The hierarchy problem: Exotic signatures from exotic approaches.

New Directions in Theoretical Physics

Edinburgh
Jan 9th 2019

Based on: Giudice, MM, 2016
Giudice, Kats, MM, Torre, Urbano 2017
Cohen, Craig, Giudice, MM 2018

Prologue:

The Hierarchy Problem...

Higgs Mechanism

- The Higgs sector of the Standard Model involves the Higgs field and the gauge fields

$$H \qquad\qquad W_\mu^a$$

- The Lagrangian for this theory is

$$\mathcal{L} = \left|(\partial_\mu + ig\sigma^a W_\mu^a)H\right|^2$$

$$+m^2(T)|H|^2 - \lambda(T)|H|^4 + \ldots$$

Higgs Mechanism

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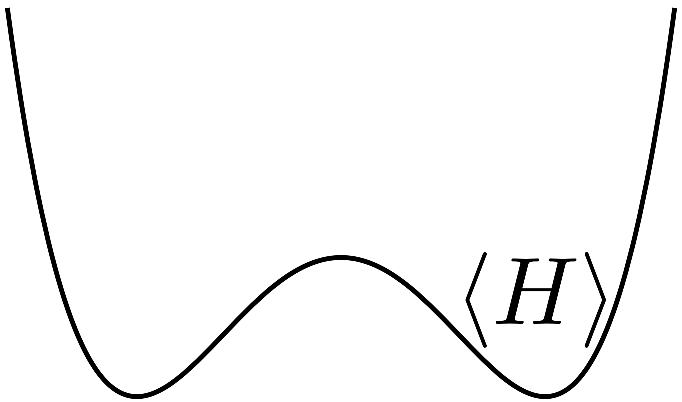
$$+m^2(T)|H|^2 - \lambda(T)|H|^4 + \ldots$$

- Below the critical temperature the mass-squared is negative:



- Gauge bosons become massive: $M_W \sim g\langle H \rangle$

Ginzburg-Landau

- The G-L Theory of superconductivity involves a complex scalar field and the photon (magnetic vector potential)

$$\Phi \qquad\qquad A$$

- The Free energy for this theory is

$$F = |(\nabla + 2ieA)\Phi|^2$$

$$+m^2(T)|\Phi|^2 + \lambda|\Phi|^4 + \ldots$$

- Where the mass depends on the temperature.

Ginzburg-Landau

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$$+m^2(T)|\Phi|^2 + \lambda|\Phi|^4 + \ldots$$

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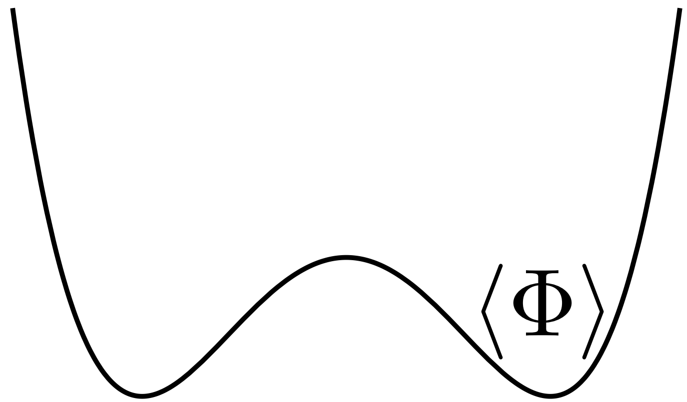

- Photon has become massive: $m_A \sim e\langle\Phi\rangle$

The Elephant in the Room

Ginzburg-Landau is just a phenomenological model, with no explanation of parameters. The macroscopic parameters follow from the detailed microscopic BCS theory (Gor'kov) and there are no surprises.

The order parameter at zero temperature is of the typical scale associated with underlying microscopic parameters.

The Elephant in the Room

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Phenomenological model parameters, such as the order parameter $\langle ee \rangle$ predicted by microscopic theory.

The order parameter at zero temperature is of the typical scale associated with underlying microscopic parameters.

The Elephant in the Room

Performing the same exercise with the Higgs field.

We can look to see if the symmetry breaking is like Ginzburg-Landau

- Direct analogy, would have no light Higgs boson: Experimentally excluded.
- Perhaps not directly analogous, but similar composite story: Study the Higgs...

The Elephant in the Room

We expect the Higgs model is phenomenological, just like G-L. But something totally different seems to be going on.

There is a hierarchy between the phenomenological model parameters and the microscopic parameters (Planck, GUT, RHN, PQ,...).

Furthermore, this hierarchy is not protected by any symmetry: Quantum corrections do not respect such a hierarchy.

The Elephant in the Room

We expect the Higgs model is phenomenological, just like G-L. But something totally different seems to b

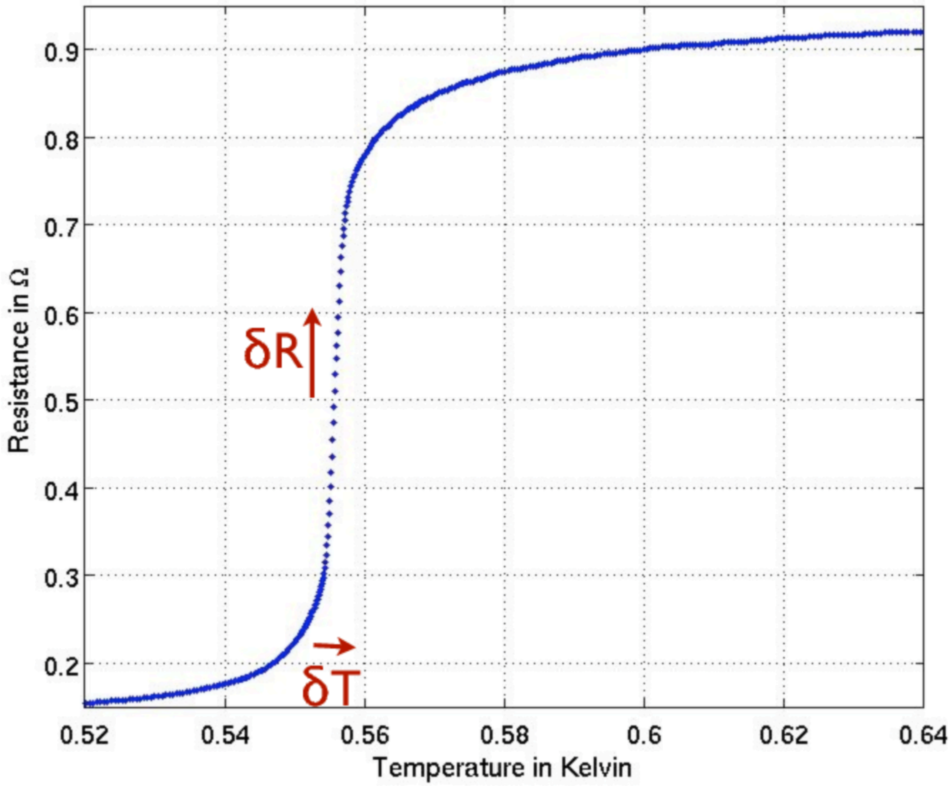
If the phenomenological model parameters, such as $\langle H \rangle$, predicted by microscopic theory, then why the hierarchy?

There is a hierarchy betw... cal model parameters and the microscop... ers (Planck, GUT, RHN, PQ,...).

Furthermore, this hierarchy is not protected by any symmetry: Quantum corrections do not respect such a hierarchy.

Fine-Tuning?

Essentially, it seems like the Universe is just like a Transition Edge Sensor:



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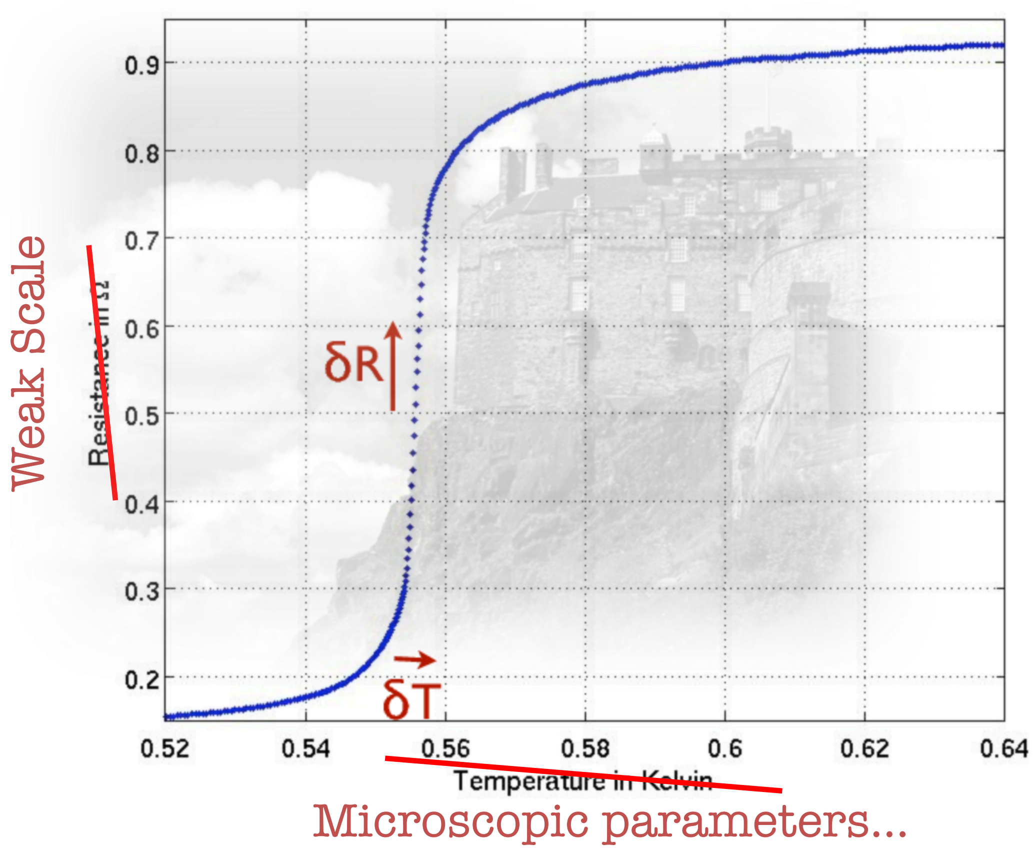

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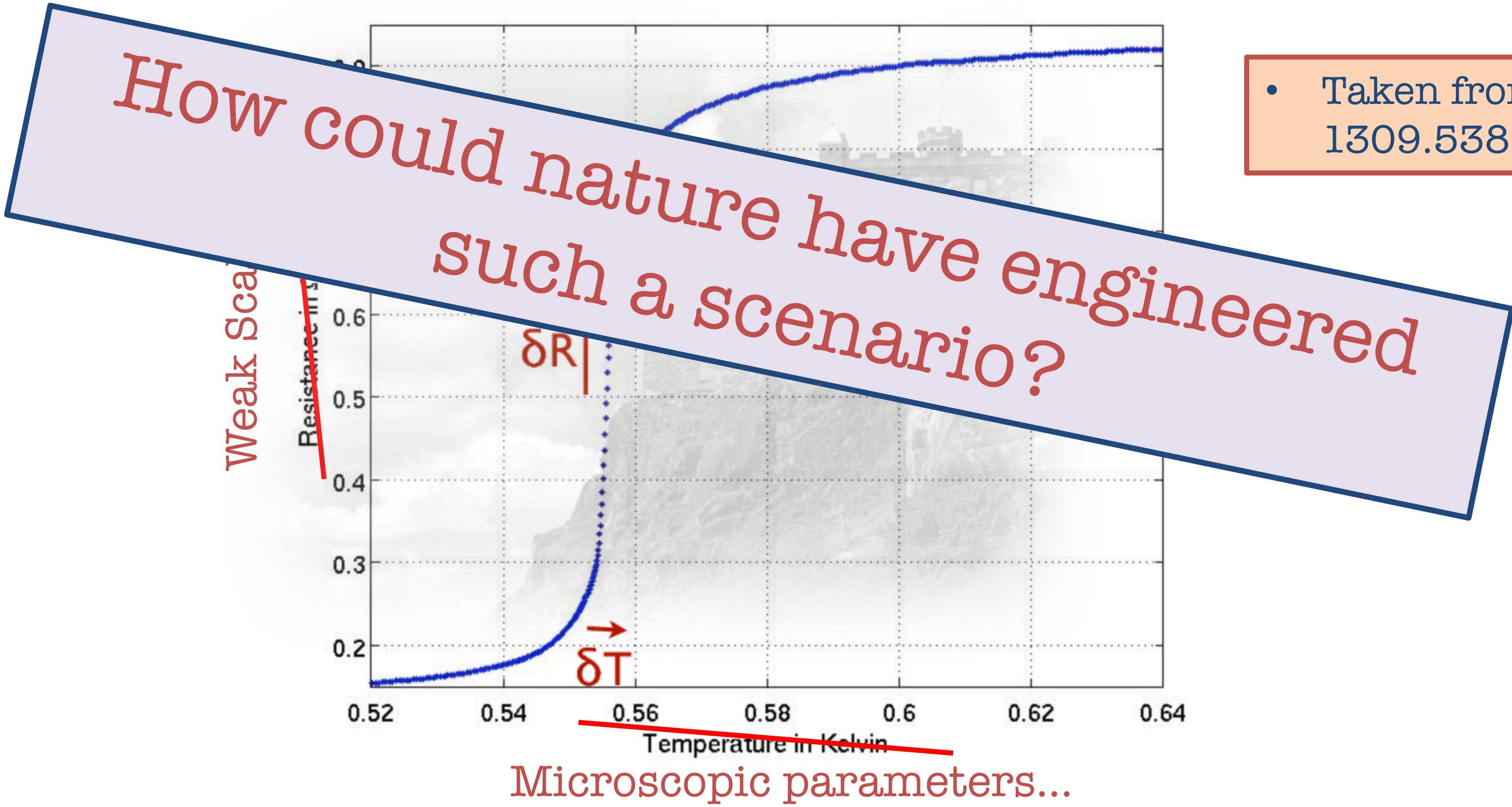

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Hierarchy Problem

Many* approaches follow three basic paradigms...

Symmetry.

Locality.

Dynamics.

This talk will cover/review two recent variations on these themes.

Hierarchy Problem

Many* approaches follow three basic paradigms...

Symmetry.

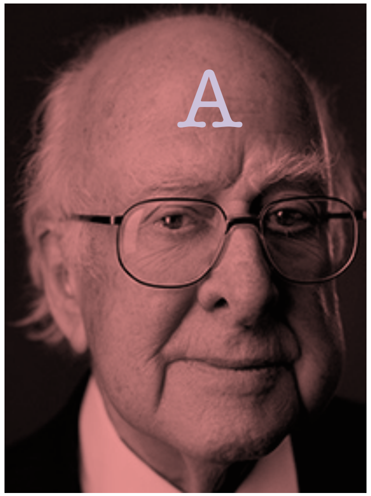
Twin Higgs

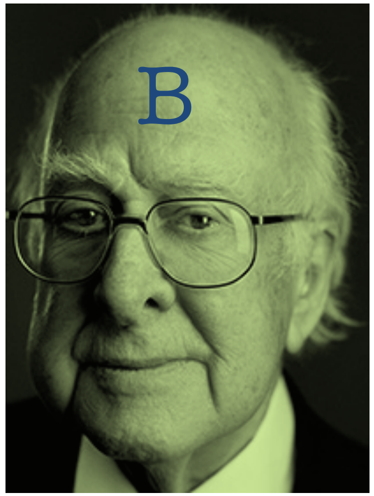
Chacko, Goh, Harnik 2005

- Take two identical copies of the Standard Model:

$$\mathrm{SM}_A \quad \xleftrightarrow{A \leftrightarrow B} \quad \mathrm{SM}_B$$

- Everything twinned.





Twin Higgs

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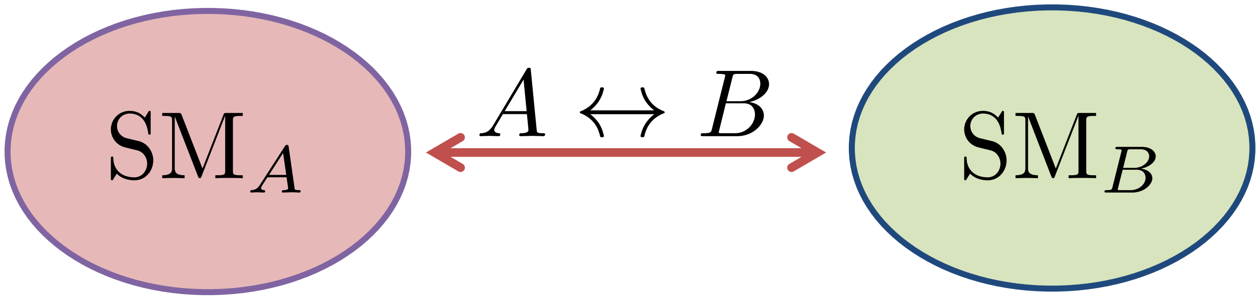


- Enhance symmetry structure to global SU(4):

Desired quartic dictated by accidental symmetry:

$$V_{\text{Higgs}} = \lambda \left(|H_A|^2 + |H_B|^2\right)^2 - \Lambda^2 \left(|H_A|^2 + |H_B|^2\right)$$

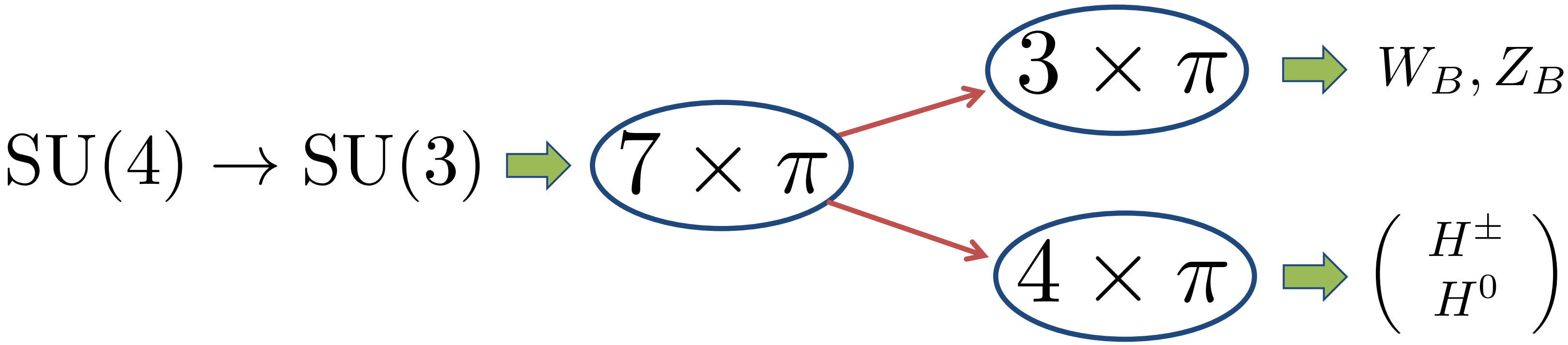
Exchange enforces equal quadratic corrections for each Higgs. Thus masses still respect SU(4) symmetry.

Twin Higgs

Chacko, Goh, Harnik 2005

- Total symmetry-breaking pattern is: $\mathrm{SU}(4) \rightarrow \mathrm{SU}(3)$

- Thus 7 pseudo-Goldstone bosons:



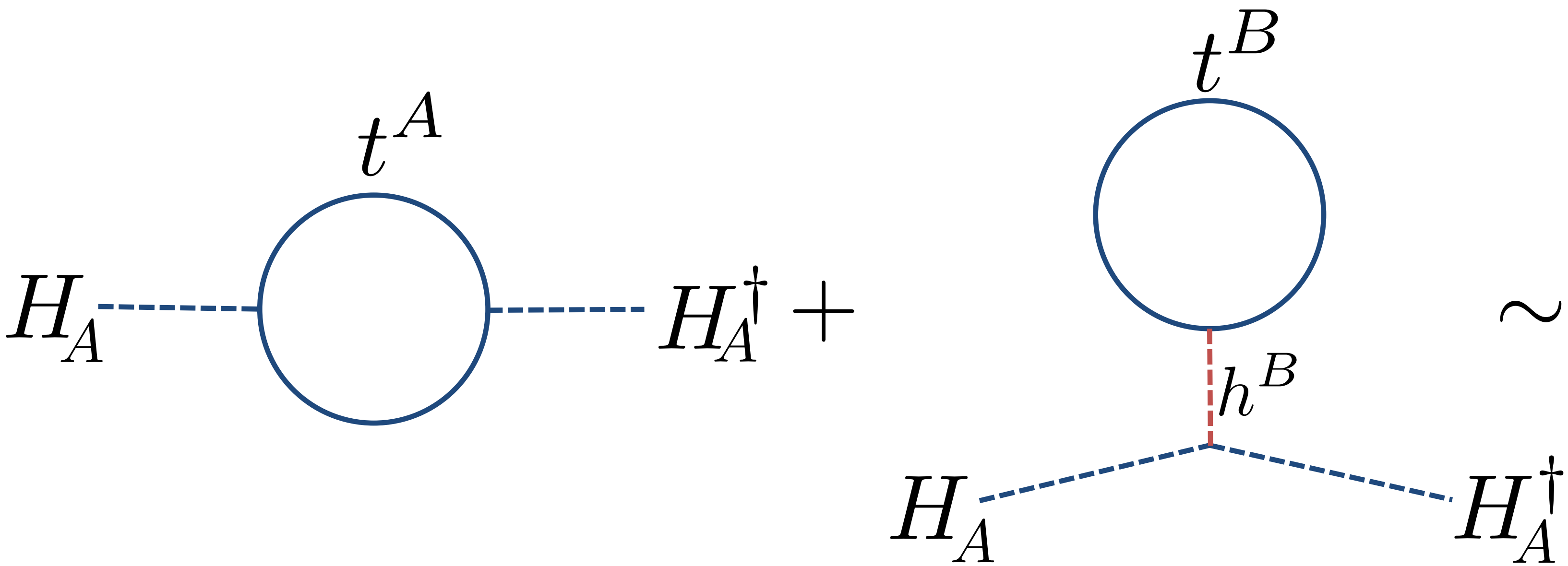
- The SM Higgs light because of the symmetry-breaking pattern!

- Hierarchy problem solved all the way up to the scale: Λ

Twin Higgs

Chacko, Goh, Harnik 2005

- In usual “quadratic divergences” parlay:



$$\sim 0 \times \Lambda^2$$

Quadratic divergences from SM top quark loops cancelled by loops of “Twin” top quarks.

- Cancellation persists for all Twin particles: Twin W-bosons, Twin gluons, etc.

Twin Higgs

Chacko, Goh, Harnik 2005

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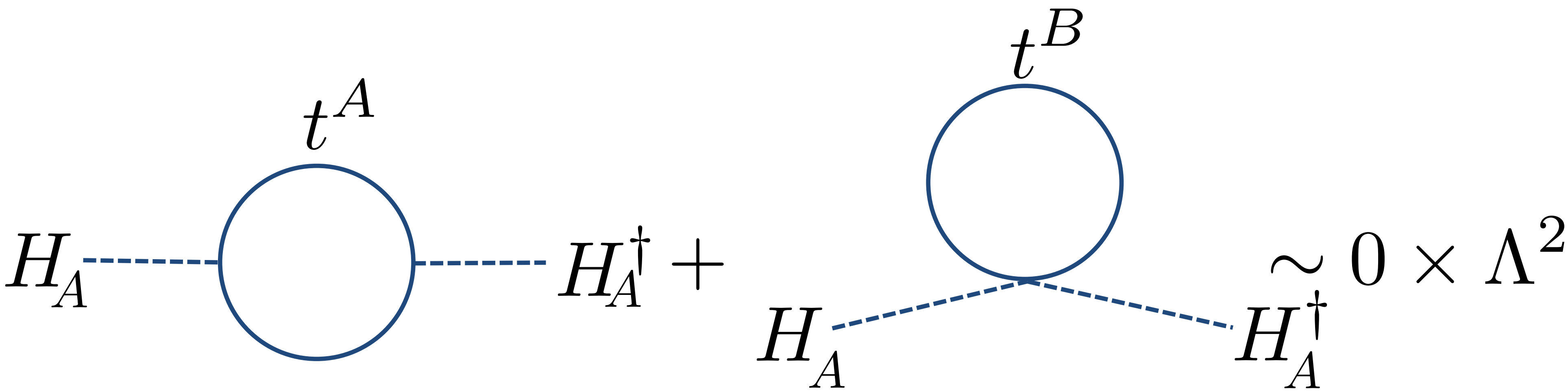

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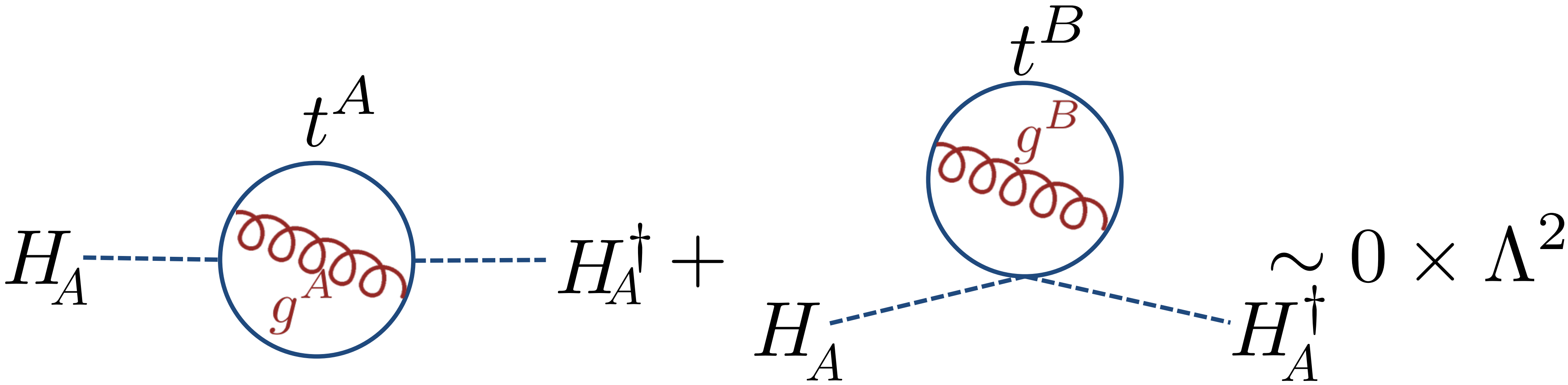

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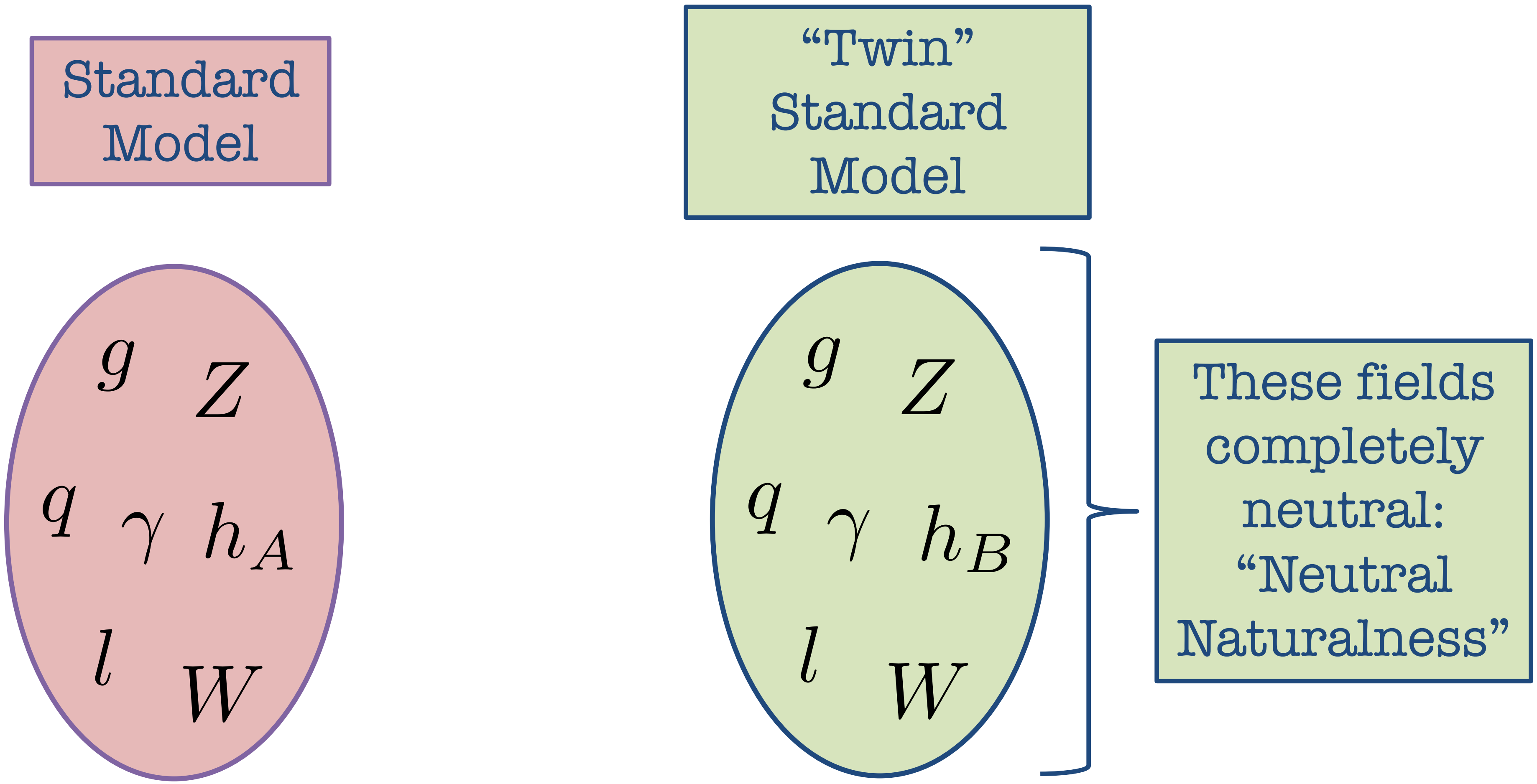
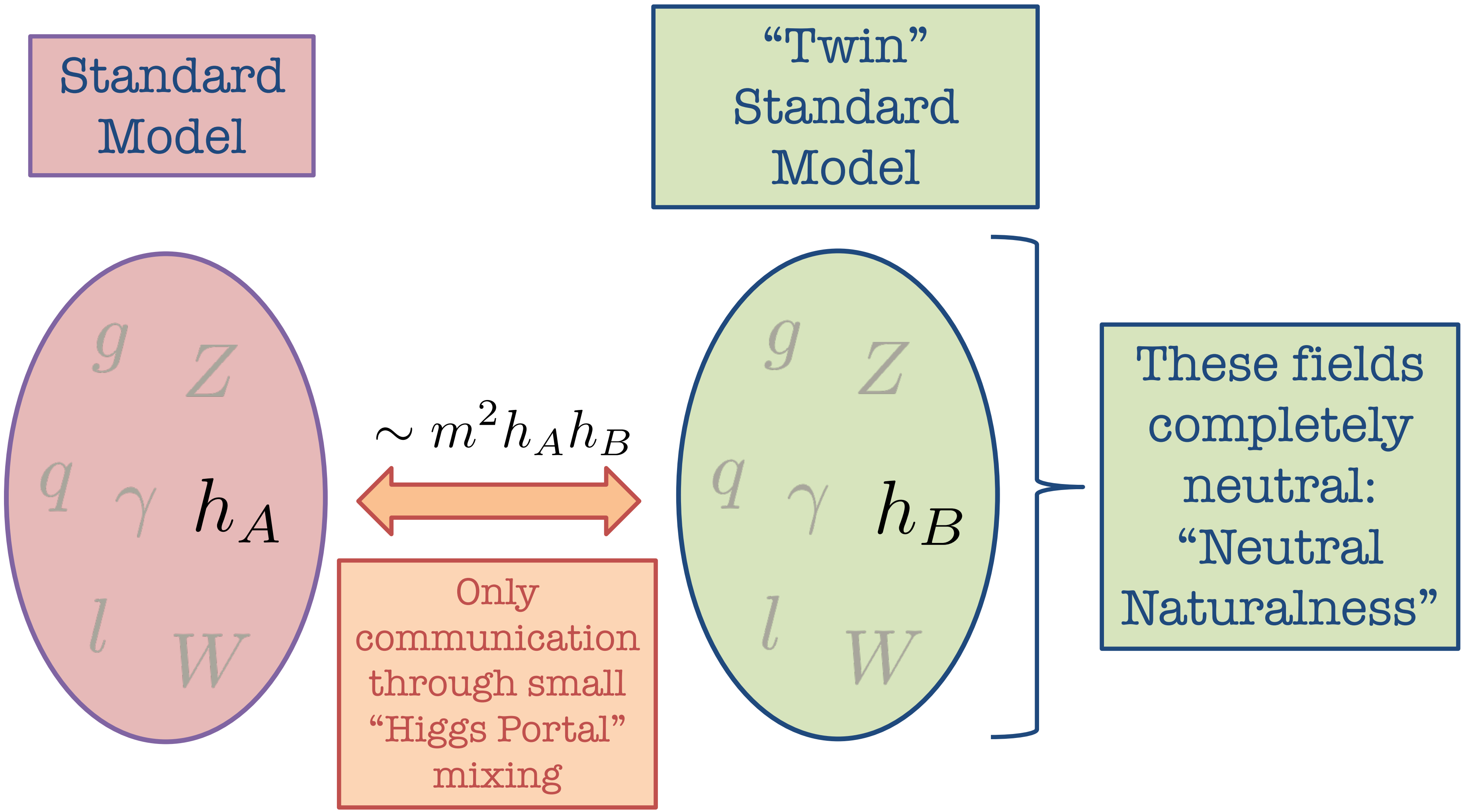


Predictions for Twin sector most robust for the Twins of the SM fields that couple most strongly to Higgs.

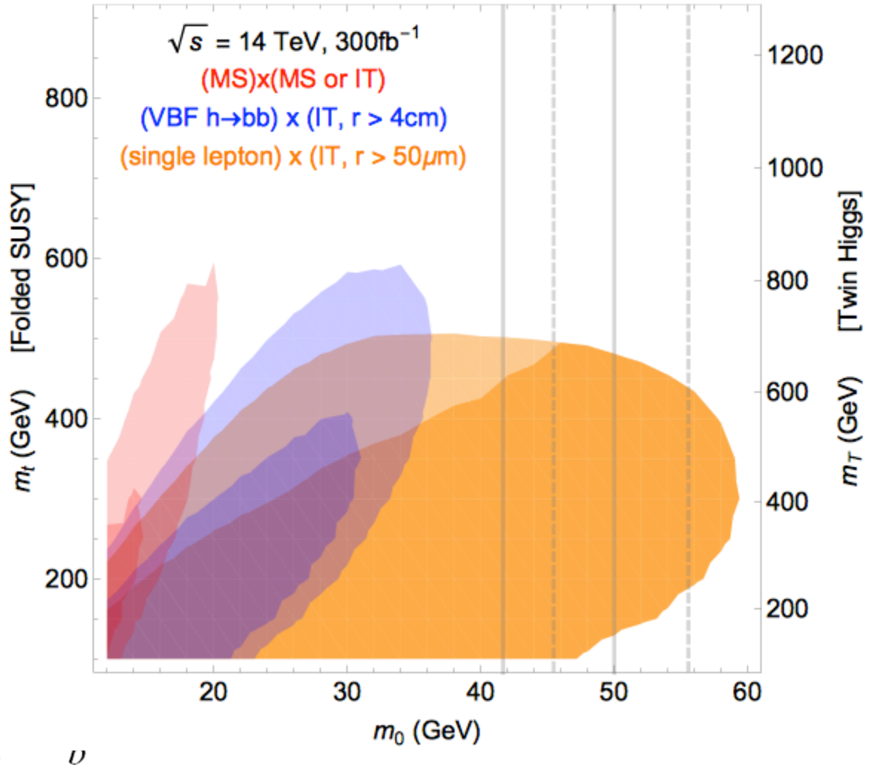


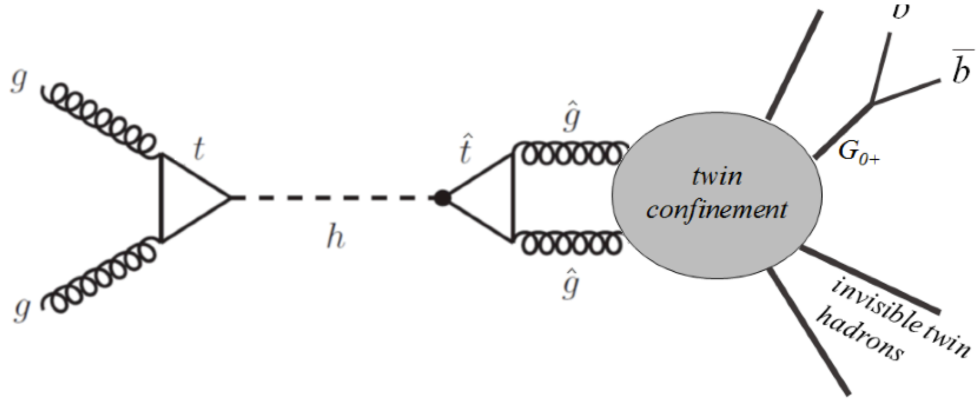
Standard Model
"Twin" Standard Model
g Z q γ h_A l W
g Z q γ h_B l W
$\sim m^2 h_A h_B$
Only communication through small "Higgs Portal" mixing
These fields completely neutral: "Neutral Naturalness"

Phenomenology

SM Higgs can decay, through the Higgs portal, to Twin gluons.

These decay back through Higgs portal.





LHC has sensitivity in future.

Hyperbolic Higgs

Craig, Cohen, Giudice, MM.

- The landscape of top partners in symmetry approaches:

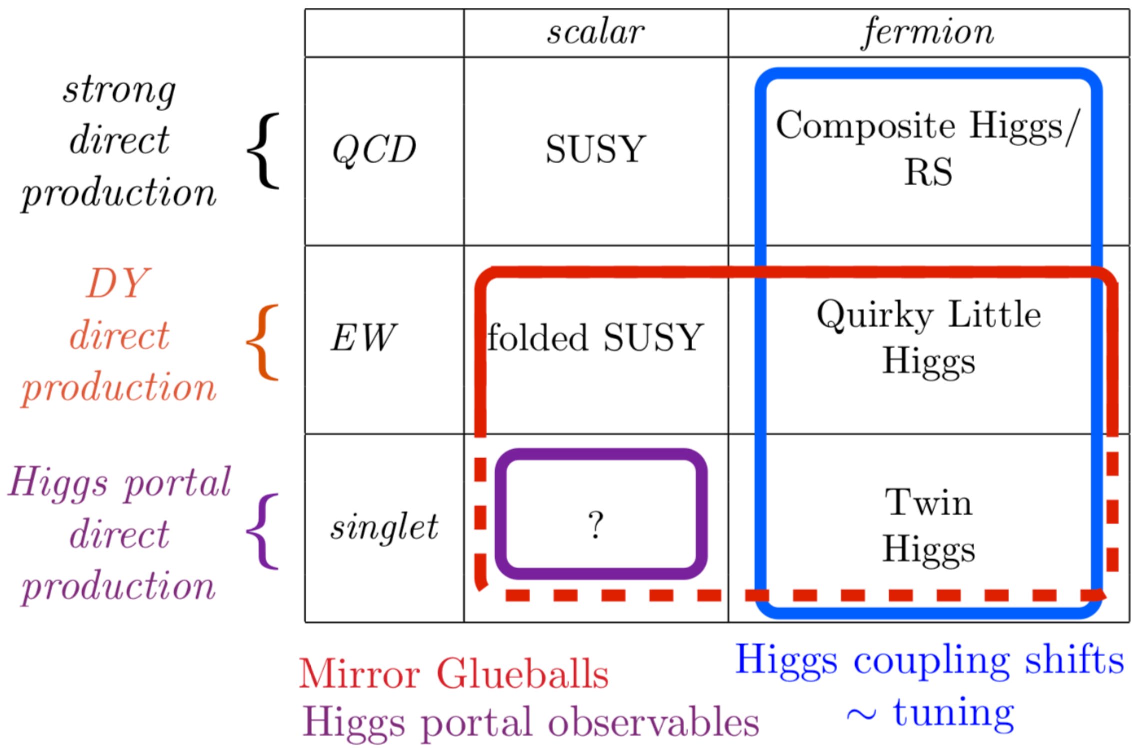


Table from Curtin and Verhaaren.

- This section: The last box.

Hyperbolic Higgs

Craig, Cohen, Giudice, MM.

- Take two identical copies of the MSSM:

- Take a large D-term with equal and opposite charges for Higgses:

$$V_{\mathcal{H}} = \frac{g_{\mathcal{H}}^2}{2} \left(|H|^2 - |H_{\mathcal{H}}|^2 \right)^2$$

This enforces that the scalar potential respects an accidental SU(2,2) symmetry. Not symmetry of theory.

Hyperbolic Higgs

Craig, Cohen, Giudice, MM.

- Remove scalar matter in A, and fermions in B:

$$\mathcal{L} = \lambda_t \, H \psi_Q \, \psi_{U^c} + \text{h.c.}$$
$$+ \lambda_t^2 \left(\left| H_{\mathcal{H}} \cdot \widetilde{Q}_{\mathcal{H}} \right|^2 + \left| H_{\mathcal{H}} \right|^2 \left| \widetilde{U}^c_{\mathcal{H}} \right|^2 \right)$$

- Quadratic corrections respect the accidental SU(2,2) symmetry:

$$V_{\mathcal{H}} = -\Lambda^2 \left(|H|^2 - |H_{\mathcal{H}}|^2 \right) + \frac{g_{\mathcal{H}}^2}{2} \left(|H|^2 - |H_{\mathcal{H}}|^2 \right)^2$$

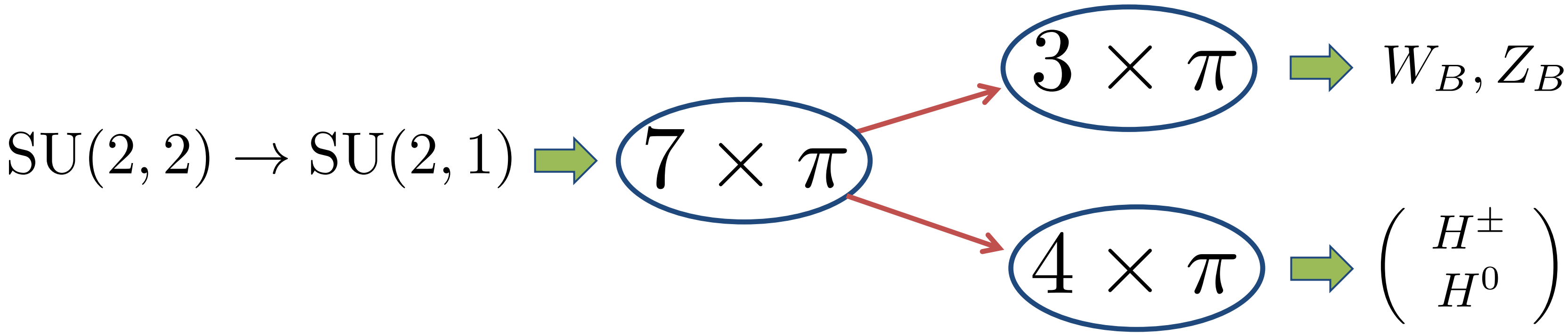
Thus, at level of one-loop corrections, scalar potential respects an accidental SU(2,2) symmetry.

Hyperbolic Higgs

- Total symmetry-breaking pattern is:

$$\mathrm{SU}(2,2) \to \mathrm{SU}(2,1)$$

- Thus 7 Quasi-Goldstone bosons:

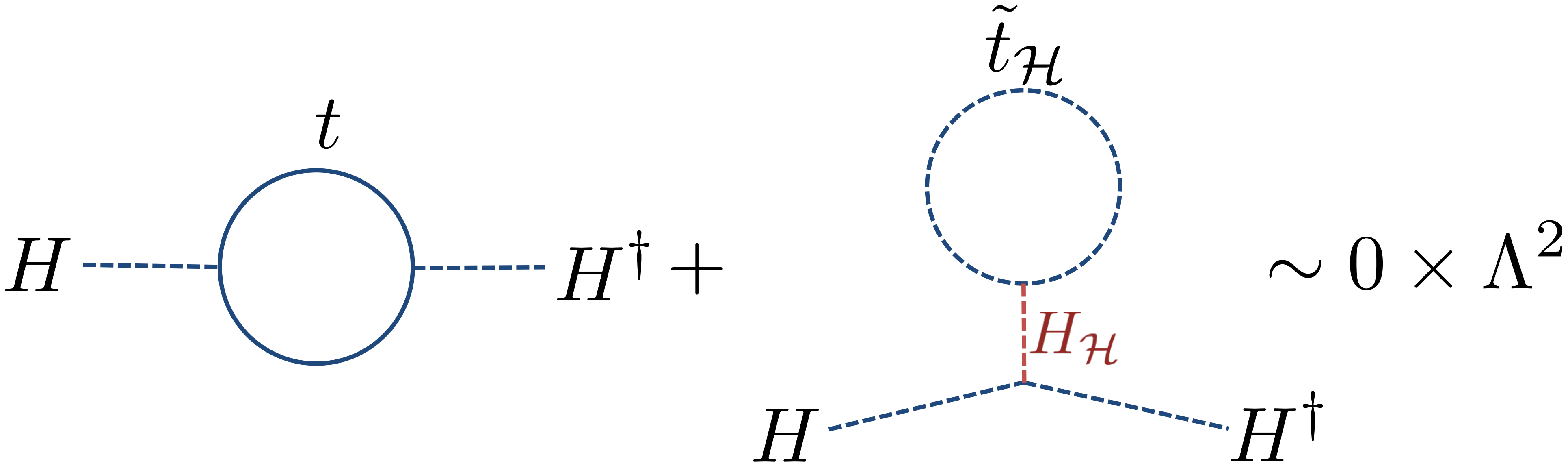


- The SM Higgs light because of the symmetry-breaking pattern!

- Higgs not really a Goldstone. More like an accidental flat direction…

Hyperbolic Higgs

- In usual "quadratic divergences" parlay:



Quadratic divergences from SM top quark loops cancelled by loops of "Hyperbolic" stop squarks.

Hyperbolic Higgs

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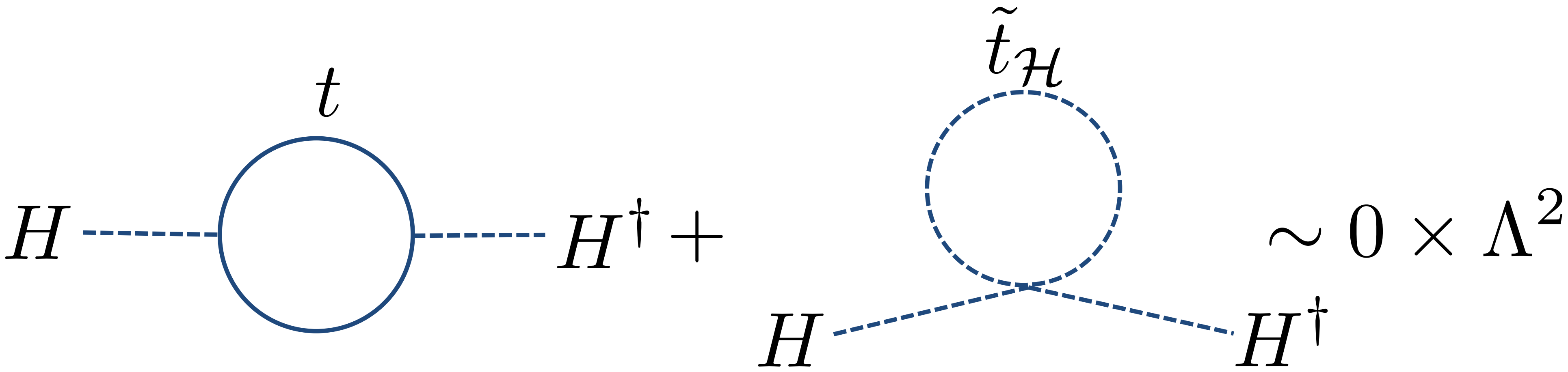

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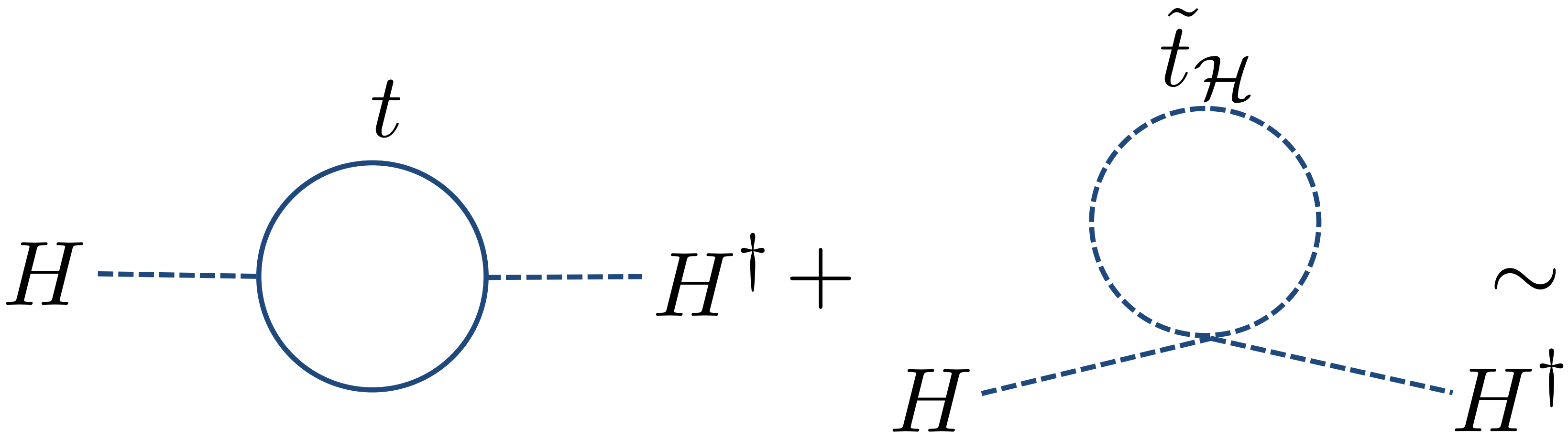


$$\sim 0 \times \Lambda^2$$

Quadratic divergences from SM top quark loops cancelled by loops of "Hyperbolic" stop squarks.

$$\mathcal{L} \sim \lambda_t \, H \, \psi_Q \, \psi_{U^c} + \text{h.c.} + \lambda_t^2 \, |H|^2 \left(\left|\tilde{t}^L_{\mathcal{H}}\right|^2 + \left|\tilde{t}^R_{\mathcal{H}}\right|^2 \right)$$

Phenomenology

One aspect could be radically different to Twin. If…

$$\langle \tilde{t}_{\mathcal{H}} \rangle \neq 0$$

Then:

- Hyperbolic QCD is broken, so no glueball signatures, no hidden sector hadronisation.
- Longitudinal modes of Hyperbolic Gluons are Top Partners!
- Radial modes of Hyperbolic Stops mix with Higgs, so Higgs becomes, partially, its own top partner!

Hierarchy Problem

Many* approaches follow three basic paradigms...

Locality.

On Masses and Scales

Masses and interaction scales are not physically equivalent. Seen by reinserting ħ into action.

$$\mathcal{L}_{\hbar \neq 1}$$

In terms of dimensionful quantities

Masses Couplings

Planck Scale

Interaction: $\mathcal{L} \sim \dfrac{h_{\mu\nu} T^{\mu\nu}}{M_P}$

Dimension: $[M_P] = \dfrac{[M_S]}{[\lambda_S]}$

UV-completion

Coupling

On Masses and Scales

Masses and interaction scales are not physically equivalent. Seen by reinserting ħ into action.

$$\mathcal{L}_{\hbar \neq 1}$$

…tities

Perhaps the huge hierarchy between weak scale and Planck scale is just due to a tiny coupling, and microscopic Planck scale physics is just around the corner...

Planck Scale

Interaction: $\mathcal{L} \sim \dfrac{h_{\mu\nu} T^{\mu\nu}}{M_P}$

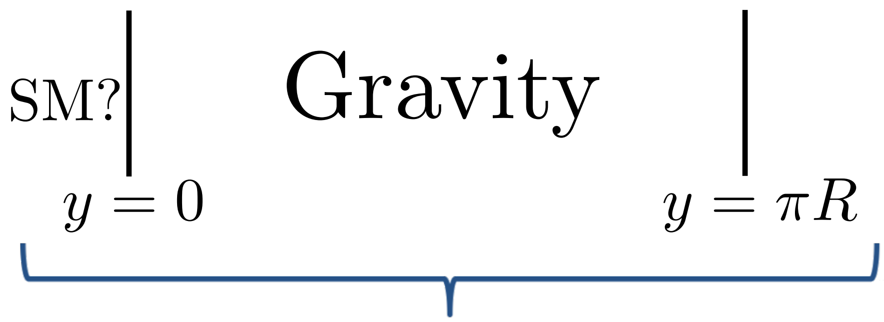
UV-complet…

Dimension: $[M_P] = \dfrac{[M_S]}{[\lambda_S]}$

Coupling

Locality

Arkani-Hamed, Dimopoulos, and Dvali discovered that large extra-dimensional scenarios may generate the required tiny effective couplings:



True cutoff at M_5.

Exercise: Out of R and M_P, construct a quantity with dimension of coupling:

$$[(M_P R)^{-1}] = [\lambda]$$

and

$$\lambda \sim (M_5/M_P)^3 \ll 1$$

Later, Randall and Sundrum showed that this can be achieved by smaller dimensions with warping.

Continuum Clockworking / Linear Dilaton Model

Short story: The continuum limit of the clockwork is a solution to Einstein's equations for gravity + dilaton (like 5D Brans-Dicke) with the metric

$$ds^2 = e^{\frac{4k|y|}{3}}(dx^2 + dy^2)$$

and it offers an extra-dimensional approach to the hierarchy problem with a very different phenomenology to RS or LED.

Proposed by Antoniadis, Arvanitaki, Dimopoulos, Giveon.

The Clockwork Metric

Put a massless scalar/graviton in this background and decompose to find 5D eigenstates (KK):

$$\phi(x,y) = \sum_{n=0}^{\infty} \frac{\tilde{\phi}_n(x)\,\psi_n(y)}{\sqrt{\pi R}}$$

SM? | Gravity |

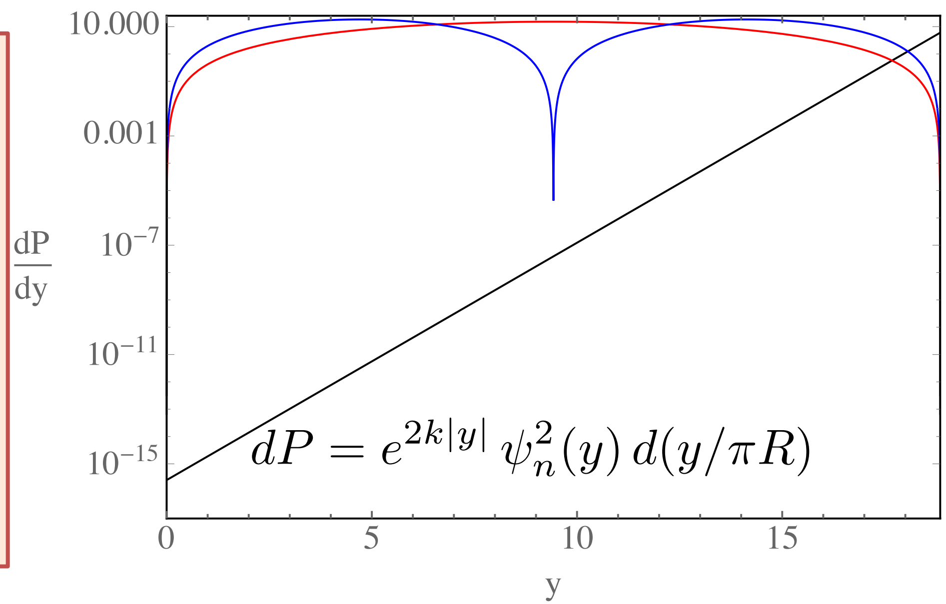
$y=0$ $\qquad$ $y=\pi R$

Find a zero-mode:

Mass: $m_0^2 = 0$

Wavefunction:

$$\psi_0(y) = \sqrt{\frac{k\pi R}{e^{2k\pi R}-1}}$$



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$$\phi(x,y) = \sum_{n=0}^{\infty} \frac{\tilde{\phi}_n(x)\,\psi_n(y)}{\sqrt{\pi R}}$$

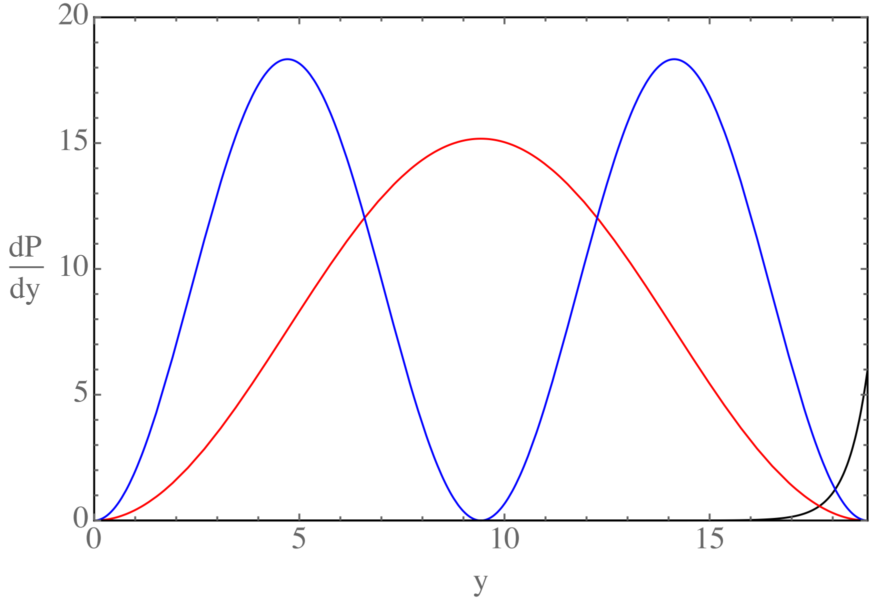
Find excited modes:

Mass: $m_n^2 = k^2 + \frac{n^2}{R^2}$

Wavefunction:

$$\psi_n(y) = \frac{n}{m_n R}\, e^{-k|y|} \left(\frac{kR}{n} \sin\frac{n|y|}{R} + \cos\frac{ny}{R} \right)$$

$dP = e^{2k|y|}\, \psi_n^2(y)\, d(y/\pi R)$



The Clockwork Metric

Put a massless scalar in this background and de… find 5D eigenstates (KK):

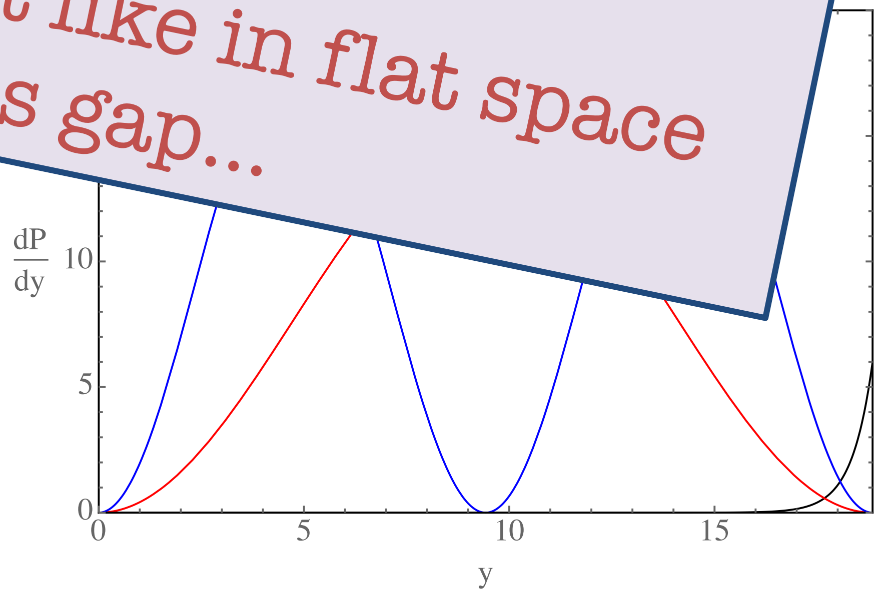
Zero mode density warped, like in warped extra dimensions…

KK mode density just like in flat space with a mass gap…

Mass: $m_n^2 = k^2 + \frac{[illegible]}{R^2}$

Wavefunction:

$$\psi_n(y) = \frac{n}{m_n R}\, e^{-k|y|} \left(\frac{kR}{n} \sin\frac{n|y|}{R} + \cos\frac{ny}{R} \right)$$



An Analogy

Is there a physical picture for what is going on?

When modes are decomposed as KK states:

$$h_{\mu\nu}(x,y) = \sum_{n=0}^{\infty} \frac{\tilde{h}^{(n)}_{\mu\nu}(x)\,\psi_n(y)}{\sqrt{\pi R}}$$

they must satisfy the following equation of motion:

$$\left(\partial_y^2 + 2k\partial_y + \partial_x^2\right)\tilde{h}^{(n)}_{\mu\nu}(x)\,\psi_n(y) = 0$$

Remind you of anything?

An Analogy

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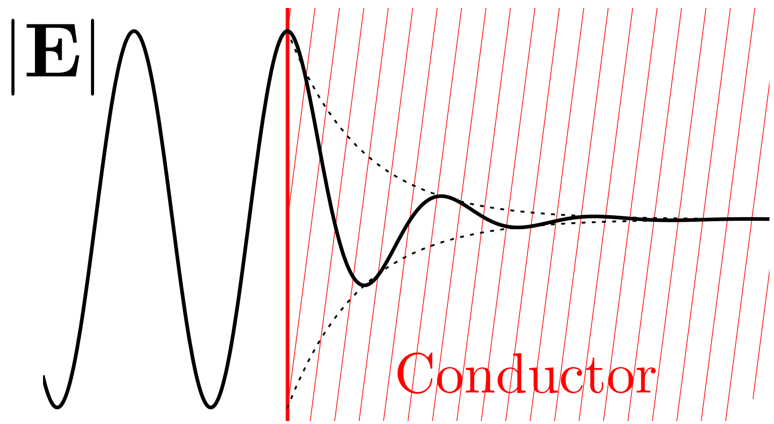
Maxwell's equations for EM wave in a conductor:

$$\left(\nabla^2 - \mu\sigma\partial_t - \mu\epsilon\partial_t^2\right)\mathbf{E} = 0$$



Phenomenology

Things get really interesting when looking to the phenomenology…

This talk: Work with Giudice, Kats, Torre, Urbano.

Previous related studies:

- Antoniadis, Arvanitaki, Dimopoulos, Giveon, 2011. (Large-k)
- Baryakhtar, 2012. (All-k)
- Cox, Gherghetta, 2012. (Dilatons)
- Giudice, Plehn, Strumia, 2004. Franceschini, Giardino, Giudice, Lodone, Strumia, 2011. (Large extra dimensions, pheno similar.)

Phenomenology

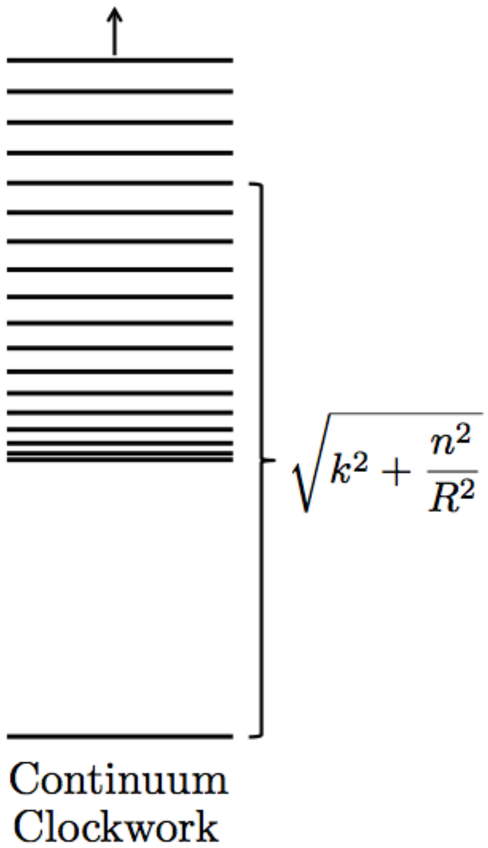
Irreducible prediction:

In this theory Planck scale is:

$$M_P \sim \sqrt{\frac{M_5^3}{k}}\, e^{k\pi R}$$

So if all other parameters at the weak scale, require:

$$kR \sim 10$$



But the mass spectrum is given by:

$$m_n \sim k\left(1 + \frac{n^2}{2(kR)^2}\right)$$

Thus the first few states will always be split by %'s, with the relative splitting decreasing for heavier modes.

This splitting is thus a key prediction of the theory.

Phenomenology

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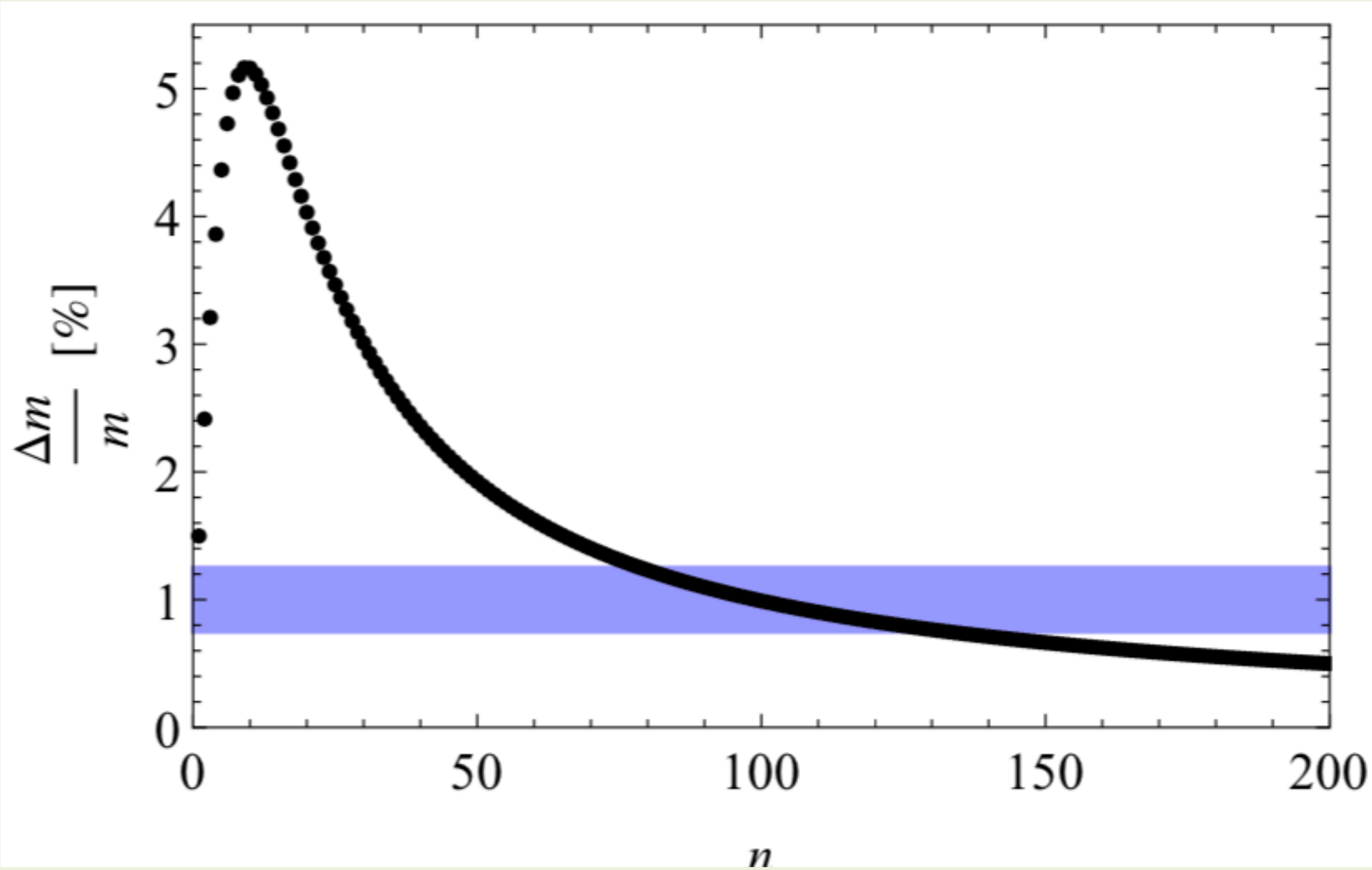
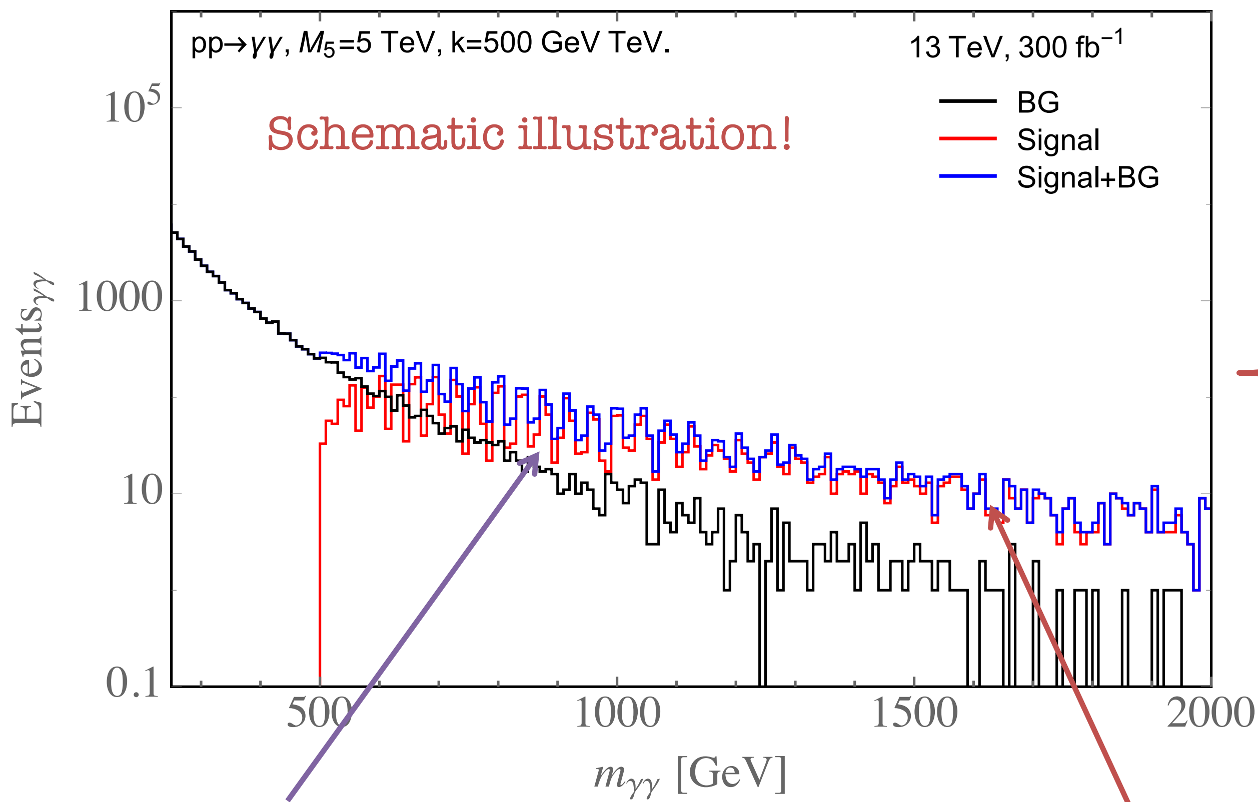
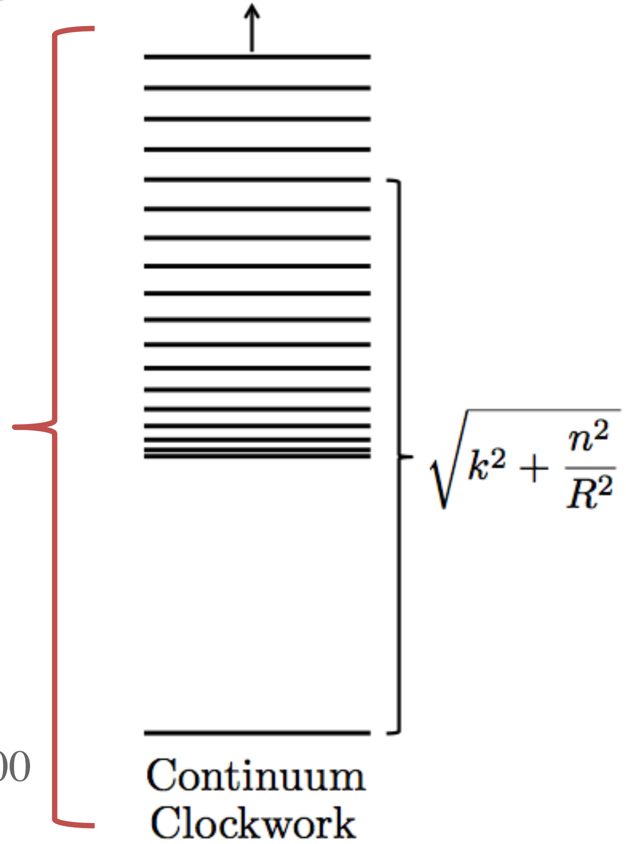
Mass splitting:



Phenomenology

At colliders would look something like:



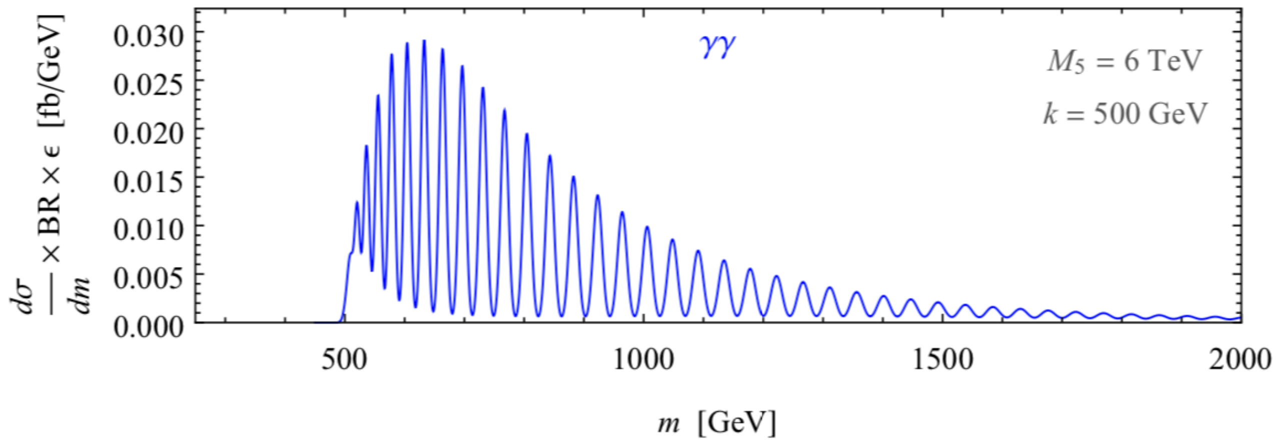


Most interestingly, due to splittings, signal appears to "oscillate". Thus get extra sensitivity by doing spectral analysis... The "power spectrum" of LHC data!

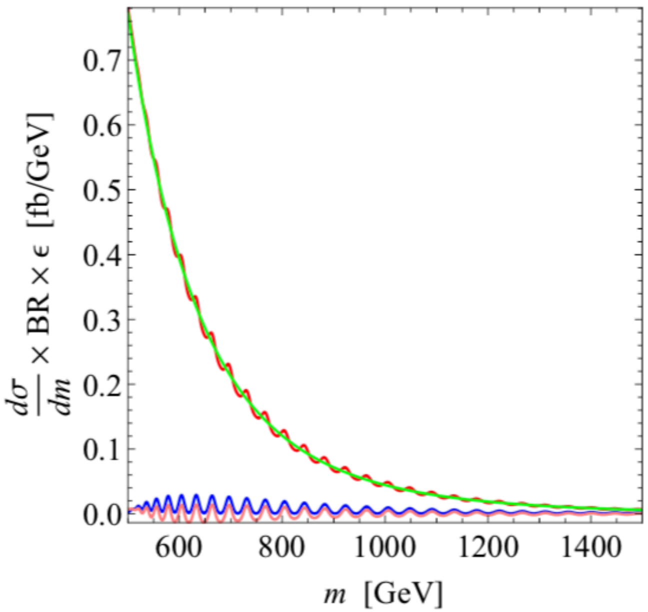
Can search for continuum spectrum at high energies. BG modelling essential...

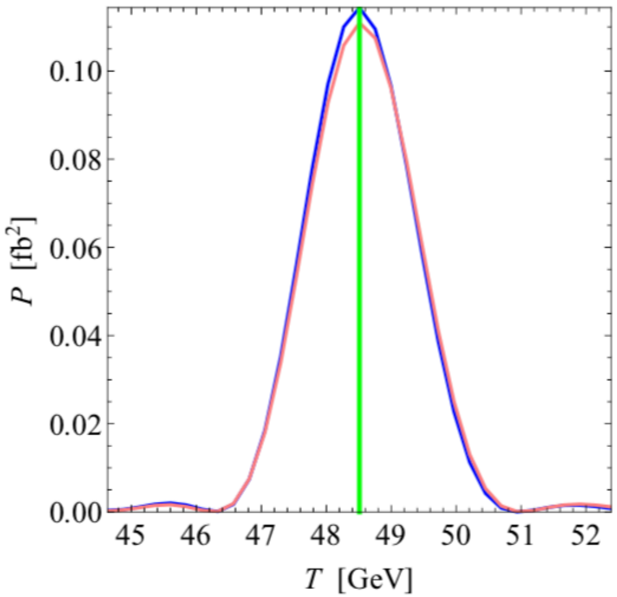
Phenomenology

Extract the oscillations, subtract off background:



And then Fourier-transform what's left over!



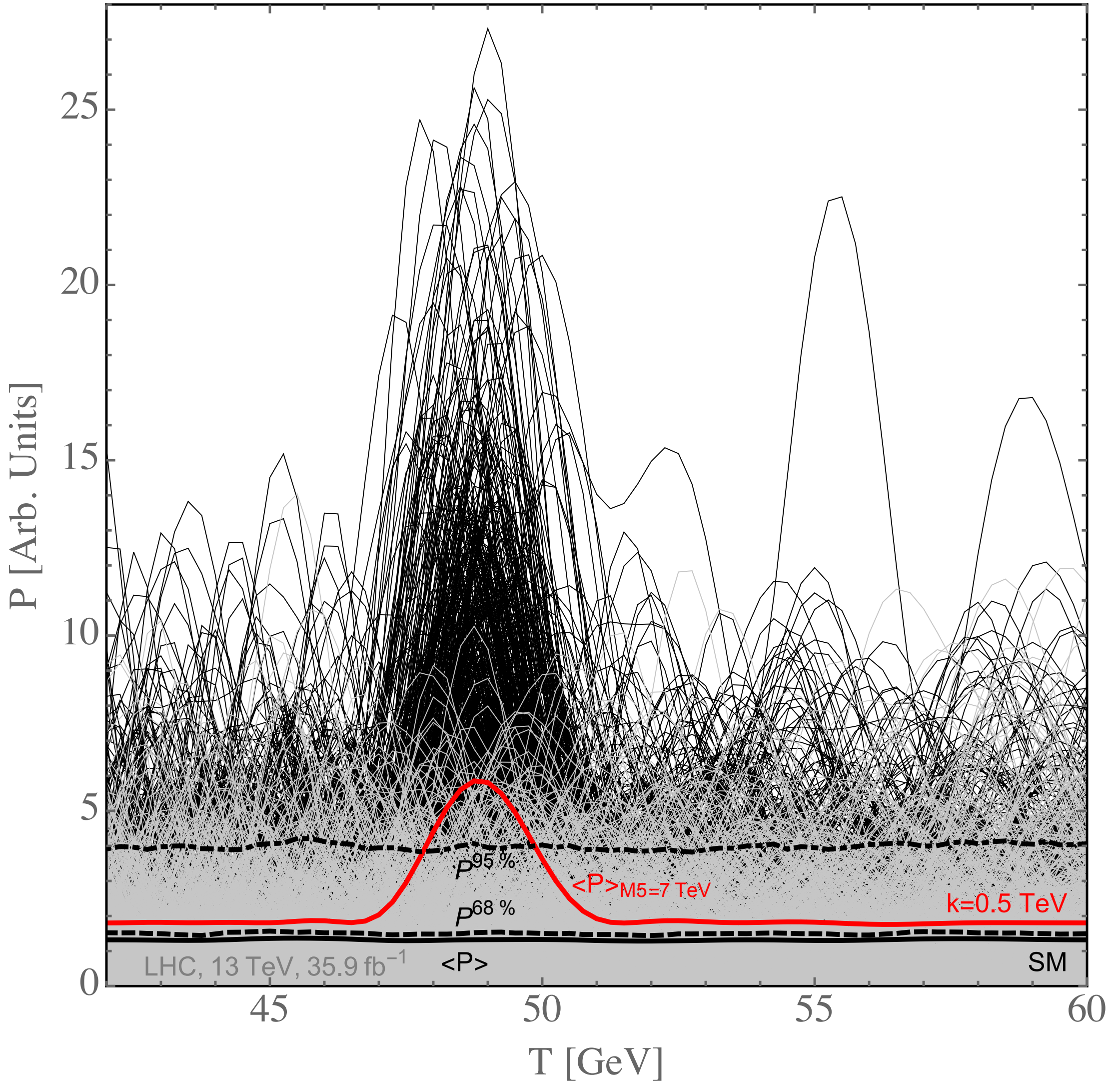


Phenomenology

With statistical fluctuations and experimental resolution included:

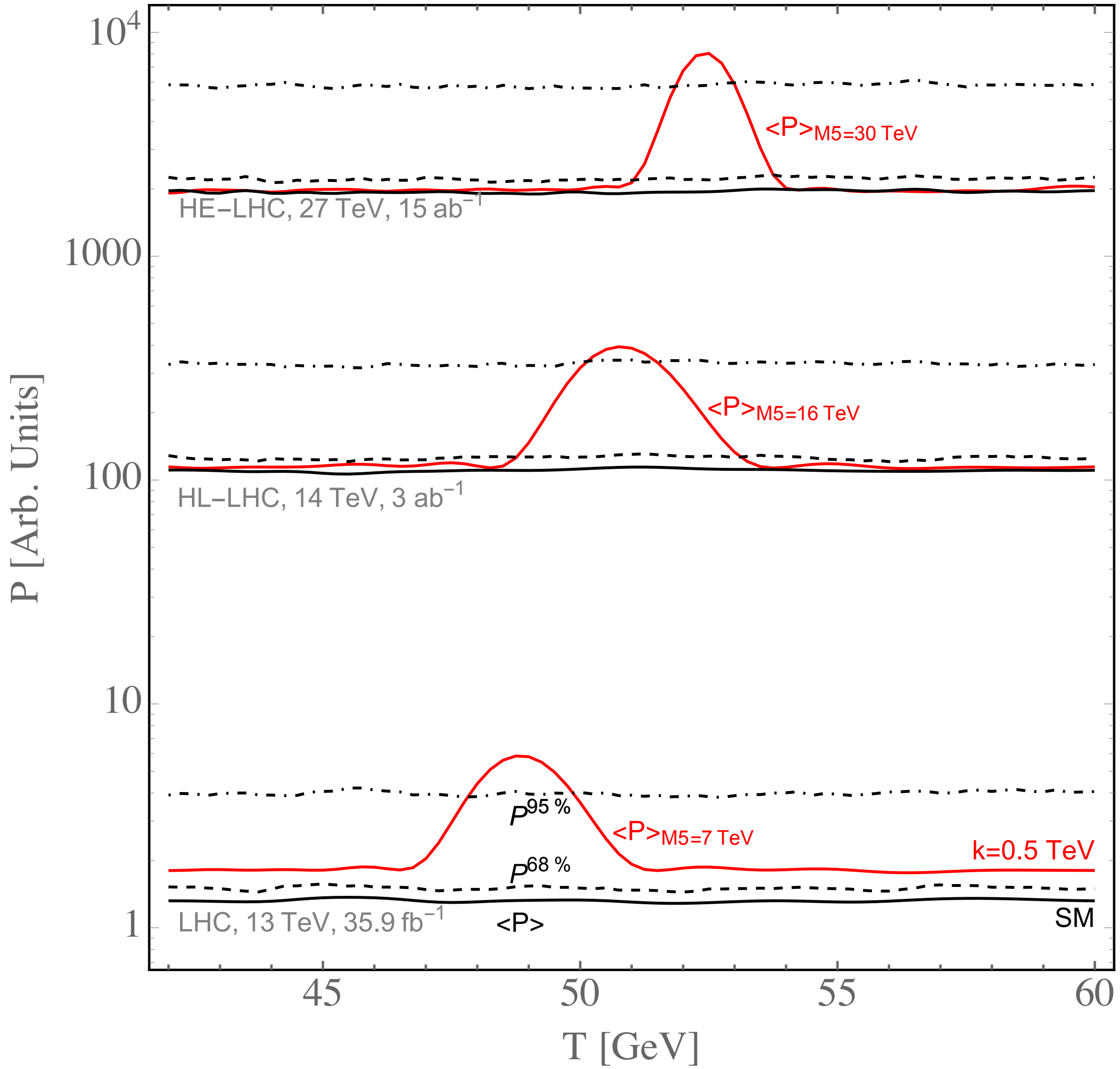
The residual power spectrum of signal+background.

The peak is at the frequency of the oscillations, which correspond to the inverse radius of the extra dimension.



Phenomenology

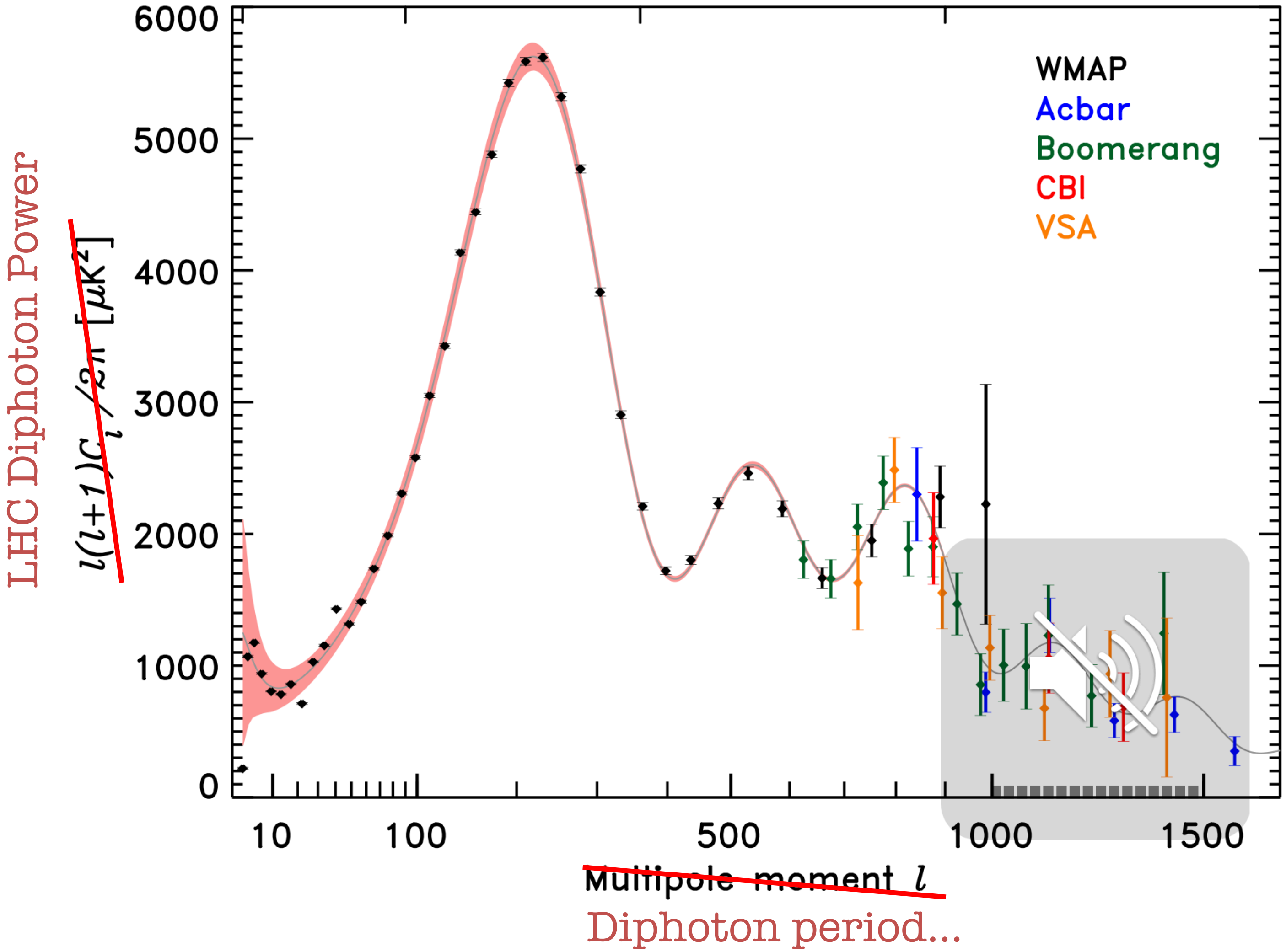
Bump hunting, but in Fourier space!



At HL-LHC and beyond could access high cutoff scales this way.

Phenomenology

Irrespective of this set of models, it would be very neat to know the LHC power spectrum!!



Summary

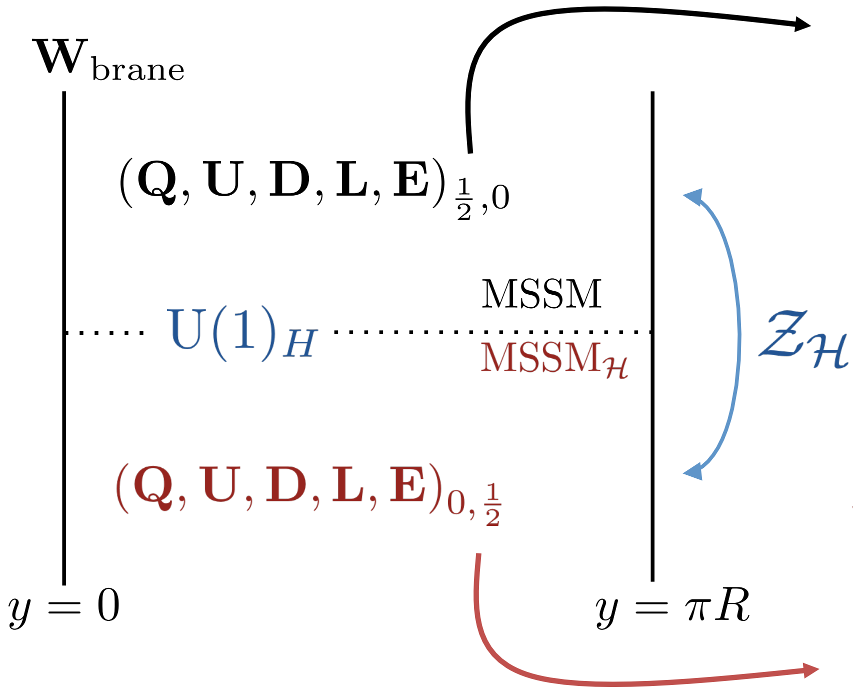
Naturalness is a strategy to search for the UV-completion of the Standard Model, and the hierarchy problem is telling us something deep.

I won't make any promises about what will or will not be seen at colliders. All bets off in my book.

Clearly we have not searched for everything yet, and exotic approaches to the hierarchy problem motivate some very <u>exotic</u> new signatures.

UV-Completion

- Scherk-Schwarz provides a natural home for the top sector. Take a flat extra dimension:



$$V_{\mathrm{CW}}(H) = \frac{1}{2}\,\mathrm{Tr} \sum_{n=-\infty}^{\infty} \int \frac{\mathrm{d}^4 p}{(2\,\pi)^4} \times \log \frac{p^2 + (n + q_B)^2/R^2 + M^2(H)}{p^2 + (n + q_F)^2/R^2 + M^2(H)}$$

$$V_{\mathrm{CW}}(H_{\mathcal{H}}) = \frac{1}{2}\,\mathrm{Tr} \sum_{n=-\infty}^{\infty} \int \frac{\mathrm{d}^4 p}{(2\,\pi)^4} \times \log \frac{p^2 + (n + q_F)^2/R^2 + M^2(H_{\mathcal{H}})}{p^2 + (n + q_B)^2/R^2 + M^2(H_{\mathcal{H}})}$$

- Scherk-Schwarz: “project out” modes and automatically give opposite sign corrections!

A Shallow Grave.

- We also need the Hyperbolic quartic. Use gauge D-term, but haven't seen a new gauge force...

$$V_{\mathrm{U}(1)_{\mathcal{H}}} \ni \frac{g_{\mathcal{H}}^2}{2}\,\xi \left(|H_{\mathcal{H}}|^2 - |H|^2 - f_{\mathcal{H}}^2\right)^2$$

- Supersymmetric breaking: D-term vanishes. Must have SUSY breaking, parameterised by

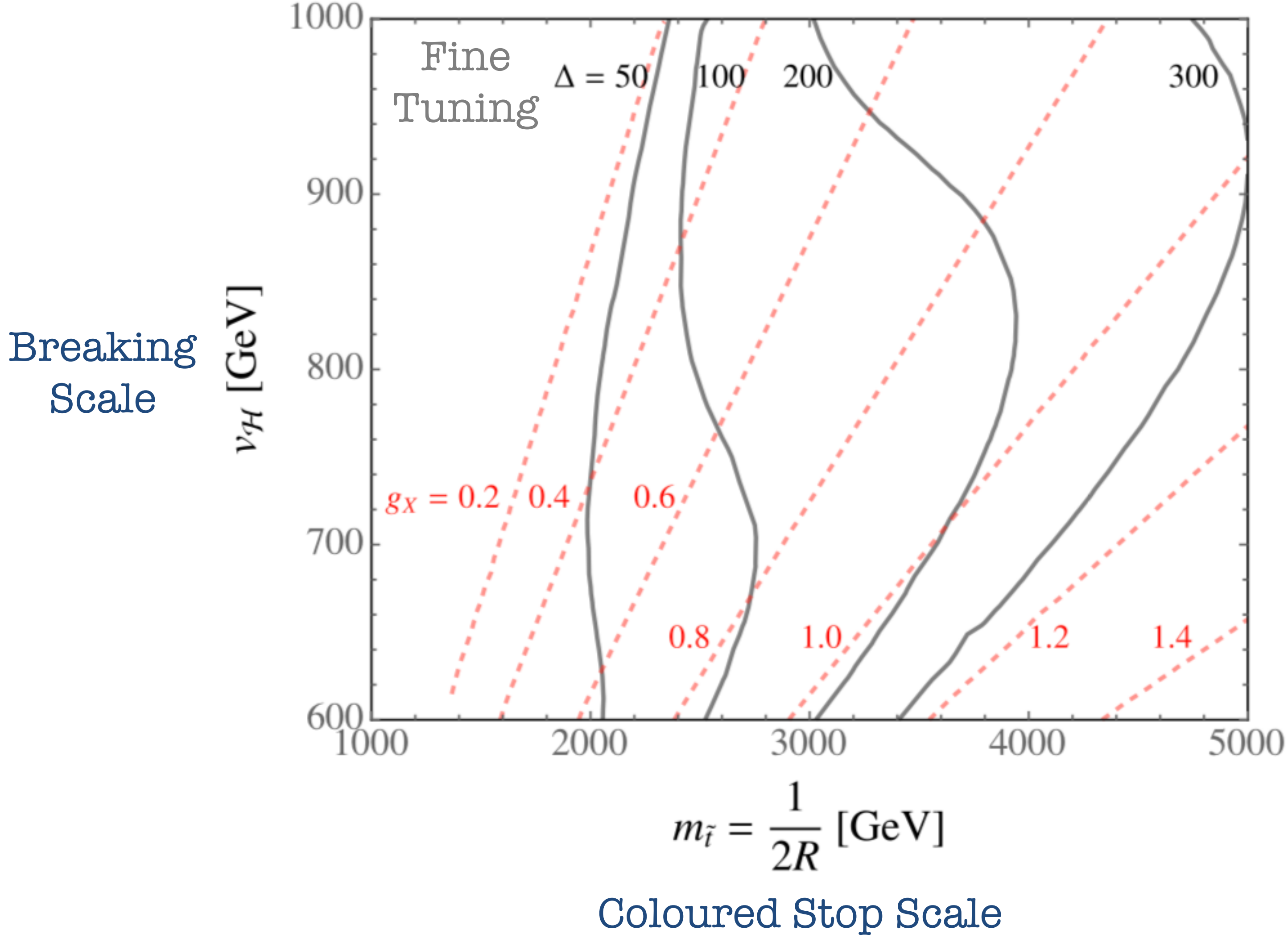
$$\xi = \left(1 - \frac{M_V^2}{M_S^2}\right)$$

- But this feeds into U(2,2) violating soft masses!

$$V_{\mathrm{U}(1)_{\mathcal{H}}} \ni -\frac{g_{\mathcal{H}}^2\, M_V^2}{16\,\pi^2}\log\left(1-\xi\right)\left(|H_{\mathcal{H}}|^2 + |H|^2\right)$$

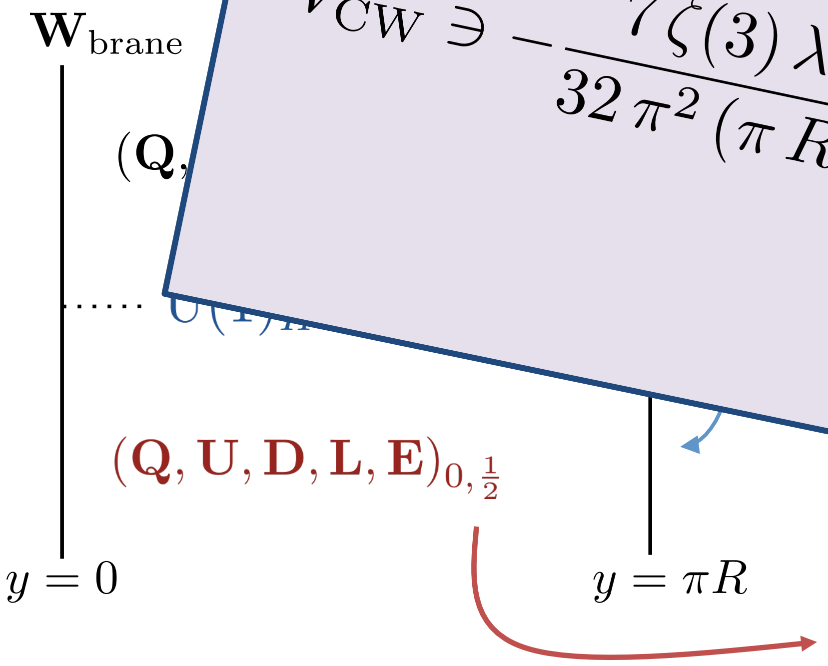
A Shallow Grave.

- This is an irreducible source of fine-tuning



UV-Completion

- Scher[...]chwarz provides a natural home for the top se[...] flat extra dimension:



$$\frac{M^2(H)}{M^2(H)}$$

$$V_{\rm CW}(H_{\mathcal{H}}) = \frac{}{2} \sum_{n=-\infty}^{\infty} \int \frac{d^4p}{} \times \log \frac{p^2 + (n+q_F)^2/R^2 + M^2(H_{\mathcal{H}})}{p^2 + (n+q_B)^2/R^2 + M^2(H_{\mathcal{H}})}$$

One-loop corrections:

$$V_{\rm CW} \ni -\frac{7\,\zeta(3)\,\lambda_t^2}{32\,\pi^2\,(\pi R)^2}\Big\{3\,|H|^2 - 3\,|H_{\mathcal{H}}|^2 - |Q_{\mathcal{H}}|^2 - 2\,|U^c_{\mathcal{H}}|^2\Big\}$$

- Scherk-Schwarz: "project out" modes and automatically give opposite sign corrections.

An Analogy

W... are decomposed as KK states:

$$\ldots \chi^{(n)}(x)\,\psi_n(y)$$

they must satis...

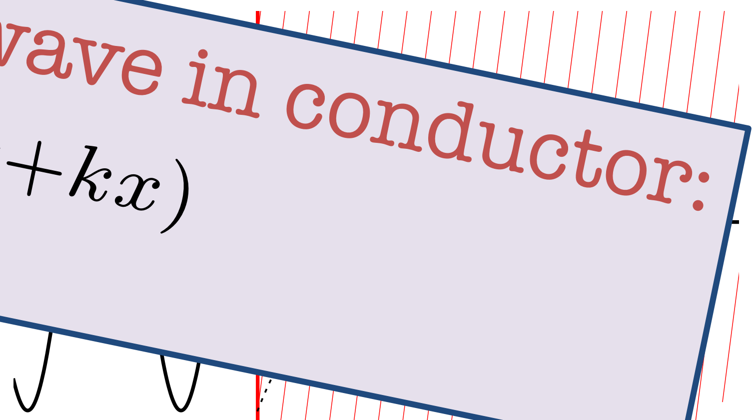
$$\left(\partial_y^2 + 2k\partial_y + \partial_x^2\right) h_{\mu\nu}^{(n)}(x) \ldots$$

General solution for stationary 4D particle:

$$\sim e^{-ky} e^{i\left(m_n t + \sqrt{m_n^2 - k^2}\, y\right)}$$

M... EM wave in a conductor:

$$\left(\nabla^2 - \mu\sigma\partial_t - \mu\epsilon\partial_t^2\right) \ldots$$



General solution for EM wave in conductor:

$$\sim e^{-\delta x} e^{i(\omega t + kx)}$$