Troubles in paradise

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Based on:
2407.11971, 2411.13633, 2504.12289, 2512.05072
(With Cotler, Dhivakar, Prohazka, Raz, Riegler, Salzer)

This is a talk about some problems with the proposal:

$$\text{Flat-space QG} \quad \longleftrightarrow \quad \text{Carrollian "CFT"}$$

Terms of engagement

"Traditional" perspective on AdS/CFT, dualities:

1. At least (sometimes only) one side is a *fundamental* description.
   ("$\mathscr{N} = 4$ defines IIB strings on $AdS_5 \times \mathbb{S}^5$".)

2. Symmetries of dual pairs match, but are sometimes (often?) *emergent*.
   (Large-radius limits, BFSS, $\mathscr{N} = 1$ flows in 4d SCFT, CMT, &c.)

3. Dualities don't merely exist; they reorganize difficult problems in useful ways.

Summary of results

1. Lagrangian Carrollian "quantum field theories" are, on their own terms, far from conventional QFT: (see also prior work esp. [de Boer et al])
   - $N_{\text{eff}} = \infty$
   - Renormalized theories only defined as scaling limits
   - Exhibit UV/IR mixing
   - interactions preclude perturbative conformal invariance
2. To the extent that flat space gravity has a Carrollian dual, the dual is
   - ~~local~~
   - perhaps ~~unitary~~

Outline:

1. Introduction

2. Quantization of Carrollian theories

3. Locality? Unitarity?

4. Final comments

Carrollian "QFT"

We can write down two-derivative Carroll-invariant Lagrangians:

$$S_E = \int du d^{d-1}x \left(\frac{1}{2}\dot{\phi}^2 - V(\phi) \right)$$

"Electric"

- Ultralocal
- Decoupled QM at each position

$$S_M = \int du d^{d-1}x \left(\chi \dot{Q} - \mathscr{H}(\chi, \nabla \chi) \right)$$

"Magnetic"

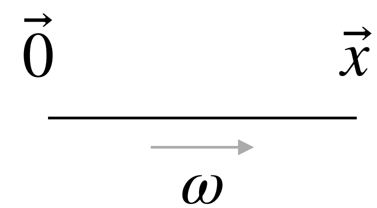
- frozen, resembles stat mech
- continuous spectrum

$$S_{int} = -\lambda \int du d^{d-1}x \, \chi \phi^2$$

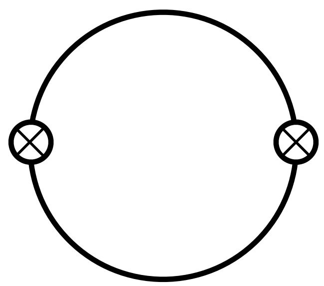
Generalizations: E/M, YM, fermions..

"Electric" - i.e. decoupled QM at each $\vec{x}$

UV-sensitivity even in the free theory:



$$= \delta^{d-1}(\vec{x})\frac{i}{\omega^2 - m^2 + i\varepsilon}$$

$\langle \phi^2 \phi^2 \rangle =$  $\propto \delta^{d-1}(\vec{0})$

Convenient approach to renormalization:

- lattice regulator, spacing a.
- rescale composites by powers of a, e.g.

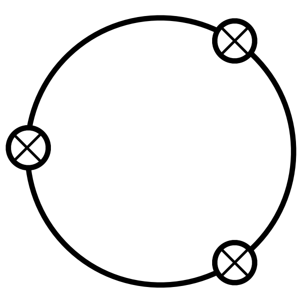
$$\tilde{\phi}^2 = a^{\frac{d-1}{2}}\phi^2$$

- then take $a \to 0$.

$$N_{\text{eff}} = \frac{1}{a^{d-1}}$$

Renormalized models have

Gaussian correlations!

 $\propto a^{\frac{d-1}{2}}$

Interacting theories specified by couplings

that vanish in $a \to 0$ limit.

$$\mathscr{L} = \frac{1}{2}\dot{\phi}^2 - \frac{1}{2}m^2\phi^2 - \frac{\lambda_3}{3!}a^{\frac{d-1}{2}}\phi^3 - \frac{\lambda_4}{4!}a^{d-1}\phi^4 - \ldots$$

UV/IR mixing akin to "field theories" of "fractons." See e.g. [Seiberg, Shao]

“Magnetic” theories

$$S_M = \int du d^{d-1}x \left(\chi \dot{Q} - \mathscr{H}(\chi, \nabla\chi)\right)$$

Free model: $\mathscr{H} = \frac{1}{2}|\vec{\nabla}\chi|^2 + \frac{1}{2}M^2\chi^2$

1. $\dot{\chi} = 0$: no dynamics; resembles stat mech

2. But, $\chi(\vec{x})$ conserved at each $\vec{x}$.

3. Continuous spectrum of $\chi(\vec{x})$/energy eigenstates.

$$\langle \chi | e^{-iHT} | \chi' \rangle = e^{-iE[\chi]} \delta[\chi - \chi']$$

Subtleties with IR divergences,

But these can be overcome.

Putting the pieces together

Carrollian theories with "electric," "magnetic" dofs behave more like "electric" theories:

- $N_{\text{eff}} = \infty$
- renormalized theories defined as scaling limits
- UV/IR mixing

One-loop self-energies depend on UV cutoff, coincident limit of magnetic correlations, &c.

No Carroll boost SSB in perturbation theory.

"QM" renormalization group flow.

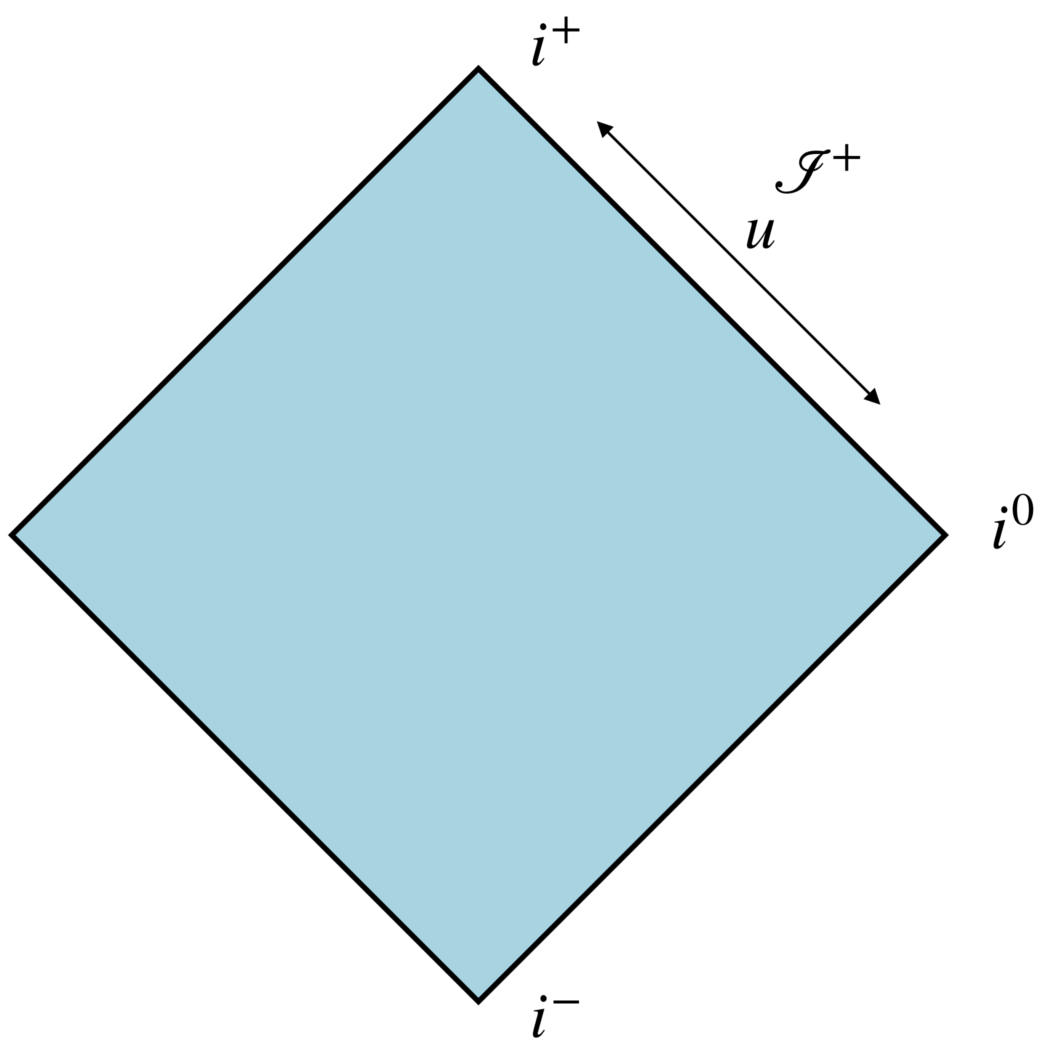
Aside: pure 3d gravity

Like JT and AdS$_3$ gravity, pure 3d gravity with $\Lambda = 0$ can be reduced to a boundary description, a magnetic Carrollian theory. [Merbis, Riegler] 2411.13633

This theory is soluble!

$$S = -\int du d\theta \left(\alpha \dot{P} + P\right) \qquad P = -\frac{1}{16\pi G}\{f(\theta,u),\theta\}$$

$$\{f(\theta),\alpha(\theta)\} \in \mathrm{BMS}_3/ISO(2,1)$$



$$\langle P | e^{-iHT} | P' \rangle = e^{-iE[P]T}\delta[P-P']$$

$$\mathrm{tr}\left(e^{-\beta H + i\omega L}\right) = e^{\frac{1}{8G}}\prod_{n=2}^{\infty}\frac{1}{|1-e^{i\omega n}|^2} \qquad \left(\frac{\omega}{2\pi}\notin\mathbb{Q}\right)$$

Extends 1-loop results of [Barnich, et al]

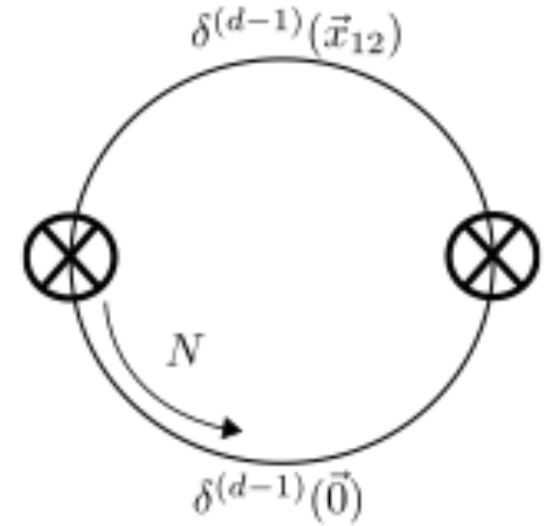
See also [Simón, Yu]

Aside: N_{eff} = finite?

With a lattice regulator, the analogue of c_T for a Carrollian theory is

$$\langle TT \rangle \sim N_{\text{eff}} = \frac{N}{a^{d-1}}$$

= QM dofs per unit cell



Non-Gaussianity organized in powers of $1/N_{\text{eff}}$.

If we want small but nonzero non-Gaussianity in the renormalized theory, we need a scaling limit with $N = (a/\ell)^{d-1}$ with ℓ fixed as $a \to 0$, i.e. $N \to 0$.

This can be done (2504.12289) in vector models, but the result is not very interesting. The point is that you have to jump through hoops like this to obtain models of weakly interacting (not just free) GFFs.

Lessons

1. Renormalized Carrollian QFT is far from textbook QFT,
   And moreover does not resemble flat-space gravity.
2. Resulting models are non-conformal, sensitive to the UV.
3. In particular,
   - no perturbative non-Gaussianity akin to 1/N
   - UV/IR mixing
   - no SSB of Carroll boosts i.e. supertranslations.

Taken together, these results give a weak no-go:

No two-derivative Carroll QFT with (i.) perturbativity and (ii.) finite N_{eff}, let alone other features of flat space gravity like ~~supertranslations~~.

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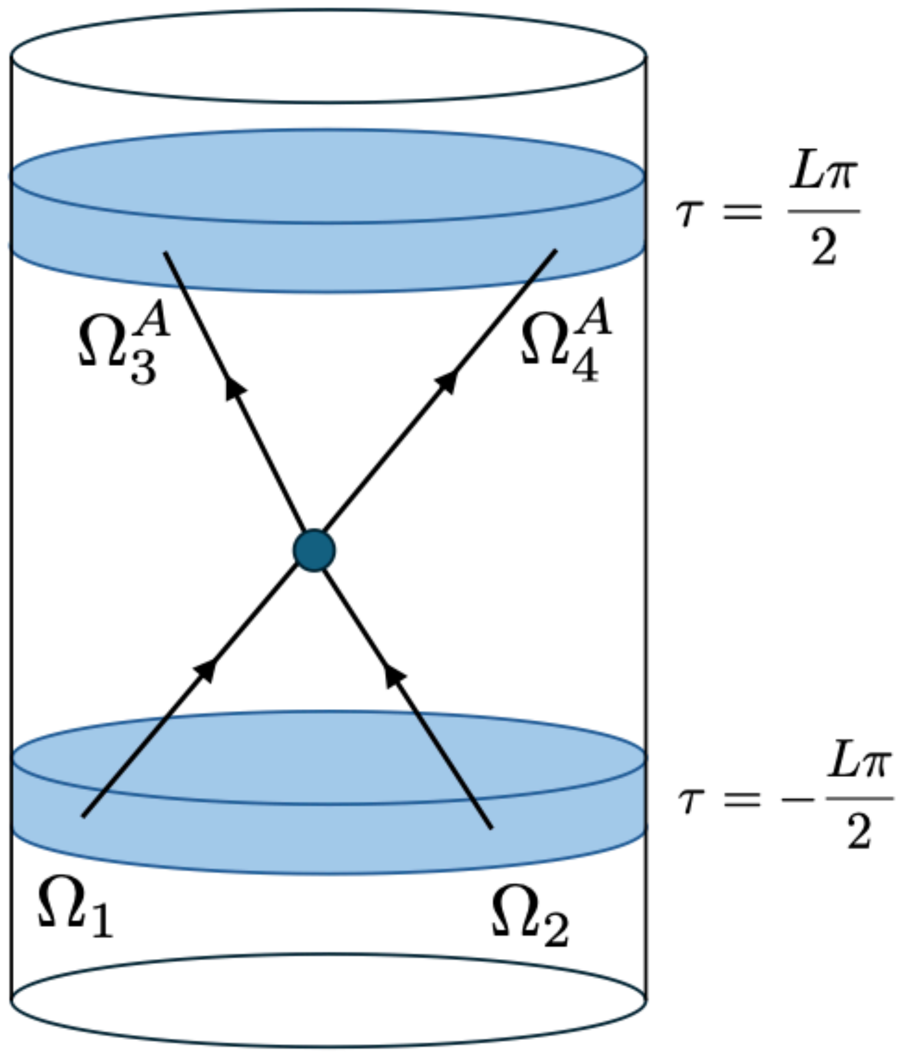
The problems in a picture



Lightning review of large radius limits:

[Giddings], [Penedones], many others

[Chester, Pufu, Yin], [Alday et al]

1. Start with a sequence of holographic CFTs (e.g. ABJM with $U(N)_1 \times U(N)_{-1}$)

2. Prepare $\begin{cases} |i\rangle = \mathcal{O}_1^\Delta(\Omega_1)\mathcal{O}_2^\Delta(\Omega_2)|0\rangle\,, \\ |f\rangle = \mathcal{O}_3^\Delta(\Omega_3^A)\mathcal{O}_4^\Delta(\Omega_4^A)|0\rangle\,, \end{cases}$

   in time-bands centred on $\tau = \mp \frac{\pi L}{2}$.

3. Take suitable scaling limits of "scattering amplitudes" $L^{\#}\langle f|i\rangle$ with $L \to \infty$, Δ/L fixed.

Non-locality

From this point of view it is obvious that $[\mathcal{O}_P, \mathcal{O}_F] \neq 0$, since P/F CFT operators are related by CFT evolution.

Using flat space *S*-matrix and accepted GKPW-like dictionary with Carrollian correlators, for essentially this reason in a putative dual we have $[\mathcal{O}(\Omega), \mathcal{O}'(\Omega')] \neq$ (contact term). (See 2512.05072 for more)

But, in a Carrollian theory, light-cone has collapsed and so this means nonvanishing commutators outside the light-cone, i.e. non-locality.

Bulk interactions ⇒ boundary non-locality

Non-unitarity?

Actually, from this point of view, perhaps the better question is:

Which, if any, of the postulates of QM hold for putative Carrollian duals?

In a parent sequence of CFTs, of course we have all that we cherish, including Hilbert space, unitary evolution, and the scattering amplitudes that become “Carrollian correlators.”

Very little of this seems to remain when interpreting integral-transformed Amplitudes as correlators.

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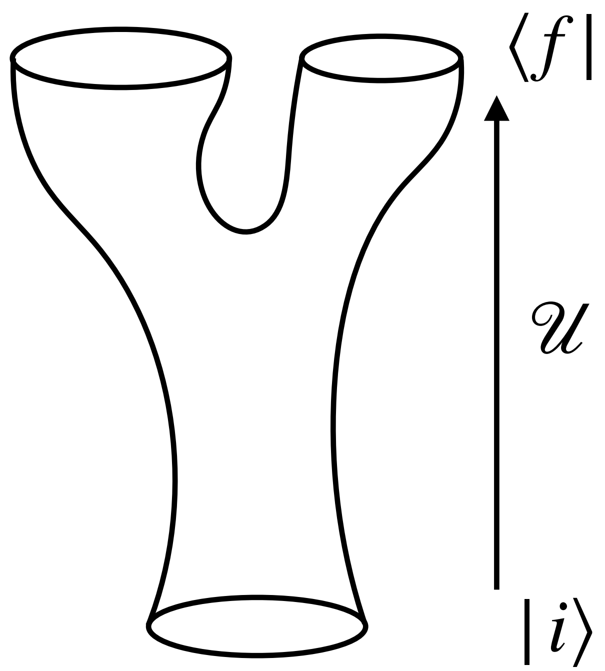
Non-AdS holography

de Sitter:

Non-unitary CFT computes no-bdy wavefunction? (dS/CFT, [Anninos et al])

What about other topologies, e.g. global dS? [Cotler, KJ, Maloney]

Evolution implies correlations between bdys, but without interactions, suggesting disorder. (dS JT: [Maldacena, Turiaci, Yang], [Cotler, KJ])



Schrödinger: [Guica, et al]

"WCFT": e.g. [Song, Strominger]

$T\bar{T}$-deformed CFT: [Zamolodchikov[2]]

} All involve non-locality

Thank you!