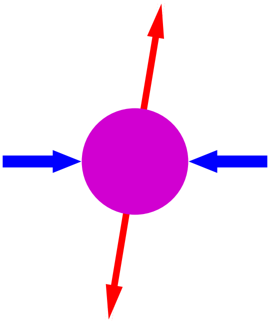


- ΔS is the azimutal angle between the sum of the two hardest jets and the sum of the two softest jets.
- This variable has no distribution at LO in collinear factorization: pairs would have to be back-to-back.
- k_T-factorization allows for the necessary momentum inbalance.

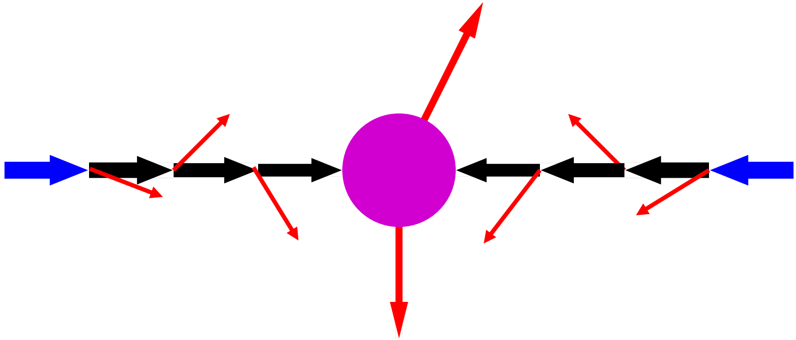
- parton level event generator, like Alpgen, Helac, MadGraph, etc.
- arbitrary processes within the standard model (including effective Higgs-gluon coupling) with several final-state particles.
- 0, 1, or 2 off-shell intial states.
- produces (partially un)weighted event files, for example in the LHEF format.
- requires LHAPDF. TMD PDFs can be provided as files containing rectangular grids, or with TMDlib Hautmann, Jung, Krämer, Mulders, Nocera, Rogers, Signori 2014.
- a calculation is steered by a single input file.
- employs an optimization stage in which the pre-samplers for all channels are optimized.
- during the generation stage several event files can be created in parallel.
- event files can be processed further by parton-shower program like CASCADE.
- (evaluation of) matrix elements now separately available, including C++ interface.

k_T from a parton shower

collinear hard scattering

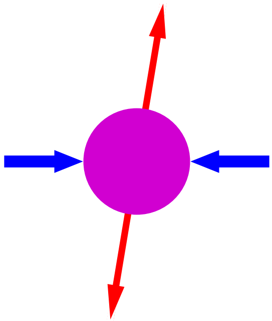


momentum inbalance by enforcing momentum conservation

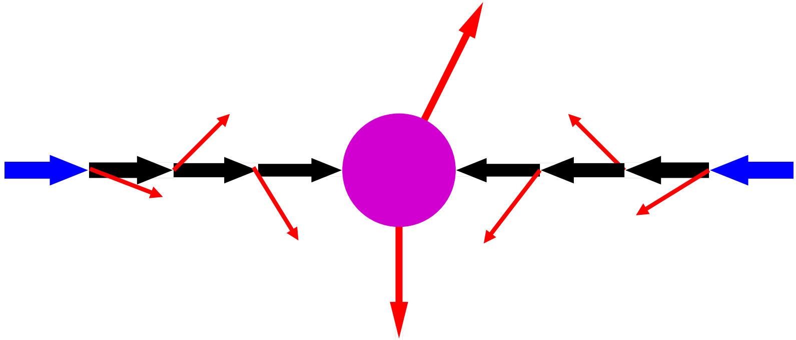


k_T from a parton shower

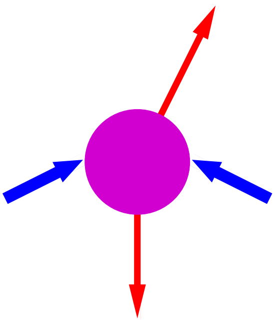
collinear hard scattering



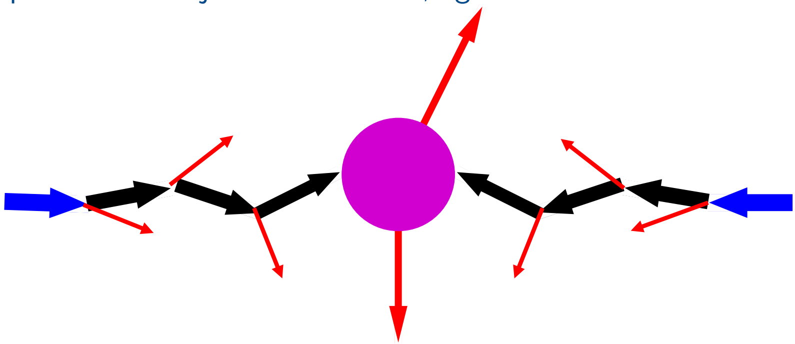
momentum inbalance by enforcing momentum conservation



"k_T" hard scattering



parton shower just unfolds PDF, eg. CCFM with CASCADE



Dijets with KaTie + CASCADE

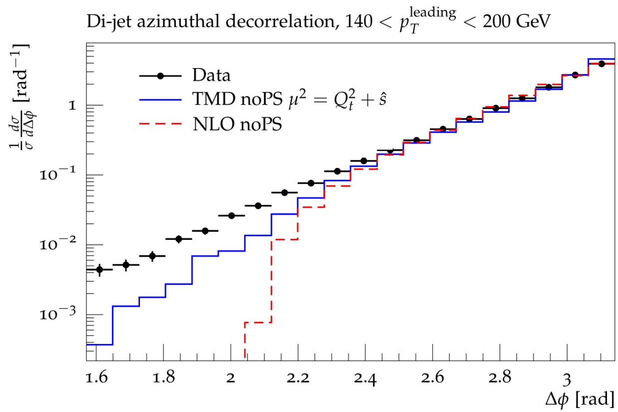
Bury, AvH, Jung, Kutak, Sapeta, Serino 2018

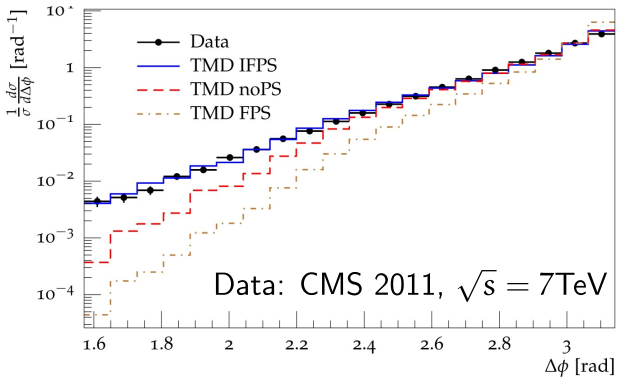
Generate parton-level events within k_T-factorization with KaTie, and shower them with CASCADE (Jung, Baranov, Deak, Grebenyuk, Hautmann, Hentschinski, Knutsson, Kraemer, Kutak, Lipatov, Zotov 2010)

KMR-type pdfs MRW-CT10nlo available in TMDlib Hautmann, Jung, Kraemer, Mulders, Nocera, Rogers, Signori

Backward-evolving shower unfolds the momentum inbalance into initial-state radiation.

Hard scale: $\mu^2 = Q_t^2 + \hat{s}$



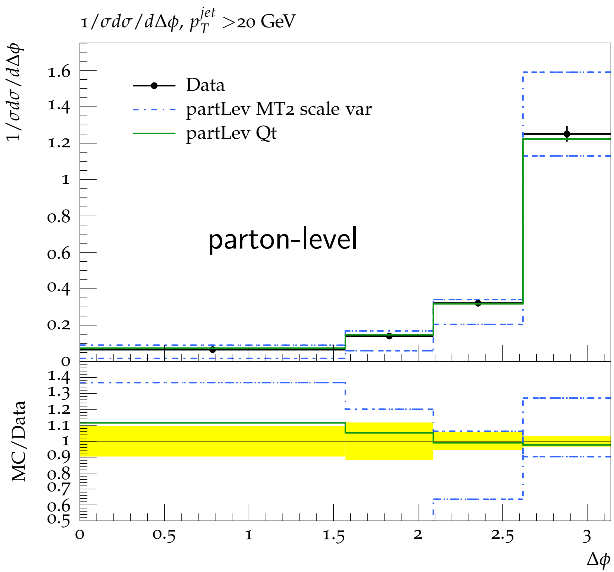


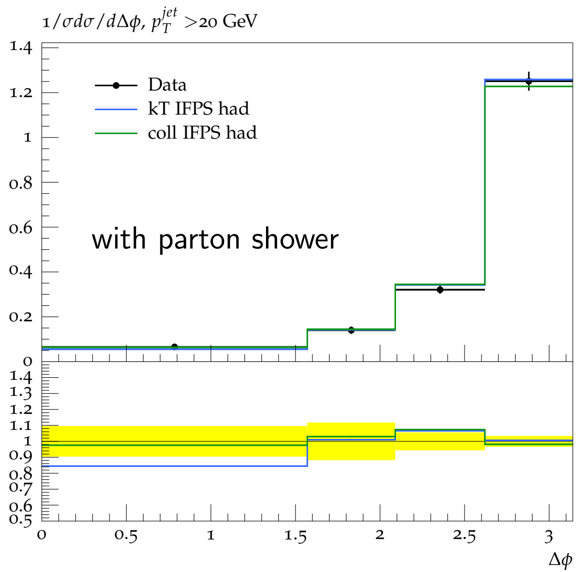
Z + j azimuthal de-correlation

Deak, AvH, Jung, Kusina, Kutak, Serino 2019

Comparison to LHCb-data at $\sqrt{s} = 7\text{TeV}$

Parton-branching TMDs PB-NLO-2018-set2 available in TMDlib





scale variation: $\{\frac{1}{2}, 2\} \times \sqrt{\mathbf{p}_{T,j}^2 + m_Z^2}$

scale $Q_t = \sqrt{\hat{s} + (\mathbf{p}_{T,j} + \mathbf{p}_{T,Z})^2}$

k_T-factorization

collinear factorization

TMD splitting functions in k_T-factorization

k_T-factorization gives the opportunity to resum large logarithms of x =(hard scale)/(total energy). Ideally, one would like to achieve this with

- a coupled system of evolution equations for unintegrated PDFs
- that allows for a smooth continuation to the large-x region
- reproduces the correct collinear (DGLAP) limit

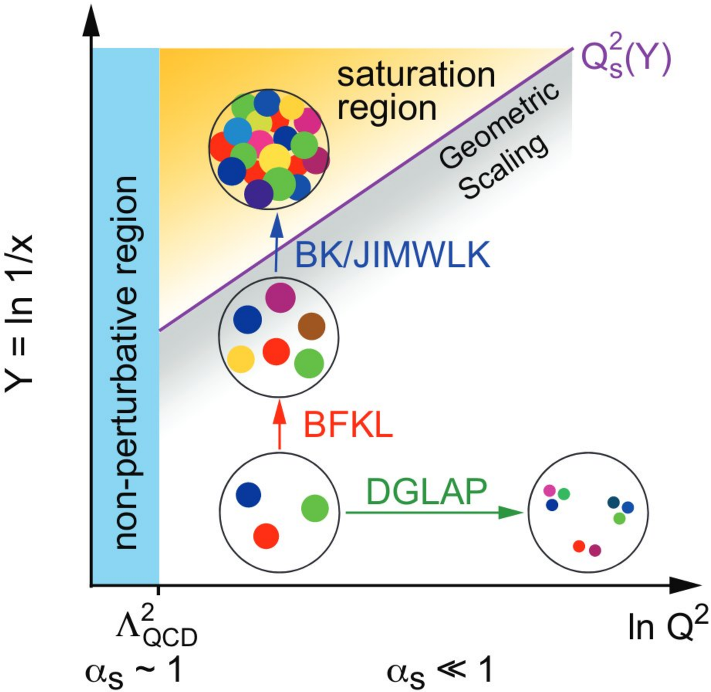
The real contribution to the necessecary k_T-dependent splitting functions have been calculated Hautmann, Henschinski, Jung 2012, Gituliar, Hentschinski, Kusina, Kutak 2016, Hentschinski, Kusina, Kutak, Serino 2018.
To get P_{gg} in particular, the formalism of Curci, Furmanski, Petronzio 1980 was extended with the necessary gauge invariant vertices and appropriate ladder-terminating projectors.

The final splitting functions feature the correct

- collinear limit (DGLAP kernels)
- high-energy limit (BFKL kernel)
- soft limit (CCFM kernel)

$$C_A \frac{\tilde{\boldsymbol{q}}^2}{\tilde{\boldsymbol{q}}^2 + z(1-z)\boldsymbol{k}^2}\left[\frac{(2-z)\tilde{\boldsymbol{q}}^2 + (z^3 - 4z^2 + 3z)\boldsymbol{k}^2}{z(1-z)\left|\tilde{\boldsymbol{q}}^2 - (1-z)^2\boldsymbol{k}^2\right|} + \frac{(2z^3 - 4z^2 + 6z - 3)\tilde{\boldsymbol{q}}^2 + z(4z^4 - 12z^3 + 9z^2 + z - 2)\boldsymbol{k}^2}{(1-z)(\tilde{\boldsymbol{q}}^2 + z(1-z)\boldsymbol{k}^2)}\right]$$

QCD evolution, dilute vs. dense



A dilute system carries a few high-x partons contributing to the hard scattering.

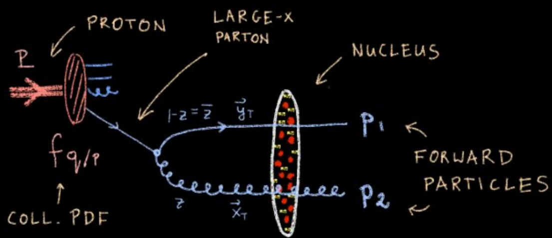
A dense system carries many low-x partons.

At high density, gluons are imagined to undergo recombination, and to saturate.

This is modeled with non-linear evolution equations, involving explicit non-vanishing k_T.

Saturation implies the turnover of the gluon density, stopping it from growing indefinitely for small x

pA (dilute-dense) collisions within CGC



$$\frac{d\sigma_{qA\to 2j}}{d^3p_1 d^3p_2} \sim \int \frac{d^2x}{(2\pi)^2}\frac{d^2x'}{(2\pi)^2}\frac{d^2y}{(2\pi)^2}\frac{d^2y'}{(2\pi)^2}\, e^{-i\vec{p}_{T1}\cdot(\vec{x}_T-\vec{x}'_T)} e^{-i\vec{p}_{T2}\cdot(\vec{y}_T-\vec{y}'_T)}$$

$$\times \psi_z^*\left(\vec{x}'_T-\vec{y}'_T\right)\psi_z\left(\vec{x}_T-\vec{y}_T\right)$$

QUARK WAVE FUNCTION

$$\times\Big\{S_x^{(6)}\left(\vec{y}_T,\vec{x}_T,\vec{y}'_T,\vec{x}'_T\right) - S_x^{(4)}\left(\vec{y}_T,\vec{x}_T,\bar{z}\vec{y}'_T+z\vec{x}'_T\right)$$
$$-S_x^{(4)}\left(\bar{z}\vec{y}_T+z\vec{x}_T,\vec{y}'_T,\vec{x}'_T\right) - S_x^{(2)}\left(\bar{z}\vec{y}_T+z\vec{x}_T,\bar{z}\vec{y}'_T+z\vec{x}'_T\right)\Big\}$$

CORRELATORS OF WILSON LINES

$$S_x^{(2)}\left(\vec{y}_T,\vec{x}_T\right) = \frac{1}{N_c}\left\langle \mathrm{Tr}\, U(\vec{y}_T)U^\dagger(\vec{x}_T)\right\rangle_x$$

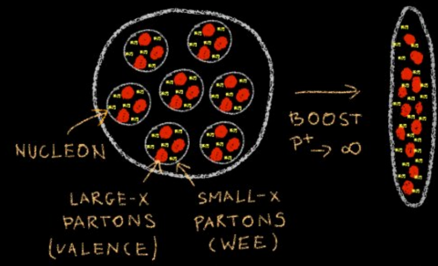
$$S_x^{(4)}\left(\vec{z}_T,\vec{y}_T,\vec{x}_T\right) = \frac{1}{2C_FN_c}\left\langle \mathrm{Tr}\left[U(\vec{z}_T)U^\dagger(\vec{y}_T)\right]\mathrm{Tr}\left[U(\vec{y}_T)U^\dagger(\vec{x}_T)\right]\right\rangle_x - S_x^{(2)}\left(\vec{z}_T,\vec{x}_T\right)$$

etc...

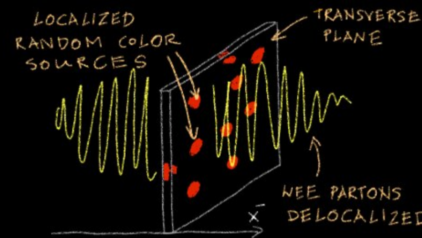
$$U(\vec{x}_T) = \mathscr{P}\exp\left\{ig\int_{-\infty}^{+\infty}dx^+\,A_a^-\left(x^+,\vec{x}_T\right)t^a\right\}$$

[C. Marquet, 2007]

COLOR FIELD OF THE NUCLEUS



[L. McLerran, R. Venugopalan, 1993]



Large-x partons — the color source for wee partons:

$$\left(D_\mu F^{\mu\nu}\right)_a\left(x^-,\vec{x}_T\right) = \delta^{\nu +}\rho_a\left(\vec{x}_T\right)\delta\left(x^-\right)$$

RANDOM DISTRIBUTION OF COLOR SOURCES

AVERAGE OVER COLOR SOURCES

GAUSSIAN FUNCTIONAL

B-JIMWLK EVOLUTION IN X

$$\mathscr{W}_x[\rho]$$

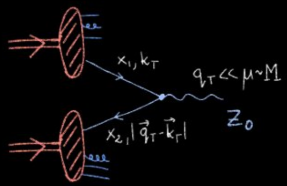
[Balitsky-Jalilian-Marian-Iancu-McLerran -Weigert-Leonidov-Kovner, 1996-2002]

P. Kotko, EPSHEP2019

TMD Factorization & TMD gluon distributions

TMD FACTORIZATION ALL-ORDER THEOREMS

- SEMI-INCLUSIVE DIS
- DRELL-YAN



$$\frac{d\sigma}{dq_T^2} \sim \int dx_1 dx_2\, \hat\sigma_{q\bar q} \int d^2k_T\, \mathcal{F}_q(x_1,k_T;\mu)\,\mathcal{F}_{\bar q}\left(x_2,|\vec k_T-\vec q_T|;\mu\right)$$

PARTONIC X-SEC. CALCULATED FROM ON-SHELL $q\bar q \to Z_0$ AMPLITUDE

TMD QUARK DISTRIBUTIONS

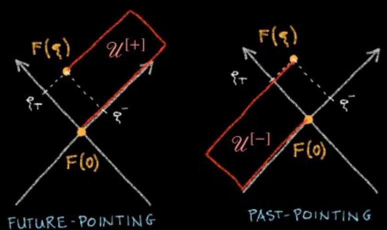
[J. Collins, D. Soper, G. Sterman, 1984; J. Collins 2011]

OPERATOR DEFINITIONS OF TMDs (we focus on GLUONS)

$$\mathcal{F}_g(x,k_T) = 2\int \frac{d\xi^+ d^2\xi_T}{(2\pi)^3P^-}\, e^{ixP^-\xi^+ - i\vec k_T\cdot\vec\xi_T}\,\langle P|\,\mathrm{Tr}\left[\hat F^{i-}\left(\xi^+,\vec\xi_T,\xi^-=0\right)\mathcal{U}_{C_1}\hat F^{i-}(0)\,\mathcal{U}_{C_2}\right]|P\rangle$$

GLUON FIELD $\hat F = F_a t^a$

GAUGE LINKS IN FUNDAMENTAL REPR.



Gauge links $\mathcal{U}_{C_1}, \mathcal{U}_{C_2}$ depend on the color structure of the hard process. They are build from two basic Wilson lines: $\mathcal{U}^{[+]}, \mathcal{U}^{[-]}$

$$\mathcal{U}^{[\pm]} = [0,(\pm\infty,\vec 0_T,0)]\;[(\pm\infty,\vec 0_T,0),(\pm\infty,\vec\xi_T,0)]\;[(\pm\infty,\vec\xi_T,0),(\xi^+,\vec\xi_T,0)]$$

STRAIGHT LINE SEGMENT $\quad [x,y] = \mathcal{P}\exp\left\{ig\int_{\overline{xy}} dz_\mu A^\mu_a(z)\,t^a\right\}$

[C. Bomhof, P. Mulders, F. Pijlman, 2004]

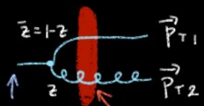
CGC & TMD

Leading twist study

P. Kotko, EPSHEP2019

FORWARD DIJET PRODUCTION IN CGC

[F. Dominguez, C. Marquet, B. Xiao, F. Yuan, 2011]



LARGE-X PARTON (x_1)

EXCHANGE OF SMALL-X GLUONS (x_2)

$\vec{k}_T = \vec{p}_{T1} + \vec{p}_{T2}$

$\vec{P}_T = \bar{z}\,\vec{p}_{T1} - z\,\vec{p}_{T2}$

$x_2 \ll x_1$

LEADING POWER LIMIT

$$\underset{\text{SATURATION SCALE}}{Q_s} \sim k_T \ll P_T \sim \underset{\text{HARD SCALE}}{\mu}$$

$$\frac{d\sigma_{pA\to 2j+X}}{dy_1 dy_2 d^2p_{T1} d^2p_{T2}} \sim \sum_{a,c,d} f_{a/p}(x_1,\mu) \sum_i \underbrace{H^{(i)}_{ag\to cd}(k_T=0)}_{\text{ON-SHELL HARD FACTORS}}\, \underbrace{\mathcal{F}^{(i)}_{ag}(x_2,k_T)}_{\text{TMD GLUON DISTRIBUTIONS (SMALL-X LIMIT)}}$$

Equivalence of leading power CGC and TMD 'factorization' was recently shown for dijet+photon process.

[T. Altinoluk, R. Boussarie, C. Marquet, P. Taels, 2018]

SMALL-X LIMIT OF TMD GLUON DISTRIBUTIONS

$$\mathcal{F}^{(i)}_{ag}(x,k_T) \sim \int \frac{d\xi^+ d^2\xi_T}{(2\pi)^3}\, \underbrace{e^{ixP^-\xi^+}}_{x\to 0} e^{-i\vec{k}_T\cdot\vec{\xi}_T} \langle P|\mathrm{Tr}\left[\hat{F}^{i-}\left(\xi^+,\vec{\xi}_T,\xi^-=0\right)\mathcal{U}_{C_1}\hat{F}^{i-}(0)\,\mathcal{U}_{C_2}\right]|P\rangle$$

DEPENDENCE ON X IS ONLY VIA THE SMALL-X EVOLUTION

For example:

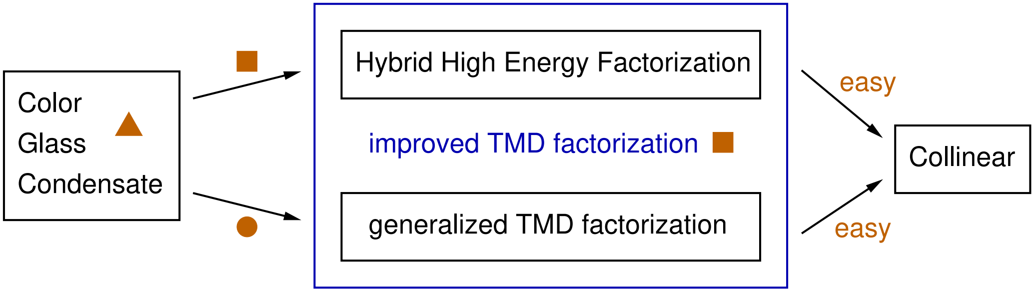
$$\mathcal{F}^{(1)}_{qg} \sim \int \frac{d^2x_T d^2y_T}{(2\pi)^4}\, k_T^2 e^{-i\vec{k}_T\cdot(\vec{x}_T-\vec{y}_T)} \langle \mathrm{Tr}\left[U(\vec{x}_T)U^\dagger(\vec{y}_T)\right]\rangle_x$$

Intensively studied:

[D. Kharzeev, Y. Kovchegov, K. Tuchin, 2003]
[B. Xiao, F. Yuan, 2010]
[F. Dominguez, C. Marquet, B. Xiao, F. Yuan, 2011]
[A. Metz, J. Zhou, 2011]
[E. Akcakaya, A. Schafer, J. Zhou, 2012]
[C. Marquet, E. Petreska, C. Roiesnel, 2016]
[I. Balitsky, A. Tarasov, 2015, 2016]
[D. Boer, P. Mulders, J. Zhou, Y. Zhou, 2017]
[C. Marquet, C. Roiesnel, P. Taels, 2018]
[Y. Kovchegov, D. Pitonyak, M. Sievert, 2017,2018]
[T. Altinoluk, R. Boussarie, 2019]

ITMD Factorization

For forward dijet production
in dilute-dense hadronic collisions



▲ McLerran, Venugopalan 1994, Iancu, Venugopalan 2003

■ Kotko, Kutak, Marquet, Petreska, Sapeta, AvH 2015

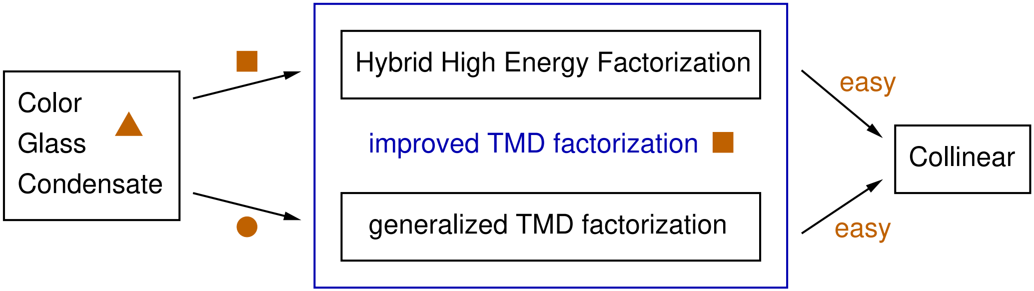
● Dominguez, Marquet, Xiao, Yuan 2011

Model interpolating between hybrid High Energy Factorization and Generalized TMD factorization and valid for kinematical regions with hard scale $\gtrsim k_T \gtrsim$ saturation scale.
Partonic cross section $d\hat{\sigma}^{(i)}_{gb}$ **depends on color-structure** i,
and is calculated with **space-like initial-state gluons**.

$$d\sigma_{AB\to X} = \int dk_T^2 \int dx_A \sum_i \int dx_B \sum_y \Phi^{(i)}_{gy}(x_A, k_T, \mu)\, f_y(x_B, \mu)\, d\hat{\sigma}^{(i)}_{gy\to X}(x_A, x_B, k_T, \mu)$$

ITMD Factorization

For forward dijet production
in dilute-dense hadronic collisions



ITMD formalism is fully obtained from the CGC formalism by taking the Wandzura-Wilczek approximation, *i.e.* neglecting all genuine twist corrections. Antinoluk, Boussarie, Kotko 2019

Model interpolating between hybrid High Energy Factorization and Generalized TMD factorization and valid for kinematical regions with hard scale $\gtrsim k_T \gtrsim$ saturation scale.
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$$d\sigma_{AB\to X} = \int dk_T^2 \int dx_A \sum_i \int dx_B \sum_y \phi^{(i)}_{gy}(x_A, k_T, \mu)\, f_y(x_B, \mu)\, d\hat{\sigma}^{(i)}_{gy\to X}(x_A, x_B, k_T, \mu)$$

ITMD Factorization for 3 or more jets

Bury, Kotko, Kutak, 2019

Color decomposition of, for example, the amplitude $\mathcal{M}^{a_1a_2i_3i_4}_{j_3j_4}$ for two gluons and two quark-anti-quark pairs in terms of color factors and partial amplitudes $\mathcal{A}_\sigma$:

$$\tilde{\mathcal{M}}^{i_1i_2i_3i_4}_{j_1j_2j_3j_4} \equiv \left(\sqrt{2}T^{a_1}\right)^{i_1}_{j_1}\left(\sqrt{2}T^{a_2}\right)^{i_2}_{j_2}\mathcal{M}^{a_1a_2i_3i_4}_{j_3j_4} = \sum_{\sigma\in S_4}\delta^{i_1}_{j_{\sigma(1)}}\delta^{i_2}_{j_{\sigma(2)}}\delta^{i_3}_{j_{\sigma(3)}}\delta^{i_4}_{j_{\sigma(4)}}\mathcal{A}_\sigma$$

The sum over colors for the squared amplitude is facilitated by a **color matrix** $C_{\tau\sigma}$

$$\mathcal{M}^{a_1a_2i_3i_4}_{j_3j_4}\mathcal{M}^{*a_1a_2j_3j_4}_{i_3i_4} = \tilde{\mathcal{M}}^{i_1i_2i_3i_4}_{j_1j_2j_3j_4}\tilde{\mathcal{M}}^{*j_1j_2j_3j_4}_{i_1i_2i_3i_4} = \sum_{\tau,\sigma}\mathcal{A}_\tau C_{\tau\sigma}\mathcal{A}^*_\sigma$$

Each element of the matrix $C_{\tau\sigma}$ is a single power of N_c (Kanaki, Papadopoulos 2002).
The cross section formula for ITMD is obtained by inserting color correlators like

$$\mathrm{TMD}_1 \times \tilde{\mathcal{M}}^{i_1i_2i_3i_4}_{j_1j_2j_3j_4}\tilde{\mathcal{M}}^{*j_1j_2j_3j_4}_{i_1i_2i_3i_4} \Rightarrow \left\langle\!\!\left\langle F^{j_1}_{i_1}\,\mathcal{U}^{k_2}_{i_2}\,\mathcal{U}^{k_3}_{i_3}\,\mathcal{U}^{k_4}_{i_4}\,F^{k_1}_{l_1}\,\mathcal{U}^{j_2}_{l_2}\,\mathcal{U}^{j_3}_{l_3}\,\mathcal{U}^{j_4}_{l_4}\right\rangle\!\!\right\rangle \tilde{\mathcal{M}}^{i_1i_2i_3i_4}_{j_1j_2j_3j_4}\tilde{\mathcal{M}}^{*l_1l_2l_3l_4}_{k_1k_2k_3k_4}$$

where the $\mathcal{U}^k_l$ are certain Wilson lines, depending on the external partons, and F^k_l the field strenght. This leads to a

TMD-valued color matrix $\mathcal{C}_{\tau\sigma}(x, k_T)$

This has been implemented in KaTie.

ALL POSSIBLE OPERATORS FOR TMD GLUON DISTRIBUTIONS

$$\mathcal{F}^{(1)}_{qg} \sim \langle P|\mathrm{Tr}\left[\hat{F}^{i-}(\xi)\,\mathcal{U}^{[-]\dagger}\hat{F}^{i-}(0)\,\mathcal{U}^{[+]}\right]|P\rangle$$

DIPOLE TMD

$$\mathcal{F}^{(2)}_{qg} \sim \langle P|\frac{\mathrm{Tr}\,\mathcal{U}^{[\square]}}{N_c}\mathrm{Tr}\left[\hat{F}^{i-}(\xi)\,\mathcal{U}^{[+]\dagger}\hat{F}^{i-}(0)\,\mathcal{U}^{[+]}\right]|P\rangle$$

$$\mathcal{F}^{(3)}_{qg} \sim \langle P|\mathrm{Tr}\left[\hat{F}^{i-}(\xi)\,\mathcal{U}^{[+]\dagger}\hat{F}^{i-}(0)\,\mathcal{U}^{[\square]}\mathcal{U}^{[+]}\right]|P\rangle$$

$$\mathcal{F}^{(1)}_{gg} \sim \langle P|\frac{\mathrm{Tr}\,\mathcal{U}^{[\square]\dagger}}{N_c}\mathrm{Tr}\left[\hat{F}^{i-}(\xi)\,\mathcal{U}^{[-]\dagger}\hat{F}^{i-}(0)\,\mathcal{U}^{[+]}\right]|P\rangle$$

$$\mathcal{F}^{(2)}_{gg} \sim \langle P|\mathrm{Tr}\left[\hat{F}^{i-}(\xi)\,\mathcal{U}^{[\square]\dagger}\right]\mathrm{Tr}\left[\hat{F}^{i-}(0)\,\mathcal{U}^{[\square]}\right]|P\rangle$$

$$\mathcal{F}^{(3)}_{gg} \sim \langle P|\mathrm{Tr}\left[\hat{F}^{i-}(\xi)\,\mathcal{U}^{[+]\dagger}\hat{F}^{i-}(0)\,\mathcal{U}^{[+]}\right]|P\rangle$$

WEISACKER-WILLIAMS TMD

$$\mathcal{F}^{(4)}_{gg} \sim \langle P|\mathrm{Tr}\left[\hat{F}^{i-}(\xi)\,\mathcal{U}^{[-]\dagger}\hat{F}^{i-}(0)\,\mathcal{U}^{[-]}\right]|P\rangle$$

$$\mathcal{F}^{(5)}_{gg} \sim \langle P|\mathrm{Tr}\left[\hat{F}^{i-}(\xi)\,\mathcal{U}^{[\square]\dagger}\mathcal{U}^{[+]\dagger}\hat{F}^{i-}(0)\,\mathcal{U}^{[\square]}\mathcal{U}^{[+]}\right]|P\rangle$$

$$\mathcal{F}^{(6)}_{gg} \sim \langle P|\frac{\mathrm{Tr}\,\mathcal{U}^{[\square]}}{N_c}\frac{\mathrm{Tr}\,\mathcal{U}^{[\square]\dagger}}{N_c}\mathrm{Tr}\left[\hat{F}^{i-}(\xi)\,\mathcal{U}^{[+]\dagger}\hat{F}^{i-}(0)\,\mathcal{U}^{[+]}\right]|P\rangle$$

$$\mathcal{F}^{(7)}_{gg} \sim \langle P|\frac{\mathrm{Tr}\,\mathcal{U}^{[\square]}}{N_c}\mathrm{Tr}\left[\hat{F}^{i-}(\xi)\,\mathcal{U}^{[\square]\dagger}\mathcal{U}^{[+]\dagger}\hat{F}^{i-}(0)\,\mathcal{U}^{[+]}\right]|P\rangle$$

WILSON LOOP → $\mathcal{U}^{[\square]} = \mathcal{U}^{[+]}\mathcal{U}^{[-]\dagger}$

[M. Bury, PK , K. Kutak, 2018]

ore jets

e $\mathcal{M}^{a_1a_2i_3i_4}_{j_3j_4}$ for two gluons and two

artial amplitudes $\mathcal{A}_\sigma$:

$$\sum_{\sigma\in S_4}\delta^{i_1}_{j_{\sigma(1)}}\delta^{i_2}_{j_{\sigma(2)}}\delta^{i_3}_{j_{\sigma(3)}}\delta^{i_4}_{j_{\sigma(4)}}\mathcal{A}_\sigma$$

litated by a **color matrix** $C_{\tau\sigma}$

$$^{j_3j_4}_{i_3i_4} = \sum_{\tau,\sigma}\mathcal{A}_\tau C_{\tau\sigma}\mathcal{A}^*_\sigma$$

N_c (Kanaki, Papadopoulos 2002).

serting color correlators like

$$\mathcal{U}^{j_2}_{i_2}\mathcal{U}^{j_3}_{i_3}\mathcal{U}^{j_4}_{i_4}\rangle\rangle\,\tilde{\mathcal{M}}^{i_1i_2i_3i_4}_{j_1j_2j_3j_4}\,\tilde{\mathcal{M}}^{*l_1l_2l_3l_4}_{k_1k_2k_3k_4}$$

he external partons, and F^k_l the field

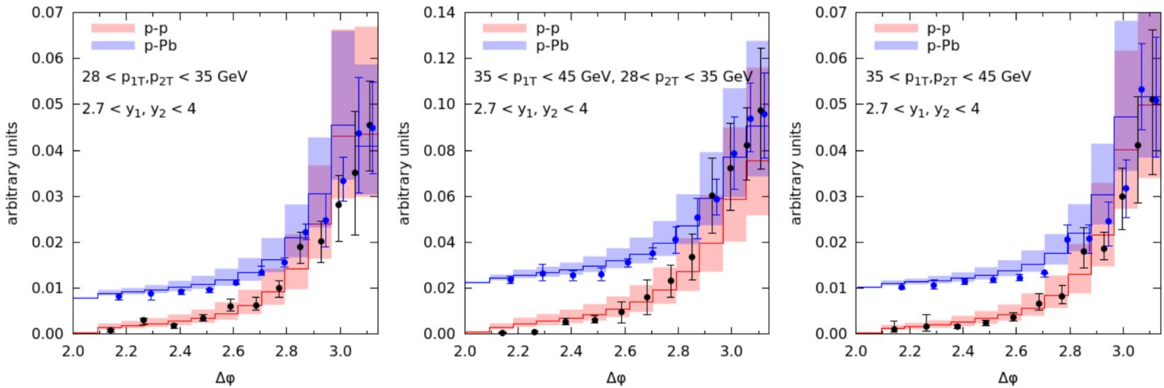
$$\mathcal{C}_{\tau\sigma}(x,k_T)$$

Saturation effects from forward jets

AvH, Kotko, Kutak, Sapeta 2019

Study of saturation using dijet production in p-p and p-pB collisions.
Angle $\Delta\varphi$ between the jets is particularly sensitive to saturation effects.

Data points from ATLAS 2019. Arbitrary normalization and relative shift to to accentuate the difference in shape between p-p and p-Pb.



Calculations where performed within ITMD factorization, using TMDs based on Kutak, Sapeta 2012. Besides saturation, the inclusion of resummed Sudakov logarithms are essential to reach this accuracy, included here via event-reweighting. Independent calculations with KaTie and LxJet (http://nz42.ifj.edu.pl/~pkotko/LxJet.html)

Towards NLO for k_T-factorization

The main obstacle are linear denominators in loop integrals and the divergecies they cause.

$$\int d^{4-2\varepsilon}\ell \, \frac{\mathcal{N}(\ell)}{p\cdot(\ell+K_0)\,(\ell+K_1)^2\,(\ell+K_3)^2\,(\ell+K_4)^2} = ?$$

In particular one would like to use a regularization that

- is manifestly Lorentz covariant
- manifestly preserves gauge invariance
- can be used incombination with dimensional regularization
- is practical

Towards NLO for k_T-factorization

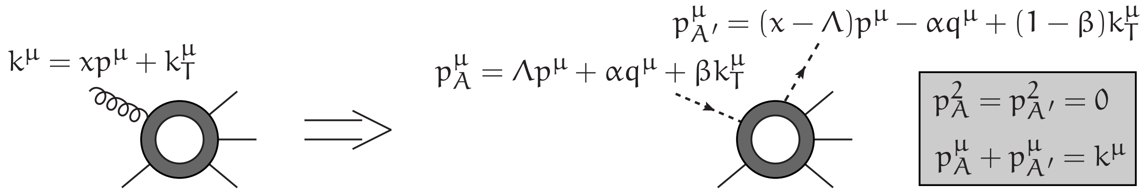
Tilted Wilson line regularization:

$$p^\mu \to p^\mu + r q^\mu \quad \text{with} \quad q^2 = 0 \;,\; p\cdot q \neq 0 \quad \text{and} \quad 0 < r \ll 1$$

Calculations using the effective action, and tilted Wilson lines:

- NLO corrections to the reggeized gluon propagator Hentschinski, Sabio Vera 2012, Chachamis, Hentschinski, Madrigal Martínez, Sabio Vera 2013
- two-loop corrections to the gluon Regge trajectory, shown to be consistent with known results Chachamis, Hentschinski, Madrigal Martínez, Sabio Vera 2013
- one-loop correction to the propagator of Reggeized quark and to the $Q\gamma\bar{q}$-vertex Nefedov, Saleev 2017
- one-loop corrections to the $Q\gamma^*\bar{q}$-vertex RHg-vertex Rgg-vertex Nefedov 2019 **Only single log(r) divergencies, no double logs or power-like divergencies**.

Regularization with auxiliary partons



$$\boxed{\begin{array}{l} p_A^2 = p_{A'}^2 = 0 \\ p_A^\mu + p_{A'}^\mu = k^\mu \end{array}}$$

where p, q are light-like with $p\cdot q > 0$, where $p\cdot k_T = q\cdot k_T = 0$, and where

$$\alpha = \frac{-\beta^2 k_T^2}{\Lambda(p+q)^2} \quad , \quad \beta = \frac{1}{1+\sqrt{1-x/\Lambda}} \quad \Longrightarrow \quad \begin{cases} p_A^2 = p_{A'}^2 = 0 \\ p_A^\mu + p_{A'}^\mu = xp^\mu + k_T^\mu \end{cases}$$

for any value of the parameter Λ. Auxiliary quark propagators become eikonal for $\Lambda \to \infty$:

$$i\,\frac{\not{p}_A + \not{K}}{(p_A+K)^2} = \frac{i\,\not{p}}{2p\cdot K} + \mathcal{O}(\Lambda^{-1})$$

- Λ-parametrization provides natural regularization for linear denominators in loop integrals.
- Taking this limit after loop integration will lead to singularities $\log\Lambda$.

$\emptyset \to Hg\,g^*$ from $\emptyset \to Hg\,q\bar{q}$

Schmidt 1997

$$m^1(g^+, q^-, \bar{q}^+) \;=\; m^0(g^+, q^-, \bar{q}^+)\,\frac{\alpha_s}{4\pi}\, r_\Gamma \left(\frac{4\pi\mu^2}{-M_H^2}\right)^\epsilon \left[N_c V_1 + \frac{1}{N_c} V_2 + n_f V_3\right] ,$$

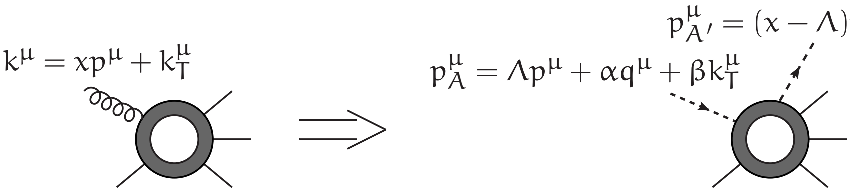
with

$$\begin{aligned}
V_1 \;=\; & \frac{1}{\epsilon^2}\left[-\left(\frac{-M_H^2}{-S_{gq}}\right)^\epsilon - \left(\frac{-M_H^2}{-S_{g\bar{q}}}\right)^\epsilon\right] + \frac{13}{6\epsilon}\left(\frac{-M_H^2}{-S_{q\bar{q}}}\right)^\epsilon \\
& - \ln\left(\frac{-S_{gq}}{-M_H^2}\right)\ln\left(\frac{-S_{q\bar{q}}}{-M_H^2}\right) - \ln\left(\frac{-S_{g\bar{q}}}{-M_H^2}\right)\ln\left(\frac{-S_{q\bar{q}}}{-M_H^2}\right) \\
& - 2\,\mathrm{Li}_2\left(1 - \frac{S_{q\bar{q}}}{M_H^2}\right) - \mathrm{Li}_2\left(1 - \frac{S_{gq}}{M_H^2}\right) - \mathrm{Li}_2\left(1 - \frac{S_{g\bar{q}}}{M_H^2}\right) \\
& + \frac{83}{18} - \frac{\delta_R}{6} + \frac{\pi^2}{3} - \frac{1}{2}\frac{S_{q\bar{q}}}{S_{g\bar{q}}} , \\
V_2 \;=\; & \left[\frac{1}{\epsilon^2} + \frac{3}{2\epsilon}\right]\left(\frac{-M_H^2}{-S_{q\bar{q}}}\right)^\epsilon + \ln\left(\frac{-S_{gq}}{-M_H^2}\right)\ln\left(\frac{-S_{g\bar{q}}}{-M_H^2}\right) \\
& + \mathrm{Li}_2\left(1 - \frac{S_{gq}}{M_H^2}\right) + \mathrm{Li}_2\left(1 - \frac{S_{g\bar{q}}}{M_H^2}\right) \\
& + \frac{7}{2} + \frac{\delta_R}{2} - \frac{\pi^2}{6} - \frac{1}{2}\frac{S_{q\bar{q}}}{S_{g\bar{q}}} , \\
V_3 \;=\; & -\frac{2}{3\epsilon}\left(\frac{-M_H^2}{-S_{q\bar{q}}}\right)^\epsilon - \frac{10}{9} .
\end{aligned}$$

$$S_{gq} = (p_A + k_g)^2 \longrightarrow 2\Lambda p\cdot k_g$$
$$S_{g\bar{q}} = (p_{A'} + k_g) \longrightarrow -2\Lambda p\cdot k_g$$
$$S_{q\bar{q}} = k_T^2$$

$$m^1(g^+, q^-, \bar{q}^+) \propto \left[\frac{1}{\epsilon} - \ln\left(\frac{-k_T^2}{\mu^2}\right)\right]\ln\Lambda + \cdots$$

Decomposition into master integrals



$$\boxed{\begin{array}{l} p_A^2 = p_{A'}^2 = 0 \\ p_A^\mu + p_{A'}^\mu = k^\mu \end{array}}$$

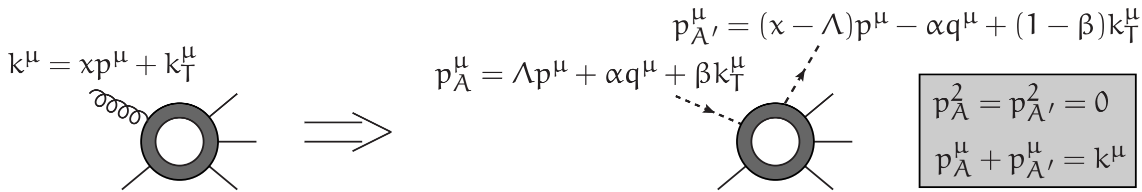
Well-known decomposition for on-shell one-loop amplitudes in terms of master integrals still holds for finite Λ.

$$\mathcal{A}^{(1)} = \int [d\ell] \frac{\mathcal{N}(\ell)}{\prod_i \mathcal{D}_i(\ell)} = \sum_{i,j,k,l} c_4(i,j,k,l)\, I_4(i,j,k,l) + \sum_{i,j,k} c_3(i,j,k)\, I_3(i,j,k)$$
$$+ \sum_{i,j} c_2(i,j)\, I_2(i,j) + \sum_i c_1(i)\, I_1(i) + \mathcal{R} + \mathcal{O}(\epsilon)$$

$$I_4(i,j,k,l) = \int [d\ell] \frac{1}{\mathcal{D}_i(\ell)\mathcal{D}_j(\ell)\mathcal{D}_k(\ell)\mathcal{D}_l(\ell)} \quad , \quad \mathcal{D}_i(\ell) = (\ell + K_i)^2 - m_i^2 + i\eta$$

The coefficients $c_4, c_3, c_2.c_1$ are determined from the *integrand*.
(di)logarithms of external invariants and Λ appear in the master integrals I_4, I_3, I_2.

Decomposition into master integrals



Well-known decomposition for on-shell one-loop amplitudes in terms of master integrals still holds for finite Λ.

$$\mathcal{A}^{(1)} = \int [d\ell]\, \frac{\mathcal{N}(\ell)}{\prod_i \mathcal{D}_i(\ell)} = \sum_{i,j,k,l} c_4(i,j,k,l)\, I_4(i,j,k,l) + \sum_{i,j,k} c_3(i,j,k)\, I_3(i,j,k) + \sum_{i,j} c_2(i,j)\, I_2(i,j) + \sum_i c_1(i)\, I_1(i) + \mathcal{R} + \mathcal{O}(\varepsilon)$$

It is not completely correct to take $\Lambda \to \infty$ in the integrand before reduction, and just replace

$$\frac{1}{2p\cdot(\ell+K)} \to \frac{\Lambda}{(\ell+\Lambda p+K)^2}$$

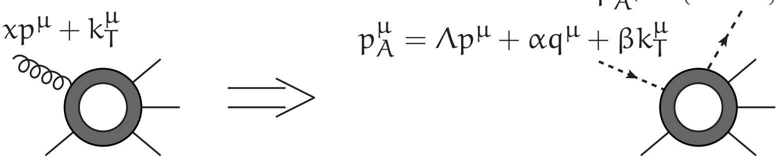
in the master integrals

Non-commuting limits

$$k^\mu = x p^\mu + k_T^\mu$$

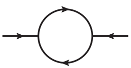
$$p_{A'}^\mu = (x-\Lambda)p^\mu - \alpha q^\mu + (1-\beta)k_T^\mu$$

$$p_A^\mu = \Lambda p^\mu + \alpha q^\mu + \beta k_T^\mu$$

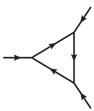


$$\boxed{\begin{array}{c} p_A^2 = p_{A'}^2 = 0 \\ p_A^\mu + p_{A'}^\mu = k^\mu \end{array}}$$

For two-point master integrals and one three-point master integrals, integration does not commute with the limit $\Lambda \to \infty$: integration "eats" a power of Λ from the denominator.



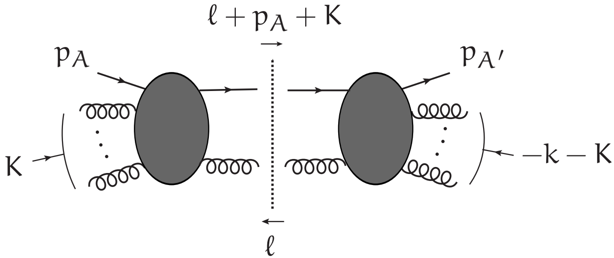
$$\Lambda p + K \quad\quad -\Lambda p - K \quad = \int \frac{[d\ell]}{\ell^2\,(\ell+\Lambda p + K)^2} \to \frac{1}{\varepsilon} + 2 - \log\left(\frac{2\Lambda p\cdot K}{-\mu^2}\right)$$



$$p_A \quad p_{A'} \quad -k \quad = \int \frac{[d\ell]}{\ell^2\,(\ell+p_A)^2\,(\ell+k)^2} \to \frac{1}{k_T^2}\left\{\frac{1}{\varepsilon^2} - \frac{1}{\varepsilon}\log\left(\frac{k_T^2}{-\mu^2}\right) + \frac{1}{2}\log^2\left(\frac{k_T^2}{-\mu^2}\right)\right\}$$

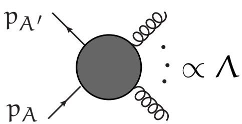
Tree-level amplitudes behave as $\propto \Lambda$; their definition involves a division by Λ before $\Lambda \to \infty$. The behavior of the integrals above indicates the possibility of Λ^2 behavior of the one-loop amplitude, and thus the occurrance of power-like divergencies in Λ in one-loop amplitudes.

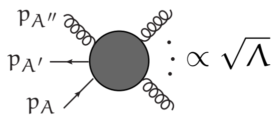
Coefficient for the bubbles



solution ℓ^μ to the cut equations
$\ell^2 = 0 \;\;,\;\; (\ell + p_A + K)^2 = 0$
is divergent: $\ell^\mu \propto \Lambda$

Coefficients for scalar integrals can be found by considering corresponding cuts on the integrand (Ossola, Papadopoulos, Pittau 2007, Ellis, Giele, Kunszt 2008). Momenta that put internal lines on-shell for a bubble are proportional to Λ. The blobs represent tree-level amplitudes with **3** divergent external momenta, which turn out to behave as $\sqrt{\Lambda}$.





So eventually coefficent$\times$scalar integral does not exhibit power-like divergencies.
This turns out to go through for the whole amplitude AvH 2017.

Thank you for your attention.