TELOS + Hadrons

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CCP-TEPP Hadrons User Workshop, 2026-04-15

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Science and Technology Facilities Council

What does TELOS do?

- Theoretical Explorations on the Lattice with Orthogonal and Symplectic groups
  - Primarily study $Sp(2N)$ groups
  - Applications to composite Higgs and other BSM model building
- Main target $Sp(4)$ theory with 2 fundamental, 3 two-index antisymmetric fermions

How does TELOS currently use Hadrons?

- Compute meson (diquark) two-point functions
  - Z_2 wall sources
  - Fundamental and antisymmetric fermions (independently)
- Issues worked around in our fork
  - Non-fundamental representations missing
    - Partial fix merged upstream
  - $N_c > 3$ does not work on GPU
    - GPU parameter space (4096 bytes) exceeded by single propagator (4096 bytes for $N_c = 4$) plus any other parameter
    - Requires disabling compilation of `Gamma3pt`, `WeakEye3pt`, and `Gauss`

What do we need?

- Excited states and singlets (this talk)
- Chimera baryons (next talk)
- Glueballs (next-to-next talk)

Excited states and singlets: current implementation

- Perform a GEVP in a basis comprising different source and sink smearing levels
- For singlets (see https://arxiv.org/pdf/2304.07191 §III):
  - Connected portion:
    - Use Z_2 wall sources
  - Disconnected part
    - Use $Z_2 \times Z_2$ noisy sources with spin and even-odd dilution
    - Use $O(100)$ noise vectors

Excited states and singlets: Hadrons requirements

- Smearing
  - APE smearing
  - Wuppertal smearing
- Distillation
  - We haven't to date used distillation for singlets, but would be very interested in deploying it

Chimera baryon: Current implementation

- From https://arxiv.org/pdf/2311.14663 §II.C
- For fundamental $Q(x)$, antisymmetric $\Psi(x)$, basic operator:
  $$\mathcal{O}_{\mathrm{CB}}^{k\ \rho}(x) = (\Gamma^1)^{\alpha\beta}(\Gamma^2)^{\rho\sigma}\Omega_{ad}\Omega_{bc}Q^{j\ a}{}_{\alpha}(x)Q^{i\ b}{}_{\beta}(x)\Psi^{k\ cd}{}_{\sigma}(x)$$
  where symplectic matrix $\Omega = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}$
- Take fermion propagators
  Fundamental: $S_Q^{ia}{}_{b\alpha\beta}(x,y) = \left\langle Q^{ia}{}_{\alpha}(x)\overline{Q^{ib}}_{\beta}(y)\right\rangle$
  Two-index antisymmetric: $S_\Psi^{k\ ab}{}_{cd\alpha\beta}(x,y) = \left\langle \Psi^{k\ ab}{}_{\alpha}(x)\overline{\Psi^{k\ cd}}_{\beta}(y)\right\rangle$
- Then propagator after contractions is
  $$\left\langle \mathcal{O}_{\mathrm{CB}}^{k\ \gamma}(x)\overline{\mathcal{O}_{\mathrm{CB}}^{k}}^{\gamma'}(y)\right\rangle = \Omega_{ad}\Omega_{bc}\Omega^{d'a'}\Omega^{b'c'}(\Gamma^1)^{\alpha\beta}(\Gamma^1)^{\alpha'\beta'}(\Gamma^2)^{\gamma\delta}(\Gamma^2)^{\gamma'\delta'}$$
  $$\times S_\Psi^{k\ cd}{}_{c'd'\ \delta\delta'}(x,y)S_Q^{2\ a}{}_{a'\ \alpha\alpha'}(x,y)S_Q^{1\ b}{}_{b'\ \beta\beta'}(x,y)$$
- Currently consider only $\Gamma^1 \in \{C\gamma^5, C\gamma^\mu\}$ (for Λ and Σ respectively), $\Gamma^2 = \mathbb{1}$
- Implemented in branch of TELOS's $Sp(2N)$ fork of HiRep

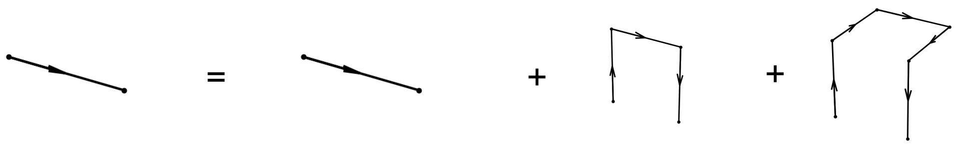
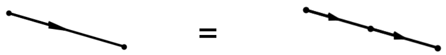
Chimera baryon: Current implementation

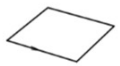
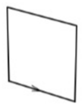
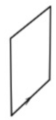
- Take fermion propagators
  Fundamental: $S_{Q\ \ b\alpha\beta}^{ia}(x,y) = \left\langle Q^{ia}{}_{\alpha}(x)\overline{Q^{ib}}_{\beta}(y)\right\rangle$
  Two-index antisymmetric: $S_{\Psi\ \ cd\alpha\beta}^{kab}(x,y) = \left\langle \Psi^{kab}{}_{\alpha}(x)\overline{\Psi^{kcd}}_{\beta}(y)\right\rangle$
- Define $C_{\mathrm{CB},\sigma\rho}^{\mu\nu,\Gamma^1}(t) \equiv \sum_{\vec{x}}\left\langle \mathcal{O}_{\mathrm{CB},\sigma}^{\mu,\Gamma^1}(x)\overline{\mathcal{O}_{CB,\rho}^{\nu,\Gamma^1}}(0)\right\rangle$
- Then $C_{\Sigma_{\mathrm{CB}},\sigma\rho}(t) \equiv \left[P_{\mu\nu}^{1/2}C_{\mathrm{CB}}^{\mu\nu,C\gamma^{\rho}}(t)\right]_{\sigma\rho}$, $C_{\Sigma_{\mathrm{CB}}^{*},\sigma\rho}(t) \equiv \left[P_{\mu\nu}^{3/2}C_{\mathrm{CB}}^{\mu\nu,\Gamma^{\rho}}(t)\right]_{\sigma\rho}$, $C_{\Lambda_{\mathrm{CB}},\sigma\rho}(t) \equiv \left[C_{\mathrm{CB}}^{\mu\nu,C\gamma^5}(t)\right]_{\sigma\rho}$
  - $P_{\mu\nu}^{1/2} \equiv \frac{1}{3}\gamma^{\mu}\gamma^{\nu}$, $P_{\mu\nu}^{3/2} \equiv \delta^{\mu\nu} - \frac{1}{3}\gamma^{\mu}\gamma^{\nu}$
- As for excited states, perform GEVP with APE and Wuppertal smearing

Chimera baryon: Hadrons requirements

- APE smearing
- Wuppertal smearing
- New contraction described above

Glueballs: Operator construction

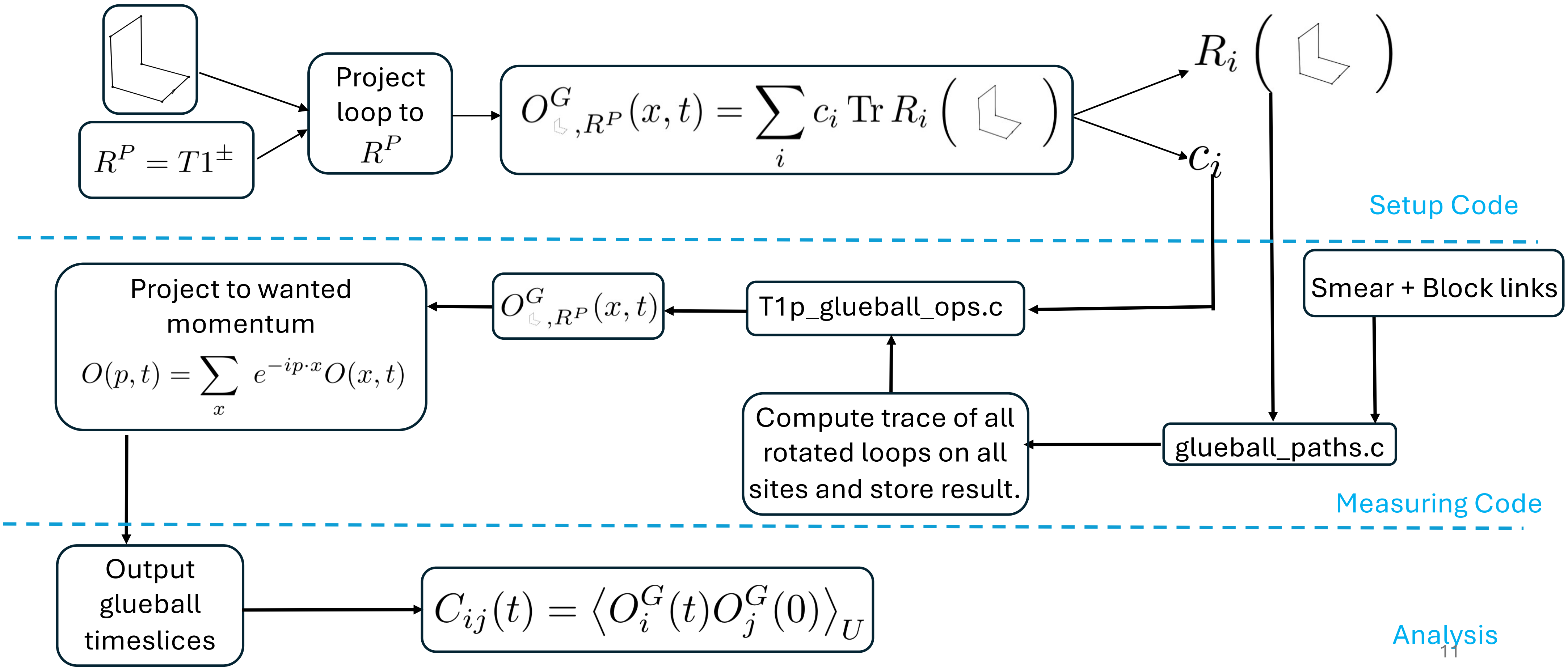
- Typical glueball studies rely on $C_{ij}(t) = \left\langle O_i^G(t) O_j^G(0) \right\rangle_U$
- Signal-to-noise ratio problem, improve overlaps with ground state with:
  1. (APE) Smearing:  + (rotations ...)
  2. Blocking: 
  3. Project input loops to $R^P \xrightarrow{a \to 0} J^P$

$O_i^G(x,t) = 1.0 \cdot \mathrm{Tr}$  $-1.0 \cdot \mathrm{Tr}$  $+2.0 \cdot \mathrm{Tr}$  $+(\ldots)$

(Smearing + Blocking): Lucini, B., Teper, M., & Wenger, U. (2004). Glueballs and k-strings in SU (N) gauge theories: Calculations with improved operators. *JHEP* , *2004*(06), 012-012.

(Projection of loops): Berg, B., & Billoire, A. (1983). Glueball spectroscopy in 4D SU(3) lattice gauge theory (I). *Nuclear Physics B*, *221*(1), 109-140.

Glueballs: Current Implementation



Glueballs: Hadrons requirements

- Improved APE Smearing, including diagonal parallel links;
- Cooling (projecting back to the gauge group)
- Blocking levels;
- A less cumbersome way of walking on the lattice with links, i.e:

{ "x" , "y", "z", "-x"} ⟶ $U_x(n) \cdot U_y(n+\hat{x}) \cdot U_z(x+\hat{x}+\hat{y}) \cdot U_x^\dagger(x+\hat{y}+\hat{z})$