Physics in strong fields:

perturbing around non-trivial vacua

Anton Ilderton

2026-04-15

Simons Collaboration on Celestial Holography Satellite Meeting

THE UNIVERSITY of EDINBURGH

Engineering and Physical Sciences Research Council

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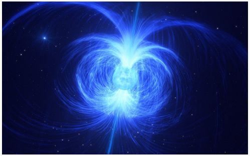
Review: Fedotov et al, Phys.Rept. 1010 (2023) 1 [arXiv:2203.00019]

Outline

1. Physical scenarios
2. Scattering in (non-) trivial vacua.
3. Examples
   - → Probing the quantum vacuum
   - → Helicity flip in YM
   - → Hawking radiation from the double copy
4. The (real!) physics of self-dual fields
   - → Focussing and depletion
   - → Double copy

1. Physical scenarios

Sources of strong fields



Strong magnetic fields: magnetar environments
ESO/L. Calcada.

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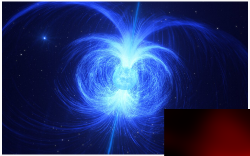

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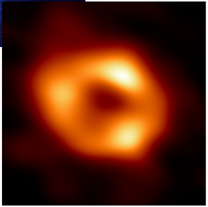
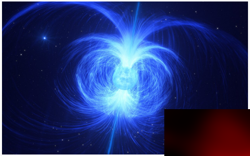


Strong gravity near massive bodies

EHT Collaboration

Sources of strong fields



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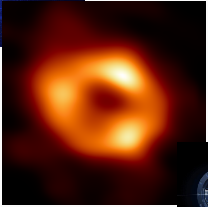

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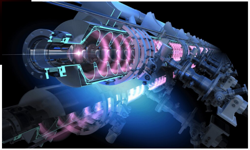

Dense bunches
$\rightarrow$ strong Coulomb

ILC international dev. team.

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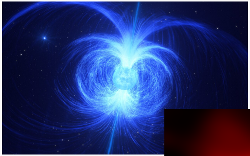

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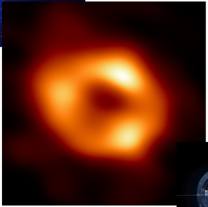

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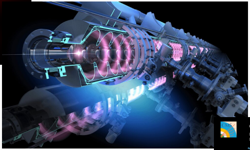

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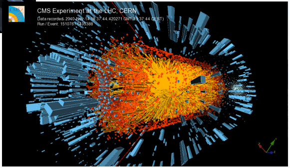
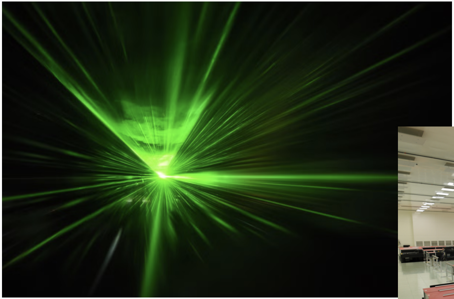


Heavy ion collisions

CMS/CERN

Modern laser pulses: huge number of photons, delivered to micron spot-size in femto-seconds.

High intensity / flux



Getty Images / The Independent

ELI NP. www.science.org

When is a laser field 'strong'? When $\xi > 1$!

- Important parameter:

$$\xi = \frac{eE}{m\omega c} = \frac{eE\lambda\!\!\!^{-}}{mc^2}$$

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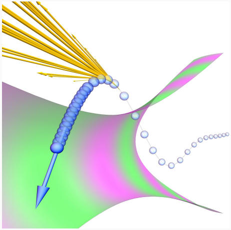
$\xi > 1$: **relativistic** physics

$\xi > 1$: multi-γ/ **nonlinear** physics

$\xi > 1$: **non-perturbative** physics

- QFT → "Strong field QED"
- Achievable $\xi \sim O(10)$, soon $\mathcal{O}(100)$.
  Mourou & Strickland, Nobel Prize 2018
- Intensity 10^{22}–10^{23} W/cm^2 @ optical



Imperial College/S.Mangles

2. Scattering in (non-) trivial vacua.

Scattering: perturbing around a trivial vacuum

- Scattering: $\langle\, \text{out}\, |S|\, \text{in}\, \rangle$ with

$$|\, \text{in}\, \rangle = \prod_{\{k,s\}} a_s^\dagger(k) |\, 0\, \rangle$$

- Number states → number states. Expand in coupling g.
- Perturbing around trivial vacuum/ Minkowski space.

- If macroscopic fields present ...

  ... not a good description of asymptotic system.

- Zero expectation for fields $\langle\, \text{in}\, |\hat{A}^\mu(x)|\, \text{in}\, \rangle = 0$.

Coherent states of light

- Coherent states $|\,\alpha\,\rangle = \mathbb{D}(\alpha)|\,0\,\rangle$

$$\mathbb{D}(\alpha) = \exp \int_k \alpha_s(k) a_s^\dagger(k) - \bar{\alpha}_s(k) a_s(k)$$

- Coherent states $\leftrightarrow$ classical solutions of Maxwell in vacuum

$$\mathcal{A}^\mu(x) := \langle\,\alpha\,|\hat{A}^\mu(x)|\,\alpha\,\rangle = \int_k \alpha_s(k)\epsilon_s^\mu\, e^{-ik\cdot x} + \bar{\alpha}_s(k)\bar{\epsilon}_s^\mu\, e^{ik\cdot x}$$

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- $S|\,\alpha$, in $\rangle \leftrightarrow$ scattering on a classical field.

- Assume: high occupation/no significant back-reaction $\implies$

Calculate amplitudes $\langle\,\alpha, \text{out}\,|S|\,\alpha, \text{in}\,\rangle$

Background fields from coherent states

- Displacement: $\mathbb{D}^\dagger(\alpha) a_s(k) \mathbb{D}(\alpha) = a_s(k) + \alpha_s(k)$.
- Amplitudes: displaces S-matrix

$$\langle\, \alpha, \mathsf{out}\, | S |\, \alpha, \mathsf{in}\, \rangle = \langle\, \mathsf{out}\, | \mathbb{D}^\dagger(\alpha) S \mathbb{D}(\alpha) |\, \mathsf{in}\, \rangle = \langle\, \mathsf{out}\, | S[\mathcal{A}] |\, \mathsf{in}\, \rangle$$

- Shift action by classical background: $eA \to eA + e\mathcal{A}$

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Scattering with coherent states ↔ scattering in backgrounds.

Franz Phys.Rev. 139 (1965) B1326, Kibble Phys.Rev. 138 (1965) B740

- QED: all vacuum solutions.

- YM & gravity: linearised vacuum solutions. (But see later!)

How do we calculate?

$$\mathcal{L} = -\frac{1}{4}F^2 + \bar{\psi}(i\partial\!\!\!/ - eA\!\!\!/ - e\mathcal{A}\!\!\!/ - m)\psi$$

- Coupling to background $\frac{e\mathcal{A}}{m} \sim \xi \gg 1$, treat exactly.

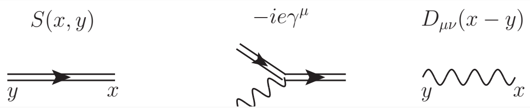
$\rightarrow$ Furry expansion. Furry, Phys.Rev. 81 (1951) 115

$$\mathcal{L} \rightarrow \underbrace{-\frac{1}{4}F^2 + \bar{\psi}(i\mathcal{D}\!\!\!\!/ - m)\psi}_{\text{quadratic}} - \underbrace{e\bar{\psi}A\!\!\!/\psi}_{\text{interaction}}$$

- Bg.-covariant derivative: $\mathcal{D}_\mu = \partial_\mu + ie\mathcal{A}_\mu$
- Coupling to dynamical photons $\sim e$, treat perturbatively.

Feynman rules

- Feynman rules:



$S(x,y) = (i\not{D} - m)^{-1} =$ 

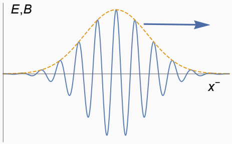
- $S(x,y)$: arbitrary functional complexity!
- Practical use for simple/symmetric fields.

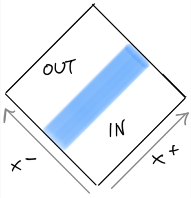
Plane waves

- QED: $A = -x^{\perp} E_{\perp}(x^{-}) \mathrm{d}x^{-}$

$$\mathrm{d}s^2 = 2\mathrm{d}x^{+}\mathrm{d}x^{-} - \mathrm{d}x^{\perp}\mathrm{d}x^{\perp}$$

$$x^{\pm} = (t \pm z)/\sqrt{2}, \quad x^{\perp} = (x, y)$$





:) $\Longrightarrow$ known exactly. Volkov Z.Phys. 94 (1951)

:) Position and velocity memory, temporal shape

:(Focussing geometry – hard problem! (See later!)

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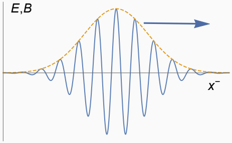
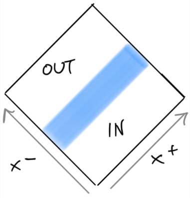




- ⌣ $\Longrightarrow\!\!=\!\!=$ known exactly. Volkov Z.Phys. 94 (1951)
- ⌣ Position and velocity memory, temporal shape
- ⌢ Focussing geometry – hard problem! (See later!)

- $e^{ip\cdot x} \rightarrow$ "Volkov wavefunction" $\qquad a'_{\perp}(x^{-}) = E_{\perp}(x^{-})$

$$\phi_p(x) = \exp\left[-iea(x^{-})\cdot x + ip\cdot x + i\int^{x^{-}} \mathrm{d}y^{-} \frac{2p\cdot ea(y^{-}) - e^2a^2(y^{-})}{2n\cdot p}\right]$$

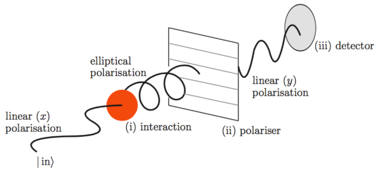
3. Examples

Example 1: probing the quantum vacuum

The quantum vacuum, exposed to intense light, is birefringent.

Toll, PhD thesis, 1952

- Probe beam/γ + intense laser.
- Probe polarisation:
  linear $\longrightarrow$ elliptic.
  Heinzl et al Opt.Comm. 267 (2006)



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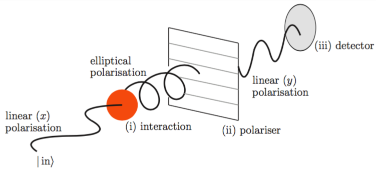

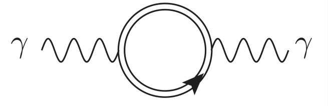

- Light by light scattering.
- Helicity can flip even if $k_\mu \to k_\mu$

- Quantum $\to$ classical, micro $\to$ macro:

  helicity flip $\to$ birefringence:

  Dinu et al, PRD 89 (2014) 125003

$$\frac{I_\perp}{I_\parallel} = |\mathcal{M}_{\text{flip}}|^2$$

Example 1: probing the quantum vacuum

$$\xi = \frac{eE}{m\omega}$$

1. "Constant crossed": $\omega \to 0$, E fixed, $\xi \to \infty$

$\to$ Refractive indices of vacuum: $n \simeq 1 + \frac{e^2}{180\pi}(11 \pm 3)\frac{e^2 E^2}{m^4}$

- Biref $\leftrightarrow$ difference of indices

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2. 'Impulsive' limit $E \to \infty$, $\omega \to \infty$, $\xi = eE/(m\omega)$ fixed:

Ilderton & Kingham, PRD 110 (2024) 105022

- $E(x^-) \sim \xi\delta(x^-)$

$$\mathbb{P}_{\text{flip}} = |\mathcal{M}_{\text{flip}}|^2 = \left(\frac{e^2}{24\pi^2} - \frac{e^2}{6\pi^2}\frac{1}{\xi\sqrt{\xi^2+4}}\tanh^{-1}\left[\frac{\xi}{\sqrt{\xi^2+4}}\right]\right)^2$$

3. Monochromatic wave: fix ω. ξ, E finite.

- No-flip (forward) amplitude. Dinu et al, PRD 89 (2014) 125003

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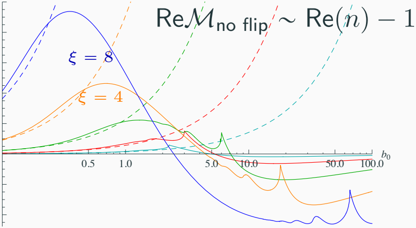
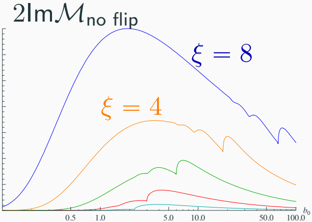
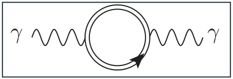
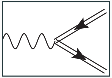




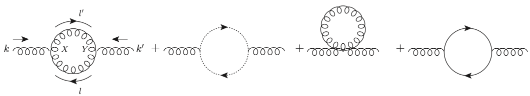




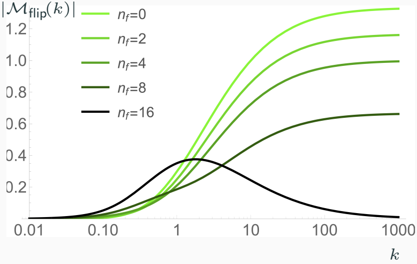
- Normal dispersion → anomalous dispersion → Re(n)<1.
- Pair creation. Borysov et al, PRD 106 (2022)
- 'Resonances' (periodicity) . Harvey et al PRL 109 (2012) 100402

Example 2: Gluon helicity flip

- YM plane waves – Cartan valued. Lift SFQED.
- SU(2) YM + n_f flavours of massless fundamental quarks



- High energy behaviour: $\mathbb{P}_{\text{flip}}(k) \sim |\mathcal{M}_{\text{flip}}(k)|^2$

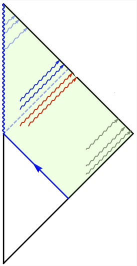


- $\mathcal{M}_{\text{flip}}(\infty) = \frac{4}{3} - \frac{n_f}{12}$
- "Flavour suppression"
- ($\beta = 0$ for $n_f = 11$)

Adamo & **Ilderton**, JHEP 06 (2019) 015

Example 3: Schwinger and Hawking

- $A_\mu = \dfrac{Q}{4\pi r} k_\mu \theta(t+r)$



$\implies$ Pair creation. Worldline:

$$\mathcal{M} = \int \mathcal{D}(z) e^{iS_{\text{cl}}[z]}$$

- Classical geodesics...

$\implies$ $$\mathcal{M} \sim \exp(-\tfrac{g^2}{2} qQ)$$

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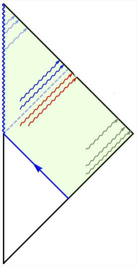
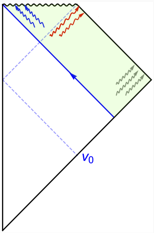
- $A_\mu = \frac{Q}{4\pi r} k_\mu \theta(t+r)$ $\xrightarrow{\text{classical D.C.}}$ • $g_{\mu\nu} = \eta_{\mu\nu} - \frac{2GM}{r} k_\mu k_\nu \theta(t+r)$

Kerr Schild, $\frac{g^2}{4\pi} \to 2G$





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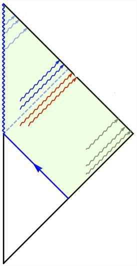

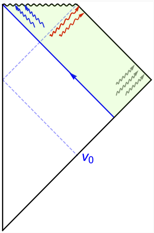

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$$\implies \quad \mathcal{M} \sim \exp(-\tfrac{g^2}{2} qQ)$$

- Geodesics double copy:

$$qQ \to M\mathcal{E}$$

- Hawking radiation

$$\xrightarrow{\text{D.C.}} \quad \mathcal{M} \sim \exp(-4\pi G M\mathcal{E})$$

Ilderton, Lindved, Rajeev, PRL 136 (2026) 081603

4. The physics of self-dual fields

Self-dual fields in QED

☺ Self-dual fields $\widetilde{F}_{\mu\nu} = \pm iF_{\mu\nu}$ – simplicities!

→ Higher loop & multiplicity. Dunne & Schubert JHEP 08 (2002) 053
Adamo, Mason, Sharma PRL 125 (2020) 041602, Bittleston & Costello 2602.17538

☹ Fields complex: $E \sim \pm iB$. Unphysical?

- What is the real physics of self-dual fields?
- Back to the coherent state picture...

Complex fields are really real fields :-)

- Scattering with different coherent states in past/future

$$\langle \beta\, , \mathsf{out}\, | S |\, \alpha\, , \mathsf{in}\, \rangle$$

Complex fields are really real fields :-)

- Scattering with different coherent states in past/future

$$\langle \beta\, , \mathsf{out}\, | S |\, \alpha\, , \mathsf{in}\, \rangle = e^{-\frac{1}{2}\int_k |\beta-\alpha|^2} \langle\, \mathsf{out}\, | S[\mathcal{A}] |\, \mathsf{in}\, \rangle$$

↔ Scattering on a complex background:

$$\mathcal{A}^\mu(x) = \int_k \alpha_s(k)\epsilon^\mu_s\, e^{-ik\cdot x} + \bar{\beta}_s(k)\bar{\epsilon}^\mu_s\, e^{ik\cdot x}$$

:) But unitary!

Complex fields are really real fields :-)

- Scattering with different coherent states in past/future

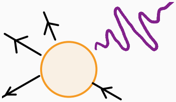
$$\langle \beta \,,\text{out}\,|S|\,\alpha\,,\text{in}\,\rangle = e^{-\frac{1}{2}\int_k |\beta-\alpha|^2}\langle\,\text{out}\,|S[\mathcal{A}]|\,\text{in}\,\rangle$$

↔ Scattering on a complex background:

$$\mathcal{A}^\mu(x) = \int_k \alpha_s(k)\epsilon_s^\mu\, e^{-ik\cdot x} + \bar{\beta}_s(k)\bar{\epsilon}_s^\mu\, e^{ik\cdot x}$$

:-) But unitary! → Absorptive processes.

$$\langle\,\text{out}\,|S|\,\alpha\,,\text{in}\,\rangle \quad \longleftrightarrow$$



Endlich et al JHEP 05 (2017) 052, **Ilderton** & Seipt PRD 97 (2018) 016007, Aoude & Ochirov JHEP 12 (2023) 103

- Self-dual now a special case.

Example 1: focussing and depletion

- Strong field QED: mostly plane wave backgrounds.
    - ? include realistic focussing geometry (pheno relevant now)
    - ? include back-reaction (not so pheno relevant yet)

- Self-dual fields allow us to model both at once.

Adamo & **Ilderton**, PRD 111 (2025) 125005

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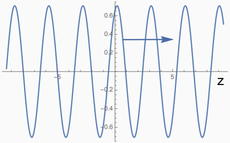
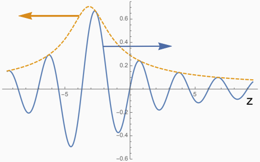
Adamo & **Ilderton**, PRD 111 (2025) 125005

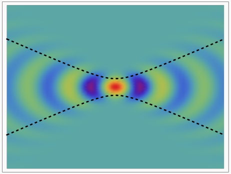
- Self-dual plane wave → self-dual 'Flying focus':

$$\exp\left[-i\omega x^-\right] \longrightarrow \frac{1}{1+ikx^+}\exp\left[-i\omega\left[x^- - \frac{ik}{2}\frac{x^\perp x^\perp}{1+ikx^+}\right]\right]$$

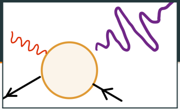


$\longrightarrow$





Example 1: focussing and depletion



- In self-dual field: can solve massless particle EOM

  Penrose JMP 10 (1969) 38, Ward & Wells CUP 1991, Mason & Woodhouse OUP 1991

✓ Exact wavefunctions $\phi_p(x)$ in fields with realistic focussing!

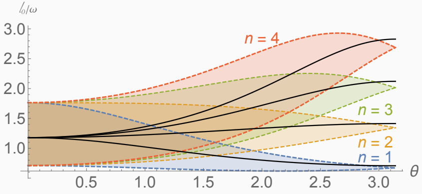
✓ Tree-level processes:

$$\langle\, q, \ell_s \,|S|\, \alpha_{FF}\,, p\,\rangle = \int \mathrm{d}^4x\, e^{i\ell\cdot x}\big[\bar{\phi}_q(x)\varepsilon_s(\ell)\cdot \overleftrightarrow{\mathcal{D}}\phi_p(x)\big]$$

- So far: total absorption. . . but work in progress.

  Adamo, **Ilderton**, Noble, *to appear 2026*

- Comparison of emission spectra:



Example 2: double-copy in self-dual fields

- Double copy of YM amps with coherent states of gluons?

$$\langle \beta, \text{out} | S_{\text{YM}} | \alpha, \text{in} \rangle$$

- Expand:

$$| \alpha, \text{in} \rangle = \mathbb{D}(\alpha) | \text{in} \rangle = \sum_n \int \frac{\alpha^n}{\sqrt{n!}} | k_1 \dots k_n, \text{in} \rangle$$

- Choice of double copy: $\varepsilon_\mu \to \varepsilon_\mu \varepsilon_\nu = \sum_P C_P \varepsilon^P_{\mu\nu}$

- $P = \{$graviton, dilaton, axion$\}$.

- Double copy vacuum amplitudes & reassemble ...

Example 2: double-copy in self-dual fields

- Amps with coherent states of gluons double copy to:

  Amplitudes with 'fat graviton' coherent-states.

  Ilderton & Lindved JHEP 09 (2025) 156

- $\langle\, \boldsymbol{\beta}, \text{out}\, | S_{\mathsf{GR}} |\, \boldsymbol{\alpha}, \text{in}\, \rangle$, where $\boldsymbol{\alpha}_P(k) = \sum_{a,s} C_P{}^s \alpha^a_s(k)$

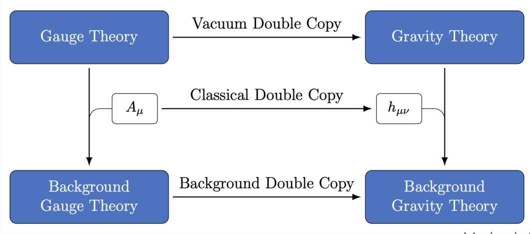
$\leftrightarrow$ Amplitudes on background metric+axion+dilaton!

$$\int_k \boldsymbol{\alpha}_P(k)\varepsilon^P_{\mu\nu}\, e^{-ik\cdot x} + \bar{\boldsymbol{\beta}}_P(k)\bar{\varepsilon}^P_{\mu\nu} e^{ik\cdot x}$$

- Classical backgrounds α inherited from amplitude prescription.

Example 2: double-copy in self-dual fields

☺ Classical double copy emerges from amplitudes.



Lindved, PhD thesis 2026

☺ Self-dual case...

Berman et al JHEP 01 (2019) 107

Brown et al PRD 109 (2024) 026009, J.-H. Kim, PRD 111 (2025) L021703

- Map relates
  - Exact vacuum solutions in YM $\leftrightarrow$ gravity
  - Scattering amplitudes on those backgrounds
  - All-order statements!

Ilderton & Lindved JHEP 09 (2025) 156

Conclusions

Conclusions

Intense lasers

- Access to quantum vacuum phenomena
- Experiments ongoing
- Theory challenges remain! Fedotov et al, Phys.Rept. 1010 (2023) 1

Connections to . . .

- Yang Mills
- Self-dual fields
- Double copy

Other topics

- Leading order self-force effects Adamo et al, PRL 131 (2023) 011601
- Amplitudes: structure beyond flat space Aoude et al arXiv: 2603.17903