

The Fine Structure Constant

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Quotations

- What is the **fine-structure constant** α ?

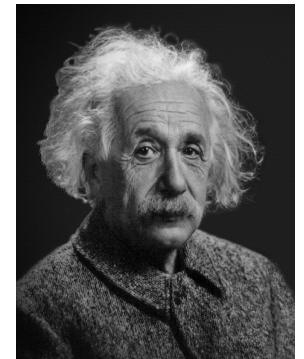
“Where does α come from; is it related to π or perhaps to e ? Nobody knows, it is one of the great damn mysteries of physics, a magic number that comes to us with no understanding by man. You might say the hand of God wrote the number and we don't know how He pushed his pencil”.



Feynman

- What is an **electron**?

“ I would just like to understand the electron.”



Einstein

Two questions related because both are about **electro-magnetism**

Constants of Mathematics

Numerics	Mathematicians	Century	Number Field
$\pi = 3.14159265358\dots$	Archimedes	3 rd BC	Real
$e = 2.71828182845\dots$	Bernoulli, Euler	17 th , 18 th	Complex
$1/\alpha = 137.035999139\dots$	Hamilton, Eddington	19 th , 20 th	Quaternion



Archimedes



Jacob
Bernoulli



Euler



Hamilton

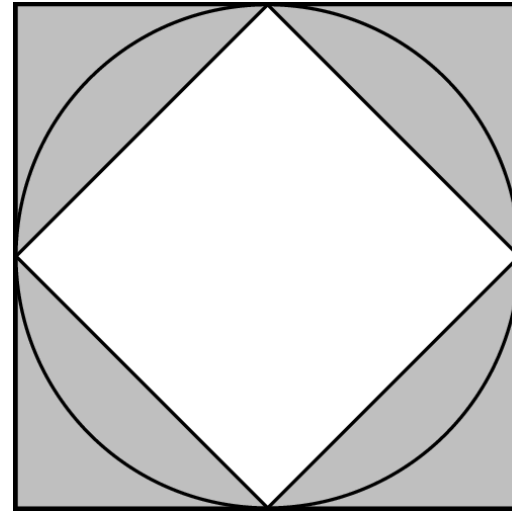


Eddington

Geometry

Regular N-gons inscribed and circumscribed to the circle

Example for N=4



Archimedes and Euler using Real and Complex numbers, let $N \rightarrow \infty$

Led to Euler's beautiful formula $e^{i\pi} = -1$

Questions

1. What is quaternionic version of Euler's formula ?
2. How does the non-commutativity of quaternions enter?
3. What replaces i ?

Answers

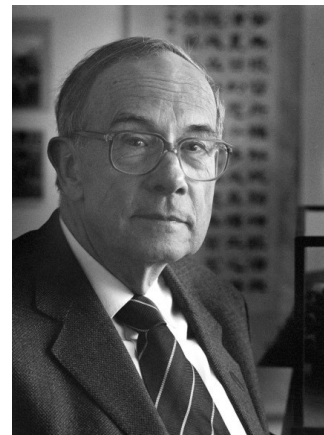
1. $e^{\frac{1}{\alpha}w} = -1$

2. Renormalize π to $1/\alpha$ by infinite iteration of exponentials, through matrix algebras and tensor products.

3. i is replaced by $w = \alpha \pi i$ in the centre of the matrix algebra.



von Neumann



Hirzebruch

Philosophical Consequences

1. Constants of Nature

No-one asks what the universe would be like if π were not equal to the numerical value 3.1415925535...

Similarly no-one should ask what the universe would be like if $1/\alpha$ were not equal to the numerical value 137.035999139...

2. Anthropic Principle

It is quite unnecessary (Occam's razor). The universe is as it is, not by accident, but because it rests on mathematical foundations and mathematics is not for sale.

Geometric Model of the Electron

(for Einstein)

Electron is modelled by a Mobius band = $\mathbb{R}P^2$ with one point removed = $\mathbb{Z}/2$ quotient of S^2 with both North & South poles removed. Spin $1/2$ about NS axis. Symmetry $Sp(1)$, broken to $U(1)$.

Mass determined by scale of S^1 got by real blow up of point in \mathbb{R}^2 . Mobius band cannot embed in \mathbb{R}^2 , but can embed as “anti-diagonal” in \mathbb{R}^4

$$\mathbb{R}^4 = \mathbb{R}^2 \times (\mathbb{R}^2)^* = \mathbb{C} \times \bar{\mathbb{C}} = \mathbb{H}$$

Model has topology (spin), symmetry and mass parameter.

Mathematical Toolbox

Mathematical Constant	π	i	$1/\alpha$	e
Field	Real	Complex	Quaternion	
Unit	+/-1	S^1	S^3	
Property	ordered	commutative	associative	
Operation	+	x	exp	
Number 2	$2 + 2 = 4$	$2 \times 2 = 4$	$2^2 = 4$	
Iteration	\sum_N	\prod_N	E_N	

References

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- [4] F.Hirzebruch, *New Topological Methods in Algebraic Geometry*, Springer (1966)
- [5] F.J.Murray and J.von Neumann, *on Rings of Operators*, Ann.of Math. (2) 37, 116-209, (1936)