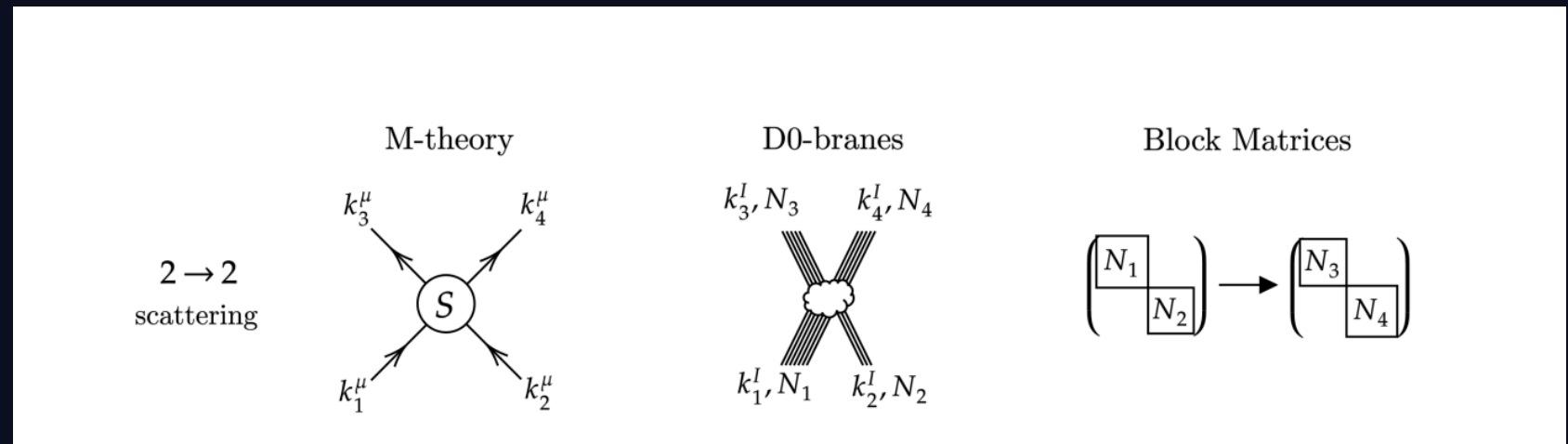


# Twisting BFSS & IKKT

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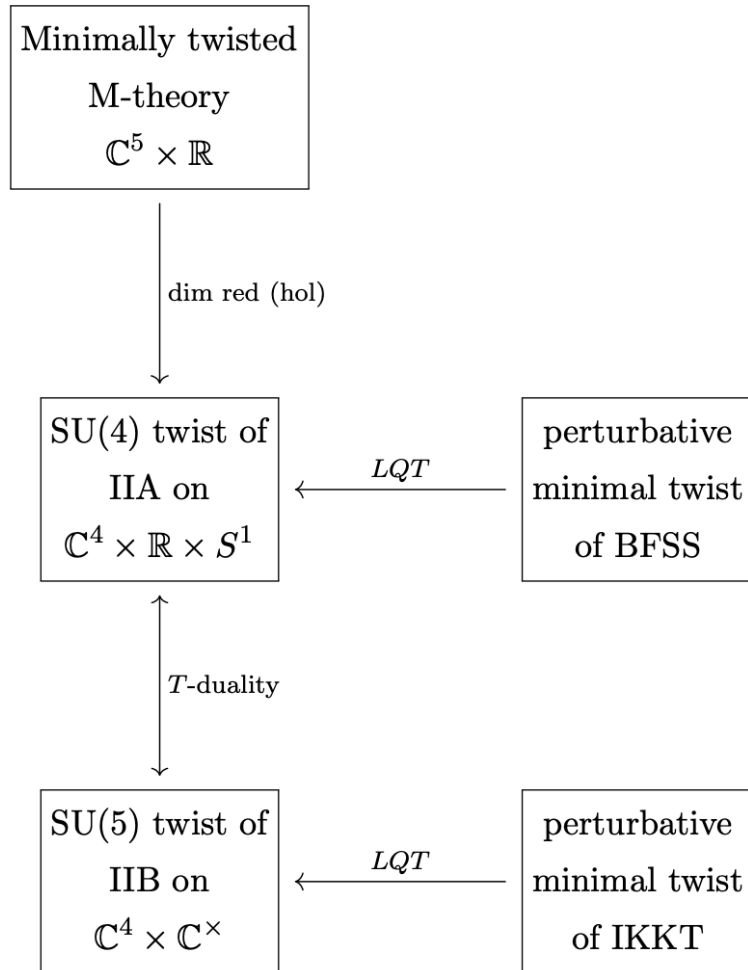
University of Washington



Based on 2602.22318 + WIP  
w/ F. Hahner

Image from Miller-Strominger-Tropper-Wang

# Outline of this talk



- 1 Motivation
- 2 BFSS, IKKT, and their BV complexes
- 3 Minimal twists and their IIB/IIA duals
- 4 Non-minimal twists and their IIB/IIA duals
- 5 Remarks and open questions

# Refine our understanding of top-down examples

DLCQ M-theory at fixed  $P^+ = N/R \Leftrightarrow U(N)$  matrix quantum mechanics

BFSS has taught us...

- 1 Nonperturbative, non-gravitational dual**  
11d scattering with fixed light-cone momentum is encoded by a quantum mechanics, not a boundary field theory.
- 2 Multiparticle bulk states are algebraic**  
Separated gravitons from block-diagonal matrices, the block sizes carry the longitudinal momentum
- 3 Infrared gravity really shows up**  
Long-distance interactions, soft theorems, asymptotic symmetries, and even evidence for 11d Lorentz symmetry have BFSS counterparts.

...but it is still hard

- A Flat space is recovered indirectly**  
Locality and Lorentz invariance must emerge from a subtle large- $N$ , infrared limit
- B The observables are scattering observables**  
But the full M-theory S-matrix from BFSS is technically hard.
- C The interesting regimes are strongly coupled**  
Black holes, generic non-BPS dynamics, and finite- $N$  corrections sit in the hardest part of the theory analytically and numerically.

Many, many classic results, as well as recent numerical studies (at finite  $T$ ), ...

Twisted holography should give a new probe of the SUSY-protected sector

# Twisted holography (as open-closed duality in topological string)

## Physical matrix models

BFSS:  $U(N)$  matrix quantum mechanics in large- $N$  limit conjecturally gives 11d M-theory in lightcone formulation.

IKKT: a proposed analogue for IIB string theory from the pointlike matrix model

## Twist by a square-zero $Q$

Pass to  $Q$ -cohomology inside the BV-BRST formalism

Keep the supersymmetry-protected subsector (perturbatively, around the trivial background).

As usual, this is a nice, coupling-insensitive subsector

## Twisted gravity side

Match to twisted IIA/IIB string theory or supergravity

infinite-dimensional symmetry algebras visible

Main payoff: the protected large- $N$  sector becomes a tractable algebraic object. At infinite- $N$ , perturbative isomorphism between twisted duals, which can be established rigorously with homological algebra methods.

# BV complexes of IKKT and BFSS

## IKKT (0d)

### Physical fields

10 bosonic matrices  $X^I$  and 16 positive-chirality fermions  $\psi_\alpha$ , all valued in  $\mathfrak{u}(N)$ .

### BV completion

Add the gauge ghost  $c$  and antifields  $X^{I*}$ ,  $\psi^*$ ,  $c^*$ .

This packages gauge symmetry and supersymmetry into one homological complex.

### Ghost numbers:

gh +1:  $c$  | gh 0:  $X^I, \psi$   
 gh -1:  $X^{I*}, \psi^*$  | gh -2:  $c^*$

## BFSS (1d)

### Physical fields

9 bosonic matrices  $X^i$ , a worldline gauge field  $A$ , and 16 fermions  $\psi_\alpha$ , again in  $\mathfrak{u}(N)$ .

### BV completion

Add  $c$  and antifields  $X^{i*}$ ,  $A^*$ ,  $\psi^*$ ,  $c^*$  again

### Ghost numbers:

gh +1:  $c$  | gh 0:  $A, X^i, \psi$   
 gh -1:  $A^*, X^{i*}, \psi^*$  | gh -2:  $c^*$

# Actions and SUSY algebras: BFSS

$$S_{BFSS} = \frac{1}{g^2} \int dt \operatorname{tr} \left( \frac{1}{2} (D_t X^i)^2 + \frac{1}{2} \psi^\alpha D_t \psi_\alpha - \frac{1}{4} [X_i, X_j]^2 - \frac{1}{2} \gamma_{\alpha\beta}^i \psi^\alpha [\psi^\beta, X_i] \right)$$

## SUSY variations

$$\delta X^i = \varepsilon^\alpha \gamma_{\alpha\beta}^i \psi^\beta$$

$$\delta \psi_\alpha = D_t X^i \gamma_i \varepsilon + \frac{1}{2} [X_i, X_j] \gamma_{\alpha\beta}^{ij} \varepsilon^\beta$$

$$\delta A = \varepsilon^\alpha \psi_\alpha$$

But the SUSY algebra only closes up to fermion equations of motion  
&  
gauge transformation with field-dependent gauge parameter.

This leads to terms in the full BV action that are quadratic in the antifields.

# Actions and SUSY algebras: IKKT

$$S = -\frac{1}{g} \text{tr} \left( \frac{1}{4} [X^I, X^J] [X_I, X_J] - \frac{1}{2} \psi \Gamma^I [X_I, \psi] \right)$$

**SUSY variations**

$$\delta X^I = \varepsilon \Gamma^I \psi \quad \text{and} \quad \delta \psi = \varepsilon \Gamma_{IJ} [X^I, X^J]$$

**Again, the SUSY algebra only closes up to fermion equations of motion  
&  
gauge transformation with field-dependent gauge parameter.**

**This leads to terms in the full BV action that are quadratic in the antifields.**

# Augmenting to BV-BRST

## Generalities

Batalin–Vilkovisky extends BRST by adjoining ghosts and antifields.

Gauge symmetry, equations of motion, and supersymmetry into one homological object

Physical information is extracted from BV cohomology.

$$S_{BV} = S_0 + S_{\text{gauge}} + S_{\text{susy}} \quad Q_{BV} = \{S_{BV}, -\}_{BV} = \delta_{\text{e.o.m.}} + \delta_{\text{gauge}} + \delta_{\epsilon}$$

e.g. BFSS. Note: in this equation,  $\epsilon$  is a bosonic ghost (cf. coupling to SUGRA, make bosonic ghosts constant)

$$S_{\text{susy}} = \text{tr}(X_I^* \epsilon \Gamma^I \psi + \frac{1}{2} \psi^* \epsilon \Gamma_{IJ} [X^I, X^J] + c^* (\epsilon \Gamma_I \epsilon) X^I + (\epsilon \Gamma^I \epsilon) (\psi^* \Gamma^I \psi^*) - 2(\epsilon \psi^*)^2)$$

## SETUP

Twisting in BV-BRST: 1. Classify  $Q = \epsilon^\alpha Q_\alpha$  s.t.  $Q^2 = 0$  (up to  $G_{Lor} \times G_R$ -orbits)

2. Specialize  $S_{BV}[\epsilon^\alpha]$ .

This deforms  $Q_{BV} \mapsto \delta_{eom} + \delta_{gauge} + \delta_Q$

3. Compute cohomology.

### IKKT

- Minimal twist:  $\epsilon\Gamma\epsilon = 0$ . Preserves  $SU(5)$
- Maximal twist:  $\epsilon\Gamma\epsilon \neq 0$ . Preserves  $Spin(7)$

### BFSS

- Minimal twist:  $\epsilon\Gamma\epsilon = 0$ . Preserves  $SU(4)$
- Maximal twist:  $\epsilon\Gamma\epsilon \neq 0$ . Preserves  $G_2$

# Minimal twists are the perturbatively richer option

- The minimal twists arise from dim red. of a holomorphic twist of 10d  $\mathcal{N} = 1$  SYM [Costello-Li]
- Spectral sequence filtered by polynomial degree
- BFSS: 0-form and 1-form operators

	$(\theta^a)^0$	$(\theta^a)^1$	$(\theta^a)^2$	$(\theta^a)^3$	$(\theta^a)^4$
1	$c$	$Z^a$	$\psi^{(2)}$	$\psi^{*(3)}$	$(X^* - A^*)$
$dt$	$(X - A)$	$\psi^{(1)}$	$\psi^{*(2)}$	$Z^{*a}$	$c^*$

$$\delta_Q(\mathcal{O}^{(1)}) = D_t \mathcal{O}^{(0)} dt$$

IKKT minimal twist

$$\alpha \in \mathbb{C}[\theta^1, \dots, \theta^5] \otimes \mathfrak{gl}_N$$

$$\int d^5\theta \operatorname{tr}(\alpha[\alpha, \alpha])$$

BFSS minimal twist

$$\alpha \in \Omega^\bullet(\mathbb{R}) \otimes_{\mathbb{R}} \mathbb{C}[\theta^1, \dots, \theta^4] \otimes \mathfrak{gl}_N$$

$$S_{BV}[\alpha] = \int dt d^5\theta \operatorname{tr}(\alpha d_t \alpha + \frac{2}{3} \alpha[\alpha, \alpha])$$

# How to get from matrix algebras to holomorphic geometry?

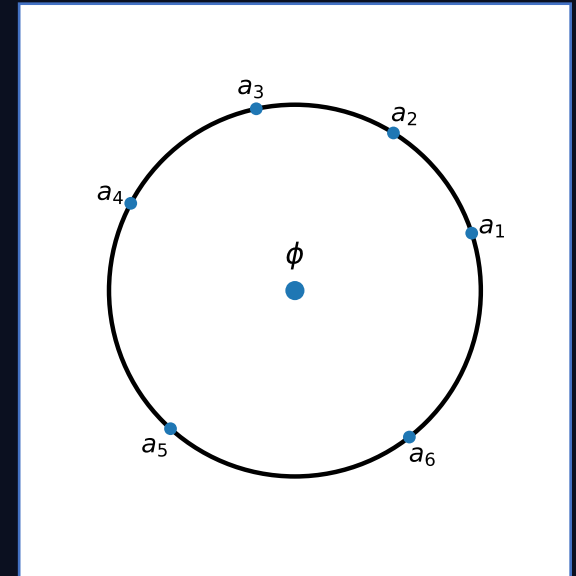
- Closed strings define multilinear operations on open string algebra called Hochschild cochains, which govern gauge invariant deformations of OSFT [Kapustin-Rozansky,...]

- If we are interested in the gauge invariants of matrix algebras in the limit  $N \rightarrow \infty$ , LQT theorem tells us it is generated by *cyclic* classes

“Single traces are closed strings”

- Koszul duality gives dual, geometric picture of this deformation data

$$\theta^i \leftrightarrow \partial_{z_i}$$



## Math recipe

Open string algebra  $A$ , w/ Hochschild cochains. Compute cyclic cochains (Connes spectral sequence), identify CC w/  $N \rightarrow \infty$ , gauge invariants of  $A$  via LQT theorem. Koszul duality helps “geometrize” CC.

# Twisted holography for minimally twisted BFSS/IKKT

IKKT

$$A = \mathbb{C}[\theta^1, \dots, \theta^5] \otimes \mathfrak{gl}_N$$

Infinite N limit:

$$CC^\bullet(A) \simeq (PV^\bullet(\mathbb{C}^5)[[t]], \bar{\partial} + t\partial)$$

SU(5)-twist of IIB

BV complex of  
fields of BCOV  
[Costello-Li]

B-model string theory on  $\mathbb{C}^5$

NB: Can get interactions as well by keeping track of brackets, not just vector spaces  
and following the trail of math from the previous slide

BFSS

$$A = \Omega^\bullet(\mathbb{R}) \otimes \mathbb{C}[\theta^1, \dots, \theta^4] \otimes \mathfrak{gl}_N$$

Infinite N limit:

$$CC^\bullet(A) \simeq (PV^\bullet(\mathbb{C}^4)[[t]], \bar{\partial} + t\partial) \otimes \Omega^\bullet(\mathbb{R}^2, d_{dR})$$

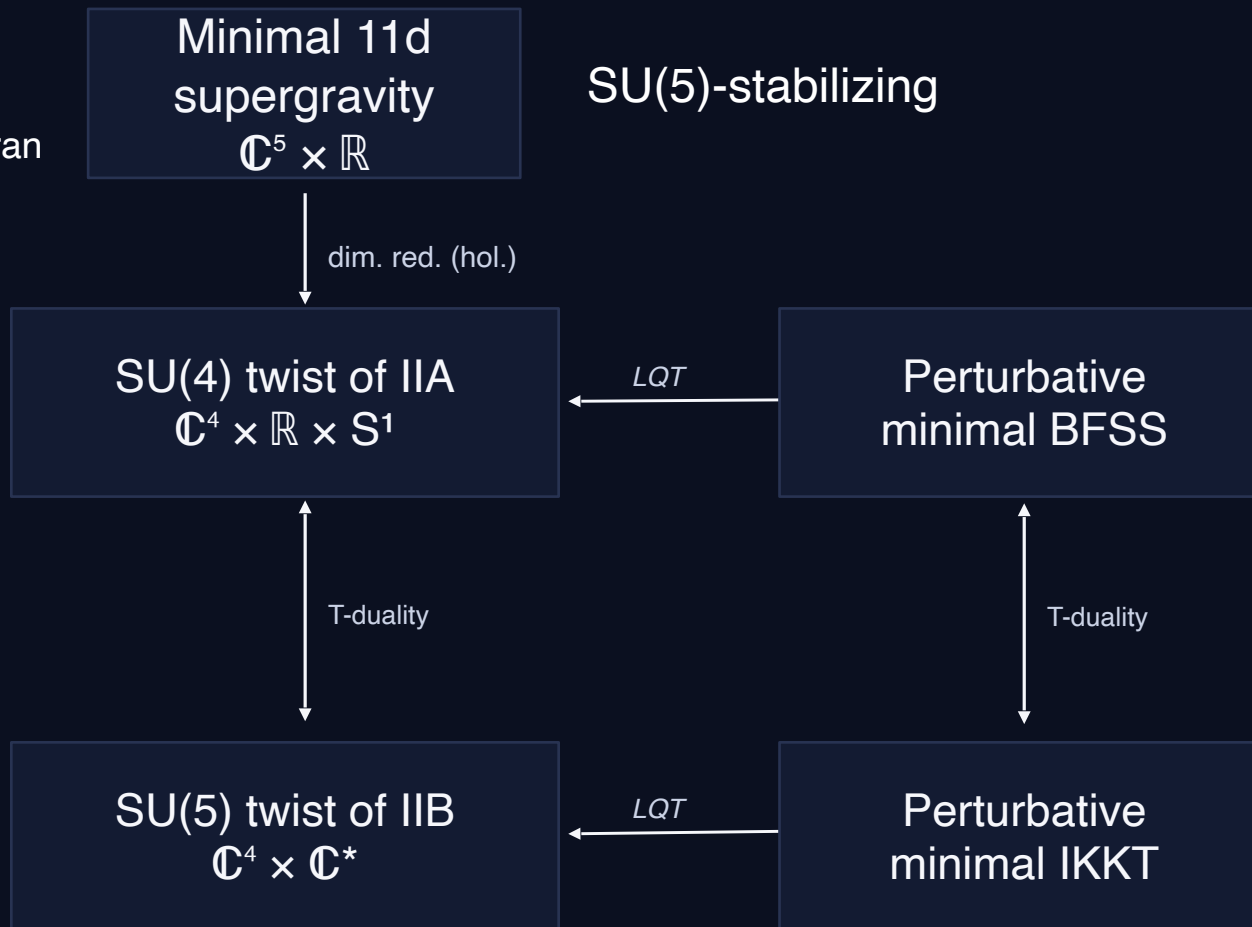
SU(4)-twist of IIA

Mixed A/B-model string theory on  $\mathbb{R}^2 \times \mathbb{C}^4$

The supergravity sector is associated to a subcomplex of the above, called “minimal BCOV theory”

# The minimal twists

[Raghavendran  
-Saber-  
Williams]



- Reduction on a holomorphic circle takes minimal M-theory to the SU(4) twist of type IIA.

- T-duality relates SU(4)-twisted IIA with SU(5)-twisted. This is compatible with the topological string theory duals

(See also [Hahner-Paquette-Raghavendran] for more on twisted 11d SUGRA)

# Maximal twists

- Maximal twists are perturbatively (locally) acyclic, so it seems like nothing interesting survives.
- Lesson from equivariant cohomology: these kinds of twists are only sensitive to *global* info in field space
- Mechanistically, one passes to gauge invariants first by hand, removing the ghost from the complex

- “Cohomology of basic subcomplex”

IKKT maximal twist

$$\text{Tr}(Z^n)$$

**Spin(7)-twist of IIB on  $\mathbb{C} \times \mathbb{R}^8$ ?**

[Hahner]: Exists in classification of IIB twisting SUSYs

BFSS maximal twist

$$\text{Tr}(Z^n) \quad \int dt \text{Tr}(\psi_+ Z^n)$$

**G2-twist of IIA on  $\mathbb{C} \times \mathbb{R}^8$ ?**

# Twisted holography for maximally twisted BFSS/IKKT

IKKT

&amp;

BFSS

In both cases, such twisting options seem to exist,  
and the naive residual translations preserved by the twist include one holomorphic direction

But: [Hahner-Paquette-Raghavendran-Saberi, to appear]  
including the fundamental string extension in the algebra kills the remaining hol'c  
direction: we recover perturbative acyclicity if we work locally in field space.

Alternatively: these twists are further twists of the minimal twist. In spacetime, one deforms BCOV by a linear superpotential.

(More on string/brane extensions for twisted theories to come)

There is probably interesting physics nonperturbatively/globally in field space.  
More on basic subcomplex TBD.

# Takeaways and open directions

## Three takeaways

1 Minimal twists of IIA/IIB supergravity match minimal twists of BFSS/IKKT respectively.

2 Maximal twists are locally acyclic, which is true both in field theory and gravity (a deformation of BCOV on  $\mathbb{C} \times \mathbb{R}^8$ ).

3 2 twists of BFSS uplift to 2 twists of 11d supergravity, but precise match only after reducing to 10d on a holomorphic circle.

## Three natural follow-ups

A Go beyond perturbation theory, beyond the trivial background (WIP), finite N stuff, branes, etc. etc.

B Could one understand the Maldacena dual of D0 brane MQM in these methods? Backreaction is non-geometric.

C On the SUGRA side, big manifest infinite dimensional symmetry algebras. Can we use them to usefully organize computations on the field theory side?

Thanks!

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