

The flat space limit of the AdS/CFT correspondence in momentum space

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Holography and flat space

- The original argument for holography was based on the entropy of black holes [’t Hooft (1992) Susskind (1994)].
- As such it should hold for any asymptotics.
- ➡ In particular, for asymptotically flat gravity.
- ➡ Observables in asymptotically flat gravity are scattering amplitudes
- ➡ ... and holography suggest that they should have a description in terms of a quantum theory in one dimension less.
- We will see how to obtain scattering amplitudes in $d + 1$ -dimensional Minkowski space from CFT correlation function in d dimensions by a taking a flat-space limit of AdS.

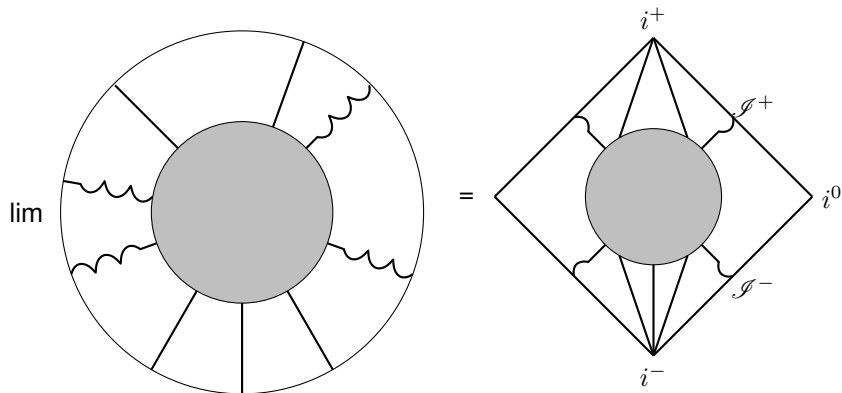
Flat-space limit

- The 'flat-space' limit of AdS was discussed since the early days of the AdS/CFT correspondence [Polchinski (1999) Susskind (1999)].
- The underlying physical picture is compelling: the physics in a small region in the centre of AdS should be the same as that of flat space.
- However, this limit is *very subtle*.
- Here we will provide a rigorous formulation of this limit.

References

- This talk is based on **on-going work** and:
R. Marotta, KS, M. Verma, **Flat space spinning massive amplitudes from momentum space CFT**, JHEP(2024), 2406.06447
- Related work:
V. Nemenli, A. Shekar, M. Verma, **From AdS to Flat Space: Massive Spin-2 Fields**, 2602.02662
- The topic has a long history:
Giddings, Penedones, Maldacena, Simmons-Duffin, Ziboedov, Komatsu, Paulos, Van Rees, Vieira, Fitzpatrick, Raju, Hijano, Neuenfield, Lipstein, McFadden, Gadde, Sharma, Pimentel, Li, Grumiller, de Gioia, Raclariu, Cassali, Gibbons, Strominger, Sleight, Taronna, Bagchi, Barnich, Kraus, Donnay, Mason, Ruzziconi, Yellshpur Srikant
- Most earlier works on flat space holography were done in **bottom-up approaches**:
 - **Celestial holography**: co-dimension two CFT in celestial sphere
 - **Carrollian holography**: co-dimension one CFT at null infinity
- Here the approach is to view **holography for flat space** as a limit of the **AdS/CFT duality**.

In pictures:



$$\lim \langle \mathcal{O}_{\Delta_1}(\vec{p}_1) \cdots \mathcal{O}_{\Delta_n}(\vec{p}_n) \rangle_{CFT_d} = \text{Scattering amplitude in } Mink_{(d+1)}$$

- Introduction
- **AdS/CFT in momentum space**
- Flat space limit
- Scattering amplitudes to all orders from AdS

- AdS gravity coupled to matter:

$$S_{AdS} = \int d^{d+1}x \sqrt{g} \left(\frac{1}{16\pi G} R + \Lambda + L_{\text{QFT}}(m_i^2, g_j) \right)$$

- $\Lambda \sim \frac{1}{\ell^2}$ is the cosmological constant, and ℓ is the AdS radius:

$$ds_{AdS}^2 = \ell^2 \left(\frac{dz^2}{z^2} + \frac{1}{z^2} d\vec{x}^2 \right)$$

where we consider **Euclidean AdS**.

- To illustrate the discussion we start with **scalar fields** with mass:

$$m^2 \ell^2 = \Delta(\Delta - d)$$

- We will comment **spinning fields** afterwards.

$$Z_{\text{EAdS}}[\varphi_{(0)}; \{\Delta_i, g_j\}; \ell] = \left\langle \exp \left(- \int_{\partial \text{EAdS}(\ell)} \varphi_{(0)} \mathcal{O} \right) \right\rangle_{\text{CFT}}$$

- CFT lives on space:

$$ds^2 = \ell^2 d\vec{x}^2$$

- Boundary conditions:

$$\Phi(z, \vec{x}) = z^{d-\Delta} \varphi_{(0)}(\vec{x}) + \dots + z^\Delta \varphi_{(\Delta)}(\vec{x}) + \dots$$

- **Momentum-space** bulk-to-boundary (Btb) propagator:

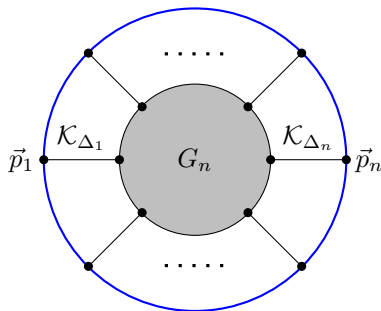
$$\mathcal{K}_\Delta(p, z) = \frac{z^{d/2} p^\beta K_\beta(pz)}{2^{\beta-1} \Gamma(\beta)}, \quad \beta = \Delta - d/2$$

- **Momentum-space** bulk-to-bulk (BtB) propagator:

$$G_\Delta(p; z, z') = \ell^{1-d} (zz')^{d/2} [K_\beta(pz) I_\beta(pz') \theta(z - z') + (z \leftrightarrow z')]$$

where $p = \sqrt{\vec{p}^2}$ and $K_\beta(qz), I_\beta(pz)$ are Bessel functions.

AdS amplitudes = CFT correlators



$$\begin{aligned} &= \langle \mathcal{O}_{\Delta_1}(\vec{p}_1) \cdots \mathcal{O}_{\Delta_n}(\vec{p}_n) \rangle \\ &= \langle \langle \mathcal{O}_{\Delta_1}(\vec{p}_1) \cdots \mathcal{O}_{\Delta_n}(\vec{p}_n) \rangle \rangle \delta(\vec{p}_1 + \cdots + \vec{p}_n) \end{aligned}$$

- G_n is (loop-level) **amputated bulk amplitude** constructed in perturbation theory using **bulk vertices** and **bulk-to-bulk propagators**.

- Introduction
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From AdS to flat space

- We reparameterize the radial coordinate

$$z = \ell e^{\tau/\ell}$$

- Then we get flat space from Euclidean AdS as follows:

$$ds^2 = \ell^2 \left(\frac{dz^2}{z^2} + \frac{1}{z^2} d\vec{x}^2 \right) \xrightarrow{\ell \rightarrow \infty} ds^2 = d\tau^2 + d\vec{x}^2 + O(1/\ell)$$

Upon Wick rotation we get Minkowski.

- One can check that the **AdS isometries** become the **flat space isometries**.
 - AdS_{d+1} isometries are also the **conformal group in d dimensions**.
 - Flat space **boosts and time translations** originate from **dilations and special conformal transformations**.

- The CFT lives at the boundary of AdS, in a space with metric:

$$ds^2 = \ell^2 d\vec{x}^2$$

- As we take $\ell \rightarrow \infty$, we focus on
 - deep IR of the CFT
 - deep interior of AdS
- When taking this limit we want to **kept fixed** the parameters of the bulk theory:

$$m_i^2, g_j \text{ fixed}$$

- Since $m_i^2 \ell^2 = \Delta(\Delta - d)$, as $\ell \rightarrow \infty$
 - $m_i^2 \rightarrow 0$, if Δ_i does not scale with ℓ .
 - to have $m_i^2 \neq 0$ one needs to have $\frac{\Delta_i}{\ell} = \text{fixed}$

Flat space limit of propagators

- The propagators involve the Bessel functions:

$$K_{\Delta - \frac{d}{2}}(pz), I_{\Delta - \frac{d}{2}}(pz).$$

- Since $z = \ell e^{\tau/\ell}$, and $\Delta = O(1)$ (massless), or $\Delta = m\ell$ (massive) one needs to two limits:

- Large argument limit (well-known):

$$I_\nu(z) \rightarrow \frac{e^z}{(2\pi z)^{\frac{1}{2}}} \quad \text{and} \quad K_\nu(z) \rightarrow \left(\frac{\pi}{2z}\right)^{\frac{1}{2}} e^{-z} \quad \text{as} \quad z \rightarrow \infty$$

- Uniform limit (less known):

$$\lim_{\nu \rightarrow \infty} K_\nu(\nu z) \simeq \left(\frac{\pi}{2\nu}\right)^{\frac{1}{2}} \frac{e^{-\nu \xi(z)}}{(1+z^2)^{\frac{1}{4}}}, \quad \lim_{\nu \rightarrow \infty} I_\nu(\nu z) \simeq \left(\frac{1}{2\pi\nu}\right)^{\frac{1}{2}} \frac{e^{\nu \xi(z)}}{(1+z^2)^{\frac{1}{4}}}$$

where

$$\xi(z) = (1+z^2)^{\frac{1}{2}} + \ln\left(\frac{z}{1+(1+z^2)^{\frac{1}{2}}}\right)$$

Flat space limit of bulk-to-boundary (Btb) propagators

- Bulk-to-boundary propagator:

$$\mathcal{K}_\Delta(pz) \stackrel{\ell \rightarrow \infty}{\equiv} \frac{1}{\sqrt{Z_m}} e^{-E\tau},$$

where $E = \sqrt{p^2 + m^2}$

- Massless case, $m^2 = 0$ (Δ does not scale with ℓ):

$$\sqrt{Z_0} \sim E^{\frac{1}{2}} p^{-\beta} \ell^{\frac{1-d}{2}} \quad (\ell \rightarrow \infty \rightarrow 0)$$

- Massive case, $m^2 \neq 0$ ($\Delta/\ell \rightarrow m$):

$$\sqrt{Z_m} \sim E^{\frac{1}{2}} p^{-\beta} \ell^{\frac{1-d}{2}} \frac{(m\ell)^{m\ell - \frac{1}{2}}}{((m+E)/2)^{m\ell}} e^{-m\ell} \quad (\ell \rightarrow \infty \rightarrow \infty)$$

Flat space linearized solutions

- Recall that the Btb propagator yields a linearized bulk equation of motion:

$$\Phi(z, p) = \mathcal{K}_\Delta(pz)\varphi_{(0)}(p) \xrightarrow{\ell \rightarrow \infty} \Phi(\tau, p) = \frac{\varphi_{(0)}(p)}{\sqrt{Z_m}} e^{-E\tau}$$

- The source $\varphi_{(0)}(p)$ on the AdS side is **arbitrary**, and for the limit to exist we need to scale the source such that:

$$\lim_{\ell \rightarrow \infty} \frac{\varphi_{(0)}(p)}{\sqrt{Z_m}} = a_{(0)} \text{ finite}$$

- This equivalent to **renormalizing the CFT operators**:

$$\mathcal{O}_R = \sqrt{Z_m} \mathcal{O}$$

and keep these operator fixed as $\ell \rightarrow \infty$.

- Upon Wick-rotation, $\tau = -it$ we find the flat space limit of the AdS linearised solution is indeed a **plane wave solution** with bulk on-shell momentum:

$$p^\mu = (E, \vec{p}) = (\sqrt{\vec{p}^2 + m^2}, \vec{p})$$

- Introduction
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- Flat space limit
- **Scattering amplitudes to all orders from AdS**

Scattering amplitudes

Scattering amplitudes:

- Depend on **on-shell** $(d+1)$ -momenta, p_i , $p_i p^i = m^2$ and **polarisation vectors**, ε_i .
- They are proportional to **momentum preserving δ -function**:

$$\mathcal{A}(p_i, \varepsilon_j) = \delta^{d+1}\left(\sum p_i\right) \hat{\mathcal{A}}(p_i, \varepsilon_j)$$

- The map from CFT d -momenta \vec{p} to **on-shell** $(d+1)$ -momenta p^i is the following:

$$p^i = (E, \vec{p}) \text{ with } E = \pm \sqrt{\vec{p}^2 + m^2}$$

\pm depends on whether particle is in-coming or out-going.

- The holographic relation is:

$$\mathcal{A}(p_i, \varepsilon_j) = \lim_{\ell \rightarrow \infty} \langle \mathcal{O}_1^R(\vec{p}_1) \cdots \mathcal{O}_n^R(\vec{p}_n) \rangle$$

2-point functions

2- and 3-point function can be analyzed **non-perturbatively** as the form of their form is uniquely fixed by Ward identities.

- The 2-point function is given by

$$\langle \mathcal{O}(p_1) \mathcal{O}(p_2) \rangle = c_\Delta p_1^{2\beta} \delta(\vec{p}_1 - \vec{p}_2)$$

where c_Δ is an (arbitrary) constant.

- Flat space limit:

$$\begin{aligned} \lim_{\ell \rightarrow \infty} \langle \mathcal{O}_R(p_1) \mathcal{O}_R(p_2) \rangle &= \lim_{\ell \rightarrow \infty} Z_m p_1^{2\beta} c_\Delta \delta(\vec{p}_1 - \vec{p}_2) \\ \Rightarrow \langle p_1 | p_2 \rangle &= \tilde{c} E \delta(\vec{p}_1 - \vec{p}_2) \end{aligned}$$

where \tilde{c} may (conventionally) chosen to be $2(2\pi)^d$.

- ➡ This is the **correct Lorentz invariant bulk inner product**.

3-point function

- The 3-point function is given by

$$\langle\langle \mathcal{O}_1(p_1) \mathcal{O}_2(p_2) \mathcal{O}_3(p_3) \rangle\rangle = g \int_0^\infty \frac{dz}{z^{d+1}} \mathcal{K}_{\Delta_1}(z, p_1) \mathcal{K}_{\Delta_2}(z, p_2) \mathcal{K}_{\Delta_3}(z, p_1)$$

This is the general solution of momentum-space conformal Ward identities [Bzowski, McFadden, KS (2013)].

- Changing variable, $z = \ell e^{\tau/\ell}$ and renormalizing the operators:

$$\lim_{\ell \rightarrow \infty} \langle\langle \mathcal{O}_1^R(p_1) \mathcal{O}_2^R(p_2) \mathcal{O}_3^R(p_3) \rangle\rangle = \int_{-\infty}^{\infty} d\tau e^{-(E_1 + E_2 + E_3)\tau}$$

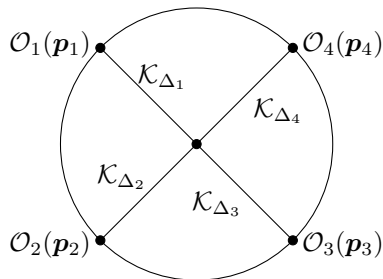
- Upon **Wick-rotation**, $\tau = -it$,

$$\lim_{\ell \rightarrow \infty} \langle\langle \mathcal{O}_1^R(p_1) \mathcal{O}_2^R(p_2) \mathcal{O}_3^R(p_3) \rangle\rangle = -2\pi i \delta(E_1 + E_2 + E_3)$$

- ➡ Thus, we recover **correctly the 3-point scattering amplitude**,

$$\mathcal{A}(p_1, p_2, p_3) = -i(2\pi)^{d+1} g \delta^{p+1}(p_1 + p_2 + p_3)$$

4-point contact diagram



$$\begin{aligned} & \langle\langle \mathcal{O}_1(\vec{p}_1) \mathcal{O}_2(\vec{p}_2) \mathcal{O}_3(\vec{p}_3) \mathcal{O}_4(\vec{p}_4) \rangle\rangle \\ &= g \int_0^\infty \frac{dz}{z^{d+1}} \prod_{i=1}^4 \mathcal{K}_{\Delta_i}(z, p_i) \end{aligned}$$

➤ The same steps as in the 3-point function yields:

$$\begin{aligned} \mathcal{A}(p_1, p_2, p_3, p_4) &= \lim_{\ell \rightarrow \infty} \langle \mathcal{O}_1^R(\vec{p}_1) \mathcal{O}_2^R(\vec{p}_2) \mathcal{O}_3^R(\vec{p}_3) \mathcal{O}_4^R(\vec{p}_4) \rangle \\ &= -i(2\pi)^{d+1} g \delta^{d+1}(p_1 + p_2 + p_3 + p_4) \end{aligned}$$

Flat space limit of bulk-to-bulk propagators

➤ Using $z = \ell e^{\tau/\ell}$, $z' = \ell e^{\tau'/\ell}$,

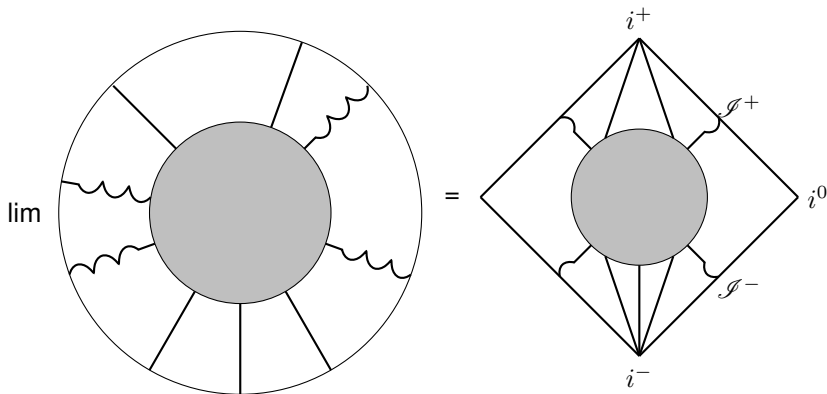
$$\begin{aligned}\lim_{\ell \rightarrow \infty} G_{\Delta}(p; z, z') &= \lim_{\ell \rightarrow \infty} \ell^{1-d} (zz')^{d/2} [K_{\beta}(pz) I_{\beta}(pz') \Theta(z - z') + (z \leftrightarrow z')] \\ &= \frac{1}{2E} \left(e^{E(\tau - \tau')} \theta(\tau - \tau') + e^{-E(\tau - \tau')} \theta(\tau' - \tau) \right)\end{aligned}$$

➤ Wick-rotating, $\tau = -it$, we find the **Feynman propagator**:

$$\lim_{\ell \rightarrow \infty} G_{\Delta}(p; z(\tau), z'(\tau')) = i \int \frac{dp^0}{2\pi} e^{-ip^0(\tau - \tau')} \frac{1}{(-(p^0)^2 + \vec{p}^2 + m^2 - i\epsilon)}$$

General case

- This proves the limit in **perturbation theory**:



- **AdS propagators** \rightarrow **flat space propagators** in the flat space limit
- **Interactions are kept fixed in the limit.**

Vector fields

- Taking the **flat-space limit** we find that the bulk linearized solution limits to

- **Massless** gauge field:

$$A_a(\tau, p) = \mathcal{A}_a e^{-E\tau}, \quad \mathcal{A}_a = (0, \pi_{\mu\nu} a^\mu), \quad a_\mu = \frac{1}{\sqrt{Z_A}} A_{(0)\mu}^\mu(p)$$

$A_{(0)\mu}^\mu(p)$ is the boundary gauge field (source) in AdS.

- One can show:

$$\mathcal{A}_a(p) = \sum_{\lambda=1}^{d-1} a^{(\lambda)}(p) \epsilon_a^{(\lambda)}, \quad \epsilon_a^{(\lambda)} = (0, \delta_i^\lambda, 0), \quad i = 1, \dots, d-1,$$

where $\epsilon_a^{(\lambda)}$ are **polarization vectors** and $a^{(\lambda)} = a^\lambda$.

- Upon quantization in the bulk $a^{(\lambda)}$ become the annihilation or creation operators of the mode with polarization vector $\epsilon_a^{(\lambda)}$.
- Similar comments apply to **massive vectors** and other spinning fields.

Flat-space limit from the CFT perspective

- We need to focus on the **deep infrared of the CFT**.
- To capture **massive fields**, we need the CFT to contain operators of dimension Δ_i that scale to infinity with a fixed slope, $\frac{\Delta_i}{\ell} = m_i$ **fixed**.
- **OPE coefficients are kept fixed**
- Operators should be renormalized, $\mathcal{O}_{\Delta_i} \rightarrow \sqrt{Z_i} \mathcal{O}_{\Delta_i}$

Conclusions/Outlook

- We provided a rigorous formulation of the flat-space limit of AdS.
- This leads to a new computation of scattering amplitudes in $(d + 1)$ -dimensions via a limit of CFT correlators in d -dimensions.
What can we learn from it?
- There is still an enormous amount to be done.
- Short scale:
 - More explicit examples ... including top-down ones.
 - What is the role of S^n in $AdS_m \times S^n$ top-down solutions?
 - Leading corrections to the flat space limit.
 - Connect with bottom-up approaches to flat-space holography.
 - ...
- Longer scale:
 - Obtain non-perturbative results for scattering amplitudes.
 - Connect with other approaches re: computation of scattering amplitudes , generalised unitarity etc.
 - ...