Cosmic Acceleration from Holographic Information Capacity

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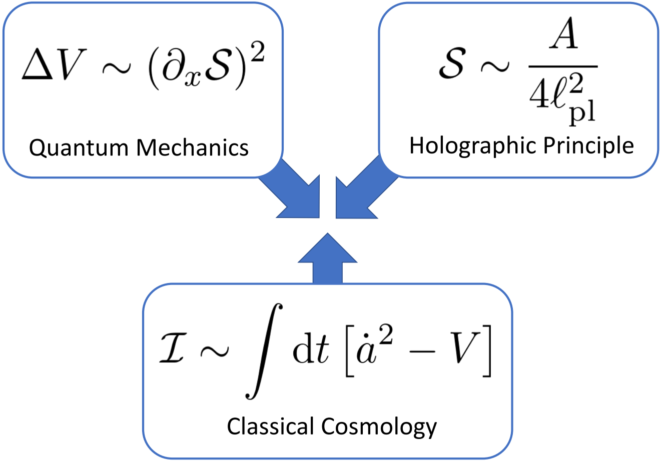
Overview


$\Delta V \sim (\partial_x \mathcal{S})^2$
Quantum Mechanics
$\mathcal{S} \sim \frac{A}{4\ell_{\mathrm{pl}}^2}$
Holographic Principle
$\mathcal{I} \sim \int \mathrm{d}t \, [\dot{a}^2 - V]$
Classical Cosmology

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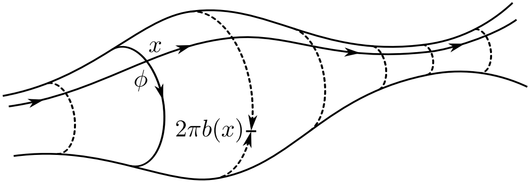
Classical Cosmology

Toy Example

Consider a (non-relativistic) particle on a curved tube:

$$ds^2 = dx^2 + [b(x)]^2 d\phi^2 \qquad\qquad (x, \phi) \in \mathbb{R} \times [0, 2\pi)$$



Want to predict $x(t)$, but don't care about $\phi(t)$.

Discarding Degrees of Freedom

Start from classical action:

$$\mathcal{I}[x,\phi] = \int dt \left[\frac{m}{2}\left(\dot{x}^2 + b^2\dot{\phi}^2\right) - V_0(x)\right]$$

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$$V_{\rm cl} = V_0 + \frac{p_\phi^2}{2mb^2}$$

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$$V_{\text{cl}} = V_0 + \frac{p_\phi^2}{2mb^2} \qquad V_{\text{qu}} = V_{\text{cl}} + \Delta V_{\text{eff}}$$

Quantum Correction

In general, the quantum correction depends on the information capacity $\mathcal{S}(x)$ of the discarded degrees of freedom:

$$\Delta V_{\text{eff}} = \frac{\hbar^2}{8m}\left[\left(1 - 4\xi\frac{d+1}{d}\right)(\partial_x\mathcal{S})^2 + 2(1-4\xi)\partial_x^2\mathcal{S}\right].$$

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$$m\partial_t^2\langle x\rangle = -\langle \partial_x V_{\text{cl}} + \partial_x \Delta V_{\text{eff}}\rangle$$

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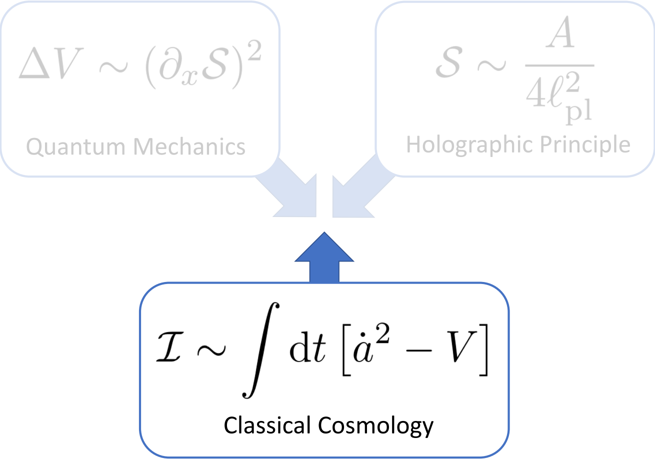
Account for this effect by including ΔV_{eff} in the action

$$\mathcal{J}[x(t)] \equiv \int dt\left[\frac{m}{2}\dot{x}^2 - V_{\text{cl}}(x) - \Delta V_{\text{eff}}(x)\right] = \mathcal{I} - \int dt\Delta V_{\text{eff}}$$

Note: we don't need detailed knowledge of discarded variables, just $\mathcal{S}(x)$.

Classical Cosmology



Classical Action for FRW Cosmology

Start from Einstein-Hilbert action:

$$\mathcal{I} = \frac{1}{2\kappa} \int_{\mathcal{M}} d^4x \sqrt{-g}\, R + \mathcal{I}_{\rm M}[g_{\mu\nu}, \psi],$$

insert FRW metric

$$ds^2 = [a(t)]^2 \left(-N(t)dt^2 + d\chi^2 + [r_k(\chi)]^2 d\Omega^2\right)$$

and discard all degrees of freedom, other than scale factor a:

$$\mathcal{I} = \frac{3\mathscr{V}_*}{\kappa} \int d\eta \left[-\left(\frac{da}{d\eta}\right)^2 + ka^2\right] + \mathcal{I}_{\rm M} \qquad \left(\eta \equiv \int dt' N(t')\right)$$

Generates Friedmann equations via variations $\delta a(t)$, $\delta N(t)$. Same form as non-relativistic particle, with $x \to a$, $t \to \eta$, and $m \to -6\mathscr{V}_*/\kappa$.

Semiclassical Action for FRW Cosmology

Accounting for the discarded degrees of freedom,

$$\mathcal{I} \to \mathcal{J} = \mathcal{I} - \int d\eta \Delta V_{\text{eff}},$$

where

$$\Delta V_{\text{eff}} = \frac{\hbar^2}{8m}\left[\left(1 - 4\xi\frac{d+1}{d}\right)(\partial_x \mathcal{S})^2 + 2(1-4\xi)\partial_x^2 \mathcal{S}\right]$$

becomes

$$\Delta V_{\text{eff}} = -\frac{4\pi^2 \ell_{\text{pl}}^4}{3\mathscr{V}_* \kappa}\left[\left(1 - 4\xi\frac{d+1}{d}\right)(\partial_a \mathcal{S})^2 + 2(1-4\xi)\partial_a^2 \mathcal{S}\right].$$

To calculate the cosmological information capacity $\mathcal{S}(a)$, we appeal to . . .

The Holographic Principle

$$\Delta V \sim (\partial_x \mathcal{S})^2$$

Quantum Mechanics

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Holographic Principle

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Classical Cosmology

Entropy and Area

Entropy of black hole is proportional to the area of its event horizon:

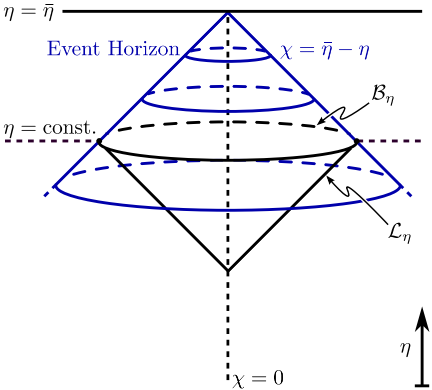
$$S_{\mathrm{BH}} = \frac{A_{\mathrm{EH}}}{4\ell_{\mathrm{pl}}^2}.$$

Roughly speaking, this is most entropy (or information) that will fit in a region of the same size.

Holographic interpretation: spacetime has microscopic (quantum gravity) degrees of freedom with information capacity

$$\mathcal{S}[\mathcal{L}] = \frac{A[\mathcal{B}]}{4\ell_{\mathrm{pl}}^2}.$$

Cosmological Information Capacity



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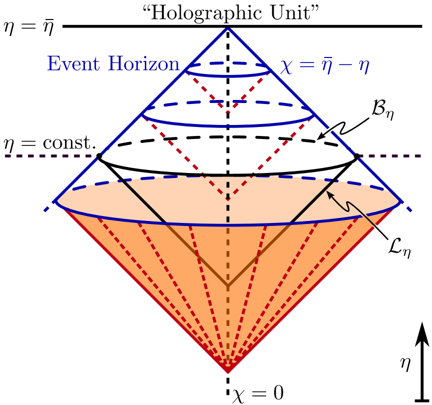

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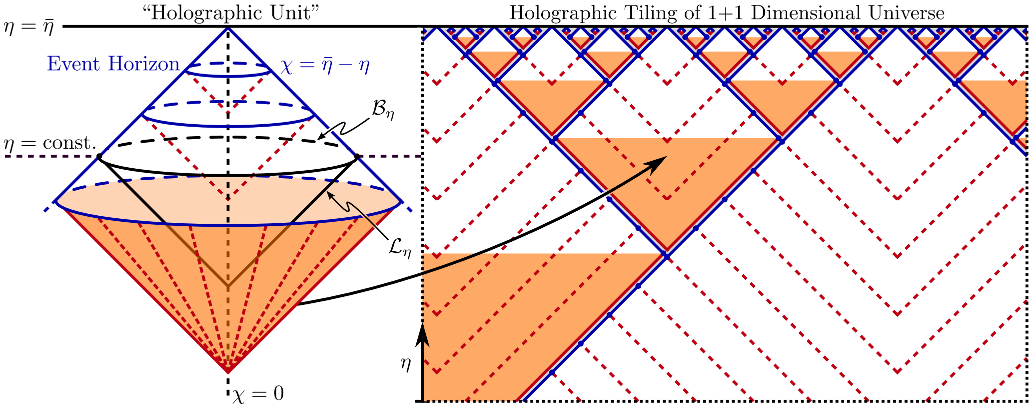

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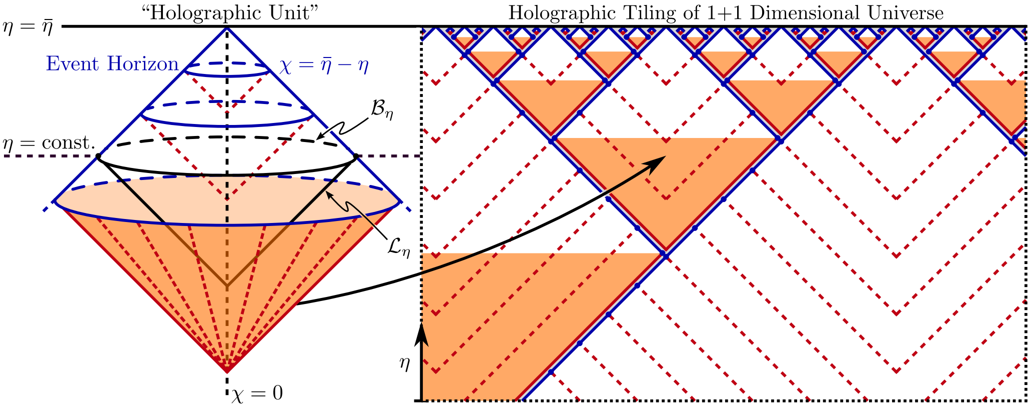

$$\mathcal{S}_{\mathrm{h}} = \frac{\mathscr{A}(\bar{\eta} - \eta)a^2}{4\ell_{\mathrm{pl}}^2} \cdot \frac{\bar{\mu}\mathscr{V}_*}{\mathscr{V}(\bar{\eta} - \eta)}$$

Semiclassical Cosmology

$$\Delta V \sim (\partial_x \mathcal{S})^2$$

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Holographic Principle

$$\mathcal{J}[a] \Rightarrow a(t)$$

Semiclassical Cosmology

$$\mathcal{I} \sim \int \mathrm{d}t \, [\dot{a}^2 - V]$$

Classical Cosmology

Semiclassical Friedman Equations

Assemble the semiclassical action

$$\mathcal{J}[a(\eta)] = \mathcal{I}[a(\eta)] - \int d\eta\, \Delta V_{\mathrm{eff}}[\mathcal{S}_{\mathrm{h}}(a,\eta)].$$

Take infinitesimal deviations $\delta a(t)$ and $\delta N(t)$ to obtain the semiclassical Friedmann equations:

Semiclassical Friedman Equations

Assemble the semiclassical action

$$\mathcal{J}[a(\eta)] = \mathcal{I}[a(\eta)] - \int d\eta\, \Delta V_{\text{eff}}[\mathcal{S}_{\text{h}}(a,\eta)].$$

Take infinitesimal deviations $\delta a(t)$ and $\delta N(t)$ to obtain the semiclassical Friedmann equations:

$$\left(\frac{da}{d\eta}\right)^2 = \frac{\kappa}{3}\rho a^4 - \left(1 + \frac{4\bar{g}}{15}\right) ka^2 + \frac{\bar{g}a^2}{(\bar{\eta}-\eta)^2} - 2\bar{g}\int_0^\eta d\eta' \frac{[a(\eta')]^2}{(\bar{\eta}-\eta')^3}$$

$$\frac{d^2 a}{d\eta^2} = \frac{\kappa}{6}(\rho - 3p)\,a^3 - \left(1 + \frac{4\bar{g}}{15}\right) ka + \frac{\bar{g}a}{(\bar{\eta}-\eta)^2}$$

where $\bar{g} \equiv \pi^2\bar{\mu}^2(1 - 4\xi\frac{d+1}{d}) \sim O(1)$ is an unknown dimensionless constant.

Cosmic Acceleration

New term in acceleration equation:

$$\frac{1}{a}\frac{d^2a}{d\tau^2} = -\frac{\kappa}{6}(\rho + 3p) + \frac{2\bar{g}}{a^4}\int_0^{\eta} d\eta' \frac{[a(\eta')]^2}{(\bar{\eta} - \eta')^3}.$$

Quantum correction generates positive acceleration, dependent on past behaviour of a.

Note: $\ell_{\rm pl}$ does not appear, $\hbar$s have cancelled, no reason to expect $\Lambda_{\rm eff} \sim 10^{120} \times \Lambda_{\rm obs}$.

Exact Solutions

To understand acceleration term, solve semiclassical Friedmann eqns.

For flat universe ($k = 0$) containing matter and radiation:

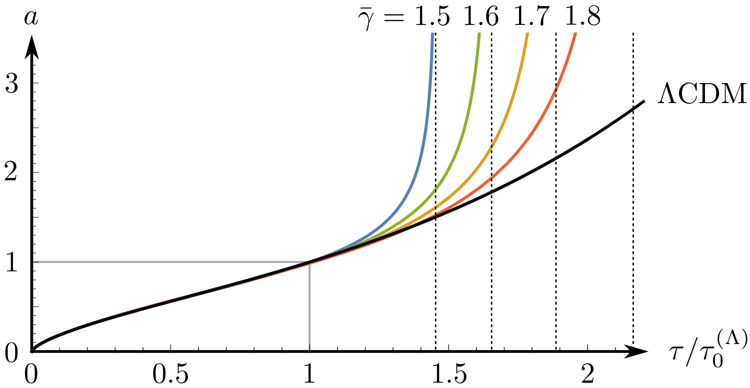
$$a = \frac{\beta_{\rm m}}{\bar{\gamma}}\left(\frac{2\bar{\gamma}u^2}{9-\bar{\gamma}^2} - \frac{u^{(1+\bar{\gamma})/2}}{3-\bar{\gamma}} + \frac{u^{(1-\bar{\gamma})/2}}{3+\bar{\gamma}}\right) - \frac{\sqrt{\beta_{\rm r}}}{\bar{\gamma}}\left(u^{(1+\bar{\gamma})/2} - u^{(1-\bar{\gamma})/2}\right)$$

$$\tau = \frac{2\bar{\eta}\beta_{\rm m}}{\bar{\gamma}\left(9-\bar{\gamma}^2\right)}\left(\frac{\bar{\gamma}}{3}\left(1-u^3\right) + u^{(3+\bar{\gamma})/2} - u^{(3-\bar{\gamma})/2}\right) + \frac{2\bar{\eta}\sqrt{\beta_{\rm r}}}{\bar{\gamma}}\left(\frac{u^{(3+\bar{\gamma})/2}}{3+\bar{\gamma}} - \frac{u^{(3-\bar{\gamma})/2}}{3-\bar{\gamma}} + \frac{2\bar{\gamma}}{9-\bar{\gamma}^2}\right)$$

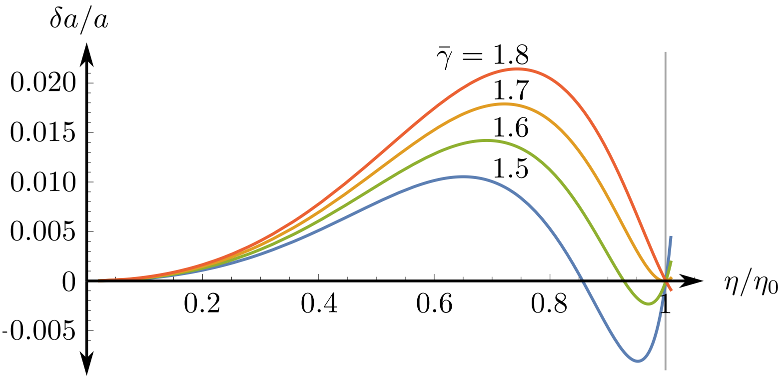
where

$$u \equiv \frac{\bar{\eta}-\eta}{\bar{\eta}}, \qquad \beta_{\rm m} \equiv \frac{\kappa\bar{\eta}^2\rho^\circ_{\rm m}a^3_\circ}{3}, \qquad \beta_{\rm r} \equiv \frac{\kappa\bar{\eta}^2\rho^\circ_{\rm r}a^4_\circ}{3}, \qquad \bar{\gamma} \equiv \sqrt{4\bar{g}+1}.$$

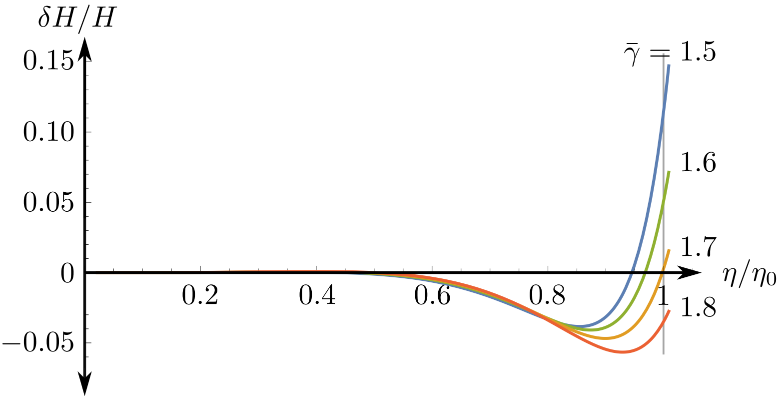
Comparison with ΛCDM



Comparison with ΛCDM: Scale Factor



Comparison with ΛCDM: Expansion Rate



Summary

- **No Dark Energy or Modified Gravity**: Obtain cosmic acceleration "for free", by treating Universe as quantum system.
- **Holographic**: Testable application of idea from quantum gravity.
- **Exact Solutions**: Predicts $a(\tau)$ from first principles.
- **Falsifiable**: One new free parameter $\bar{\gamma}$, cannot mimic Λ exactly.
- **Passes Current Tests**: $\delta a/a|_{z\sim 1} \sim 1\%$ vs ΛCDM.
- **Phantom**: Provides $w < -1$ acceleration without issue.
- **Future Work**: Compare with actual data! Explain H_0 tension?
- **Future Work**: Is inflation responsible for $\Lambda_{\rm obs} \sim 10^{-120}/\ell_{\rm pl}^2$?