

# Jets in the High Energy Limit

Ideas from Regge Theory & BFKL, and their use in HEJ

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# Overview of Talk

1. Basic ideas from Regge Theory
2. Scaling of QCD Amplitudes in the Multi-Regge-Kinematic limit
3. Connection to BFKL
4. From Amplitudes to All-Order Corrections
5. Recent results
6. Comparison to Data and hopes for new measurements

# Regge theory

**Regge theory** describes scattering from a **central potential** in terms of the projections on Legendre polynomial and states of **definite orbital angular momentum**

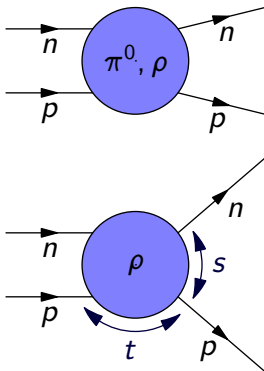
The analysis of **scattering amplitudes** in terms of Regge Theory: Regge (1959)

$$\mathcal{M} = \sum_i \Gamma_i(t) (s)^j$$

At **large energies**  $s$ , the contribution from particle of **highest spin  $j$**  dominates

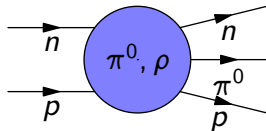
$$\mathcal{M} \rightarrow \Gamma(t) (s)^j$$

**Regge limit:**  $s \gg -t$  or  $s \gg p_t^2$



# Multi-Regge theory

**Large  $s$**  of course leads to the possibility of **multi-particle production**

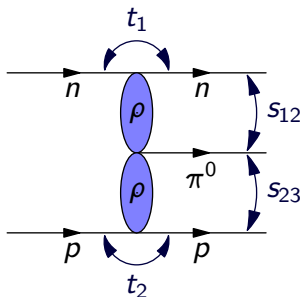


**Multi-Regge limit:**

Brower, DeTAR, Weis (1974)

$$s_{12}, s_{23} \gg p_{t_i}^2, |t_i|, \quad |t_i| \sim |t_j|, \quad |p_{t_i}| \sim |p_{t_j}|$$

$$\mathcal{M} = s_{12}^j s_{23}^j \Gamma(t_1, t_2, s/(s_{12}s_{23}))$$



This framework obtains **good descriptions of hadronic processes**.  
Based primarily on **analyticity of scattering matrix**.

# Application to QCD amplitudes using BFKL

The inclusive partonic scattering of on-shell states is calculated to leading  $\log(s)$  accuracy as

$$\begin{aligned} d\hat{\sigma}(p_a, p_b) &= \Gamma_a(\mathbf{p}_a) f(\mathbf{p}_a, -\mathbf{p}_b, \Delta) \Gamma_b(\mathbf{p}_b) \\ \text{BFKL eqn: } \frac{\partial}{\partial y} f(q_a, q_b, y) &= \int d^{D-2} \mathbf{q} K(q_a, \mathbf{q}) f(\mathbf{q}, q_b, y) \end{aligned}$$

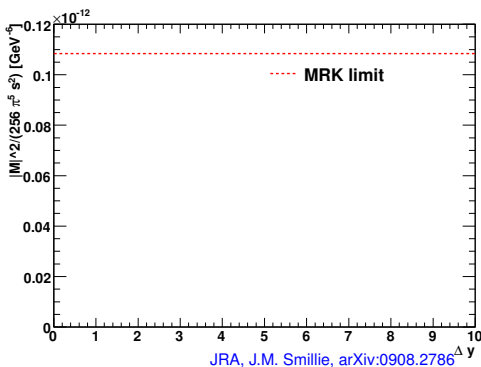
Reconstructs the FKL amplitudes that VDD found directly from QCD

$$|\overline{\mathcal{M}}_{gg \rightarrow g \dots g}|^2 \longrightarrow \frac{4 \hat{s}^2}{N_C^2 - 1} \frac{g^2 C_A}{|p_{1\perp}|^2} \left( \prod_{i=2}^{n-1} \frac{4 g^2 C_A}{|p_{i\perp}|^2} \right) \frac{g^2 C_A}{|p_{n\perp}|^2}.$$

# Comparison of 3-jet hard scattering matrix elements

Study just a slice in phase space, and compare full tree-level with  $\alpha_s^3$ -approximation from resummation:

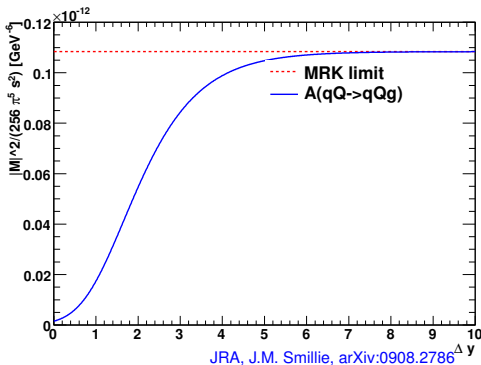
40GeV jets in Mercedes star (transverse) configuration. Rapidities at  $-\Delta y, 0, \Delta y$ .



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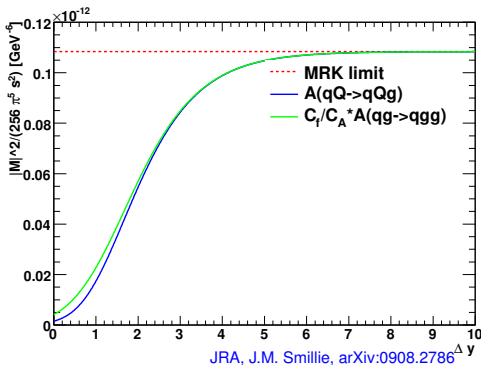
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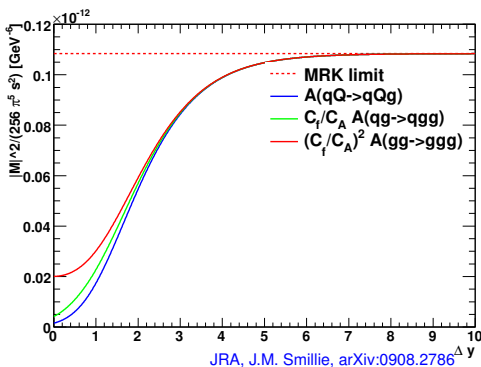




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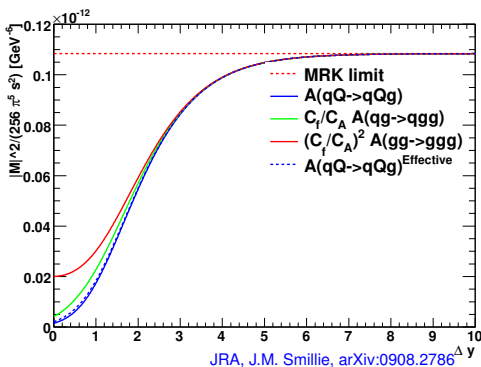
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High Energy Jets (HEJ):

- 1) Inspiration from Fadin&Lipatov: dominance by  $t$ -channel colour octet exchange
- 2) No kinematic approximations in invariants
- 3) Accurate definition of currents (coupling through  $t$ -channel exchange)
- 4) Full gauge invariance.



Use **Regge insight** for **amplitudes** in SM scattering cross sections

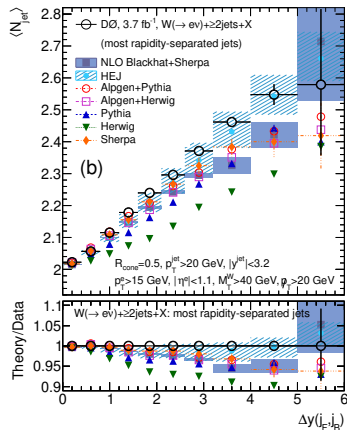
High Energy Jets:

- **Factorisation of matrix elements** using **currents**
- systematic **power expansion of QCD amplitudes**
- all-order **leading and sub-leading logarithmic corrections**
- matching, **results** . . .

Will try to highlight connection to BFKL, amplitudes etc.

# Why We Should Develop a Better Understanding

- $(n + 1)$ -jet rates are not necessarily small compared to  $n$ -jet rates within the kinematic regions of interest
- For some analyses, the “**difficult regions**” of phase space include significant contributions from  $n, n + 1, n + 2, \dots$  jet-rates (so beyond what NLO can describe) are exactly the regions of interest  
**VBF Higgs production, Vector Boson Scattering,...**

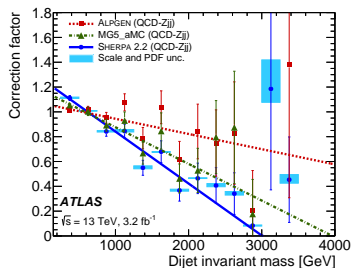


D0, arXiv:1302.6508

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**VBF Higgs** production, **Vector Boson Scattering**,...



**Huge variations** in the perturbative description of regions of large  $\hat{s}$

arXiv:1703.04362, 1709.10264

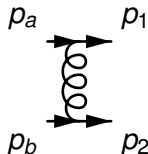
## Calculations of amplitudes for HEJ

# Scattering of qQ-Helicity States

Start by describing quark scattering. Simple matrix element for

$$q(a)Q(b) \rightarrow q(1)Q(2): \quad j_\mu^{1a} = \bar{\psi}_1 \gamma_\mu \psi_a, \quad j_\nu^{2b} = \bar{\psi}_2 \gamma_\nu \psi_b$$

$$M_{q^- Q^- \rightarrow q^- Q^-} = j_\mu^{1a} \frac{g^{\mu\nu}}{t} j_\nu^{2b}$$



**t-channel factorised:** Contraction of (local) currents across  $t$ -channel pole.  $\|S_{qQ \rightarrow qQ}\|^2 = \sum j_\mu^{1a} g^{\mu\nu} j_\nu^{2b} = \hat{s}^2 + \hat{u}^2$

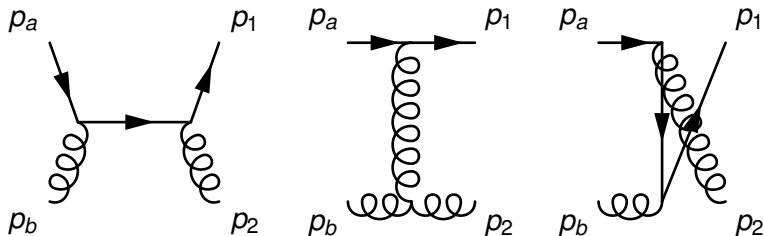
$$\begin{aligned} \left| \overline{\mathcal{M}}_{qQ \rightarrow qQ}^t \right|^2 &= \frac{1}{4 (N_C^2 - 1)} \|S_{qQ \rightarrow qQ}\|^2 \\ &\cdot \left( g^2 C_F \frac{1}{t_1} \right) \cdot \left( g^2 C_F \frac{1}{t_2} \right). \end{aligned}$$

$|\mathcal{M}| \propto \hat{s}^1$  for a gluon exchange !

J.M.Smillie and JRA: arXiv:0908.2786

# Quark-Gluon Scattering

“What happens in  $2 \rightarrow 2$ -processes with gluons? Surely the  $t$ -channel factorisation is spoiled!”



Direct calculation ( $q^- g^- \rightarrow q^- g^-$ ):

$$\mathcal{M} = \frac{g^2}{\hat{t}} \times \frac{p_{2\perp}^*}{|p_{2\perp}|} \left( t_{ae}^2 t_{e1}^b \sqrt{\frac{p_b^-}{p_2^-}} - t_{ae}^b t_{e1}^2 \sqrt{\frac{p_2^-}{p_b^-}} \right) j_{b2}^\mu j_\mu^{1a}$$

J.M.Smillie and JRA

Complete  $t$ -channel factorisation! Same scaling as qQ scattering



# Quark-Gluon Scattering

The  $t$ -channel current generated by a gluon in  $qg$  scattering is that generated by a quark, but with a colour factor

$$\frac{1}{2} \left( C_A - \frac{1}{C_A} \right) \left( \frac{p_b^-}{p_2^-} + \frac{p_2^-}{p_b^-} \right) + \frac{1}{C_A}$$

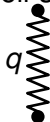
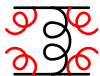
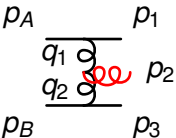
instead of  $C_F$ . Tends to  $C_A$  in the MRK limit.

Similar results for e.g.  $g^+g^- \rightarrow g^+g^-$  (well-defined  $t$ -channel):  
**Exact, complete  $t$ -channel factorisation.**

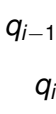
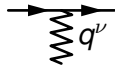
By using the formalism of **current-current scattering**, we get a better description of the  $t$ -channel pole than by using just the MRK kinematic limit of BFKL.

# Building Blocks for an Amplitude

Identification of the **dominant contributions** to the **perturbative series** in the limit of well-separated particles



$$\frac{1}{q^2} \exp(\hat{\alpha}(q)\Delta y)$$



$$\mu V^\mu(q_{i-1}, q_i)$$

$$j^\nu = \bar{\psi}\gamma^\nu\psi$$

$$V^\rho(q_1, q_2) = -(q_1 + q_2)^\rho$$

$$+ \frac{p_A^\rho}{2} \left( \frac{q_1^2}{p_2 \cdot p_A} + \frac{p_2 \cdot p_B}{p_A \cdot p_B} + \frac{p_2 \cdot p_n}{p_A \cdot p_n} \right) + p_A \leftrightarrow p_1$$

$$- \frac{p_B^\rho}{2} \left( \frac{q_2^2}{p_2 \cdot p_B} + \frac{p_2 \cdot p_A}{p_B \cdot p_A} + \frac{p_2 \cdot p_1}{p_A \cdot p_1} \right) - p_B \leftrightarrow p_3.$$

# Performing the Explicit Resummation

**Analytic subtraction** of soft divergence from real radiation:

$$|\mathcal{M}_t^{p_a p_b \rightarrow p_0 p_1 p_2 p_3}|^2 \xrightarrow{\mathbf{p}_1^2 \rightarrow 0} \left( \frac{4g_s^2 C_A}{\mathbf{p}_1^2} \right) |\mathcal{M}_t^{p_a p_b \rightarrow p_0 p_2 p_3}|^2$$

Integrate over the soft part  $\mathbf{p}_1^2 < \lambda^2$  of phase space in  $D = 4 + 2\epsilon$  dimensions

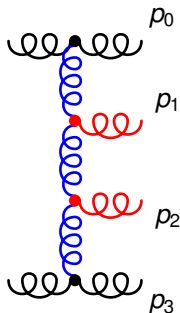
$$\begin{aligned} & \int_0^\lambda \frac{d^{2+2\epsilon} \mathbf{p} dy_1}{(2\pi)^{2+2\epsilon} 4\pi} \left( \frac{4g_s^2 C_A}{\mathbf{p}^2} \right) \mu^{-2\epsilon} \\ &= \frac{4g_s^2 C_A}{(2\pi)^{2+2\epsilon} 4\pi} \Delta y_{02} \frac{\pi^{1+\epsilon}}{\Gamma(1+\epsilon)} \frac{1}{\epsilon} (\lambda^2/\mu^2)^\epsilon \end{aligned}$$

Pole in  $\epsilon$  cancels with that from the **virtual corrections**

$$\frac{1}{t_1} \rightarrow \frac{1}{t_1} \exp(\hat{\alpha}(t)\Delta y_{02}) \quad \hat{\alpha}(t) = -\frac{g_s^2 C_A \Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}} \frac{2}{\epsilon} \left( \mathbf{q}^2/\mu^2 \right)^\epsilon.$$

# Expression for the Regularised Amplitude

$$\begin{aligned}
 \overline{|\mathcal{M}_{\text{HEJ}}^{\text{reg}}(\{\mathbf{p}_i\})|^2} &= \frac{1}{4(N_C^2 - 1)} \|\mathcal{S}_{f_1 f_2 \rightarrow f_1 f_2}\|^2 \cdot \left(g^2 K_{f_1} \frac{1}{t_1}\right) \cdot \left(g^2 K_{f_2} \frac{1}{t_{n-1}}\right) \\
 &\cdot \prod_{i=1}^{n-2} \left( g^2 C_A \left( \frac{-1}{t_i t_{i+1}} V^\mu(q_i, q_{i+1}) V_\mu(q_i, q_{i+1}) - \frac{4}{\mathbf{p}_i^2} \theta(\mathbf{p}_i^2 < \lambda^2) \right) \right) \\
 &\cdot \prod_{j=1}^{n-1} \exp[\omega^0(q_j, \lambda)(y_{j-1} - y_j)], \quad \omega^0(q_j, \lambda) = -\frac{\alpha_s N_C}{\pi} \log \frac{\mathbf{q}_j^2}{\lambda^2}.
 \end{aligned}$$



# All-Order Summed (and Matched) Cross Section

The cross section is calculated as the sum over the phase space integrals of the explicit  $n$ -body phase space

$$\begin{aligned}\sigma_{2j}^{\text{sum,match}} &= \sum_{n=2}^{\infty} \sum_{f_1, f_2} \prod_{i=1}^n \left( \int \frac{d^2 \mathbf{p}_{i\perp}}{(2\pi)^3} \int \frac{dy_i}{2} \right) \frac{|\mathcal{M}_{\text{HEJ}}^{f_1 f_2 \rightarrow f_1 g \dots g f_2}(\{\mathbf{p}_i\})|^2}{\hat{s}^2} \\ &\times \mathcal{O}_{2j}(\{\mathbf{p}_i\}) \times \sum_m \mathcal{O}_{mj}^e(\{\mathbf{p}_i\}) w_{m\text{-jet}} \\ &\times x_a f_{A, f_1}(x_a, Q_a) x_b f_{B, f_2}(x_b, Q_b) (2\pi)^4 \delta^2 \left( \sum_{i=1}^n \mathbf{p}_{i\perp} \right).\end{aligned}$$

**Matching** to fixed order (tree-level so far) is obtained by clustering the  $n$ -parton phase space point into  $m$ -jet momenta and multiply by the ratio of full to approximate matrix element:

$$w_{m\text{-jet}} \equiv \frac{|\mathcal{M}^{f_1 f_2 \rightarrow f_1 g \dots g f_2}(\{\mathbf{p}_{\mathcal{J}_1}(\{\mathbf{p}_i\})\})|^2}{|\mathcal{M}^{t, f_1 f_2 \rightarrow f_1 g \dots g f_2}(\{\mathbf{p}_{\mathcal{J}_1}(\{\mathbf{p}_i\})\})|^2}.$$

## Summary: All-Orders, Regularisation, etc.

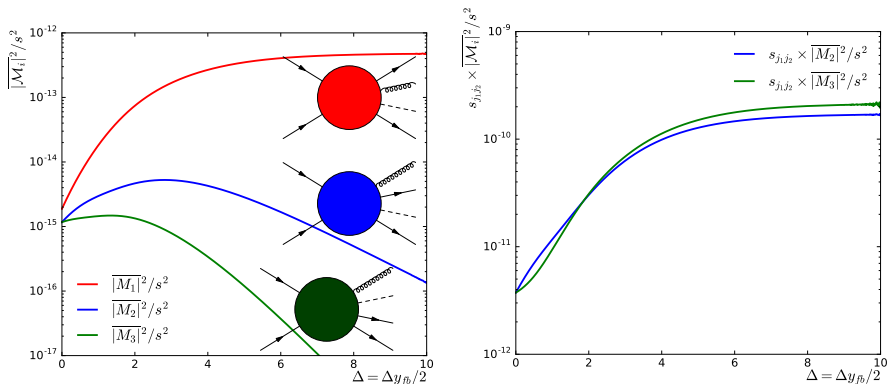
- Have prescription for  $2 \rightarrow n$  matrix element, including virtual corrections: Lipatov Ansatz  $1/t \rightarrow 1/t \exp(-\omega(t)\Delta y_{ij})$
- Organisation of cancellation of IR (soft) divergences is easy
- Can **calculate** the **sum over the  $n$ -particle phase space** explicitly ( $n \sim 30$ ) to get the **all-order corrections** (just as if one had provided all the  $N^{30} LO$  matrix elements and a regularisation procedure)
- **Merge**  $n$ -jet tree-level MEs (by merging  $m$ -parton momenta to  $n$  hard jet-momenta) where these can be evaluated in reasonable time
- HEJ merged with **parton shower** (Pythia) [JRA, Brooks, Lönnblad, arXiv:1712.00178](#)

Captures the same logarithms as BFKL.

Lorentz-invariant, gauge-invariant, respects crossing-symmetry in “impact factors”, can be matched to high-multiplicity fixed-order matrix elements etc.

# Factorisation For $H_{+ \geq 2\text{jets}}$ Too

The factorisation extends to dijet processes involving Higgs bosons, W, Z and photon production too.



The **scaling** for different kinematic evaluations of the same amplitude is exactly as predicted by Regge theory applied to the **planar graph** connecting the rapidity-ordered configuration.

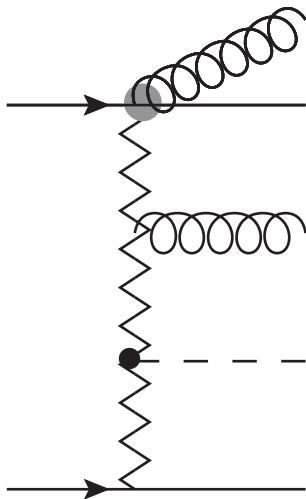
M. Heil, A. Maier, J.M. Smillie, JRA,

# Subleading Corrections and Quark Mass Dependence

The first **subleading correction** can be included by **relaxing** the constraint between two emissions:  
**Quasi Multi Regge Kinematics**

Effective currents can be extracted from the full amplitude, now dependent on more momenta. These currents obey **crossing** ( $q \rightarrow qgg^*$  related to  $g \rightarrow q\bar{q}g^*$ )

Simplicity of the amplitudes allow for calculation of **full quark mass dependence** of Higgs coupling (requires just triangles and boxes).



M. Heil, A. Maier, J.M. Smillie, JRA, arXiv:1706.01002,1812.08072



# Calculation of First Set of Subleading Corrections

$$\mathcal{M}_{\text{tree } qQ \rightarrow gqQ}^{\text{HEJ}} = -g_s^3 T_{2b}^d \frac{j_{\mu}^{\text{uno } cd}(p_1, p_g, p_a) j^{\mu}(p_2, p_b)}{t_{b2}}$$

$$j^{\text{uno } \mu cd}(p_1, p_g, p_a) = i \varepsilon_{g\nu} \left( T_{1i}^c T_{ia}^d (U_1^{\mu\nu} - L^{\mu\nu}) + T_{1i}^d T_{ia}^c (U_2^{\mu\nu} + L^{\mu\nu}) \right).$$

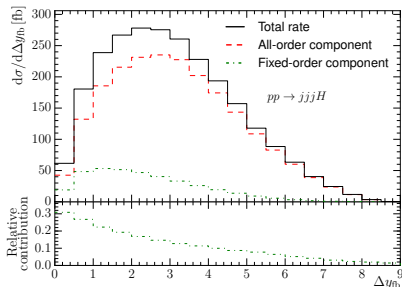
$$U_1^{\mu\nu} = \frac{1}{s_{1g}} (j_{1g}^{\nu} j_{1a}^{\mu} + 2p_1^{\nu} j_{1a}^{\mu})$$

$$U_2^{\mu\nu} = \frac{1}{t_{ag}} (2j_{1a}^{\mu} p_a^{\nu} - j_{1g}^{\nu} j_{1a}^{\mu})$$

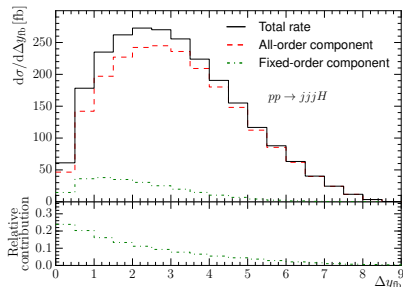
$$L^{\mu\nu} = \frac{1}{t_{a1}} \left( -2p_g^{\mu} j_{1a}^{\nu} + 2p_g \cdot j_{1a} g^{\mu\nu} + (q_1 + q_2)^{\nu} j_{1a}^{\mu} + \frac{t_{b2}}{2} j_{1a}^{\mu} \left( \frac{p_2^{\nu}}{p_g \cdot p_2} + \frac{p_b^{\nu}}{p_g \cdot p_b} \right) \right).$$

Gauge-invariant. Respects crossing symmetry.

# Impact of Subleading Corrections



Leading logs only + matching

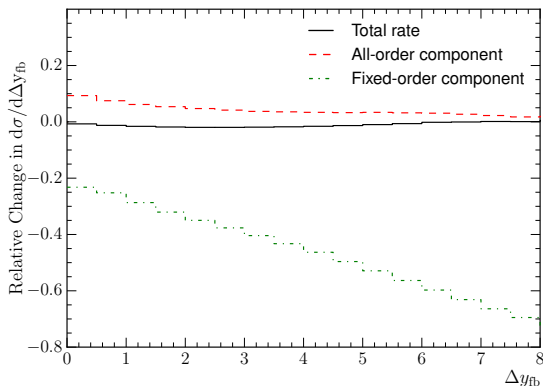


LL, one unordered emission + matching

The **all-order component** of the cross section **increases**, the impact from **matching is decreased**, but the **total result** changes only **little**.

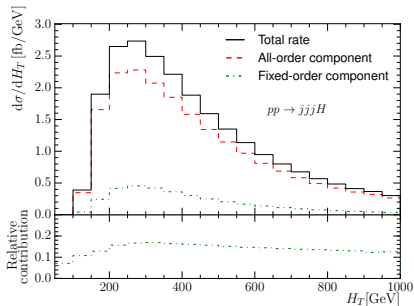
M. Heil, A. Maier, J.M. Smillie, JRA, arXiv:1706.01002

# Subleading Corrections as Expected

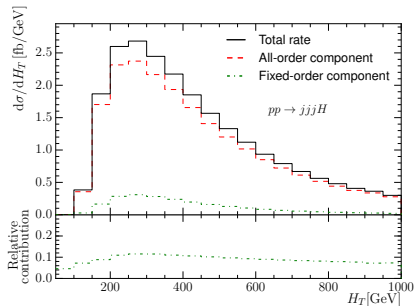


The fixed-order component of the cross section is **systematically reduced** by a component growing with  $\Delta y_{fb}$  (as expected, of course).

# Impact of Subleading Corrections for $H_T$



Leading logs only + matching

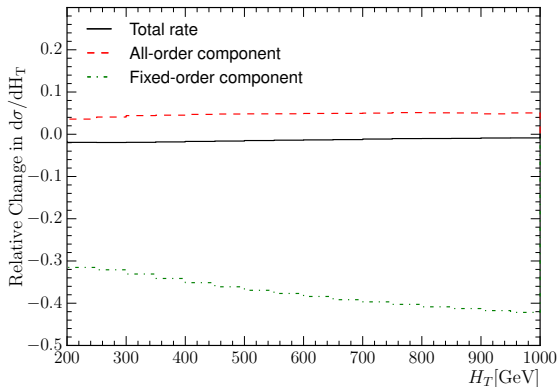


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JRA, A. Maier et al., arXiv:1706.01002

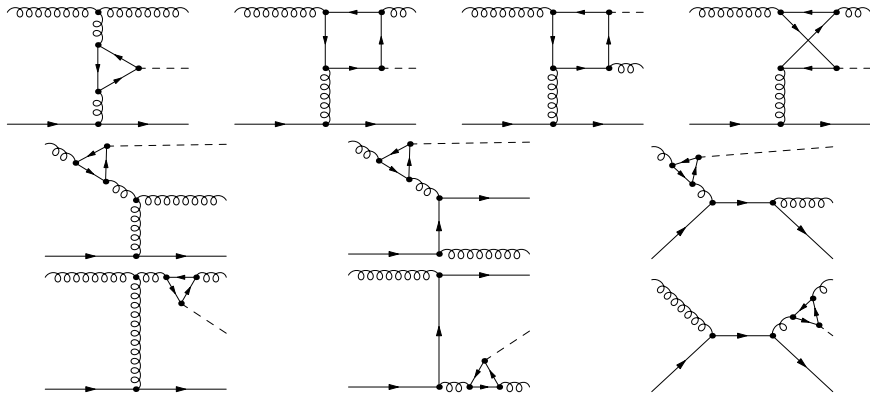
# Subleading Corrections for $H_T$



The all-order and fixed-order component of the cross section is **changed uniformly**, but their **sum changes only little**.

Conclusion: The **all-order perturbative treatment** is very **stable**.

# Finite quark mass effects

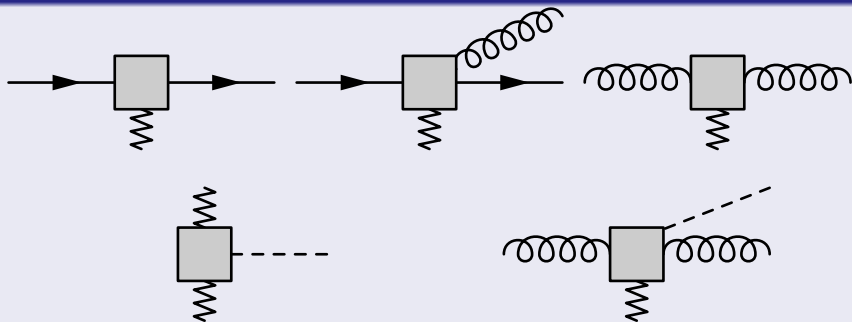


Del Duca, Kilgore, Oleari, Schmidt, Zeppenfeld, hep-ph/0105129

Effective HEJ-currents with full mass dependence can be extracted similarly to simple  $qg$ -scattering.

# Finite quark mass effects

Building pieces in HEJ (currents) arxiv:1812.08072



Effective HEJ-currents with full mass dependence can be extracted similarly to simple  $qg$ -scattering.

# Matching & merging with Fixed Order

$$\begin{aligned}
 \sigma_{2j}^{\text{resum, match}} &= \sum_{f_1, f_2} \sum_m \prod_{j=1}^m \left( \int_{p_{j\perp}^B=0}^{p_{j\perp}^B=\infty} \frac{d^2 \mathbf{p}_{j\perp}^B}{(2\pi)^3} \int \frac{dy_j^B}{2} \right) (2\pi)^4 \delta^{(2)} \left( \sum_{k=1}^m \mathbf{p}_{k\perp}^B \right) \\
 &\cdot x_a^B f_a(x_a^B, Q_a^B) x_b^B f_b(x_b^B, Q_b^B) \frac{|\mathcal{M}^B|^2}{(\hat{s}^B)^2} \\
 &\cdot \overline{|\mathcal{M}_{\text{HEJ}}^{\text{tree}}|}^{-2} (2\pi)^{-4+3m} 2^m \frac{(\hat{s}^B)^2}{x_a^B f_{a,f_1}(x_a^B, Q_a^B) x_b^B f_{b,f_2}(x_b^B, Q_b^B)} \\
 &\cdot \sum_{n=2}^{\infty} \int_{p_{1\perp}=\cdot 9p_{j,\perp}}^{p_{1\perp}=\infty} \frac{d^2 \mathbf{p}_{1\perp}}{(2\pi)^3} \int_{p_{n\perp}=\cdot 9p_{j,\perp}}^{p_{n\perp}=\infty} \frac{d^2 \mathbf{p}_{n\perp}}{(2\pi)^3} \prod_{i=2}^{n-1} \int_{p_{i\perp}=\lambda}^{p_{i\perp}=\infty} \frac{d^2 \mathbf{p}_{i\perp}}{(2\pi)^3} (2\pi)^4 \delta^{(2)} \left( \sum_{k=1}^n \mathbf{p}_{k\perp} \right) \\
 &\cdot \tau_y \prod_{i=1}^n \left( \int \frac{dy_i}{2} \right) \mathcal{O}_{mj}^e \left( \prod_{l=1}^{m-1} \delta^{(2)}(\mathbf{p}_{\mathcal{J}_l\perp}^B - \mathbf{j}_{l\perp}) \right) \left( \prod_{l=1}^m \delta(y_{\mathcal{J}_l}^B - y_{\mathcal{J}_l}) \right) \mathcal{O}_{2j}(\{\rho_i\}) \\
 &\cdot x_a f_{a,f_1}(x_a, Q_a) x_b f_{b,f_2}(x_b, Q_b) \frac{\overline{|\mathcal{M}_{\text{HEJ}}^{f_1 f_2 \rightarrow f_1 g \dots g f_2}(\{\rho_i\})|}^2}{\hat{s}^2}.
 \end{aligned}$$

- Generate Fixed Order, explore all of higher-order phase space
- ⇒ computationally efficient, merging limited by Fixed Order ( $\sim 5$  jets)

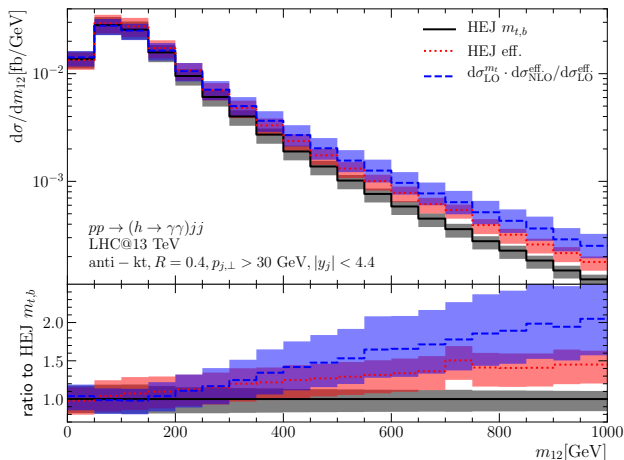


# Matching & merging with Fixed Order

$$\begin{aligned}
 \sigma_{2j}^{\text{resum, match}} = & \sum_{f_1, f_2} \sum_m \prod_{j=1}^m \left( \int_{p_{j\perp}^B=0}^{p_{j\perp}^B=\infty} \frac{d^2 \mathbf{p}_{j\perp}^B}{(2\pi)^3} \int \frac{dy_j^B}{2} \right) (2\pi)^4 \delta^{(2)} \left( \sum_{k=1}^m \mathbf{p}_{k\perp}^B \right) && \text{Fixed Order} \\
 & \cdot x_a^B f_a(x_a^B, Q_a^B) x_b^B f_b(x_b^B, Q_b^B) \frac{|\mathcal{M}^B|^2}{(\hat{s}^B)^2} \\
 & \cdot |\mathcal{M}_{\text{HEJ}}^{\text{tree}}|^{-2} (2\pi)^{-4+3m} 2^m \frac{(\hat{s}^B)^2}{x_a^B f_{a,f_1}(x_a^B, Q_a^B) x_b^B f_{b,f_2}(x_b^B, Q_b^B)} && \text{Matching All order} \\
 & \cdot \sum_{n=2}^{\infty} \int_{p_{1\perp}=\dots=p_{j\perp}}^{p_{1\perp}=\dots=p_{j\perp}=\infty} \frac{d^2 \mathbf{p}_{1\perp}}{(2\pi)^3} \dots \int_{p_{n\perp}=\dots=p_{j\perp}}^{p_{n\perp}=\dots=p_{j\perp}=\infty} \frac{d^2 \mathbf{p}_{n\perp}}{(2\pi)^3} \prod_{i=2}^{n-1} \int_{p_{i\perp}=\lambda}^{p_{i\perp}=\infty} \frac{d^2 \mathbf{p}_{i\perp}}{(2\pi)^3} (2\pi)^4 \delta^{(2)} \left( \sum_{k=1}^n \mathbf{p}_{k\perp} \right) \\
 & \cdot \mathbf{T}_y \prod_{i=1}^n \left( \int \frac{dy_i}{2} \right) \mathcal{O}_{mj}^e \left( \prod_{l=1}^{m-1} \delta^{(2)}(\mathbf{p}_{\mathcal{J}_l}^B - \mathbf{j}_{l\perp}) \right) \left( \prod_{l=1}^m \delta(y_{\mathcal{J}_l}^B - y_{\mathcal{J}_l}) \right) \mathcal{O}_{2j}(\{p_i\}) \\
 & \cdot x_a f_{a,f_1}(x_a, Q_a) x_b f_{b,f_2}(x_b, Q_b) \frac{|\mathcal{M}_{\text{HEJ}}^{f_1 f_2 \rightarrow f_1 g \dots g f_2}(\{p_i\})|^2}{\hat{s}^2} .
 \end{aligned}$$

- Generate Fixed Order, explore all of higher-order phase space  
 $\Rightarrow$  computationally efficient, merging limited by Fixed Order ( $\sim 5$  jets)

# Spectrum in Invariant dijet mass



⇒ After VBF-cuts:  $\sigma_{\text{HEJ}}^{m_t \rightarrow \infty} \approx 1.1 \cdot \sigma_{\text{HEJ}} \approx 0.5 \cdot \sigma_{\text{NLO}}$

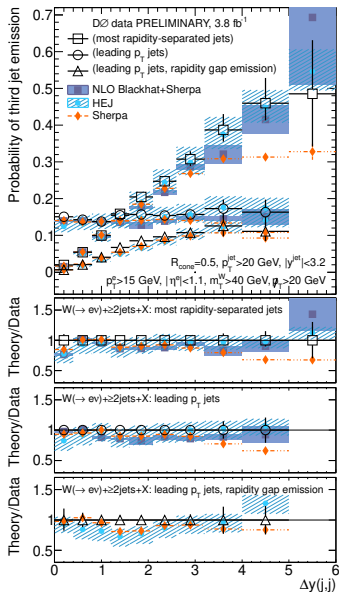
⇒ HEJ suppresses the region of large  $m_{jj}$  compared to fixed order.

## Comparison to Data

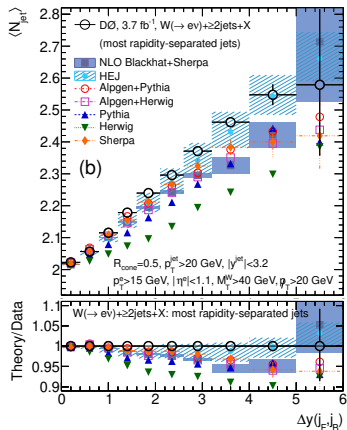
D0 measurement of the probability of at least one additional jet when requiring just a  $W$  in association with two jets. Probability measured vs. rapidity separation of

- 1 the two most rapidly separated jets
- 2 the two hardest (in pt) jets
- 3 the two hardest (in pt) jets, counting additional jets only in the rapidity interval between the two hardest jets

Good agreement between data and HEJ for all observables - effects are even more pronounced at the LHC.



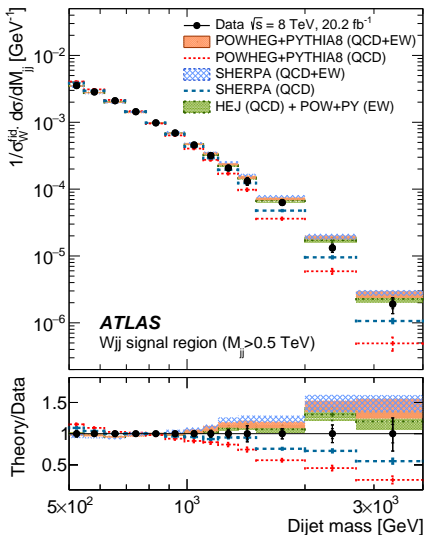
Number of jets measured as a function of the maximum rapidity span of jets. Differences in predictions more pronounced.



# Electroweak $W_{jj}$ -production

Electroweak  $W_{jj}$  production.  
Background from  $\alpha_s^2 \alpha$ -component.  
Compare NLO+PS  
(Powheg+Pythia, Sherpa) and  
HEJ+EW(Powheg)

- QCD contribution decreases at large dijet mass, but remains significant
- NLO+PS slightly overshoot

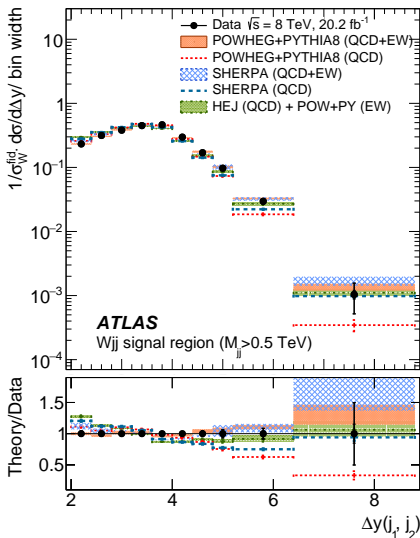


# Electroweak $W_{jj}$ -production

Electroweak  $W_{jj}$  production.  
Background from  $\alpha_s^2 \alpha$ -component.  
Compare NLO+PS  
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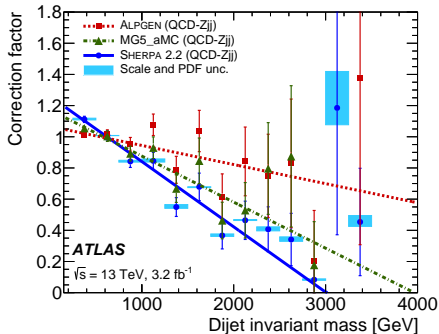
Similar conclusions for  $\Delta y_{j_1 j_2}$



# Electroweak Zjj-production

Electroweak Zjj production. Background from  $\alpha_s^2\alpha$ -component. Compare Alpgen, MG5\_aMC, SHERPA in control region.

Bin-by-bin corrections factors very different from unity are necessary in order to “describe” the control regions, and are then used to subtract the QCD contributions in order to analyse the EW process.



arXiv: 1709.10264



- The phase space explored by the LHC can enhance higher-order terms, which hinders the convergence of the fixed-order expansion
- The effect is present in LHC data (also Tevatron)
- Sizeable effects and implications for analyses of Higgs VBF and Vector-Boson-Scattering
- HEJ systematically includes the logarithmic corrections to all orders. Next-to-leading logarithmic corrections are being included.
- Public code, documentation, sample analyses, . . .

<http://hej.web.cern.ch>