Towards a double-copy picture in AdS

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1

One language for gauge and gravity

Introduction

- Gauge and gravity theories: crucial in our understanding of physical phenomena
- Gauge theories: weak, strong, electromagnetic interactions
- Gravity: spacetime, macroscopic evolution of the Universe

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Beyond standard QFT...

History suggests giving up on traditional approaches and welcoming new principles to find a unified framework!

Introduction

1) String theory

   1d objects, different vibrational modes $\rightarrow$ spectrum

   - **gauge** bosons (photons, gluons): massless excitations of open strings
   - **gravitons** : massless excitations of closed strings

Introduction

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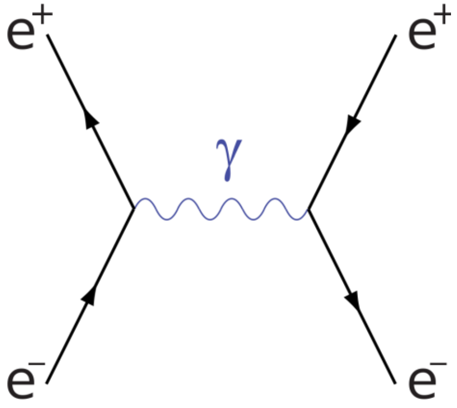
2) Double-copy

   Gravity **from** Yang-Mills^2

   Not manifest in the Lagrangian. Focus on objects closely related to observable quantities.

Powerful frameworks to explore the perturbative structure of QFTs.

Scattering amplitudes



DIAGRAMS:
history of a scattering event

- Scattering amplitudes
  - dynamics
  - test predictions / explore conjectures
  - uncover structure and reveal symmetries

- Surprising properties encode deep lessons about QFT and gravity.

- One of the most remarkable properties is the **double-copy**.

Double-copy: a direct construction

Gauge theory amplitude where all the particles are in the adjoint color representation.

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Claim 1: Tree-level gauge theory amplitudes can be **rearranged** to display a duality between color and kinematics.
[Bern, Carrasco, Johansson]

EXISTENCE STATEMENT !

$$A_n^{YM} = g^{n-2} \sum_{diags\ i} \frac{c_i n_i}{D_i} \qquad c_i + c_j + c_k = 0 \implies n_i + n_j + n_k = 0$$

Full, color-dressed

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Full, color-dressed

Claim 2: Get gravity amplitudes by using 2 copies of the gauge theory diagram **numerators**.

$$c_i \to \tilde{n}_i$$

The BCJ/CK setup is an on-shell, momentum-conserving amplitude statement!

Double-copy

- Compute amplitudes in one theory using amplitudes from **simpler** theories.
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- At tree-level, avoid numerators and use **color-ordered** amplitude:

$$\underbrace{A_n^{L\otimes R}}_{\text{double-copy}} = \sum_{\alpha,\beta} \underbrace{A_n^L[\alpha]}_{\text{single copy}} \boxed{K_n[\alpha|\beta]} \underbrace{A_n^R[\beta]}_{\text{single copy}}$$

KERNEL

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KERNEL

- The kernel cancels double poles and supplies missing simple poles.

Example

s,t = Mandelstam variables (Lorentz-invariant combinations of external momenta)

FIELD THEORY KERNEL

$$A_4^{GR} = \boxed{\frac{st}{s+t}} \left(A_4^{YM}[1234]\right)^2$$

4 gravitons (coupling-stripped)

4 gluons (color-stripped)

- Tree-level 4-graviton scattering from by 4-gluon scattering, bypassing GR.
- True for any number of external states: "Gravity from Yang-Mills^2"

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- Tree-level 4-graviton scattering from by 4-gluon scattering, bypassing GR.
- True for any number of external states: "Gravity from Yang-Mills^2"
- Dynamics of *weakly-coupled* gauge/gravity, described by the same kinematical building blocks!

The double-copy has its origin in **string theory**.
[Kawai, Lewellen, Tye, «KLT»,1985]

In the infinite tension limit, we learn about QFTs!

String amplitudes in flat space

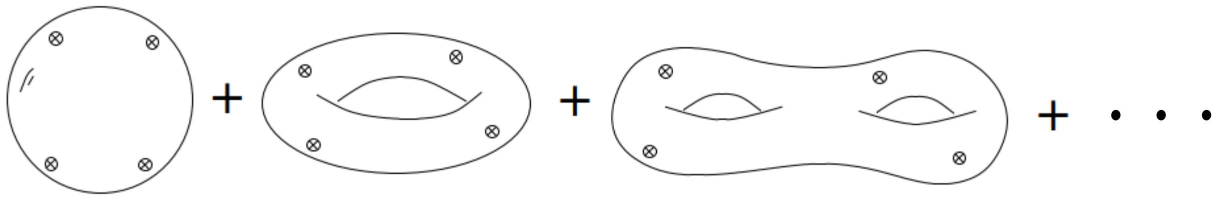
$$A_{(n)}(\Lambda_i, p_i) = \sum_{\text{topologies}} g_s^{-\chi} \frac{1}{\text{Vol}} \int DX Dg \, e^{-S_{\text{Poly}}} \prod_{i=1}^{n} V_{\Lambda_i}(p_i)$$

Parameters

g_s (string coupling constant)

α' (size of the string)

- The worldsheet encodes the interactions!
- **Genus** expansion over punctured Riemann surfaces with **vertex operator** insertions (asymptotic states).

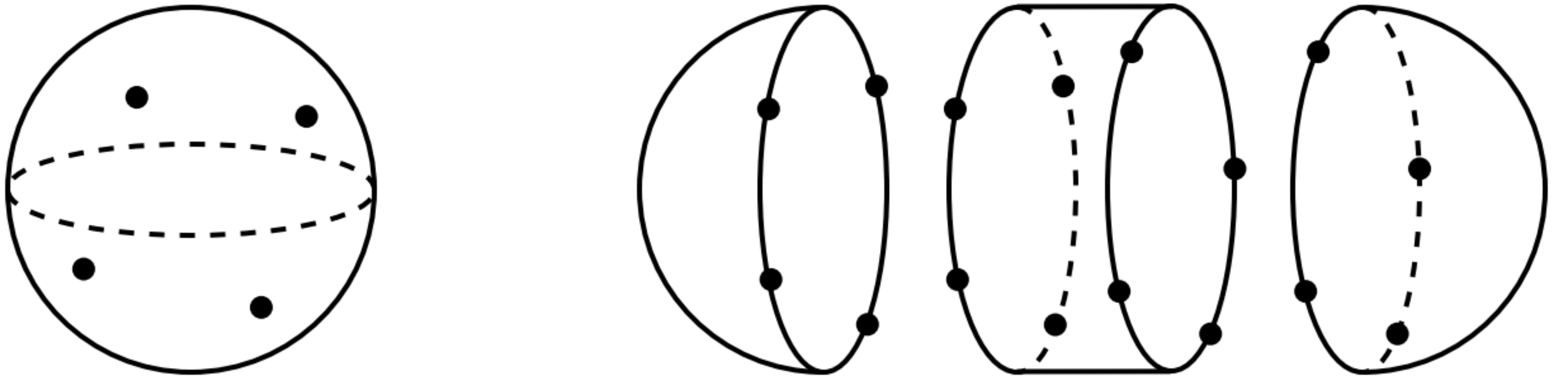


- Tree-level string amplitudes: punctured spheres.

KLT relations in flat space

- Closed-string amplitude: correlator of vertex operators on the sphere
- Open-string amplitude: correlator of vertex operators on the disk boundary

Closed string amplitudes as bilinears of open string amplitudes



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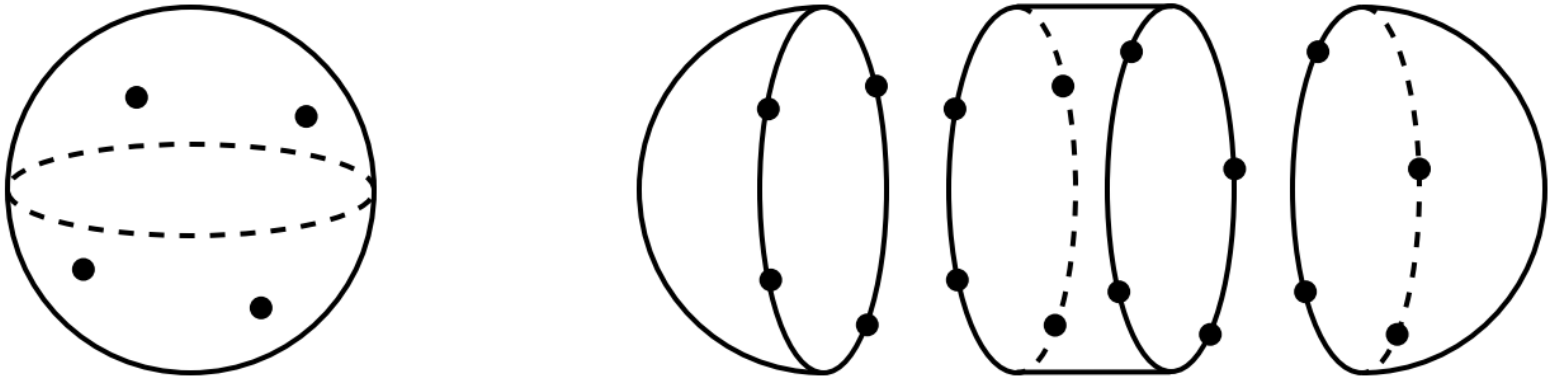
KERNEL

Picture adapted from Mizera, 2016.

KLT relations in flat space



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Closed string as “left × right” = holomorphic factorization on the sphere

KERNEL: L/R *kinematic* data, no additional punctures
[pairing matrix between 2 orderings]

Picture adapted from Mizera, 2016.

Example

STRING THEORY KERNEL

$$A_4^{Virasoro-Shapiro} = \boxed{\frac{\sin(\pi\alpha' s)\sin(\pi\alpha' t)}{\sin(\pi\alpha'(s+t))}} (A_4^{Veneziano}[1234])^2$$

4 gravitons (coupling-stripped)

4 gluons (color-stripped)

Closed from Open^2

$\alpha' \to 0$

Gravity from Yang-Mills^2

What about curved spacetimes?

What changes when flat-space momentum kinematics is lost?

What about curved spacetimes?

- String amplitudes
  $\longrightarrow$ the background controls the local worldsheet dynamics
  $\longrightarrow$ difficulties with standard formulations

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- String amplitudes
  → the background controls the local worldsheet dynamics
  → difficulties with standard formulations

CONFORMAL BOUNDARY

In our favor,

→ AdS/CFT: string scattering amplitudes on AdS
from CFT correlators of the dual boundary theory

→ Exact limits
(flat space, high energy [Alday, Hansen, MN, Virally, Zhou])

(Some) results

- AdS 3pt amplitudes [Farrow, Lipstein, McFadden, ...]
- AdS color-kinematics duality [Armstrong, Lipstein, Mei, Alvayrak, Kharel, Meltzer, Alday, Behan, Ferrero, Zhou, ...], but no sensible double-copy amplitudes
- Plane waves [Adamo, Casali, Mason, Nekovar, ...]
- Ambitwistor strings [Mason, Skinner, ...]
- Double-copy for *reduced* 4-point amplitudes [Zhou]
- Also, Celestial/Carrollian [Casali, Puhm, Sharma, ...],
  twistor theory [Adamo, Klisch, White, ...];
  classical double-copy [Monteiro, O'Connell, ...];
  ...

Double-copy in curved spacetime

Focusing on AdS:

- *systematic* formulation of double-copy
- prescription for building blocks for gravitational amplitudes

How much of the flat space structure persists?

3

Building blocks of string amplitudes in AdS

The AdS Virasoro-Shapiro amplitude
[Alday, Hansen, ...]

$$A(S,T) = \sum_{k=1}^{\infty} \left(\frac{\alpha'}{R^2}\right)^k A^{(k)}(S,T)$$ EXPANSION AROUND FLAT SPACE

$$= \int d^2z |z|^{-2S}|1-z|^{-2T} W_0(z,\bar{z}) \left(1 + \frac{S^2}{R^2} W_3(z,\bar{z}) + \frac{S^4}{R^4} W_6(z,\bar{z}) + \dots\right)$$

$$W_0(z,\bar{z}) = \frac{1}{2\pi U^2 |z|^2 |1-z|^2}$$

- AdS corrections as **genus 0 worldsheet integrals** with SVMPLs.
- Same structure in different backgrounds!
  [Alday, Giribet, Hansen, Chester, Zhong,...]

Multiple polylogarithms

$$L_{0^p}(z) = \frac{\log^p z}{p!}$$

$$L_{0^{n-1}1} = -\mathrm{Li}_n(z)$$

- Labelled by a word *w*, whose length we call **weight**.

- Recursively defined by $$\frac{d}{dz} L_{0w}(z) = \frac{1}{z} L_w(z)\,, \qquad \frac{d}{dz} L_{1w}(z) = \frac{1}{z-1} L_w(z)$$

$$L_e(z) = 1 \qquad \lim_{z\to 0} L_w(z) = 0$$ e = empty word

- Special functions of a single complex variable, multi-valued (branch cuts at 0,1).

- Combinations of polylogs such that all the branch cuts cancel. [Brown]

$$\mathcal{L}_w(z) = sv\left(L_w(z)\right)$$

$$\log|z|^2 = \log z + \log \bar{z}$$

Flat space 4pt building blocks

Closed strings $$\beta_{\mathbb{C}}(s,t) = \int |z|^{2s-2}|1-z|^{2t-2} d^2 z = \frac{\Gamma(s)\Gamma(t)\Gamma(1-s-t)}{\Gamma(s+t)\Gamma(1-s)\Gamma(1-t)}$$

Open strings $$\beta(s,t) = \int_0^1 x^{s-1}(1-x)^{t-1} dx = \frac{\Gamma(s)\Gamma(t)}{\Gamma(s+t)}$$

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- In a small s, t expansion, they are related by a single-valued map:

$$\zeta_{\mathrm{sv}}(2n) = 0$$
$$\zeta_{\mathrm{sv}}(2n+1) = 2\,\zeta(2n+1)$$

LINEAR

$$\beta_{\mathbb{C}}(s,t) = sv(\beta(s,t))$$

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KERNEL

QUADRATIC

- KLT relations.

In flat space, **explicit answers** for string amplitudes from the **worldsheet** theory

⇒ study properties and relations

Going into AdS?

Ansatz = insertions of (SV) MPLs

- CFT data agree with integrability and localization results! ✅
- High energy limits confirm the full proposal ✅

AdS 4pt building blocks [Alday, MN, Sangaré]

Closed strings

$$I_w(s,t) = \int |z|^{2s-2}|1-z|^{2t-2}\mathcal{L}_w(z)d^2z$$

SVMPL

Open strings

$$J_w(s,t) = \int_0^1 x^{s-1}(1-x)^{t-1}L_w(x)dx$$

MPL

w = empty word reduces to flat space

AdS 4pt building blocks [Alday, MN, Sangaré]

- Can be computed in a low-energy expansion.

$$J_w(s,t) = \text{poles} + \sum_{p,q=0} s^p t^q \sum_{W \in 0^p \sqcup 1^q \sqcup w} \left(L_{0W}(1) - L_{1W}(1) \right)$$

sv

$$I_w(s,t) = \text{poles} + \sum_{p,q=0} s^p t^q \sum_{W \in 0^p \sqcup 1^q \sqcup w} \left(\mathcal{L}_{0W}(1) - \mathcal{L}_{1W}(1) \right)$$

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$$\xrightarrow{sv}$$

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.

- Explicit results up to weight 4 for open strings.
  (Aomoto-Gelfand hypergeometrics)

- More generally, compute I-integrals via KLT for SVMPL integrals!

Holomorphic factorization

[Alday, Chester, Hansen, Zhong, 2024]

$$A_{\text{closed}}(s,t) = \int |z|^{2s-2}|1-z|^{2t-2} \underline{\sum_i^n F_i(z)G_i(\overline{z})d^2 z}$$

SINGLE-VALUED

$$= \frac{1}{2\pi i}\sum_i^n \int_0^1 x^{s-1}(1-x)^{t-1}G_i(x)dx \int_1^\infty y^{s-1}\text{Disc}_1\left[(1-y)^{t-1}F_i(y)\right]dy$$

$$\text{Disc}_1\left[f(y)\right] = f(y+i\epsilon) - f(y-i\epsilon)\ , \quad y>1$$

With SVMPL insertions, factorization into products of 1d MPL integrals!

4

Towards a double-copy picture in AdS

AdS 4pt building blocks [Alday, MN, Sangaré]

Closed strings $$I_w(s,t) = \int |z|^{2s-2}|1-z|^{2t-2}\mathcal{L}_w(z)d^2z$$ **SVMPL**

Open strings $$J_w(s,t) = \int_0^1 x^{s-1}(1-x)^{t-1}L_w(x)dx$$ **MPL**

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Open strings $$J_w(s,t) = \int_0^1 x^{s-1}(1-x)^{t-1}L_w(x)dx$$ **MPL**

The generalizations of the Euler and complex beta functions are related by a single-valued map:

$$I_w(s,t) = sv\left(J_w(s,t)\right) = \sum_{w_1,w_2} J_{w_1}(s,t)\, \boxed{K_w^{w_1,w_2}(s,t)}\, J_{w_2}(s,t)$$

KERNEL

Open strings in AdS

$$J_w(s,t) = \int_0^1 x^{s-1}(1-x)^{t-1} L_w(x) dx$$

- Introduce non-commutative variables e_0, e_1 associated with the letters 0 and 1.
- Define the **generating function** of MPLs:

$$L(e_0, e_1; x) = L_e(x) + L_0(x)e_0 + L_1(x)e_1 + L_{00}(x)e_0^2 + L_{01}(x)e_0e_1 + L_{10}(x)e_1e_0 + \ldots$$

- Correspondingly, introduce a generating function for the J-integrals:

$$\mathcal{J}(s,t;e_0,e_1) = \int_0^1 x^{s-1}(1-x)^{t-1} L(e_0, e_1; x) dx$$

$$= J_e(s,t) + J_0(s,t)e_0 + J_1(s,t)e_1 + J_{00}(s,t)e_0^2 + J_{01}(s,t)e_0e_1 + \cdots$$

Closed strings in AdS

$$I_w(s,t) = \int |z|^{2s-2}|1-z|^{2t-2}\mathcal{L}_w(z)d^2z$$

- Define the generating function of SVMPLs.

REVERSED

$$\mathcal{L}(e_0,e_1;z) = L(e_0,e_1;z)L^{R}(e_0,e_1';\bar{z})$$

DEFORMED VARIABLE $e_1' = e_1 - 2\zeta(3)[e_0+e_1,[e_1,[e_0,e_1]]] + \cdots$

- Correspondingly, introduce a generating function for the I-integrals:

$$\mathcal{I}(s,t;e_0,e_1) = \int |z|^{2s-2}|1-z|^{2t-2}\mathcal{L}(e_0,e_1;z)d^2z$$

- Aim: find relation to the generating function of open strings in AdS.

KLT relations in AdS

$$\mathcal{I}(s,t;e_0,e_1) = \mathcal{J}(s,t;e_0,e_1)\boxed{\mathcal{K}(s,t;e_0,e_1)}\mathcal{J}^R(s,t;e_0,e_1')$$

KERNEL : expansion in the non-commutative variables

$$\mathcal{K}(s,t;e_0,e_1) = \kappa(s,t) + \kappa_0(s,t)e_0 + \kappa_1(s,t)e_1 + \cdots$$

- The AdS kernel starts with the flat space KLT Kernel and contains only trigonometric functions in s,t.

- Can be computed from direct computation to a given order in e_0, e_1 or from holomorphic factorization **to all orders**!

KLT relations in AdS

- The AdS KLT kernel takes a remarkably simple form,

$$\mathcal{K}(s,t;e_0,e_1) = -\frac{1}{2\pi i}\left(1+\frac{e^{2i\pi s}M_0}{1-e^{2i\pi s}M_0}+\frac{e^{2i\pi t}M_1}{1-e^{2i\pi t}M_1}\right)^{-1}$$

where the **monodromy** matrices govern the discontinuities of MPLs. [Brown]

$$M_0 = e^{2\pi i e_0}, \quad M_1 = Z(e_1,e_0)e^{2\pi i e_1}Z(e_0,e_1)$$

- Non-commutative word-valued kernel.
  (acting linearly on weight-w MPLs by mixing words)

KLT relations in AdS

More explicitly, to all orders:

$$\mathcal{K}^{-1} = \pi\left(\cot\left(\pi(s{+}e_0)\right) + Z(e_1,e_0)\cot\left(\pi(t{+}e_1)\right) Z(e_0,e_1)\right)$$

$$= \pi(\cot(\pi s) + \cot(\pi t)) + \cdots$$

GENERATING FUNCTION OF MZVs

FLAT SPACE

Generating functions: the natural language?

The AdS VS amplitude, as a curvature expansion around flat space, **generates** *certain* I-integrals!

$$A(s,t) = \sum_k \left(\frac{\alpha'}{R^2}\right)^k A^{(k)}(s,t) = \underline{A^{(0)}(s,t)} + \frac{\alpha'}{R^2}\underline{A^{(1)}(s,t)} + \dots$$

FLAT SPACE VIRASORO-SHAPIRO

FIRST CURVATURE CORRECTION

$$\mathcal{I}(s,t;e_0,e_1) = \underline{I_e(s,t)} + \cdots + \underline{(e_0^3\, I_{000}(s,t) + e_0 e_1 e_0\, I_{010}(s,t) + \dots)} + \dots$$

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$$\mathcal{I}(s,t;e_0,e_1) = I_e(s,t) + \cdots + (e_0^3\, I_{000}(s,t) + e_0 e_1 e_0\, I_{010}(s,t) + \dots) + \dots$$

- Yes, natural language to extend flat space KLT to AdS!

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- Yes, natural language to extend flat space KLT to AdS!
- Need to project onto the physical amplitude!
  For example, «inner-product scheme». [Alday, Sangaré, Pitombo]

Generating functions: the natural language?

Our work prompted many interesting studies!

- Recent works:
  Associators for AdS string amplitude building blocks [Baume]
  Twisted de Rham theory for string double copy in AdS [Kakkad, Ochirov, Zhang]

- Work in progress:
  Motivic coaction [Schlotterer,...]
  More on twisted de Rham [Tao, Zhang,...]

➡ Robust geometric structure underlying string perturbation theory on curved backgrounds (fully general!)

Key takeaways

- Special representations restore a flat-like structure for amplitudes in AdS!
- Single-valuedness as guiding principle.
- Emergent **worldsheet** picture (*universality*).

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- Special representations restore a flat-like structure for amplitudes in AdS!
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⇒ A unifying mathematical **structure**.

- Tree-level **building blocks**
- Analytic AdS corrections

Future directions

- The *inverse* of the kernel is remarkably simple, as in flat space. [CHY, Mizera]

  AdS corrections to $\mathbf{BAS}_{\alpha'}$?

- Kernel from *monodromy* of open-string integrals.

  Monodromy for AdS ***amplitudes***. [Alday, Pitombo, Sangaré]

  Worldsheet derivation?

- AdS double-copy for specific **amplitudes** in specific theories?

Future directions: higher points

[MN, Pitombo, Sangaré, Tao, wip]

- Color ordering

$$J_{1234}(s_{12}, s_{23}; e_{12}, e_{23}) = \int_0^1 dx\, x^{s_{12}}(1-x)^{s_{23}} L(x; e_{12}, e_{23})$$

- Proposal for building blocks: **multivariable MPLs**

$$\mathcal{J}_{13245} = \int_{0<z_2<z_3<1} dz_2 dz_3 z_2^{s_{12}} z_3^{s_{13}} z_{32}^{s_{23}} (1-z_2)^{s_{24}} (1-z_3)^{s_{34}} L_{\{0,1,z_2\}}(z_3) L_{\{0,1\}}(z_2)$$

- Properties (IBP,...)
- **n-pt** mondromy relations
- Bootstrap?

Thank you for your attention!

Maria Nocchi